Introduction to Nuclear Fusion as An Energy Source

Po-Yu Chang

Institute of Space and Plasma Sciences, National Cheng Kung University

2024 spring semester

Wednesday 9:10-12:00

Materials:

https://capst.ncku.edu.tw/PGS/index.php/teaching/

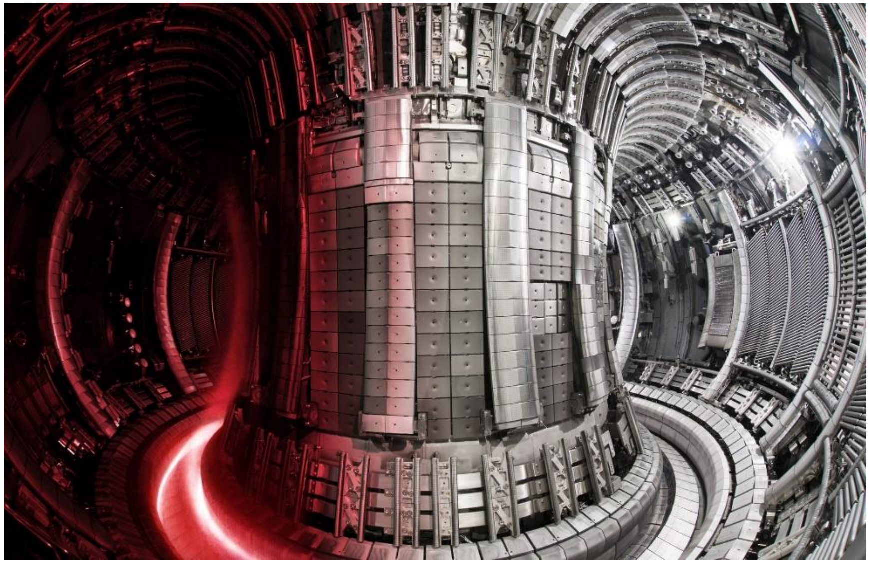
Online courses:

https://nckucc.webex.com/nckucc/j.php?MTID=ma76b50f97b1c6d72db61de9eaa9f0b27

2024/2/21 updated

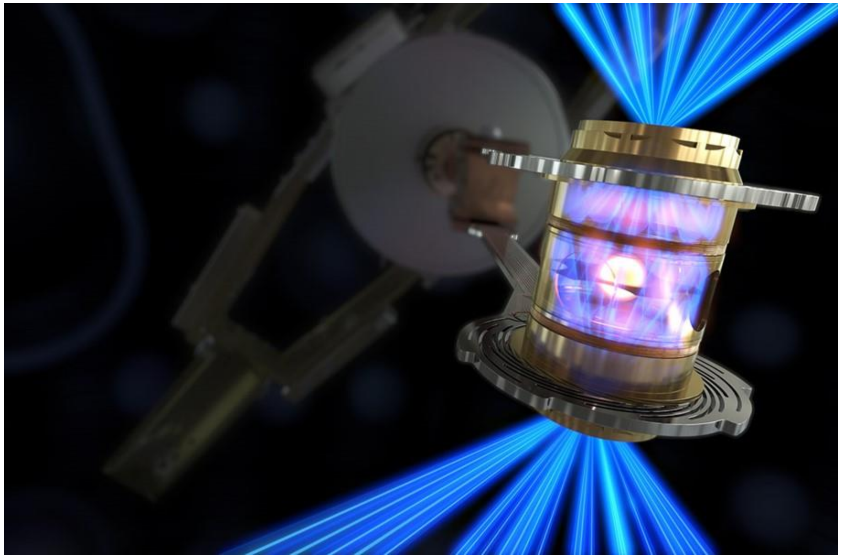
Significant breakthrough is achieved recently

- **Magnetic confinement fusion (MCF)**



- **On 2024/2/(8), record-breaking 69.26 megajoules of sustained fusion energy in Joint European Torus (JET) facility in Oxford demonstrates powerplant potential and strengthens case for ITER.**

- **Inertial confinement fusion (ICF)**

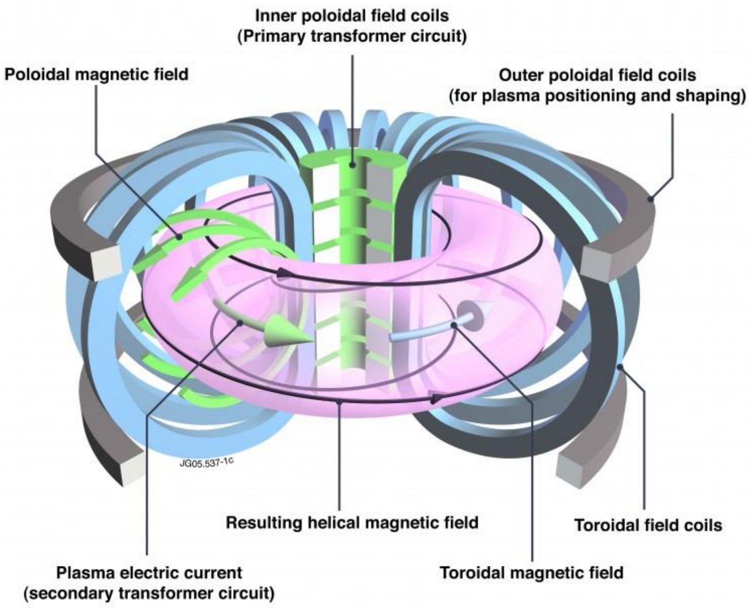


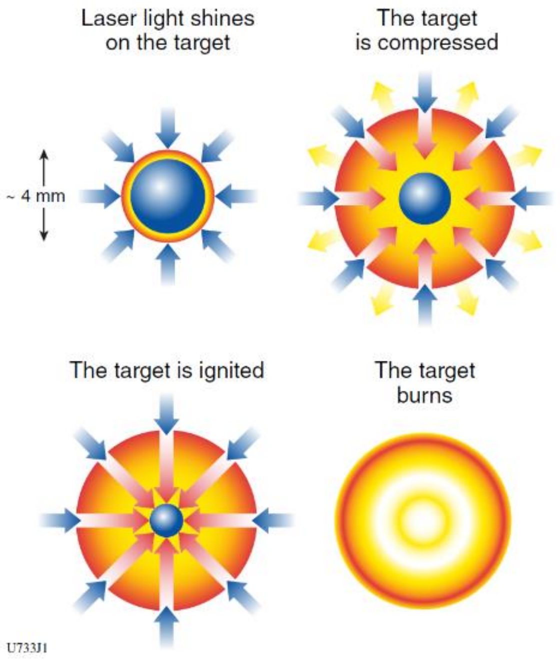
- **National Ignition Facility (NIF) demonstrated a gain grater than 1 for the first time on 2022/12/5. The yield of 3.15 MJ from the 2.05-MJ input laser energy, i.e., Q=1.5.**

https://ccfe.ukaea.uk/resources/#gallery
https://www.science.org/content/article/historic-explosion-long-sought-fusion-breakthrough

Nuclear fusion as an energy source is being developed

- **Magnetic confinement fusion (MCF)**
- **Inertial confinement fusion (ICF)**



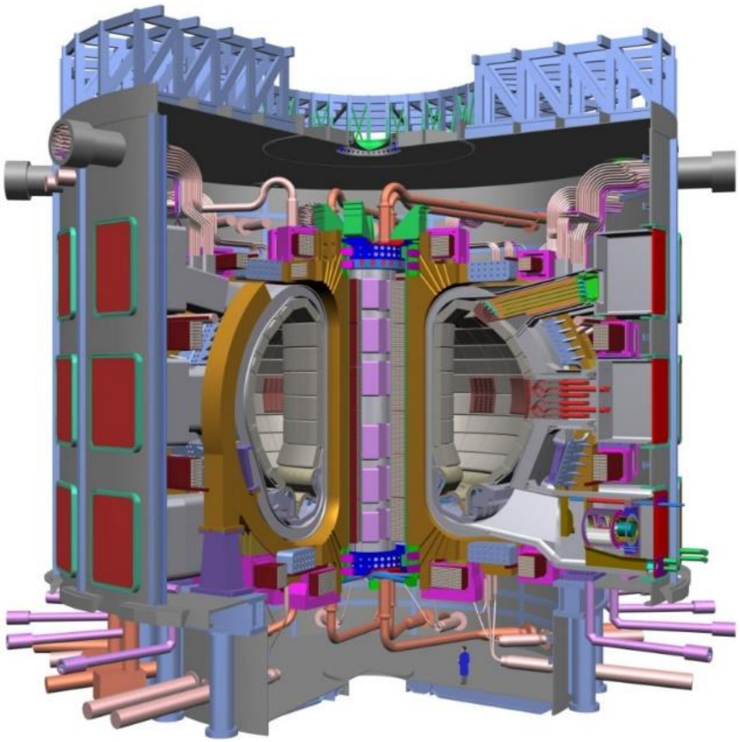


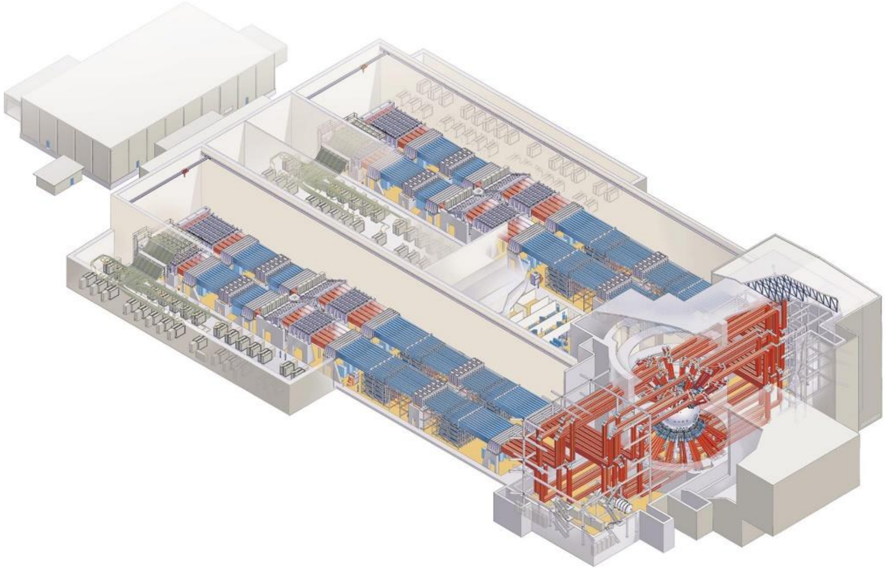
---

https://www.euro-fusion.org/2011/09/tokamak-principle-2/
Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester

