Introduction to Nuclear Fusion as An Energy Source

Po-Yu Chang

Institute of Space and Plasma Sciences, National Cheng Kung University

Lecture 2

2025 spring semester

Tuesday 9:00-12:00

Materials:

https://capst.ncku.edu.tw/PGS/index.php/teaching/

Online courses:

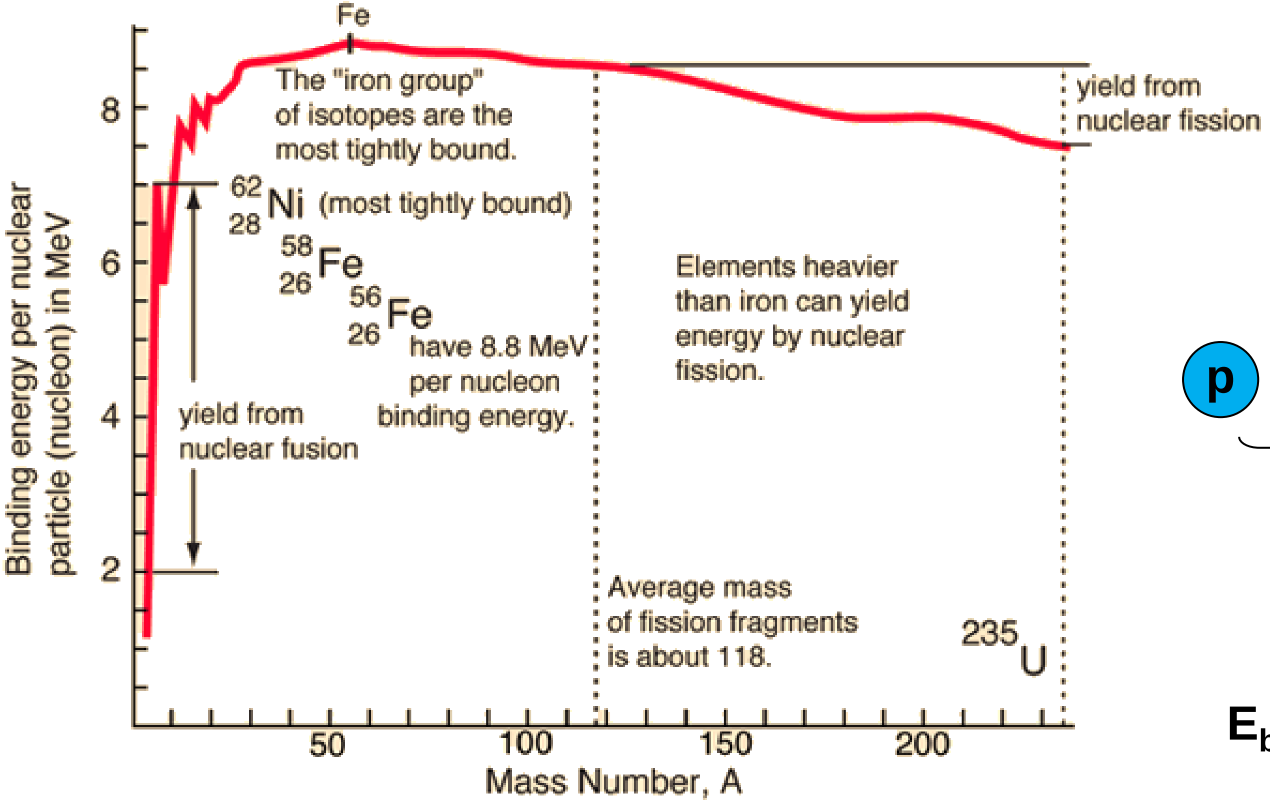
https://nckucc.webex.com/nckucc/j.php?MTID=mf1a33a5dab5eb71de9da4380ae888592

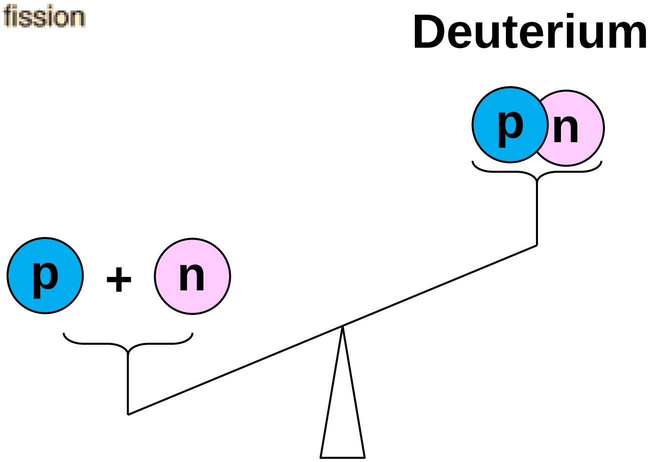
2025/2/25 updated

Course Outline

- Brief background reviews
  - Electromagnetics
  - Plasma physics
- Introduction to nuclear fusion
  - Nuclear binding energy (Fission vs Fusion)
  - Fusion reaction physics
  - Some important fusion reactions (Cross section)
    - Main controlled fusion fuels
    - Advanced fusion fuels
  - Maxwell-averaged fusion reactivities

The “iron group” of isotopes are the most tightly bound





$m_p + m_n > m_D$

$E_{binding} = [(m_p + m_n) - m_D]c^2$

$$Q = \left(\sum_i m_i - \sum_f m_f\right) c^2$$

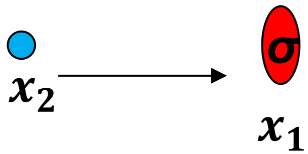
$$\Delta m = z m_p + (A - z) m_n - m$$

- **Binding energy:** $B = \Delta m c^2$
- **Output energy:** $Q = \sum_f B_f - \sum_i B_i$

http://hyperphysics.phy-astr.gsu.edu/hbase/nucene/nucbin.html

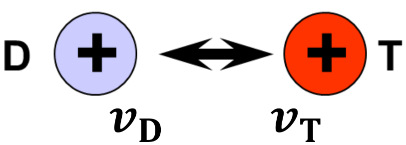
Cross section measures the probability per pair of particles for the occurrence of the reaction

$x_1 + x_2 \rightarrow x_3 + x_4$



- **The hard sphere cross section:**
  σ **≈ *πR*^{2} where R ≈ 5x10^{-15} m is the nuclear radius, i.e.,**
  σ **= *0.8 x 10*$^{-28}$ m^2 ≈ 1 barm. (barm ≡ 10^{-28} m^2)**

- **Classical cross section:**

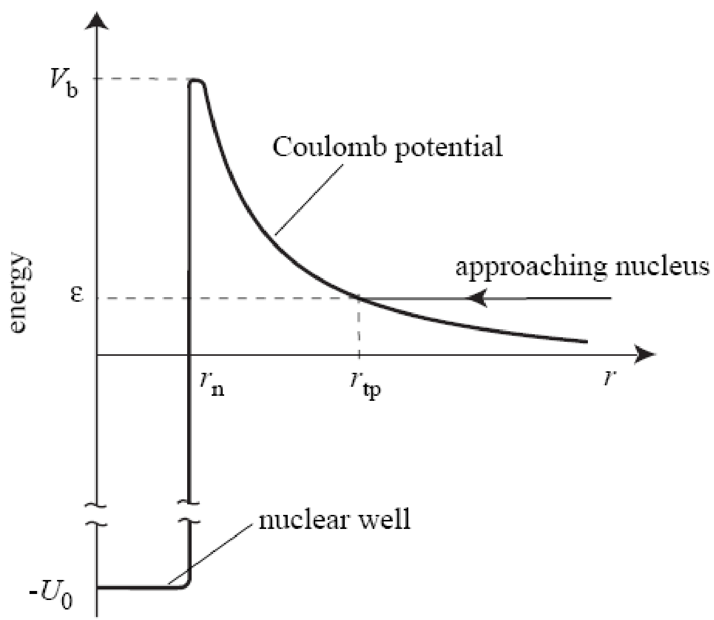


$$\frac{m_\mathrm{D}}{2}{v_\mathrm{D}}^2 + \frac{m_\mathrm{T}}{2}{v_\mathrm{T}}^2 \geq \frac{e^2}{4\pi\epsilon_o R}$$

- **Let** $v = \left|\vec{v}_D - \vec{v}_T\right|$

$$v_\mathrm{D} = \frac{m_\mathrm{T}}{m_\mathrm{D} + m_\mathrm{T}} v \qquad v_\mathrm{T} = \frac{m_\mathrm{D}}{m_\mathrm{D} + m_\mathrm{T}} v$$

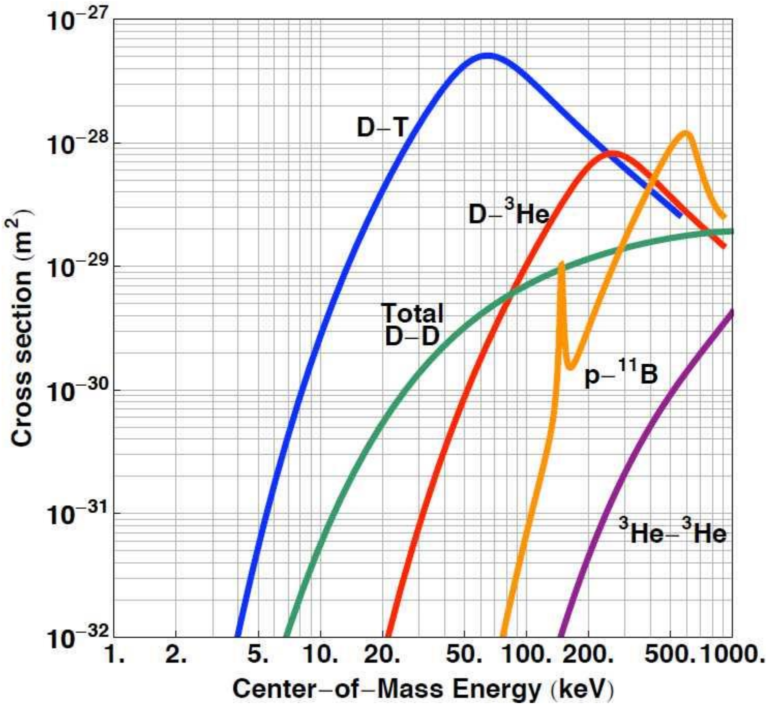
$$\frac{m_\mathrm{r}}{2} v^2 \geq \frac{e^2}{4\pi\epsilon_o R} \qquad m_\mathrm{r} = \frac{m_\mathrm{D} m_\mathrm{T}}{m_\mathrm{D} + m_\mathrm{T}}$$



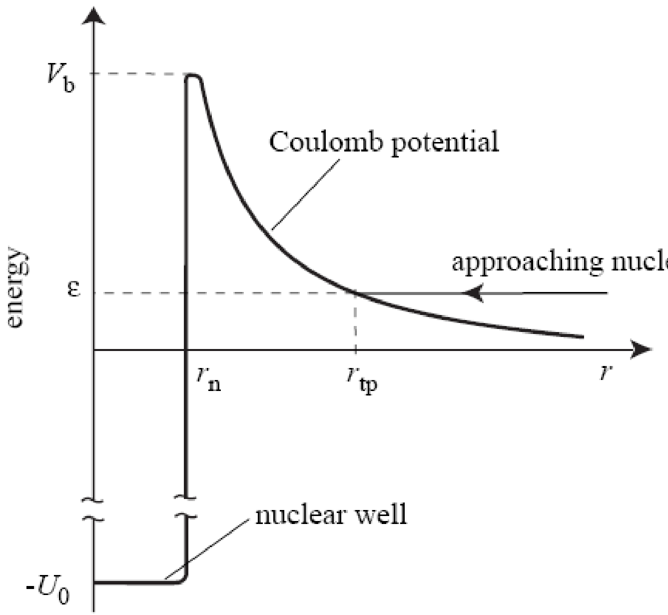
- **Classical kinetic energy required for fusion is**

  $K_{\mathrm{c.m.}} > 288\ \mathrm{keV}$!!!

Cross section of fusion reaction is much larger than the classical approach



- **Classical kinetic energy required for fusion is** $K_{\mathrm{c.m.}} > 288\ \mathrm{keV}$ **!!!**
- **DT cross section has a peak of ~5 barns at ~60 keV.**
- **$\sigma_{DT} \approx 100\sigma_{DD}$ @ 20 keV.**



D + D → T + p
→ He^3 + n

D + T → He^4 + n

D + He^3 → He^4 + p

p + B^{11} → 3He^4

---

https://i.stack.imgur.com/wXQD5.jpg
Santarius, J. F., "Fusion Space Propulsion – A Shorter Time Frame Than You Think", JANNAF, Monterey, 5-8 December 2005.

Flux of incident particles reduces after collisions

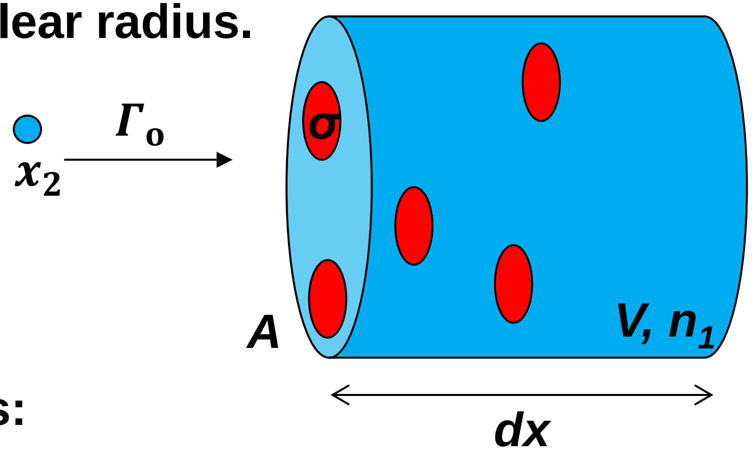
$x_1 + x_2 \rightarrow x_3 + x_4$

- Cross section: $\sigma \approx \pi R^2$ where R is the nuclear radius.

$$V = Adx$$

$$N_1 = n_1 V = n_1 A\mathrm{dx}$$

$$A_{\mathrm{Target}} = N_1\sigma = \sigma n_1 A\mathrm{dx}$$



- Fraction of total area blocked by targets is:

$$dF = \frac{\sigma N_1}{A} = \frac{\sigma n_1 A dx}{A} = \sigma n_1 dx$$

$$\frac{dF}{dx} = \sigma n_1$$

- Flux of incident particles (x_1) is Γ_o

$$-d\Gamma = dF\Gamma = \sigma n_1 \Gamma dx$$

$$\frac{-d\Gamma}{\Gamma} = \sigma n_1 dx$$

$$\Gamma = \Gamma_o \exp\left(-\frac{x}{\lambda_{\mathrm{mfp}}}\right)$$

- Mean free path:

$$\lambda_{\mathrm{mfp}} = \frac{1}{n_1\sigma}$$

- Collision frequency:

$$\nu = \frac{1}{\tau}, \tau = \frac{\lambda_{\mathrm{mfp}}}{v} = \frac{1}{n_1\sigma v}$$

Reactions happen when collision happen

- **Reaction rate R_{12}: number of fusion collisions/reactions per unit volume per unit time.**
- **In the time $dt=dx/v$, $n_2 Adx$ incident particles will pass through the target volume.**
- **The number having a collisions is:** $dF(n_2 Adx)$
- **The volumetric reaction rate R_{12}, i.e., the number of reaction per unit time and per unit volume is:**

$$R_{12} = \frac{dF(n_2 Adx)}{Adxdt} = \sigma n_1 n_2 \frac{dx}{dt} = n_1 n_2 \sigma v$$

- **The fusion power density (W/m^3) is:** $S_f = E_f n_1 n_2 \sigma v \; (W/m^3)$
- **For DT fusion, E_f=17.6 MeV.**
- **For a particle population with a distribution function in velocity space:**

$$n = \int d\vec{v}\, f(\vec{r}, \vec{v}, t)$$

- **Therefore,**

$$n_1 \to d\vec{v}_1 f_1(\vec{r}, \vec{v}_1, t) \qquad n_2 \to d\vec{v}_2 f_2(\vec{r}, \vec{v}_2, t) \qquad v \to \left|\vec{v}_1 - \vec{v}_2\right|$$

$$R_{12} = \int f_1(\vec{v}_1) f_1(\vec{v}_2) \sigma\left(\left|\vec{v}_1 - \vec{v}_2\right|\right)\left|\vec{v}_1 - \vec{v}_2\right| d\vec{v}_1\, d\vec{v}_2$$

The fusion power density needs to consider the distribution function of particles

$$R_{12} = \int f_1(\vec{v}_1) f_1(\vec{v}_2) \sigma\left(\left|\vec{v}_1 - \vec{v}_2\right|\right)\left|\vec{v}_1 - \vec{v}_2\right| d\,\vec{v}_1\, d\,\vec{v}_2 = n_1 n_2 \langle \sigma v \rangle$$

$$\langle \sigma v \rangle \equiv \frac{\int f_1(\vec{v}_1) f_1(\vec{v}_2) \sigma(|\vec{v}_1 - \vec{v}_2|)|\vec{v}_1 - \vec{v}_2| d\,\vec{v}_1\, d\,\vec{v}_2}{\int f_1(\vec{v}_1) f_1(\vec{v}_2) d\,\vec{v}_1\, d\,\vec{v}_2}$$

$$= \frac{\int f_1(\vec{v}_1) f_1(\vec{v}_2) \sigma(|\vec{v}_1 - \vec{v}_2|)|\vec{v}_1 - \vec{v}_2| d\,\vec{v}_1\, d\,\vec{v}_2}{n_1 n_2}$$

