

Introduction to Nuclear Fusion as An Energy Source



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Institute of Space and Plasma Sciences, National Cheng Kung University

2026 spring semester

Tuesday 9:00-12:00

Materials:

<https://capst.ncku.edu.tw/PGS/index.php/teaching/>

Online courses:

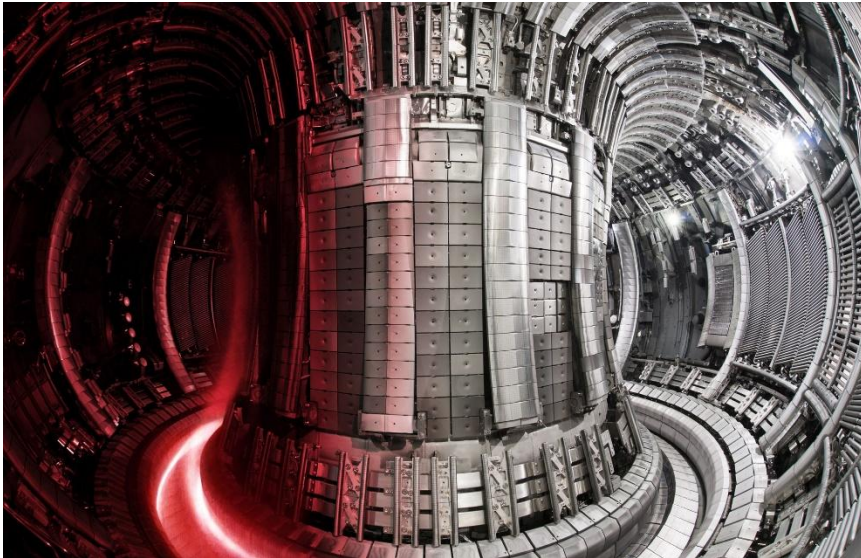
<https://reurl.cc/MMnkOL>



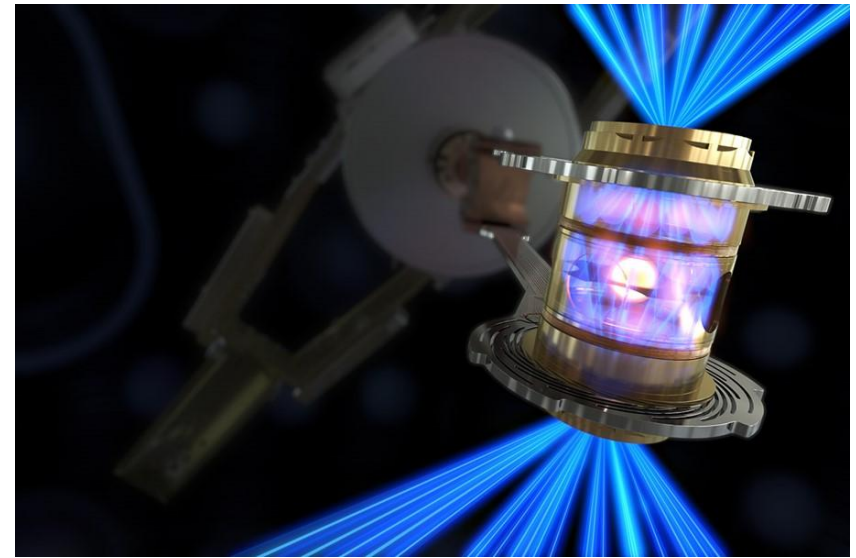
Significant breakthrough is achieved recently



- **Magnetic confinement fusion (MCF)**
- **Inertial confinement fusion (ICF)**



- **On 2024/2/(8), record-breaking 69.26 megajoules of sustained fusion energy in Joint European Torus (JET) facility in Oxford demonstrates powerplant potential and strengthens case for ITER.**



- **National Ignition Facility (NIF) demonstrated a gain greater than 1 for the first time on 2022/12/5. The yield of 3.15 MJ from the 2.05-MJ input laser energy, i.e., $Q=1.5$.**

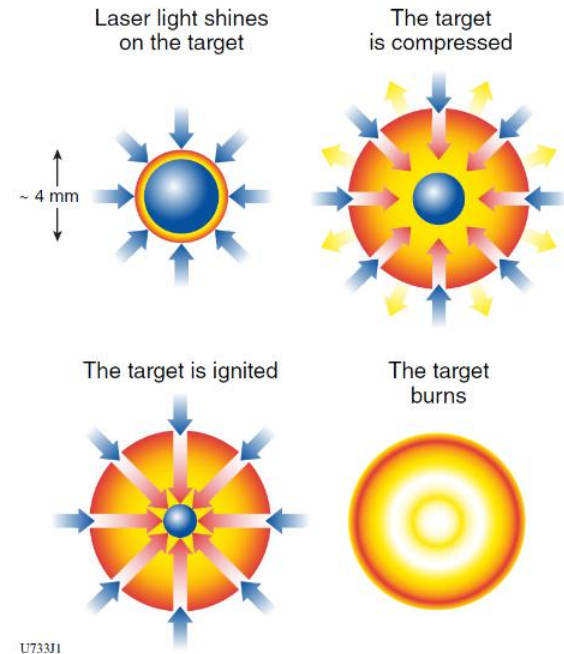
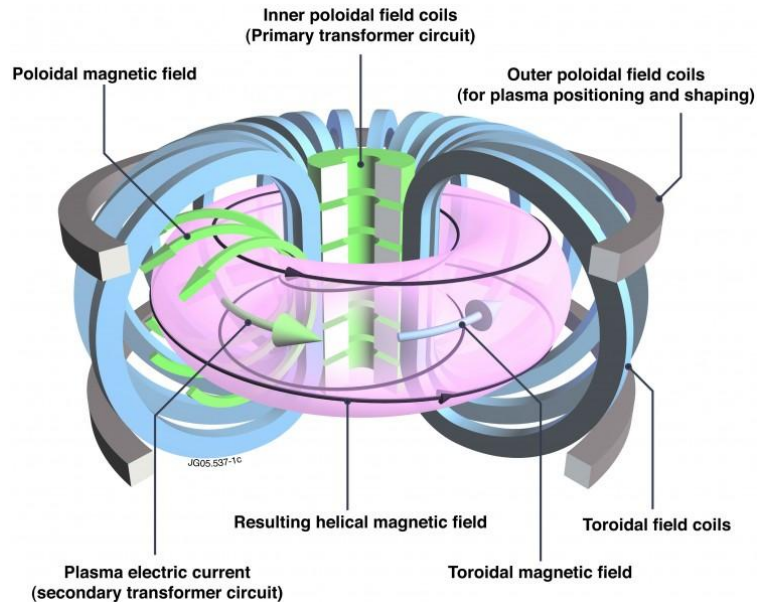
<https://ccfe.ukaea.uk/resources/#gallery>

<https://www.science.org/content/article/historic-explosion-long-sought-fusion-breakthrough>

Nuclear fusion as an energy source is being developed



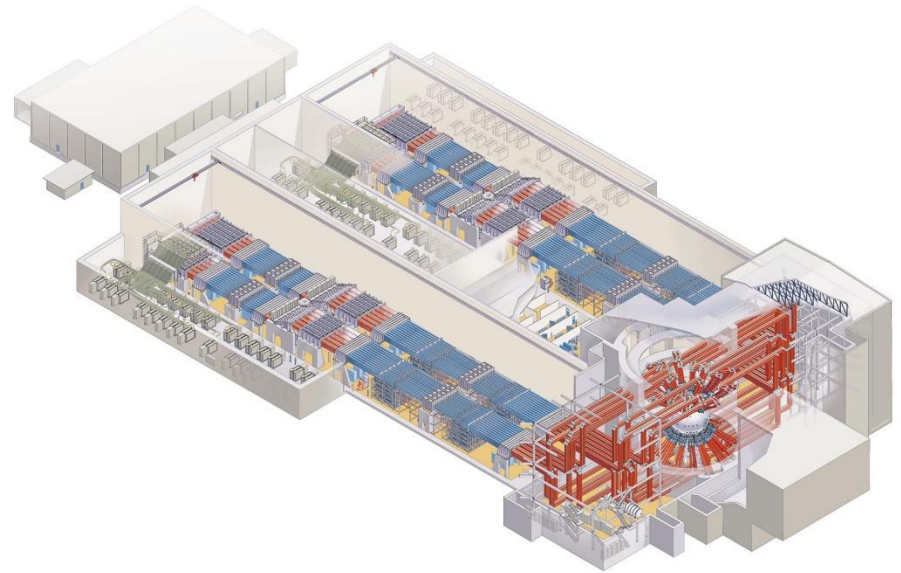
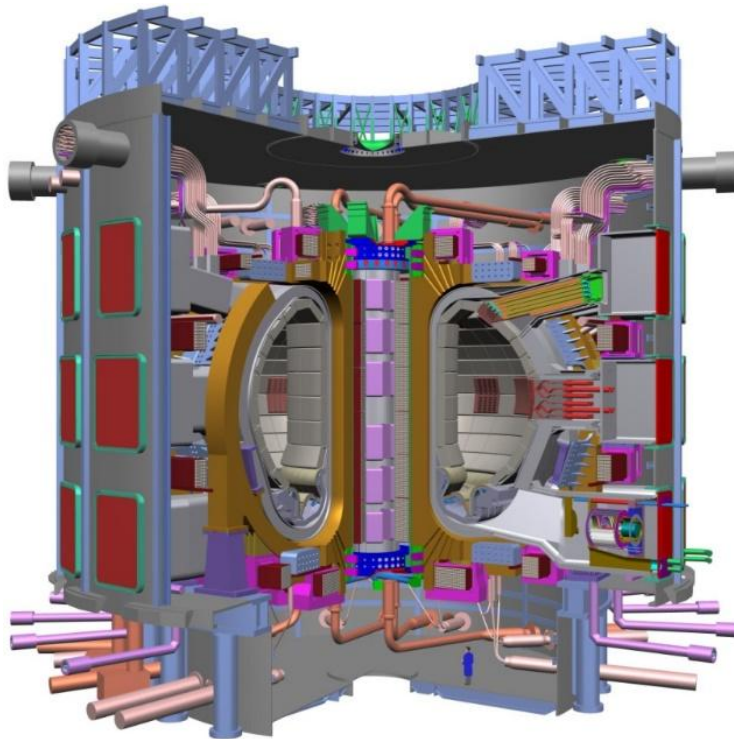
- Magnetic confinement fusion (MCF)
- Inertial confinement fusion (ICF)



Nuclear fusion as an energy source is being developed



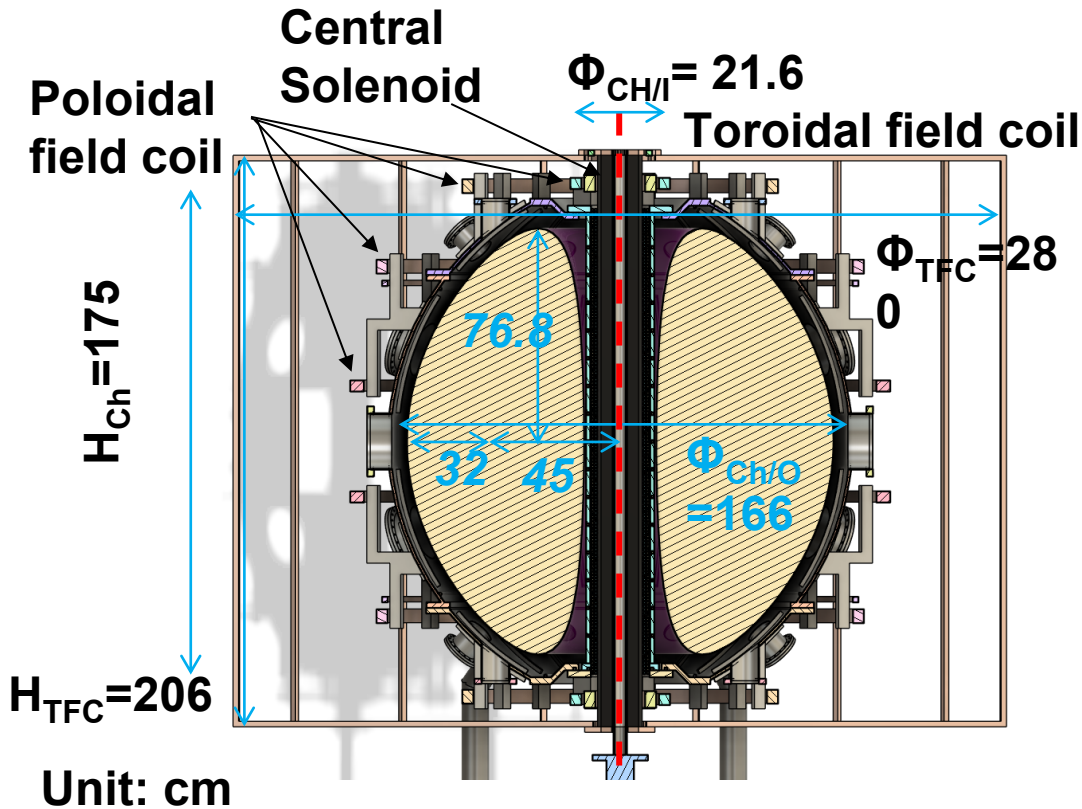
- Magnetic confinement fusion (MCF)
- Inertial confinement fusion (ICF)



