Introduction to Nuclear Fusion as An Energy Source

Po-Yu Chang

Institute of Space and Plasma Sciences, National Cheng Kung University

Lecture 3

2025 spring semester

Tuesday 9:00-12:00

Materials:

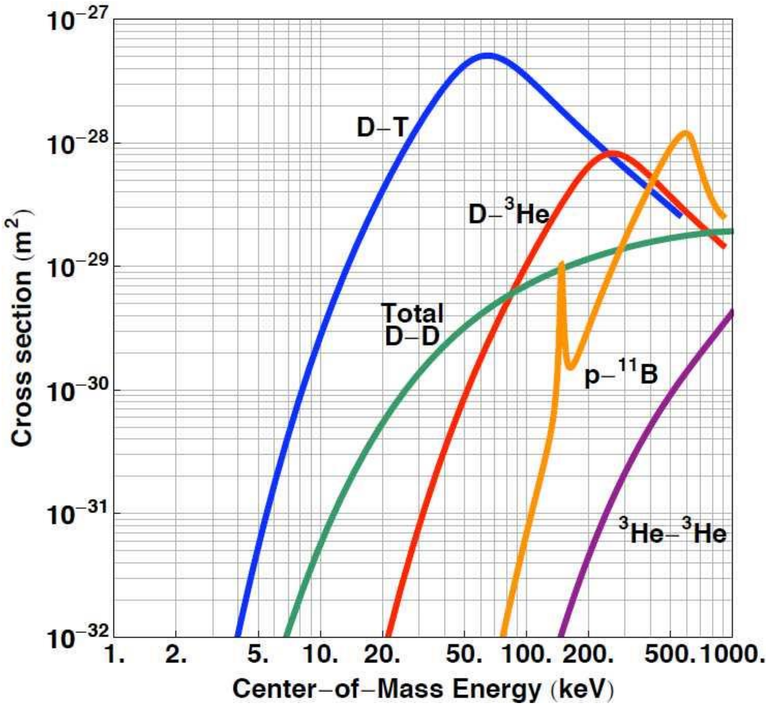
https://capst.ncku.edu.tw/PGS/index.php/teaching/

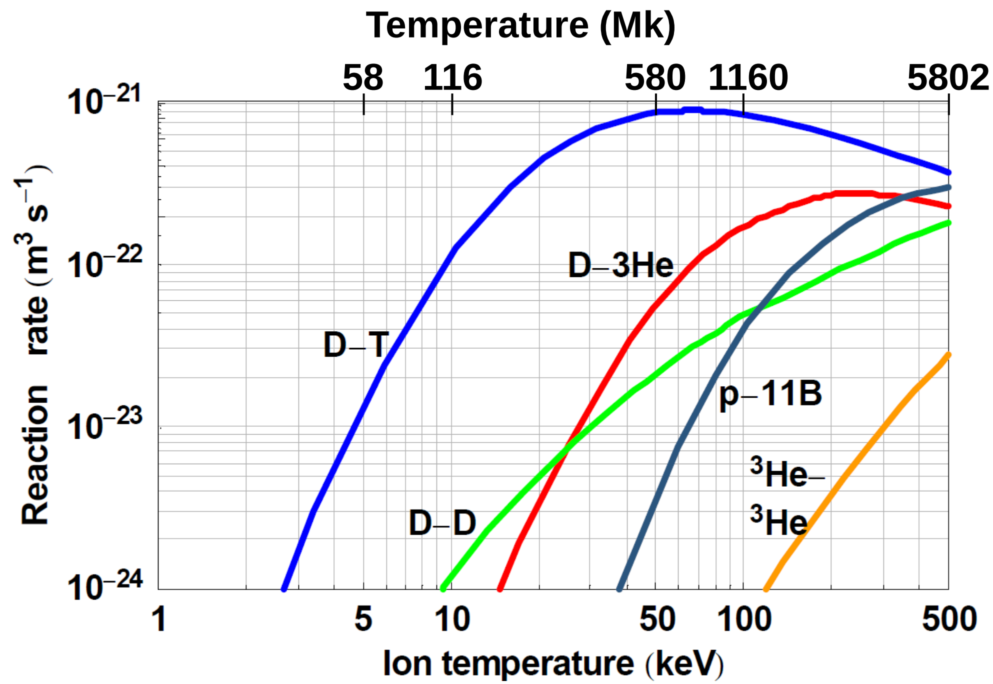
Online courses:

https://nckucc.webex.com/nckucc/j.php?MTID=mf1a33a5dab5eb71de9da4380ae888592

2025/3/5 updated

Fusion doesn't come easy

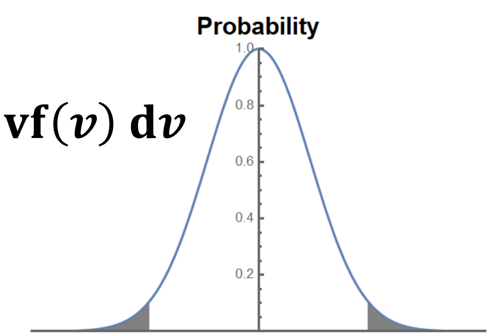




- **The DT fusion reactivity is maximum at T ≈ 64 keV**
- **@ T = 10 keV,** $<\sigma v>_{DT} \approx 100 <\sigma v>_{DD}$

- **Reaction rate:**

$$<\sigma v> = \int \sigma(v) v f(v)\, dv$$



https://i.stack.imgur.com/wXQD5.jpg
Santarius, J. F., "Fusion Space Propulsion – A Shorter Time Frame Than You Think", JANNAF, Monterey, 5-8 December 2005.

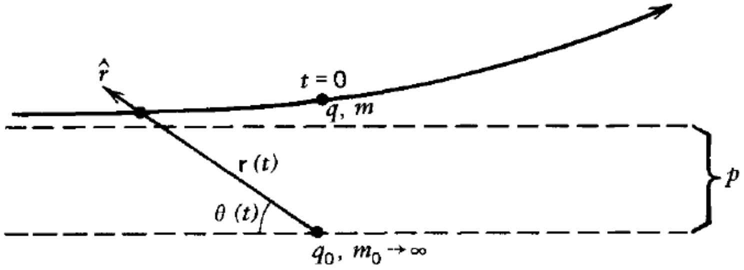
Course Outline

- Brief background reviews
  - Electromagnetics
  - Plasma physics
- **Introduction to nuclear fusion**
  - **Nuclear binding energy (Fission vs Fusion)**
  - **Fusion reaction physics**
  - **Some important fusion reactions (Cross section)**
    - **Main controlled fusion fuels**
    - **Advanced fusion fuels**
  - **Maxwell-averaged fusion reactivities**

Course Outline

- **Introduction to nuclear fusion (cont.)**
  - **Collisions (Bremsstrahlung radiation)**
  - **Columb scattering. Cross section of the Columb scattering**
  - **Beam-target fusion vs thermonuclear fusion**
  - **Lawson criteria, ignition conditions**
  - **Magnetic confinement fusion (MCF) vs Inertial confinement fusion (ICF)**

Charged particles collide with each other through coulomb collisions



$$\mathrm{m}v_{\perp} = \int_{-\infty}^{\infty} dt\, \mathrm{F}_{\perp}(t)$$

- **Coulomb force:**

$$m\ddot{\vec{r}} = \frac{qq_0}{r^2}\hat{r}$$

$$F_{\perp} = \frac{qq_0}{p^2}\sin^3\theta$$

- **Relation between θ and t is**

$$x = -r\cos\theta = -\frac{p\cos\theta}{\sin\theta} = v_0 t$$

- **Therefore,**

$$v_{\perp} = \frac{qq_0}{mv_0 p}\int_0^{\pi} d\theta \sin\theta = \frac{2qq_0}{mv_0 p} \equiv \frac{v_0 p_0}{p}$$

where $$p_0 \equiv \frac{2qq_0}{m{v_0}^2}$$

- **Note that this is valid only when** $v_{\perp} << v_0$ **, i.e.,** $p >> p_0$ **.**

Cumulative effect of many small angle collisions is more important than large angle collisions

- **Consider a variable Δx that is the sum of many small random variables Δx_i, i=1,2,3,…,N,**

$$\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 + \cdots + \Delta x_N = \sum_{i=1}^{N} \Delta x_i$$

- **Suppose** $< \Delta x_i > = < \Delta x_i \Delta x_j >_{i \neq j} = 0$

$$\langle (\Delta x)^2 \rangle = \left\langle \left(\sum_{i=1}^{N} \Delta x_i \right)^2 \right\rangle = \sum_{i=1}^{N} \langle (\Delta x_i)^2 \rangle = N \langle (\Delta x_i)^2 \rangle$$

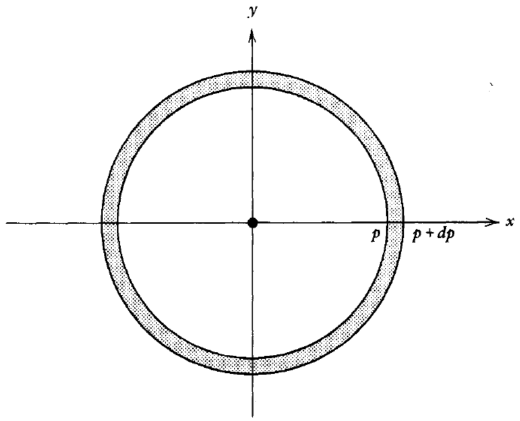
- **For one collision:**

$$\langle {v_\perp}^2 \rangle = \langle (\Delta v_x)^2 \rangle + \left\langle \left(\Delta v_y\right)^2 \right\rangle = \frac{{v_0}^2 {p_0}^2}{p^2} \qquad \langle (\Delta v_x)^2 \rangle = \left\langle \left(\Delta v_y\right)^2 \right\rangle = \frac{1}{2} \frac{{v_0}^2 {p_0}^2}{p^2}$$

- **The total velocity in** $\hat{x}$

$$\left\langle \left(\Delta {v_x}^{tot}\right)^2 \right\rangle = N \langle (\Delta v_x)^2 \rangle = \frac{N}{2} \frac{{v_0}^2 {p_0}^2}{p^2}$$

The collision frequency can be obtained by integrating all the possible impact parameter



- Number of collisions in a time interval:

$$dN = n_0 2\pi p\, dp\, v_0\, dt$$

i.e., $$\frac{dN}{dt} = 2\pi p\, dp\, n_0 v_0$$

- Therefore

$$\frac{d}{dt}\left\langle \left(\Delta v_{\mathrm{x}}{}^{\mathrm{tot}}\right)^2 \right\rangle = \frac{1}{2}\frac{{v_0}^2{p_0}^2}{p^2}\frac{dN}{dt}$$

$$= \pi\, n_0 {v_0}^3 {p_0}^2 \frac{dp}{p}$$

$$\frac{d}{dt}\left\langle \left(\Delta_{\perp}{}^{\mathrm{tot}}\right)^2 \right\rangle = 2\frac{d}{dt}\left\langle \left(\Delta v_{\mathrm{x}}{}^{\mathrm{tot}}\right)^2 \right\rangle$$

$$= 2\pi n_0 {v_0}^3 {p_0}^2 \int_{p_{\min}}^{p_{\max}} \frac{dp}{p}$$

$$= 2\pi n_0 {v_0}^3 {p_0}^2 \ln\left(\frac{p_{\max}}{p_{\min}}\right)$$

$$\approx 2\pi n_0 {v_0}^3 {p_0}^2 \ln\left(\frac{\lambda_{\mathrm{D}}}{|p_0|}\right)$$

$$\approx 2\pi n_0 {v_0}^3 {p_0}^2 \ln \Lambda$$

- Note that

$$\lambda_{\mathrm{D}} \approx \left(\frac{KT_{\mathrm{e}}}{4\pi n_0 e^2}\right)^{1/2}$$

$$\frac{\lambda_{\mathrm{D}}}{|p_0|} \approx \frac{\lambda_{\mathrm{D}} m_{\mathrm{e}} {v_{\mathrm{e}}}^2}{2e^2} \approx \frac{\lambda_{\mathrm{D}} KT_{\mathrm{e}}}{e^2} \approx 4\pi n_0 {\lambda_{\mathrm{D}}}^3$$

$$\approx \Lambda$$

Comparison between the mean free path and the system size L determines the regime of the plasma

- A reasonable definition for the scattering time due to small angle collisions is the time it takes $\left\langle\left(\Delta v_\perp^{\text{tot}}\right)^2\right\rangle$ to equal v_0^2. The collision frequency ν_c due to small-angle collisions:

$$\frac{d}{dt}\left\langle\left(\Delta_\perp^{\text{tot}}\right)^2\right\rangle \approx 2\pi n_0 v_0^3 p_0^2 \ln\Lambda \approx v_0^2 \nu_c, \quad p_0 \equiv \frac{2qq_0}{m_e v_0^2} \Rightarrow \nu_c = \frac{8\pi n_0 e^4 \ln\Lambda}{m_e^2 v_0^3}$$

- With more careful derivation, the collisional time is obtained:

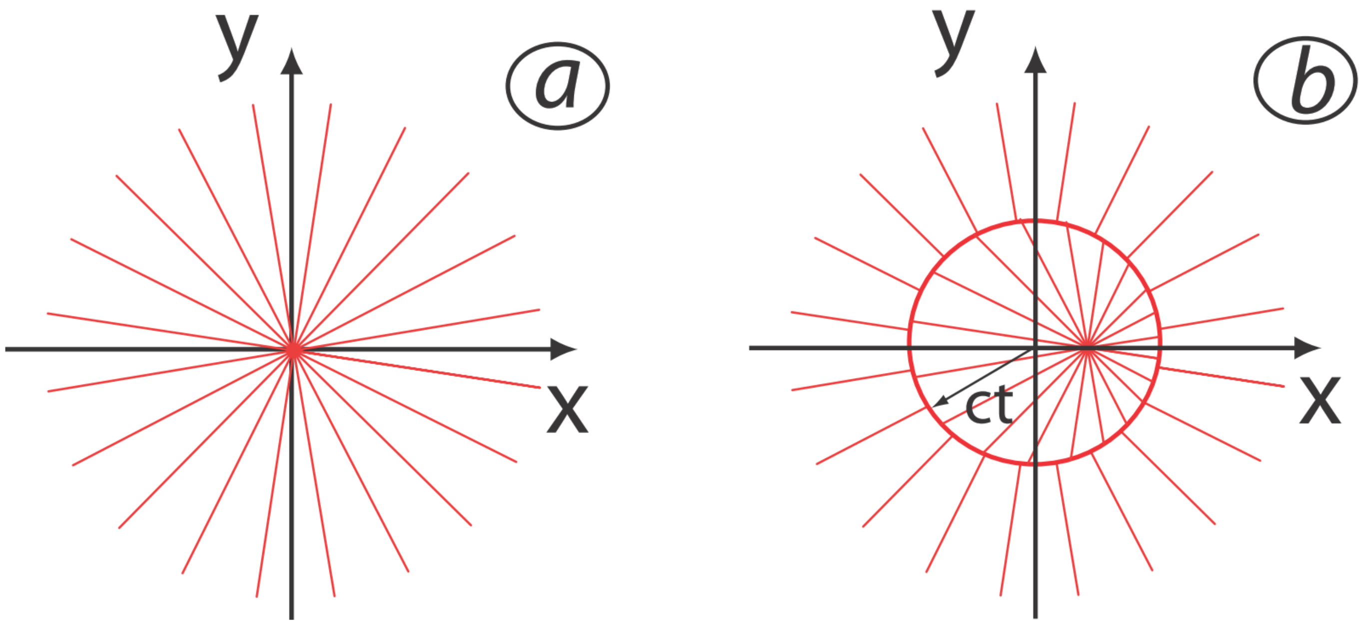
$$\tau_e^{-1} = \nu_c = \frac{4\sqrt{2\pi} n e^4 \ln\Lambda}{3\sqrt{m_e}(KT_e)^{3/2}}$$