- **The fusion power density (W/m^3) is:**

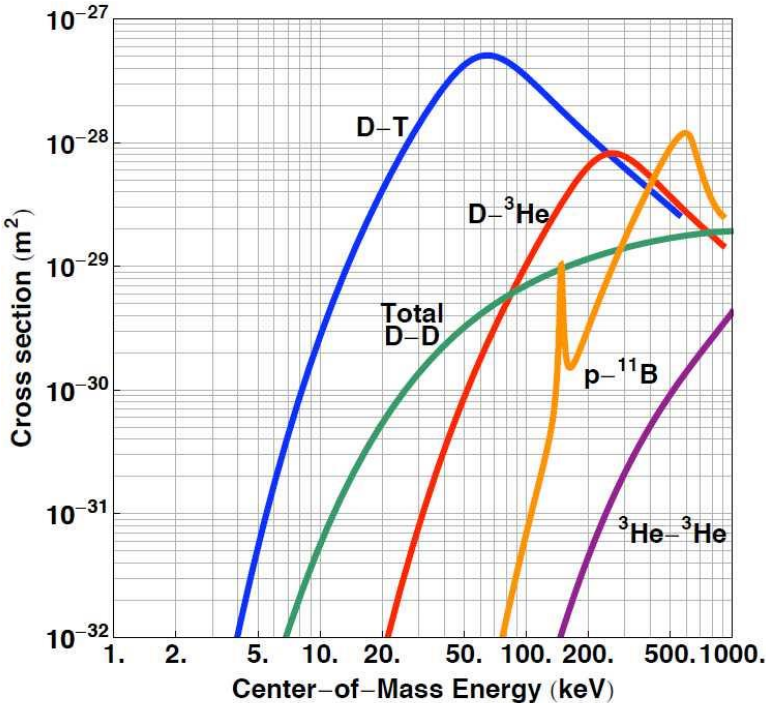
$$S_\mathrm{f} = E_\mathrm{f} n_1 n_2 \langle \sigma v \rangle \left(\mathrm{W/m^3}\right)$$

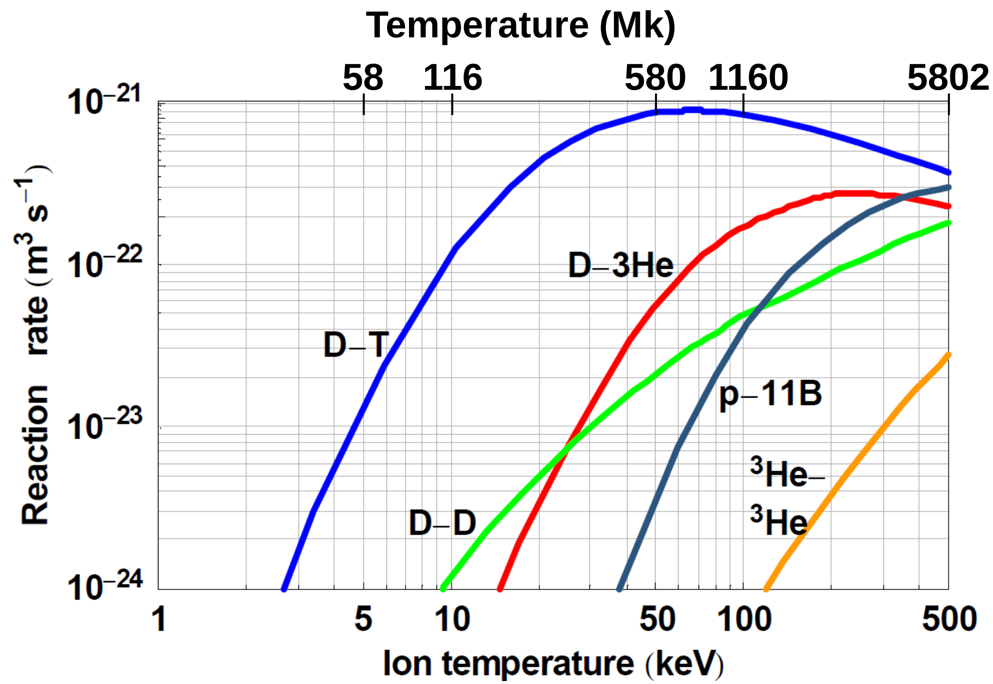
- **Optimum concentration of DT fusion is 50-50.**

$$S_\mathrm{f} = E_\mathrm{f} n_\mathrm{D} n_\mathrm{T} \langle \sigma v \rangle \qquad n_D = k n_o \qquad n_T = (1-k) n_o$$

$$S_\mathrm{f} = E_\mathrm{f} k(1-k) {n_\mathrm{o}}^2 \text{ which peak at } k = 0.5 \;.$$

Fusion doesn’t come easy

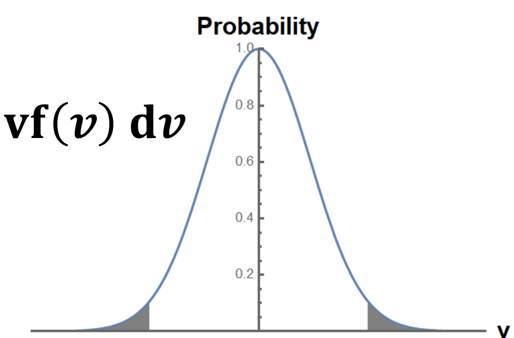




- **The DT fusion reactivity is maximum at T ≈ 64 keV**
- **@ T = 10 keV,** $< \sigma v >_{DT} \approx 100 < \sigma v >_{DD}$

- **Reaction rate:**

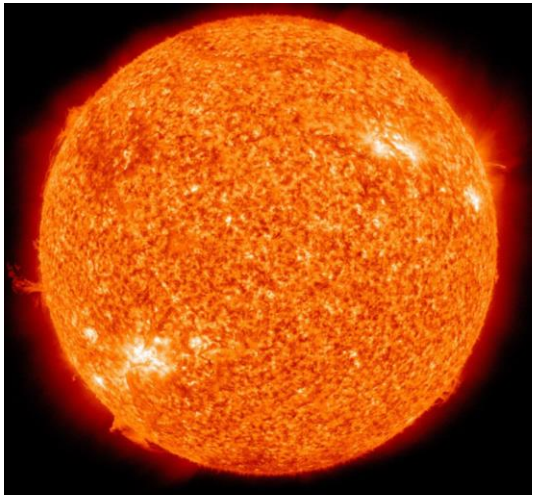
$$< \sigma v > = \int \sigma(v) v f(v)\, dv$$

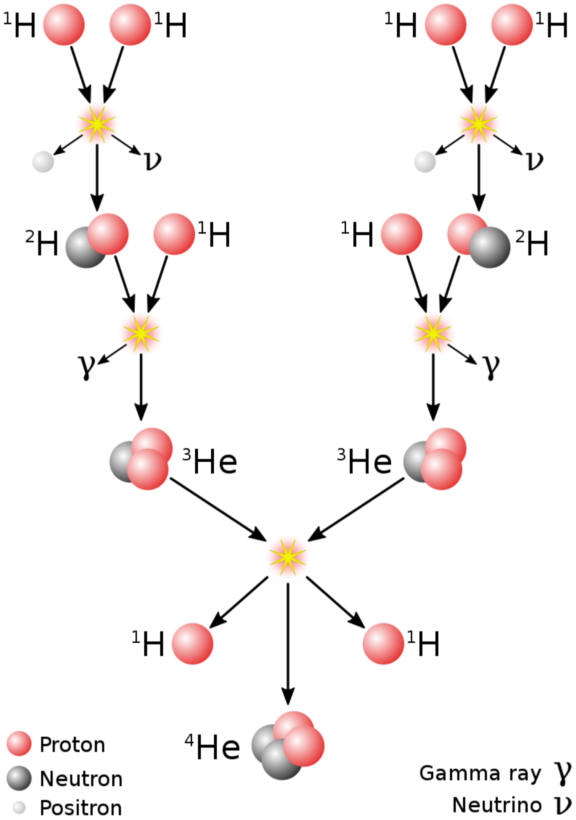


https://i.stack.imgur.com/wXQD5.jpg
Santarius, J. F., “Fusion Space Propulsion – A Shorter Time Frame Than You Think”, JANNAF, Monterey, 5-8 December 2005.

Fusion in the sun provides the energy

- **Proton-proton chain in sun or smaller**

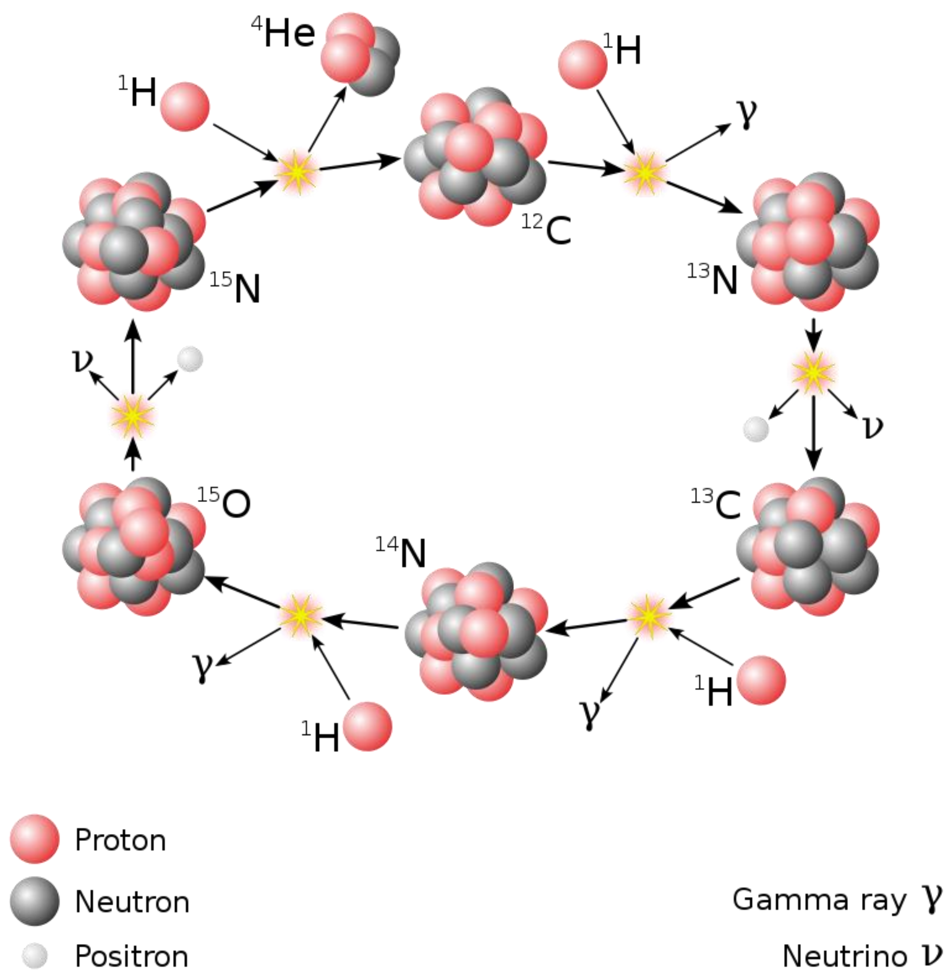




- **Particles are confined by the gravity.**

https://en.wikipedia.org/wiki/Sun
https://en.wikipedia.org/wiki/Nuclear_fusion

In heavy sun, the fusion reaction is the CNO cycle



https://en.wikipedia.org/wiki/Nuclear_fusion

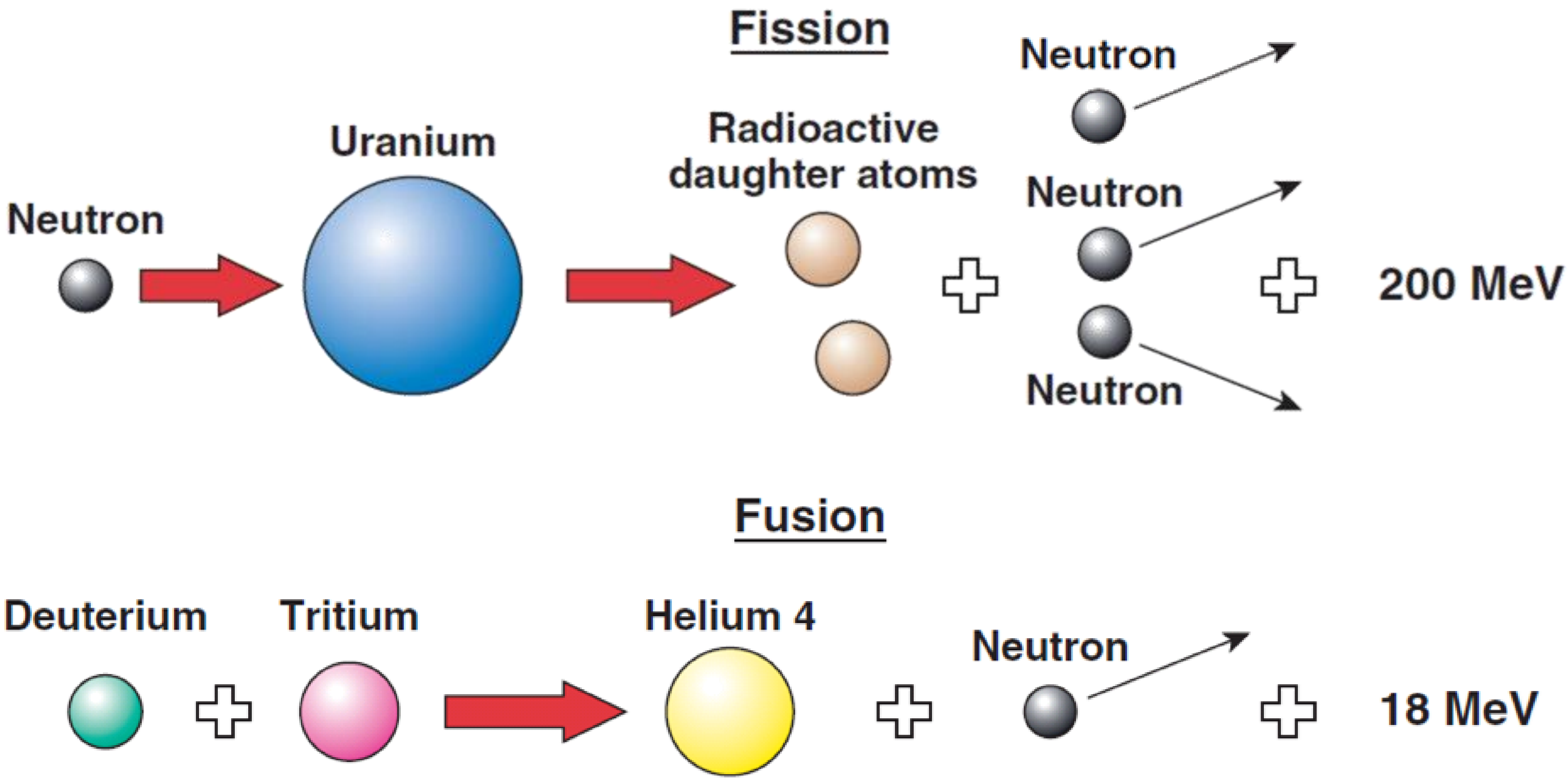
The cross section of proton-proton chain is much smaller than D T fusion

Reaction	$\sigma_{10\ keV}$ (barn)	$\sigma_{100\ keV}$ (barn)	σ_{max} (barn)	ε_{max} (keV)
$D+T \rightarrow \alpha+n$	2.72×10^{-2}	3.43	5.0	64
$D+D \rightarrow T+p$	2.81×10^{-4}	3.3×10^{-2}	0.06	1250
$D+D \rightarrow {}^3He+n$	2.78×10^{-4}	3.7×10^{-2}	0.11	1750
$T+T \rightarrow \alpha+2n$	7.90×10^{-4}	3.4×10^{-2}	0.16	1000
$D+{}^3He \rightarrow \alpha+p$	2.2×10^{-7}	0.1	0.9	250
$p+{}^6Li \rightarrow \alpha+{}^3He$	6×10^{-10}	7×10^{-3}	0.22	1500
$p+{}^{11}B \rightarrow 3\alpha$	(4.6×10^{-17})	3×10^{-4}	1.2	550
$p+p \rightarrow D+e^{+}+v$	(3.6×10^{-26})	(4.4×10^{-25})		
$p+{}^{12}C \rightarrow {}^{13}N+\gamma$	(1.9×10^{-26})	2.0×10^{-10}	1.0x10.4	400
${}^{12}C+{}^{12}C$ (all branches)		(5.0×10^{-103})		

- **"( )" are theoretical values while others are measured values.**

The Physics of Inertial Fusion, by Stefano Atzeni and Jürgen Meyer-Ter-Vehn

Nuclear fusion and fission release energy through energetic neutrons



(3.5MeV) (14.1MeV)

Nuclear fusion provides more energy per atomic mass unit (amu) than nuclear fission

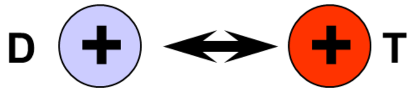
- Fusion of D+T: $$\frac{Q}{A} = \frac{17.6\text{MeV}}{(3+2)\text{amu}} = 3.5\frac{\text{MeV}}{\text{amu}}$$
- Fission of ^{235}U+n: $$\frac{Q}{A} = \frac{200\text{MeV}}{(235+1)\text{amu}} = 0.85\frac{\text{MeV}}{\text{amu}}$$