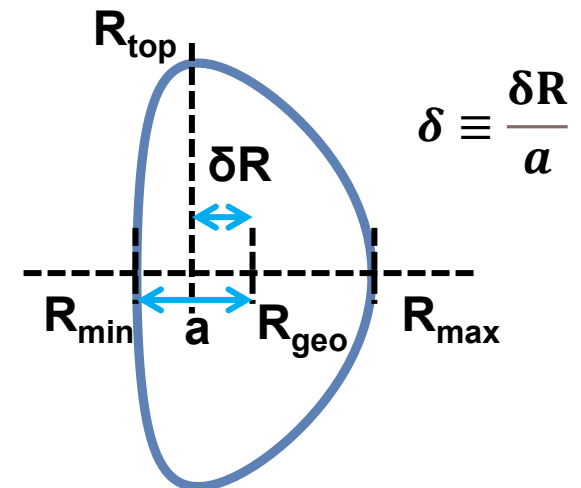
<https://www.iter.org>

<https://zh.wikipedia.org/wiki/國家點火設施>

Formosa Integrated Research Spherical Tokamak (FIRST)



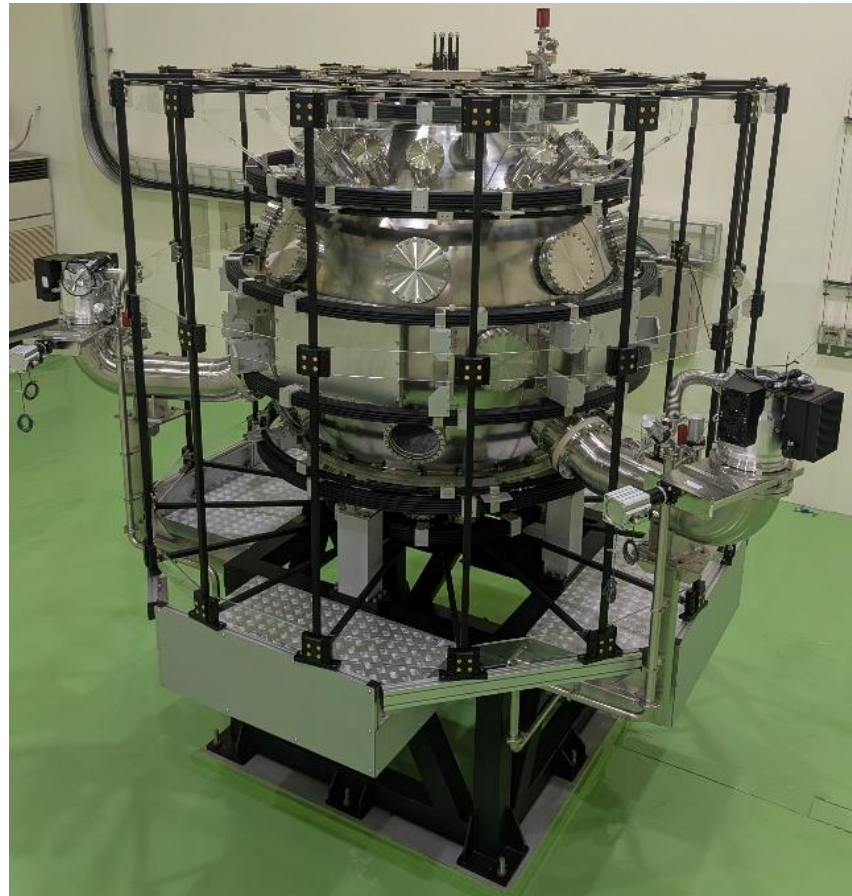
- Parameters:
 - $R = 45$ cm
 - $a = 32$ cm
 - Aspect Ratio = $R/a = 1.41$
 - Elongation $\kappa = b/a = 2.4$
 - Triangularity $\delta = 0.5$
 - $T \sim 100$ eV
 - $B_T \sim 0.1 \sim 0.5$ T
 - $I_p \sim 100$ kA



$$r = R + a \cos(\theta + \delta \sin(\theta)) \quad z = a \kappa \sin(\theta)$$

- We welcome anyone interested in fusion research to join us!

The vacuum chamber of FIRST was delivered



Course Outline



- **Brief background reviews**
 - **Electromagnetics**
 - **Plasma physics**
- **Introduction to nuclear fusion**
 - **Nuclear binding energy (Fission vs Fusion)**
 - **Fusion reaction physics**
 - **Some important fusion reactions (Cross section)**
 - **Main controlled fusion fuels**
 - **Advanced fusion fuels**
 - **Maxwell-averaged fusion reactivities**

Course Outline



- **Introduction to nuclear fusion (cont.)**
 - **Collisions (Bremsstrahlung radiation)**
 - **Columb scattering. Cross section of the Columb scattering**
 - **Beam-target fusion vs thermonuclear fusion**
 - **Lawson criteria, ignition conditions**
 - **Magnetic confinement fusion (MCF) vs Inertial confinement fusion (ICF)**

Course Outline



- **Magnetic confinement fusion (MCF)**
 - Gyro motion, MHD
 - 1D equilibrium (z pinch, theta pinch)
 - Drift: ExB drift, grad B drift, and curvature B drift
 - Tokamak, Stellarator (toroidal field, poloidal field)
 - Magnetic flux surface
 - 2D axisymmetric equilibrium of a torus plasma: Grad-Shafranov equation.
 - Stability (Kink instability, sausage instability, Safety factor Q)
 - Central-solenoid (CS) start-up (discharge) and current drive
 - CS-free current drive: electron cyclotron current drive, bootstrap current.
 - Auxiliary Heating: ECRH, Ohmic heating, Neutral beam injection.

Course Outline



- **Inertial confinement fusion (ICF)**
 - **Plasma frequency and critical density**
 - **Direct- and indirect- drive**
 - **Laser generated pressure (Inverse bremsstrahlung and Ablation pressure)**
 - **Burning fraction, why compressing a capsule?**
 - **Implosion dynamics**
 - **Shock (Compression with different adiabat)**
 - **Laser pulse shape**
 - **Rocket model, shell velocity**
 - **Laser-plasma interaction (Stimulated Raman Scattering, SRS; Stimulated Brillouin Scattering, SBS; Two-plasmon decay)**
 - **Instabilities (Rayleigh-taylor instability, Kelvin-Helmholtz instability, Richtmeyer-Meshkov instability)**

Course Outline



- **Innovation Fusion scheme**
- **Status of fusion research in Taiwan**
 - **Formosa Integrated Research Spherical Tokamak (FIRST)**

References



- **The physics of inertial fusion, by Stefano Atzeni and Jürgen Meyer-Ter-Vehn**
- **Nuclear fusion, by Edward Morse**
- **Ideal magnetohydrodynamics, by Jeffrey P. Freidberg**
- **Introduction to plasma theory, by Dwight R. Nicholson**
- **Introduction to plasma physics and controlled fusion, by Francis F. Chen**
- **Principles of plasma physics for engineers and scientists, by Umran S. Inan and Marek Golkowski**
- **Introduction to plasma physics, by Gurnett and Bhattacharjee**
- **The physics of plasma, by T. J. M. Boyd and J. J. Sanderson**
- **Principles of plasma physics, by Krall and Trivelpiece**
- **NRL Plasma Formulary, Naval Research Laboratory, 2013 by J. D. Huba**

Grading



- Homework 20 % (Particle trajectories in a Tokamak; Equilibrium state calculation. **Programing is needed!**)
- Midterm 40 % @ 5/7 (Potential)
- Final exam 40 % @ 6/11 (Potential)
- After final exam:
 - 6/18 Introduction to Formosa Integrated Research Spherical Tokamak (FIRST), First Tokamak being developed in Taiwan
 - 6/25 Q&A

• No class: 3/19

Course Outline



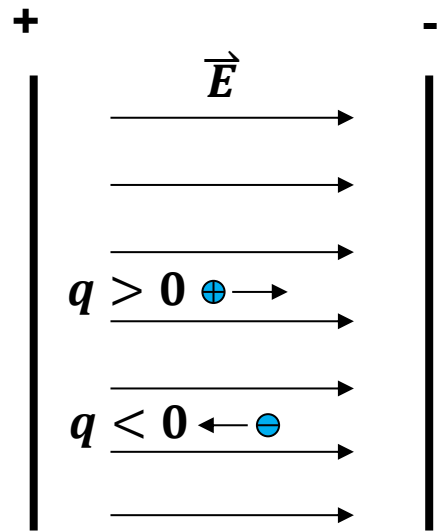
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- **Introduction to nuclear fusion**
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Charged particles are accelerated due to Lorentz force under electromagnetic fields

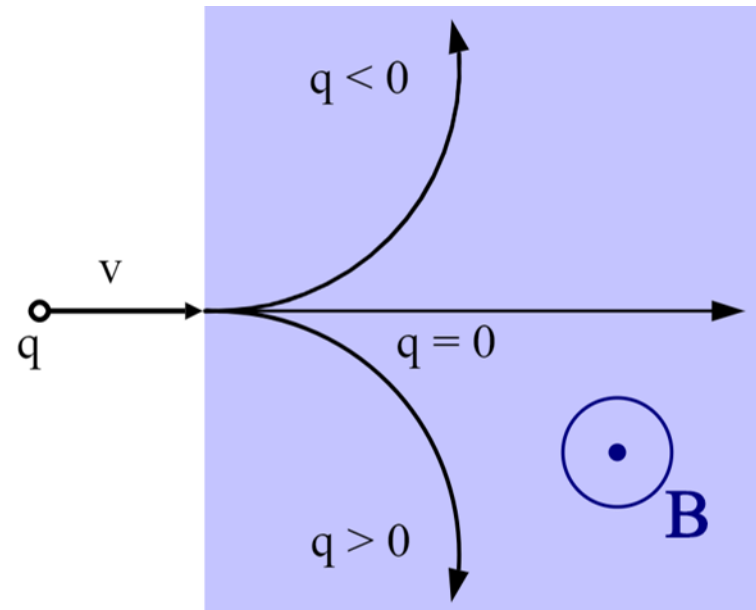


- Lorentz force: $\vec{F} = m \vec{a} = q \vec{E} + q \vec{v} \times \vec{B}$ $m \frac{d\vec{v}}{dt} = q \vec{E} + q \vec{v} \times \vec{B}$

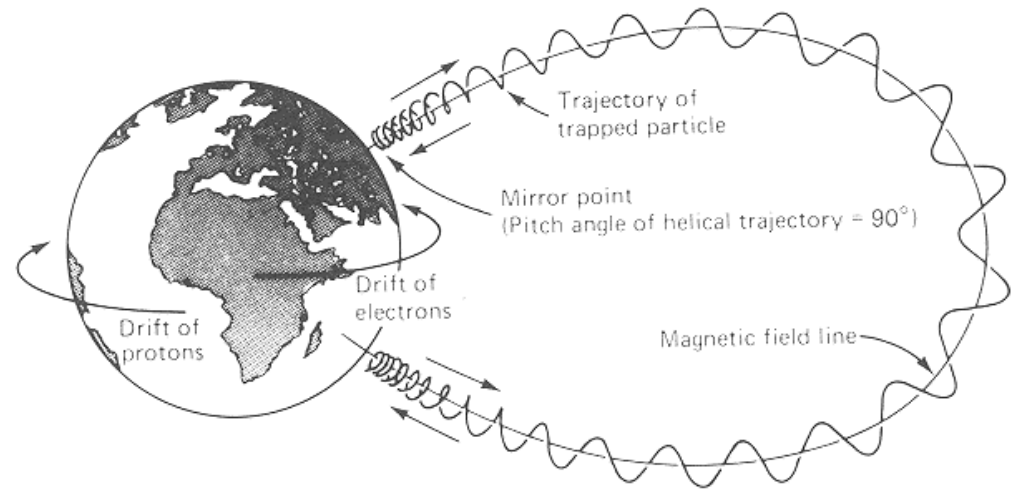
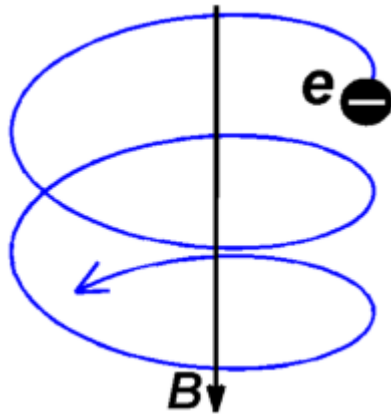
- Force under electric fields