- Mean free path: $l_{\text{mfp}} = v_e \tau_e$

$$\begin{cases} l_{\text{mfp}} < L & \text{Fluid Theory} \\ l_{\text{mfp}} > L & \text{Kinetic Theory} \end{cases}$$

Electromagnetic wave is radiated when a charge particle is accelerated

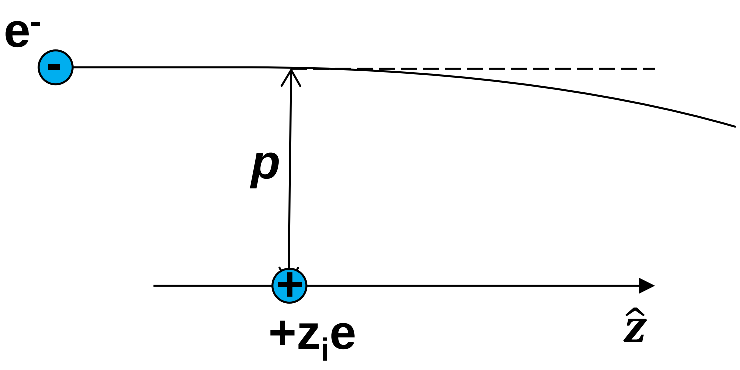
- **The retarded potentials are the electromagnetic potentials for the electromagnetic field generated by time-varying electric current or charge distributions in the past.**



https://jseldredphysics.files.wordpress.com/2018/04/lecture18.pdf

Bremsstrahlung emission

- When an electron collides with a nucleus via coulomb interaction, the electron is accelerated and thus radiates, called Bremsstrahlung radiation.
- In the non-relativistic limit, the Bremsstrahlung radiation power is:



$$P_{\mathrm{B,e1,i1}} = \frac{e^2}{6\pi\epsilon_o}\frac{\dot{v}^2}{c^3}$$

p: Impact parameter

- The coulomb force experienced by the electron is:

$$\dot{v} = \frac{F}{m_e} = \frac{ze^2}{4\pi\epsilon_o m_e r^2} = \frac{ze^2}{4\pi\epsilon_o m_e[p^2 + (vt)^2]} \approx \frac{ze^2}{4\pi\epsilon_o m_e p^2}$$

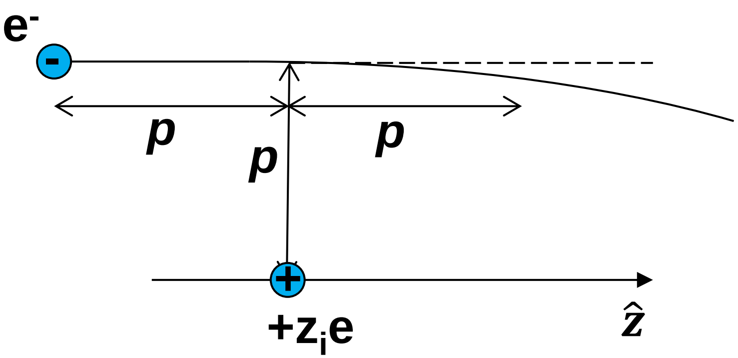
=>

$$P_{\mathrm{B,e1,i1}} = \frac{z^2 e^6}{96\pi^3{\epsilon_o}^3 c^3 {m_e}^2}\frac{1}{p^4} \quad \mathrm{(W)}$$

Bremsstrahlung emission

- The electron begins to accelerate when it is about a distance *p* from the ion. It continuous to accelerate until it travels a distance *p* away from the ion.

$$\Delta t = \frac{2p}{v}$$



- Therefore, the energy loss by one electron colliding one ion is:

$$E_{\mathrm{B,e1,i1}} \approx P_{\mathrm{B,e1,i1}} \Delta t = \frac{z^2 e^6}{48\pi^3 \epsilon_o{}^3 c^3 m_e{}^2} \frac{1}{v p^3} \quad \mathrm{(J)}$$

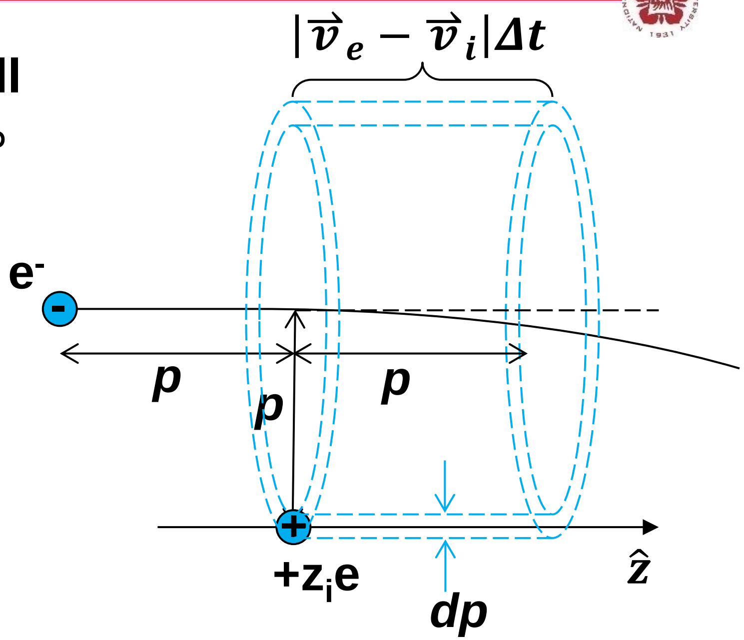
- With careful integration:

$$E_{\mathrm{B,e1,i1}} = \int_{-\infty}^{\infty} P_{\mathrm{B,e1,i1}}\, \mathrm{d}t = \frac{2z^2 e^6}{3(4\pi\epsilon_o)^3 m_e{}^2 c^3} \int_{-\infty}^{\infty} \frac{1}{[p^2 + (vt)^2]^2}\, dt$$

$$= \frac{\pi z^2 e^6}{3(4\pi\epsilon_o)^3 m_e{}^2 c^3} \frac{1}{v p^3}$$

Bremsstrahlung emission

- To consider the electron colliding with all ions with impact parameter *p* from 0 to ∞ and include the distribution function of ions $f_i(v_i)$.



- Number of ions within the cylinder collided with the incident electron is:

$$N_i = n_i 2\pi p\, dp|\vec{v}_e - \vec{v}_i|\Delta t$$

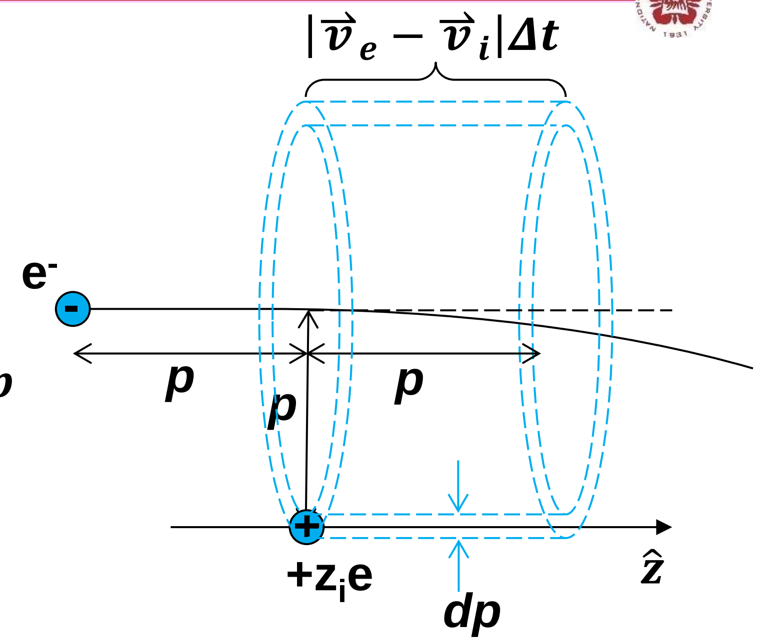
$$= \int \int_0^\infty f_i(\vec{v}_i)|\vec{v}_e - \vec{v}_i|\Delta t 2\pi p\, dp\, d\vec{v}_i$$

- The averaged radiation power is:

$$\bar{P}_{B,e1} = \frac{\bar{E}_{B,e1,i1} N_i}{\Delta t} = \int \int_0^\infty \frac{\bar{E}_{B,e1,i1} f_i(\vec{v}_i)|\vec{v}_e - \vec{v}_i|\Delta t 2\pi p\, dp\, d\vec{v}_i}{\Delta t}$$

$$= \int d\vec{v}_i \int_0^\infty \bar{E}_{B,e1,i1}|\vec{v}_e - \vec{v}_i| f_i(\vec{v}_i) 2\pi p\, dp$$

Bremsstrahlung emission



- Power of the electron colliding with all ions with impact parameter *p* from 0 to ∞ and include the distribution function of ions $f_i(v_i)$:

$$\bar{P}_{B,e1} = \int d\vec{v}_\mathrm{i} \int_0^\infty \bar{E}_{\mathrm{B,e1,i1}} |\vec{v}_\mathrm{e} - \vec{v}_\mathrm{i}| f_\mathrm{i}(\vec{v}_\mathrm{i}) 2\pi p \, dp$$

- In addition, we need to consider the distribution function of electrons $f_e(v_e)$. The total power loss is:

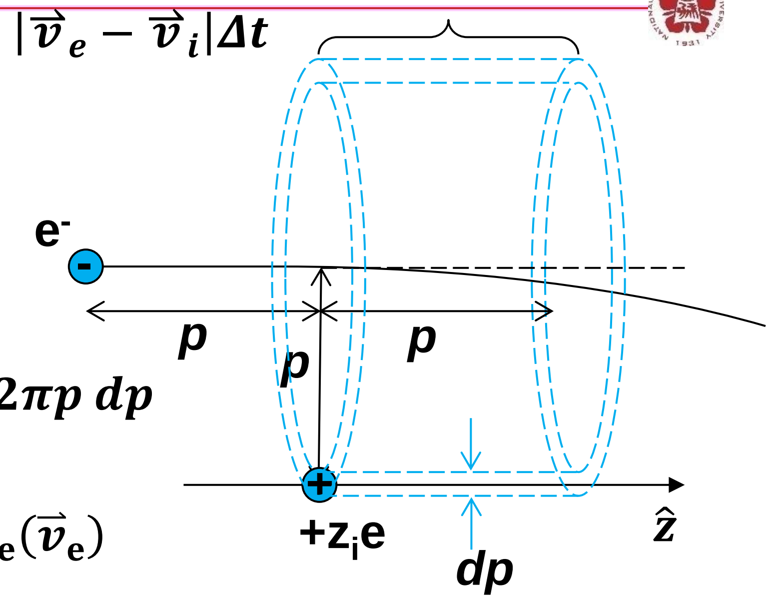
$$\bar{P}_B = \int d\vec{v}_\mathrm{i} \int d\vec{v}_\mathrm{e} \int_0^\infty \bar{E}_{\mathrm{B,e1,i1}} |\vec{v}_\mathrm{e} - \vec{v}_\mathrm{i}| f_\mathrm{i}(\vec{v}_\mathrm{i}) f_\mathrm{e}(\vec{v}_\mathrm{e}) 2\pi p \, dp$$

- Since $|v_e| >> |v_i|$, $|\vec{v}_\mathrm{e} - \vec{v}_\mathrm{i}| \approx v_\mathrm{e}$.
- In addition: $$\int f_i(\vec{v}_i) \, \mathrm{d}\, \vec{v}_i \equiv n_i$$

$$d\,\vec{v}_e = dv_x dv_y dv_z = v_\mathrm{e}^2 \sin\theta \, dv d\theta d\phi \to 4\pi v_\mathrm{e}^2 dv_\mathrm{e}$$

$$f_\mathrm{e} = n_\mathrm{e} \left(\frac{m_\mathrm{e}}{2\pi \mathrm{T_e}}\right)^{3/2} \exp\left(-\frac{m_\mathrm{e} v_\mathrm{e}^2}{2T_\mathrm{e}}\right)$$

Bremsstrahlung emission



$$\bar{P}_B = \int d\vec{v}_\mathrm{i} \int d\vec{v}_\mathrm{e} \int_0^\infty \bar{E}_{\mathrm{B,e1,i1}} |\vec{v}_\mathrm{e} - \vec{v}_\mathrm{i}| f_\mathrm{i}(\vec{v}_\mathrm{i}) f_\mathrm{e}(\vec{v}_\mathrm{e}) 2\pi p\, dp$$

$$= 2\pi \int f_\mathrm{i}(\vec{v}_\mathrm{i})\, d\vec{v}_i \int 4\pi {v_\mathrm{e}}^2 dv_\mathrm{e} \int_0^\infty p\, dp E_{\mathrm{B,e1,i1}} v_\mathrm{e} f_\mathrm{e}(\vec{v}_\mathrm{e})$$

$$= 8\pi^2 n_\mathrm{i} \int {v_\mathrm{e}}^3 dv_\mathrm{e} \int_0^\infty p\, dp E_{\mathrm{B,e1,i1}} n_\mathrm{e} \left(\frac{m_\mathrm{e}}{2\pi \mathrm{T_e}}\right)^{3/2} \exp\left(-\frac{m_\mathrm{e} {v_\mathrm{e}}^2}{2T_\mathrm{e}}\right)$$

$$= 8\pi^2 n_\mathrm{i} n_\mathrm{e} \left(\frac{m_\mathrm{e}}{2\pi \mathrm{T_e}}\right)^{3/2} \int_0^\infty {v_\mathrm{e}}^3 dv_\mathrm{e} \int_0^\infty p\, dp \left(\frac{z^2 e^6}{48\pi^3 {\epsilon_\mathrm{o}}^3 c^3 {m_\mathrm{e}}^2} \frac{1}{v_\mathrm{e} p^3}\right) \exp\left(-\frac{m_\mathrm{e} {v_\mathrm{e}}^2}{2T_\mathrm{e}}\right)$$

$$= 8\pi^2 n_\mathrm{i} n_\mathrm{e} \left(\frac{z^2 e^6}{48\pi^3 {\epsilon_\mathrm{o}}^3 c^3 {m_\mathrm{e}}^2}\right) \left(\frac{m_\mathrm{e}}{2\pi \mathrm{T_e}}\right)^{3/2} \int_0^\infty {v_\mathrm{e}}^2 \exp\left(-\frac{m_\mathrm{e} {v_\mathrm{e}}^2}{2T_\mathrm{e}}\right) dv_\mathrm{e} \int_0^\infty \frac{dp}{p^2}$$