Nuclear fusion as an energy source is being developed

- **Magnetic confinement fusion (MCF)**
- **Inertial confinement fusion (ICF)**

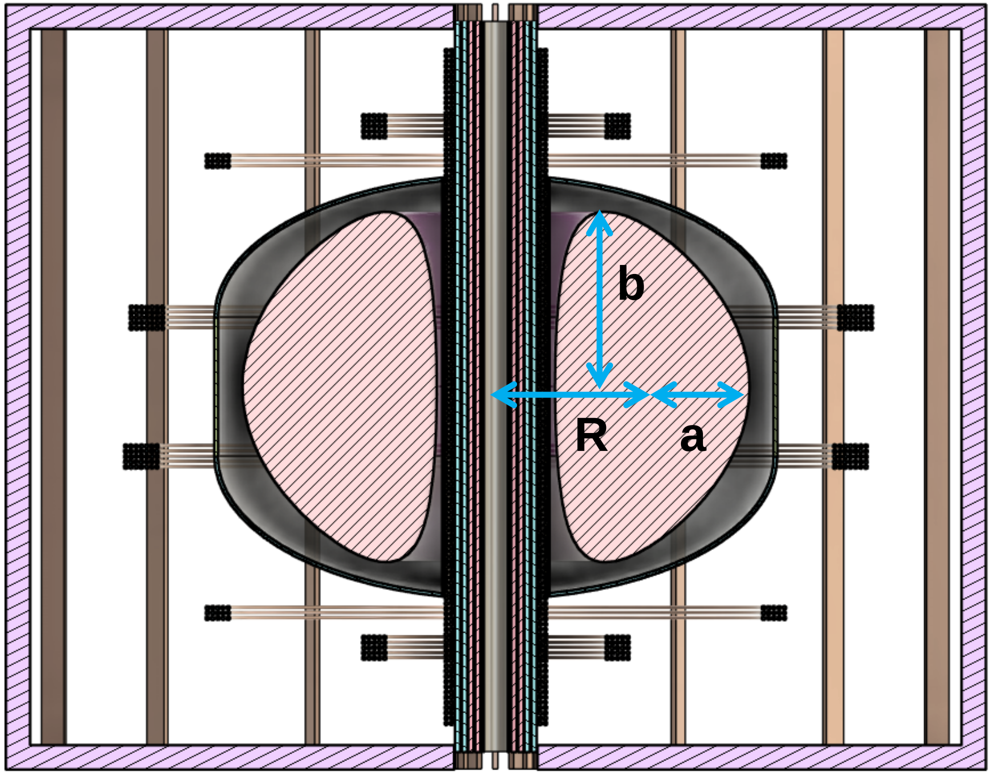




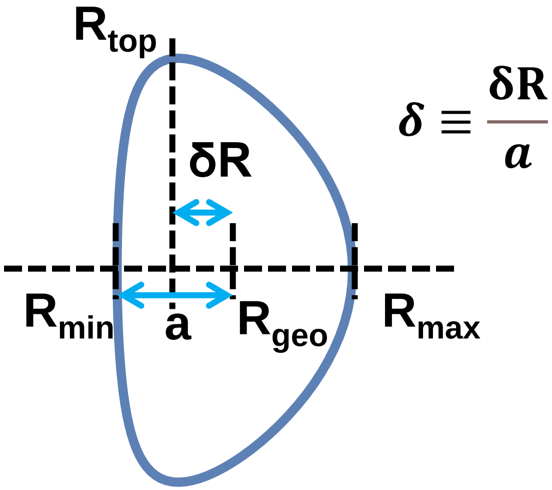
---

https://www.iter.org
https://zh.wikipedia.org/wiki/國家點火設施

Formosa Integrated Research Spherical Tokamak (FIRST)



- **Parameters:**
  - **R=45 cm**
  - **a=28 cm**
  - **Aspect Raio = R/a=1.61**
  - **Elongation κ=b/a=1.8**
  - **Triangularity δ=0.5**
  - **T~100 eV**
  - **B_T~0.5 T**
  - **I_p~100 kA**



$$\delta \equiv \frac{\delta R}{a}$$

$$r = R + a\mathrm{Cos}\big(\theta + \delta \mathrm{Sin}(\theta)\big)$$

$$z = a\kappa \mathrm{Sin}(\theta)$$

- **We welcome anyone interested in fusion research to join us!**

Course Outline

- Brief background reviews
  - Electromagnetics
  - Plasma physics
- Introduction to nuclear fusion
  - Nuclear binding energy (Fission vs Fusion)
  - Fusion reaction physics
  - Some important fusion reactions (Cross section)
    - Main controlled fusion fuels
    - Advanced fusion fuels
  - Maxwell-averaged fusion reactivities

Course Outline

- **Introduction to nuclear fusion (cont.)**
  - **Collisions (Bremsstrahlung radiation)**
  - **Columb scattering. Cross section of the Columb scattering**
  - **Beam-target fusion vs thermonuclear fusion**
  - **Lawson criteria, ignition conditions**
  - **Magnetic confinement fusion (MCF) vs Inertial confinement fusion (ICF)**

Course Outline

- **Magnetic confinement fusion (MCF)**
  - **Gyro motion, MHD**
  - **1D equilibrium (z pinch, theta pinch)**
  - **Drift: ExB drift, grad B drift, and curvature B drift**
  - **Tokamak, Stellarator (toroidal field, poloidal field)**
  - **Magnetic flux surface**
  - **2D axisymmetric equilibrium of a torus plasma: Grad-Shafranov equation.**
  - **Stability (Kink instability, sausage instability, Safety factor Q)**
  - **Central-solenoid (CS) start-up (discharge) and current drive**
  - **CS-free current drive: electron cyclotron current drive, bootstrap current.**
  - **Auxiliary Heating: ECRH, Ohmic heating, Neutral beam injection.**

Course Outline

- Inertial confinement fusion (ICF)
  - Plasma frequency and critical density
  - Direct- and indirect- drive
  - Laser generated pressure (Inverse bremsstrahlung and Ablation pressure)
  - Burning fraction, why compressing a capsule?
  - Implosion dynamics
  - Shock (Compression with different adiabat)
  - Laser pulse shape
  - Rocket model, shell velocity
  - Laser-plasma interaction (Stimulated Raman Scattering, SRS; Stimulated Brillouin Scattering, SBS; Two-plasmon decay)
  - Instabilities (Rayleigh-taylor instability, Kelvin-Helmholtz instability, Richtmeyer-Meshkov instability)

Course Outline

- **Innovation Fusion scheme**
- **Status of fusion research in Taiwan**
  - **Formosa Integrated Research Spherical Tokamak (FIRST)**

References

- The physics of inertial fusion, by Stefano Atzeni and Jürgen Meyer-Ter-Vehn
- Nuclear fusion, by Edward Morse
- Ideal magnetohydrodynamics, by Jeffrey P. Freidberg
- Introduction to plasma theory, by Dwight R. Nicholson
- Introduction to plasma physics and controlled fusion, by Francis F. Chen
- Principles of plasma physics for engineers and scientists, by Umran S. Inan and Marek Golkowski
- Introduction to plasma physics, by Gurnett and Bhattacharjee
- The physics of plasma, by T. J. M. Boyd and J. J. Sanderson
- Principles of plasma physics, by Krall and Trivelpiece
- NRL Plasma Formulary, Naval Research Laboratory, 2013 by J. D. Huba

Grading

- **Midterm 40 % @ 4/17**
- **Final exam 60 % @ 6/19**

- **No class: 2/28, 3/6.**
- **Online class: 3/20.**

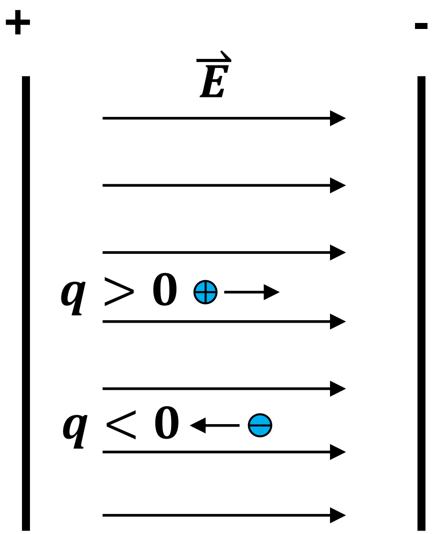
Course Outline

- **Brief background reviews**
  - **Electromagnetics**
  - **Plasma physics**
- Introduction to nuclear fusion
  - Nuclear binding energy (Fission vs Fusion)
  - Fusion reaction physics
  - Some important fusion reactions (Cross section)
    - Main controlled fusion fuels
    - Advanced fusion fuels
  - Maxwell-averaged fusion reactivities