- Force under magnetic fields



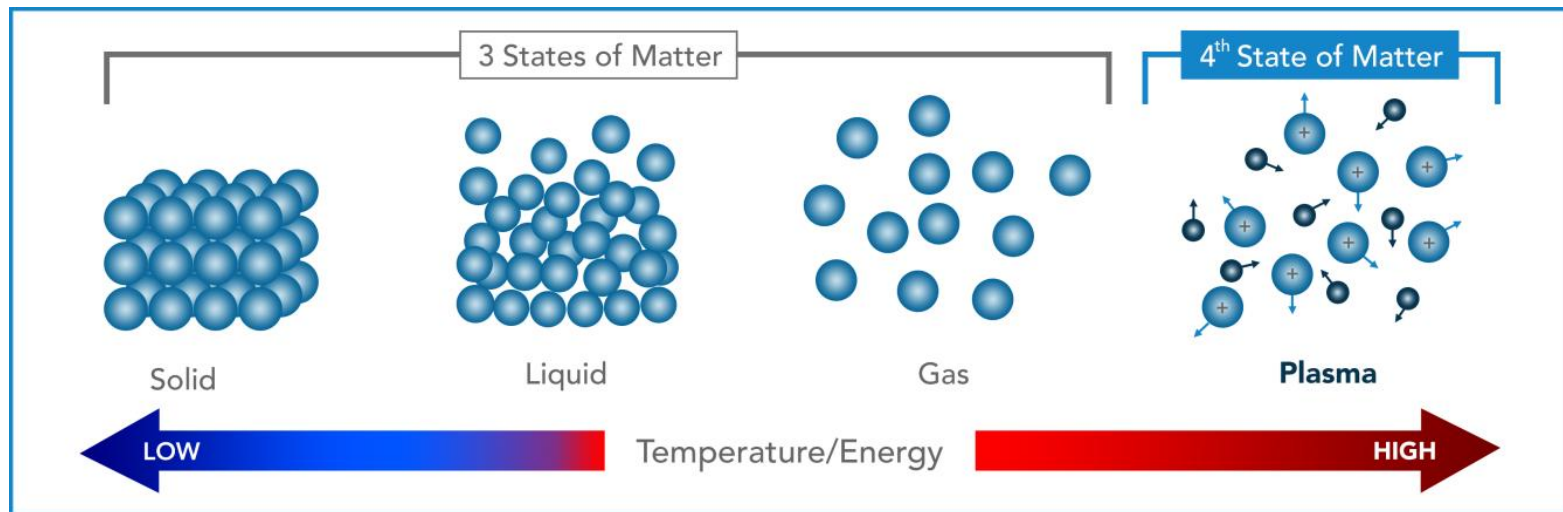
Charged particles gyro around magnetic field lines



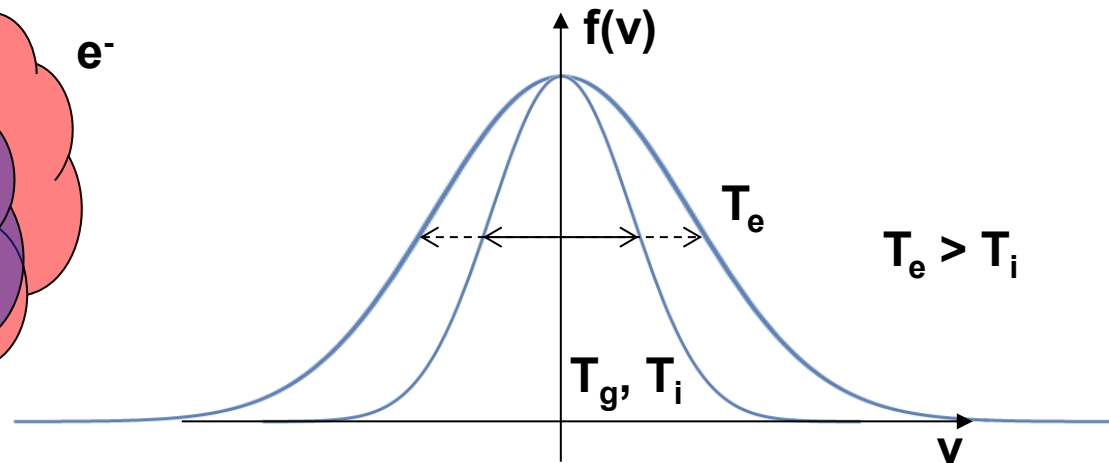
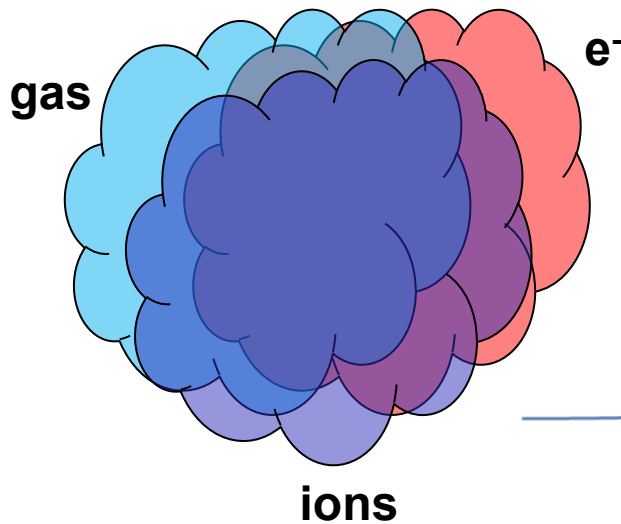
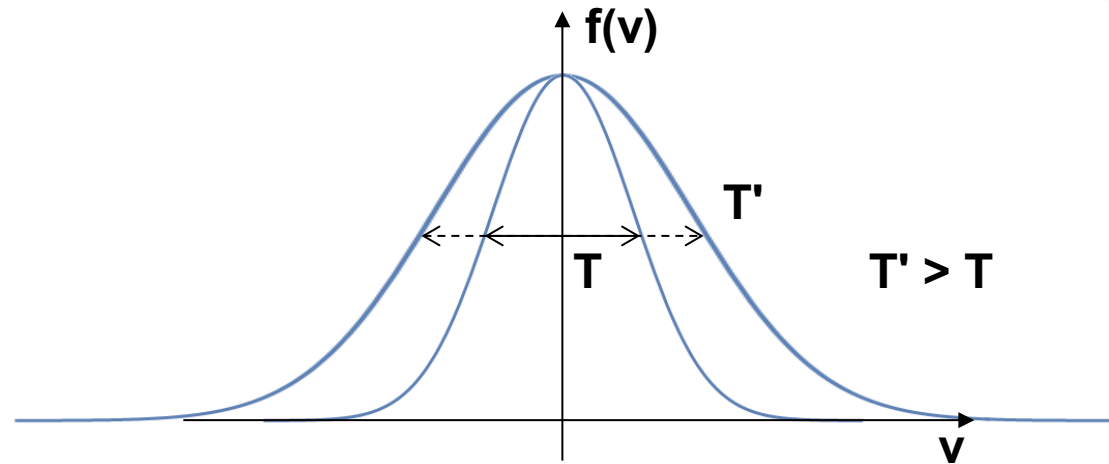
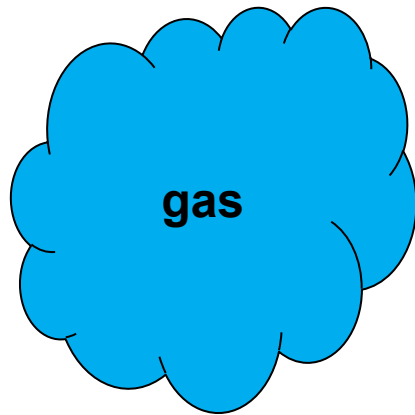
http://www.ipp.cas.cz/vedecka_struktura_uvp/tokamak/tokamak_compass/diagnostics/mikrovlne-diagnostiky/ece-ebw-radiometr.html

<http://www.ssg.sr.unh.edu/tof/Smart/Students/lees/periods.html>

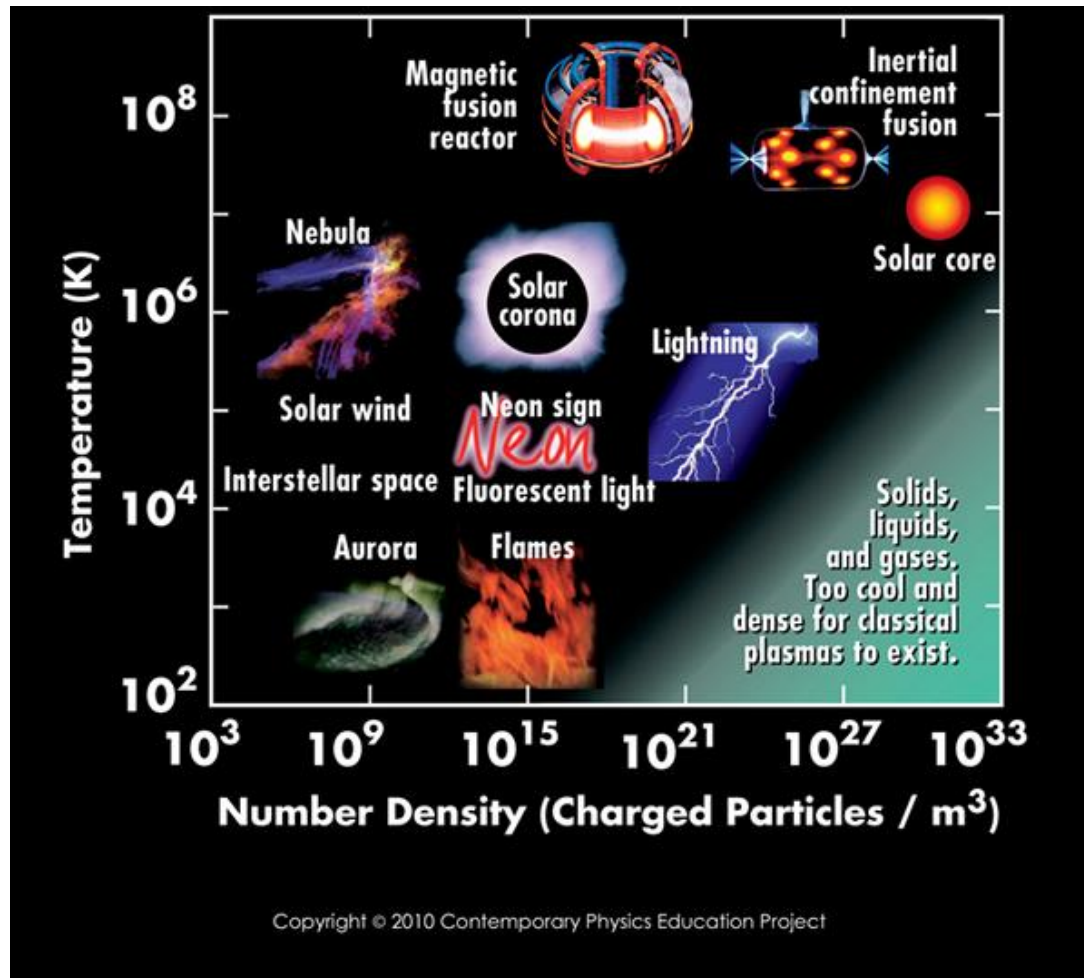
Plasma is the 4th state of matter



In plasma, there are ions, electrons, and neutral gas



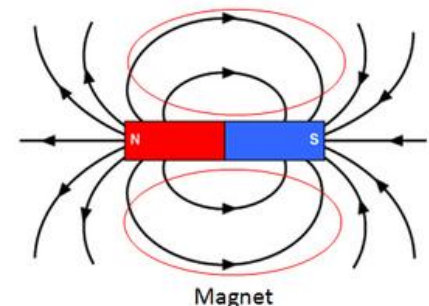
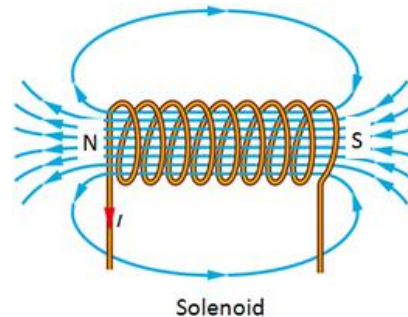
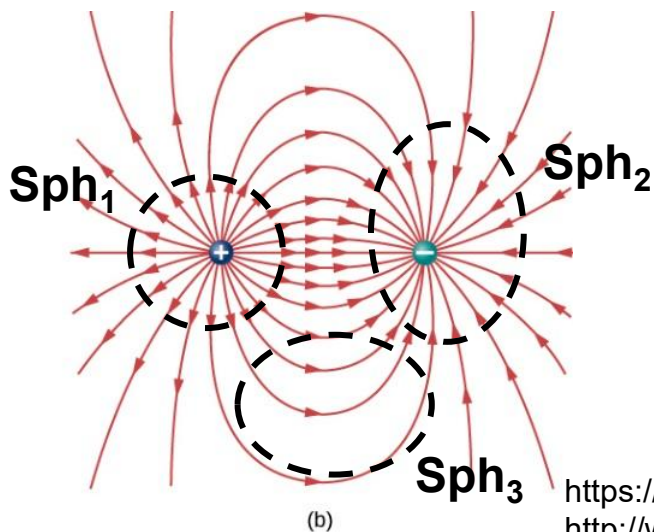
Nuclear fusion occurs at a very high temperature region