Bremsstrahlung emission

- **Notice that we are using classical physics. We are not taking account of quantum effects which happen on a length scale of deBroglie wavelength $\Delta x = \hbar/(m_e v)$. Therefore, we have $p_{\min} = \hbar/(m_e v)$.**

$$\int_0^\infty \frac{dp}{p^2} \rightarrow \int_{p_{\min}}^\infty \frac{dp}{p^2} = \frac{1}{p_{min}} = \frac{m_e v_e}{\hbar} = \frac{2\pi m_e v_e}{h}$$

$$\overline{P}_B = 8\pi^2 n_i n_e \left(\frac{z^2 e^6}{48\pi^3 \epsilon_o{}^3 c^3 m_e{}^2}\right)\left(\frac{m_e}{2\pi T_e}\right)^{3/2} \int_0^\infty v_e{}^2 \exp\left(-\frac{m_e v_e{}^2}{2T_e}\right) dv_e \int_0^\infty \frac{dp}{p^2}$$

$$= 8\pi^2 n_i n_e \left(\frac{z^2 e^6}{48\pi^3 \epsilon_o{}^3 c^3 m_e{}^2}\right)\left(\frac{m_e}{2\pi T_e}\right)^{3/2} \frac{2\pi m_e}{h} \int_0^\infty v_e{}^3 \exp\left(-\frac{m_e v_e{}^2}{2T_e}\right) dv_e$$

- **With** $\int_0^\infty x^3 e^{-x^2}\, dx = \frac{1}{2}$ **,** **a better value:** $\left(\frac{2^{1/2}}{3\pi^{5/2}}\right)$

$$\overline{P}_B = \left(\frac{2^{1/2}}{6\pi^{3/2}}\right)\left(\frac{e^6}{\epsilon_o{}^3 c^3 h m_e{}^{3/2}}\right) z^2 n_i n_e T_e{}^{1/2} \quad \left(\frac{W}{m^3}\right)$$

Bremsstrahlung emission

- For multiple ion species: n_j, z_j

$$\bar{P}_B = \left(\frac{2^{1/2}}{3\pi^{5/2}}\right)\left(\frac{e^6}{\epsilon_o{}^3 c^3 h m_e{}^{3/2}}\right) n_e T_e{}^{1/2} \sum_j z_j^2 n_{i,j} \left(\frac{W}{m^3}\right)$$

$$= \left(\frac{2^{1/2}}{3\pi^{5/2}}\right)\left(\frac{e^6}{\epsilon_o{}^3 c^3 h m_e{}^{3/2}}\right) Z_{eff} n_e^2 T_e{}^{1/2} \left(\frac{W}{m^3}\right)$$

where $Z_{eff} \equiv \frac{\sum_j z_j{}^2 n_j}{n_e} = \frac{\sum_j z_j{}^2 n_j}{\sum_j z_j n_j}$ $\quad n_e = \sum_j z_j n_j$

$$\bar{P}_B = 5.35 \times 10^{-37} Z_{eff} n^2_{e\,(m^{-3})} T^{1/2}_{e\,(keV)} \left(\frac{W}{m^3}\right)$$

$$\bar{P}_B \equiv C_B Z_{eff} n^2_{e\,(m^{-3})} T^{1/2}_{e\,(keV)} \left(\frac{W}{m^3}\right)$$

Ignition condition (Lawson criterion) revision

- **Steady state 0-D power balance:**

$S_\alpha + S_h = S_B + S_k$

S_h: external heating

S_α: α particle heating

$$D + T \longrightarrow He^4\ (3.5\ \mathrm{MeV}) + n\ (14.1\ \mathrm{MeV})$$

$$S_f = E_f n_1 n_2 \langle \sigma v \rangle \left(\mathrm{W/m^3}\right) \quad n_D = n_T = \frac{1}{2} n$$

$$S_\alpha = \frac{1}{4} E_\alpha n^2 \langle \sigma v \rangle = \frac{1}{16} E_\alpha \frac{p^2 \langle \sigma v \rangle}{T^2}$$

E_α = 3.5 MeV

$$p = p_e + p_i = 2p_e = 2n_e T \equiv 2nT$$

S_B: Bremsstrahlung radiation

$$S_B = C_B Z_{\mathrm{eff}} n^2_{\mathrm{e}\,(\mathrm{m}^{-3})} T^{1/2}_{\mathrm{e}\,(\mathrm{keV})} \left(\frac{W}{m^3}\right)$$

$$= \frac{1}{4} C_B \frac{p^2}{T^{3/2}}$$

S_k: heat conduction lost

$$S_\kappa = \frac{3}{2} \frac{p}{\tau}$$

$$\frac{1}{16} E_\alpha \frac{p^2 \langle \sigma v \rangle}{T^2} \geqslant \frac{1}{4} C_B \frac{p^2}{T^{3/2}} + \frac{3}{2} \frac{p}{\tau}$$

Ignition condition (Lawson criterion) revision

- **Steady state 0-D power balance:**

$$S_\alpha + S_h = S_B + S_k$$

$$\frac{1}{16} E_\alpha \frac{p^2 \langle \sigma v \rangle}{T^2} \geqslant \frac{1}{4} C_B \frac{p^2}{T^{3/2}} + \frac{3}{2} \frac{p}{\tau}$$

$$p\tau \geqslant \frac{6}{\frac{1}{4} E_\alpha \frac{\langle \sigma v \rangle}{T^2} - C_B \frac{1}{T^{3/2}}}$$

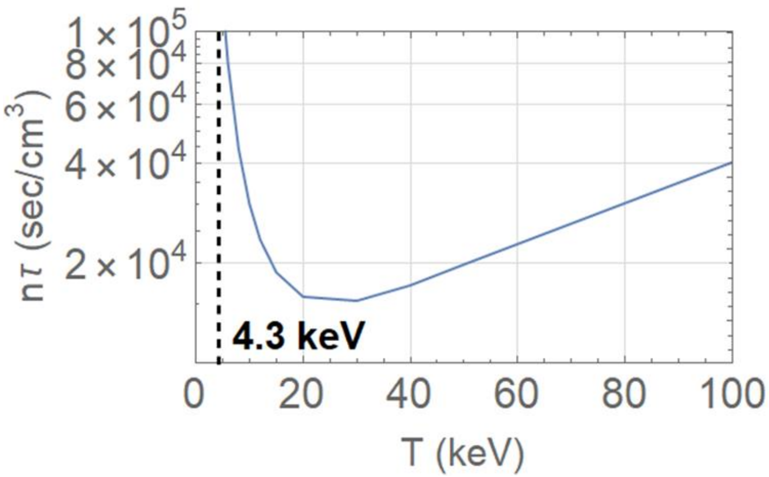
$$p = p_e + p_i = 2p_e = 2n_e T \equiv 2nT$$

$$nT\tau > \frac{3T^2}{\frac{1}{4} \langle \sigma v \rangle E_\alpha - C_B \sqrt{T}}$$

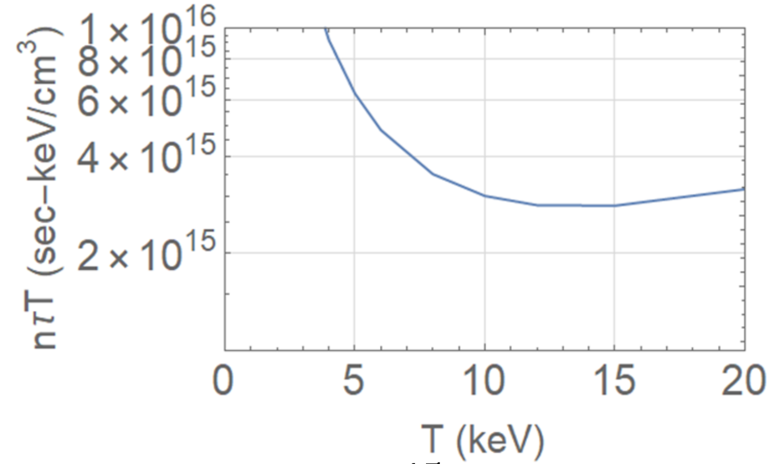
$$n\tau > \frac{3T}{\frac{1}{4} \langle \sigma v \rangle E_\alpha - C_B \sqrt{T}}$$

Temperature needs to be greater than ~5 keV to ignite

$$n\tau > \frac{3T}{\frac{1}{4}\langle\sigma v\rangle\epsilon_\alpha - C_B\sqrt{T}}$$



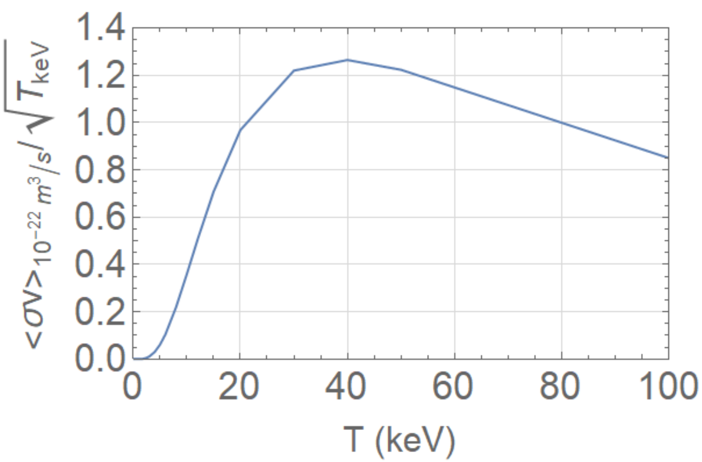
$$n\tau > 2 \times 10^4 \ \mathrm{sec/cm^3}$$



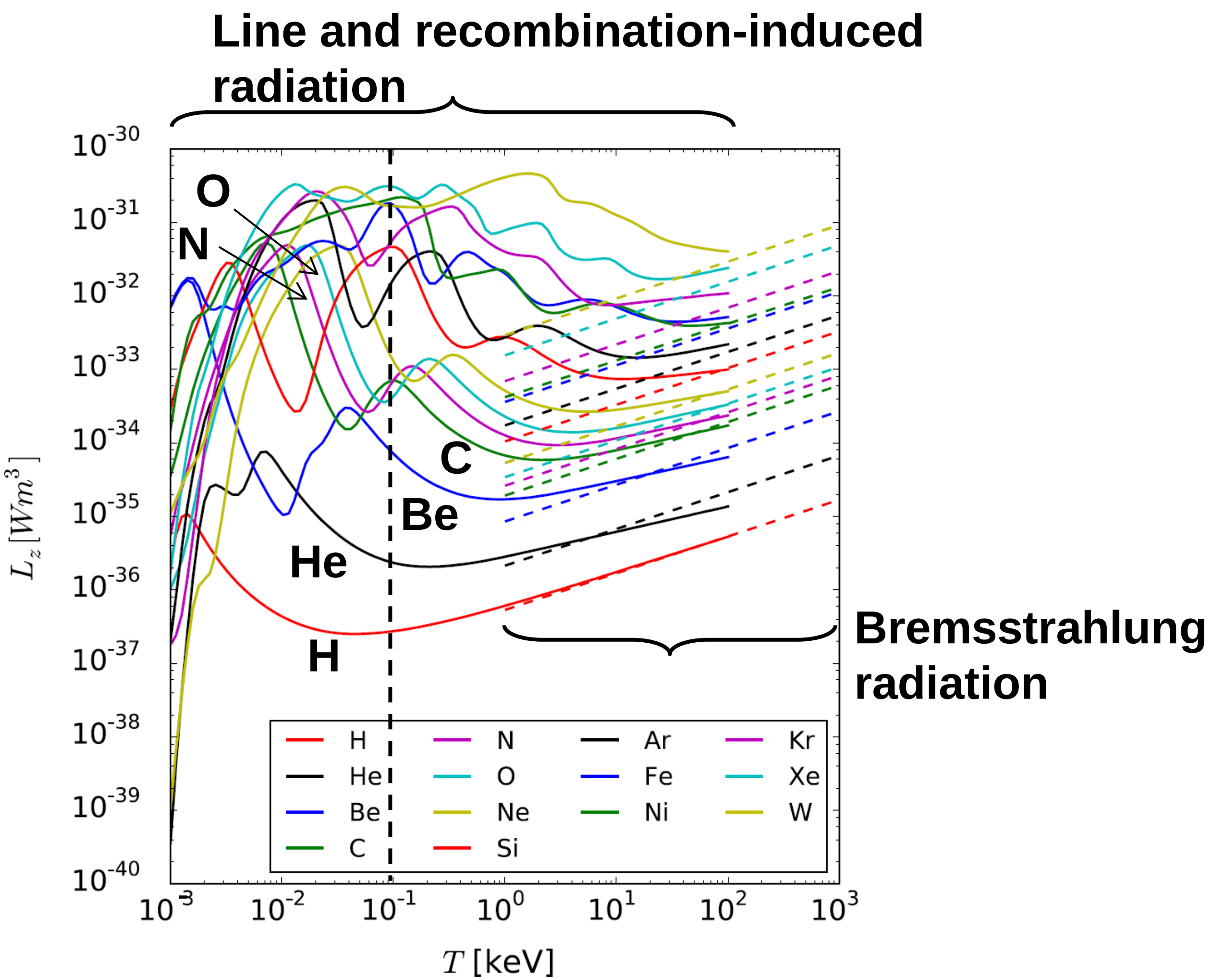
$$nT\tau > 3.5 \times 10^{15} \ \mathrm{keV - sec/cm^3}$$

$S_\alpha > S_B$ $$\frac{1}{4}E_\alpha n^2\langle\sigma v\rangle > C_B n^2 T^{1/2}$$

$$\frac{\langle\sigma v\rangle}{T^{1/2}} > \frac{4C_B}{E_\alpha} \qquad T > 4.3 \ \mathrm{keV}$$



Temperature of 100 eV is the threshold of radiation barrier by impurities



H. Lux, etc., Fusion Eng. Des. **42**, 101 (2015)

Under what conditions the plasma keeps itself hot?