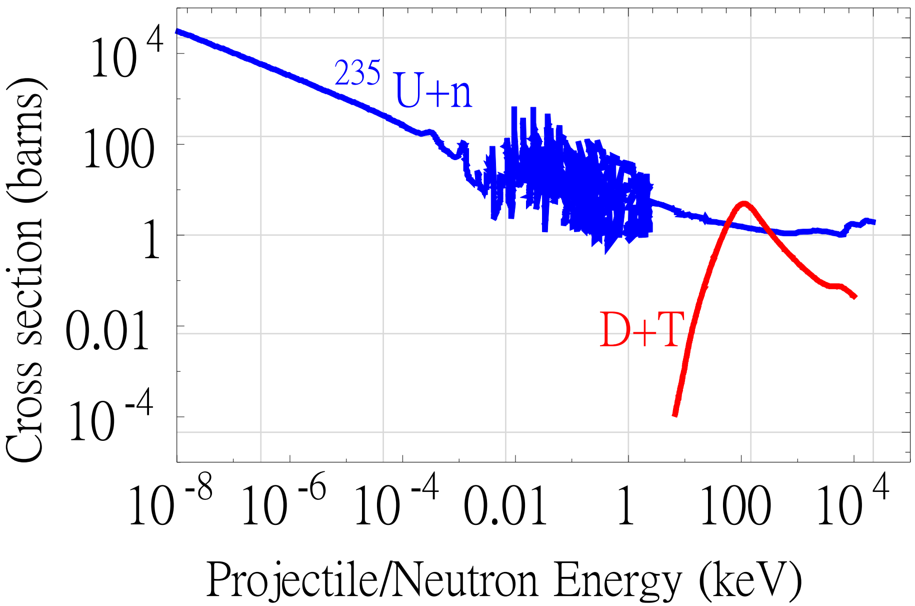
	Half-life (years)
U235	$7.04\text{x}10^{8}$
U238	$4.47\text{x}10^{9}$
…	
Tritium	12.3

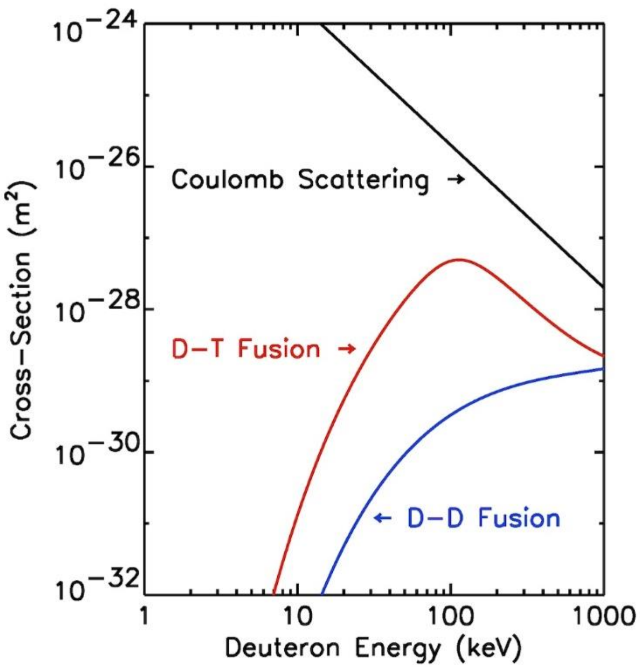
$$n + Li^{6} \rightarrow He^{4} + T$$

Fusion is much harder than fission

- **Fission:** $n +^{235}_{92} U \rightarrow^{236}_{92} U \rightarrow^{144}_{56} Ba +^{89}_{36} Kr + 3n + 177\ \mathrm{MeV}$
- **Fusion:** $D + T \rightarrow He^4\,(3.5\ \mathrm{MeV}) + n\,(14.1\ \mathrm{MeV})$



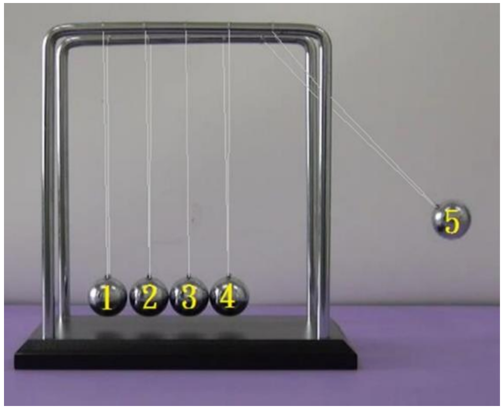




https://www6.lehigh.edu/~eus204/lab/PCL_fusion.php#x1-10096

Fast neutrons are slowed down due to the collisions

- **A moderator is used to slow down fast neutrons but not to absorb neutrons.**
- **For m_M~m_N, the energy decrement is higher. Therefore, H slows down neutron most efficiently.**
- **However, H + n → D, i.e., H absorbs neutrons.**



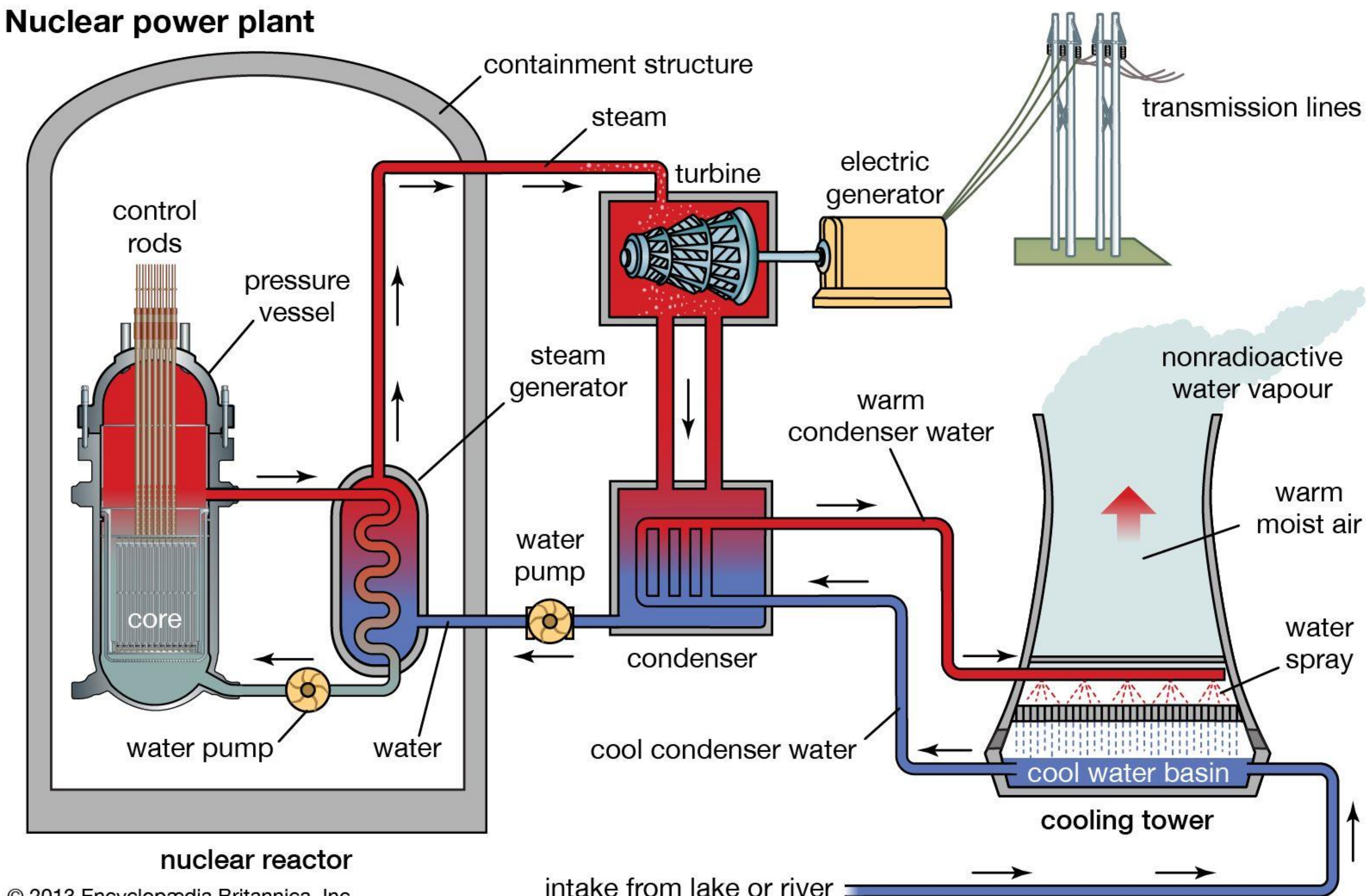
Neutron (m_N) ⇒ **Atom** (m_M)

- **The best option is the D in the heavy water (D_2O).**

	Energy decrement	Neutron scattering cross section (σ_s) (Barns)	Neutron absorption cross section (σ_a) (Barns)
H	1	49 (H_2O)	0.66 (H_2O)
D	0.7261	10.6 (D_2O)	0.0013 (D_2O)
C	0.1589	4.7 (Graphite)	0.0035 (Graphite)

https://en.wikipedia.org/wiki/Neutron_moderator#cite_note-Weston-4
https://energyeducation.ca/encyclopedia/Neutron_moderator#cite_note-3

Nuclear power plant



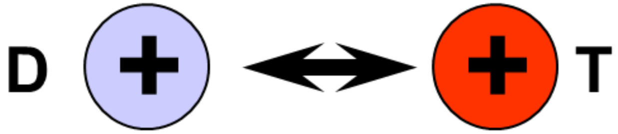
https://www.britannica.com/technology/nuclear-power

Comparison between nuclear fission and nuclear fusion

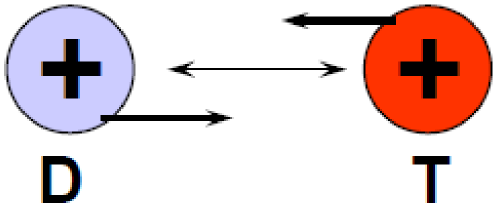
	Nuclear Fission	Nuclear Fusion
Chain reaction	Yes	No
Melt down	Possible	Impossible
Nuclear waste	High radiative	Low radiative / None

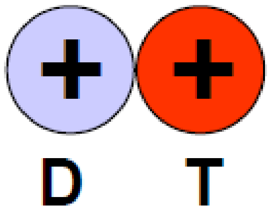
A “hot plasma” at 100M ⁰C is needed

- **Probability for fusion reactions to occur is low at low temperatures due to the coulomb repulsion force.**



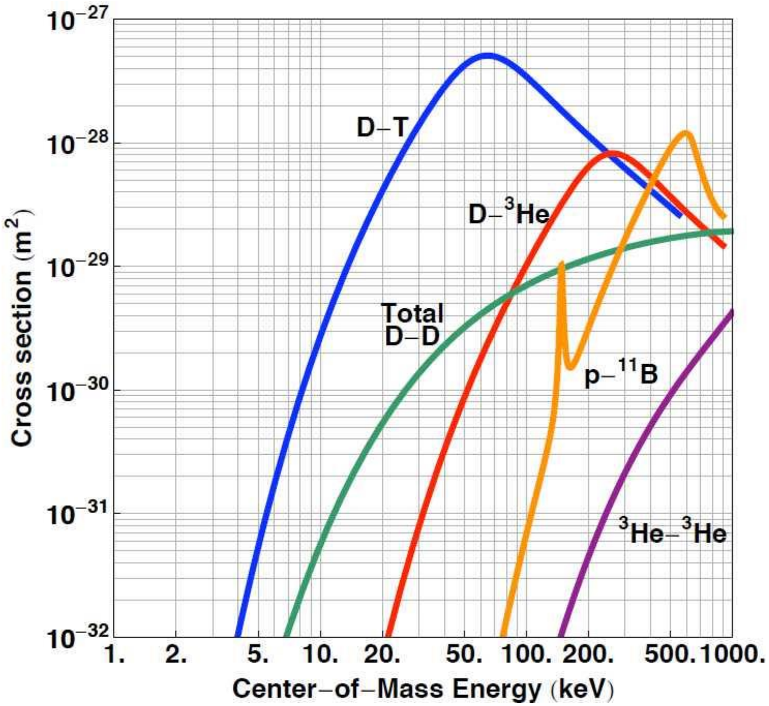
- **If the ions are sufficiently hot, i.e., large random velocity, they can collide by overcoming coulomb repulsion**

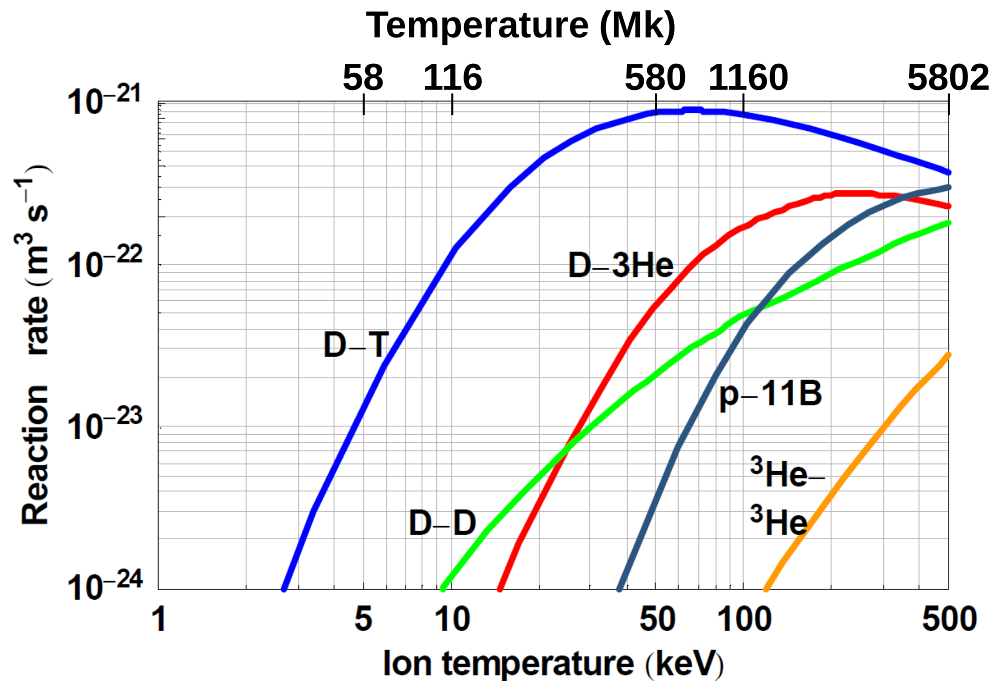




*R. Betti, HEDSA HEDP Summer School, 2015

Fusion doesn’t come easy

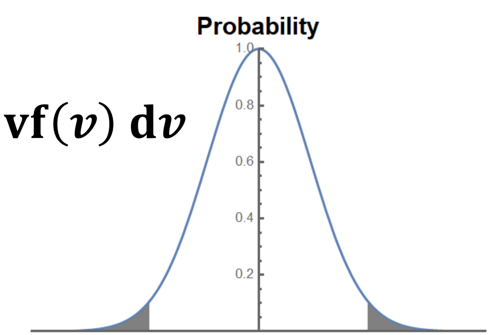




- The DT fusion reactivity is maximum at T ≈ 64 keV
- @ T = 10 keV, $<\sigma v>_{DT} \approx 100 <\sigma v>_{DD}$

- **Reaction rate:**

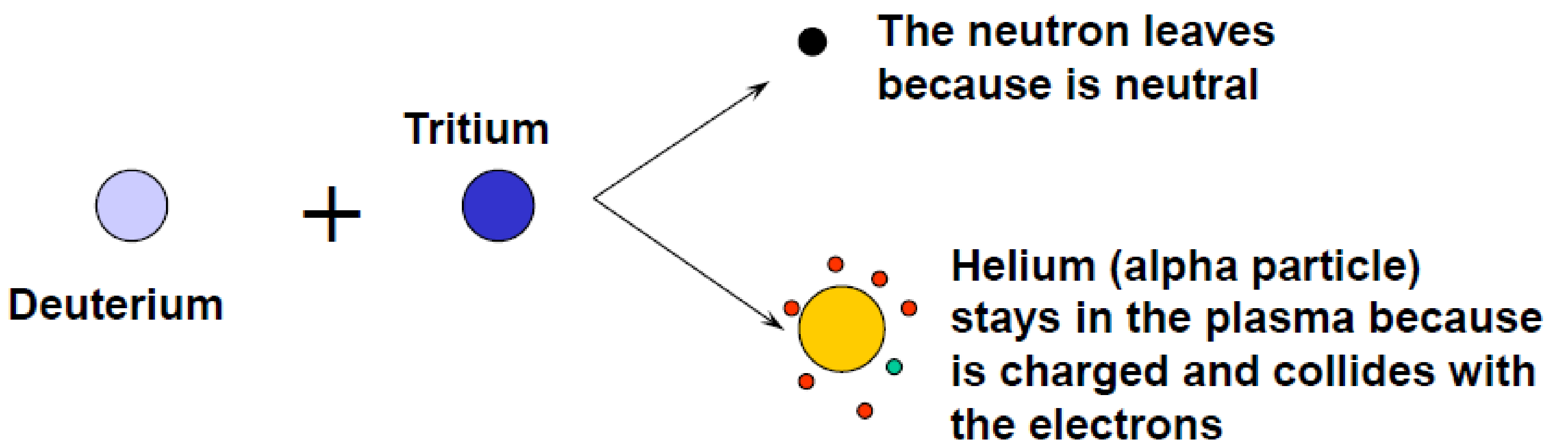
$$<\sigma v> = \int \sigma(v) v f(v)\, dv$$



https://i.stack.imgur.com/wXQD5.jpg
Santarius, J. F., “Fusion Space Propulsion – A Shorter Time Frame Than You Think”, JANNAF, Monterey, 5-8 December 2005.

