Introduction to Nuclear Fusion as An Energy Source

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Institute of Space and Plasma Sciences, National Cheng Kung University

2025 spring semester

Tuesday 9:00-12:00

Materials:

https://capst.ncku.edu.tw/PGS/index.php/teaching/

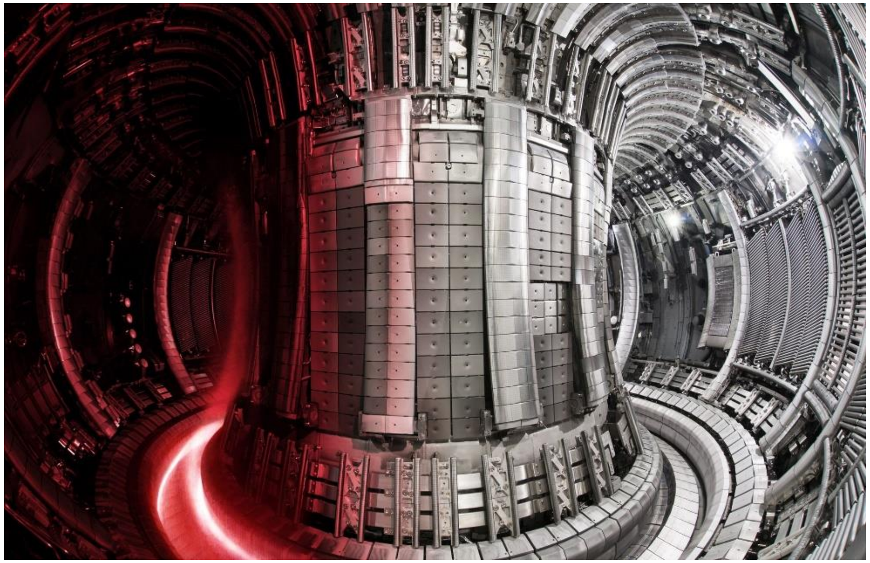
Online courses:

https://nckucc.webex.com/nckucc/j.php?MTID=mf1a33a5dab5eb71de9da4380ae888592

2025/2/18 updated

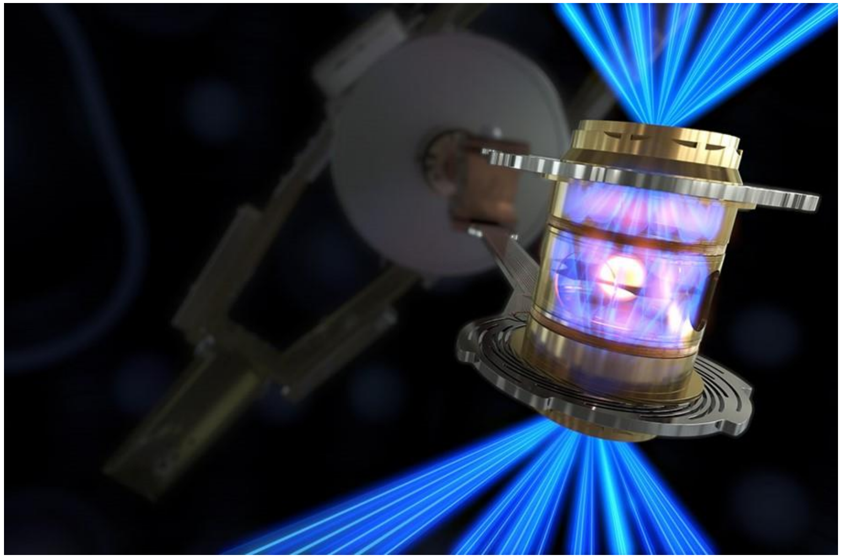
Significant breakthrough is achieved recently

- **Magnetic confinement fusion (MCF)**



- **On 2024/2/(8), record-breaking 69.26 megajoules of sustained fusion energy in Joint European Torus (JET) facility in Oxford demonstrates powerplant potential and strengthens case for ITER.**

- **Inertial confinement fusion (ICF)**

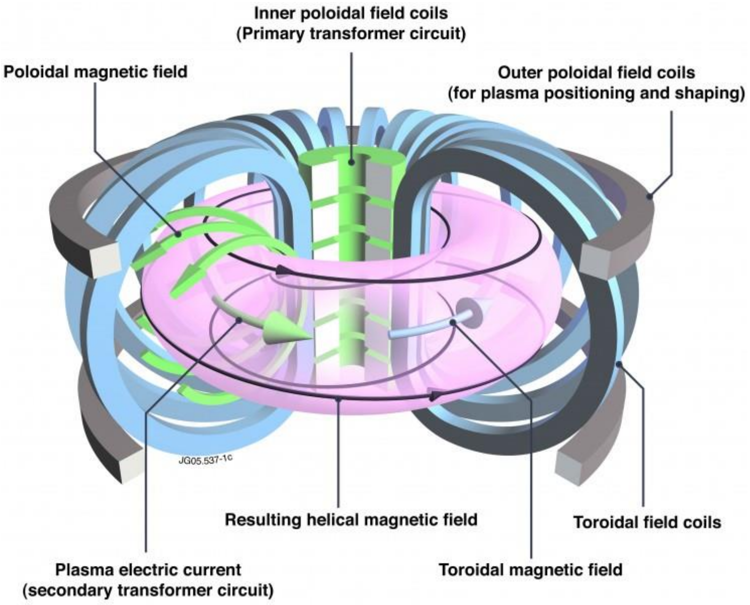


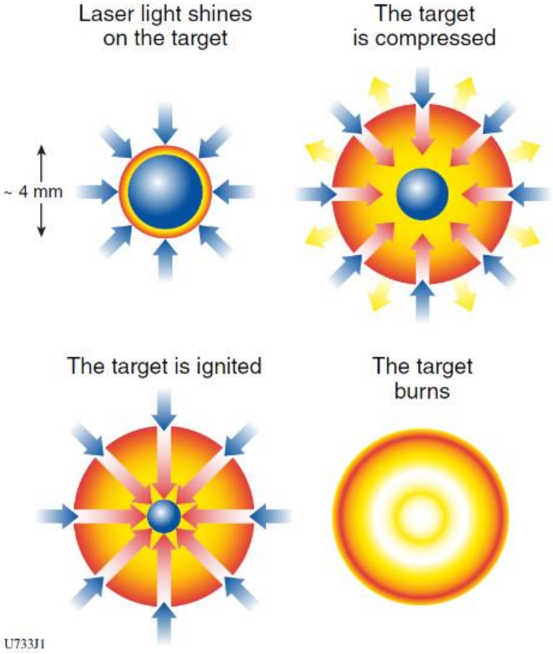
- **National Ignition Facility (NIF) demonstrated a gain grater than 1 for the first time on 2022/12/5. The yield of 3.15 MJ from the 2.05-MJ input laser energy, i.e., Q=1.5.**

https://ccfe.ukaea.uk/resources/#gallery
https://www.science.org/content/article/historic-explosion-long-sought-fusion-breakthrough

Nuclear fusion as an energy source is being developed

- **Magnetic confinement fusion (MCF)**
- **Inertial confinement fusion (ICF)**





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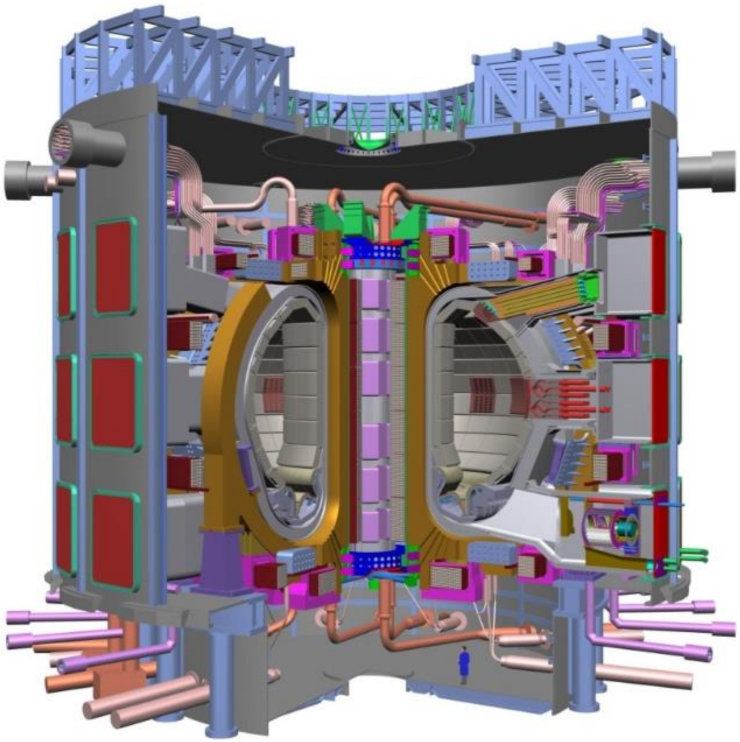
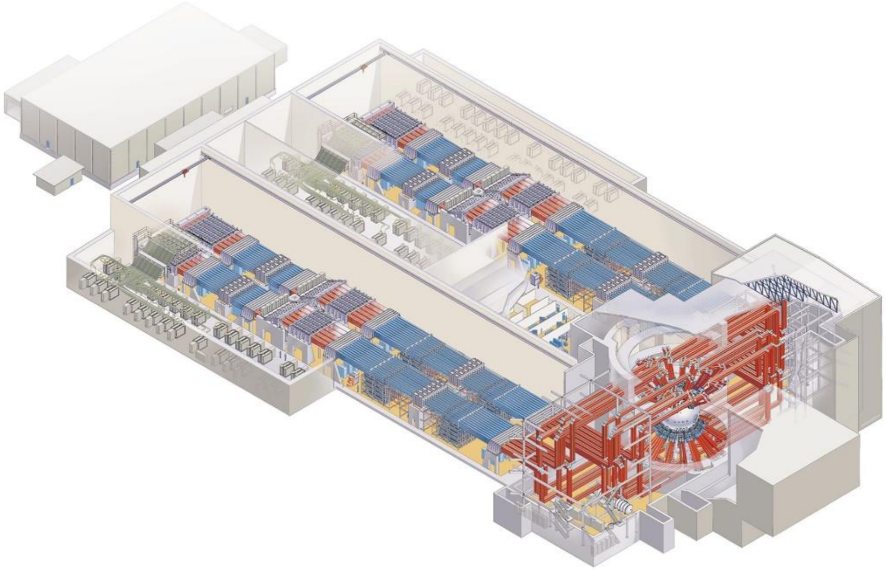
https://www.euro-fusion.org/2011/09/tokamak-principle-2/
Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester

Nuclear fusion as an energy source is being developed

- **Magnetic confinement fusion (MCF)**
- **Inertial confinement fusion (ICF)**





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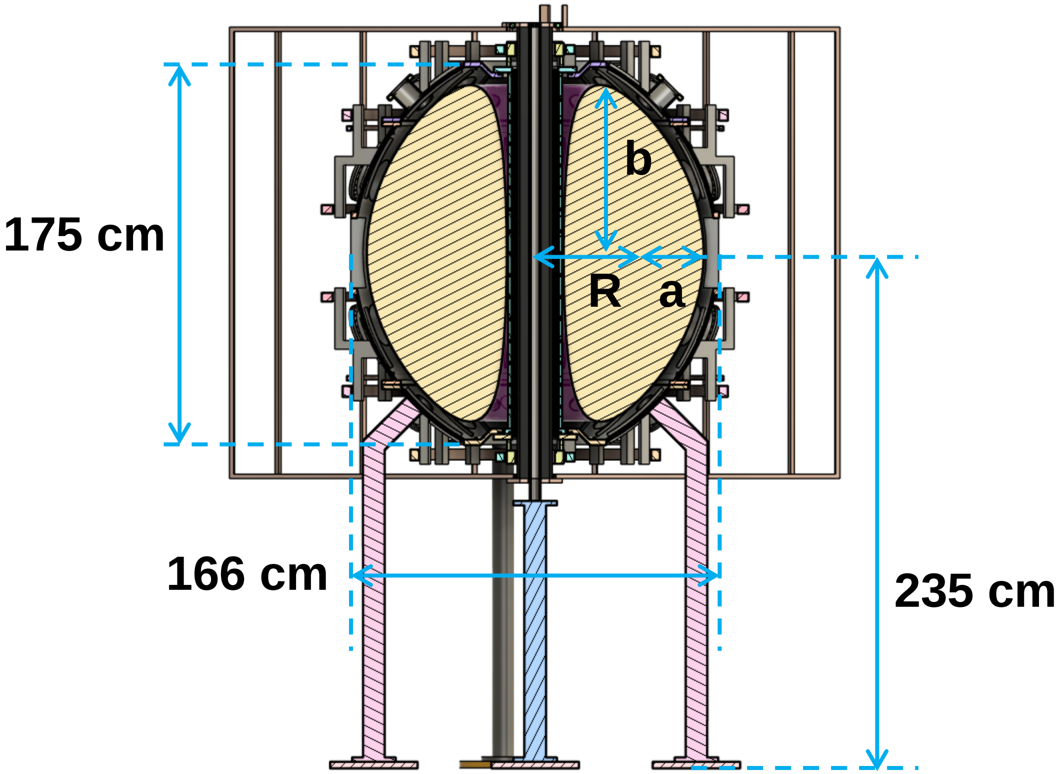
https://www.iter.org
https://zh.wikipedia.org/wiki/國家點火設施

Formosa Integrated Research Spherical Tokamak (FIRST)