- **Steady state 0-D power balance:**

  $S_\alpha + S_h = S_B + S_k$

  S_α: α particle heating

  S_h: external heating

  S_B: Bremsstrahlung radiation

  S_k: heat conduction lost

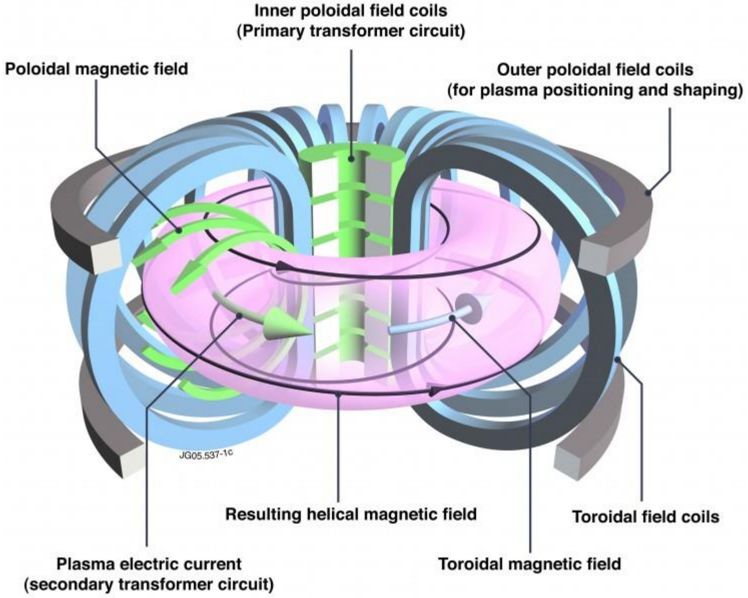
Ignition condition: Pτ > 10 atm-s = 10 Gbar - ns

- **P: pressure, or called energy density**
- **τ is confinement time**

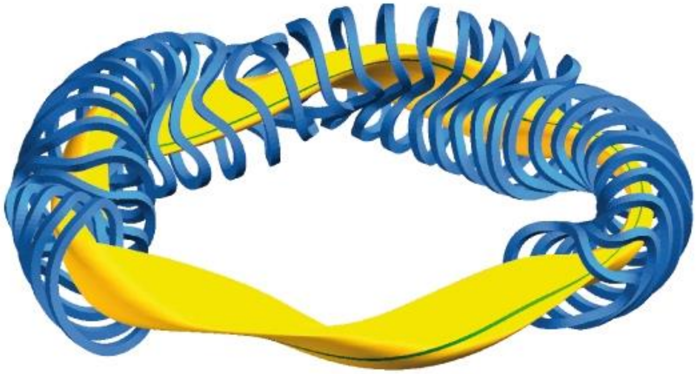
The plasma is too hot to be contained

- **Solution 1: Magnetic confinement fusion (MCF), use a magnetic field to contain it. P~atm, τ~sec, T~10 keV (10^8 ⁰C)**

Tokamak



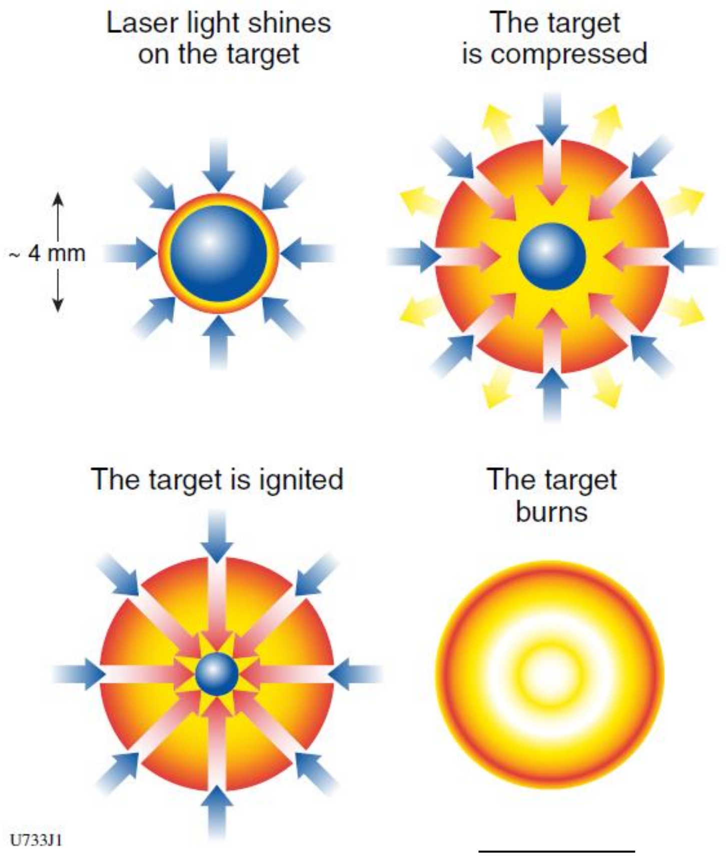
Stellarator



https://www.euro-fusion.org/2011/09/tokamak-principle-2/
https://en.wikipedia.org/wiki/Stellarator

Don’t confine it!

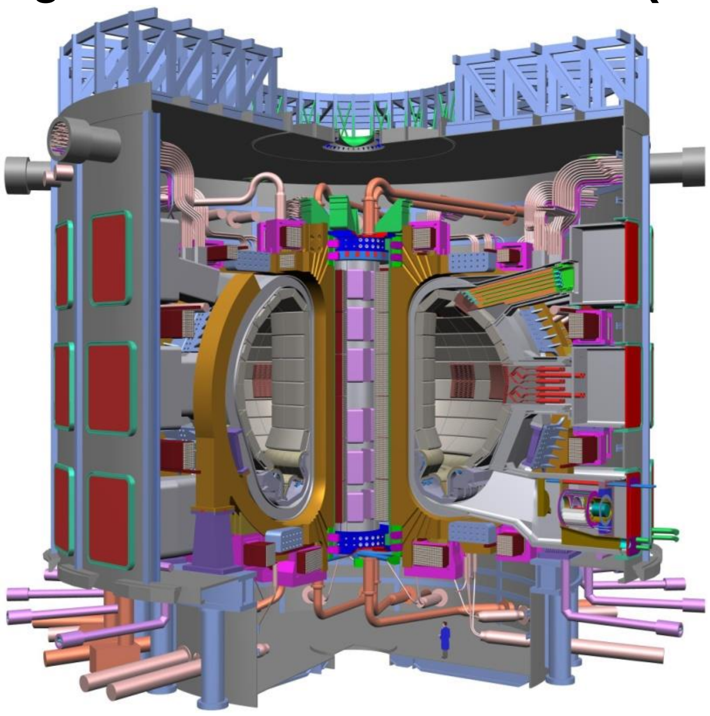
- **Solution 2: Inertial confinement fusion (ICF). Or you can say it is confined by its own inertia: P~Gigabar, τ~nsec, T~10 keV (10^8 ⁰C)**



Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester

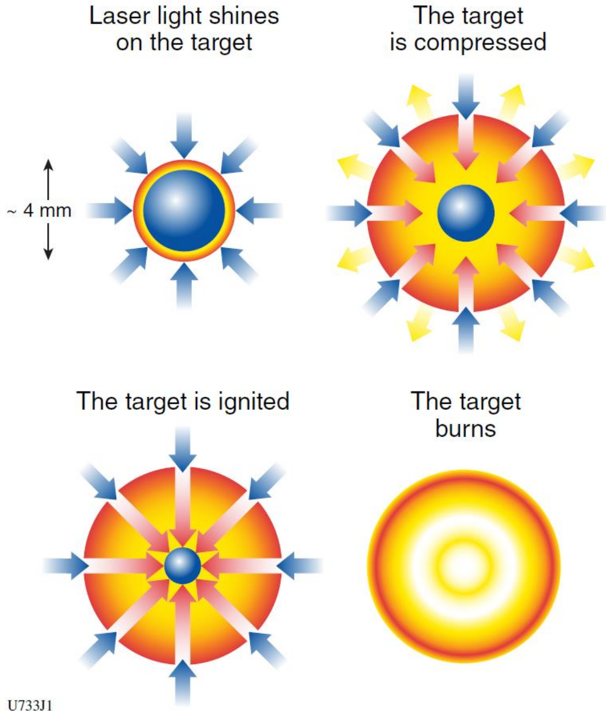
To control? Or not to control?

- **Magnetic confinement fusion (MCF)**



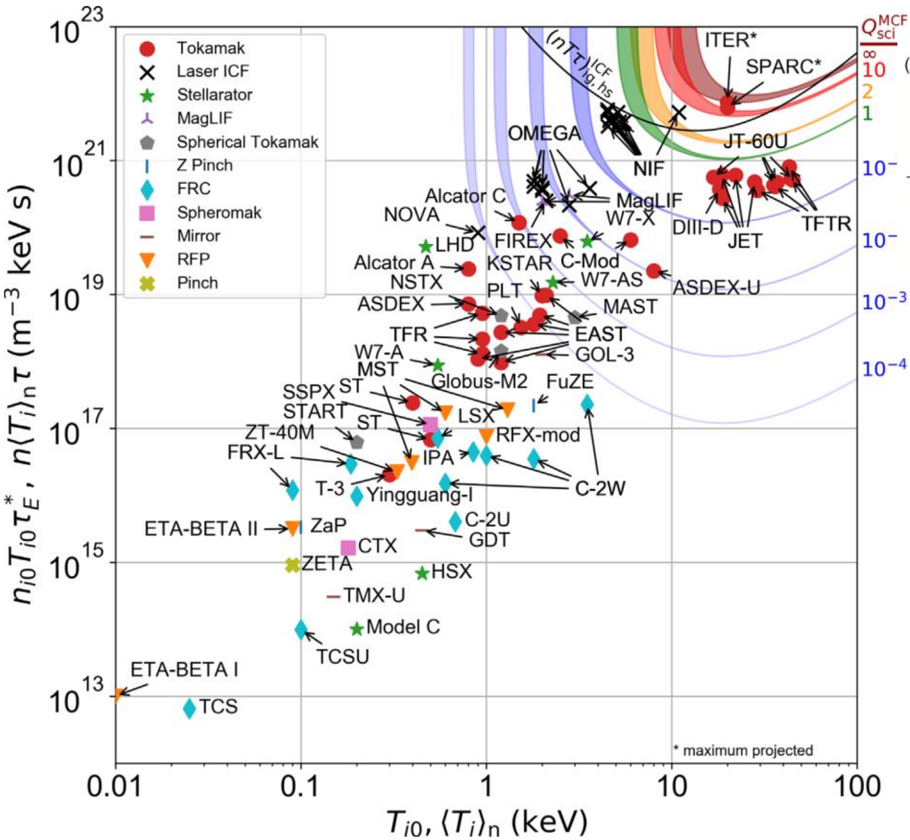
  - **Plasma is confined by toroidal magnetic field.**

- **Inertial confinement fusion (ICF)**

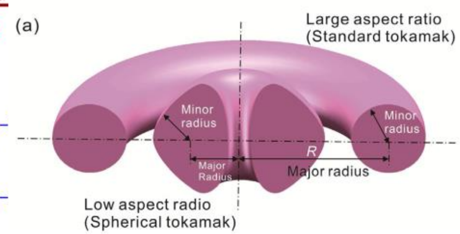


  - **A DT ice capsule filled with DT gas is imploded by laser.**

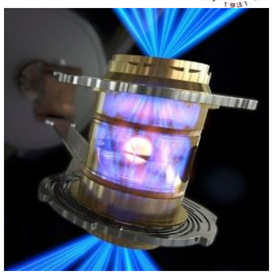
We are closed to ignition!



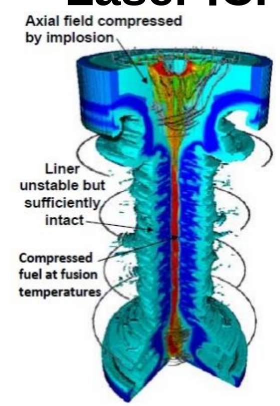
MCF | ICF



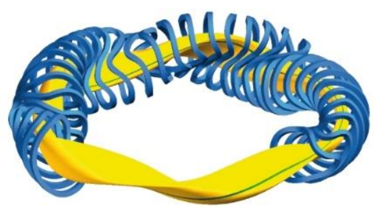
(Spherical) tokamak



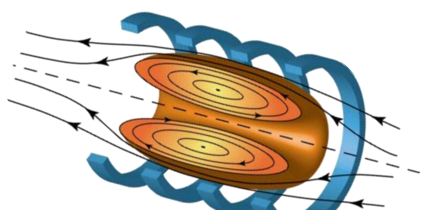
Laser ICF



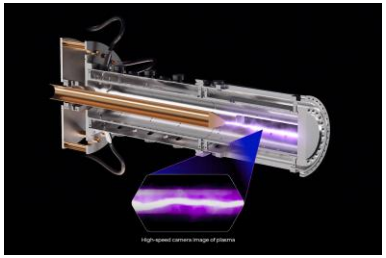
MagLIF



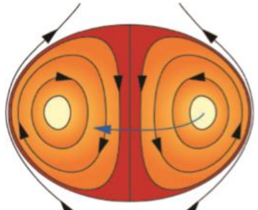
Stellerator



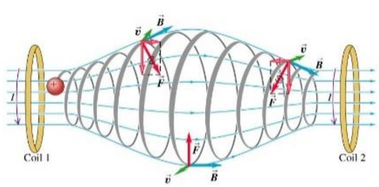
FRC



Sheared-flow Z pinch



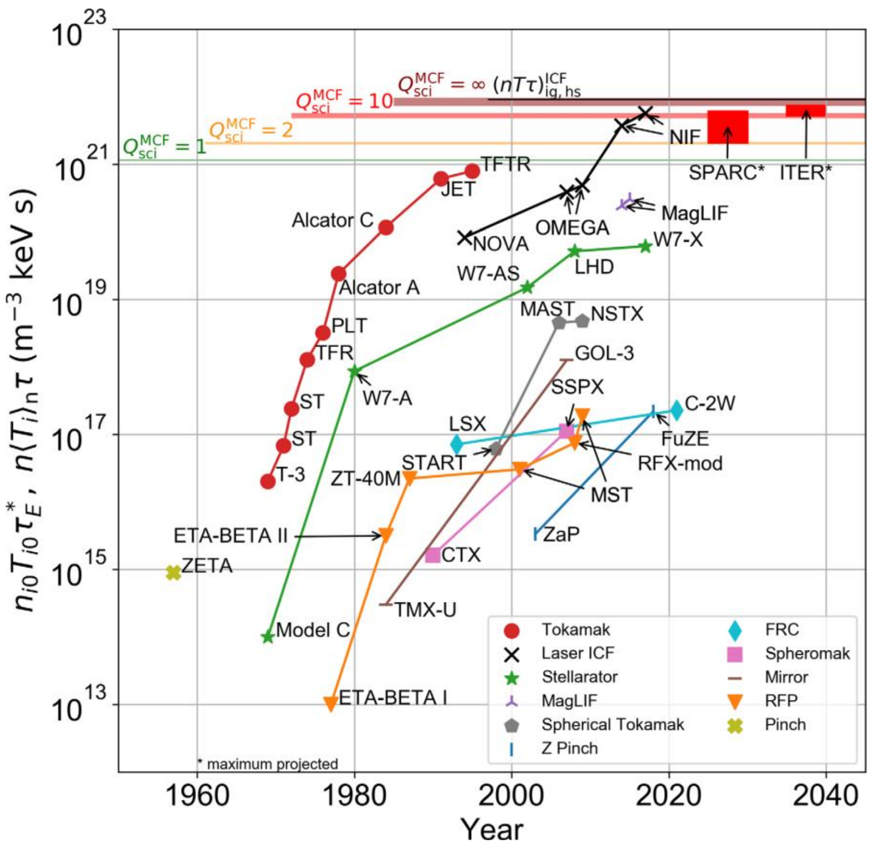
Spheromak



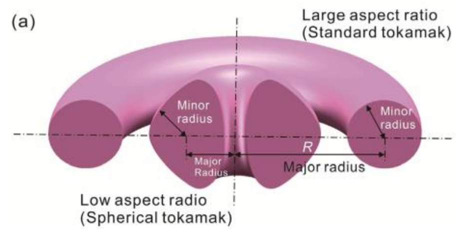
Mirror

Samuel E. Wurzel and Scott C. Hsu, Phys. Plasmas, **29**, 062103 (2022)

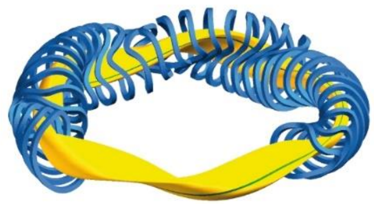
We are closed to ignition!