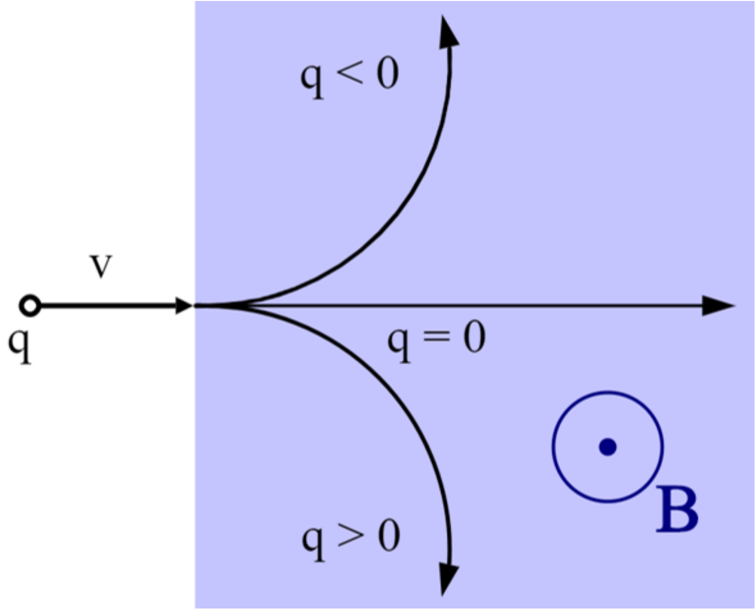
Charged particles are accelerated due to Lorentz force under electromagnetic fields

- **Lorentz force:** $\vec{F} = m\,\vec{a} = q\,\vec{E} + q\,\vec{v} \times \vec{B}$ $\quad m\dfrac{d\,\vec{v}}{dt} = q\,\vec{E} + q\,\vec{v} \times \vec{B}$

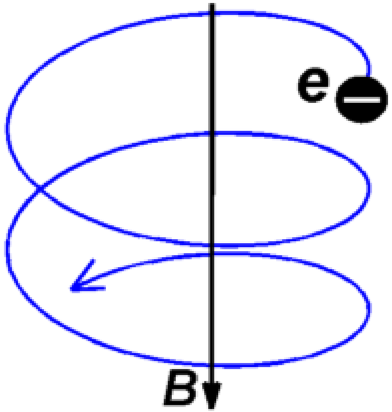
- **Force under electric fields**

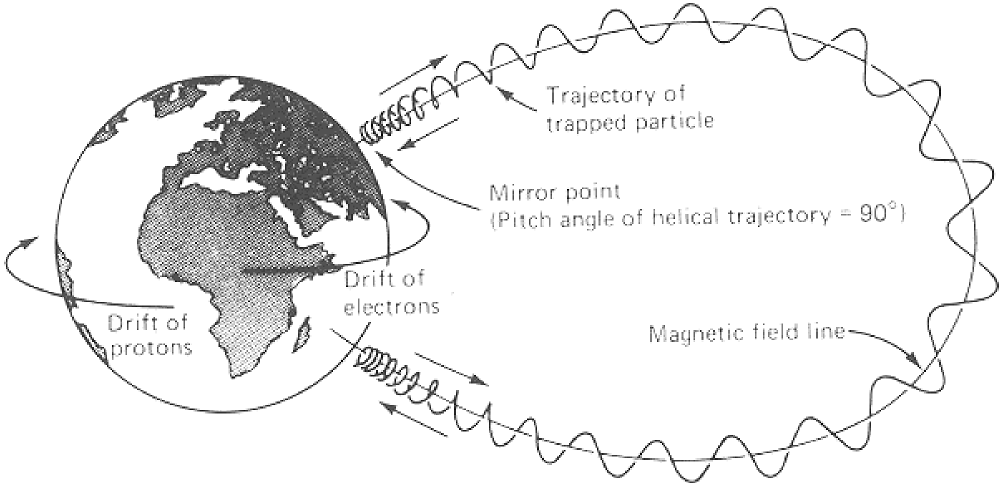


- **Force under magnetic fields**



Charged particles gyro around magnetic field lines

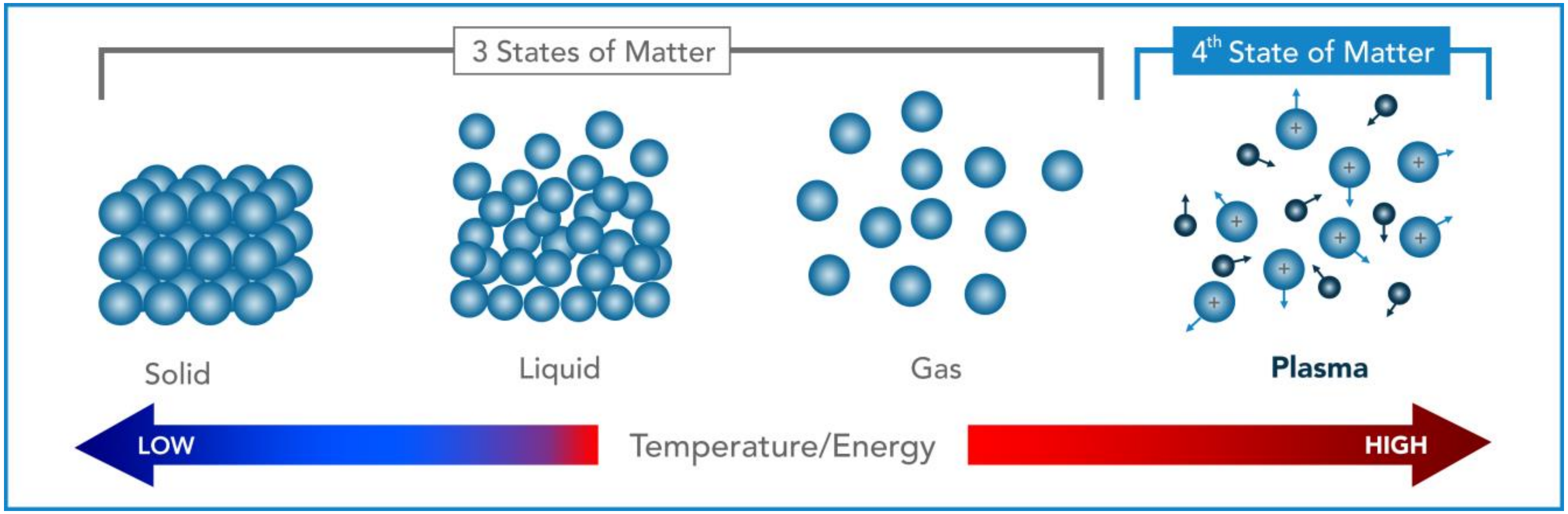




---

http://www.ipp.cas.cz/vedecka_struktura_ufp/tokamak/tokamak_compass/diagnostics/mikrovlnne-diagnostiky/ece-ebw-radiometr.html
http://www-ssg.sr.unh.edu/tof/Smart/Students/lees/periods.html