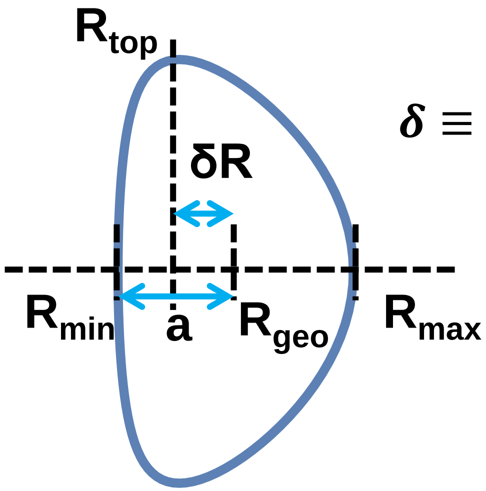


$$r = R + a\mathrm{Cos}\left(\theta + \delta \mathrm{Sin}(\theta)\right)$$

$$z = a\kappa \mathrm{Sin}(\theta)$$

- **We welcome anyone interested in fusion research to join us!**

- **Parameters:**
  - **R=45 cm**
  - **a=32 cm**
  - **Aspect Raio = R/a=1.41**
  - **Elongation κ=b/a=2.4**
  - **Triangularity δ=0.5**
  - **T~100 eV**
  - **B_T~0.1 ~ 0.5 T**
  - **I_p~100 kA**



$$\delta \equiv \frac{\delta R}{a}$$

Course Outline

- Brief background reviews
  - Electromagnetics
  - Plasma physics
- Introduction to nuclear fusion
  - Nuclear binding energy (Fission vs Fusion)
  - Fusion reaction physics
  - Some important fusion reactions (Cross section)
    - Main controlled fusion fuels
    - Advanced fusion fuels
  - Maxwell-averaged fusion reactivities

Course Outline

- Introduction to nuclear fusion (cont.)
  - Collisions (Bremsstrahlung radiation)
  - Columb scattering. Cross section of the Columb scattering
  - Beam-target fusion vs thermonuclear fusion
  - Lawson criteria, ignition conditions
  - Magnetic confinement fusion (MCF) vs Inertial confinement fusion (ICF)

Course Outline

- Magnetic confinement fusion (MCF)
  - Gyro motion, MHD
  - 1D equilibrium (z pinch, theta pinch)
  - Drift: ExB drift, grad B drift, and curvature B drift
  - Tokamak, Stellarator (toroidal field, poloidal field)
  - Magnetic flux surface
  - 2D axisymmetric equilibrium of a torus plasma: Grad-Shafranov equation.
  - Stability (Kink instability, sausage instability, Safety factor Q)
  - Central-solenoid (CS) start-up (discharge) and current drive
  - CS-free current drive: electron cyclotron current drive, bootstrap current.
  - Auxiliary Heating: ECRH, Ohmic heating, Neutral beam injection.

Course Outline

- **Inertial confinement fusion (ICF)**
  - **Plasma frequency and critical density**
  - **Direct- and indirect- drive**
  - **Laser generated pressure (Inverse bremsstrahlung and Ablation pressure)**
  - **Burning fraction, why compressing a capsule?**
  - **Implosion dynamics**
  - **Shock (Compression with different adiabat)**
  - **Laser pulse shape**
  - **Rocket model, shell velocity**
  - **Laser-plasma interaction (Stimulated Raman Scattering, SRS; Stimulated Brillouin Scattering, SBS; Two-plasmon decay)**
  - **Instabilities (Rayleigh-taylor instability, Kelvin-Helmholtz instability, Richtmeyer-Meshkov instability)**

Course Outline

- **Innovation Fusion scheme**
- **Status of fusion research in Taiwan**
  - **Formosa Integrated Research Spherical Tokamak (FIRST)**

References

- The physics of inertial fusion, by Stefano Atzeni and Jürgen Meyer-Ter-Vehn
- Nuclear fusion, by Edward Morse
- Ideal magnetohydrodynamics, by Jeffrey P. Freidberg
- Introduction to plasma theory, by Dwight R. Nicholson
- Introduction to plasma physics and controlled fusion, by Francis F. Chen
- Principles of plasma physics for engineers and scientists, by Umran S. Inan and Marek Golkowski
- Introduction to plasma physics, by Gurnett and Bhattacharjee
- The physics of plasma, by T. J. M. Boyd and J. J. Sanderson
- Principles of plasma physics, by Krall and Trivelpiece
- NRL Plasma Formulary, Naval Research Laboratory, 2013 by J. D. Huba

Grading

- Midterm 40 % @ 4/15
- Final exam 60 % @ 6/3
- After final exam:
  - 6/10 Introduction to Formosa Integrated Research Spherical Tokamak (FIRST), First Tokamak being developed in Taiwan
  - 6/17 Q&A

- No class: 4/8

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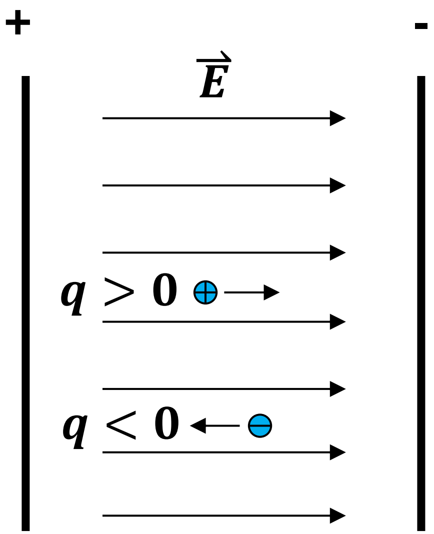
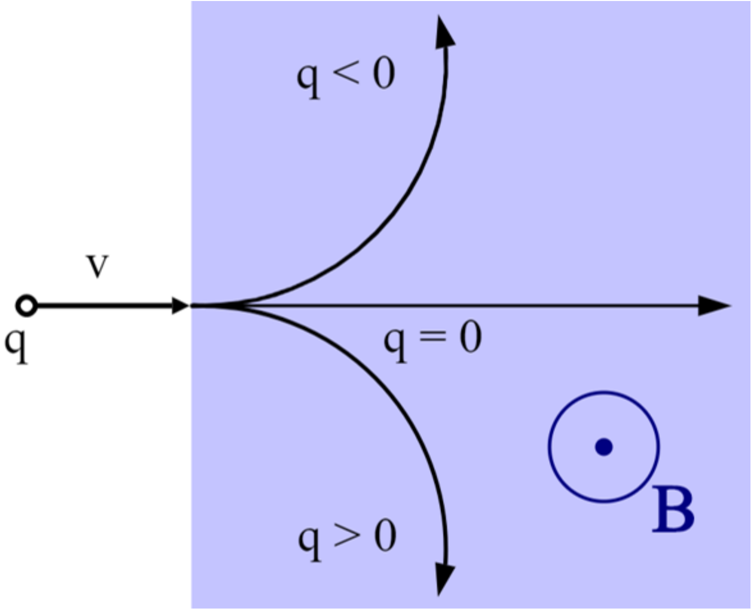
Charged particles are accelerated due to Lorentz force under electromagnetic fields

- **Lorentz force:** $\vec{F} = m\,\vec{a} = q\,\vec{E} + q\,\vec{v} \times \vec{B}$ $\qquad m\dfrac{d\,\vec{v}}{dt} = q\,\vec{E} + q\,\vec{v} \times \vec{B}$

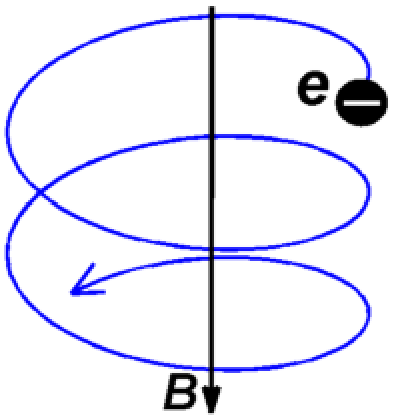
- **Force under electric fields**

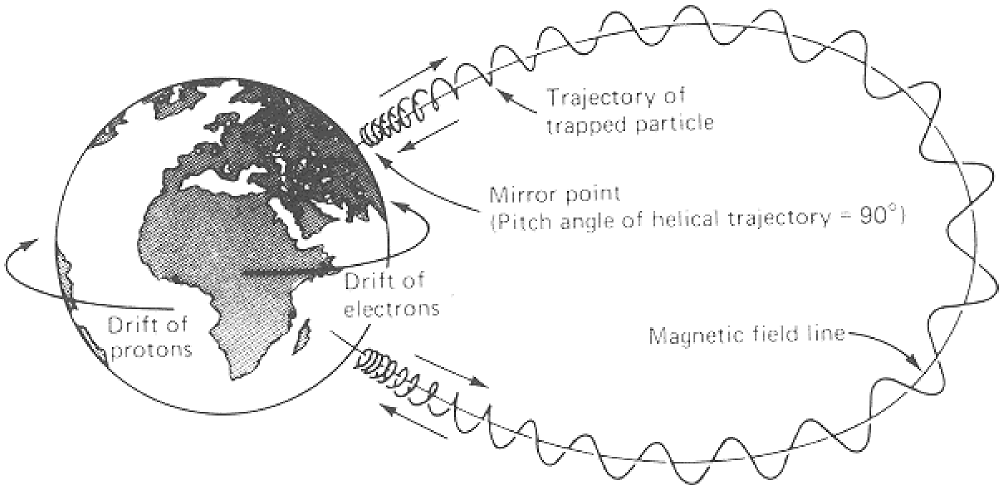


- **Force under magnetic fields**



Charged particles gyro around magnetic field lines





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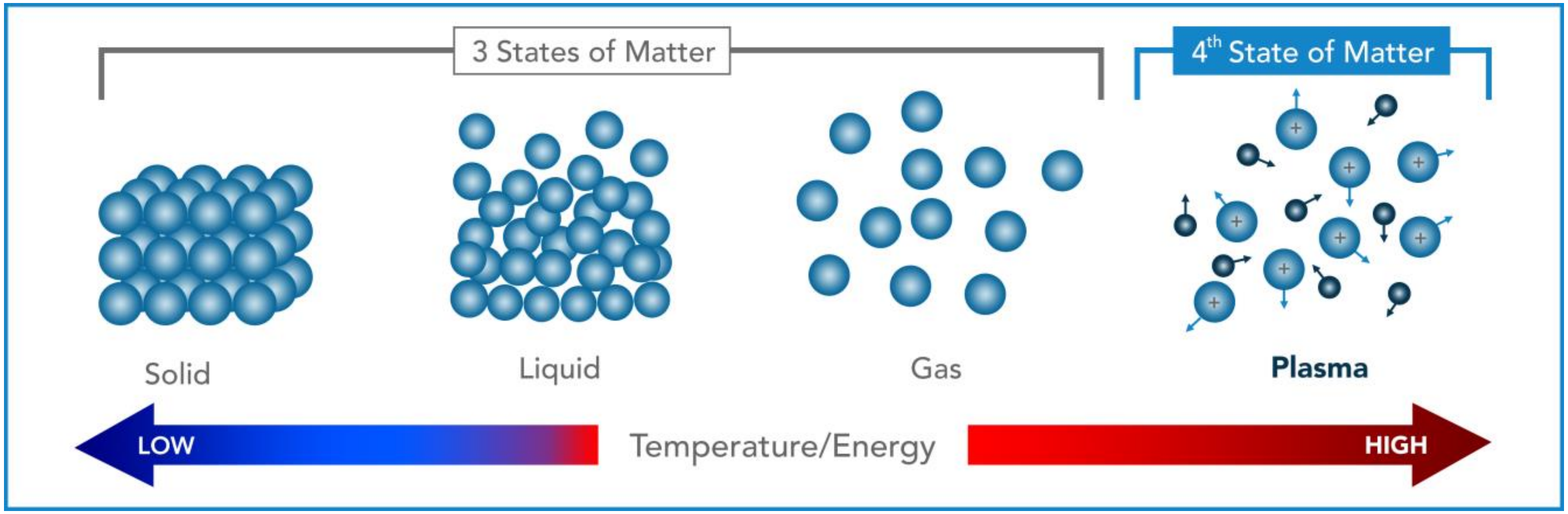
http://www.ipp.cas.cz/vedecka_struktura_ufp/tokamak/tokamak_compass/diagnostics/mikrovlnne-diagnostiky/ece-ebw-radiometr.html
http://www-ssg.sr.unh.edu/tof/Smart/Students/lees/periods.html