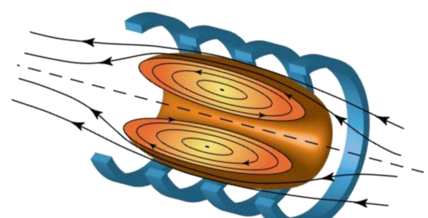
MCF | **ICF**



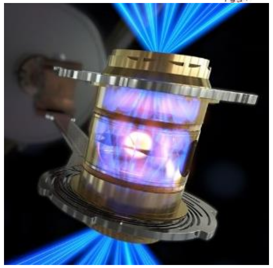
(Spherical) tokamak



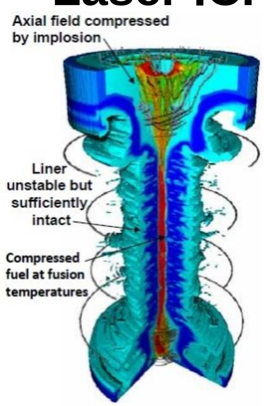
Stellerator



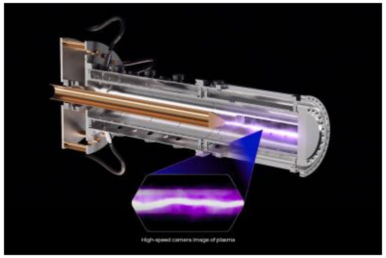
FRC



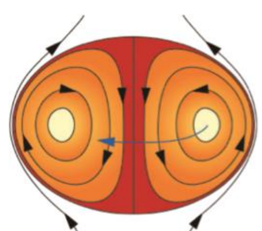
Laser ICF



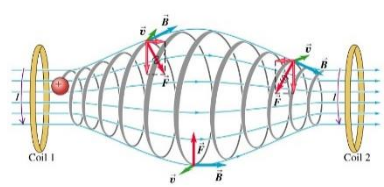
MagLIF



Sheared-flow Z pinch



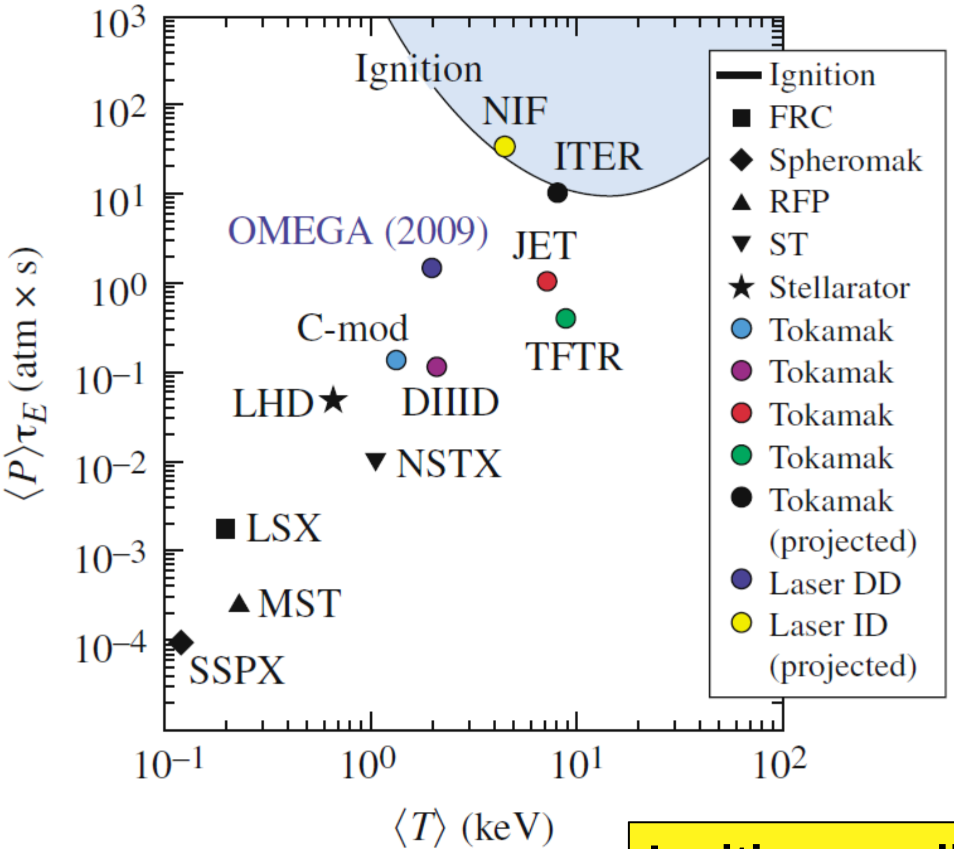
Spheromak

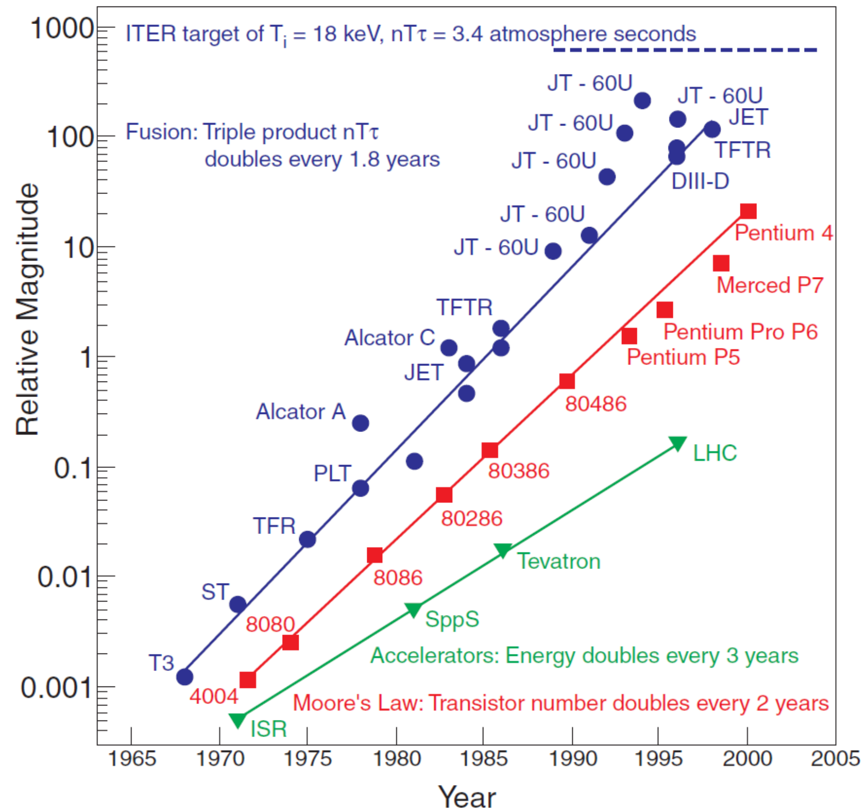


Mirror

Samuel E. Wurzel and Scott C. Hsu, Phys. Plasmas, **29**, 062103 (2022)

We are closed to ignition!





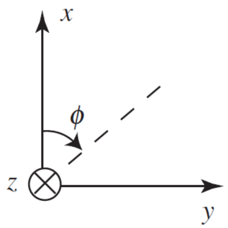
Ignition condition: Pτ > 10 atm-s = 10 Gbar - ns

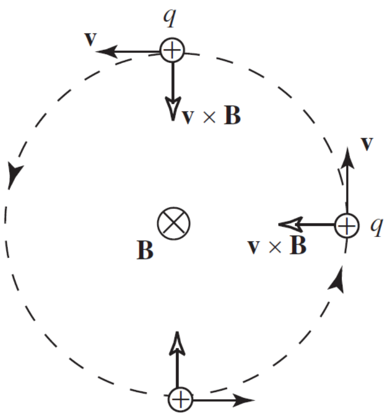
A. J. Webster, Phys. Educ. **38**, 135 (2003)
R. Betti, etc., Phys. Plasmas, **17**, 058102 (2010)

Course Outline

- **Magnetic confinement fusion (MCF)**
  - **Gyro motion, MHD**
  - **1D equilibrium (z pinch, theta pinch)**
  - **Drift: ExB drift, grad B drift, and curvature B drift**
  - **Tokamak, Stellarator (toroidal field, poloidal field)**
  - **Magnetic flux surface**
  - **2D axisymmetric equilibrium of a torus plasma: Grad-Shafranov equation.**
  - **Stability (Kink instability, sausage instability, Safety factor Q)**
  - **Central-solenoid (CS) start-up (discharge) and current drive**
  - **CS-free current drive: electron cyclotron current drive, bootstrap current.**
  - **Auxiliary Heating: ECRH, Ohmic heating, Neutral beam injection.**

Charged particles gyro around the magnetic field line





$$m\frac{d\vec{v}}{dt} = q\,\vec{v} \times \vec{B}$$

- Assuming $\vec{B} = B\hat{z}$ and the electron oscillates in x-y plane

$$m\dot{v}_x = qBv_y$$

$$m\dot{v}_y = -qBv_x$$

$$m\dot{v}_z = 0 \qquad v_z = v_{||} = \text{constant}$$

$$\ddot{v}_x = -\frac{qB}{m}\dot{v}_y = -\left(\frac{qB}{m}\right)^2 v_x$$

$$\ddot{v}_y = -\frac{qB}{m}\dot{v}_x = -\left(\frac{qB}{m}\right)^2 v_y$$

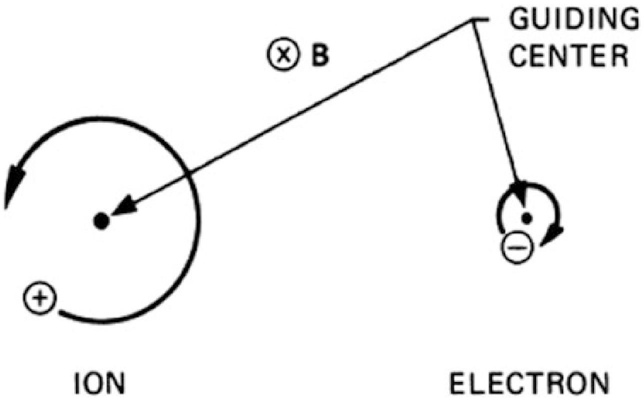
$$\omega_c \equiv \frac{|q|B}{m}$$ Cyclotron frequency or gyrofrequency

$$\ddot{v}_x + {\omega_c}^2 v_x = 0$$

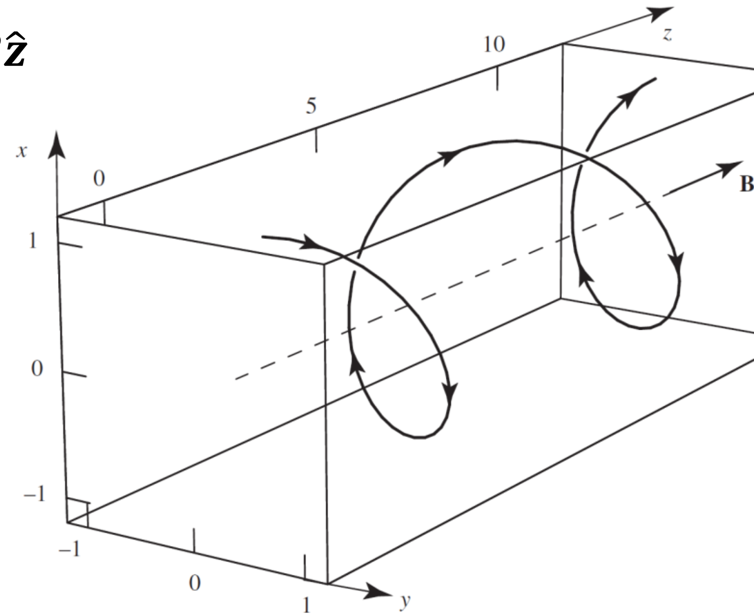
$$\ddot{v}_y + {\omega_c}^2 v_y = 0$$

$$v_x = v_\perp \cos(\pm\omega_c t + \psi)$$
$$v_y = -v_\perp \sin(\pm\omega_c t + \psi)$$
$$v_z = v_{||}$$

Charged particles spiral around the magnetic field line



$\vec{B} = B\hat{z}$



$$v_x = v_\perp \cos(\pm\omega_c t + \psi)$$
$$v_y = v_\perp \sin(\pm\omega_c t + \psi)$$
$$v_z = v_{||}$$

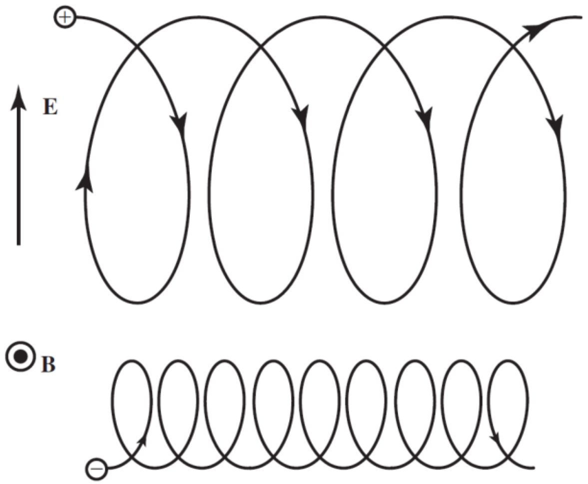
$$x = \mp r_c sin(\pm\omega_c t + \psi) + (x_o - r_c sin\psi)$$
$$y = \pm r_c cos(\pm\omega_c t + \psi) + (y_o + r_c cos\psi)$$
$$z = z_o + v_{||}t$$

$$\omega_c \equiv \frac{|q|B}{m}$$

$$\left|\frac{m{v_\perp}^2}{r}\right| = \left|q\,\vec{v} \times \vec{B}\right| = |qv_\perp B|$$

$$r_c = \frac{v_\perp}{\omega_c} = \frac{mv_\perp}{|q|B}$$ **Larmor radius or gyroradius**