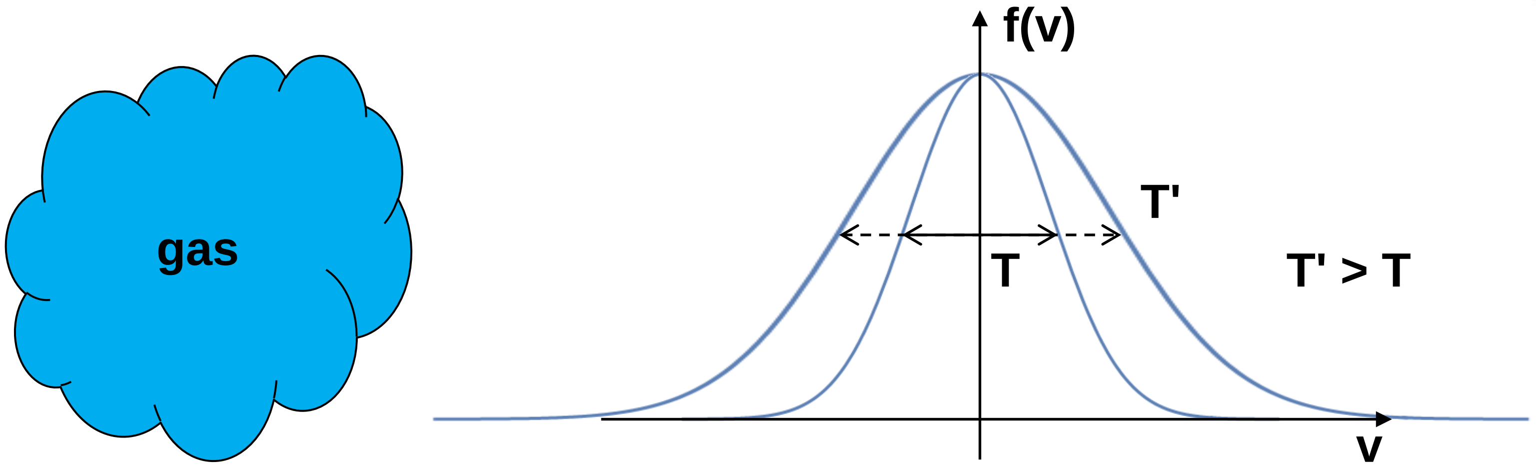
Plasma is the 4th state of matter

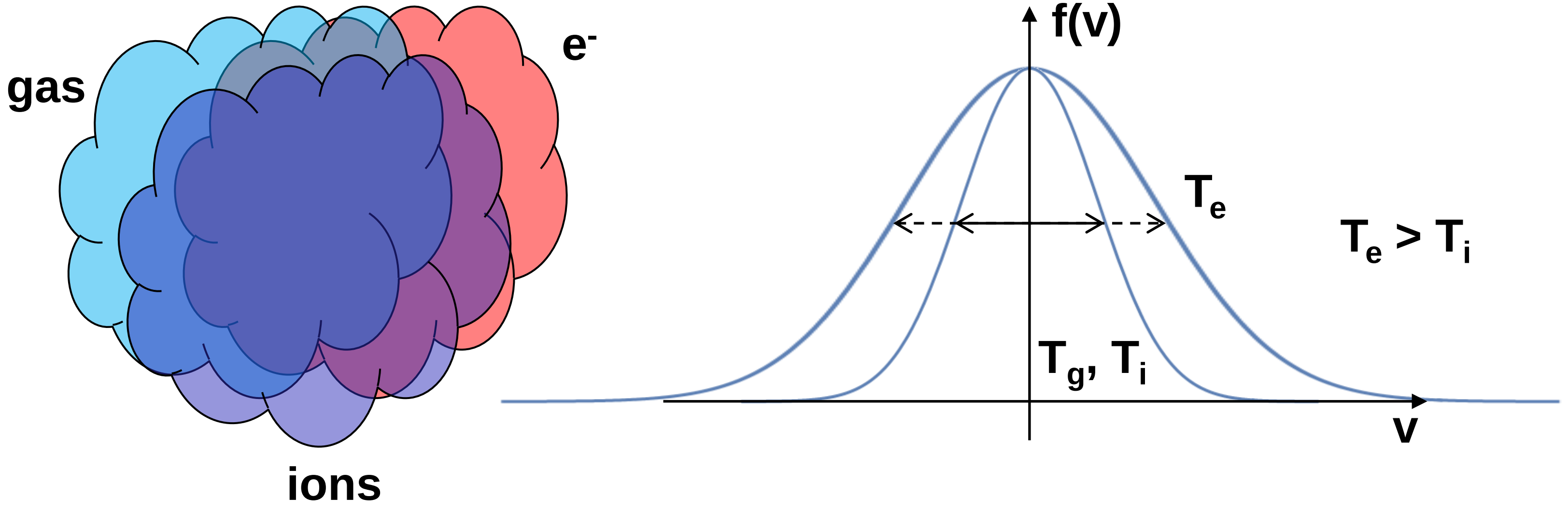


---

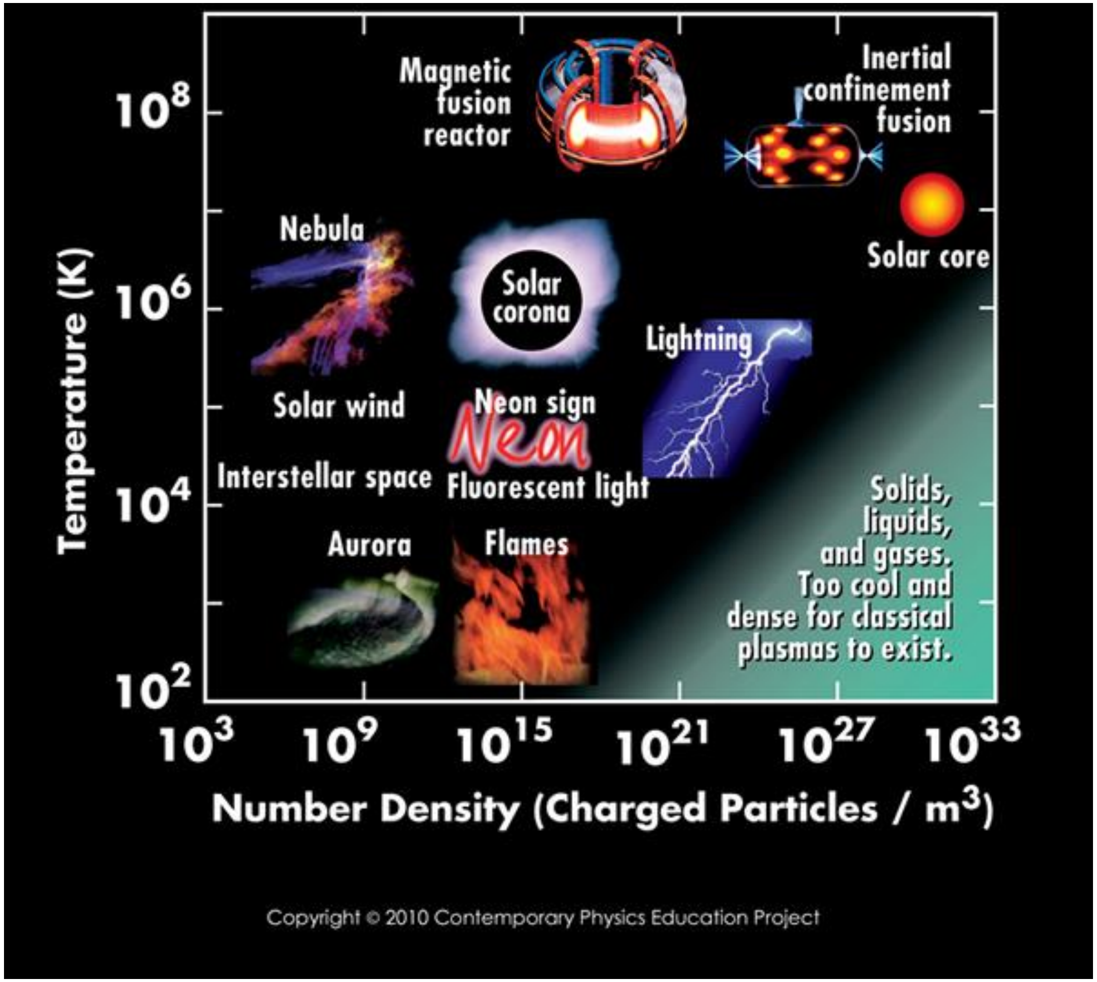
http://tetronics.com/our-technology/what-is-plasma/

In plasma, there are ions, electrons, and neutral gas





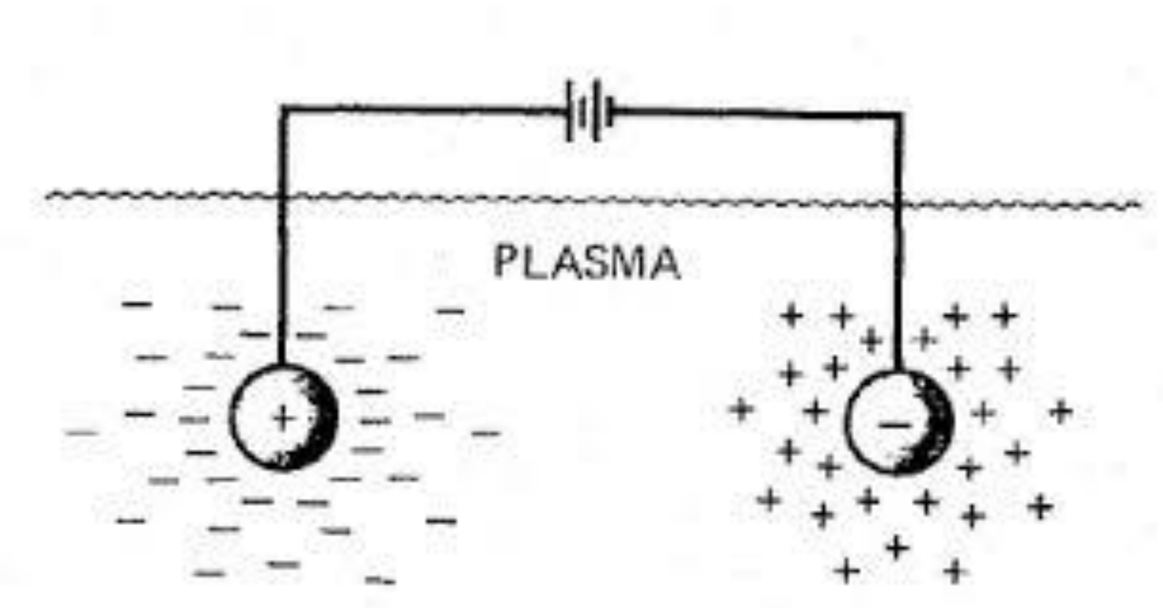
Nuclear fusion occurs at a very high temperature region



---

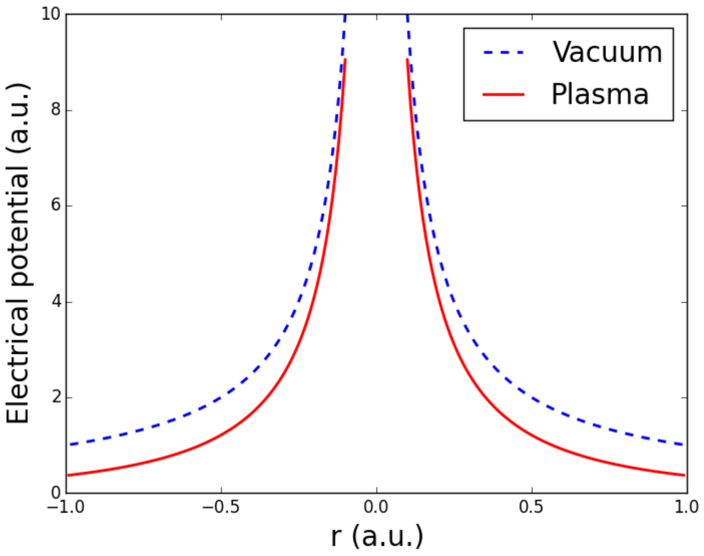
https://lasers.llnl.gov/science/understanding-the-universe/plasma-physics