Maxwell's equations

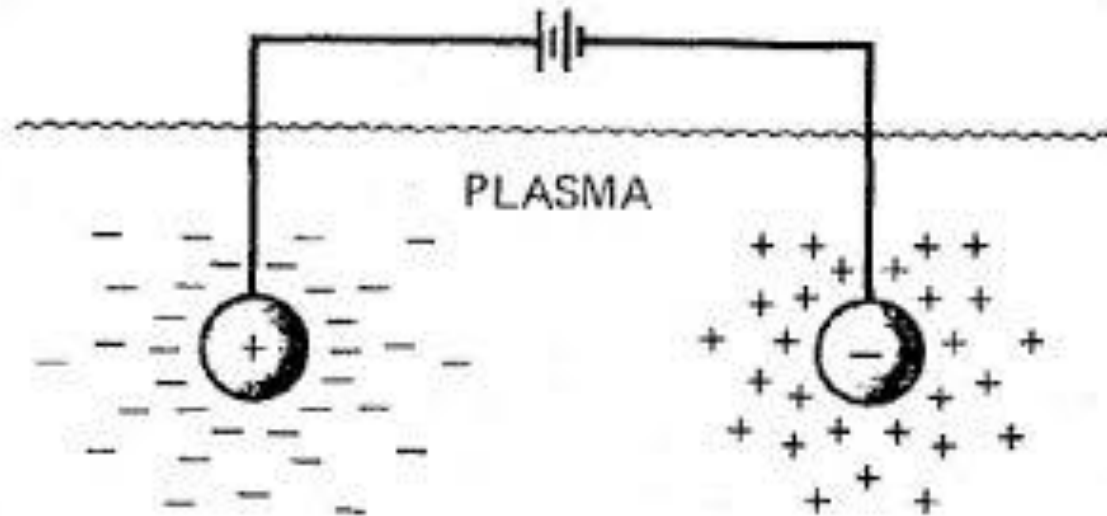


- **Gauss's law:** $\nabla \cdot \vec{E} = \frac{\rho_q}{\epsilon_0}$ $\nabla \times \vec{E} = 0$ $\vec{E} = -\nabla\phi$ **Potential** • **Poisson's eq.:**
- **No name:** $\nabla \cdot \vec{B} = 0$ $\vec{B} = -\nabla \times \vec{A}$ **Vector Potential** • $\nabla^2 \phi = -\frac{\rho_q}{\epsilon_0}$
- **Faraday's law:** $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ • **Laplace's eq.:**
- **Ampère's law:** $\nabla \times \vec{B} = \mu_0 \vec{j} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$ **Displacement current** $\nabla^2 \phi = 0$
- **Lorentz force:** $\vec{F} = m \vec{a} = q \vec{E} + q \vec{v} \times \vec{B}$

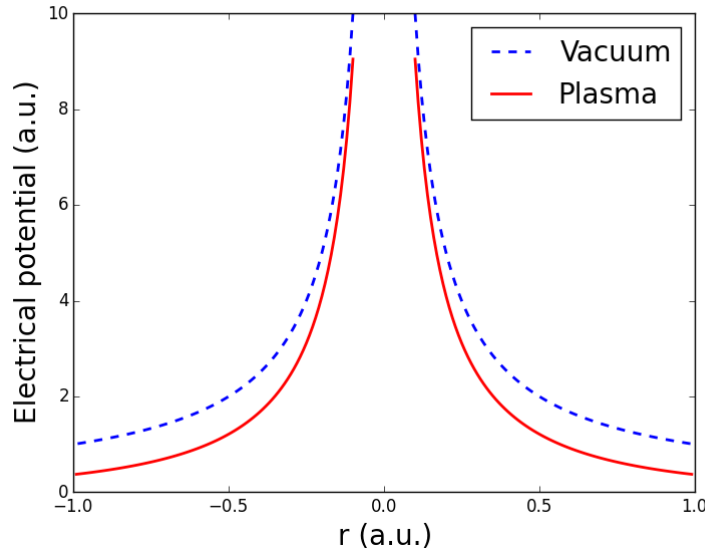


<https://pressbooks.online.ucf.edu/osuniversityphysics2/chapter/electric-field-lines/>
<http://www.excelatphysics.com/magnetic-field-pattern.html>

A test ion in the plasma gathers a shielding cloud that tends to cancel its own charge



Debye shielding is a phenomenon such that the potential due to a test charge in a plasma falls off much faster than in vacuum



$$\nabla^2 \phi = -\frac{\rho_q}{\epsilon_0}$$

- Vacuum potential:

$$\phi = \frac{\phi_0}{r}$$

- Poisson's equation:

$$\nabla^2 \phi = 4\pi e(n_e - n_i) - 4\pi q_T \delta(\vec{r})$$

- Density profile:

$$n_e = n_0 \exp\left(\frac{e\phi}{KT_e}\right), \quad n_i = n_0$$

- For $\vec{r} \neq 0$ and assuming $\frac{e\phi}{T_e} \ll 1$

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\phi}{dr} \right) = \frac{4\pi n_0 e^2}{KT_e} \phi$$

$$\phi = \frac{\phi_0}{r} \exp\left(-\frac{r}{\lambda_D}\right) \quad \lambda_D \approx \left(\frac{KT_e}{4\pi n e^2}\right)^{1/2}$$

Magnetohydrodynamics description of plasma



- **Continuity eq:** $\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{v}) = 0$
- **Momentum eq:** $\rho_m \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = \rho_q \vec{E} + \vec{j} \times \vec{B} - \nabla \cdot \vec{P}$
- **Ohm's law:** $\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B})$
- **Equation of state:** $\frac{d}{dt} (P \rho_m^{-\gamma}) = 0$

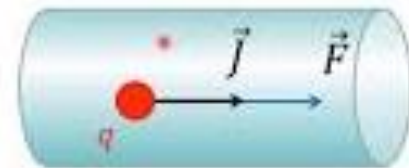
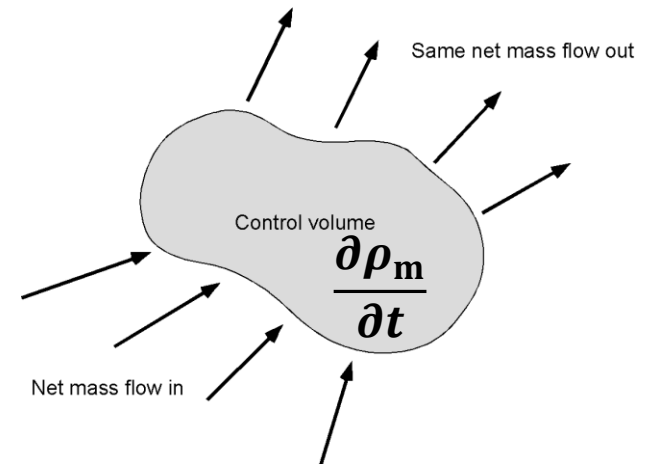
- **Maxwell's eqs:**

$$\nabla \cdot \vec{E} = \frac{\rho_q}{\epsilon_0}$$

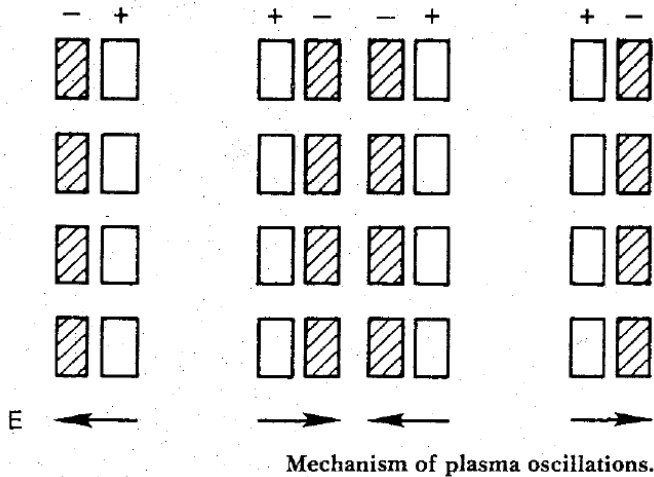
$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$



Electron plasma frequency is the characteristic frequency such that electrons oscillate around their equilibrium positions



- Assumption:

$$\vec{\nabla} \equiv \hat{x} \frac{\partial}{\partial x}, \quad \vec{E} = \hat{x} E, \quad \vec{v}_e = \hat{x} v,$$

$$\vec{\nabla} \times \vec{E} = 0, \quad \vec{E} = -\vec{\nabla} \phi$$

- Continuity and momentum equation for electron:

$$\frac{\partial n_e}{\partial t} + \vec{\nabla} \cdot (n_e \vec{v}_e) = 0$$

$$m_e n_e \left[\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e \cdot \vec{\nabla}) \vec{v}_e \right] = -e n_e \vec{E}$$

- Gauss' law:

$$\frac{\partial E}{\partial x} = 4\pi e (n_i - n_e)$$

Electron plasma frequency is obtained by linearizing the hydrodynamic equations



- The oscillation is assumed to be small:

$$n_e = n_0 + n_1, \vec{E} = \vec{E}_0 + \vec{E}_1, v_e = v_0 + v_1$$

where

$$\frac{\partial n_0}{\partial x} = v_0 = \vec{E}_0 = 0$$

$$\frac{\partial n_0}{\partial t} = \frac{\partial v_0}{\partial t} = \frac{\partial \vec{E}_0}{\partial t} = 0$$

- Linearization:

$$m_e \frac{\partial v_1}{\partial t} = -eE_1$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} = 0$$

$$\frac{\partial E_1}{\partial x} = -4\pi en_1$$

- Plane wave solution:

$$\eta_1 = \hat{\eta} \exp[i(kx - \omega t)]$$

$$\eta_1 = v_1, n_1, E_1$$

- Substitute into the previous equations:

$$-im_e \omega v_1 = -eE_1$$

$$-i\omega n_1 = -n_0 ikv_1$$

$$ikE_1 = -4\pi en_1$$

- Electron plasma frequency is obtained by eliminating n_1 and E_1 :

$$\omega_{pe} \equiv \omega = \left(\frac{4\pi n_e e^2}{m_e} \right)^{1/2}$$

Plasma β is the ratio between hydro pressure and magnetic pressure



- Momentum equation in Magnetohydrodynamics (MHD) approach:

$$\rho_m \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = \vec{j} \times \vec{B} - \nabla \cdot \vec{P} \Rightarrow \vec{j} \times \vec{B} - \nabla P$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \quad \text{w/ low freq. } (\omega \ll \omega_{pe})$$

$$\vec{j} \times \vec{B} = \frac{1}{\mu_0} (\vec{\nabla} \times \vec{B}) \times \vec{B} = \frac{1}{\mu_0} \left[(\vec{B} \cdot \vec{\nabla}) \vec{B} - \frac{1}{2} \vec{\nabla} B^2 \right]$$

$$\rho_m \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = -\nabla \left(P + \frac{B^2}{2\mu_0} \right) + \frac{1}{\mu_0} (\vec{B} \cdot \vec{\nabla}) \vec{B}$$

- Magnetic pressure:

$$\frac{B^2}{2\mu_0}$$

- Magnetic tension:

$$\frac{1}{\mu_0} (\vec{B} \cdot \vec{\nabla}) \vec{B}$$

$$\beta \equiv \frac{P}{B^2/2\mu_0}$$

Magnetohydrodynamics description of plasma w/ low-freq. and long-wavelength approximation



- Continuity eq: $\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{v}) = 0$ w/ long wavelength ($\lambda \gg \lambda_d$)
- Momentum eq: $\rho_m \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = \cancel{\rho_q \vec{E}} + \vec{j} \times \vec{B} - \nabla \cdot \vec{P}$
- Ohm's law: $\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B})$
- Equation of state: $\frac{d}{dt} (P \rho_m^{-\gamma}) = 0$

- Maxwell's eqs:

$$\nabla \cdot \vec{E} = \frac{\rho_q}{\epsilon_0} \approx 0 \quad \text{w/ long wavelength (} \lambda \gg \lambda_d \text{)} \Rightarrow \text{quasi neutral}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \cancel{\epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}}$$

w/ low freq. ($\omega \ll \omega_{pe}$)

Magnetohydrodynamics description of plasma w/ low-freq. and long-wavelength approximation



- Continuity eq: $\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{v}) = 0$
- Momentum eq: $\rho_m \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = \vec{j} \times \vec{B} - \nabla \cdot \vec{P}$
- Ohm's law: $\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B})$
- Equation of state: $\frac{d}{dt} (P \rho_m^{-\gamma}) = 0$
- Maxwell's eqs:

$$\begin{aligned} \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} &= \mu_0 \vec{j} \quad \Rightarrow \quad \nabla \cdot \vec{j} = 0 \end{aligned}$$