Charge particles drift across magnetic field lines when an electric field not parallel to the magnetic field occurs



$$m\frac{d}{dt}\left(\vec{v}_{\mathrm{E}} + \vec{v}_{\mathrm{ac}}(t)\right) = q\left[\hat{x}E_{\perp} + \left(\vec{v}_{\mathrm{E}} + \vec{v}_{\mathrm{ac}}(t)\right) \times \hat{z}B\right]$$

$$m\frac{d\,\vec{v}_{\mathrm{ac}}(t)}{dt} = q\left[\hat{x}E_{\perp} + \vec{v}_{\mathrm{E}} \times \hat{z}B + \vec{v}_{\mathrm{ac}}(t) \times \hat{z}B\right]$$

No E field case: $m\frac{d\,\vec{v}}{dt} = q\,\vec{v} \times \vec{B}$

$$\hat{x}E_{\perp} + \vec{v}_{\mathrm{E}} \times \hat{z}B = 0$$

$$\left(\vec{C} \times \vec{B}\right) \times \vec{A} = \left(\vec{A}\cdot\vec{C}\right)\vec{B} - \left(\vec{A}\cdot\vec{B}\right)\vec{C}$$

$\times\,\hat{z}B$

$$\vec{E} = \vec{E}_{\perp} + \hat{z}E_{||} = \hat{x}E_{\perp} + \hat{z}E_{||}$$

$$\vec{v}_{\mathrm{E}} = \frac{\hat{x}E_{\perp} \times \hat{z}B}{B^2} = \frac{\vec{E} \times \vec{B}}{B^2}$$

ExB drift velocity

$$m\frac{dv_{||}}{dt} = qE_{||}$$

$$m\frac{d\,\vec{v}_{\mathrm{ac}}(t)}{dt} = q\,\vec{v}_{\mathrm{ac}}(t) \times \hat{z}B$$

Gyro motion

$$m\frac{d\,\vec{v}_{\perp}}{dt} = q\left(\hat{x}E_{\perp} + \vec{v}_{\perp} \times \hat{z}B\right)$$

$$\vec{v}(t) = \hat{z}v_{||}(t) + \vec{v}_{\mathrm{E}} + \vec{v}_{\mathrm{ac}}(t)$$

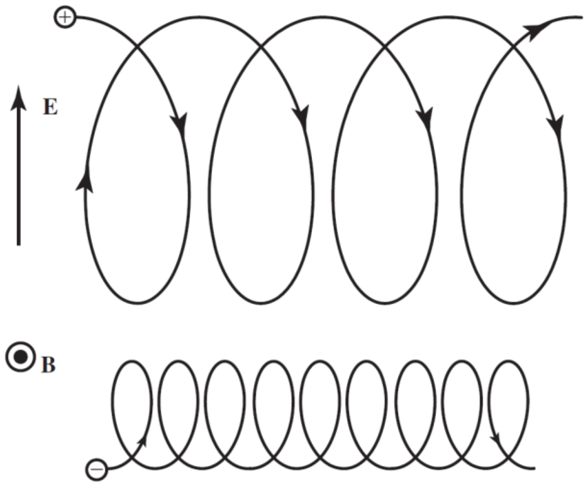
$$v_{||}(t) = \frac{qE_{||}}{m}t + v_{||,o}$$

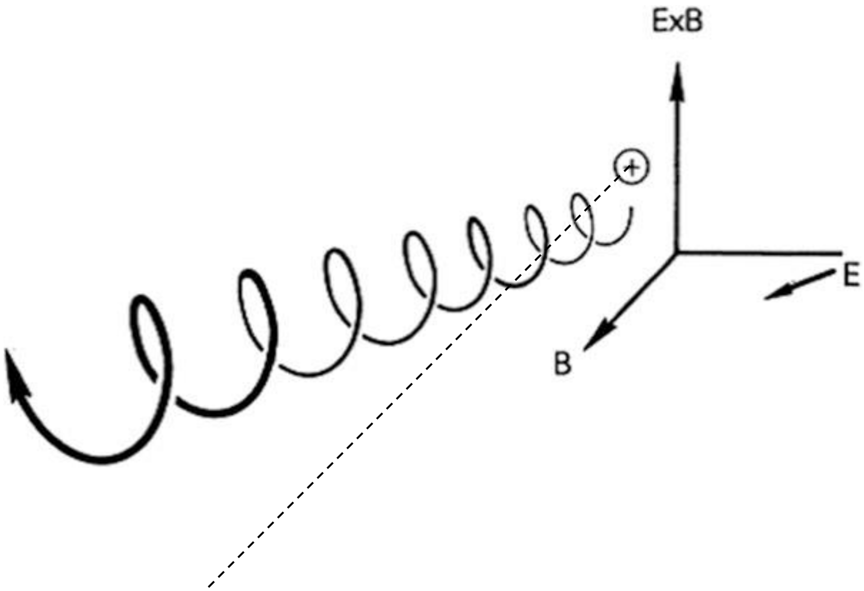
$$\langle\vec{v}(t)\rangle = \frac{1}{T_c}\int_0^{T_c}\vec{v}(t)\,dt = \hat{z}v_{||}(t) + \vec{v}_E$$

$$\vec{v}_{\perp}(t) = \vec{v}_{\mathrm{E}} + \vec{v}_{\mathrm{ac}}(t)$$

- Electrons and ions drift in the same direction.

No current is generated in ExB drift





$$\langle \vec{v}(t) \rangle = \frac{1}{T_c} \int_0^{T_c} \vec{v}(t)\, dt = \hat{z} v_{||}(t) + \vec{v}_E$$

$$\vec{v}_{\mathrm{E}} = \frac{\hat{x} E_{\perp} \times \hat{z} B}{B^2} = \frac{\vec{E} \times \vec{B}}{B^2}$$ **ExB drift velocity**

- **Electrons and ions drift in the same direction.**

Charge particles drift across magnetic field lines when an external field not parallel to the magnetic field occurs

$$\vec{E} = \vec{E}_\perp + \hat{z}E_{||} = \hat{x}E_\perp + \hat{z}E_{||}$$

$$m\frac{dv_{||}}{dt} = qE_{||}$$

$$m\frac{d\,\vec{v}_\perp}{dt} = q(\hat{x}E_\perp + \vec{v}_\perp \times \hat{z}B)$$

$$\langle\vec{v}\,(t)\rangle = \frac{1}{T_c}\int_0^{T_c}\vec{v}\,(t)\,dt = \hat{z}v_{||}(t) + \vec{v}_E$$

$$\vec{v}_{\mathrm{E}} = \frac{\hat{x}E_\perp \times \hat{z}B}{B^2} = \frac{\vec{E}\times\vec{B}}{B^2}$$

ExB drift velocity

$$\vec{F} = \vec{F}_\perp + \hat{z}F_{||} = \hat{x}F_\perp + \hat{z}F_{||}$$

$$m\frac{dv_{||}}{dt} = F_{||}$$

$$m\frac{d\,\vec{v}_\perp}{dt} = q\left(\hat{x}\frac{F_\perp}{q} + \vec{v}_\perp \times \hat{z}B\right)$$

$$\langle\vec{v}\,(t)\rangle = \frac{1}{T_c}\int_0^{T_c}\vec{v}\,(t)\,dt = \hat{z}v_{||}(t) + \vec{v}_F$$

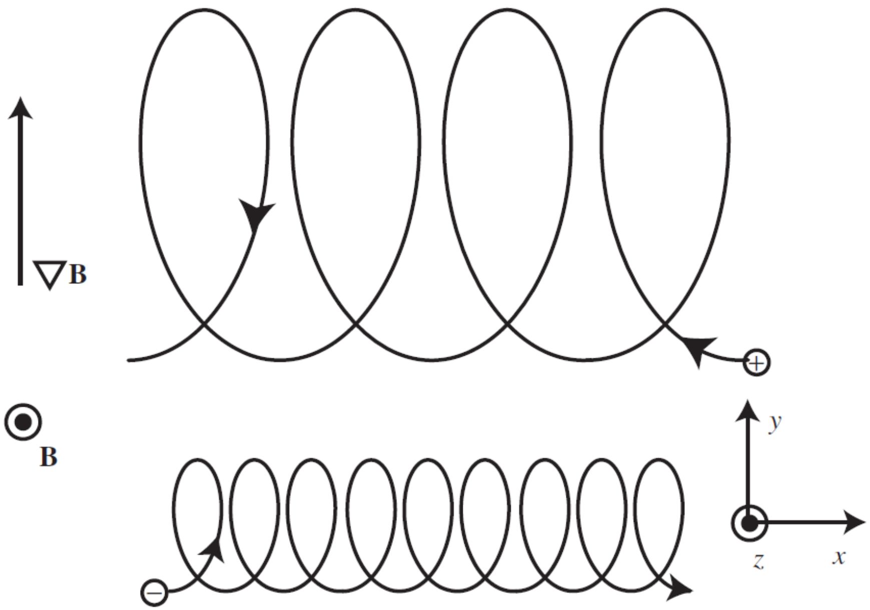
$$\vec{v}_{\mathrm{F}} = \frac{\hat{x}(F_\perp/q)\times\hat{z}B}{B^2} = \frac{1}{q}\frac{\vec{F}\times\vec{B}}{B^2}$$

Gravitational drift velocity

- **Electrons and ions drift in the opposite directions in the gravitational drift. Therefore, currents are generated.**

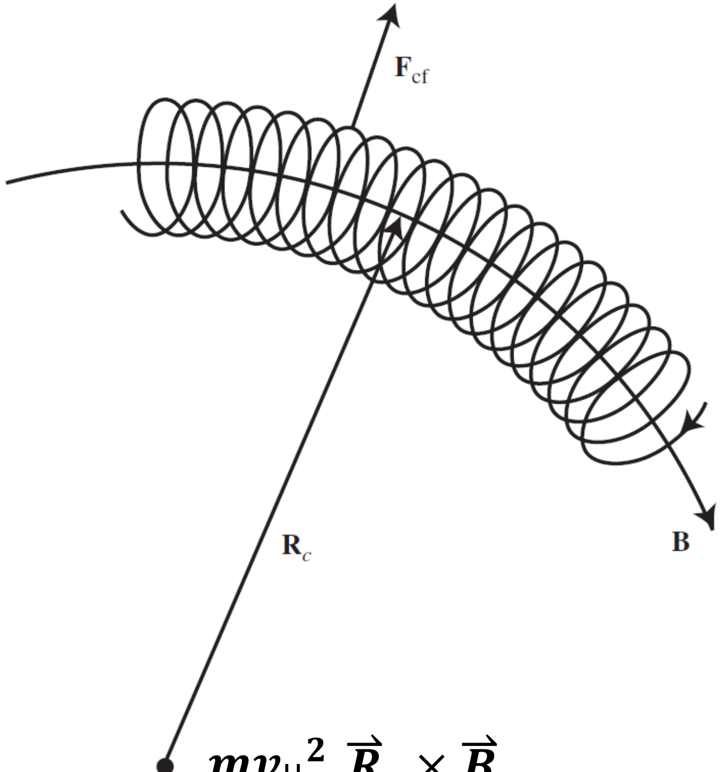
Charge particles drift across magnetic field lines when the magnetic field is not uniform or curved

- **Gradient-B drift**



$$\vec{v}_{\nabla} = \frac{m{v_{\perp}}^2}{2q}\frac{\vec{B}\times\nabla B}{B^3}$$

- **Curvature drift**

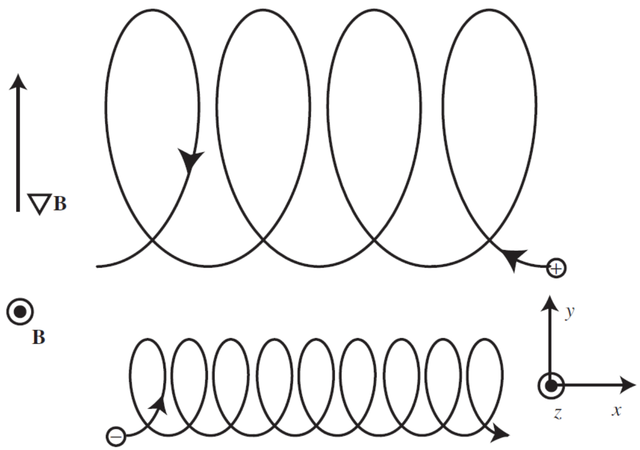


$$\vec{v}_{R} = \frac{m{v_{||}}^2}{2q}\frac{\vec{R}_c\times\vec{B}}{R_c B^2}$$

$$\vec{v}_{\text{total}} = \vec{v}_{\text{R}} + \vec{v}_{\nabla} = \frac{\vec{B}\times\nabla B}{\omega_c B^2}\left({v_{||}}^2 + \frac{1}{2}{v_{\perp}}^2\right) = \frac{m}{q}\frac{\vec{R}_c\times\vec{B}}{{R_c}^2 B^2}\left({v_{||}}^2 + \frac{1}{2}{v_{\perp}}^2\right)$$

Gradient-B drift

Charge particles drift across magnetic field lines when the magnetic field is not uniform or curved



- In the case with no gradient B

$$x_c = \mp r_c \sin(\pm\omega_c t + \psi)$$

$$y_c = \pm r_c \cos(\pm\omega_c t + \psi)$$

$$v_x = v_\perp \cos(\pm\omega_c t + \psi)$$

$$v_y = -v_\perp \sin(\pm\omega_c t + \psi)$$

$$\vec{F} = q(\vec{v} \times \vec{B}) = \hat{x} q v_y B_z - \hat{y} q v_x B_z$$

$$\simeq \hat{x} q v_y \left(B_o + y\frac{\partial B_z}{\partial y}\right) - \hat{y} q v_x \left(B_o + y\frac{\partial B_z}{\partial y}\right)$$

$$B_z(y) = B_o + y\frac{\partial B_z}{\partial y} + y^2 \frac{1}{2}\frac{\partial^2 B_z}{\partial y^2} + \ldots$$

$$F_x = q v_y \left(B_o + y\frac{\partial B_z}{\partial y}\right)$$

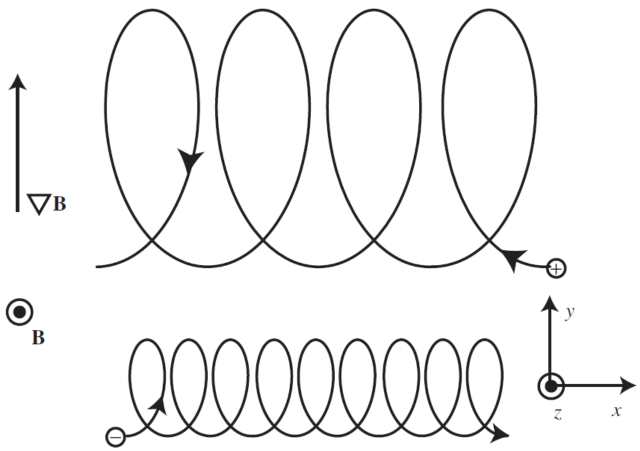
$$F_y = -q v_x \left(B_o + y\frac{\partial B_z}{\partial y}\right)$$

$$F_x \simeq -q v_\perp \sin(\pm\omega_c t + \psi) \times \left(B_o \pm r_c \cos(\pm\omega_c t + \psi)\frac{\partial B_z}{\partial y}\right)$$

$$F_y = -q v_\perp \cos(\pm\omega_c t + \psi) \times \left(B_o \pm r_c \cos(\pm\omega_c t + \psi)\frac{\partial B_z}{\partial y}\right)$$

$$\vec{v}_F = \frac{1}{q}\frac{\vec{F} \times \vec{B}}{B^2}$$

Charge particles drift across magnetic field lines when the magnetic field is not uniform



$$F_x \simeq -qv_\perp \sin(\pm\omega_c t + \psi)\left(B_o \pm r_c \cos(\pm\omega_c t + \psi)\frac{\partial B_z}{\partial y}\right)$$

$$F_y \simeq -qv_\perp \cos(\pm\omega_c t + \psi)\left(B_o \pm r_c \cos(\pm\omega_c t + \psi)\frac{\partial B_z}{\partial y}\right)$$