A test ion in the plasma gathers a shielding cloud that tends to cancel its own charge



Francis F. Chen, \Introduction to plasma physics and controlled fusion"

Debye shielding is a phenomenon such that the potential due to a test charge in a plasma falls off much faster than in vacuum



- **Vacuum potential:**

$$\boldsymbol{\phi = \frac{\phi_0}{r}}$$

- **Poisson's equation:**

$$\nabla^2\phi = 4\pi e(n_\mathrm{e} - n_\mathrm{i}) - 4\pi q_\mathrm{T}\delta(\vec{r})$$

- **Density profile:**

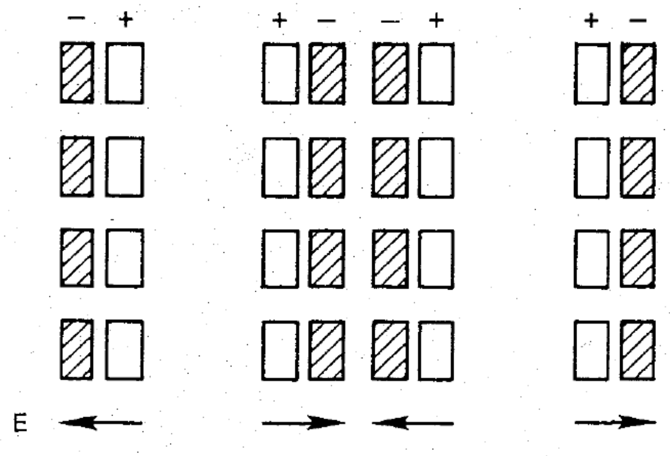
$$n_\mathrm{e} = n_0 \exp\left(\frac{e\phi}{KT_\mathrm{e}}\right) \quad , \quad n_\mathrm{i} = n_0$$

- **For $\vec{r} \neq 0$ and assuming $\frac{e\phi}{T_\mathrm{e}} \ll 1$**

$$\frac{1}{r^2}\frac{d}{\mathrm{dr}}\left(r^2\frac{\mathrm{d}\phi}{\mathrm{dr}}\right) = \frac{4\pi n_0 e^2}{KT_\mathrm{e}}\phi$$

$$\phi = \frac{\phi_0}{r}\exp\left(-\frac{r}{\lambda_\mathrm{D}}\right) \qquad \lambda_D \approx \left(\frac{KT_\mathrm{e}}{4\pi n e^2}\right)^{1/2}$$

Electron plasma frequency is the characteristic frequency such that electrons oscillate around their equilibrium positions



Mechanism of plasma oscillations.

- **Assumption:**

$$\vec{\nabla} \equiv \hat{x}\frac{\partial}{\partial x}\ ,\ \vec{E} = \hat{x}E\ ,\ \ \vec{v}_{\mathrm{e}} = \hat{x}v\ \ ,$$

$$\vec{\nabla} \times \vec{E} = 0\ ,\ \vec{E} = -\vec{\nabla}\phi$$

- **Continuity and momentum equation for electron:**

$$\frac{\partial n_{\mathrm{e}}}{\partial t} + \vec{\nabla}\cdot(n_{\mathrm{e}}\vec{v}_{\mathrm{e}}) = 0$$

$$m_{\mathrm{e}}n_{\mathrm{e}}\left[\frac{\partial \vec{v}_{\mathrm{e}}}{\partial t} + \left(\vec{v}_{\mathrm{e}}\cdot\vec{\nabla}\right)\vec{v}_{\mathrm{e}}\right] = -en_{\mathrm{e}}\vec{E}$$

- **Gauss' law:**

$$\frac{\partial E}{\partial x} = 4\pi\mathrm{e}(n_{\mathrm{i}} - n_{\mathrm{e}})$$

Electron plasma frequency is obtained by linearizing the hydrodynamic equations

- **The oscillation is assumed to be small:**

$$n_e = n_0 + n_1,\ \vec{E} = \vec{E}_0 + \vec{E}_1,\ v_e = v_0 + v_1$$

where

$$\frac{\partial n_0}{\partial x} = v_0 = \vec{E}_0 = 0$$

$$\frac{\partial n_0}{\partial t} = \frac{\partial v_0}{\partial t} = \frac{\partial \vec{E}_0}{\partial t} = 0$$

- **Linearization:**

$$m_e \frac{\partial v_1}{\partial t} = -\mathrm{e}\mathrm{E}_1$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} = 0$$

$$\frac{\partial E_1}{\partial x} = -4\pi \mathrm{e} \mathrm{n}_1$$

- **Plane wave solution:**

$$\eta_1 = \hat{\eta} \exp[i(kx - \omega t)]$$

$$\eta_1 = v_1, n_1, E_1$$

- **Substitute into the previous equations:**

$$-i m_e \omega v_1 = -e E_1$$

$$-i\omega n_1 = -n_0 i k v_1$$

$$i k E_1 = -4\pi e n_1$$

- **Electron plasma frequency is obtained by eliminating n_1 and E_1:**

$$\omega_{pe} \equiv \omega = \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2}$$

Plasma β is the ratio between hydro pressure and magnetic pressure

- **Momentum equation in Magnetohydrodynamics (MHD) approach:**

$$\rho_{\mathrm{m}}\left[\frac{\partial \vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\,\vec{v}\right]=\vec{j}\times\vec{B}-\nabla\cdot\overleftrightarrow{P}\Rightarrow\vec{j}\times\vec{B}-\nabla P$$

$$\nabla\times\vec{B}=\mu_{\mathrm{o}}\,\vec{j}$$

$$\vec{j}\times\vec{B}=\frac{1}{\mu_{\mathrm{o}}}\left(\vec{\nabla}\times\vec{B}\right)\times\vec{B}=\frac{1}{\mu_{\mathrm{o}}}\left[\left(\vec{B}\cdot\vec{\nabla}\right)\vec{B}-\frac{1}{2}\,\vec{\nabla}\,B^2\right]$$

$$\rho_{\mathrm{m}}\left[\frac{\partial \vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\,\vec{v}\right]=-\nabla\left(P+\frac{B^2}{2\mu_{\mathrm{o}}}\right)+\frac{1}{\mu_o}\left(\vec{B}\cdot\vec{\nabla}\right)\vec{B}$$

- **Magnetic pressure:** $\dfrac{B^2}{2\mu_{\mathrm{o}}}$

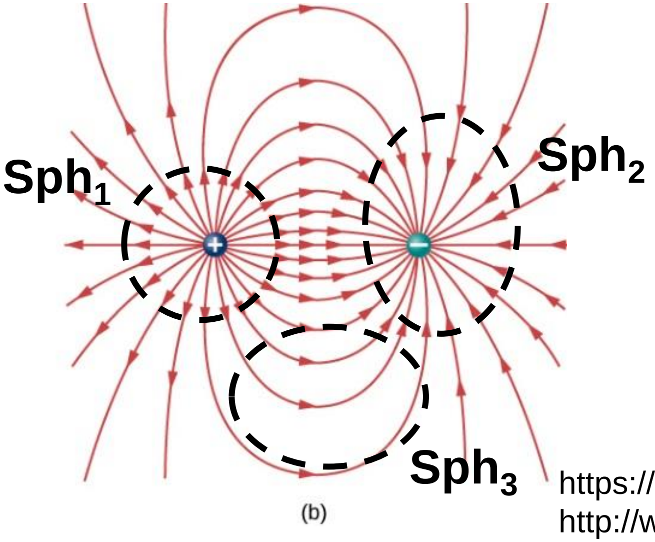
$$\beta\equiv\frac{P}{B^2/2\mu_{\mathrm{o}}}$$

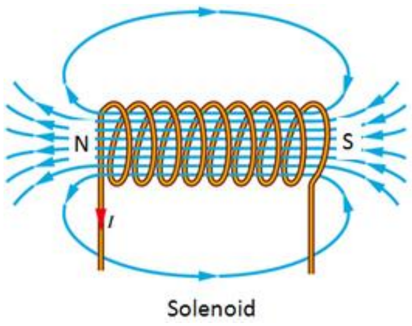
- **Magnetic tension:** $\dfrac{1}{\mu_o}\left(\vec{B}\cdot\vec{\nabla}\right)\vec{B}$

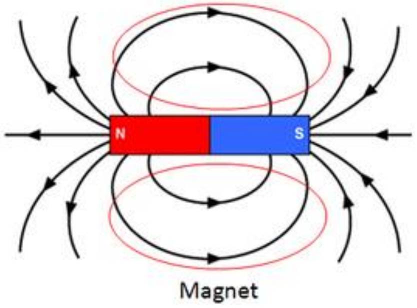
Maxwell’s equations

- **Gauss’s law:** $\nabla \cdot \vec{E} = \frac{\rho_q}{\epsilon_o}$
- **No name:** $\nabla \cdot \vec{B} = 0$
- **Faraday’s law:** $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- **Ampére’s law:** $\nabla \times \vec{B} = \mu_o \vec{j} + \epsilon_o \mu_o \frac{\partial \vec{E}}{\partial t}$
- **Lorentz force:** $\vec{F} = m\,\vec{a} = q\,\vec{E} + q\,\vec{v} \times \vec{B}$

Displacement current







https://pressbooks.online.ucf.edu/osuniversityphysics2/chapter/electric-field-lines/
http://www.excelatphysics.com/magnetic-field-pattern.html