It takes a lot of energy or power to keep the plasma at 100M ^{0}C

- **Let the plasma do it itself!**



- **The α-particles heat the plasma.**

*R. Betti, HEDSA HEDP Summer School, 2015

Under what conditions the plasma keeps itself hot?

- **Steady state 0-D power balance:**

$S_\alpha + S_h = S_B + S_k$

S_α: **α particle heating**

S_h: **external heating**

S_B: **Bremsstrahlung radiation**

S_k: **heat conduction lost**

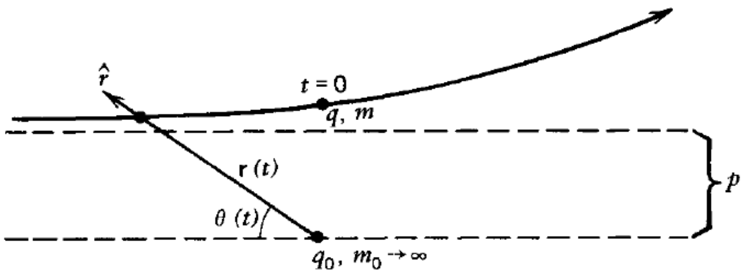
Ignition condition: Pτ > 10 atm-s = 10 Gbar - ns

- **P: pressure, or called energy density**
- **τ is confinement time**

Course Outline

- Introduction to nuclear fusion (cont.)
  - Collisions (Bremsstrahlung radiation)
  - Columb scattering. Cross section of the Columb scattering
  - Beam-target fusion vs thermonuclear fusion
  - Lawson criteria, ignition conditions
  - Magnetic confinement fusion (MCF) vs Inertial confinement fusion (ICF)

Charged particles collide with each other through coulomb collisions



$$\mathrm{m}v_\perp = \int_{-\infty}^{\infty} dt\, \mathrm{F}_\perp(t)$$

- **Coulomb force:**

$$m\ddot{\vec{r}} = \frac{qq_0}{r^2}\hat{r}$$

$$F_\perp = \frac{qq_0}{p^2}\sin^3\theta$$

- **Relation between θ and t is**

$$x = -r\cos\theta = -\frac{b\cos\theta}{\sin\theta} = v_0 t$$

- **Therefore,**

$$v_\perp = \frac{qq_0}{mv_0 p}\int_0^{\pi} d\theta \sin\theta = \frac{2qq_0}{mv_0 p} \equiv \frac{v_0 p_0}{p}$$

where $$p_0 \equiv \frac{2qq_0}{m{v_0}^2}$$

- **Note that this is valid only when** $v_\perp \ll v_0$ **, i.e.,** $p \gg p_0$ **.**

Cumulative effect of many small angle collisions is more important than large angle collisions

- **Consider a variable Δx that is the sum of many small random variables Δx_i, i=1,2,3,…,N,**

$$\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 + \cdots + \Delta x_N = \sum_{i=1}^{N} \Delta x_i$$

- **Suppose** $< \Delta x_i > = < \Delta x_i \Delta x_j >_{i \neq j} = 0$

$$\left\langle (\Delta x)^2 \right\rangle = \left\langle \left(\sum_{i=1}^{N} \Delta x_i \right)^2 \right\rangle = \sum_{i=1}^{N} \left\langle (\Delta x_i)^2 \right\rangle = N \left\langle (\Delta x_i)^2 \right\rangle$$

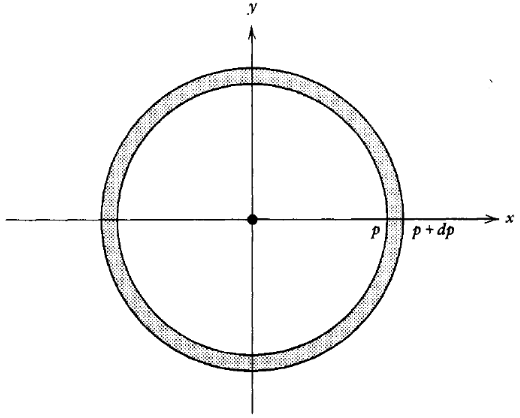
- **For one collision:**

$$\left\langle {v_\perp}^2 \right\rangle = \left\langle (\Delta v_x)^2 \right\rangle + \left\langle \left(\Delta v_y\right)^2 \right\rangle = \frac{{v_0}^2 {p_0}^2}{p^2} \qquad \left\langle (\Delta v_x)^2 \right\rangle = \left\langle \left(\Delta v_y\right)^2 \right\rangle = \frac{1}{2} \frac{{v_0}^2 {p_0}^2}{p^2}$$

- **The total velocity in** $\hat{x}$

$$\left\langle \left(\Delta {v_x}^{tot}\right)^2 \right\rangle = N \left\langle (\Delta v_x)^2 \right\rangle = \frac{N}{2} \frac{{v_0}^2 {p_0}^2}{p^2}$$

The collision frequency can be obtained by integrating all the possible impact parameter



- Number of collisions in a time interval:

$$dN = n_0 2\pi p\, dp\, v_0\, dt$$

i.e., $$\frac{dN}{dt} = 2\pi p\, dp\, n_0 v_0$$

- Therefore

$$\frac{d}{dt}\left\langle\left(\Delta v_{\mathrm{x}}{}^{\mathrm{tot}}\right)^2\right\rangle = \frac{1}{2}\,\frac{v_0{}^2 p_0{}^2}{p^2}\frac{dN}{dt}$$

$$= \pi\, n_0 v_0{}^3 p_0{}^2 \frac{dp}{p}$$

$$\frac{d}{dt}\left\langle\left(\Delta_{\perp}{}^{\mathrm{tot}}\right)^2\right\rangle = 2\frac{d}{dt}\left\langle\left(\Delta v_{\mathrm{x}}{}^{\mathrm{tot}}\right)^2\right\rangle$$

$$= 2\pi n_0 v_0{}^3 p_0{}^2 \int_{p_{\min}}^{p_{\max}} \frac{dp}{p}$$

$$= 2\pi n_0 v_0{}^3 p_0{}^2 \ln\left(\frac{p_{\max}}{p_{\min}}\right)$$

$$\approx 2\pi n_0 v_0{}^3 p_0{}^2 \ln\left(\frac{\lambda_{\mathrm{D}}}{|p_0|}\right)$$

$$\approx 2\pi n_0 v_0{}^3 p_0{}^2 \ln\Lambda$$

- Note that

$$\lambda_{\mathrm{D}} \approx \left(\frac{KT_{\mathrm{e}}}{4\pi n_0 e^2}\right)^{1/2}$$

$$\frac{\lambda_{\mathrm{D}}}{|p_0|} \approx \frac{\lambda_{\mathrm{D}} m_{\mathrm{e}} v_{\mathrm{e}}{}^2}{2e^2} \approx \frac{\lambda_{\mathrm{D}} KT_{\mathrm{e}}}{e^2} \approx 4\pi n_0 \lambda_{\mathrm{D}}{}^3$$

$$\approx \Lambda$$

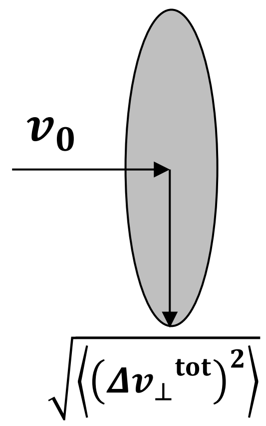
Comparison between the mean free path and the system size L determines the regime of the plasma

- **A reasonable definition for the scattering time due to small angle collisions is the time it takes $\left\langle \left(\Delta v_{\perp}{}^{\mathrm{tot}}\right)^2 \right\rangle$ to equal $v_0{}^2$. The collision frequency ν_c due to small-angle collisions:**

$$\frac{d}{dt}\left\langle \left(\Delta_{\perp}{}^{\mathrm{tot}}\right)^2 \right\rangle \approx 2\pi n_0 v_0{}^3 p_0{}^2 \ln\Lambda \approx v_0{}^2 \nu_{\mathrm{c}}, \quad p_0 \equiv \frac{2qq_0}{m_{\mathrm{e}} v_0{}^2} \Rightarrow \nu_c = \frac{8\pi n_0 e^4 \ln\Lambda}{m_{\mathrm{e}}{}^2 v_0{}^3}$$

- **With more careful derivation, the collisional time is obtained:**

$$\tau_{\mathrm{e}}{}^{-1} = \nu_{\mathrm{c}} = \frac{4\sqrt{2\pi}\, n e^4 \ln\Lambda}{3\sqrt{m_{\mathrm{e}}}\,(KT_{\mathrm{e}})^{3/2}}$$

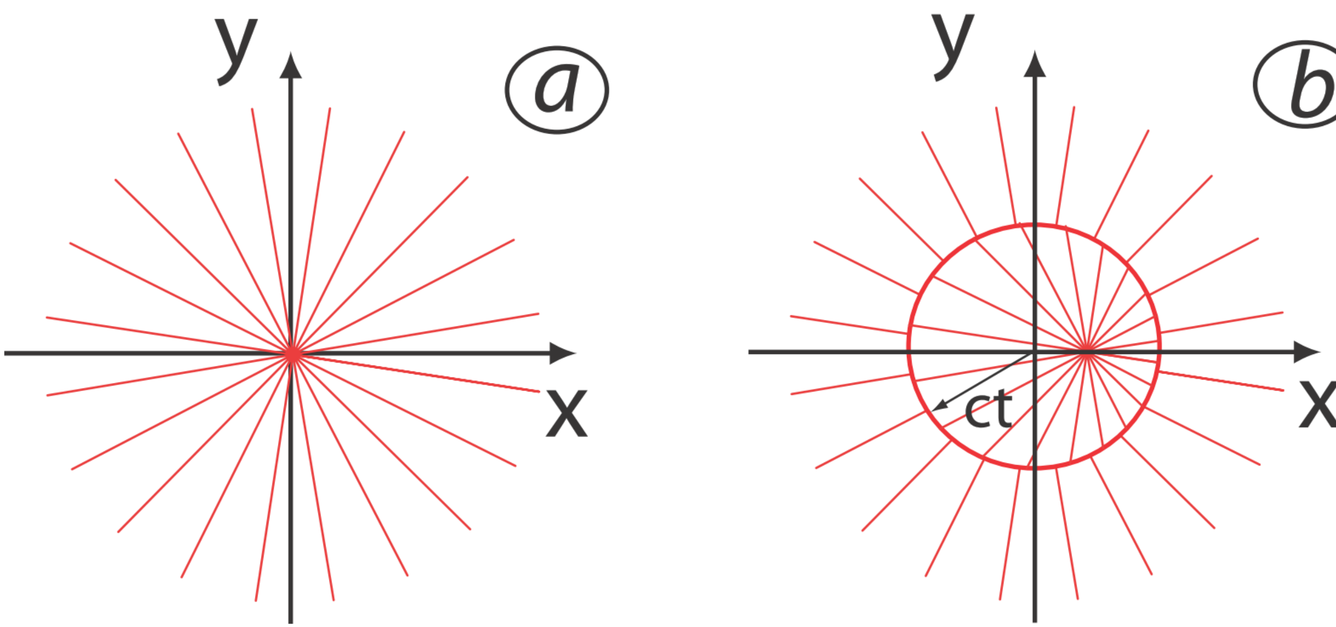


- **Mean free path:** $l_{\mathrm{mfp}} = v_{\mathrm{e}} \tau_{\mathrm{e}}$

$$\begin{cases} l_{\mathrm{mfp}} < L & \textbf{Fluid Theory} \\ l_{\mathrm{mfp}} > L & \textbf{Kinetic Theory} \end{cases}$$

Electromagnetic wave is radiated when a charge particle is accelerated

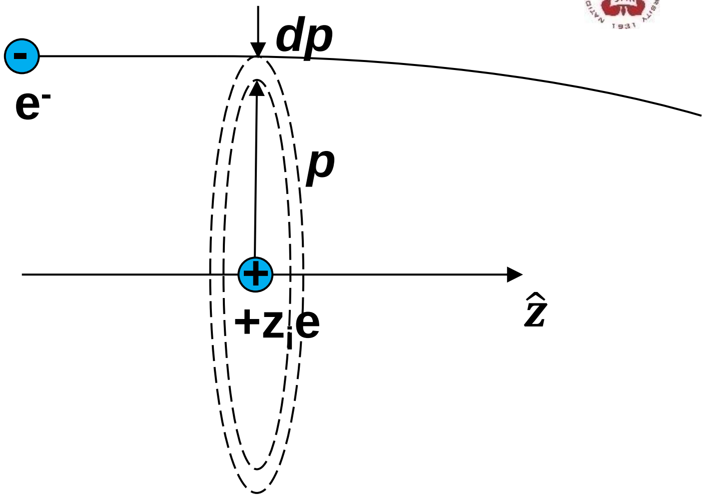
- **The retarded potentials are the electromagnetic potentials for the electromagnetic field generated by time-varying electric current or charge distributions in the past.**



https://jseldredphysics.files.wordpress.com/2018/04/lecture18.pdf

Bremsstrahlung emission

- **When an electron collides with a nucleus via coulomb interaction, the electron is accelerated and thus radiates, called Bremsstrahlung radiation.**
- **In the non-relativistic limit, the Bremsstrahlung radiation power is:**



$$P_{\mathrm{B,e1,i1}} = \frac{e^2}{6\pi\epsilon_o}\frac{\dot{v}^2}{c^3}$$

p: **Impact parameter**

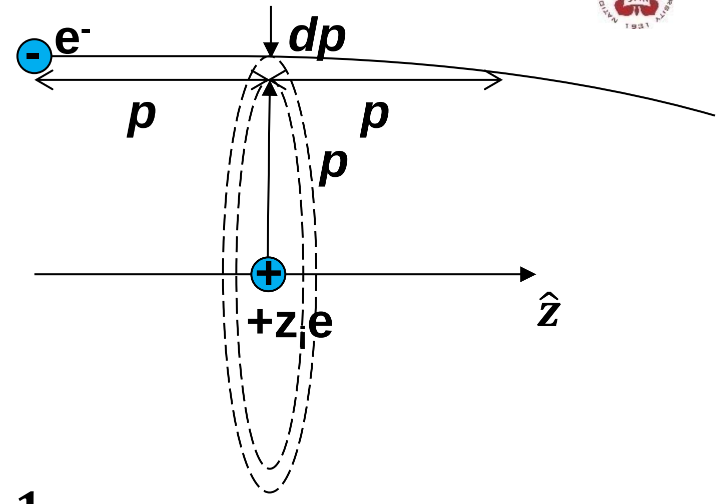
- **The coulomb force experienced by the electron is:**

$$\dot{v} = \frac{F}{m_\mathrm{e}} = \frac{\mathrm{z}e^2}{4\pi\epsilon_o m_\mathrm{e} r^2} = \frac{ze^2}{4\pi\epsilon_o m_\mathrm{e}[p^2+(vt)^2]} \approx \frac{\mathrm{z}e^2}{4\pi\epsilon_o m_\mathrm{e} p^2}$$

=>

$$P_{\mathrm{B,e1,i1}} = \frac{z^2 e^6}{96\pi^3\epsilon_o{}^3 c^3 m_\mathrm{e}{}^2}\frac{1}{p^4}\quad (\mathrm{W})$$

Bremsstrahlung emission



- The electron begins to accelerate when it is about a distance *b* from the ion. It continuous to accelerate until it travels a distance *p* away from the ion.

$$\Delta t = \frac{2p}{v}$$

- Therefore, the energy loss by one electron is:

$$E_{\mathrm{B,e1,i1}} \approx P_{\mathrm{B,e1,i1}} \Delta t = \frac{z^2 e^6}{48\pi^3 \epsilon_o{}^3 c^3 m_e{}^2} \frac{1}{v p^3} \quad \mathrm{(J)}$$

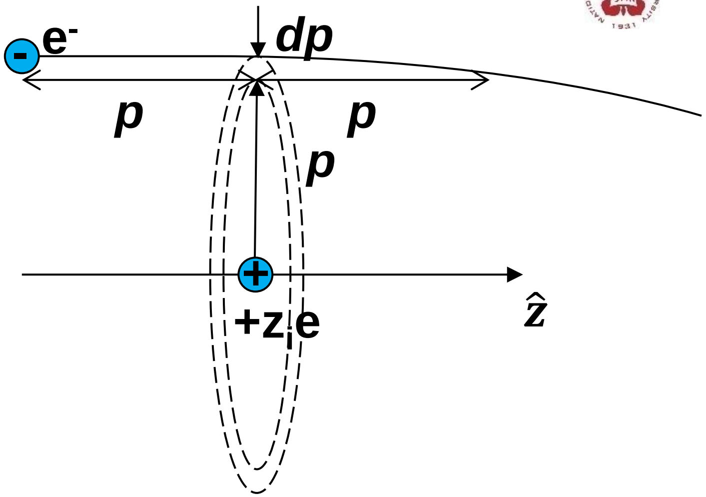
- With careful integration:

$$E_{\mathrm{B,e1,i1}} = \int_{-\infty}^{\infty} P_{\mathrm{B,e1,i1}}\, \mathrm{d}t = \frac{2 z^2 e^6}{3(4\pi\epsilon_o)^3 m_e{}^2 c^3} \int_{-\infty}^{\infty} \frac{1}{[p^2 + (vt)^2]^2}\, dt$$

$$= \frac{\pi z^2 e^6}{3(4\pi\epsilon_o)^3 m_e{}^2 c^3} \frac{1}{v p^3}$$

Bremsstrahlung emission

- To consider the electron colliding with all ions with impact parameter p from 0 to ∞ and include the distribution function of ions $f_i(v_i)$.