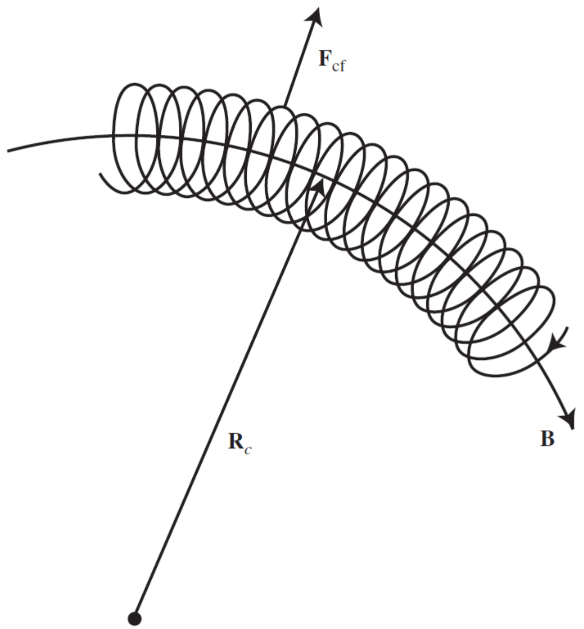
$$\langle F_x \rangle = 0$$

$$\langle F_y \rangle = \mp \frac{qv_\perp r_c}{2}\frac{\partial B_z}{\partial y} = -\frac{m{v_\perp}^2}{2B}\frac{\partial B_z}{\partial y} \qquad r_c = \frac{v_\perp}{\omega_c} \quad \omega_c \equiv \frac{|q|B}{m}$$

$$\vec{v}_F = \frac{1}{q}\frac{\vec{F}\times\vec{B}}{B^2} \qquad \vec{v}_\nabla = \frac{1}{q}\frac{\langle F_y\rangle \hat{y}\times\hat{z}B_z}{{B_z}^2} = -\frac{m{v_\perp}^2}{2qB_z}\frac{\partial B_z}{\partial y}\hat{x}$$

- More general: $\vec{v}_\nabla = \frac{m{v_\perp}^2}{2q}\frac{\vec{B}\times\nabla B}{B^3}$

Charge particles drift across magnetic field lines when the magnetic field line is curved

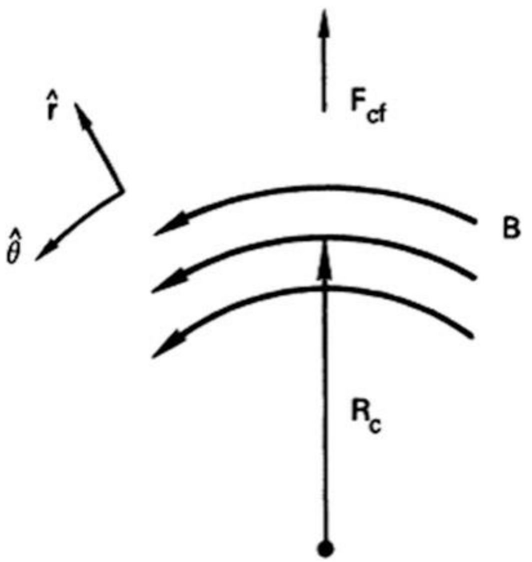


$$\vec{F}_{cf} = m{v_{||}}^2 \frac{\vec{R}_c}{{R_c}^2}$$

$$\vec{v}_F = \frac{1}{q}\frac{\vec{F} \times \vec{B}}{B^2}$$

$$\vec{v}_R = \frac{1}{q}\frac{\vec{F}_{cf} \times \vec{B}}{B^2} = \frac{m{v_{||}}^2}{2q}\frac{\vec{R}_c \times \vec{B}}{R_c B^2}$$

Charge particles drift across magnetic field lines when the magnetic field is not uniform or curved



$$\vec{B} = B\hat{\theta}$$

$$\nabla B = \nabla B \hat{r}$$

Cylindrical coordinate

$$\vec{v}_\nabla = \frac{m{v_\perp}^2}{2q}\frac{\vec{B}\times\nabla B}{B^3} \qquad \vec{v}_R = \frac{m{v_{||}}^2}{2q}\frac{\vec{R}_c\times\vec{B}}{R_c B^2}$$

$$\nabla\times\vec{B} = 0$$

$$\left(\nabla\times\vec{B}\right)_r = \frac{1}{r}\frac{\partial B_z}{\partial\theta} - \frac{\partial B_\theta}{\partial z} = 0 \qquad \left(\nabla\times\vec{B}\right)_\theta = \frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} = 0$$

$$\left(\nabla\times\vec{B}\right)_z = \frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) - \frac{1}{r}\frac{\partial B_r}{\partial\theta}$$

$$\nabla\times\vec{B} = \left(\nabla\times\vec{B}\right)_z = \frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) = 0 \qquad B_\theta \propto \frac{1}{r}$$

$$\frac{\nabla|B|}{|B|} = -\frac{\vec{R}_c}{{R_c}^2}$$

$$\vec{v}_{total} = \vec{v}_R + \vec{v}_\nabla = \frac{\vec{B}\times\nabla B}{\omega_c B^2}\left({v_{||}}^2 + \frac{1}{2}{v_\perp}^2\right) = \frac{m}{q}\frac{\vec{R}_c\times\vec{B}}{{R_c}^2 B^2}\left({v_{||}}^2 + \frac{1}{2}{v_\perp}^2\right)$$

- **Electrons and ions drift in the opposite directions in the grad-B and curvature drifts. Therefore, currents are generated.**

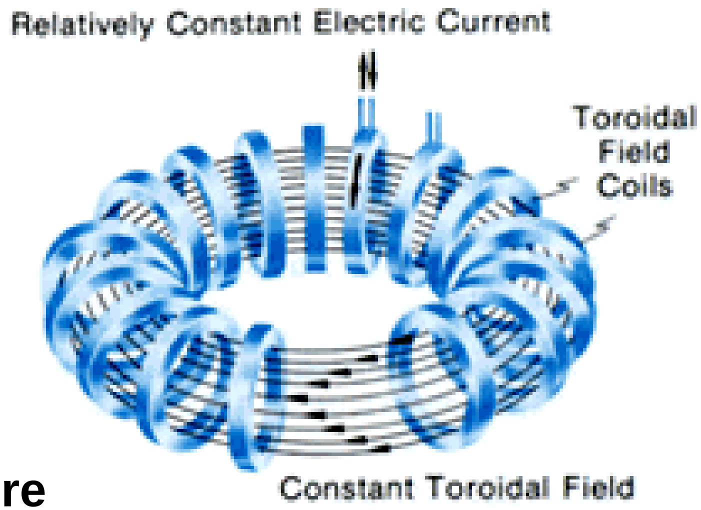
Quick summary of different drifts

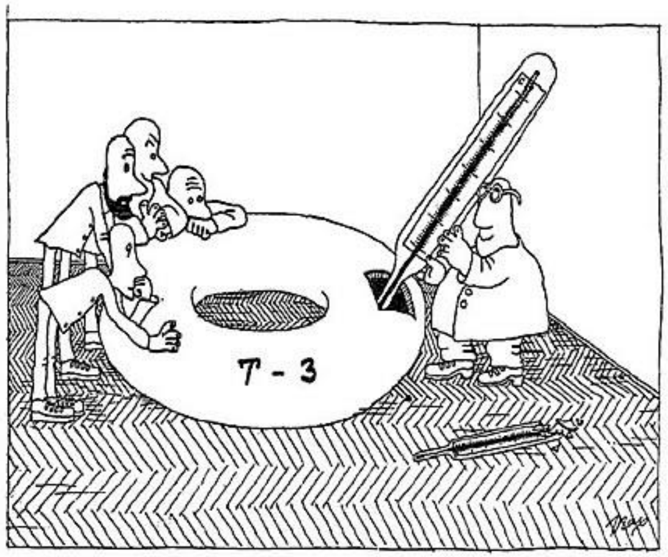
- **ExB drift:** $\vec{v}_{\mathrm{E}} = \dfrac{\vec{E} \times \vec{B}}{B^2}$ **Independent to charge**
- **Gravitational drift:** $\vec{v}_{\mathrm{F}} = \dfrac{1}{q} \dfrac{\vec{F} \times \vec{B}}{B^2}$ **Depended on charge**
- **Grad-B drift:** $\vec{v}_{\nabla} = \dfrac{m v_{\perp}{}^2}{2q} \dfrac{\vec{B} \times \nabla B}{B^3}$ **Depended on charge**
- **Curvature drift:** $\vec{v}_{R} = \dfrac{m v_{||}{}^2}{2q} \dfrac{\vec{R}_{\mathrm{c}} \times \vec{B}}{R_{\mathrm{c}} B^2}$ **Depended on charge**
- **Non-uniform B drift:**

$$\vec{v}_{\mathrm{total}} = \vec{v}_{\mathrm{R}} + \vec{v}_{\nabla} = \frac{\vec{B} \times \nabla B}{\omega_{\mathrm{c}} B^2} \left(v_{||}{}^2 + \frac{1}{2} v_{\perp}{}^2 \right) = \frac{m}{q} \frac{\vec{R}_{\mathrm{c}} \times \vec{B}}{R_{\mathrm{c}}{}^2 B^2} \left(v_{||}{}^2 + \frac{1}{2} v_{\perp}{}^2 \right)$$

Plasma can be confined in a doughnut-shaped chamber with toroidal magnetic field

- **Tokamak - "toroidal chamber with magnetic coils" (тороидальная камера с магнитными катушками)**





Nature

Measurement of the Electron Temperature by Thomson Scattering in Tokamak T3

by

N. J. PEACOCK
D. C. ROBINSON
M. J. FORREST
P. D. WILCOCK
UKAEA Research Group, Culham Laboratory, Abingdon, Berkshire

V. V. SANNIKOV
I. V. Kurchatov Institute, Moscow

Electron temperatures of 100 eV up to 1 keV and densities in the range $1\text{–}3 \times 10^{13}$ cm^{-3} have been measured by Thomson scattering on Tokamak T3. These results agree with those obtained by other techniques where direct comparison has been possible.

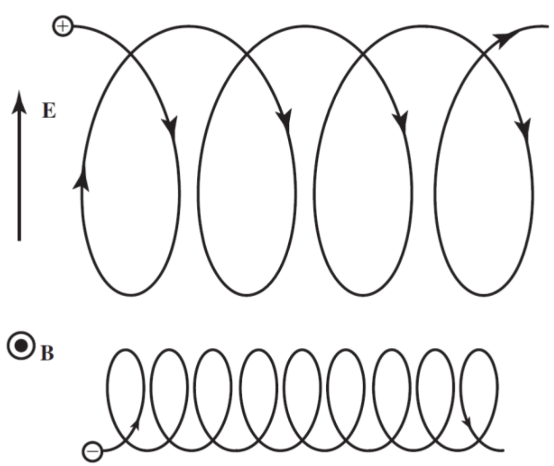
T_e = 100 ~ 1 keV

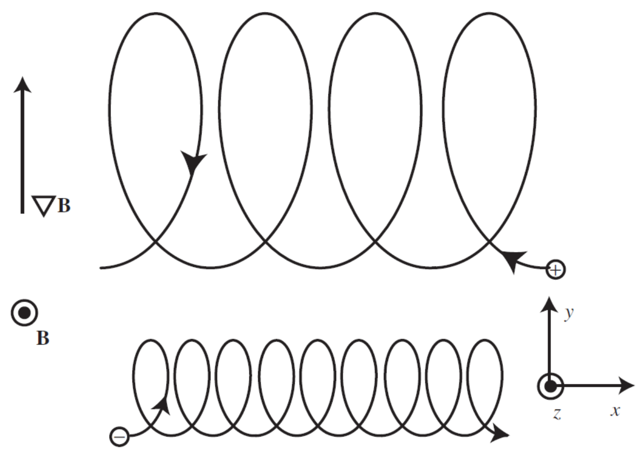
n_e = 1-3 x 10^{13} cm^{-3}

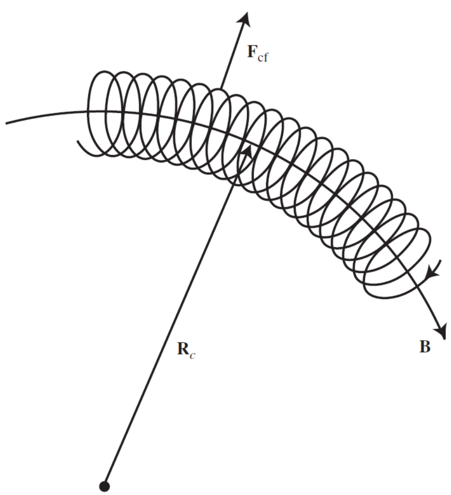
https://www.iter.org/mach/tokamak
https://en.wikipedia.org/wiki/Tokamak#cite_ref-4
Drawing from the talk “Evolution of the Tokamak” given in 1988 by B.B. Kadomtsev at Culham.
N. J. Peacock,et al., Nature **224**, 488 (1969)

Quick summary of different drifts

- **ExB drift:** $\vec{v}_E = \dfrac{\vec{E} \times \vec{B}}{B^2}$ **Independent to charge**
- **Grad-B drift:** $\vec{v}_\nabla = \dfrac{m{v_\perp}^2}{2q} \dfrac{\vec{B} \times \nabla B}{B^3}$ **Depended on charge**
- **Curvature drift:** $\vec{v}_R = \dfrac{m{v_{||}}^2}{q} \dfrac{\vec{R}_c \times \vec{B}}{{R_c}^2 B^2}$ **Depended on charge**