Magnetohydrodynamics description of plasma

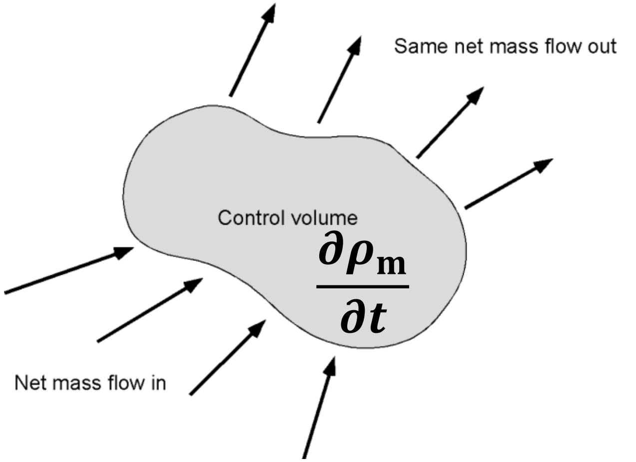
- **Continuity eq:** $$\frac{\partial \rho_{\mathrm{m}}}{\partial t} + \nabla \cdot (\rho_m \vec{v}) = 0$$
- **Momentum eq:** $$\rho_{\mathrm{m}} \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = \rho_{\mathrm{q}} \vec{E} + \vec{j} \times \vec{B} - \nabla \cdot \overleftrightarrow{P}$$
- **Ohm's law:** $$\vec{j} = \sigma\left(\vec{E} + \vec{v} \times \vec{B}\right)$$
- **Equation of state:** $$\frac{d}{dt}\left(P {\rho_{\mathrm{m}}}^{-\gamma}\right) = 0$$
- **Maxwell's eqs:**

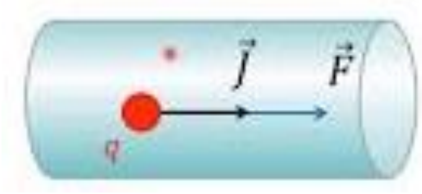
$$\nabla \cdot \vec{E} = \frac{\rho_{\mathrm{q}}}{\epsilon_o}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_{\mathrm{o}} \vec{j} + \epsilon_{\mathrm{o}} \mu_{\mathrm{o}} \frac{\partial \vec{E}}{\partial t}$$





https://eaglepubs.erau.edu/introductiontoaerospaceflightvehicles/chapter/conservation-of-mass-continuity-equation/
https://www.youtube.com/watch?v=lu0Ep8_Gp8U

Magnetohydrodynamics description of plasma w/ low-freq. and long-wavelength approximation

- **Continuity eq:** $\frac{\partial \rho_{\mathrm{m}}}{\partial t} + \nabla \cdot (\rho_m \vec{v}) = 0$ **w/ long wavelength ($\lambda >> \lambda_d$)**
- **Momentum eq:** $\rho_{\mathrm{m}} \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = \cancel{\rho_{\mathrm{q}} \vec{E}} + \vec{j} \times \vec{B} - \nabla \cdot \overleftrightarrow{P}$
- **Ohm's law:** $\vec{j} = \sigma\left(\vec{E} + \vec{v} \times \vec{B}\right)$
- **Equation of state:** $\frac{d}{dt}\left(P {\rho_{\mathrm{m}}}^{-\gamma}\right) = 0$
- **Maxwell's eqs:**

$$\nabla \cdot \vec{E} = \frac{\rho_{\mathrm{q}}}{\epsilon_o} \approx 0$$ **w/ long wavelength ($\lambda >> \lambda_d$) => quasi neutral**

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_{\mathrm{o}} \vec{j} + \cancel{\epsilon_{\mathrm{o}} \mu_{\mathrm{o}} \frac{\partial \vec{E}}{\partial t}}$$

w/ low freq. ($\omega << \omega_{\mathrm{pe}}$)

Magnetohydrodynamics description of plasma w/ low-freq. and long-wavelength approximation

- **Continuity eq:** $\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{v}) = 0$
- **Momentum eq:** $\rho_m \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = \vec{j} \times \vec{B} - \nabla \cdot \overleftrightarrow{P}$
- **Ohm's law:** $\vec{j} = \sigma (\vec{E} + \vec{v} \times \vec{B})$
- **Equation of state:** $\frac{d}{dt}(P {\rho_m}^{-\gamma}) = 0$
- **Maxwell's eqs:**

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

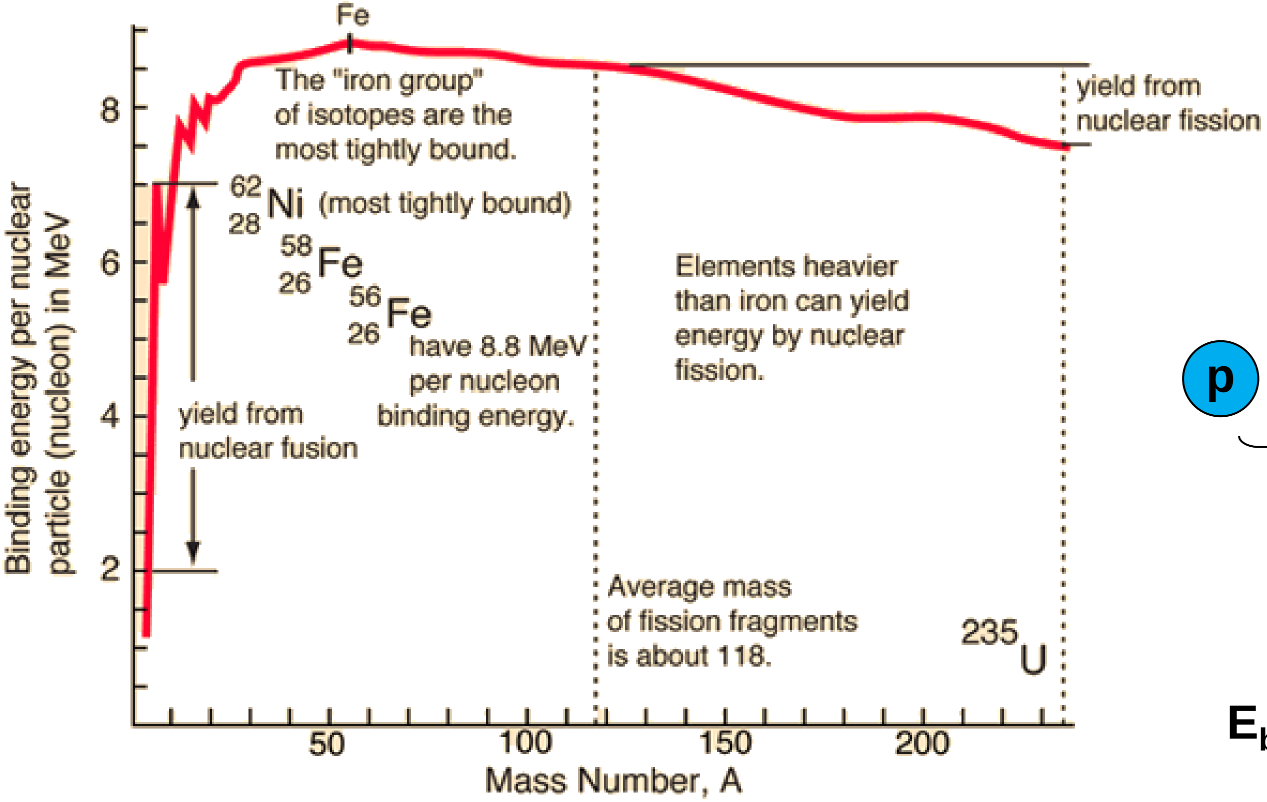
$$\nabla \times \vec{B} = \mu_o \vec{j} \quad \underset{\nabla \cdot}{\Rightarrow} \quad \nabla \cdot \vec{j} = 0$$

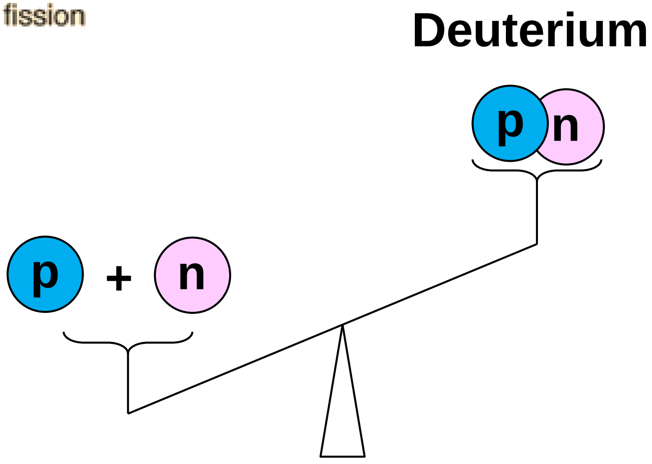
- **Force balance condition:** $0 = \vec{j} \times \vec{B} - \nabla \cdot \overleftrightarrow{P}$

Course Outline

- Brief background reviews
  - Electromagnetics
  - Plasma physics
- Introduction to nuclear fusion
  - Nuclear binding energy (Fission vs Fusion)
  - Fusion reaction physics
  - Some important fusion reactions (Cross section)
    - Main controlled fusion fuels
    - Advanced fusion fuels
  - Maxwell-averaged fusion reactivities

The “iron group” of isotopes are the most tightly bound





$$m_p + m_n > m_D$$

$$E_{binding} = [(m_p + m_n) - m_D]c^2$$

$$Q = \left(\sum_i m_i - \sum_f m_f\right) c^2$$

$$\Delta m = z m_p + (A - z) m_n - m$$

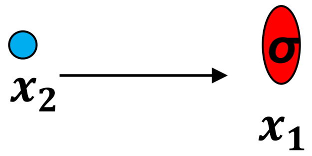
- **Binding energy:** $B = \Delta m c^2$
- **Output energy:** $Q = \sum_f B_f - \sum_i B_i$