$$\bar{P}_{B,e1} = \int d\vec{v}_\mathrm{i} \int_0^\infty \bar{E}_{\mathrm{B,e1,i1}} |\vec{v}_\mathrm{e} - \vec{v}_\mathrm{i}| f_\mathrm{i}(\vec{v}_\mathrm{i}) 2\pi p \, dp$$

- In addition, we need to consider the distribution function of electrons $f_e(v_e)$. The total power loss is:

$$\bar{P}_B = \int d\vec{v}_\mathrm{i} \int d\vec{v}_\mathrm{e} \int_0^\infty \bar{E}_{\mathrm{B,e1,i1}} |\vec{v}_\mathrm{e} - \vec{v}_\mathrm{i}| f_\mathrm{i}(\vec{v}_\mathrm{i}) f_\mathrm{e}(\vec{v}_\mathrm{e}) 2\pi p \, dp$$

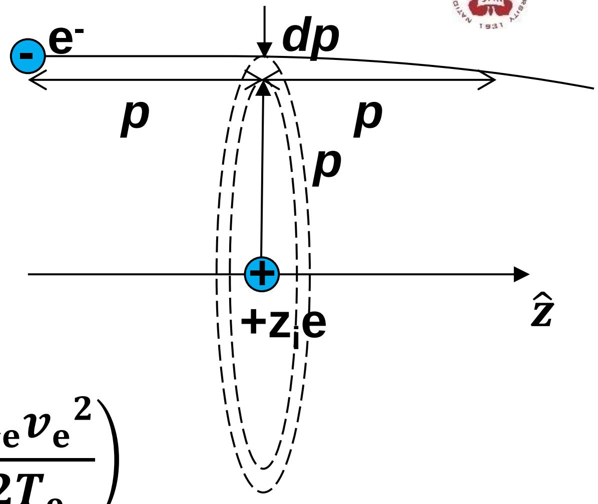
- Since $|v_e| >> |v_i|$, $|\vec{v}_\mathrm{e} - \vec{v}_\mathrm{i}| \approx v_\mathrm{e}$.
- In addition:

$$\int f_i(\vec{v}_i) \, \mathrm{d}\, \vec{v}_i \equiv n_i$$

$$d\,\vec{v}_e = dv_x dv_y dv_z = v_\mathrm{e}{}^2 \sin\theta \, dv d\theta d\phi \rightarrow 4\pi v_\mathrm{e}{}^2 dv_\mathrm{e}$$

$$f_\mathrm{e} = n_\mathrm{e} \left(\frac{m_\mathrm{e}}{2\pi \mathrm{T_e}}\right)^{3/2} \exp\left(-\frac{m_\mathrm{e} v_\mathrm{e}{}^2}{2T_\mathrm{e}}\right)$$

Bremsstrahlung emission



$$\overline{P}_B = \int d\vec{v}_\mathrm{i} \int d\vec{v}_\mathrm{e} \int_0^\infty \overline{E}_{\mathrm{B,e1,i1}} |\vec{v}_\mathrm{e} - \vec{v}_\mathrm{i}| f_\mathrm{i}(\vec{v}_\mathrm{i}) f_\mathrm{e}(\vec{v}_\mathrm{e}) 2\pi p\, dp$$

$$= 2\pi \int f_\mathrm{i}(\vec{v}_\mathrm{i})\, d\vec{v}_i \int 4\pi {v_\mathrm{e}}^2 dv_\mathrm{e} \int_0^\infty p\, dp E_{\mathrm{B,e1,i1}} v_\mathrm{e} f_\mathrm{e}(\vec{v}_\mathrm{e})$$

$$= 8\pi^2 n_\mathrm{i} \int {v_\mathrm{e}}^3 dv_\mathrm{e} \int_0^\infty p\, dp E_{\mathrm{B,e1,i1}} n_\mathrm{e} \left(\frac{m_\mathrm{e}}{2\pi \mathrm{T_e}}\right)^{3/2} \exp\left(-\frac{m_\mathrm{e} {v_\mathrm{e}}^2}{2T_\mathrm{e}}\right)$$

$$= 8\pi^2 n_\mathrm{i} n_\mathrm{e} \left(\frac{m_\mathrm{e}}{2\pi \mathrm{T_e}}\right)^{3/2} \int_0^\infty {v_\mathrm{e}}^3 dv_\mathrm{e} \int_0^\infty p\, dp \left(\frac{z^2 e^6}{48\pi^3 {\epsilon_\mathrm{o}}^3 c^3 {m_\mathrm{e}}^2} \frac{1}{v_\mathrm{e} p^3}\right) \exp\left(-\frac{m_\mathrm{e} {v_\mathrm{e}}^2}{2T_\mathrm{e}}\right)$$

$$= 8\pi^2 n_\mathrm{i} n_\mathrm{e} \left(\frac{z^2 e^6}{48\pi^3 {\epsilon_\mathrm{o}}^3 c^3 {m_\mathrm{e}}^2}\right) \left(\frac{m_\mathrm{e}}{2\pi \mathrm{T_e}}\right)^{3/2} \int_0^\infty {v_\mathrm{e}}^2 \exp\left(-\frac{m_\mathrm{e} {v_\mathrm{e}}^2}{2T_\mathrm{e}}\right) dv_\mathrm{e} \int_0^\infty \frac{dp}{p^2}$$

Bremsstrahlung emission

- **Notice that we are using classical physics. We are not taking account of quantum effects which happen on a length scale of deBroglie wavelength *Δx* = *ℏ/(m*$_e$*v)* . Therefore, we have *p*$_{min}$ = *ℏ/(m*$_e$*v).***

$$\int_0^\infty \frac{dp}{p^2} \rightarrow \int_{p_{\min}}^\infty \frac{dp}{p^2} = \frac{1}{p_{min}} = \frac{m_e v_e}{\hbar} = \frac{2\pi m_e v_e}{h}$$

$$\bar{P}_B = 8\pi^2 n_\mathrm{i} n_\mathrm{e} \left(\frac{z^2 e^6}{48\pi^3 \epsilon_\mathrm{o}{}^3 c^3 m_\mathrm{e}{}^2}\right)\left(\frac{m_\mathrm{e}}{2\pi \mathrm{T_e}}\right)^{3/2} \int_0^\infty v_\mathrm{e}{}^2 \exp\left(-\frac{m_\mathrm{e} v_\mathrm{e}{}^2}{2T_\mathrm{e}}\right) dv_\mathrm{e} \int_0^\infty \frac{dp}{p^2}$$

$$= 8\pi^2 n_\mathrm{i} n_\mathrm{e} \left(\frac{z^2 e^6}{48\pi^3 \epsilon_\mathrm{o}{}^3 c^3 m_\mathrm{e}{}^2}\right)\left(\frac{m_\mathrm{e}}{2\pi \mathrm{T_e}}\right)^{3/2} \frac{2\pi m_e}{h} \int_0^\infty v_\mathrm{e}{}^3 \exp\left(-\frac{m_\mathrm{e} v_\mathrm{e}{}^2}{2T_\mathrm{e}}\right) dv_\mathrm{e}$$

- **With** $\int_0^\infty x^3 e^{-x^2}\, \mathrm{d}x = \frac{1}{2}$, **a better value:** $\left(\frac{2^{1/2}}{3\pi^{5/2}}\right)$

$$\bar{P}_B = \left(\frac{2^{1/2}}{6\pi^{3/2}}\right)\left(\frac{e^6}{\epsilon_\mathrm{o}{}^3 c^3 h m_\mathrm{e}{}^{3/2}}\right) z^2 n_\mathrm{i} n_\mathrm{e} \mathrm{T_e}{}^{1/2} \quad \left(\frac{W}{m^3}\right)$$

Bremsstrahlung emission

- For multiple ion species: n_j, z_j

$$\bar{P}_B = \left(\frac{2^{1/2}}{3\pi^{5/2}}\right)\left(\frac{e^6}{\epsilon_o{}^3 c^3 h m_e{}^{3/2}}\right) n_e T_e{}^{1/2} \sum_j z_j^2 n_{i,j} \left(\frac{W}{m^3}\right)$$

$$= \left(\frac{2^{1/2}}{3\pi^{5/2}}\right)\left(\frac{e^6}{\epsilon_o{}^3 c^3 h m_e{}^{3/2}}\right) z_{eff}^2 n_e^2 T_e{}^{1/2} \left(\frac{W}{m^3}\right)$$

where $\quad Z_{eff} \equiv \frac{\sum_j z_j{}^2 n_j}{n_e} = \frac{\sum_j z_j{}^2 n_j}{\sum_j z_j n_j} \qquad n_e = \sum_j z_j n_j$

$$\bar{P}_B = 5.35 \times 10^{-37} Z_{eff} n_{e\,(m^{-3})}^2 T_{e\,(keV)}^{1/2} \left(\frac{W}{m^3}\right)$$

$$\bar{P}_B \equiv C_B Z_{eff} n_{e\,(m^{-3})}^2 T_{e\,(keV)}^{1/2} \left(\frac{W}{m^3}\right)$$

Ignition condition (Lawson criterion) revision

- **Steady state 0-D power balance:**

$S_\alpha + S_h = S_B + S_k$

S_h: external heating

S_α: α particle heating

$$D + T \longrightarrow He^4(3.5\ \mathrm{MeV}) + n(14.1\ \mathrm{MeV})$$

$$S_\alpha = \frac{1}{4} E_\alpha n^2 \langle \sigma v \rangle = \frac{1}{16} E_\alpha \frac{p^2 \langle \sigma v \rangle}{T^2}$$

E_α = 3.5 MeV

$$p = p_\mathrm{e} + p_\mathrm{i} = 2p_\mathrm{e} = 2n_e T \equiv 2nT$$

S_B: Bremsstrahlung radiation

$$S_B = C_\mathrm{B} Z_\mathrm{eff} n^2_{\mathrm{e}\ (\mathrm{m}^{-3})} T^{1/2}_{\mathrm{e}\ (\mathrm{keV})} \left(\frac{W}{m^3}\right)$$

$$= \frac{1}{4} C_\mathrm{B} \frac{p^2}{T^{3/2}}$$

S_k: heat conduction lost

$$S_\kappa = \frac{3}{2} \frac{p}{\tau}$$

$$\frac{1}{16} E_\alpha \frac{p^2 \langle \sigma v \rangle}{T^2} \geqslant \frac{1}{4} C_\mathrm{B} \frac{p^2}{T^{3/2}} + \frac{3}{2} \frac{p}{\tau}$$

Ignition condition (Lawson criterion) revision

- **Steady state 0-D power balance:**

$$S_\alpha + S_h = S_B + S_k$$

$$\frac{1}{16} E_\alpha \frac{\cancel{p^2}\langle\sigma v\rangle}{T^2} \geqslant \frac{1}{4} C_B \frac{\cancel{p^2}}{T^{3/2}} + \frac{3}{2}\frac{\cancel{p}}{\tau}$$

$$p\tau \geqslant \frac{6}{\frac{1}{4} E_\alpha \frac{\langle\sigma v\rangle}{T^2} - C_B \frac{1}{T^{3/2}}}$$

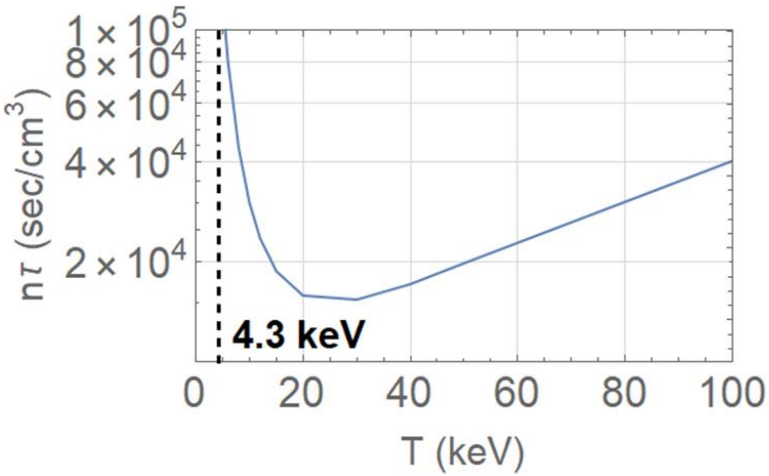
$$p = p_e + p_i = 2p_e = 2n_e T \equiv 2nT$$

$$nT\tau > \frac{3T^2}{\frac{1}{4}\langle\sigma v\rangle E_\alpha - C_B\sqrt{T}}$$

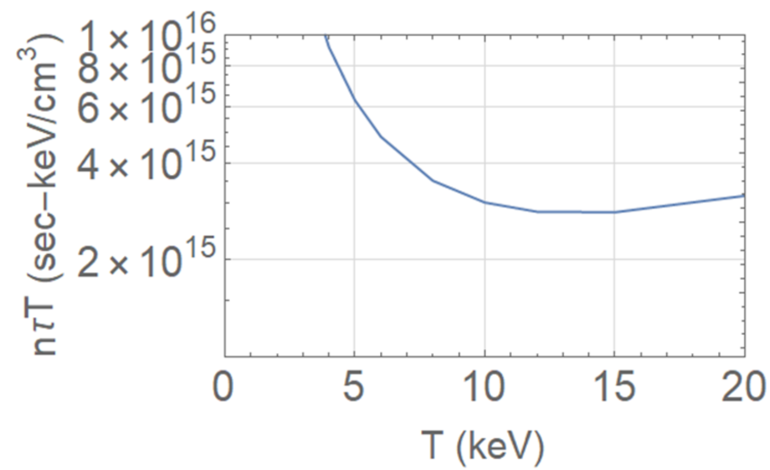
$$n\tau > \frac{3T}{\frac{1}{4}\langle\sigma v\rangle E_\alpha - C_B\sqrt{T}}$$

Temperature needs to be greater than ~5 keV to ignite

$$n\tau > \frac{3T}{\frac{1}{4}\langle \sigma v \rangle E_\alpha - C_B\sqrt{T}}$$



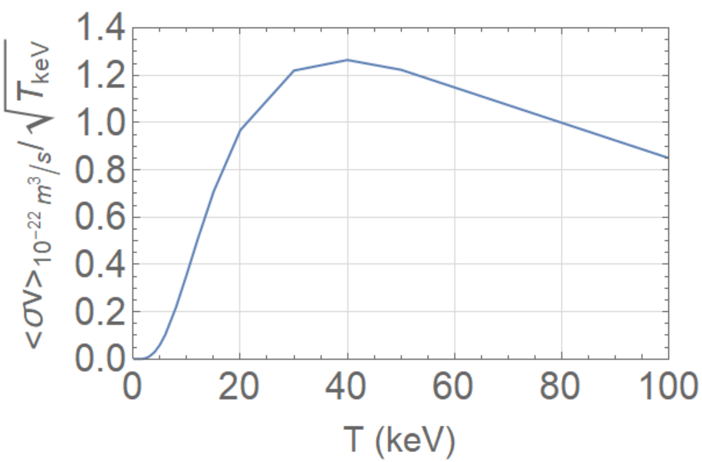
$$n\tau > 2 \times 10^4 \ \mathrm{sec/cm^3}$$



$$nT\tau > 3.5 \times 10^{15} \ \mathrm{keV-sec/cm^3}$$

S$_\alpha$>S$_B$ $\quad \frac{1}{4}E_\alpha n^2\langle \sigma v \rangle > C_B n^2 T^{1/2}$

$$\frac{\langle \sigma v \rangle}{T^{1/2}} > \frac{4C_B}{E_\alpha} \qquad T > 4.3 \ \mathrm{keV}$$



Under what conditions the plasma keeps itself hot?