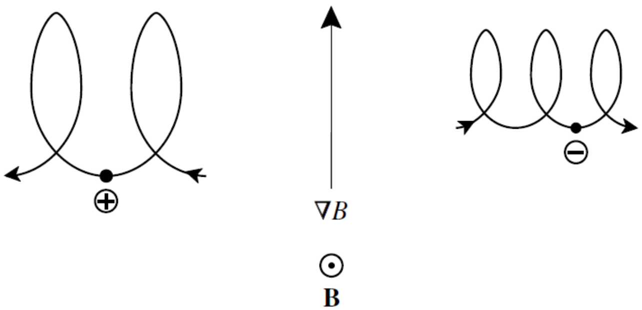




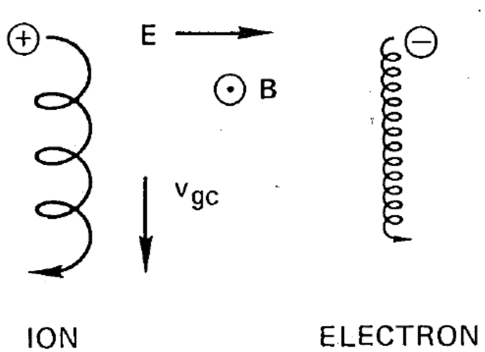


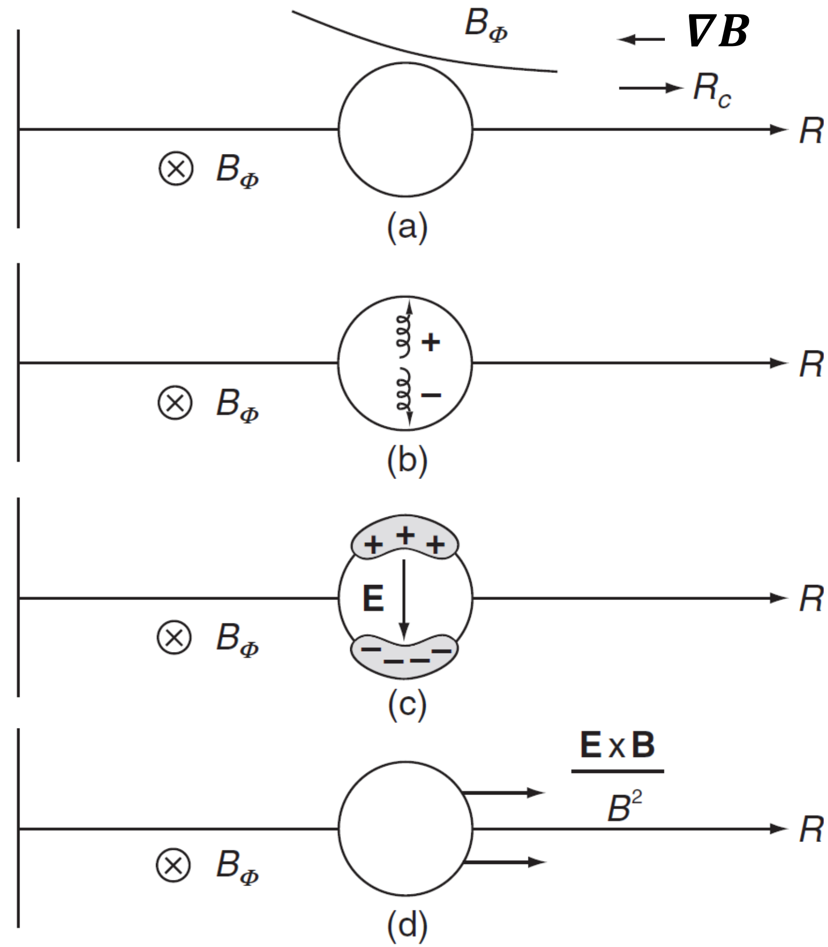
Charged particles drift across field lines

- **Grad-B drift**



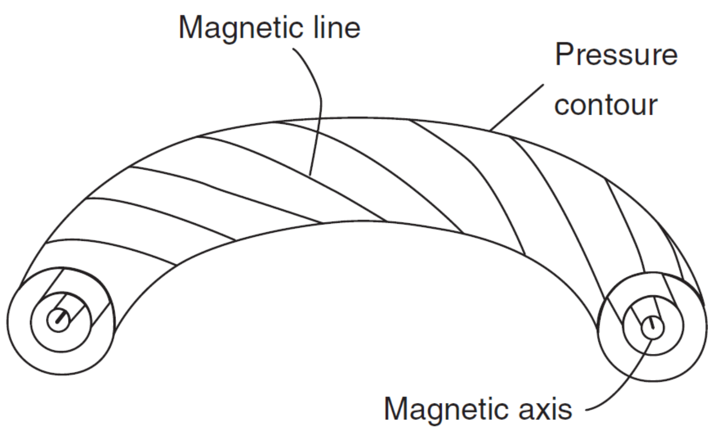
- **ExB drift**

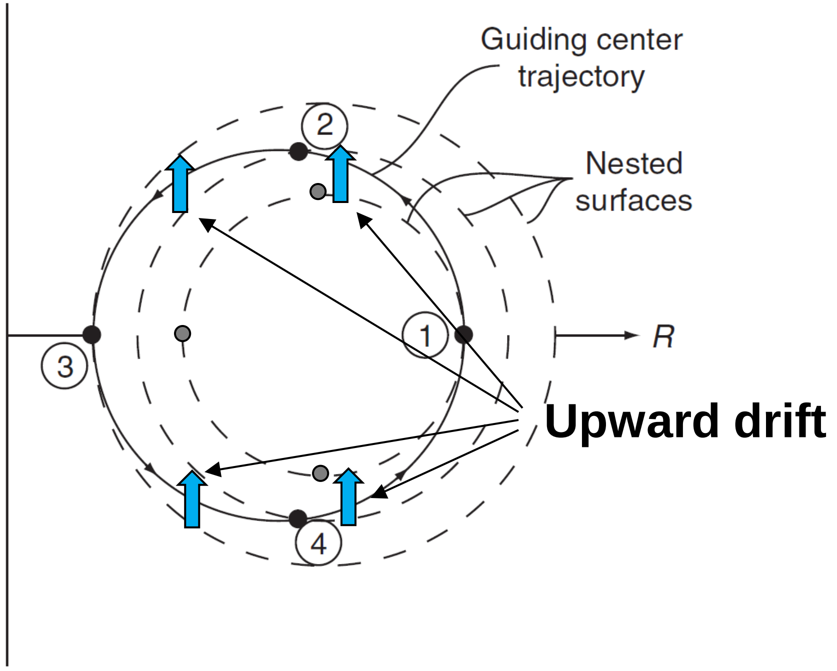




J. P. Freidberg, Ideal Magnetohydrodynamics

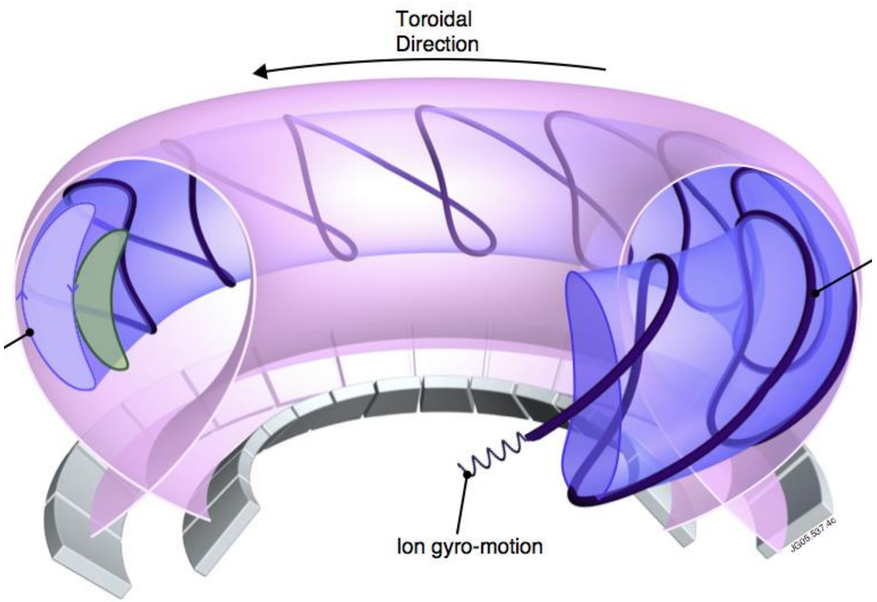
The particle drifts back to the original position if a small poloidal field is superimposed on the toroidal field

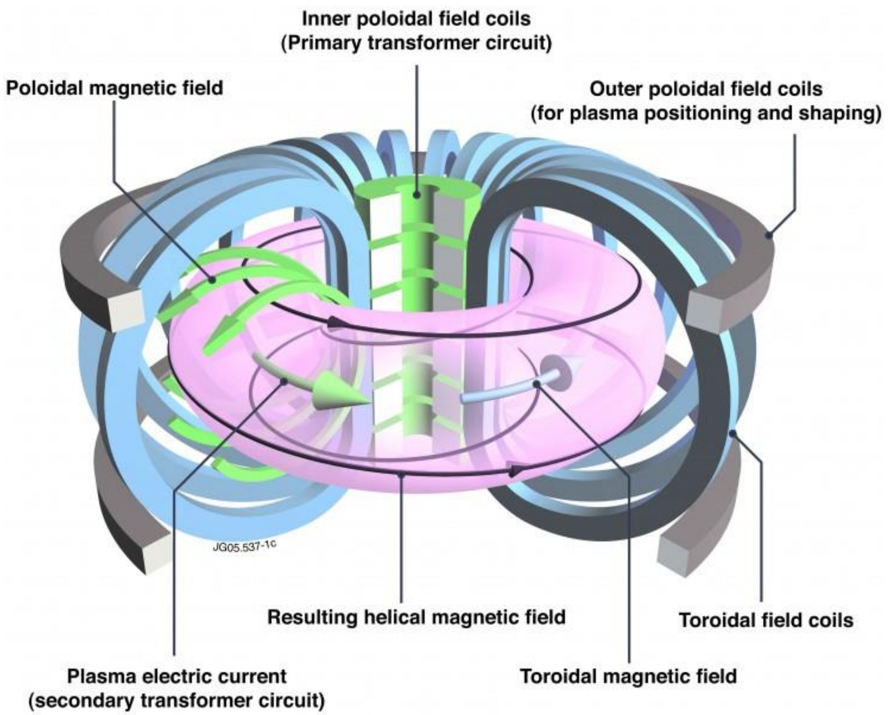




• Points with no drift

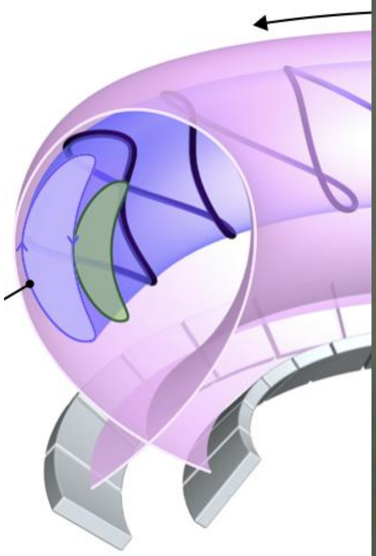
A poloidal magnetic field is required to reduce the drift across field lines

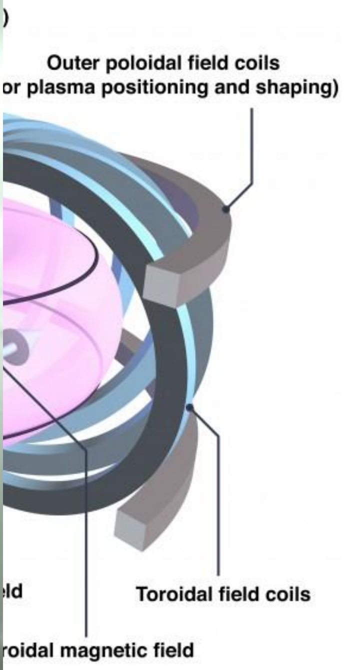




https://www.davidpace.com/keeping-fusion-plasmas-hot/
https://www.euro-fusion.org/2011/09/tokamak-principle-2/

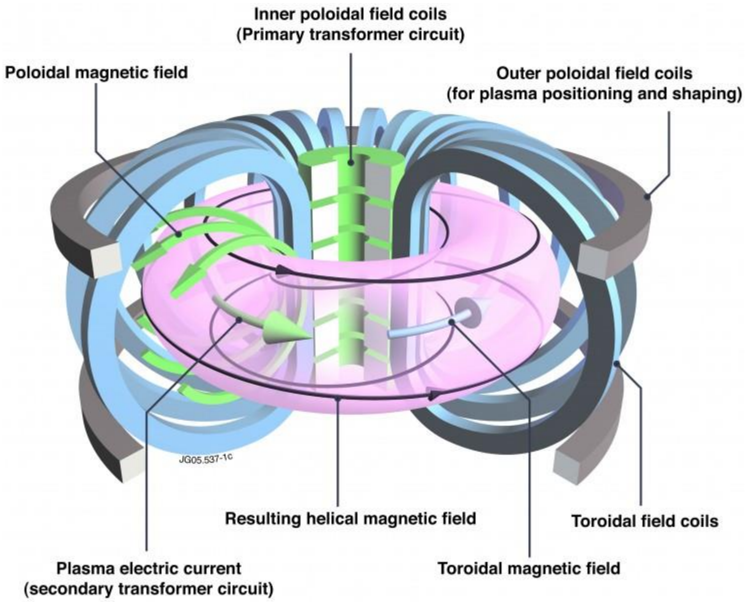
A poloidal magnetic field is required to reduce the drift across field lines



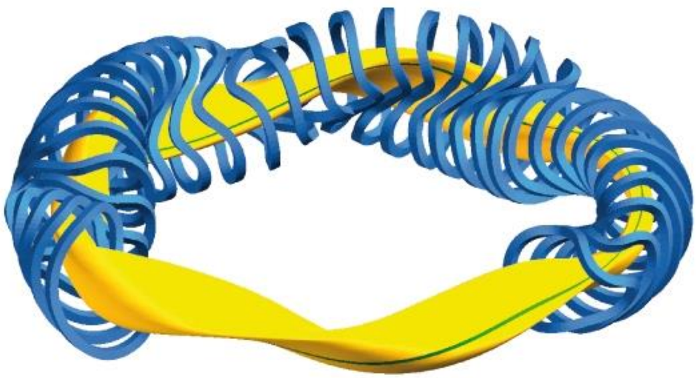


Stellarator uses twisted coil to generate poloidal magnetic field

Tokamak



Stellarator



https://www.euro-fusion.org/2011/09/tokamak-principle-2/
https://en.wikipedia.org/wiki/Stellarator