---

http://hyperphysics.phy-astr.gsu.edu/hbase/nucene/nucbin.html

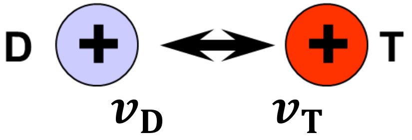
Cross section measures the probability per pair of particles for the occurrence of the reaction

$x_1 + x_2 \rightarrow x_3 + x_4$



- **The hard sphere cross section:**
  $\sigma \approx \pi R^2$ where R ≈ 5x10^{-15} m is the nuclear radius, i.e.,
  $\sigma = 0.8 \times 10^{-28}$ m^2 ≈ 1 barm. (barm ≡ 10^{-28} m^2)

- **Classical cross section:**

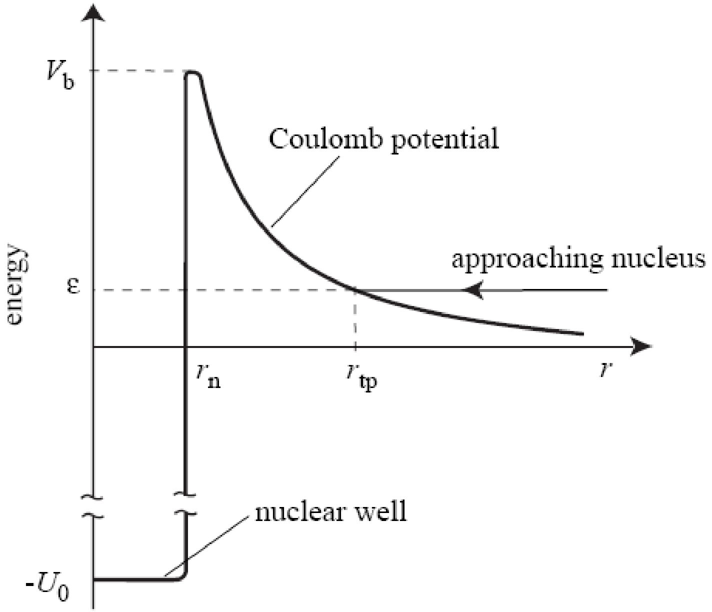


$$\frac{m_\mathrm{D}}{2} v_\mathrm{D}{}^2 + \frac{m_\mathrm{T}}{2} v_\mathrm{T}{}^2 \geq \frac{e^2}{4\pi\epsilon_\mathrm{o} R}$$

- **Let** $v = \left|\vec{v}_D - \vec{v}_T\right|$

$$v_\mathrm{D} = \frac{m_\mathrm{T}}{m_\mathrm{D} + m_\mathrm{T}} v \qquad v_\mathrm{T} = \frac{m_\mathrm{D}}{m_\mathrm{D} + m_\mathrm{T}} v$$

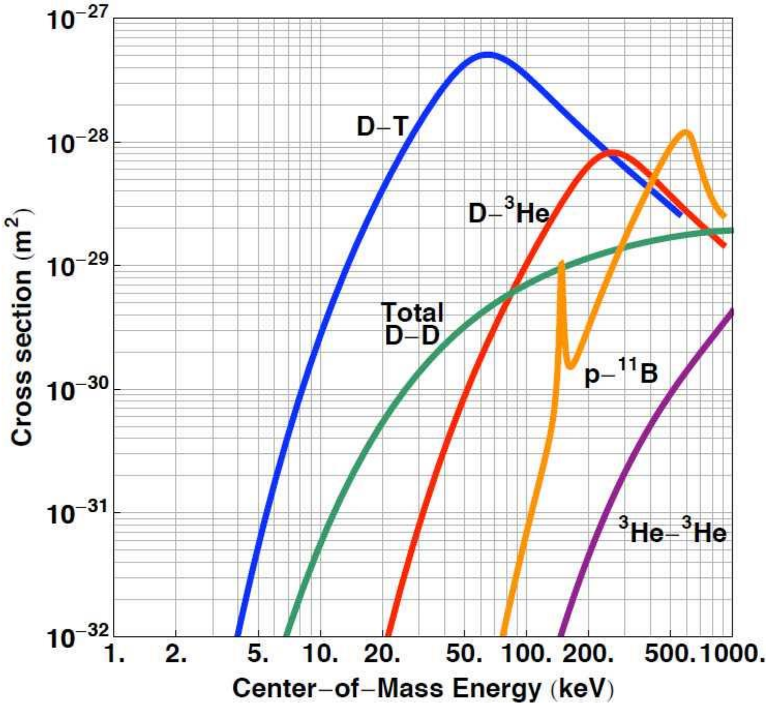
$$\frac{m_\mathrm{r}}{2} v^2 \geq \frac{e^2}{4\pi\epsilon_\mathrm{o} R} \qquad m_\mathrm{r} = \frac{m_\mathrm{D} m_\mathrm{T}}{m_\mathrm{D} + m_\mathrm{T}}$$



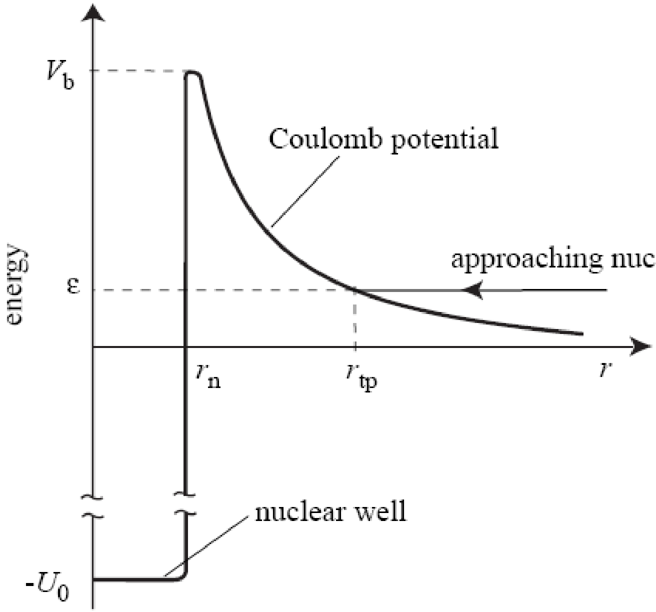
- **Classical kinetic energy required for fusion is**

$$K_{\mathrm{c.m.}} > 288\ \mathrm{keV} \quad !!!$$

Cross section of fusion reaction is much larger than the classical approach



- Classical kinetic energy required for fusion is $K_{\text{c.m.}} > 288\ \mathbf{keV}$!!!
- DT cross section has a peak of ~5 barns at 120 keV.
- $\sigma_{DT} \approx 100\sigma_{DD}$ @ 20 keV.



$$D + D \rightarrow T + p$$
$$\rightarrow He^3 + n$$

$$D + T \rightarrow He^4 + n$$

$$D + He^3 \rightarrow He^4 + p$$

$$p + B^{11} \rightarrow 3He^3$$

---

https://i.stack.imgur.com/wXQD5.jpg
Santarius, J. F., “Fusion Space Propulsion – A Shorter Time Frame Than You Think”, JANNAF, Monterey, 5-8 December 2005.

Flux of incident particles reduces after collisions

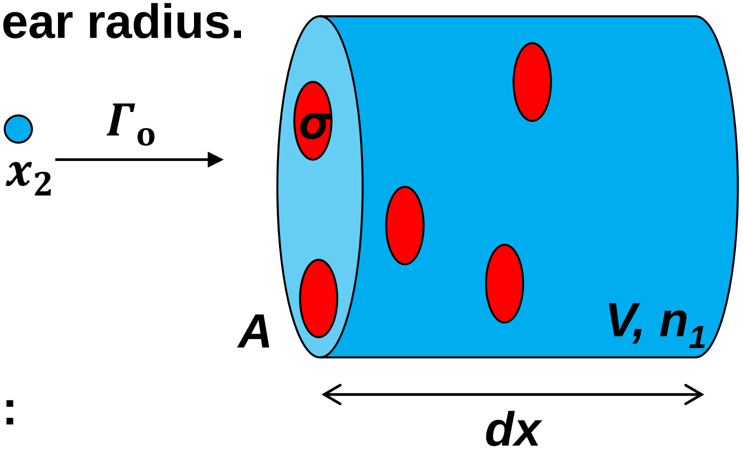
$$x_1 + x_2 \rightarrow x_3 + x_4$$

- Cross section: $\sigma \approx \pi R^2$ where R is the nuclear radius.

$$V = Adx$$

$$N_1 = n_1 V = n_1 A\mathrm{dx}$$

$$A_{\mathrm{Target}} = N_1\sigma = \sigma n_1 A\mathrm{dx}$$



- Fraction of total area blocked by targets is:

$$dF = \frac{\sigma N_1}{A} = \frac{\sigma n_1 A dx}{A} = \sigma n_1 dx$$

$$\frac{dF}{dx} = \sigma n_1$$

- Flux of incident particles (x_1) is Γ_o

$$-d\Gamma = dF\Gamma = \sigma n_1 \Gamma dx$$

$$\frac{-d\Gamma}{\Gamma} = \sigma n_1 dx$$

$$\Gamma = \Gamma_o \exp\left(-\frac{x}{\lambda_{\mathrm{mfp}}}\right)$$

- Mean free path:

$$\lambda_{\mathrm{mfp}} = \frac{1}{n_1\sigma}$$

- Collision frequency:

$$\nu = \frac{1}{\tau}, \tau = \frac{\lambda_{\mathrm{mfp}}}{v} = \frac{1}{n_1\sigma v}$$