Plasma is the 4th state of matter



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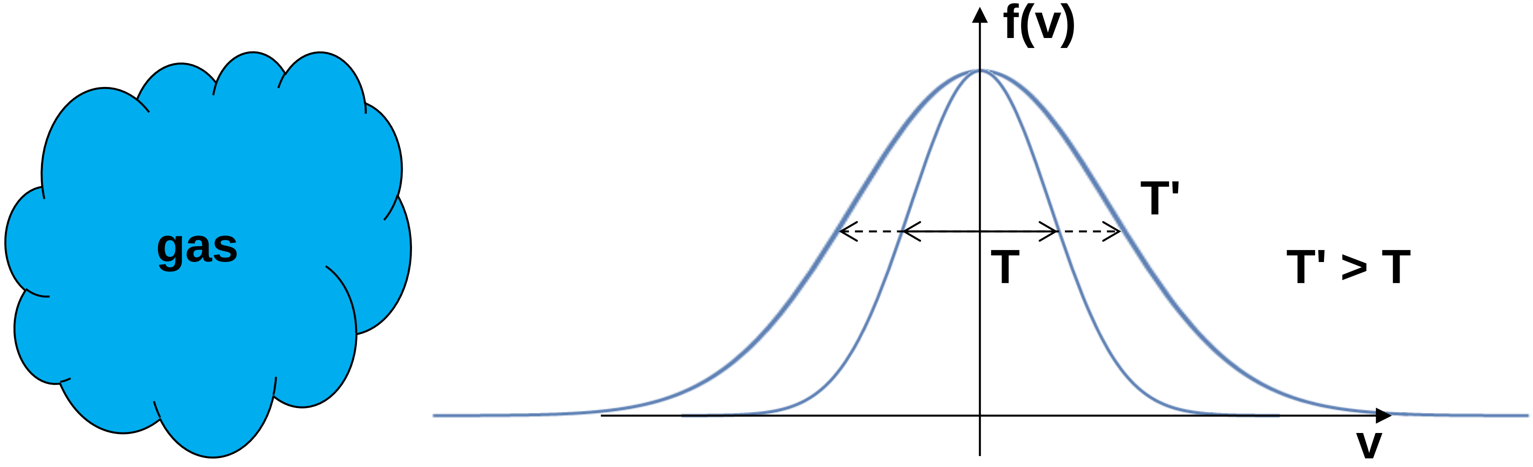
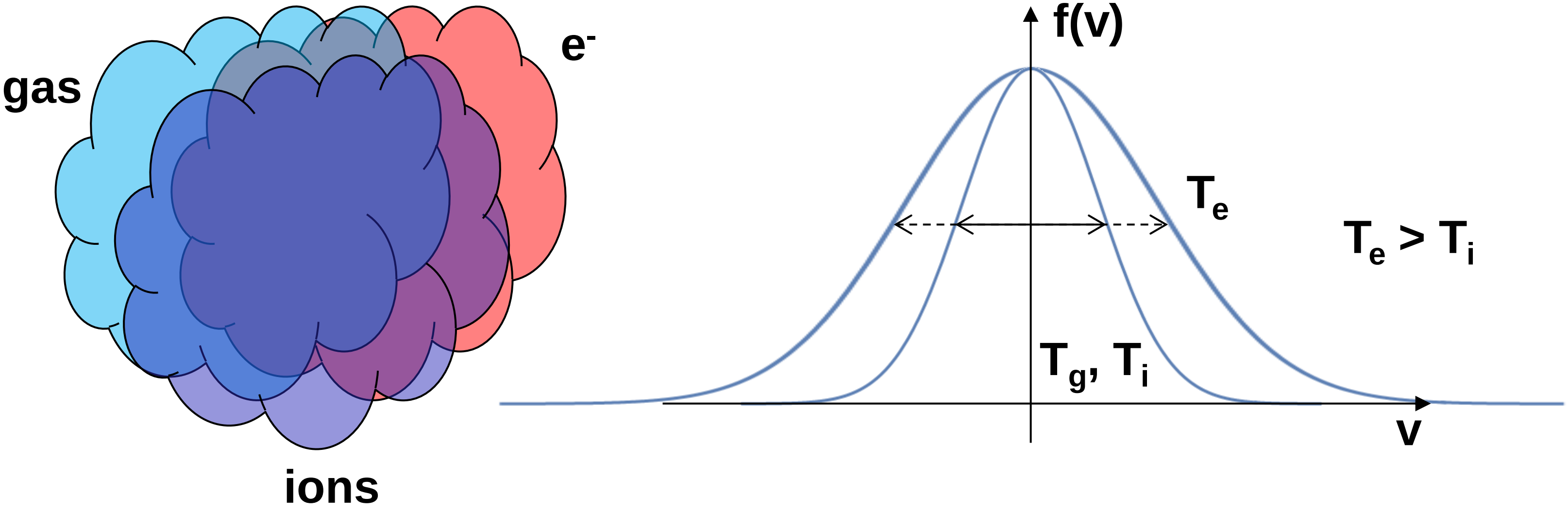
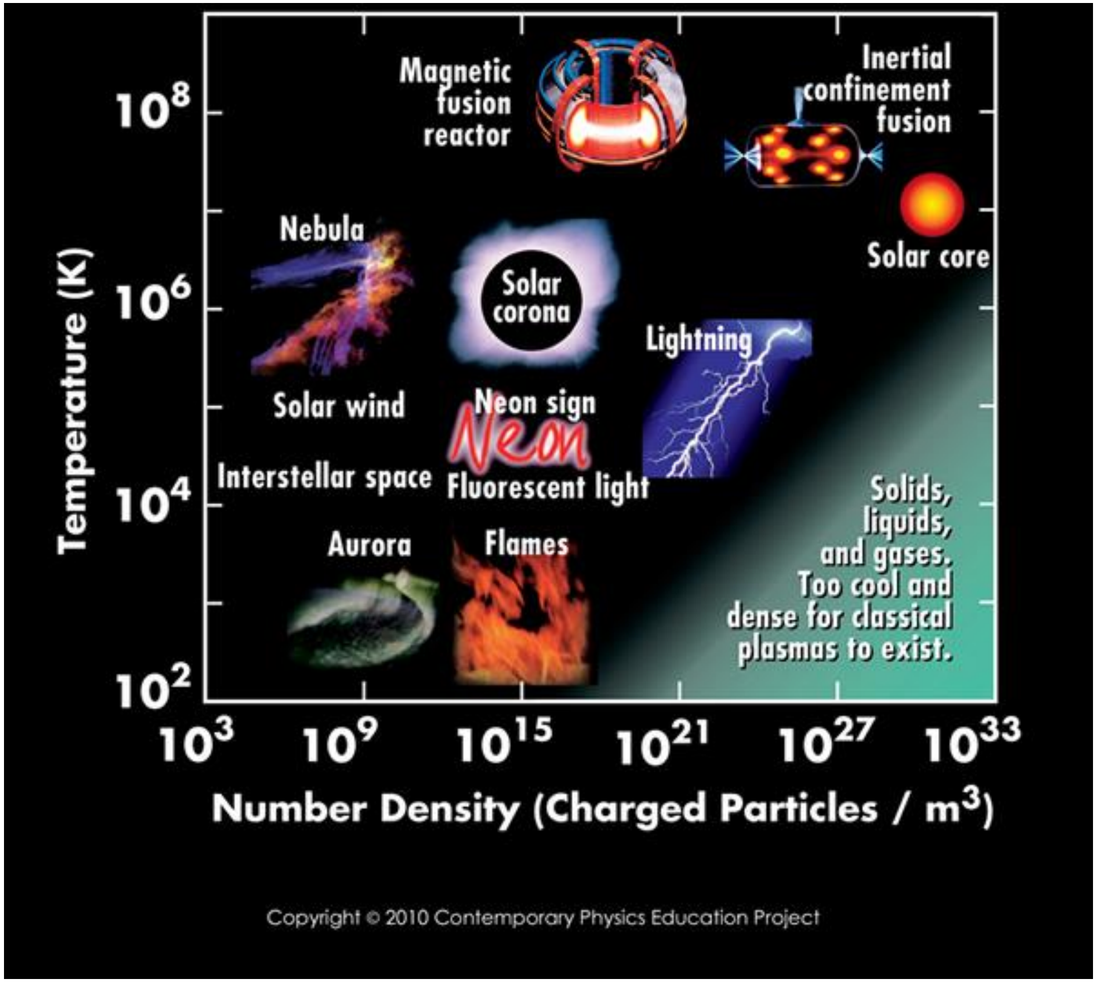
http://tetronics.com/our-technology/what-is-plasma/

In plasma, there are ions, electrons, and neutral gas





Nuclear fusion occurs at a very high temperature region



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https://lasers.llnl.gov/science/understanding-the-universe/plasma-physics

Maxwell’s equations

- **Gauss’s law:** $\nabla \cdot \vec{E} = \frac{\rho_q}{\epsilon_o}$ $\qquad \nabla \times \vec{E} = 0$ $\qquad \vec{E} = -\nabla \phi$
- **No name:** $\nabla \cdot \vec{B} = 0$ $\qquad \vec{B} = -\nabla \times \vec{A}$
- **Faraday’s law:** $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- **Ampére’s law:** $\nabla \times \vec{B} = \mu_o \vec{j} + \epsilon_o \mu_o \frac{\partial \vec{E}}{\partial t}$
- **Lorentz force:** $\vec{F} = m\,\vec{a} = q\,\vec{E} + q\,\vec{v} \times \vec{B}$

Potential

Vector Potential

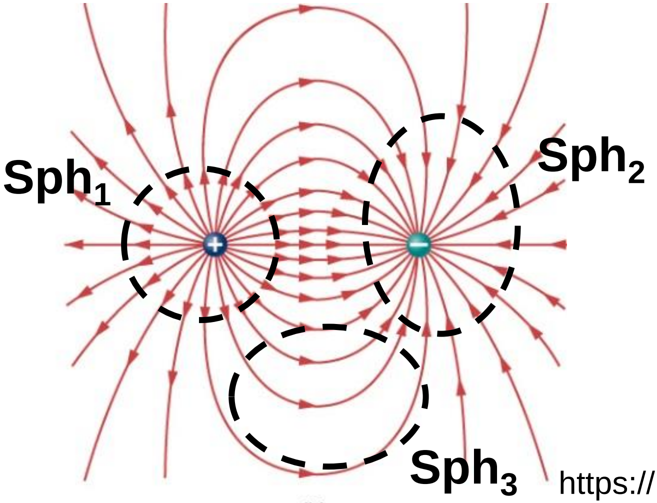
Displacement current

- **Poisson’s eq.:**

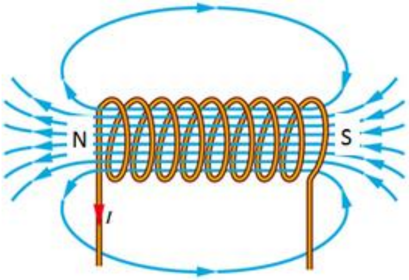
$$\nabla^2 \phi = -\frac{\rho_q}{\epsilon_o}$$

- **Laplace’s eq.:**

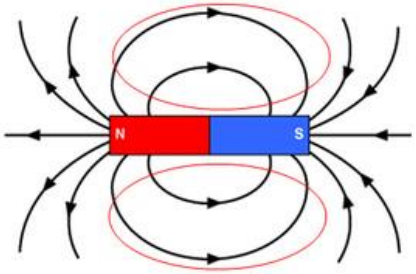
$$\nabla^2 \phi = 0$$



(b)



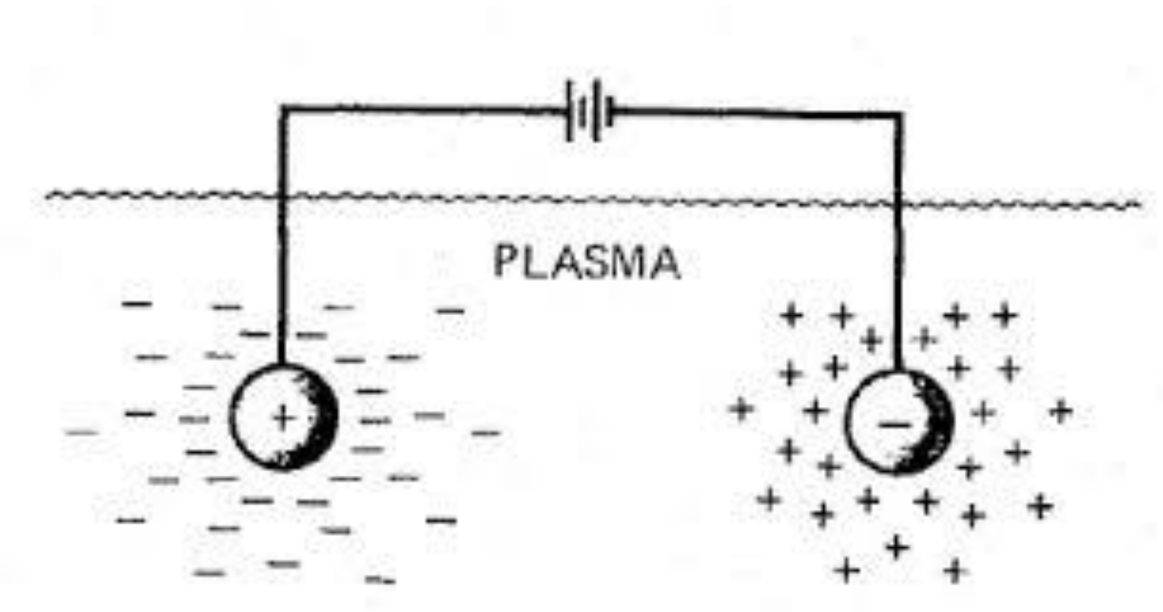
Solenoid



Magnet

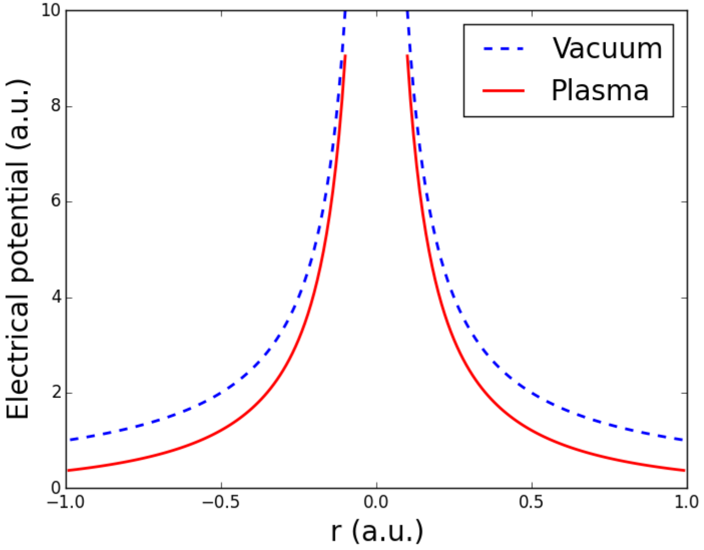
https://pressbooks.online.ucf.edu/osuniversityphysics2/chapter/electric-field-lines/
http://www.excelatphysics.com/magnetic-field-pattern.html