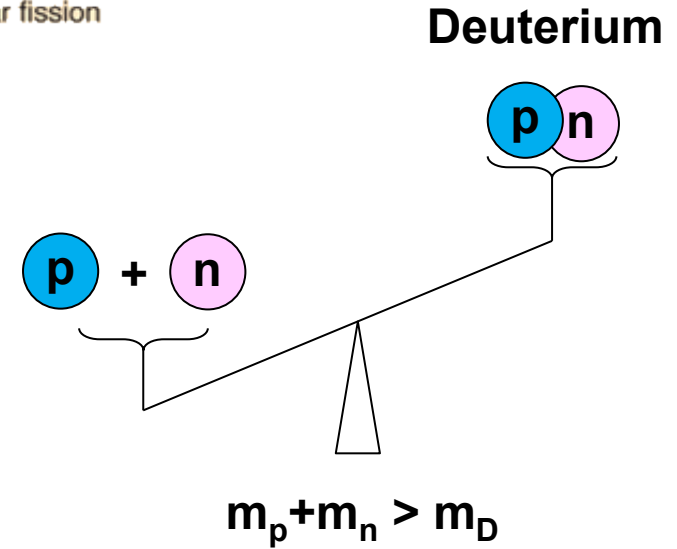
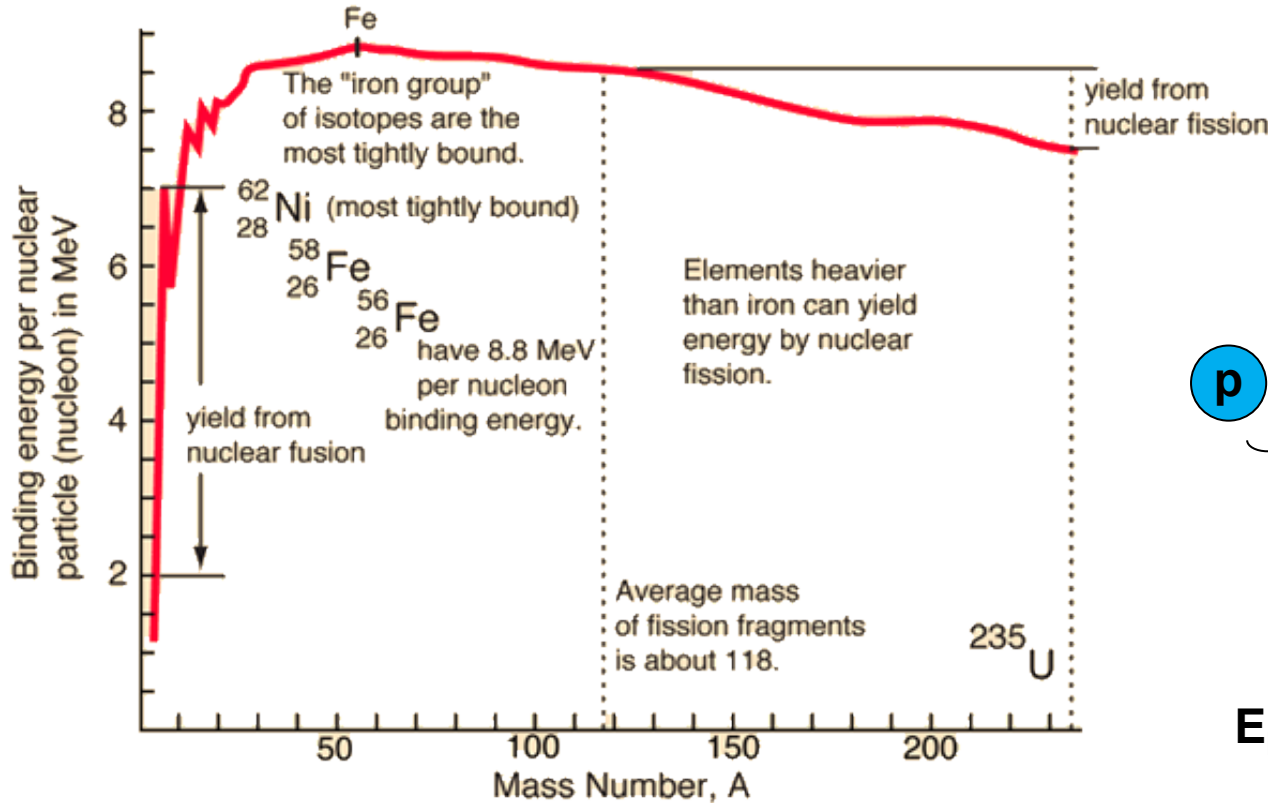
- Force balance condition: $0 = \vec{j} \times \vec{B} - \nabla \cdot \vec{P}$

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The “iron group” of isotopes are the most tightly bound



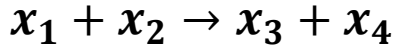
$$E_{\text{binding}} = [(m_p + m_n) - m_D]c^2$$

$$Q = \left(\sum_i m_i - \sum_f m_f \right) c^2$$

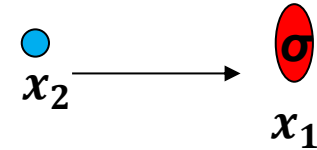
$$\Delta m = z m_p + (A - z) m_n - m$$

- Binding energy: $B = \Delta m c^2$
- Output energy: $Q = \sum_f B_f - \sum_i B_i$

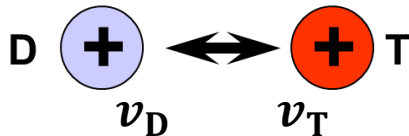
Cross section measures the probability per pair of particles for the occurrence of the reaction



- The hard sphere cross section:
 $\sigma \approx \pi R^2$ where $R \approx 5 \times 10^{-15}$ m is the nuclear radius, i.e.,
 $\sigma = 0.8 \times 10^{-28} \text{ m}^2 \approx 1 \text{ barn}$. (barn $\equiv 10^{-28} \text{ m}^2$)



- Classical cross section:

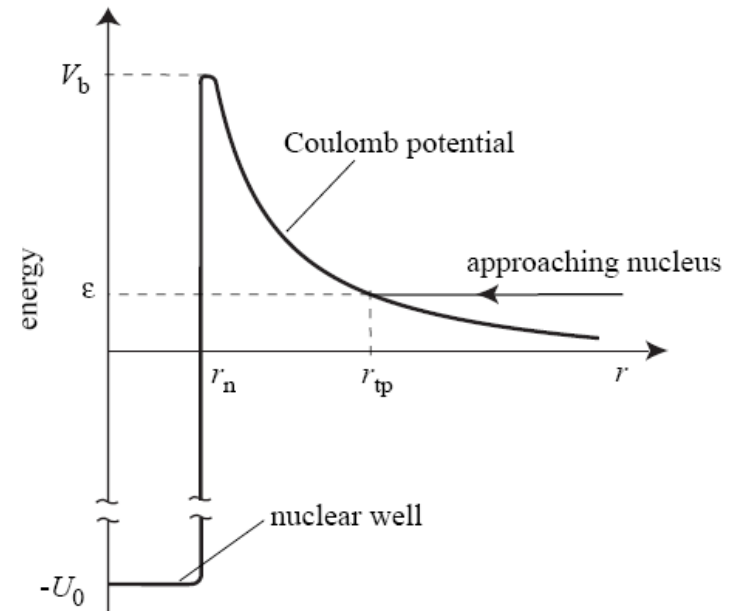


$$\frac{m_D}{2} v_D^2 + \frac{m_T}{2} v_T^2 \geq \frac{e^2}{4\pi\epsilon_0 R}$$

- Let $v = |\vec{v}_D - \vec{v}_T|$

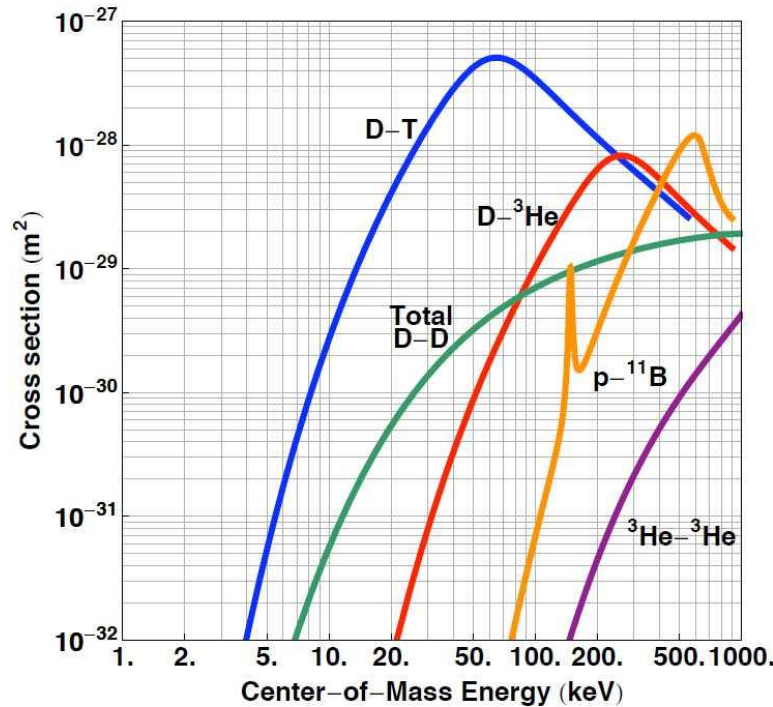
$$v_D = \frac{m_T}{m_D + m_T} v \quad v_T = \frac{m_D}{m_D + m_T} v$$

$$\frac{m_T}{2} v^2 \geq \frac{e^2}{4\pi\epsilon_0 R} \quad m_T = \frac{m_D m_T}{m_D + m_T}$$

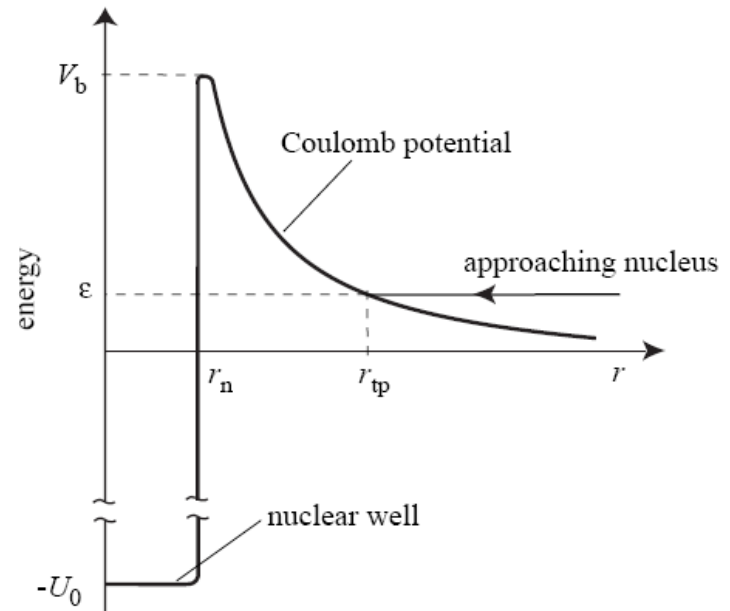
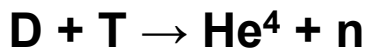
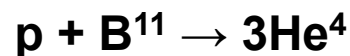
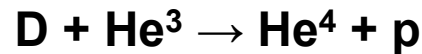
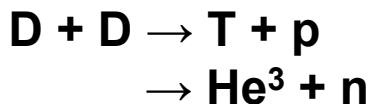


- Classical kinetic energy required for fusion is
 $K_{c.m.} > 288 \text{ keV} \quad !!!$

Cross section of fusion reaction is much larger than the classical approach



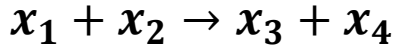
- Classical kinetic energy required for fusion is $K_{c.m.} > 288 \text{ keV} !!!$
- DT cross section has a peak of ~ 5 barns at $E_{neD} = 120 \text{ keV}$.
- $\sigma_{DT} \approx 100\sigma_{DD} @ 20 \text{ keV}$.



<https://i.stack.imgur.com/wXQD5.jpg>

Santarius, J. F., "Fusion Space Propulsion – A Shorter Time Frame Than You Think", JANNAF, Monterey, 5-8 December 2005.

Flux of incident particles reduces after collisions

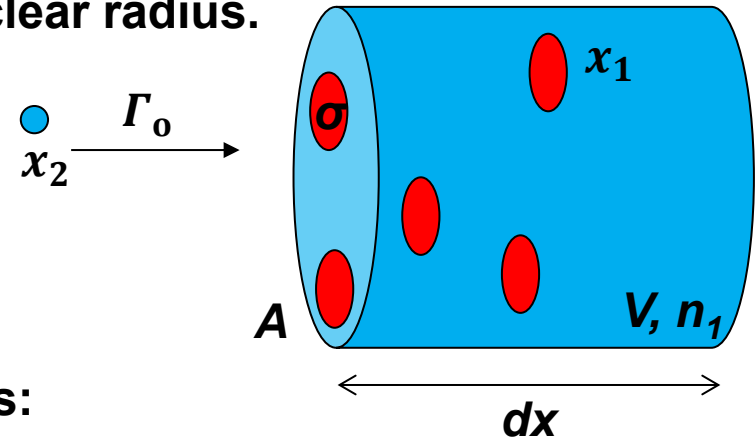


- **Cross section:** $\sigma \approx \pi R^2$ where R is the nuclear radius.

$$V = A dx$$

$$N_1 = n_1 V = n_1 A dx$$

$$A_{\text{Target}} = N_1 \sigma = \sigma n_1 A dx$$



- **Fraction of total area blocked by targets is:**

$$dF = \frac{\sigma N_1}{A} = \frac{\sigma n_1 A dx}{A} = \sigma n_1 dx$$

$$\frac{dF}{dx} = \sigma n_1$$

- **Flux of incident particles (x_2) is Γ_0**

$$-d\Gamma = dF\Gamma = \sigma n_1 \Gamma dx$$

$$\frac{-d\Gamma}{\Gamma} = \sigma n_1 dx \quad \Gamma = \Gamma_0 \exp\left(-\frac{x}{\lambda_{\text{mfp}}}\right)$$

- **Mean free path:**

$$\lambda_{\text{mfp}} = \frac{1}{n_1 \sigma}$$

- **Collision frequency:**

$$\nu = \frac{1}{\tau}, \tau = \frac{\lambda_{\text{mfp}}}{v} = \frac{1}{n_1 \sigma v}$$

Reactions happen when collision happen



- Reaction rate R_{12} : number of fusion collisions/reactions per unit volume per unit time.
- In the time $dt=dx/v$, $n_2 A dx$ incident particles will pass through the target volume.
- The number having a collisions is: $dF(n_2 A dx)$
- The volumetric reaction rate R_{12} , i.e., the number of reaction per unit time and per unit volume is:

$$R_{12} = \frac{dF(n_2 A dx)}{A dx dt} = \sigma n_1 n_2 \frac{dx}{dt} = n_1 n_2 \sigma v \quad \left(\frac{dF}{dx} = \sigma n_1 \right)$$

- The fusion power density (W/m^3) is: $S_f = E_f n_1 n_2 \sigma v$ (W/m^3)
- For DT fusion, $E_f=17.6$ MeV.
- For a particle population with a distribution function in velocity space:

$$n = \int d\vec{v} f(\vec{r}, \vec{v}, t)$$

- Therefore,

$$n_1 \rightarrow d\vec{v}_1 f_1(\vec{r}, \vec{v}_1, t) \quad n_2 \rightarrow d\vec{v}_2 f_2(\vec{r}, \vec{v}_2, t) \quad v \rightarrow |\vec{v}_1 - \vec{v}_2|$$

$$R_{12} = \int f_1(\vec{v}_1) f_2(\vec{v}_2) \sigma \left(|\vec{v}_1 - \vec{v}_2| \right) |\vec{v}_1 - \vec{v}_2| d\vec{v}_1 d\vec{v}_2$$