- **Steady state 0-D power balance:**

$$S_\alpha + S_h = S_B + S_k$$

S_α: α particle heating

S_h: external heating

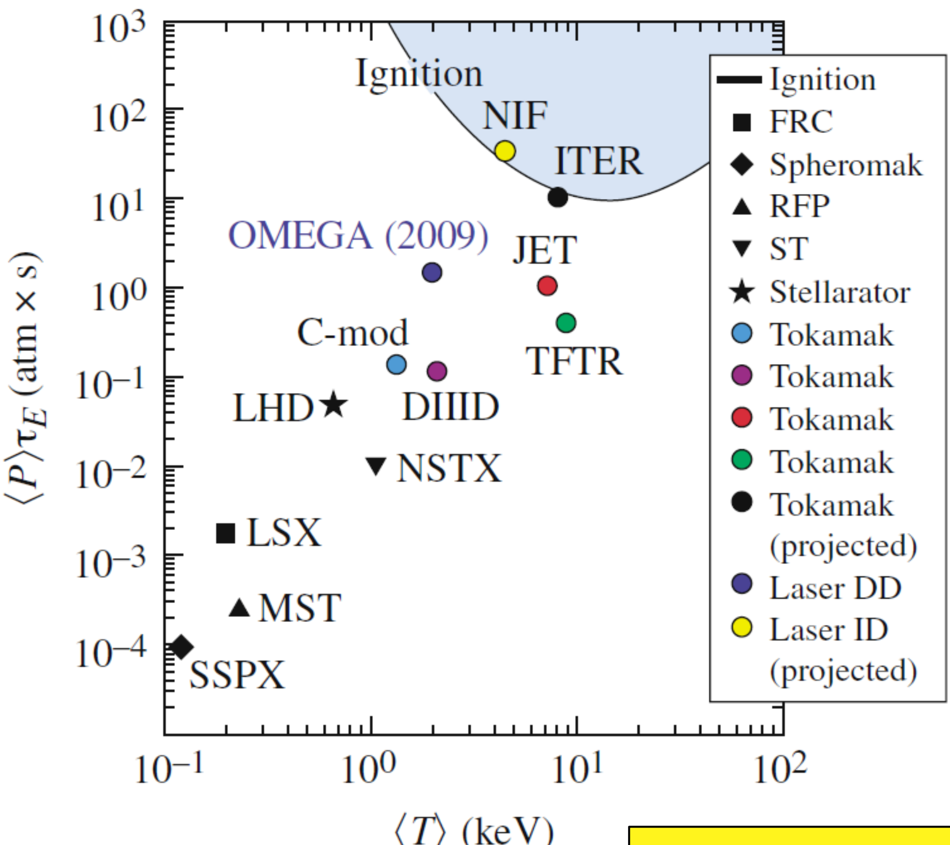
S_B: Bremsstrahlung radiation

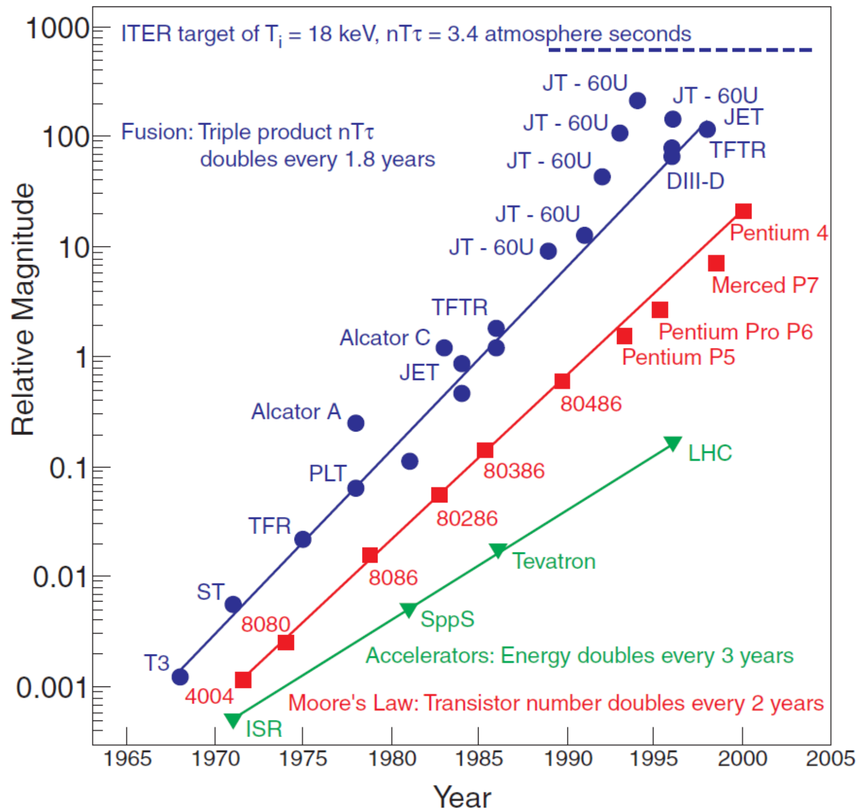
S_k: heat conduction lost

Ignition condition: $P\tau > 10$ atm-s = 10 Gbar - ns

- **P: pressure, or called energy density**
- **τ is confinement time**

We are closed to ignition!





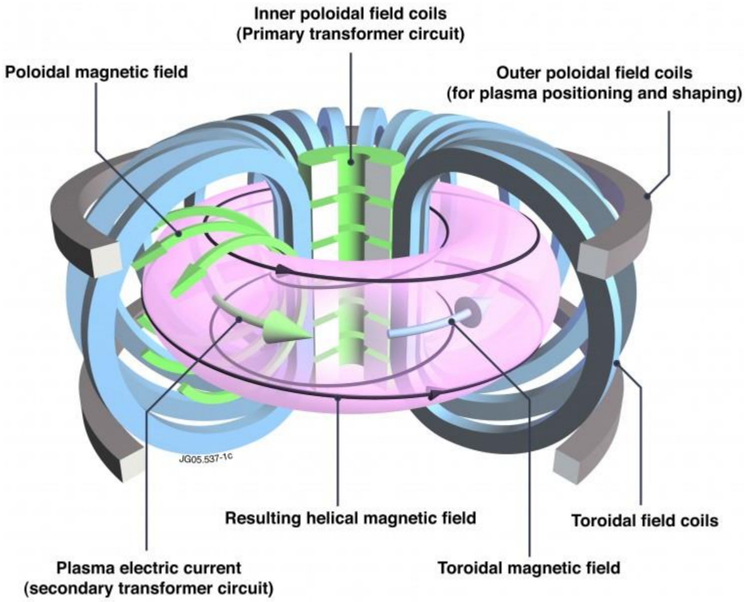
Ignition condition: Pτ > 10 atm-s = 10 Gbar - ns

A. J. Webster, Phys. Educ. **38**, 135 (2003)
R. Betti, etc., Phys. Plasmas, **17**, 058102 (2010)

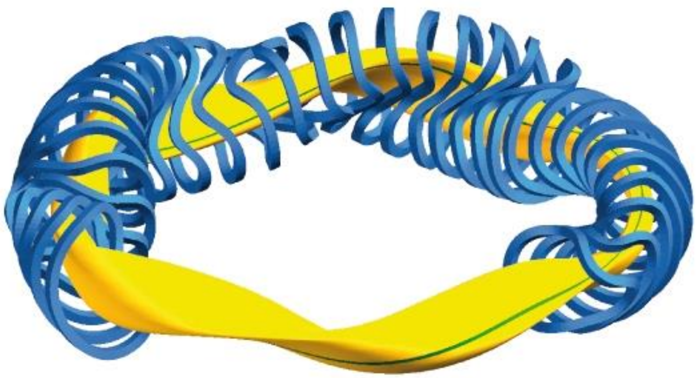
The plasma is too hot to be contained

- **Solution 1: Magnetic confinement fusion (MCF), use a magnetic field to contain it. P~atm, τ~sec, T~10 keV (10^8 ºC)**

Tokamak



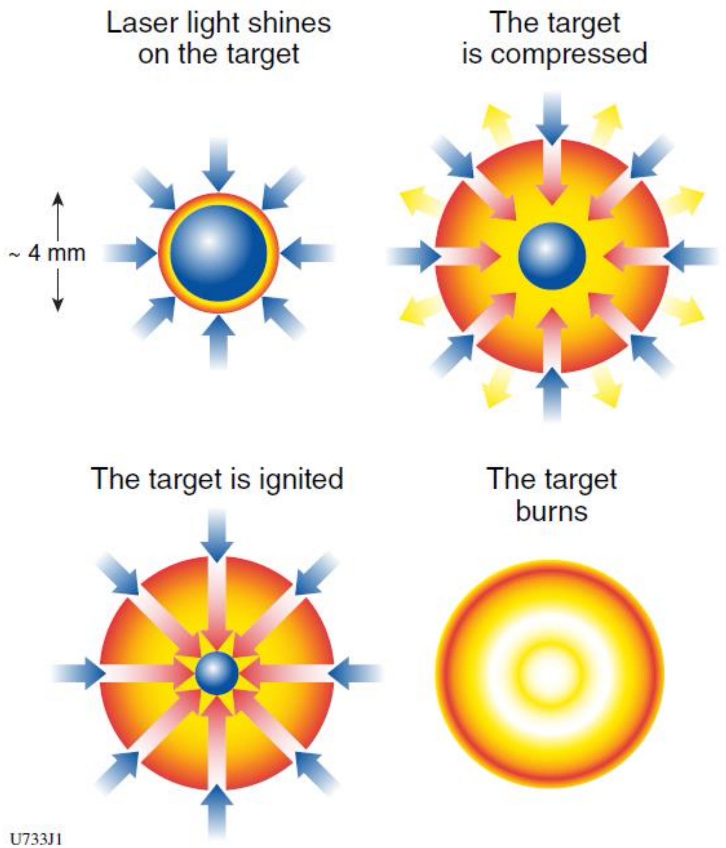
Stellarator



https://www.euro-fusion.org/2011/09/tokamak-principle-2/
https://en.wikipedia.org/wiki/Stellarator

Don’t confine it!

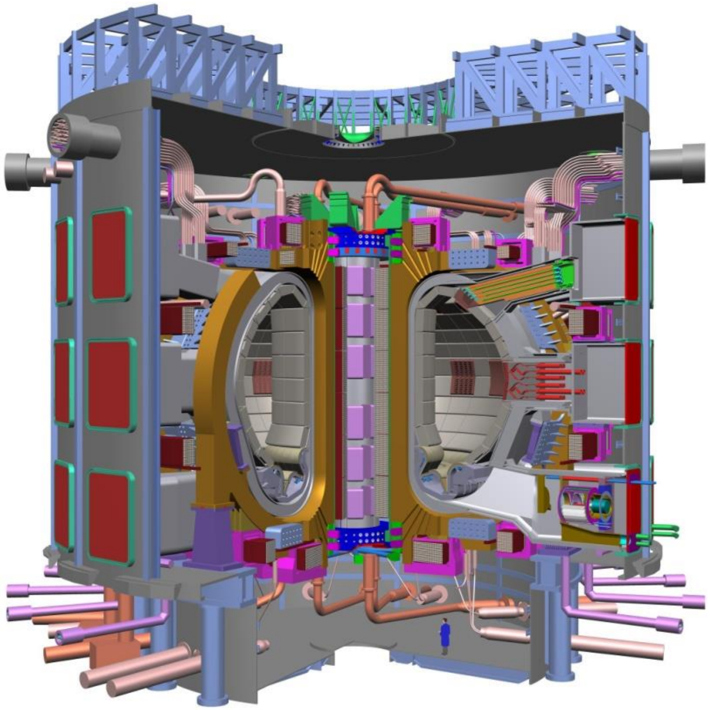
- **Solution 2: Inertial confinement fusion (ICF). Or you can say it is confined by its own inertia: P~Gigabar, τ~nsec, T~10 keV (10^8 ⁰C)**



Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester

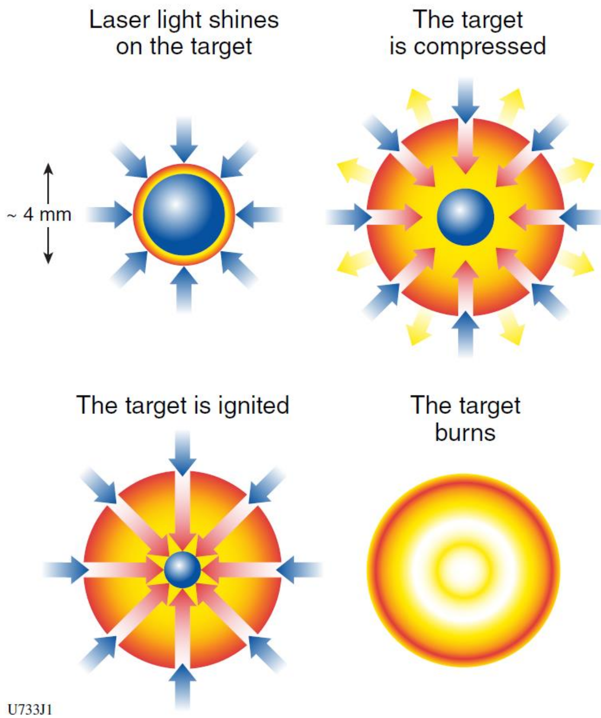
To control? Or not to control?

- **Magnetic confinement fusion (MCF)**



  - **Plasma is confined by toroidal magnetic field.**

- **Inertial confinement fusion (ICF)**



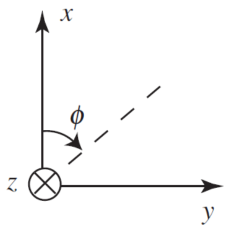
  - **A DT ice capsule filled with DT gas is imploded by laser.**

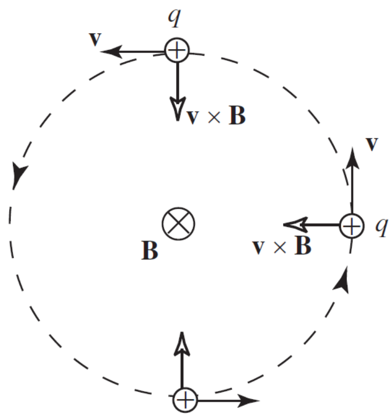
Laboratory for Laser Energetics, University of Rochester is a pioneer in laser fusion

Course Outline

- **Magnetic confinement fusion (MCF)**
  - **Gyro motion, MHD**
  - **1D equilibrium (z pinch, theta pinch)**
  - **Drift: ExB drift, grad B drift, and curvature B drift**
  - **Tokamak, Stellarator (toroidal field, poloidal field)**
  - **Magnetic flux surface**
  - **2D axisymmetric equilibrium of a torus plasma: Grad-Shafranov equation.**
  - **Stability (Kink instability, sausage instability, Safety factor Q)**
  - **Central-solenoid (CS) start-up (discharge) and current drive**
  - **CS-free current drive: electron cyclotron current drive, bootstrap current.**
  - **Auxiliary Heating: ECRH, Ohmic heating, Neutral beam injection.**

Charged particles gyro around the magnetic field line





$$m\frac{d\vec{v}}{dt} = q\,\vec{v} \times \vec{B}$$

- Assuming $\vec{B} = B\hat{z}$ and the electron oscillates in x-y plane

$$m\dot{v}_x = qBv_y$$

$$m\dot{v}_y = -qBv_x$$

$$m\dot{v}_z = 0 \qquad v_z = v_{||} = \text{constant}$$

$$\ddot{v}_x = -\frac{qB}{m}\dot{v}_y = -\left(\frac{qB}{m}\right)^2 v_x$$

$$\ddot{v}_y = -\frac{qB}{m}\dot{v}_x = -\left(\frac{qB}{m}\right)^2 v_y$$

$$\omega_c \equiv \frac{|q|B}{m}$$ Cyclotron frequency or gyrofrequency

$$\ddot{v}_x + \omega_c{}^2 v_x = 0$$

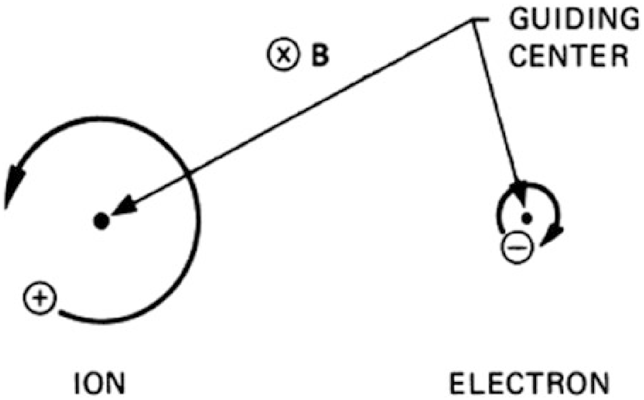
$$\ddot{v}_y + \omega_c{}^2 v_y = 0$$

$$v_x = v_\perp \cos(\pm\omega_c t + \psi)$$

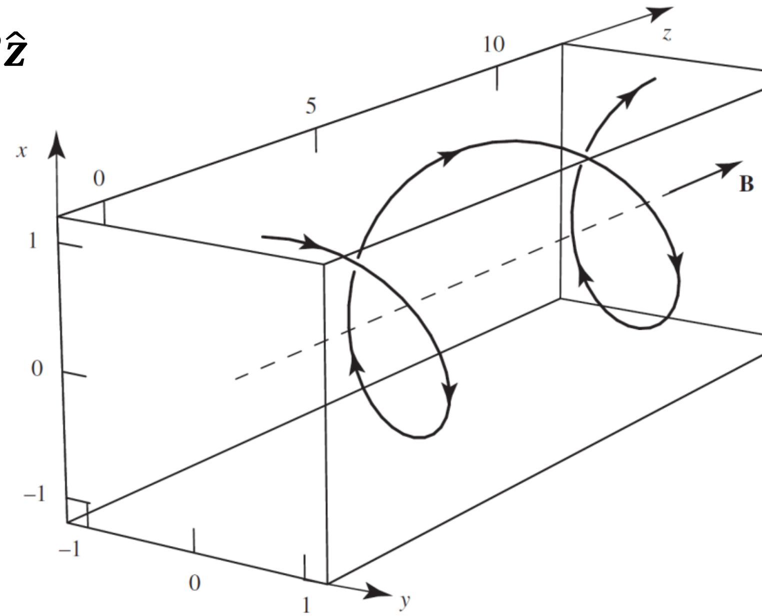
$$v_y = -v_\perp \sin(\pm\omega_c t + \psi)$$

$$v_z = v_{||}$$

Charged particles spiral around the magnetic field line



$$\vec{B} = B\hat{z}$$



$$v_x = v_\perp \cos(\pm\omega_c t + \psi)$$

$$v_y = v_\perp \sin(\pm\omega_c t + \psi)$$

$$v_z = v_{||}$$

$$\omega_c \equiv \frac{|q|B}{m}$$

$$\left|\frac{m{v_\perp}^2}{r}\right| = \left|q\,\vec{v} \times \vec{B}\right| = |qv_\perp B|$$