A test ion in the plasma gathers a shielding cloud that tends to cancel its own charge



Francis F. Chen, \Introduction to plasma physics and controlled fusion"

Debye shielding is a phenomenon such that the potential due to a test charge in a plasma falls off much faster than in vacuum



$$\nabla^2\phi = -\frac{\rho_q}{\epsilon_o}$$

- **Vacuum potential:**

$$\phi = \frac{\phi_0}{r}$$

- **Poisson’s equation:**

$$\nabla^2\phi = 4\pi e(n_e - n_i) - 4\pi q_T\delta(\vec{r})$$

- **Density profile:**

$$n_e = n_0 \exp\left(\frac{e\phi}{KT_e}\right) \quad , \quad n_i = n_0$$

- **For $\vec{r} \neq 0$ and assuming $\frac{e\phi}{T_e} \ll 1$**

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = \frac{4\pi n_0 e^2}{KT_e}\phi$$

$$\phi = \frac{\phi_0}{r}\exp\left(-\frac{r}{\lambda_D}\right) \qquad \lambda_D \approx \left(\frac{KT_e}{4\pi n e^2}\right)^{1/2}$$

Magnetohydrodynamics description of plasma

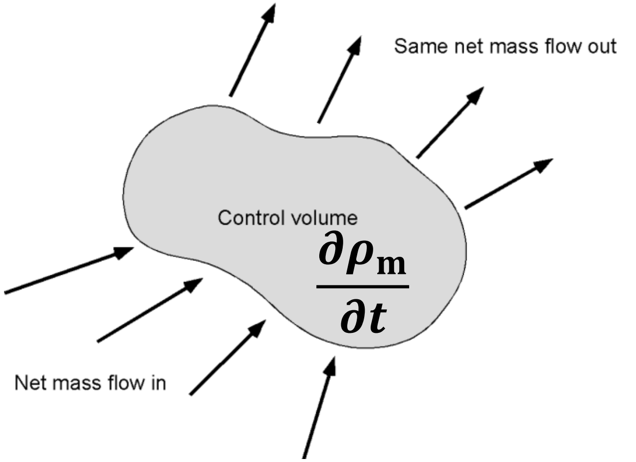
- **Continuity eq:** $\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{v}) = 0$
- **Momentum eq:** $\rho_m \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = \rho_q \vec{E} + \vec{j} \times \vec{B} - \nabla \cdot \overleftrightarrow{P}$
- **Ohm’s law:** $\vec{j} = \sigma\left(\vec{E} + \vec{v} \times \vec{B}\right)$
- **Equation of state:** $\frac{d}{dt}\left(P {\rho_m}^{-\gamma}\right) = 0$
- **Maxwell’s eqs:**

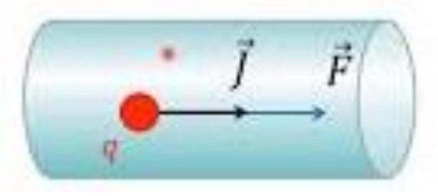
$$\nabla \cdot \vec{E} = \frac{\rho_q}{\epsilon_o}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

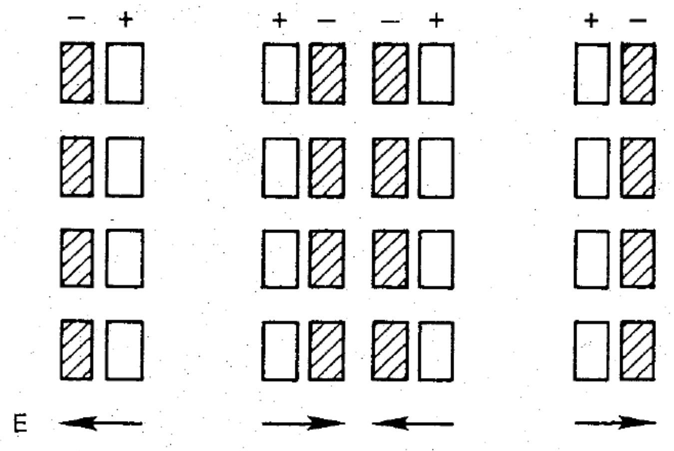
$$\nabla \times \vec{B} = \mu_o \vec{j} + \epsilon_o \mu_o \frac{\partial \vec{E}}{\partial t}$$





https://eaglepubs.erau.edu/introductiontoaerospaceflightvehicles/chapter/conservation-of-mass-continuity-equation/
https://www.youtube.com/watch?v=lu0Ep8_Gp8U

Electron plasma frequency is the characteristic frequency such that electrons oscillate around their equilibrium positions



Mechanism of plasma oscillations.

- **Assumption:**

$$\vec{\nabla} \equiv \hat{x}\frac{\partial}{\partial x}\ , \quad \vec{E} = \hat{x}E\ , \quad \vec{v}_{\mathrm{e}} = \hat{x}v \quad ,$$

$$\vec{\nabla} \times \vec{E} = 0 \ \text{’}\ \vec{E} = -\vec{\nabla}\phi$$

- **Continuity and momentum equation for electron:**

$$\frac{\partial n_{\mathrm{e}}}{\partial t} + \vec{\nabla}\cdot\left(n_{\mathrm{e}}\vec{v}_{\mathrm{e}}\right) = 0$$

$$m_{\mathrm{e}} n_{\mathrm{e}}\left[\frac{\partial \vec{v}_{\mathrm{e}}}{\partial t} + \left(\vec{v}_{\mathrm{e}}\cdot\vec{\nabla}\right)\vec{v}_{\mathrm{e}}\right] = -e n_{\mathrm{e}}\vec{E}$$

- **Gauss' law:**

$$\frac{\partial E}{\partial x} = 4\pi e\left(n_{\mathrm{i}} - n_{\mathrm{e}}\right)$$

Electron plasma frequency is obtained by linearizing the hydrodynamic equations

- **The oscillation is assumed to be small:**

$$n_e = n_0 + n_1,\ \vec{E} = \vec{E}_0 + \vec{E}_1,\ v_e = v_0 + v_1$$

where

$$\frac{\partial n_0}{\partial x} = v_0 = \vec{E}_0 = 0$$

$$\frac{\partial n_0}{\partial t} = \frac{\partial v_0}{\partial t} = \frac{\partial \vec{E}_0}{\partial t} = 0$$

- **Linearization:**

$$m_e \frac{\partial v_1}{\partial t} = -eE_1$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} = 0$$

$$\frac{\partial E_1}{\partial x} = -4\pi e n_1$$

- **Plane wave solution:**

$$\eta_1 = \hat{\eta}\exp[i(kx - \omega t)]$$

$$\eta_1 = v_1, n_1, E_1$$

- **Substitute into the previous equations:**

$$-im_e \omega v_1 = -eE_1$$

$$-i\omega n_1 = -n_0 ikv_1$$

$$ikE_1 = -4\pi e n_1$$

- **Electron plasma frequency is obtained by eliminating n$_1$ and E$_1$:**

$$\omega_{pe} \equiv \omega = \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2}$$

Plasma β is the ratio between hydro pressure and magnetic pressure

- **Momentum equation in Magnetohydrodynamics (MHD) approach:**

$$\rho_{\mathrm{m}}\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\,\vec{v}\right] = \vec{j}\times\vec{B} - \nabla\cdot\overleftrightarrow{P} \Rightarrow \vec{j}\times\vec{B} - \nabla P$$

$$\nabla\times\vec{B} = \mu_{\mathrm{o}}\,\vec{j} + \cancel{\epsilon_{\mathrm{o}}\mu_{\mathrm{o}}\frac{\partial \vec{E}}{\partial t}}$$ **w/ low freq. ($\omega << \omega_{\mathrm{pe}}$)**

$$\vec{j}\times\vec{B} = \frac{1}{\mu_{\mathrm{o}}}\left(\vec{\nabla}\times\vec{B}\right)\times\vec{B} = \frac{1}{\mu_{\mathrm{o}}}\left[\left(\vec{B}\cdot\vec{\nabla}\right)\vec{B} - \frac{1}{2}\,\vec{\nabla}\,B^2\right]$$

$$\rho_{\mathrm{m}}\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\,\vec{v}\right] = -\nabla\left(P + \frac{B^2}{2\mu_{\mathrm{o}}}\right) + \frac{1}{\mu_o}\left(\vec{B}\cdot\vec{\nabla}\right)\vec{B}$$

- **Magnetic pressure:** $\frac{B^2}{2\mu_{\mathrm{o}}}$

$$\beta \equiv \frac{P}{B^2/2\mu_{\mathrm{o}}}$$

- **Magnetic tension:** $\frac{1}{\mu_o}\left(\vec{B}\cdot\vec{\nabla}\right)\vec{B}$

Magnetohydrodynamics description of plasma w/ low-freq. and long-wavelength approximation

- **Continuity eq:** $\frac{\partial \rho_{\mathrm{m}}}{\partial t} + \nabla \cdot (\rho_m \vec{v}) = 0$ **w/ long wavelength ($\lambda >> \lambda_d$)**
- **Momentum eq:** $\rho_{\mathrm{m}} \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\, \vec{v} \right] = \cancel{\rho_{\mathrm{q}} \vec{E}} + \vec{j} \times \vec{B} - \nabla \cdot \overleftrightarrow{P}$
- **Ohm's law:** $\vec{j} = \sigma\left(\vec{E} + \vec{v} \times \vec{B}\right)$
- **Equation of state:** $\frac{d}{dt}\left(P {\rho_{\mathrm{m}}}^{-\gamma}\right) = 0$
- **Maxwell's eqs:**

$\nabla \cdot \vec{E} = \frac{\rho_{\mathrm{q}}}{\epsilon_o} \approx 0$ **w/ long wavelength ($\lambda >> \lambda_d$) => quasi neutral**

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_{\mathrm{o}} \vec{j} + \cancel{\epsilon_{\mathrm{o}} \mu_{\mathrm{o}} \frac{\partial \vec{E}}{\partial t}}$$

w/ low freq. ($\omega << \omega_{\mathrm{pe}}$)

Magnetohydrodynamics description of plasma w/ low-freq. and long-wavelength approximation

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- **Equation of state:** $$\frac{d}{dt} \left(P {\rho_{\mathrm{m}}}^{-\gamma} \right) = 0$$
- **Maxwell's eqs:**

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_{\mathrm{o}} \vec{j} \qquad \Rightarrow \qquad \nabla \cdot \vec{j} = 0$$

$\nabla \cdot$

- **Force balance condition:** $0 = \vec{j} \times \vec{B} - \nabla \cdot \overleftrightarrow{P}$

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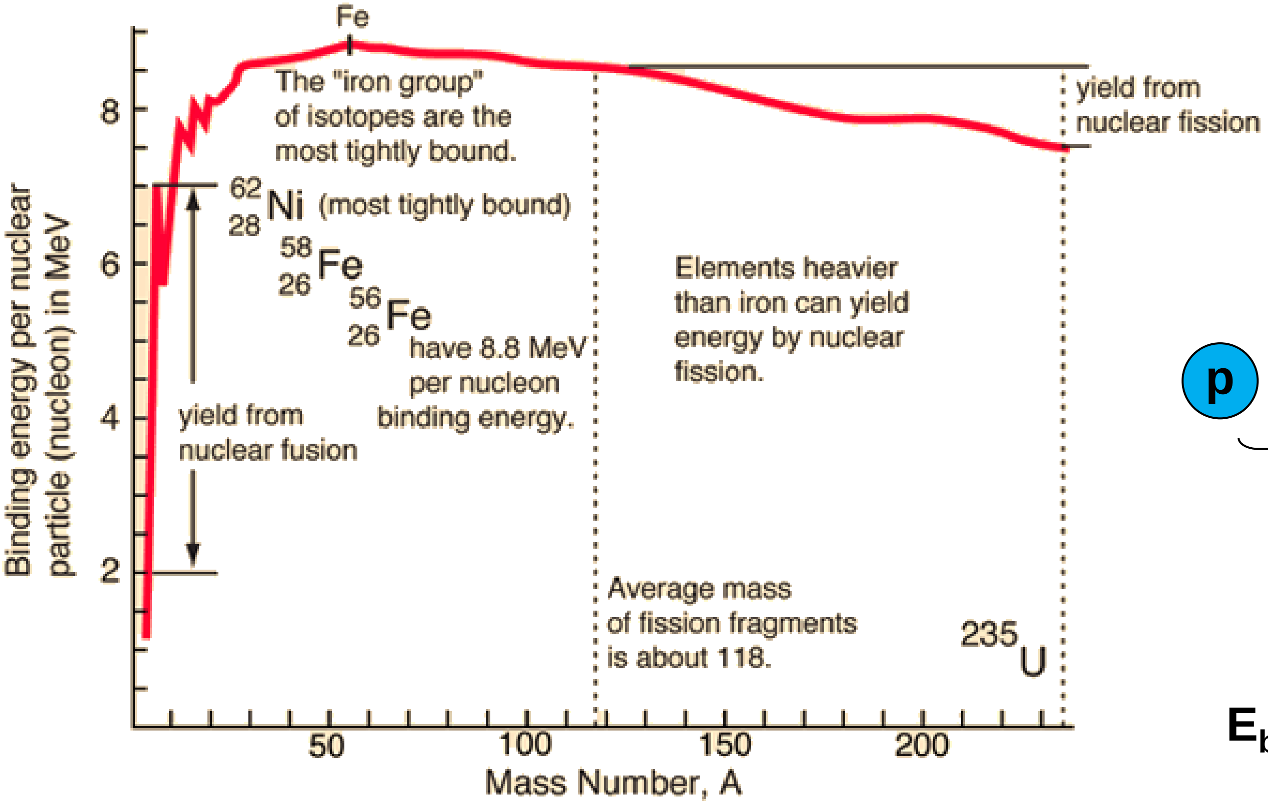
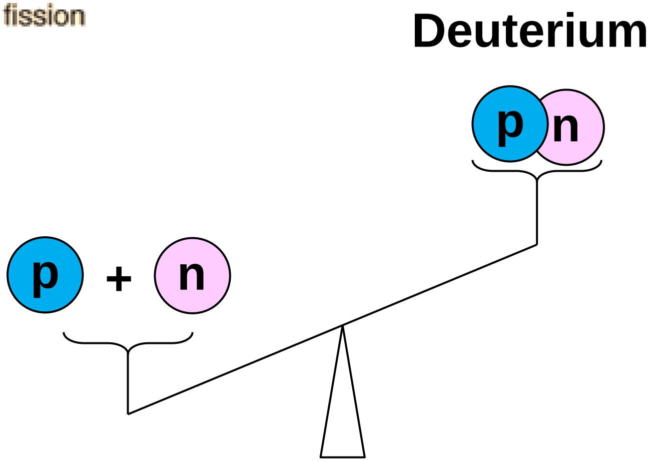
The “iron group” of isotopes are the most tightly bound