The fusion power density needs to consider the distribution function of particles



$$R_{12} = \int f_1(\vec{v}_1) f_2(\vec{v}_2) \sigma(|\vec{v}_1 - \vec{v}_2|) |\vec{v}_1 - \vec{v}_2| d\vec{v}_1 d\vec{v}_2 = n_1 n_2 \langle \sigma v \rangle$$

$$\begin{aligned} \langle \sigma v \rangle &\equiv \frac{\int f_1(\vec{v}_1) f_2(\vec{v}_2) \sigma(|\vec{v}_1 - \vec{v}_2|) |\vec{v}_1 - \vec{v}_2| d\vec{v}_1 d\vec{v}_2}{\int f_1(\vec{v}_1) f_2(\vec{v}_2) d\vec{v}_1 d\vec{v}_2} \\ &= \frac{\int f_1(\vec{v}_1) f_2(\vec{v}_2) \sigma(|\vec{v}_1 - \vec{v}_2|) |\vec{v}_1 - \vec{v}_2| d\vec{v}_1 d\vec{v}_2}{n_1 n_2} \end{aligned}$$

- The fusion power density (W/m^3) is:

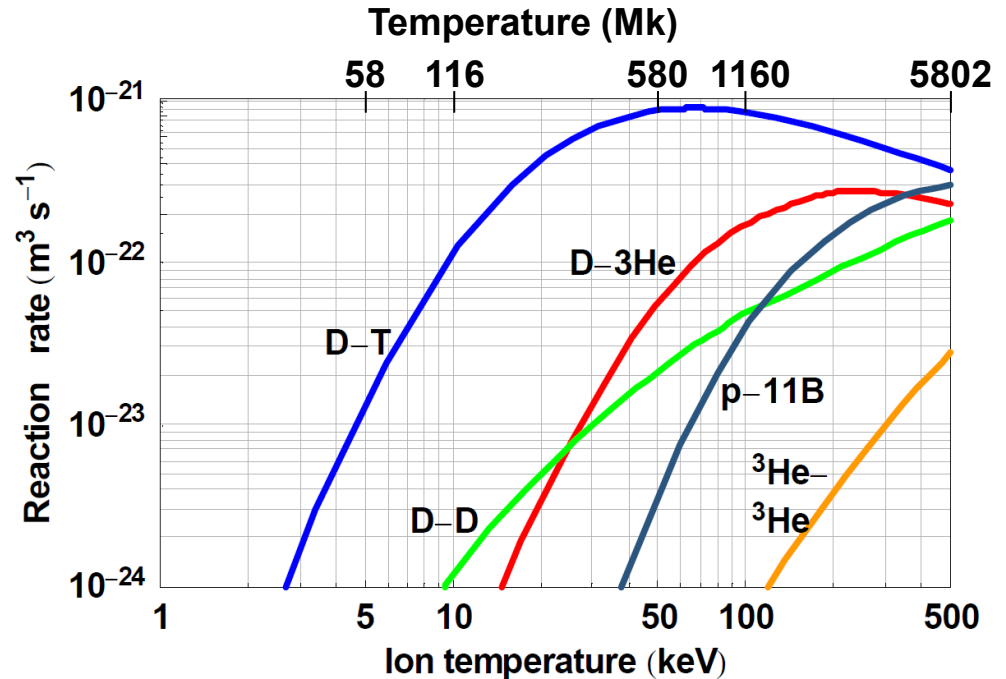
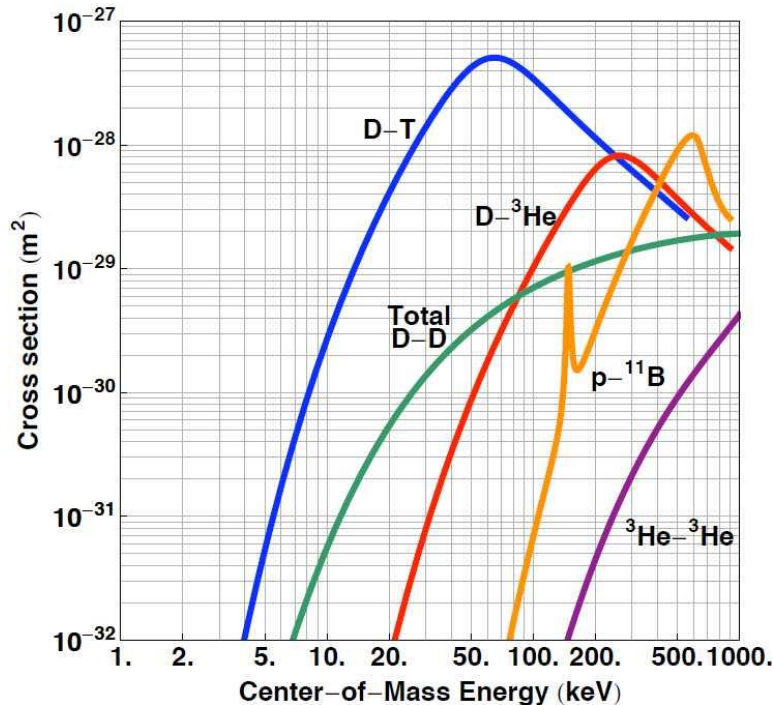
$$S_f = E_f n_1 n_2 \langle \sigma v \rangle (\text{W/m}^3)$$

- Optimum concentration of DT fusion is 50-50.

$$S_f = E_f n_D n_T \langle \sigma v \rangle \quad n_D = k n_0 \quad n_T = (1 - k) n_0$$

$$S_f = E_f k(1 - k) n_0^2 \text{ which peak at } k = 0.5 .$$

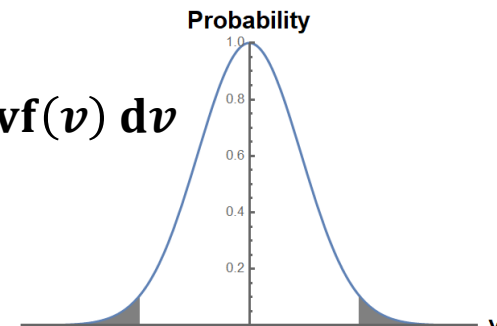
Fusion doesn't come easy



- The DT fusion reactivity is maximum at $T \approx 64$ keV
- @ $T = 10$ keV, $\langle \sigma v \rangle_{DT} \approx 100 \langle \sigma v \rangle_{DD}$

- Reaction rate:

$$\langle \sigma v \rangle = \int \sigma(v) v f(v) dv$$



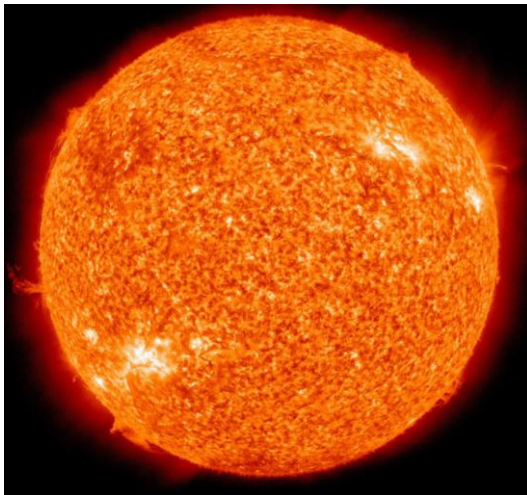
<https://i.stack.imgur.com/wXQD5.jpg>

Santarius, J. F., "Fusion Space Propulsion – A Shorter Time Frame Than You Think", JANNAF, Monterey, 5-8 December 2005.

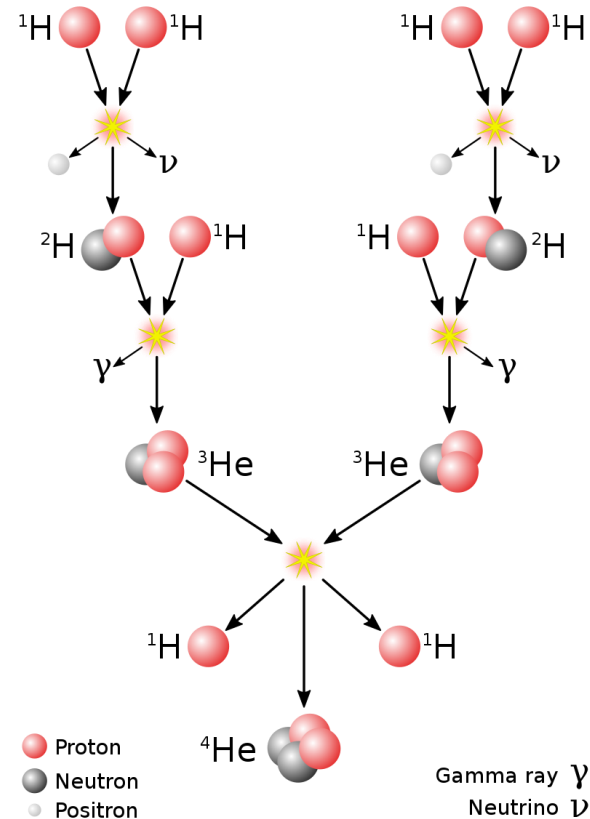
Fusion in the sun provides the energy



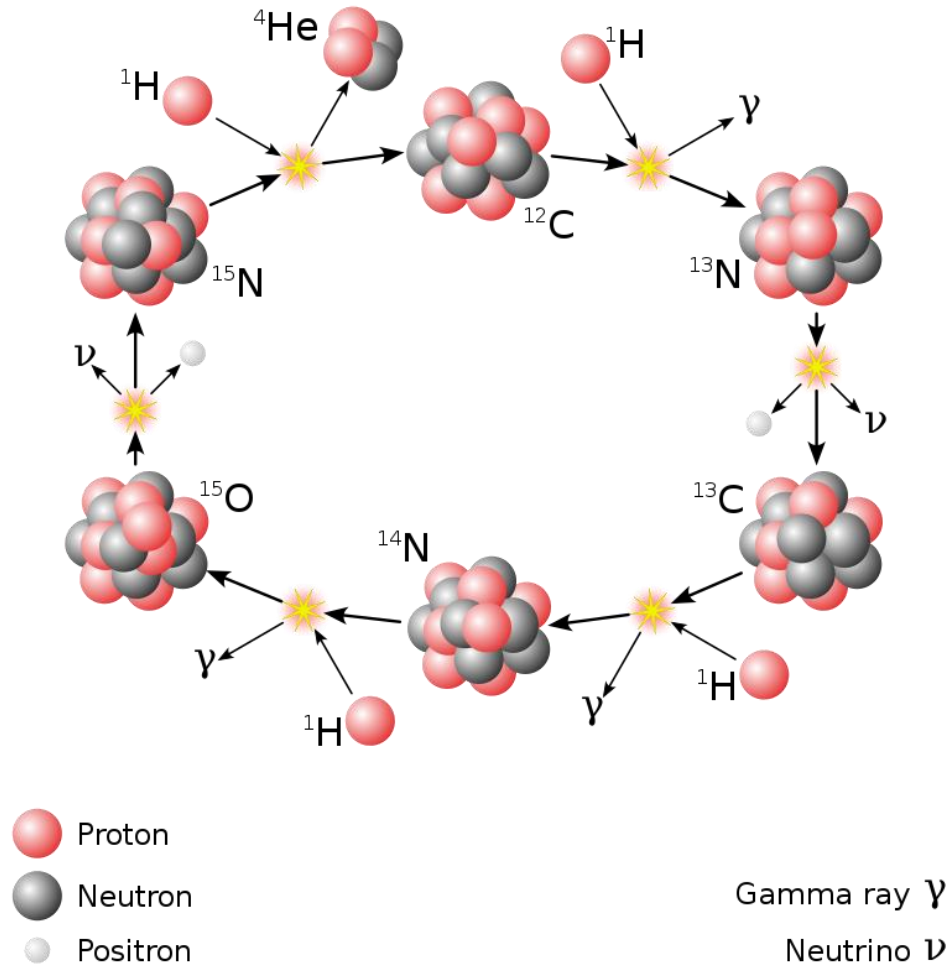
- Proton-proton chain in sun or smaller



- Particles are confined by the gravity.