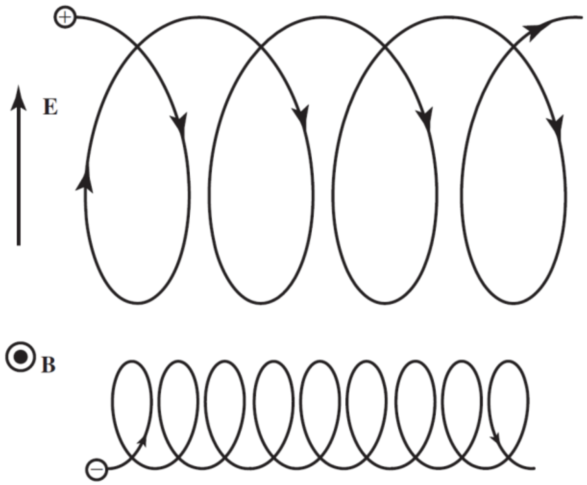
$$r_c = \frac{v_\perp}{\omega_c} = \frac{mv_\perp}{|q|B}$$ **Larmor radius or gyroradius**

$$x = \mp r_c sin(\pm\omega_c t + \psi) + (x_o - r_c sin\psi)$$

$$y = \pm r_c cos(\pm\omega_c t + \psi) + (y_o + r_c cos\psi)$$

$$z = z_o + v_{||}t$$

Charge particles drift across magnetic field lines when an electric field not parallel to the magnetic field occurs



$$m\frac{d}{dt}\left(\vec{v}_{\mathrm{E}} + \vec{v}_{\mathrm{ac}}(t)\right) = q\left[\hat{x}E_\perp + \left(\vec{v}_{\mathrm{E}} + \vec{v}_{\mathrm{ac}}(t)\right)\times\hat{z}B\right]$$

$$m\frac{d\,\vec{v}_{\mathrm{ac}}(t)}{dt} = q\left[\hat{x}E_\perp + \vec{v}_{\mathrm{E}}\times\hat{z}B + \vec{v}_{\mathrm{ac}}(t)\times\hat{z}B\right]$$

No E field case: $m\frac{d\,\vec{v}}{dt} = q\,\vec{v}\times\vec{B}$

$$\hat{x}E_\perp + \vec{v}_{\mathrm{E}}\times\hat{z}B = 0$$

$$\boxed{\vec{v}_{\mathrm{E}} = \frac{\hat{x}E_\perp\times\hat{z}B}{B^2} = \frac{\vec{E}\times\vec{B}}{B^2}}$$

ExB drift velocity

$$\vec{E} = \vec{E}_\perp + \hat{z}E_{||} = \hat{x}E_\perp + \hat{z}E_{||}$$

$$m\frac{dv_{||}}{dt} = qE_{||}$$

$$m\frac{d\,\vec{v}_\perp}{dt} = q\left(\hat{x}E_\perp + \vec{v}_\perp\times\hat{z}B\right)$$

$$v_{||}(t) = \frac{qE_{||}}{m}t + v_{||,o}$$

$$\vec{v}_\perp(t) = \vec{v}_{\mathrm{E}} + \vec{v}_{\mathrm{ac}}(t)$$

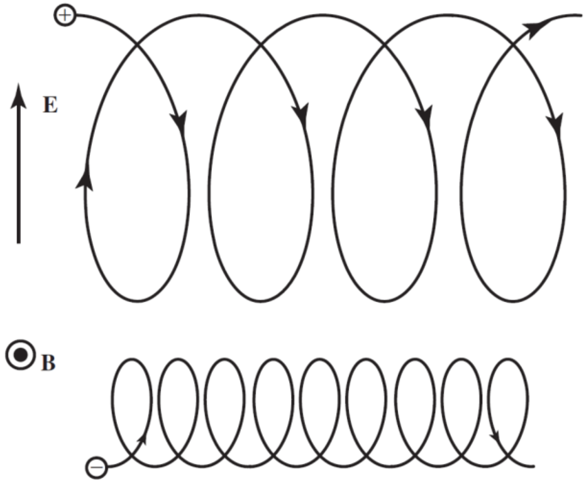
$$m\frac{d\,\vec{v}_{\mathrm{ac}}(t)}{dt} = q\,\vec{v}_{\mathrm{ac}}(t)\times\hat{z}B$$ Gyro motion

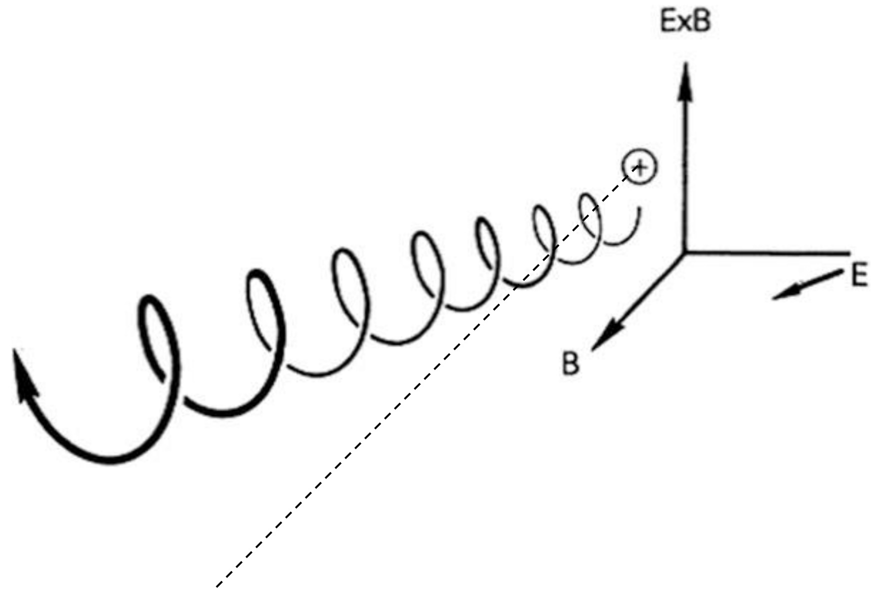
$$\vec{v}(t) = \hat{z}v_{||}(t) + \vec{v}_{\mathrm{E}} + \vec{v}_{\mathrm{ac}}(t)$$

$$\langle\vec{v}(t)\rangle = \frac{1}{T_c}\int_0^{T_c}\vec{v}(t)\,dt = \hat{z}v_{||}(t) + \vec{v}_E$$

- **Electrons and ions drift in the same direction.**

No current is generated in ExB drift





$$\langle \vec{v}(t) \rangle = \frac{1}{T_c}\int_0^{T_c} \vec{v}(t)\,dt = \hat{z}v_{||}(t) + \vec{v}_E$$

$$\vec{v}_{\mathrm{E}} = \frac{\hat{x}E_{\perp} \times \hat{z}B}{B^2} = \frac{\vec{E} \times \vec{B}}{B^2}$$ **ExB drift velocity**

- **Electrons and ions drift in the same direction.**

Charge particles drift across magnetic field lines when an external field not parallel to the magnetic field occurs

$$\vec{E} = \vec{E}_\perp + \hat{z}E_{||} = \hat{x}E_\perp + \hat{z}E_{||}$$

$$m\frac{dv_{||}}{dt} = qE_{||}$$

$$m\frac{d\,\vec{v}_\perp}{dt} = q(\hat{x}E_\perp + \vec{v}_\perp \times \hat{z}B)$$

$$\langle \vec{v}\,(t)\rangle = \frac{1}{T_c}\int_0^{T_c} \vec{v}\,(t)\,dt = \hat{z}v_{||}(t) + \vec{v}_E$$

$$\vec{v}_\mathrm{E} = \frac{\hat{x}E_\perp \times \hat{z}B}{B^2} = \frac{\vec{E}\times\vec{B}}{B^2}$$

ExB drift velocity

$$\vec{F} = \vec{F}_\perp + \hat{z}F_{||} = \hat{x}F_\perp + \hat{z}F_{||}$$

$$m\frac{dv_{||}}{dt} = F_{||}$$

$$m\frac{d\,\vec{v}_\perp}{dt} = q\left(\hat{x}\frac{F_\perp}{q} + \vec{v}_\perp \times \hat{z}B\right)$$

$$\langle \vec{v}\,(t)\rangle = \frac{1}{T_c}\int_0^{T_c} \vec{v}\,(t)\,dt = \hat{z}v_{||}(t) + \vec{v}_F$$

$$\vec{v}_\mathrm{F} = \frac{\hat{x}(F_\perp/q)\times\hat{z}B}{B^2} = \frac{1}{q}\frac{\vec{F}\times\vec{B}}{B^2}$$

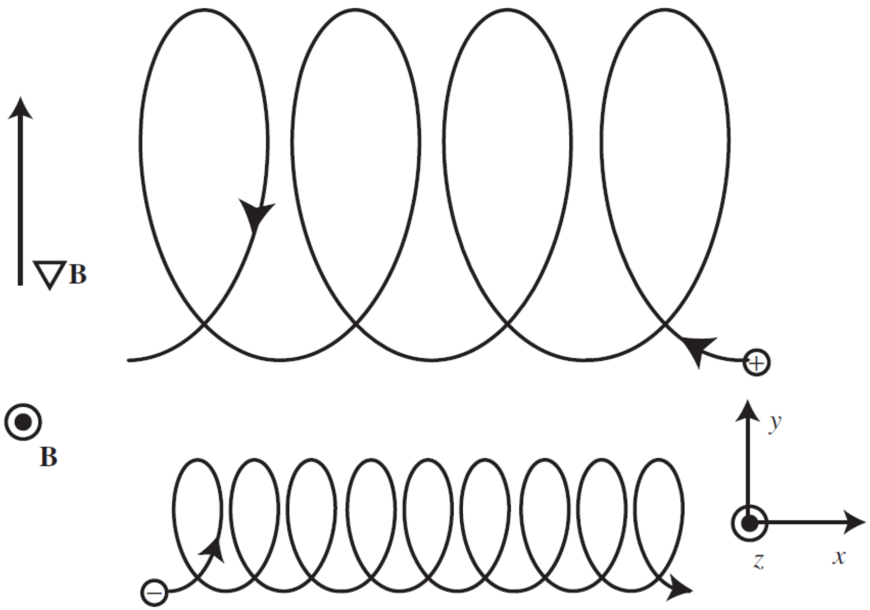
Gravitational drift velocity

- **Electrons and ions drift in the opposite directions in the gravitational drift. Therefore, currents are generated.**

Drift in non-uniform B fields

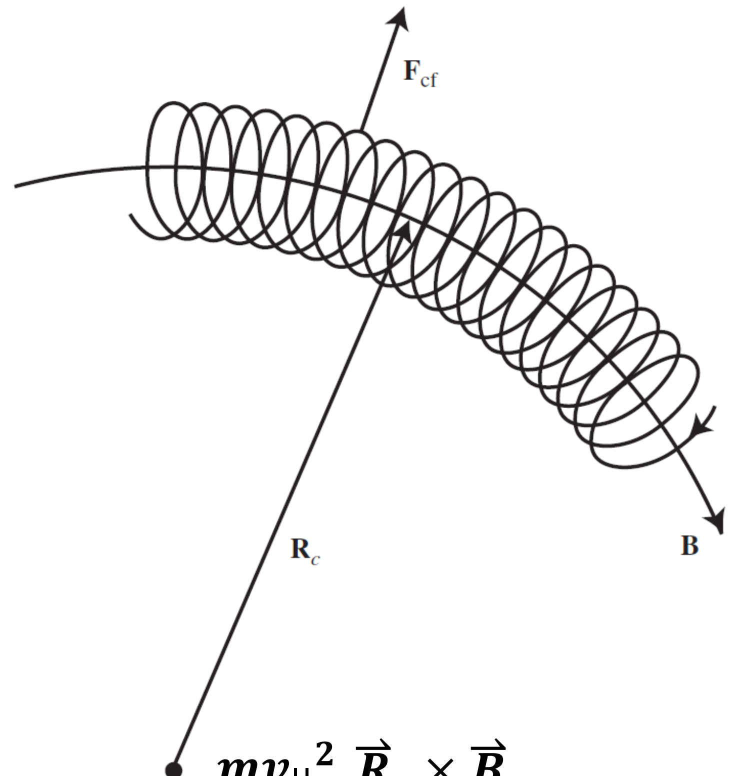
Charge particles drift across magnetic field lines when the magnetic field is not uniform or curved

- **Gradient-B drift**



$$\vec{v}_{\nabla} = \frac{m{v_{\perp}}^2}{2q}\frac{\vec{B}\times\nabla B}{B^3}$$

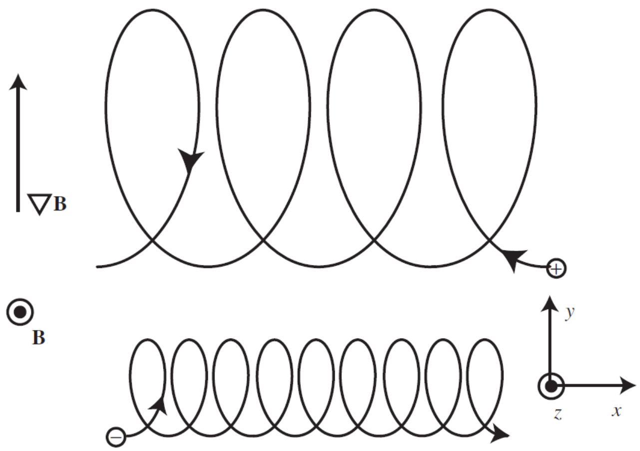
- **Curvature drift**



$$\vec{v}_R = \frac{m{v_{||}}^2}{2q}\frac{\vec{R}_c\times\vec{B}}{R_c B^2}$$

$$\vec{v}_{\text{total}} = \vec{v}_{\text{R}} + \vec{v}_{\nabla} = \frac{\vec{B}\times\nabla B}{\omega_c B^2}\left({v_{||}}^2 + \frac{1}{2}{v_{\perp}}^2\right) = \frac{m}{q}\frac{\vec{R}_c\times\vec{B}}{{R_c}^2 B^2}\left({v_{||}}^2 + \frac{1}{2}{v_{\perp}}^2\right)$$

Charge particles drift across magnetic field lines when the magnetic field is not uniform or curved



- In the case with no gradient B

$$x_c = \mp r_c \sin(\pm\omega_c t + \psi)$$

$$y_c = \pm r_c \cos(\pm\omega_c t + \psi)$$

$$v_x = v_\perp \cos(\pm\omega_c t + \psi)$$

$$v_y = -v_\perp \sin(\pm\omega_c t + \psi)$$

$$\vec{F} = q(\vec{v} \times \vec{B}) = \hat{x} q v_y B_z - \hat{y} q v_x B_z$$

$$\simeq \hat{x} q v_y \left(B_o + y\frac{\partial B_z}{\partial y}\right) - \hat{y} q v_x \left(B_o + y\frac{\partial B_z}{\partial y}\right)$$

$$B_z(y) = B_o + y\frac{\partial B_z}{\partial y} + y^2 \frac{1}{2}\frac{\partial^2 B_z}{\partial y^2} + \ldots$$

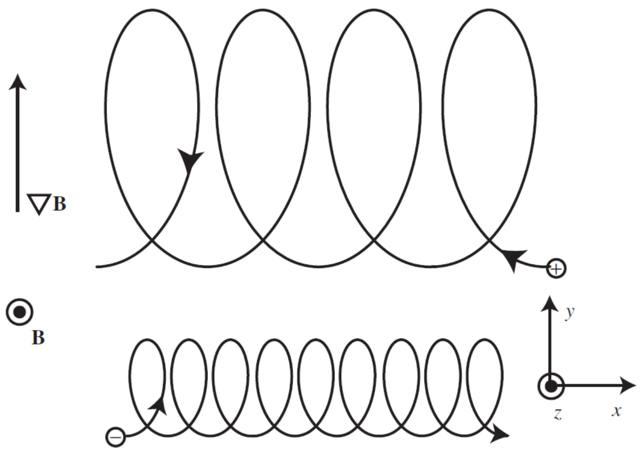
$$F_x = q v_y \left(B_o + y\frac{\partial B_z}{\partial y}\right)$$

$$F_y = -q v_x \left(B_o + y\frac{\partial B_z}{\partial y}\right)$$

$$F_x \simeq -q v_\perp \sin(\pm\omega_c t + \psi) \times \left(B_o \pm r_c \cos(\pm\omega_c t + \psi)\frac{\partial B_z}{\partial y}\right)$$

$$F_y = -q v_\perp \cos(\pm\omega_c t + \psi) \times \left(B_o \pm r_c \cos(\pm\omega_c t + \psi)\frac{\partial B_z}{\partial y}\right)$$

Charge particles drift across magnetic field lines when the magnetic field is not uniform



$$F_x \simeq -qv_\perp \sin(\pm\omega_c t + \psi)\left(B_o \pm r_c \cos(\pm\omega_c t + \psi)\frac{\partial B_z}{\partial y}\right)$$

$$F_y \simeq -qv_\perp \cos(\pm\omega_c t + \psi)\left(B_o \pm r_c \cos(\pm\omega_c t + \psi)\frac{\partial B_z}{\partial y}\right)$$