$$m_p + m_n > m_D$$

$$E_{binding} = [(m_p + m_n) - m_D]c^2$$

$$Q = \left(\sum_i m_i - \sum_f m_f\right) c^2$$

$$\Delta m = z m_p + (A - z) m_n - m$$

- **Binding energy:** $B = \Delta m c^2$
- **Output energy:** $Q = \sum_f B_f - \sum_i B_i$

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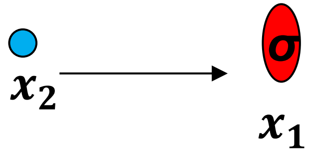
http://hyperphysics.phy-astr.gsu.edu/hbase/nucene/nucbin.html

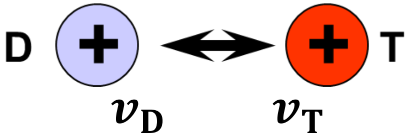
Cross section measures the probability per pair of particles for the occurrence of the reaction

$$x_1 + x_2 \rightarrow x_3 + x_4$$



- **The hard sphere cross section:**
  ***σ ≈ πR*^{2} where R ≈ 5x10^{-15} m is the nuclear radius, i.e.,**
  ***σ = 0.8 x 10*$^{-28}$ m^2 ≈ 1 barm. (barm ≡ 10^{-28} m^2)**

- **Classical cross section:**

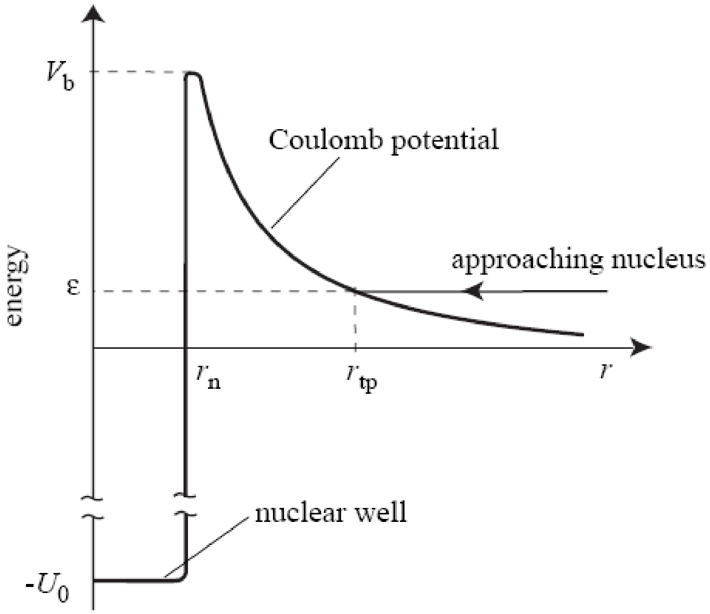


$$\frac{m_D}{2} v_D{}^2 + \frac{m_T}{2} v_T{}^2 \geq \frac{e^2}{4\pi\epsilon_o R}$$

- **Let** $v = \left| \vec{v}_D - \vec{v}_T \right|$

$$v_D = \frac{m_T}{m_D + m_T} v \qquad v_T = \frac{m_D}{m_D + m_T} v$$

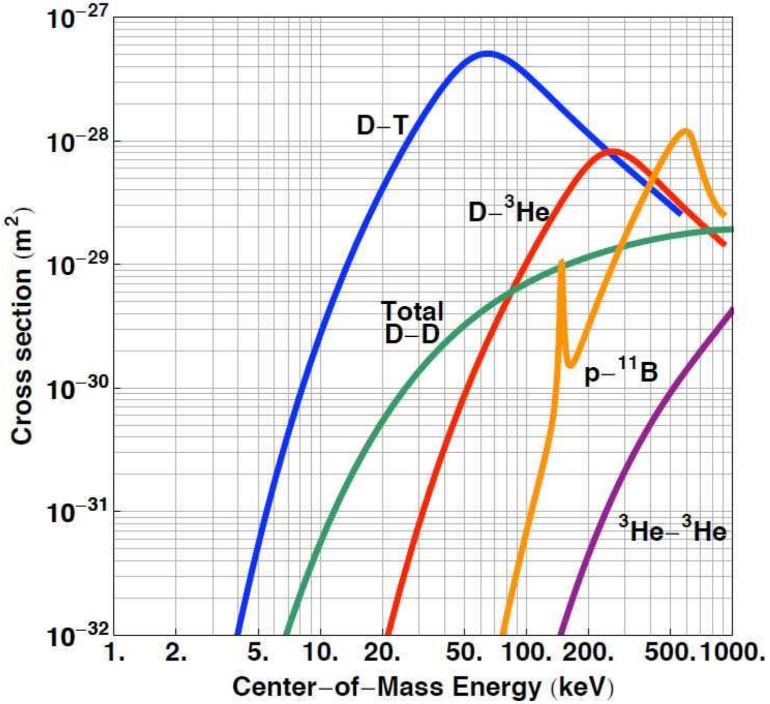
$$\frac{m_r}{2} v^2 \geq \frac{e^2}{4\pi\epsilon_o R} \qquad m_r = \frac{m_D m_T}{m_D + m_T}$$



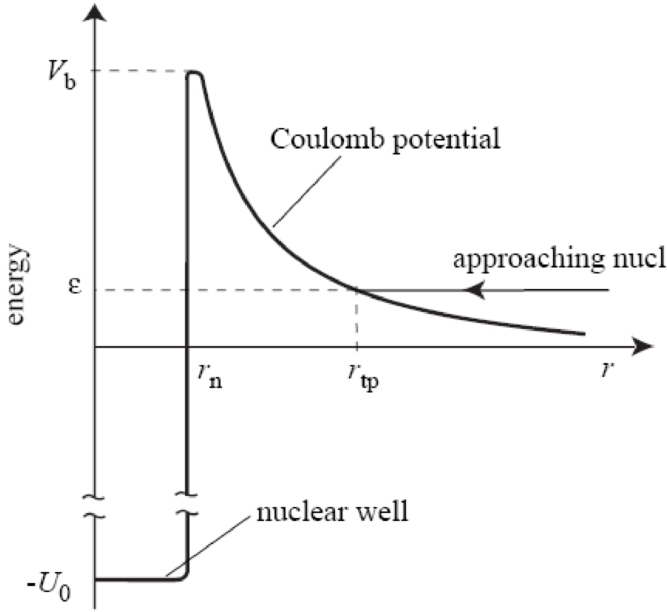
- **Classical kinetic energy required for fusion is**

$$K_{c.m.} > 288\ \text{keV} \quad !!!$$

Cross section of fusion reaction is much larger than the classical approach



- **Classical kinetic energy required for fusion is $K_{\text{c.m.}} > 288\ \text{keV}$!!!**
- **DT cross section has a peak of ~5 barns at 120 keV.**
- **$\sigma_{DT} \approx 100\sigma_{DD}$ @ 20 keV.**



$D + D \rightarrow T + p$

$\rightarrow He^3 + n$

$D + T \rightarrow He^4 + n$

$D + He^3 \rightarrow He^4 + p$

$p + B^{11} \rightarrow 3He^3$

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https://i.stack.imgur.com/wXQD5.jpg

Santarius, J. F., “Fusion Space Propulsion – A Shorter Time Frame Than You Think”, JANNAF, Monterey, 5-8 December 2005.

Flux of incident particles reduces after collisions

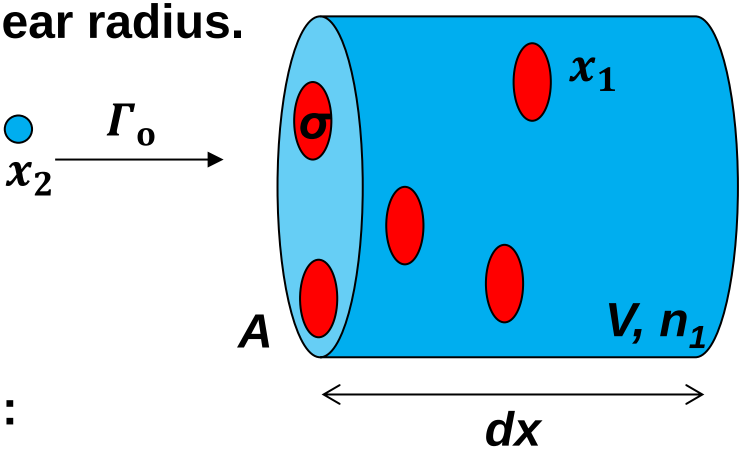
$$x_1 + x_2 \rightarrow x_3 + x_4$$

- Cross section: $\sigma \approx \pi R^2$ where R is the nuclear radius.

$$V = Adx$$

$$N_1 = n_1 V = n_1 A\mathrm{dx}$$

$$A_{\mathrm{Target}} = N_1\sigma = \sigma n_1 A\mathrm{dx}$$



- Fraction of total area blocked by targets is:

$$dF = \frac{\sigma N_1}{A} = \frac{\sigma n_1 A dx}{A} = \sigma n_1 dx$$

$$\frac{dF}{dx} = \sigma n_1$$

- Flux of incident particles (x_2) is Γ_{o}

$$-d\Gamma = dF\Gamma = \sigma n_1 \Gamma dx$$

$$\frac{-d\Gamma}{\Gamma} = \sigma n_1 dx$$

$$\Gamma = \Gamma_o \exp\left(-\frac{x}{\lambda_{\mathrm{mfp}}}\right)$$

- Mean free path:

$$\lambda_{\mathrm{mfp}} = \frac{1}{n_1\sigma}$$

- Collision frequency:

$$\nu = \frac{1}{\tau}, \tau = \frac{\lambda_{\mathrm{mfp}}}{v} = \frac{1}{n_1\sigma v}$$

Reactions happen when collision happen

- Reaction rate R_{12}: number of fusion collisions/reactions per unit volume per unit time.
- In the time $dt=dx/v$, n_2Adx incident particles will pass through the target volume.
- The number having a collisions is: $dF(n_2Adx)$
- The volumetric reaction rate R_{12}, i.e., the number of reaction per unit time and per unit volume is:

$$R_{12}=\frac{dF(n_2Adx)}{Adxdt}=\sigma n_1 n_2\frac{dx}{dt}=n_1 n_2\sigma v \qquad \left(\frac{dF}{dx}=\sigma n_1\right)$$

- The fusion power density ($\mathrm{W/m^3}$) is: $S_\mathrm{f}=E_\mathrm{f}n_1 n_2\sigma \mathrm{v}\ \left(\mathrm{W/m^3}\right)$
- For DT fusion, E_f=17.6 MeV.
- For a particle population with a distribution function in velocity space:

$$n=\int d\,\vec{v}\,f(\vec{r}\,,\vec{v}\,,t)$$

- Therefore,

$$n_1\rightarrow d\,\vec{v}_1\,f_1(\vec{r}\,,\vec{v}_1\,,t)\qquad n_2\rightarrow d\,\vec{v}_2\,f_2(\vec{r}\,,\vec{v}_2\,,t)\qquad v\rightarrow\left|\vec{v}_1-\vec{v}_2\right|$$

$$R_{12}=\int f_1(\vec{v}_1)f_1(\vec{v}_2)\sigma\left(\left|\vec{v}_1-\vec{v}_2\right|\right)\left|\vec{v}_1-\vec{v}_2\right|d\,\vec{v}_1\,d\,\vec{v}_2$$