In heavy sun, the fusion reaction is the CNO cycle



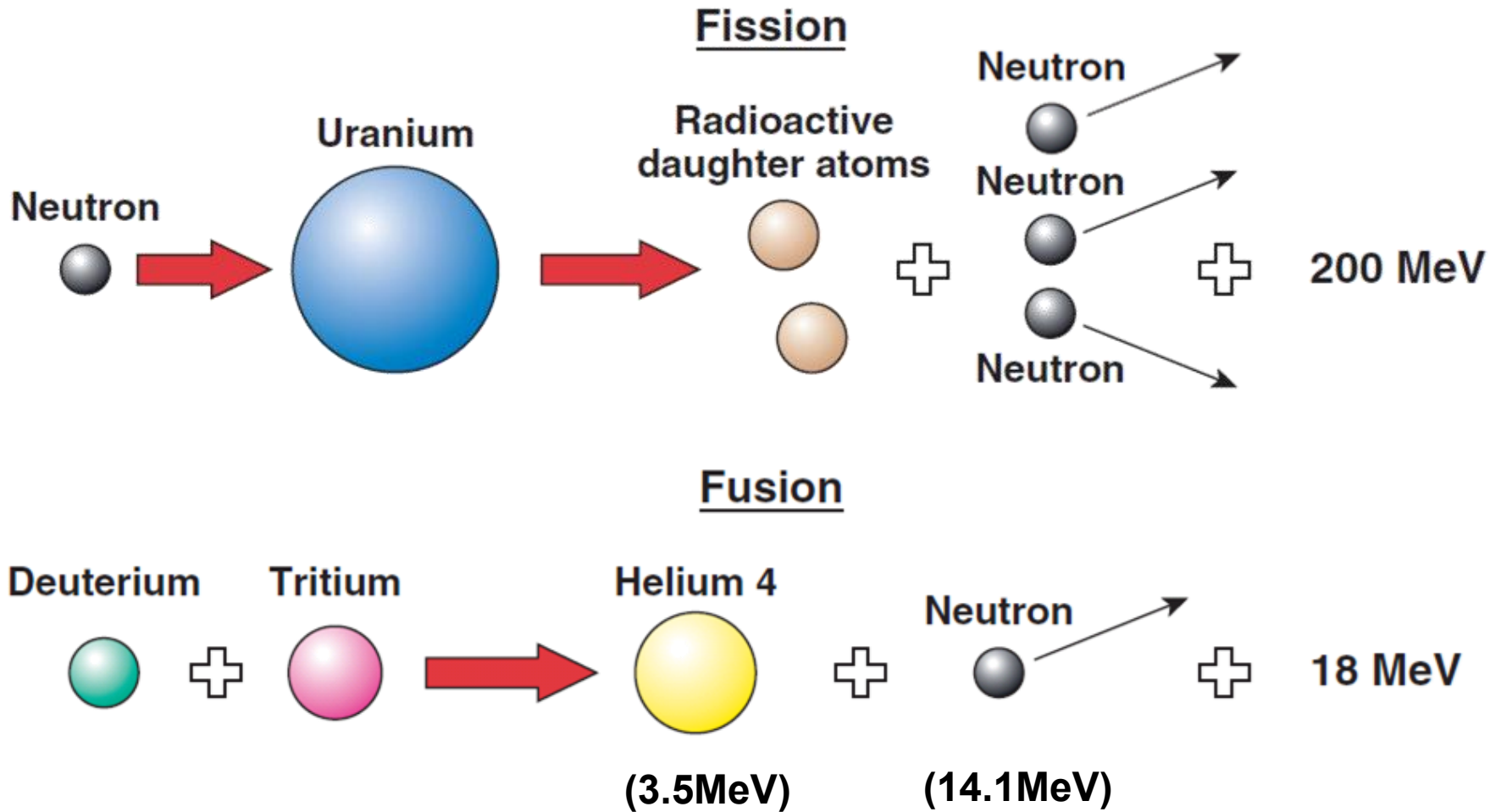
The cross section of proton-proton chain is much smaller than D T fusion



Reaction	$\sigma_{10 \text{ keV}}$ (barn)	$\sigma_{100 \text{ keV}}$ (barn)	σ_{max} (barn)	ϵ_{max} (keV)
$\text{D}+\text{T}\rightarrow\alpha+\text{n}$	2.72×10^{-2}	3.43	5.0	64
$\text{D}+\text{D}\rightarrow\text{T}+\text{p}$	2.81×10^{-4}	3.3×10^{-2}	0.06	1250
$\text{D}+\text{D}\rightarrow{}^3\text{He}+\text{n}$	2.78×10^{-4}	3.7×10^{-2}	0.11	1750
$\text{T}+\text{T}\rightarrow\alpha+2\text{n}$	7.90×10^{-4}	3.4×10^{-2}	0.16	1000
$\text{D}+{}^3\text{He}\rightarrow\alpha+\text{p}$	2.2×10^{-7}	0.1	0.9	250
$\text{p}+{}^6\text{Li}\rightarrow\alpha+{}^3\text{He}$	6×10^{-10}	7×10^{-3}	0.22	1500
$\text{p}+{}^{11}\text{B}\rightarrow 3\alpha$	(4.6×10^{-17})	3×10^{-4}	1.2	550
$\text{p}+\text{p}\rightarrow\text{D}+\text{e}^++\text{v}$	(3.6×10^{-26})	(4.4×10^{-25})		
$\text{p}+{}^{12}\text{C}\rightarrow{}^{13}\text{N}+\gamma$	(1.9×10^{-26})	2.0×10^{-10}	1.0×10^4	400
${}^{12}\text{C}+{}^{12}\text{C}$ (all branches)		(5.0×10^{-103})		

- “()” are theoretical values while others are measured values.

Nuclear fusion and fission release energy through energetic neutrons

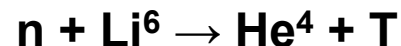


Nuclear fusion provides more energy per atomic mass unit (amu) than nuclear fission

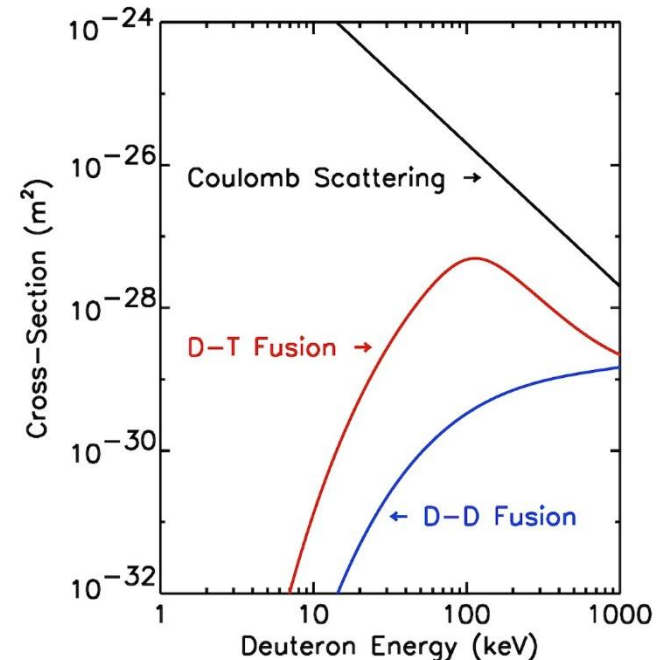
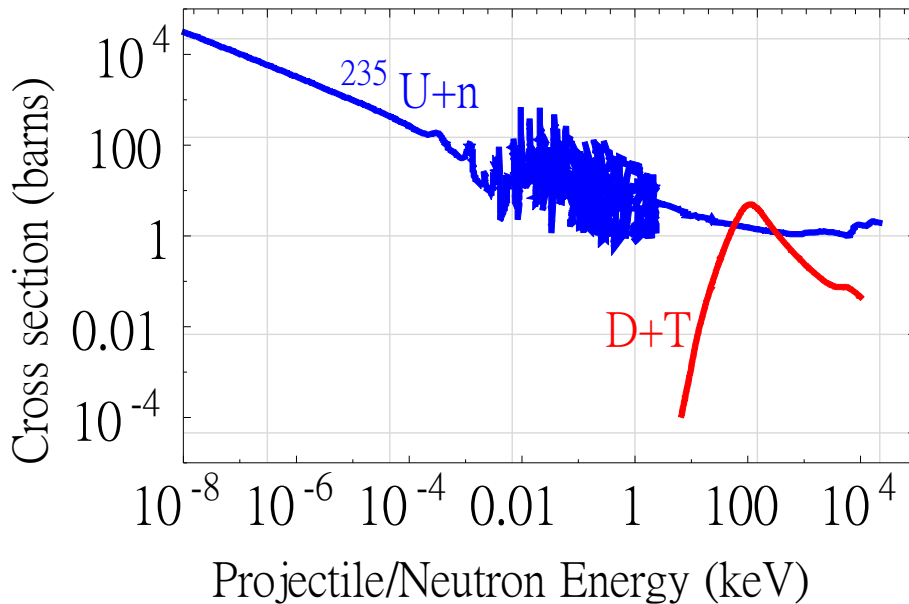
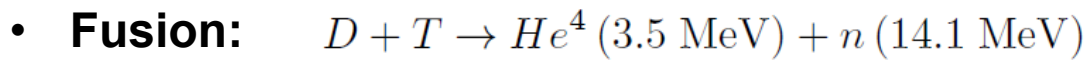
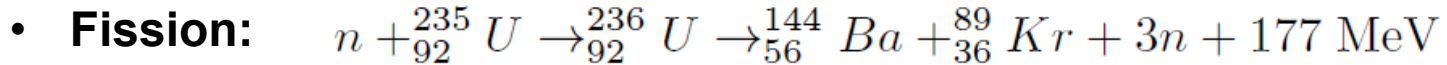


- Fusion of D+T: $\frac{Q}{A} = \frac{17.6\text{MeV}}{(3 + 2)\text{amu}} = 3.5 \frac{\text{MeV}}{\text{amu}}$
- Fission of $^{235}\text{U}+\text{n}$: $\frac{Q}{A} = \frac{200\text{MeV}}{(235 + 1)\text{amu}} = 0.85 \frac{\text{MeV}}{\text{amu}}$

	Half-life (years)
U235	7.04×10^8
U238	4.47×10^9
...	
Tritium	12.3



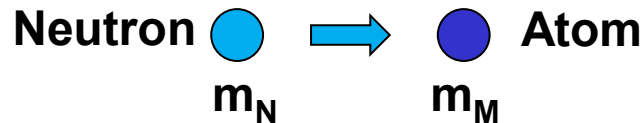
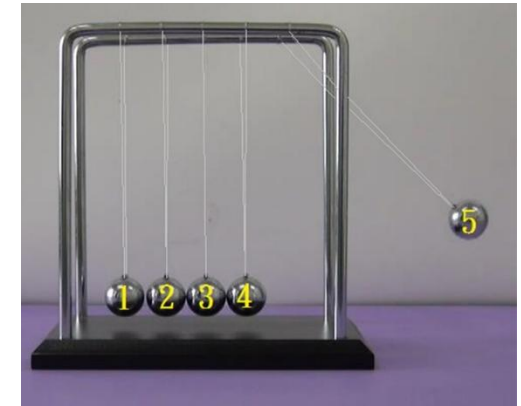
Fusion is much harder than fission



Fast neutrons are slowed down due to the collisions



- A moderator is used to slow down fast neutrons but not to absorb neutrons.
- For $m_M \sim m_N$, the energy decrement is higher. Therefore, H slows down neutron most efficiently.
- However, $H + n \rightarrow D$, i.e., H absorbs neutrons.



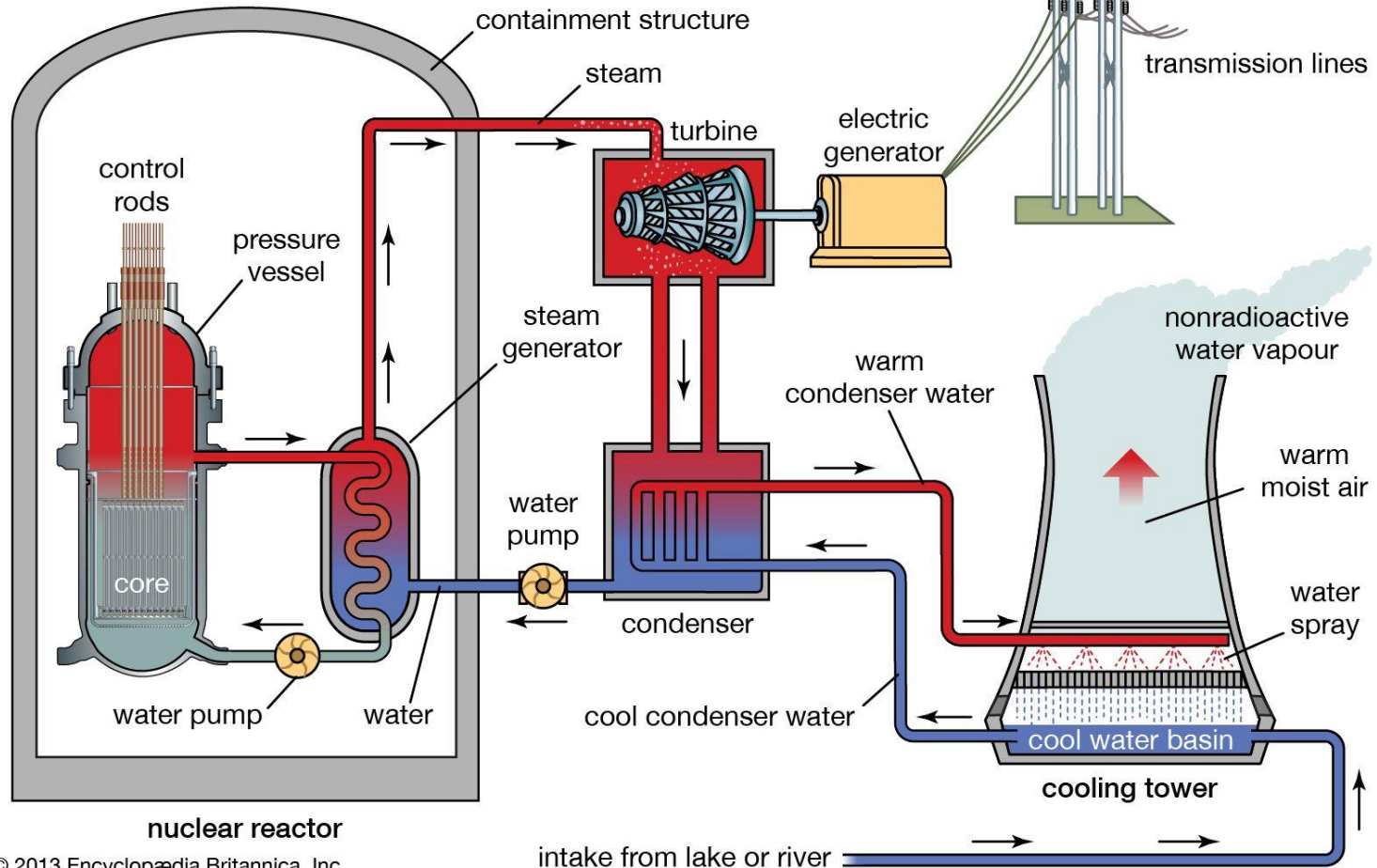
- The best option is the D in the heavy water (D_2O).