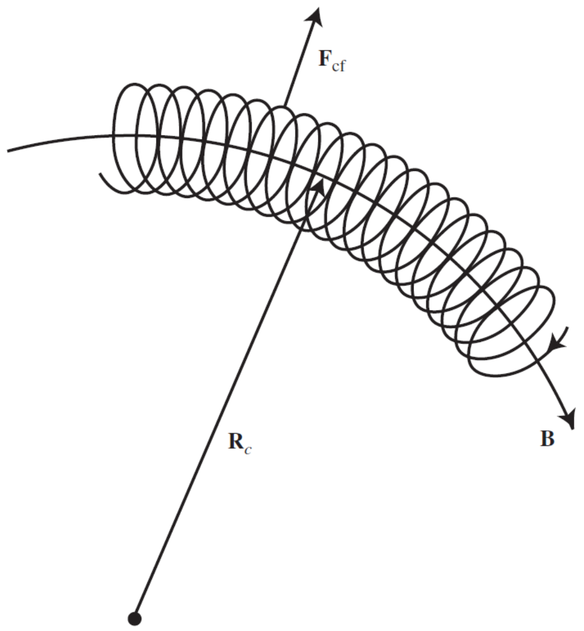
$$\langle F_x \rangle = 0$$

$$\langle F_y \rangle = \mp \frac{qv_\perp r_c}{2}\frac{\partial B_z}{\partial y} = -\frac{m{v_\perp}^2}{2B}\frac{\partial B_z}{\partial y} \qquad r_c = \frac{v_\perp}{\omega_c} \quad \omega_c \equiv \frac{|q|B}{m}$$

$$\vec{v}_F = \frac{1}{q}\frac{\vec{F}\times\vec{B}}{B^2} \qquad \vec{v}_\nabla = \frac{1}{q}\frac{\langle F_y\rangle\hat{y}\times\hat{z}B_z}{{B_z}^2} = -\frac{m{v_\perp}^2}{2qB_z}\frac{\partial B_z}{\partial y}\hat{x}$$

- **More general:** $\vec{v}_\nabla = \frac{m{v_\perp}^2}{2q}\frac{\vec{B}\times\nabla B}{B^3}$

Charge particles drift across magnetic field lines when the magnetic field line is curved

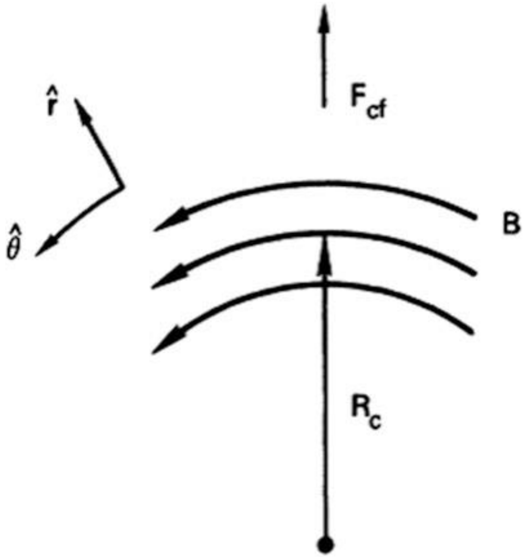


$$\vec{F}_{cf} = m v_{||}^2 \frac{\vec{R}_c}{R_c^{\,2}}$$

$$\vec{v}_{\mathrm{F}} = \frac{1}{q}\frac{\vec{F} \times \vec{B}}{B^2}$$

$$\vec{v}_R = \frac{1}{q}\frac{\vec{F}_{cf} \times \vec{B}}{B^2} = \frac{m v_{||}^2}{2q}\frac{\vec{R}_c \times \vec{B}}{R_c B^2}$$

Charge particles drift across magnetic field lines when the magnetic field is not uniform or curved



$$\vec{v}_\nabla = \frac{m v_\perp^{\,2}}{2q}\frac{\vec{B}\times\nabla B}{B^3} \qquad \vec{v}_R = \frac{m v_{||}^{\,2}}{2q}\frac{\vec{R}_c\times\vec{B}}{R_c B^2}$$

$$\nabla\times\vec{B} = 0 \qquad \left(\nabla\times\vec{B}\right)_r = \left(\nabla\times\vec{B}\right)_\theta = 0$$

$$\left(\nabla\times\vec{B}\right)_r = \frac{1}{r}\frac{\partial \cancel{B_z}}{\partial\theta} - \cancel{\frac{\partial B_\theta}{\partial z}} \qquad \left(\nabla\times\vec{B}\right)_\theta = \frac{\partial \cancel{B_r}}{\partial z} - \frac{\partial \cancel{B_z}}{\partial r}$$

$$\left(\nabla\times\vec{B}\right)_z = \frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) - \frac{1}{r}\frac{\partial \cancel{B_r}}{\partial\theta}$$

$$\vec{B} = B\hat{\theta}$$

$$\nabla B = \nabla B\hat{r}$$

$$\nabla\times\vec{B} = \left(\nabla\times\vec{B}\right)_z = \frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) = 0 \qquad B_\theta \propto \frac{1}{r}$$

$$\frac{\nabla|B|}{|B|} = -\frac{\vec{R}_c}{R_c^{\,2}}$$

Cylindrical coordinate

$$\vec{v}_{\text{total}} = \vec{v}_R + \vec{v}_\nabla = \frac{\vec{B}\times\nabla B}{\omega_c B^2}\left(v_{||}^{\,2} + \frac{1}{2}v_\perp^{\,2}\right) = \frac{m}{q}\frac{\vec{R}_c\times\vec{B}}{R_c^{\,2}B^2}\left(v_{||}^{\,2} + \frac{1}{2}v_\perp^{\,2}\right)$$

- **Electrons and ions drift in the opposite directions in the grad-B and curvature drifts. Therefore, currents are generated.**

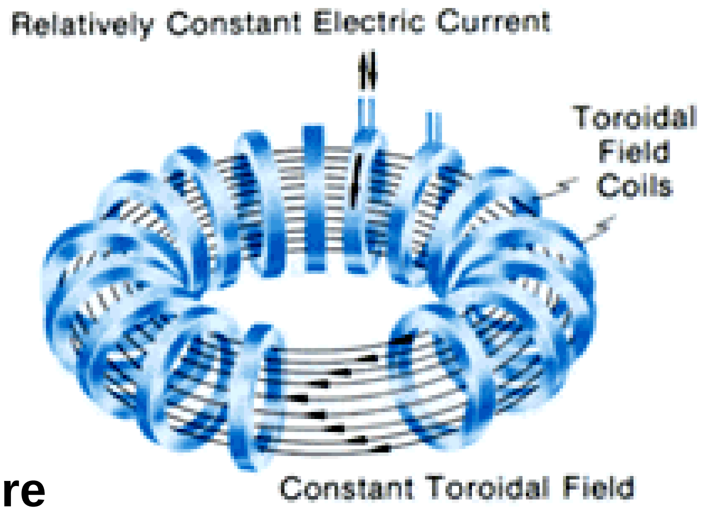
Quick summary of different drifts

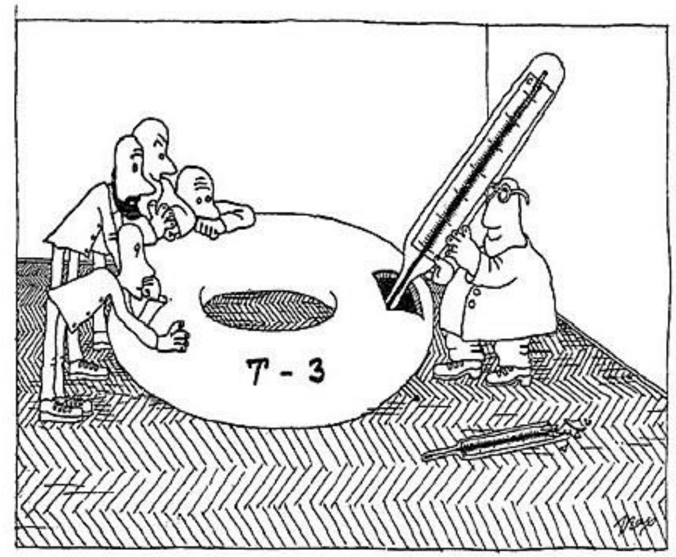
- **ExB drift:** $\vec{v}_{\mathrm{E}} = \dfrac{\vec{E} \times \vec{B}}{B^2}$ **Independent to charge**
- **Gravitational drift:** $\vec{v}_{\mathrm{F}} = \dfrac{1}{q} \dfrac{\vec{F} \times \vec{B}}{B^2}$ **Depended on charge**
- **Grad-B drift:** $\vec{v}_{\nabla} = \dfrac{m{v_\perp}^2}{2q} \dfrac{\vec{B} \times \nabla B}{B^3}$ **Depended on charge**
- **Curvature drift:** $\vec{v}_{R} = \dfrac{m{v_{||}}^2}{2q} \dfrac{\vec{R}_{\mathrm{c}} \times \vec{B}}{R_{\mathrm{c}} B^2}$ **Depended on charge**
- **Non-uniform B drift:**

$$\vec{v}_{\mathrm{total}} = \vec{v}_{\mathrm{R}} + \vec{v}_{\nabla} = \frac{\vec{B} \times \nabla B}{\omega_{\mathrm{c}} B^2} \left({v_{||}}^2 + \frac{1}{2} {v_\perp}^2 \right) = \frac{m}{q} \frac{\vec{R}_{\mathrm{c}} \times \vec{B}}{{R_{\mathrm{c}}}^2 B^2} \left({v_{||}}^2 + \frac{1}{2} {v_\perp}^2 \right)$$

Plasma can be confined in a doughnut-shaped chamber with toroidal magnetic field

- **Tokamak - "toroidal chamber with magnetic coils" (тороидальная камера с магнитными катушками)**





Nature

Measurement of the Electron Temperature by Thomson Scattering in Tokamak T3

by

N. J. PEACOCK
D. C. ROBINSON
M. J. FORREST
P. D. WILCOCK
UKAEA Research Group, Culham Laboratory, Abingdon, Berkshire

V. V. SANNIKOV
I. V. Kurchatov Institute, Moscow

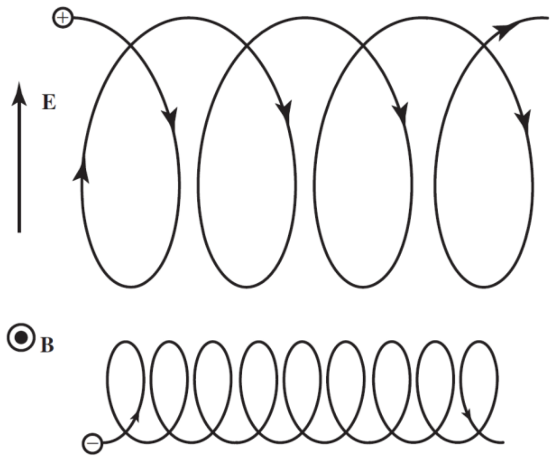
Electron temperatures of 100 eV up to 1 keV and densities in the range 1–3×10^{13} cm^{-3} have been measured by Thomson scattering on Tokamak T3. These results agree with those obtained by other techniques where direct comparison has been possible.

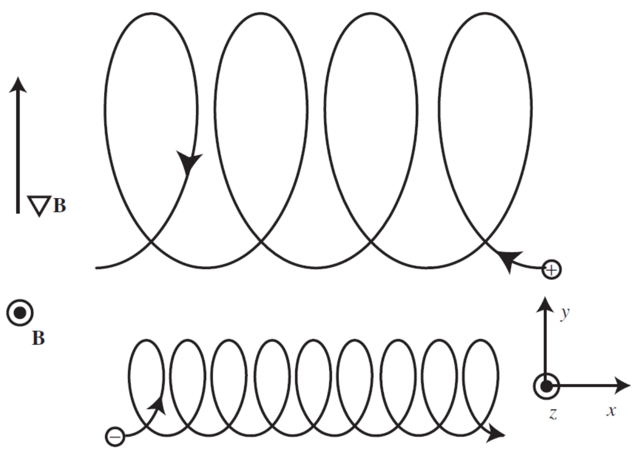
T_e **= 100 ~ 1 keV**
n_e **= 1-3 x 10^{13} cm^{-3}**

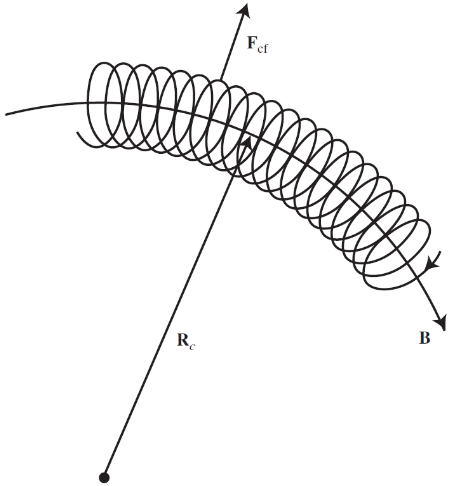
https://www.iter.org/mach/tokamak
https://en.wikipedia.org/wiki/Tokamak#cite_ref-4
Drawing from the talk "Evolution of the Tokamak" given in 1988 by B.B. Kadomtsev at Culham.
N. J. Peacock,et al., Nature **224**, 488 (1969)

Quick summary of different drifts

- **ExB drift:** $\vec{v}_{\mathrm{E}} = \dfrac{\vec{E} \times \vec{B}}{B^2}$ **Independent to charge**
- **Grad-B drift:** $\vec{v}_{\nabla} = \dfrac{m{v_\perp}^2}{2q} \dfrac{\vec{B} \times \nabla B}{B^3}$ **Depended on charge**
- **Curvature drift:** $\vec{v}_R = \dfrac{m{v_{||}}^2}{q} \dfrac{\vec{R}_c \times \vec{B}}{{R_c}^2 B^2}$ **Depended on charge**

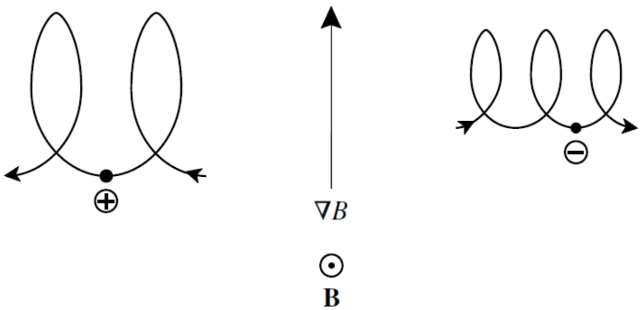




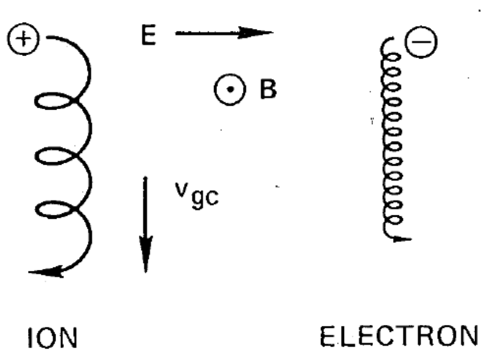


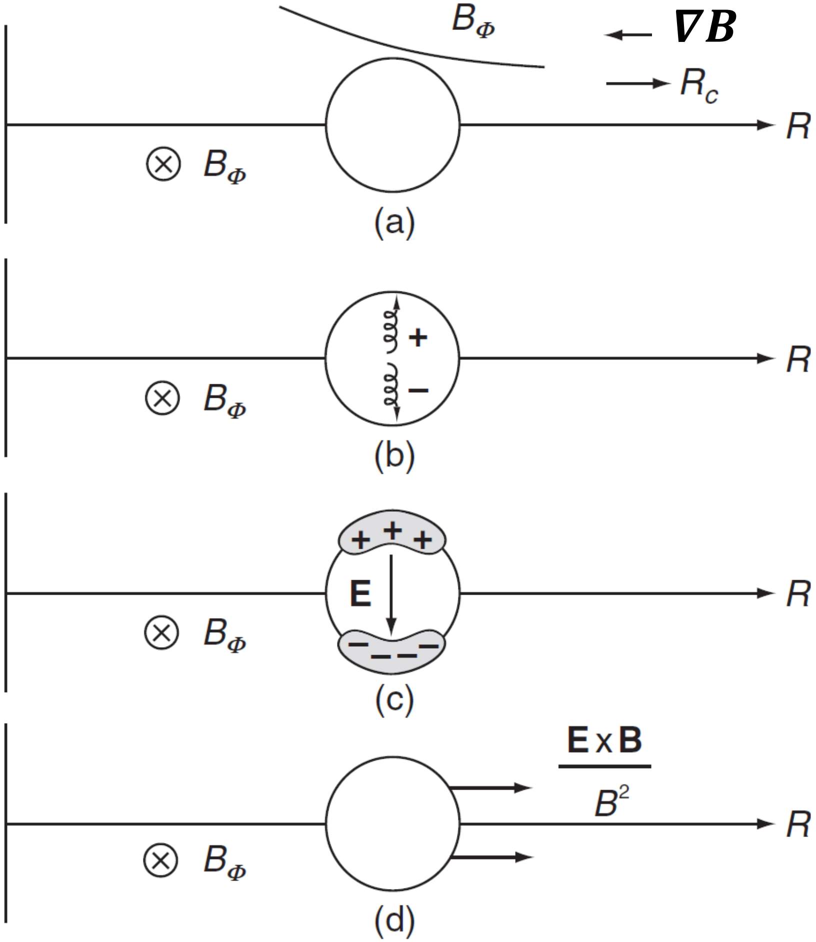
Charged particles drift across field lines

- **Grad-B drift**



- **ExB drift**





J. P. Freidberg, Ideal Magnetohydrodynamics