The fusion power density needs to consider the distribution function of particles

$$R_{12} = \int f_1(\vec{v}_1) f_1(\vec{v}_2) \sigma\left(\left|\vec{v}_1 - \vec{v}_2\right|\right) \left|\vec{v}_1 - \vec{v}_2\right| d\,\vec{v}_1\, d\,\vec{v}_2 = n_1 n_2 \langle \sigma \mathrm{v} \rangle$$

$$\langle \sigma \mathrm{v} \rangle \equiv \frac{\int f_1(\vec{v}_1) f_1(\vec{v}_2) \sigma(|\vec{v}_1 - \vec{v}_2|) |\vec{v}_1 - \vec{v}_2| d\,\vec{v}_1\, d\,\vec{v}_2}{\int f_1(\vec{v}_1) f_1(\vec{v}_2) d\,\vec{v}_1\, d\,\vec{v}_2}$$

$$= \frac{\int f_1(\vec{v}_1) f_1(\vec{v}_2) \sigma(|\vec{v}_1 - \vec{v}_2|) |\vec{v}_1 - \vec{v}_2| d\,\vec{v}_1\, d\,\vec{v}_2}{n_1 n_2}$$

- **The fusion power density (W/m^3) is:**

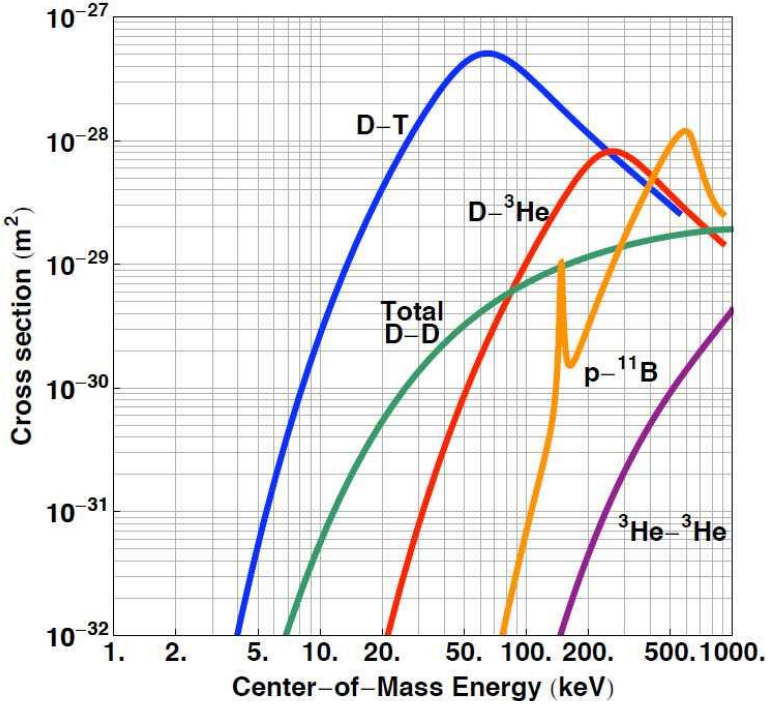
$$S_{\mathrm{f}} = E_{\mathrm{f}} n_1 n_2 \langle \sigma \mathrm{v} \rangle \left(\mathrm{W/m^3}\right)$$

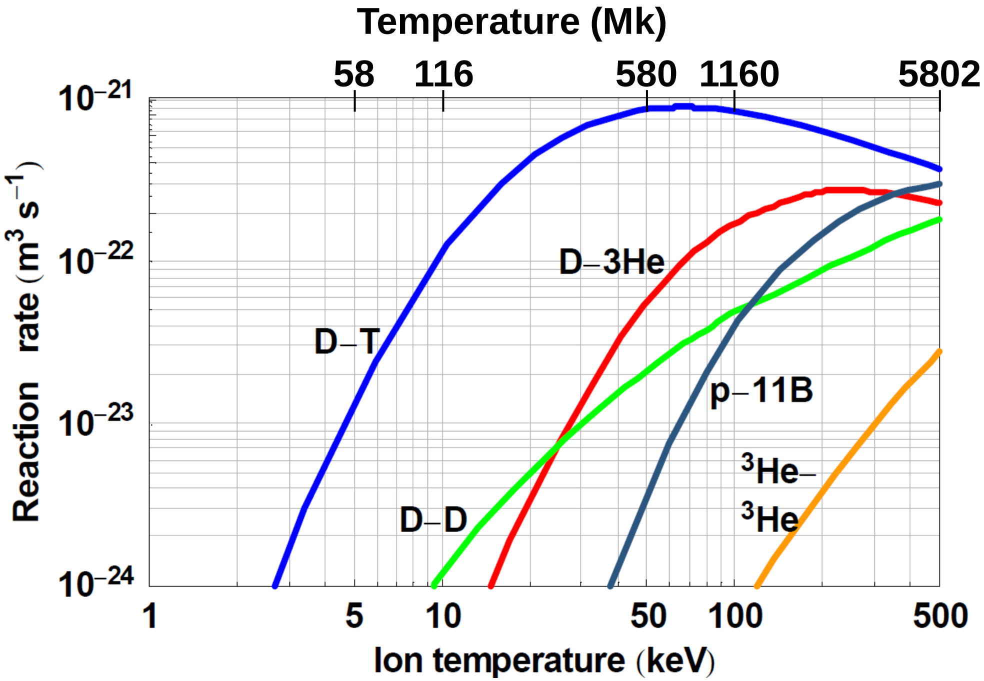
- **Optimum concentration of DT fusion is 50-50.**

$$S_{\mathrm{f}} = E_{\mathrm{f}} n_{\mathrm{D}} n_{\mathrm{T}} \langle \sigma \mathrm{v} \rangle \qquad n_D = k n_o \qquad n_T = (1-k) n_o$$

$$S_{\mathrm{f}} = E_{\mathrm{f}} k (1-k) {n_{\mathrm{o}}}^2 \text{ which peak at } k = 0.5\ .$$

Fusion doesn’t come easy

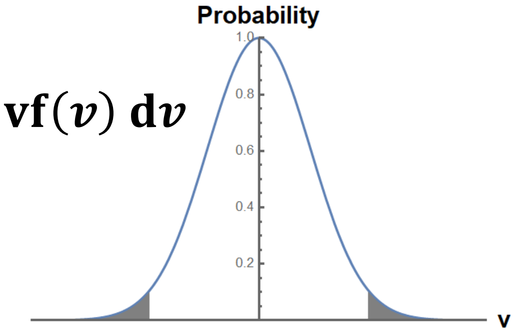




- **The DT fusion reactivity is maximum at T ≈ 64 keV**
- **@ T = 10 keV,** $< \boldsymbol{\sigma v} >_{DT} \approx 100 < \boldsymbol{\sigma v} >_{DD}$

- **Reaction rate:**

$$< \boldsymbol{\sigma v} >= \int \boldsymbol{\sigma}(\boldsymbol{v})\mathbf{v}\mathbf{f}(\boldsymbol{v})\ \mathbf{d}\boldsymbol{v}$$



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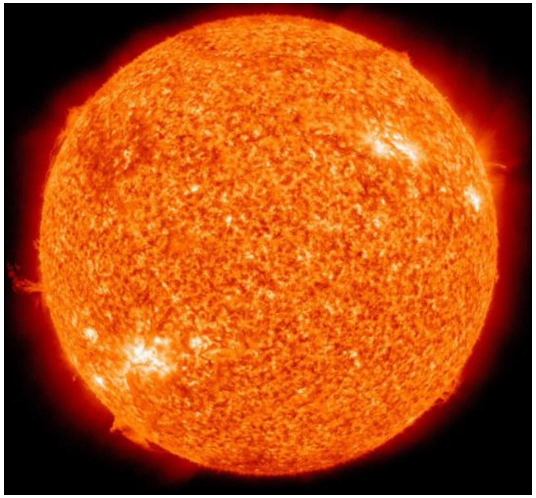
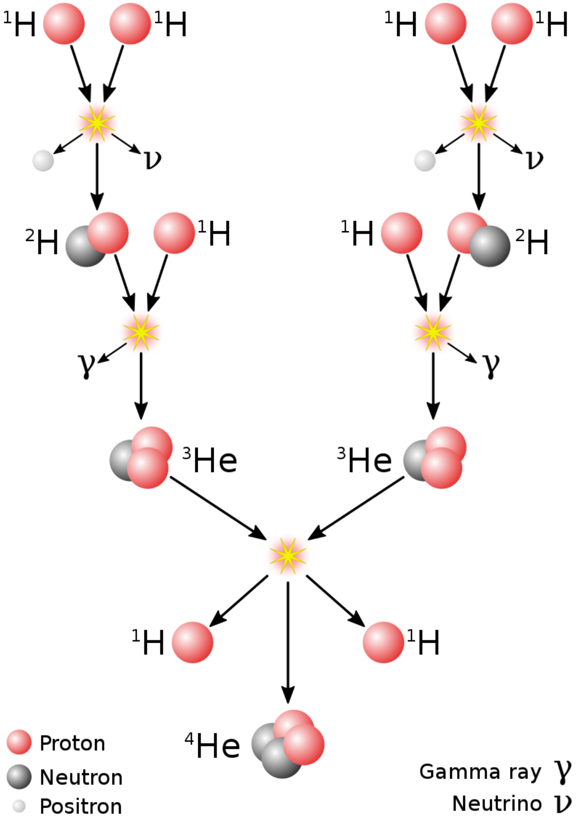
https://i.stack.imgur.com/wXQD5.jpg
Santarius, J. F., “Fusion Space Propulsion – A Shorter Time Frame Than You Think”, JANNAF, Monterey, 5-8 December 2005.