	Energy decrement	Neutron scattering cross section (σ_s) (Barns)	Neutron absorption cross section (σ_a) (Barns)
H	1	49 (H_2O)	0.66 (H_2O)
D	0.7261	10.6 (D_2O)	0.0013 (D_2O)
C	0.1589	4.7 (Graphite)	0.0035 (Graphite)

Nuclear power plant



Nuclear power plant



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Comparison between nuclear fission and nuclear fusion

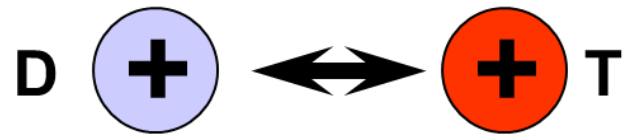


	Nuclear Fission	Nuclear Fusion
Chain reaction	Yes	No
Melt down	Possible	Impossible
Nuclear waste	High radiative	Low radiative / None

A “hot plasma” at 100M °C is needed



- Probability for fusion reactions to occur is low at low temperatures due to the coulomb repulsion force.



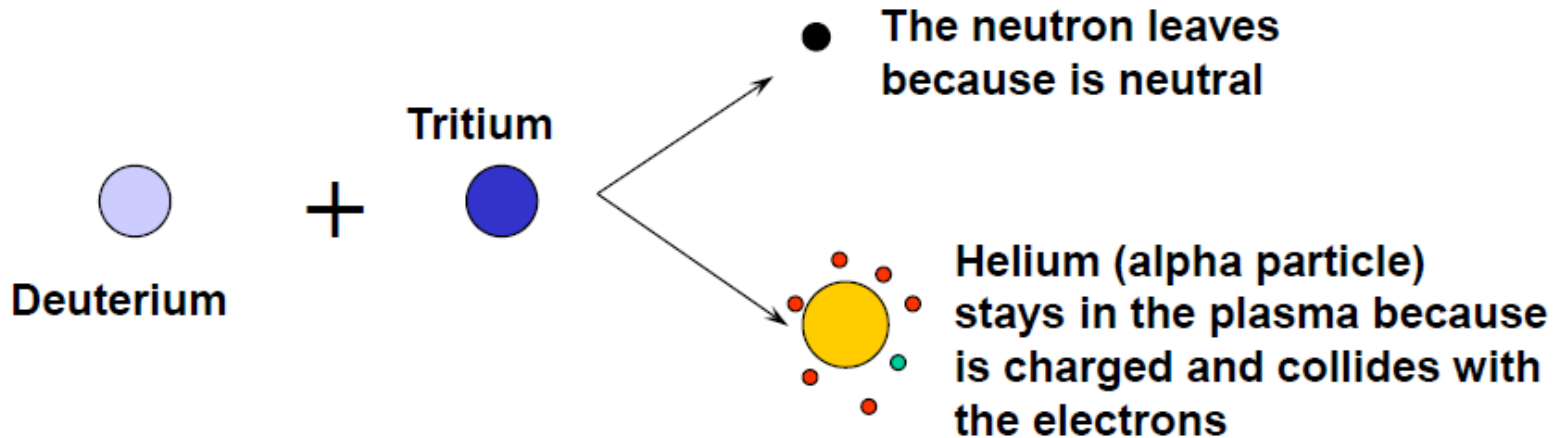
- If the ions are sufficiently hot, i.e., large random velocity, they can collide by overcoming coulomb repulsion



It takes a lot of energy or power to keep the plasma at 100M °C



- Let the plasma do it itself!



- The α -particles heat the plasma.

Under what conditions the plasma keeps itself hot?



- **Steady state 0-D power balance:**

$$S_{\alpha} + S_h = S_B + S_k$$

S_{α} : α particle heating

S_h : external heating

S_B : Bremsstrahlung radiation

S_k : heat conduction lost

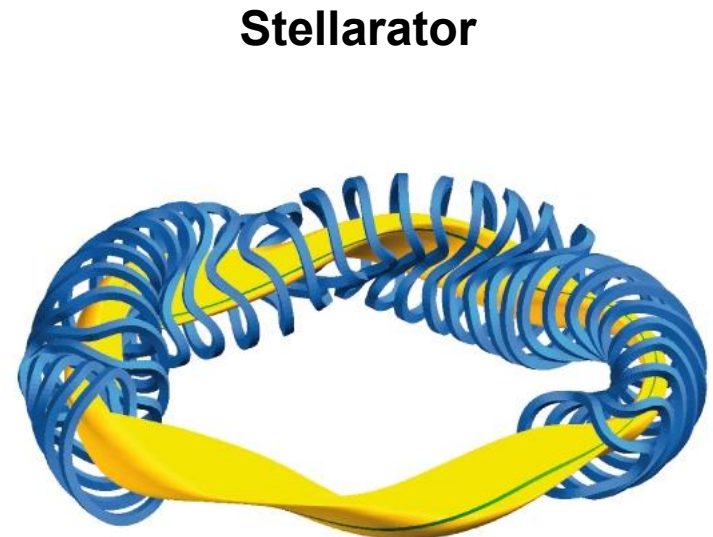
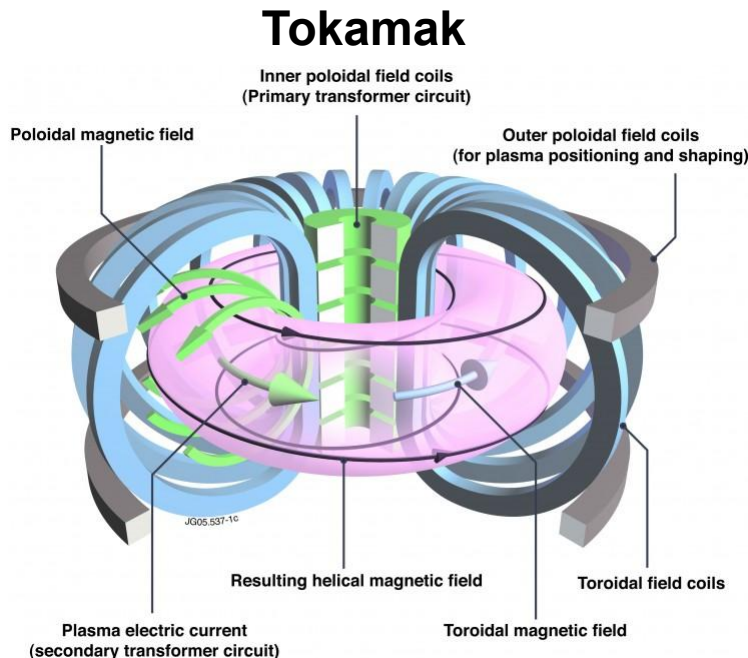
Ignition condition: $P\tau > 10 \text{ atm-s} = 10 \text{ Gbar} \cdot \text{ns}$

- **P: pressure, or called energy density**
- **τ is confinement time**

The plasma is too hot to be contained



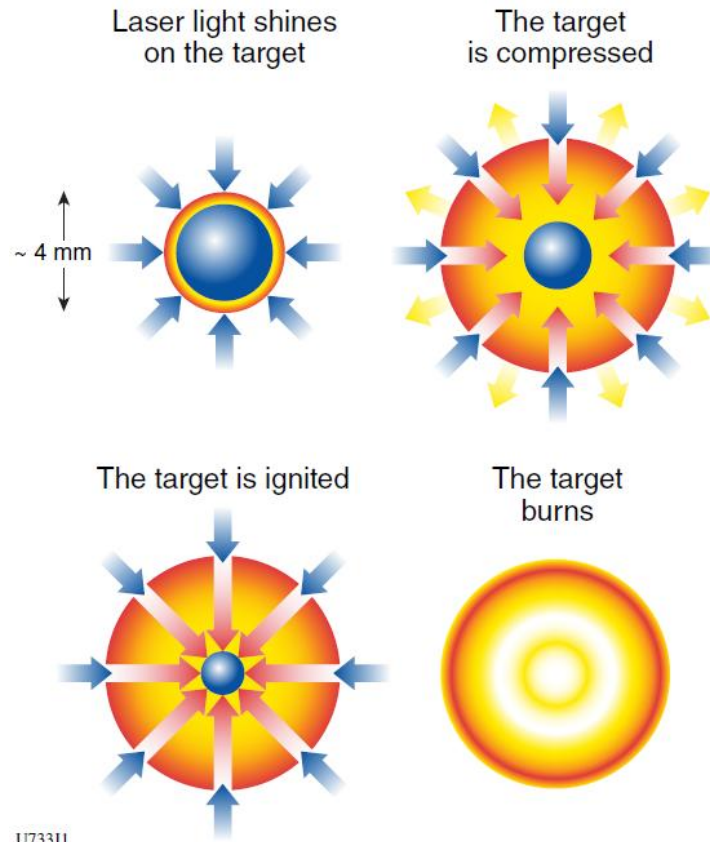
- **Solution 1: Magnetic confinement fusion (MCF), use a magnetic field to contain it. $P \sim \text{atm}$, $\tau \sim \text{sec}$, $T \sim 10 \text{ keV}$ ($10^8 \text{ }^\circ\text{C}$)**



Don't confine it!



- **Solution 2: Inertial confinement fusion (ICF). Or you can say it is confined by its own inertia: P~Gigabar, τ ~nsec, T~10 keV (10^8 °C)**

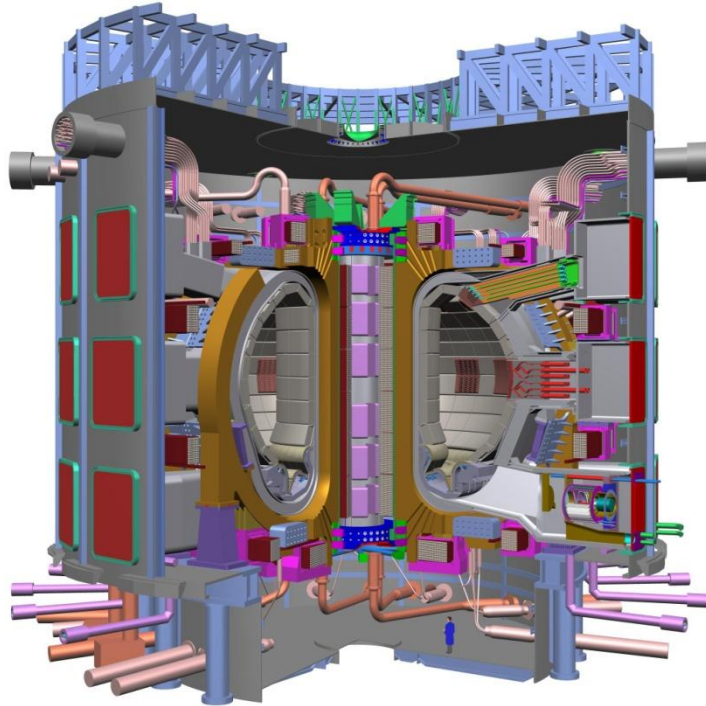


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To control? Or not to control?

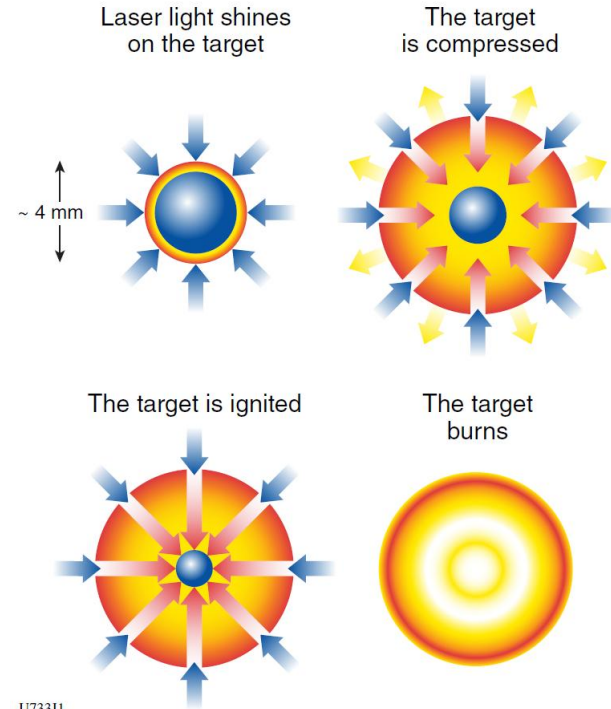


- **Magnetic confinement fusion (MCF)**



- **Plasma is confined by toroidal magnetic field.**

- **Inertial confinement fusion (ICF)**



- **A DT ice capsule filled with DT gas is imploded by laser.**