Fusion in the sun provides the energy

- **Proton-proton chain in sun or smaller**

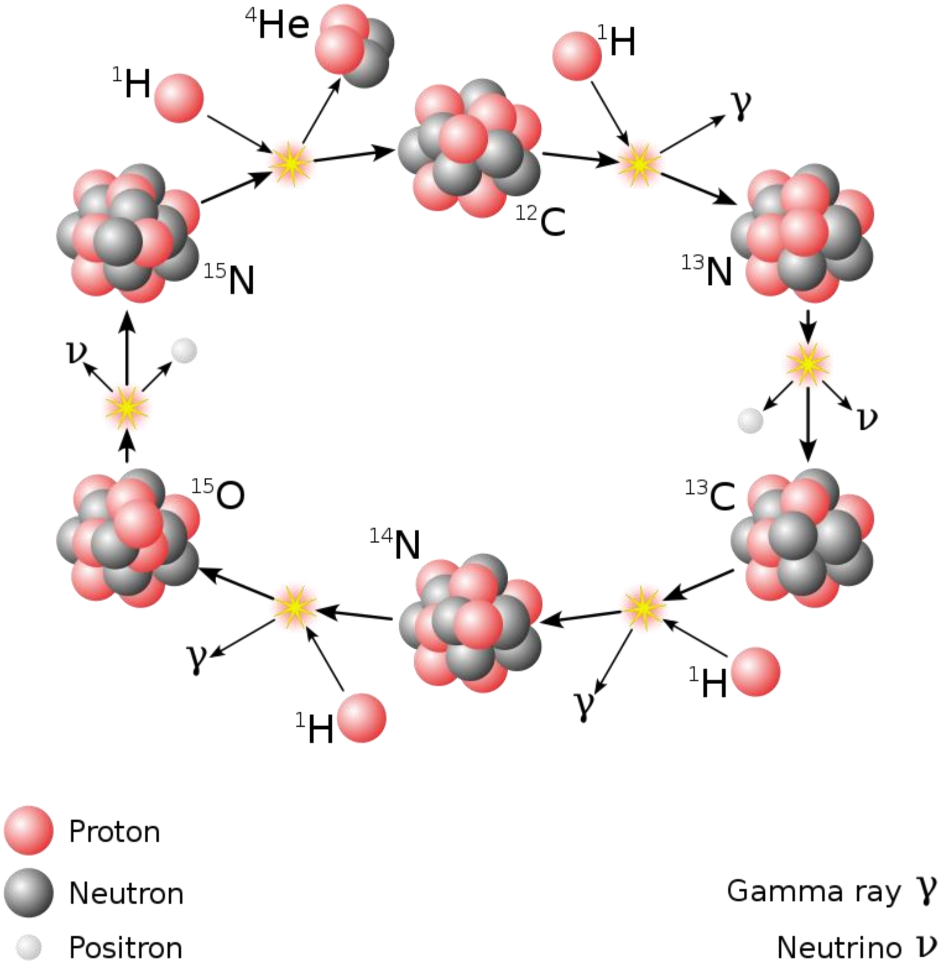


- **Particles are confined by the gravity.**



https://en.wikipedia.org/wiki/Sun
https://en.wikipedia.org/wiki/Nuclear_fusion

In heavy sun, the fusion reaction is the CNO cycle



https://en.wikipedia.org/wiki/Nuclear_fusion

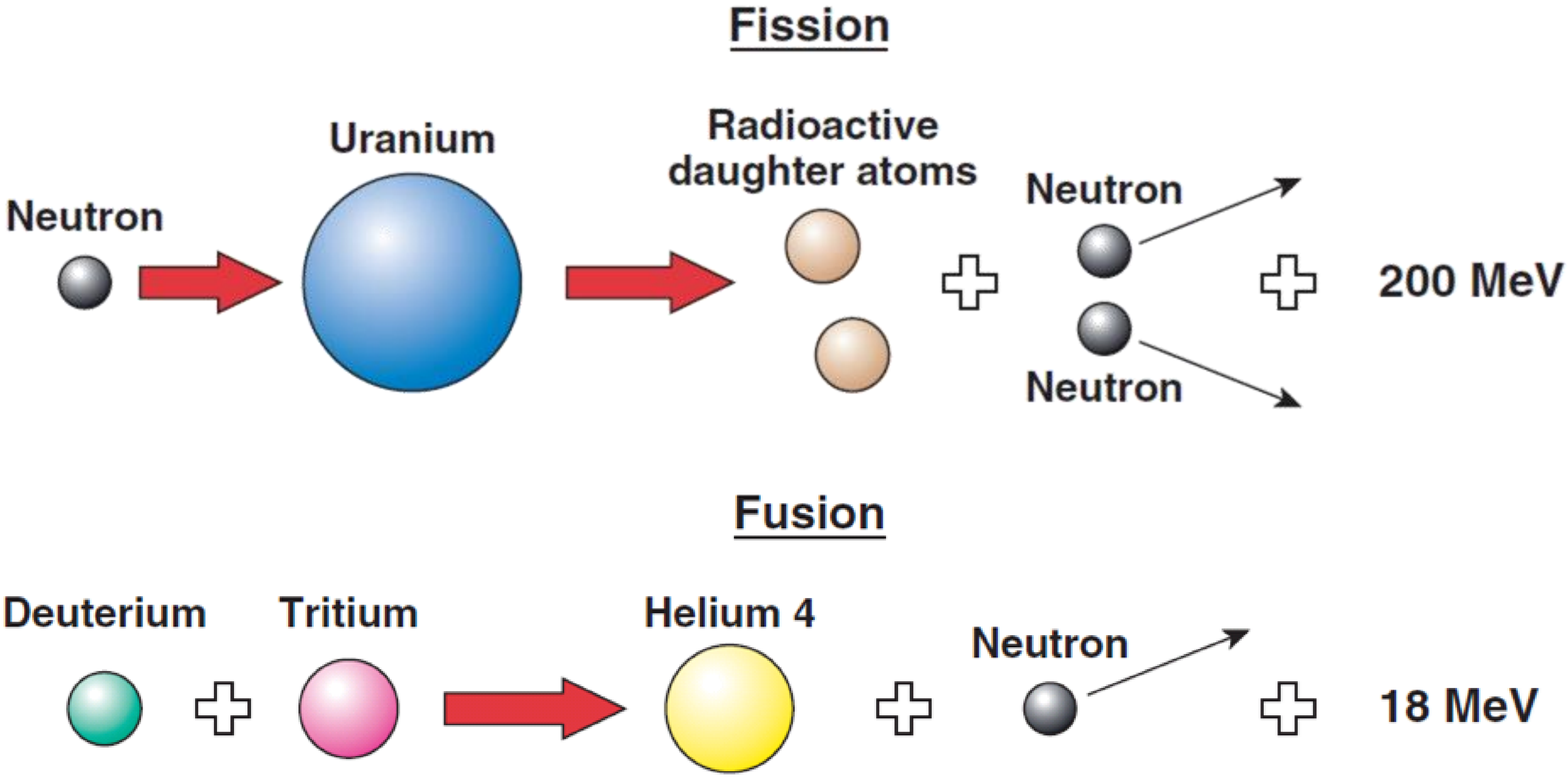
The cross section of proton-proton chain is much smaller than D T fusion

Reaction	$\sigma_{10\ \text{keV}}$ (barn)	$\sigma_{100\ \text{keV}}$ (barn)	σ_{max} (barn)	ε_{max} (keV)
$D+T \rightarrow \alpha+n$	2.72×10^{-2}	3.43	5.0	64
$D+D \rightarrow T+p$	2.81×10^{-4}	3.3×10^{-2}	0.06	1250
$D+D \rightarrow {}^3He+n$	2.78×10^{-4}	3.7×10^{-2}	0.11	1750
$T+T \rightarrow \alpha+2n$	7.90×10^{-4}	3.4×10^{-2}	0.16	1000
$D+{}^3He \rightarrow \alpha+p$	2.2×10^{-7}	0.1	0.9	250
$p+{}^6Li \rightarrow \alpha+{}^3He$	6×10^{-10}	7×10^{-3}	0.22	1500
$p+{}^{11}B \rightarrow 3\alpha$	(4.6×10^{-17})	3×10^{-4}	1.2	550
$p+p \rightarrow D+e^++v$	(3.6×10^{-26})	(4.4×10^{-25})		
$p+{}^{12}C \rightarrow {}^{13}N+\gamma$	(1.9×10^{-26})	2.0×10^{-10}	1.0x10.4	400
${}^{12}C+{}^{12}C$ (all branches)		(5.0×10^{-103})		

- "( )" are theoretical values while others are measured values.

The Physics of Inertial Fusion, by Stefano Atzeni and Jürgen Meyer-Ter-Vehn

Nuclear fusion and fission release energy through energetic neutrons



(3.5MeV) (14.1MeV)

Nuclear fusion provides more energy per atomic mass unit (amu) than nuclear fission

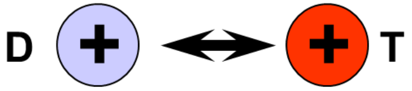
- Fusion of D+T: $$\frac{Q}{A} = \frac{17.6\mathrm{MeV}}{(3+2)\mathrm{amu}} = 3.5\,\frac{\mathrm{MeV}}{\mathrm{amu}}$$
- Fission of ^{235}U+n: $$\frac{Q}{A} = \frac{200\mathrm{MeV}}{(235+1)\mathrm{amu}} = 0.85\,\frac{\mathrm{MeV}}{\mathrm{amu}}$$

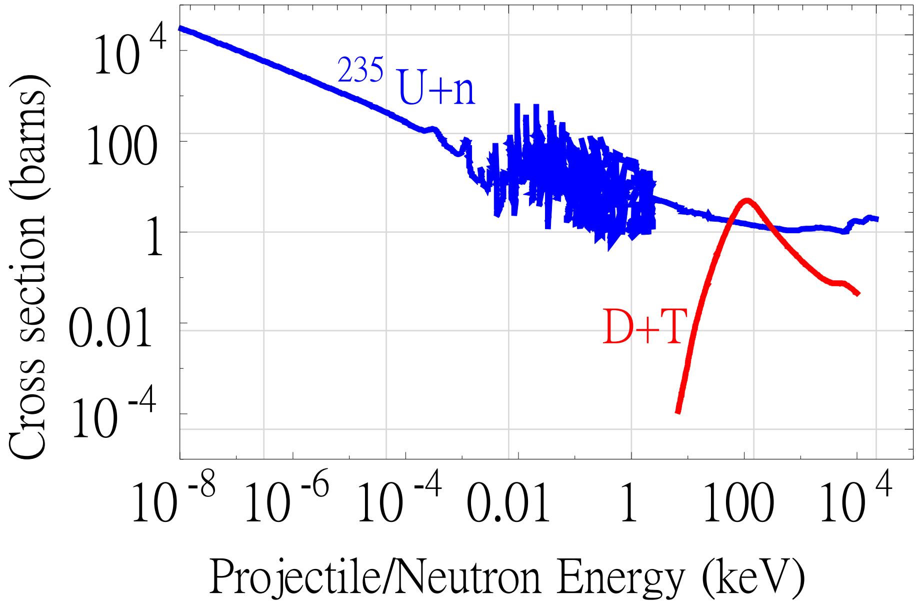
	Half-life (years)
U235	$7.04\mathrm{x}10^{8}$
U238	$4.47\mathrm{x}10^{9}$
…	
Tritium	12.3

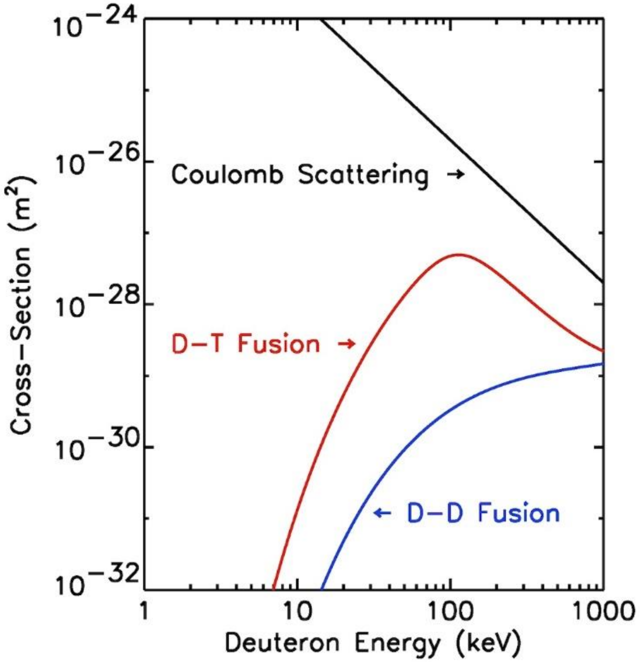
n + Li^{6} → He^{4} + T

Fusion is much harder than fission

- **Fission:** $n +{}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3n + 177\ \text{MeV}$
- **Fusion:** $D + T \rightarrow He^4\,(3.5\ \text{MeV}) + n\,(14.1\ \text{MeV})$

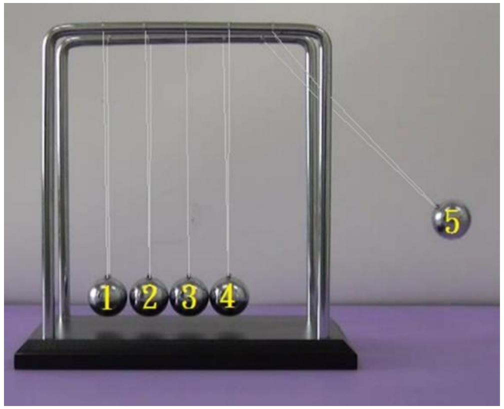




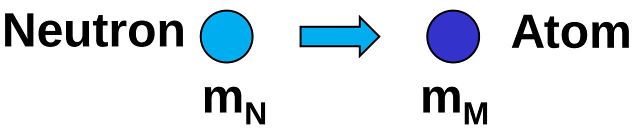


https://www6.lehigh.edu/~eus204/lab/PCL_fusion.php#x1-10096

Fast neutrons are slowed down due to the collisions



- A moderator is used to slow down fast neutrons but not to absorb neutrons.
- For $m_M \sim m_N$, the energy decrement is higher. Therefore, H slows down neutron most efficiently.
- However, H + n → D, i.e., H absorbs neutrons.

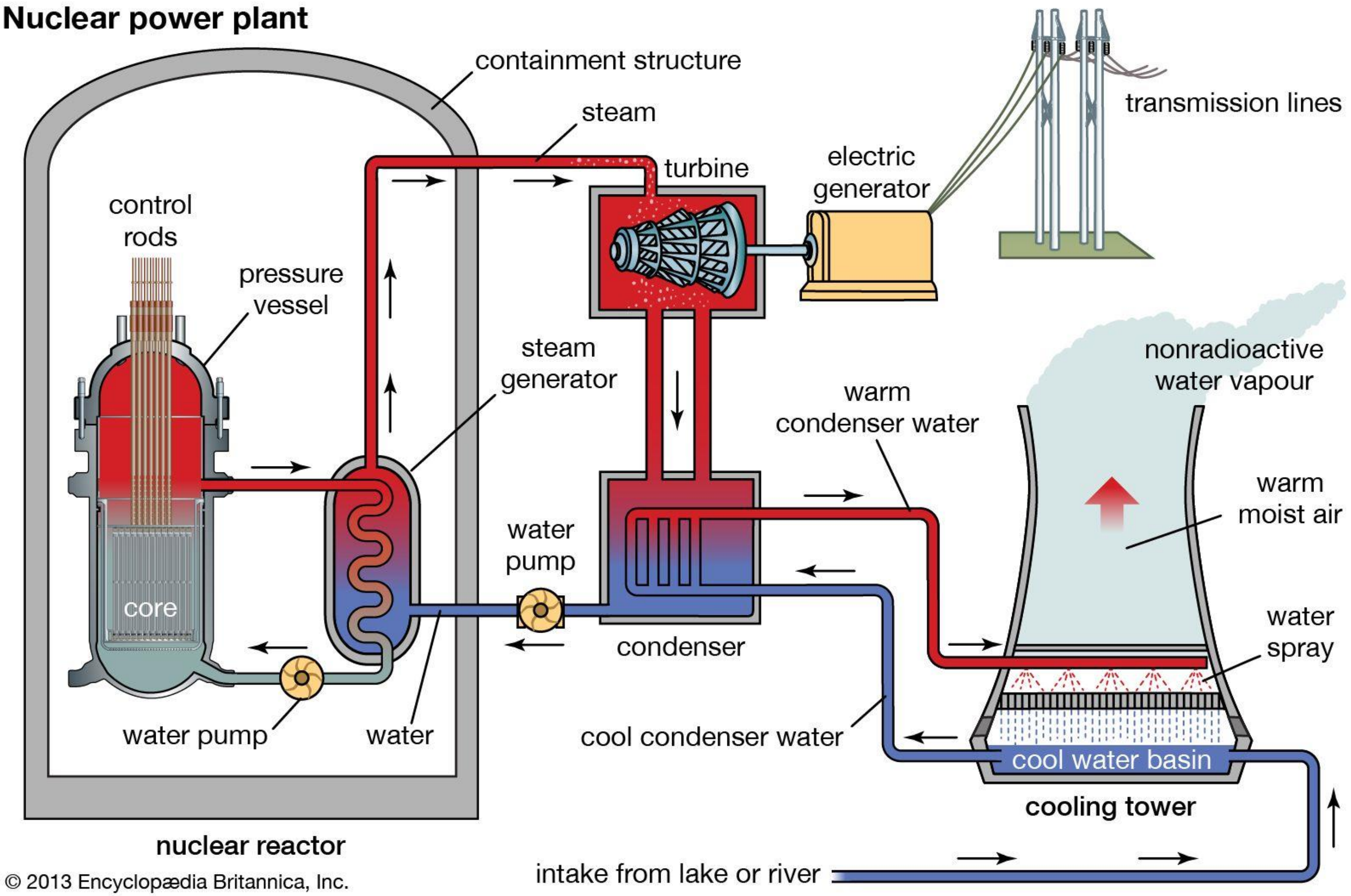

Neutron
Atom
m_N
m_M

- The best option is the D in the heavy water (D_2O).

	Energy decrement	Neutron scattering cross section (σ_s) (Barns)	Neutron absorption cross section (σ_a) (Barns)
H	1	49 (H_2O)	0.66 (H_2O)
D	0.7261	10.6 (D_2O)	0.0013 (D_2O)
C	0.1589	4.7 (Graphite)	0.0035 (Graphite)

https://en.wikipedia.org/wiki/Neutron_moderator#cite_note-Weston-4
https://energyeducation.ca/encyclopedia/Neutron_moderator#cite_note-3

Nuclear power plant



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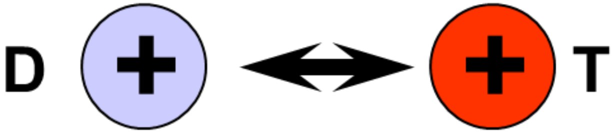
https://www.britannica.com/technology/nuclear-power

Comparison between nuclear fission and nuclear fusion

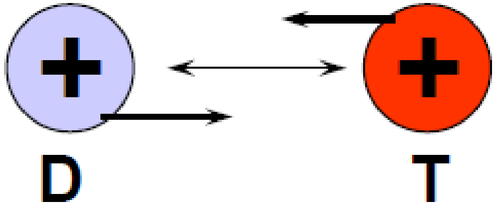
	Nuclear Fission	Nuclear Fusion
Chain reaction	**Yes**	**No**
Melt down	**Possible**	**Impossible**
Nuclear waste	**High radiative**	**Low radiative / None**

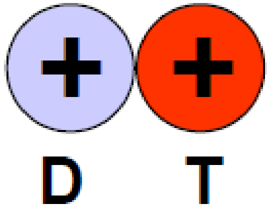
A “hot plasma” at 100M ⁰C is needed

- Probability for fusion reactions to occur is low at low temperatures due to the coulomb repulsion force.



- If the ions are sufficiently hot, i.e., large random velocity, they can collide by overcoming coulomb repulsion





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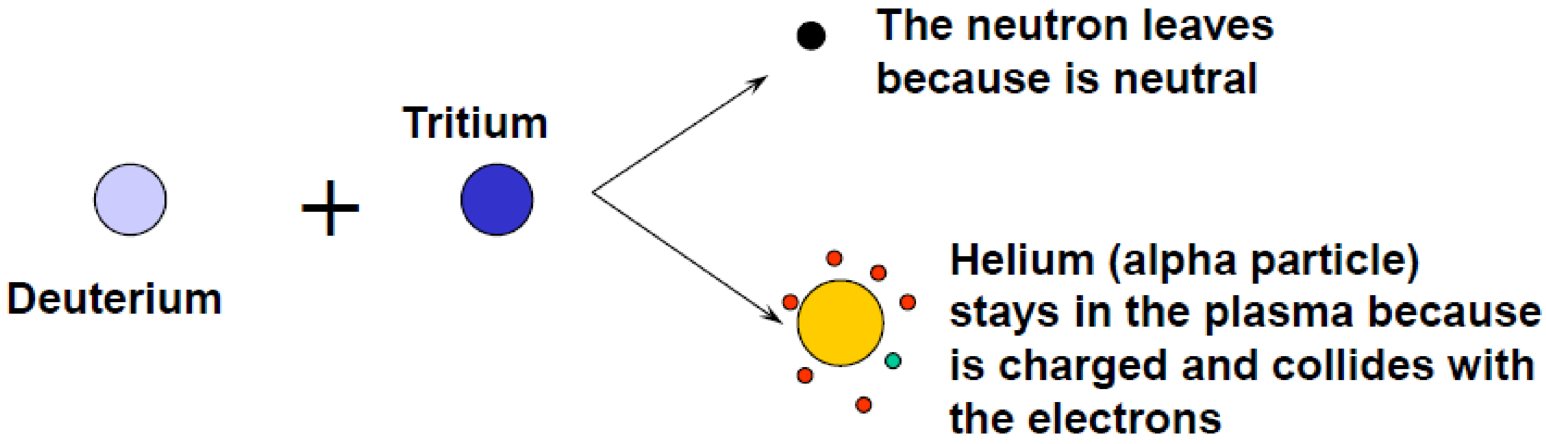
*R. Betti, HEDSA HEDP Summer School, 2015

It takes a lot of energy or power to keep the plasma at 100M ⁰C

- **Let the plasma do it itself!**



- **The α-particles heat the plasma.**

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*R. Betti, HEDSA HEDP Summer School, 2015

Under what conditions the plasma keeps itself hot?

- **Steady state 0-D power balance:**

$S_\alpha+S_h=S_B+S_k$

S_α: α particle heating

S_h: external heating

S_B: Bremsstrahlung radiation

S_k: heat conduction lost

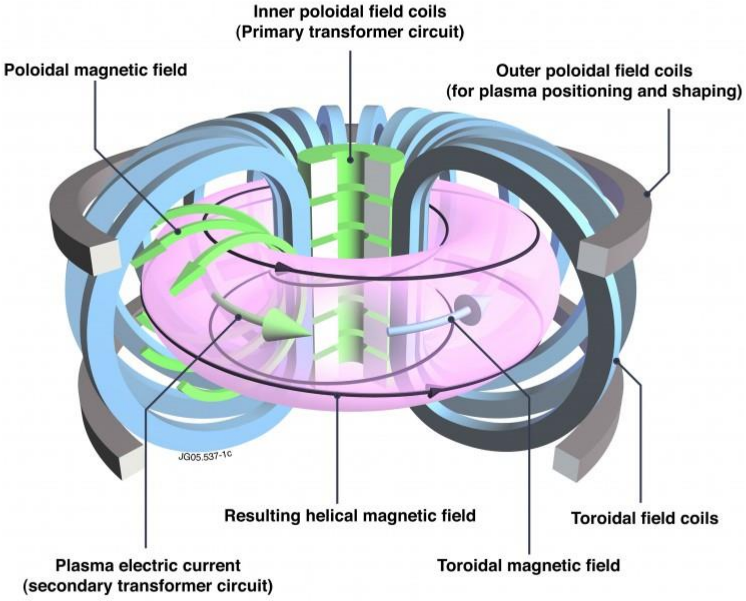
Ignition condition: Pτ > 10 atm-s = 10 Gbar - ns

- **P: pressure, or called energy density**
- **τ is confinement time**

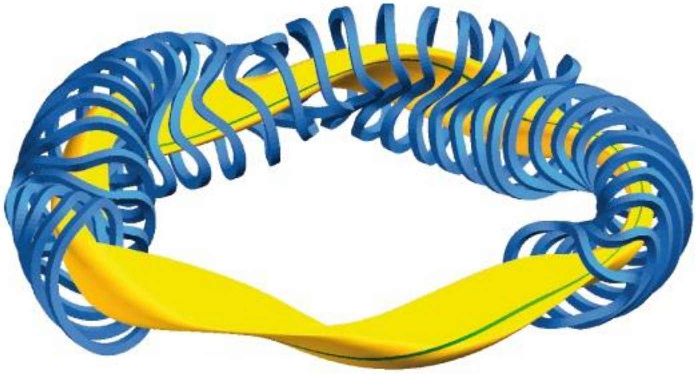
The plasma is too hot to be contained

- **Solution 1: Magnetic confinement fusion (MCF), use a magnetic field to contain it. P~atm, τ~sec, T~10 keV (10^8 ⁰C)**

Tokamak



Stellarator



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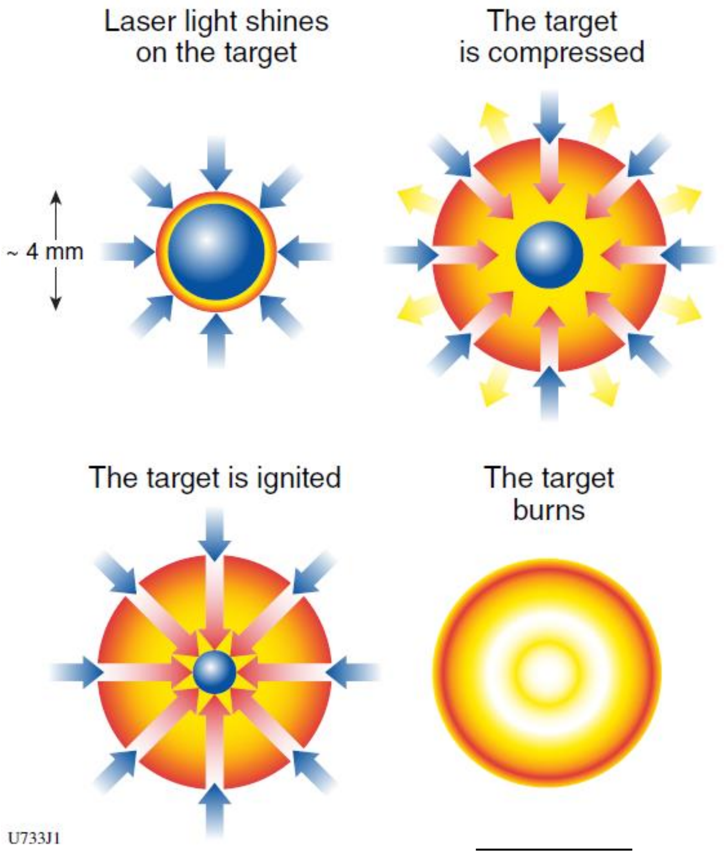
https://www.euro-fusion.org/2011/09/tokamak-principle-2/
https://en.wikipedia.org/wiki/Stellarator

Don’t confine it!

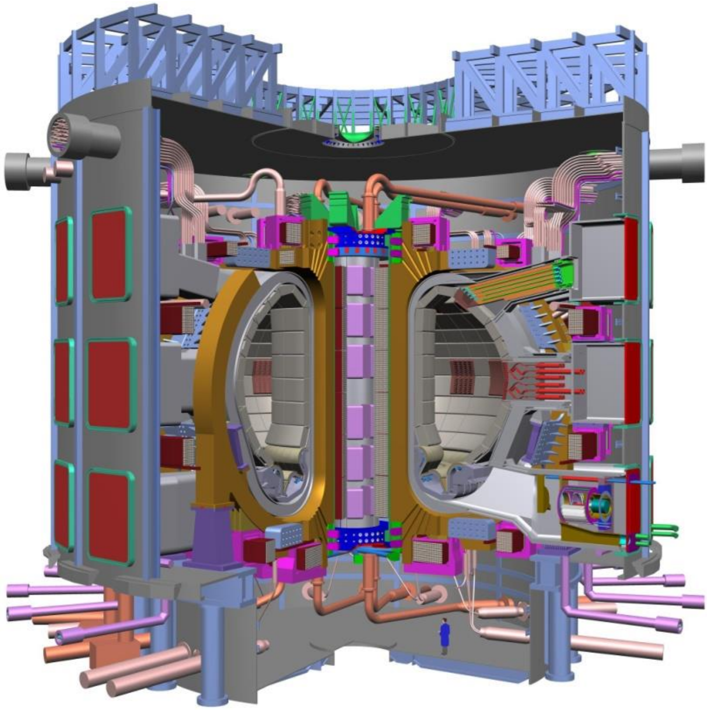
- **Solution 2: Inertial confinement fusion (ICF). Or you can say it is confined by its own inertia: P~Gigabar, τ~nsec, T~10 keV (10^8 ⁰C)**



Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester

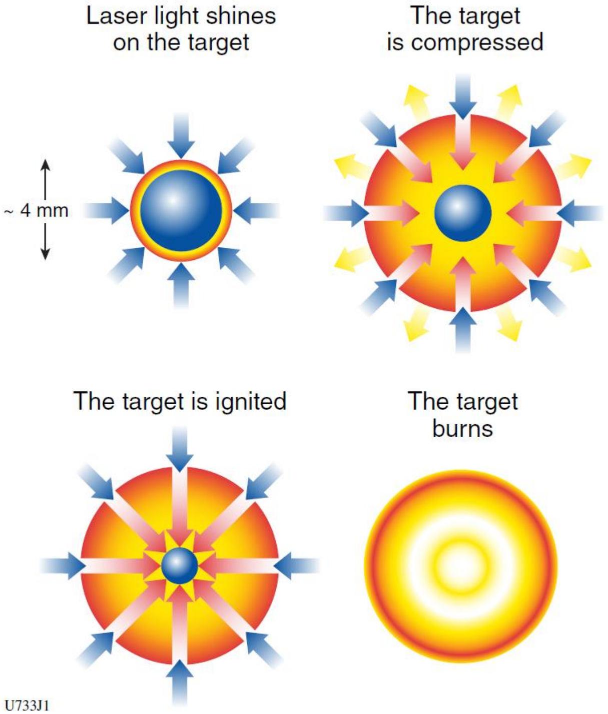
To control? Or not to control?

- **Magnetic confinement fusion (MCF)**



- **Plasma is confined by toroidal magnetic field.**

- **Inertial confinement fusion (ICF)**



- **A DT ice capsule filled with DT gas is imploded by laser.**