Reactions happen when collision happen

- **Reaction rate *R_{12}*: number of fusion collisions/reactions per unit volume per unit time.**
- **In the time *dt*=*dx*/*v* , *n_2Adx* incident particles will pass through the target volume.**
- **The number having a collisions is:** $\boldsymbol{dF(n_2Adx)}$
- **The volumetric reaction rate *R_{12}*, i.e., the number of reaction per unit time and per unit volume is:**

$$\boldsymbol{R_{12} = \frac{dF(n_2Adx)}{Adxdt} = \sigma n_1 n_2 \frac{dx}{dt} = n_1 n_2 \sigma v}$$

- **The fusion power density (W/m^3) is:** $\boldsymbol{S_\mathrm{f} = E_\mathrm{f} n_1 n_2 \mathrm{\sigma v} \ \left(\mathrm{W/m^3}\right)}$
- **For DT fusion, *E_f*=17.6 MeV.**
- **For a particle population with a distribution function in velocity space:**

$$\boldsymbol{n = \int d\,\vec{v}\, f(\vec{r}\,, \vec{v}\,, t)}$$

- **Therefore,**

$$\boldsymbol{n_1 \rightarrow d\,\vec{v}_1\, f_1(\vec{r}\,, \vec{v}_1\,, t) \qquad n_2 \rightarrow d\,\vec{v}_2\, f_2(\vec{r}\,, \vec{v}_2\,, t) \qquad v \rightarrow \left|\vec{v}_1 - \vec{v}_2\right|}$$

$$\boldsymbol{R_{12} = \int f_1(\vec{v}_1) f_1(\vec{v}_2) \sigma\left(\left|\vec{v}_1 - \vec{v}_2\right|\right) \left|\vec{v}_1 - \vec{v}_2\right| d\,\vec{v}_1\, d\,\vec{v}_2}$$

The fusion power density needs to consider the distribution function of particles

$$R_{12} = \int f_1(\vec{v}_1) f_1(\vec{v}_2) \sigma\left(\left|\vec{v}_1 - \vec{v}_2\right|\right)\left|\vec{v}_1 - \vec{v}_2\right| d\,\vec{v}_1\, d\,\vec{v}_2 = n_1 n_2 \langle \sigma v \rangle$$

$$\langle \sigma v \rangle \equiv \frac{\int f_1(\vec{v}_1) f_1(\vec{v}_2) \sigma(|\vec{v}_1 - \vec{v}_2|)|\vec{v}_1 - \vec{v}_2| d\,\vec{v}_1\, d\,\vec{v}_2}{\int f_1(\vec{v}_1) f_1(\vec{v}_2) d\,\vec{v}_1\, d\,\vec{v}_2}$$

$$= \frac{\int f_1(\vec{v}_1) f_1(\vec{v}_2) \sigma(|\vec{v}_1 - \vec{v}_2|)|\vec{v}_1 - \vec{v}_2| d\,\vec{v}_1\, d\,\vec{v}_2}{n_1 n_2}$$

- **The fusion power density (W/m^3) is:**

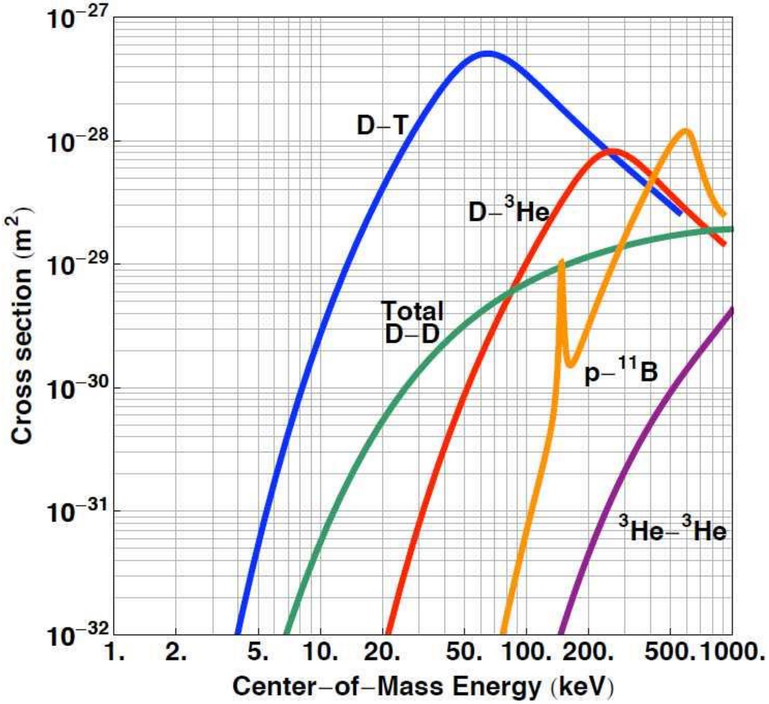
$$S_\mathrm{f} = E_\mathrm{f} n_1 n_2 \langle \sigma v \rangle \left(\mathrm{W/m^3}\right)$$

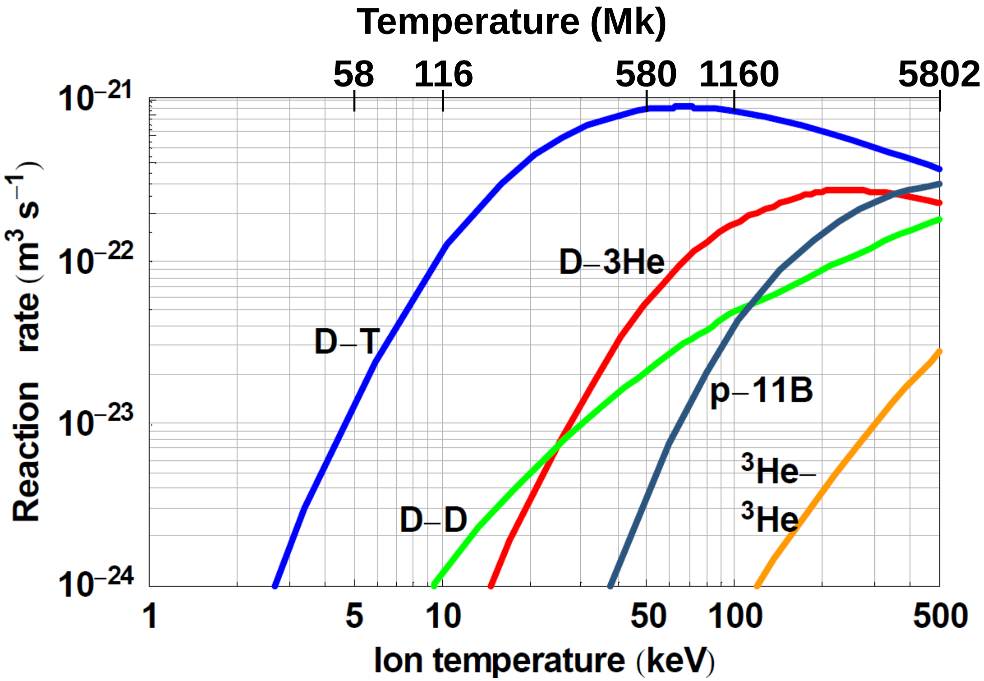
- **Optimum concentration of DT fusion is 50-50.**

$$S_\mathrm{f} = E_\mathrm{f} n_\mathrm{D} n_\mathrm{T} \langle \sigma v \rangle \qquad n_D = k n_o \qquad n_T = (1-k) n_o$$

$$S_\mathrm{f} = E_\mathrm{f} k (1-k) {n_\mathrm{o}}^2 \text{ which peak at } k = 0.5 \ .$$

Fusion doesn’t come easy

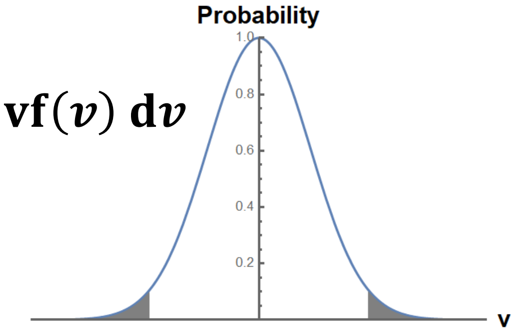




- **The DT fusion reactivity is maximum at T ≈ 64 keV**
- **@ T = 10 keV,** $< \sigma v >_{DT} \approx 100 < \sigma v >_{DD}$

- **Reaction rate:**

$$< \sigma v >= \int \sigma(v) v f(v)\, dv$$



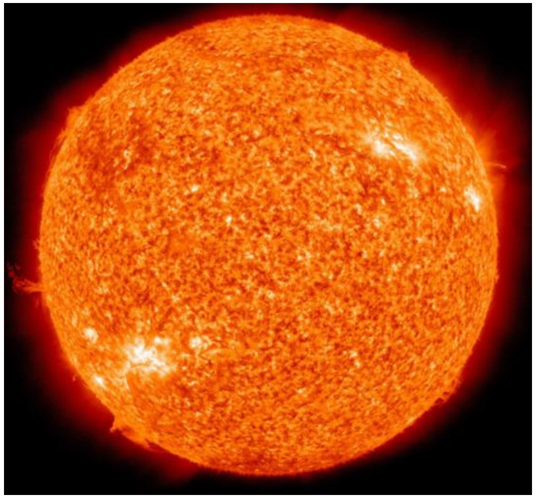
---

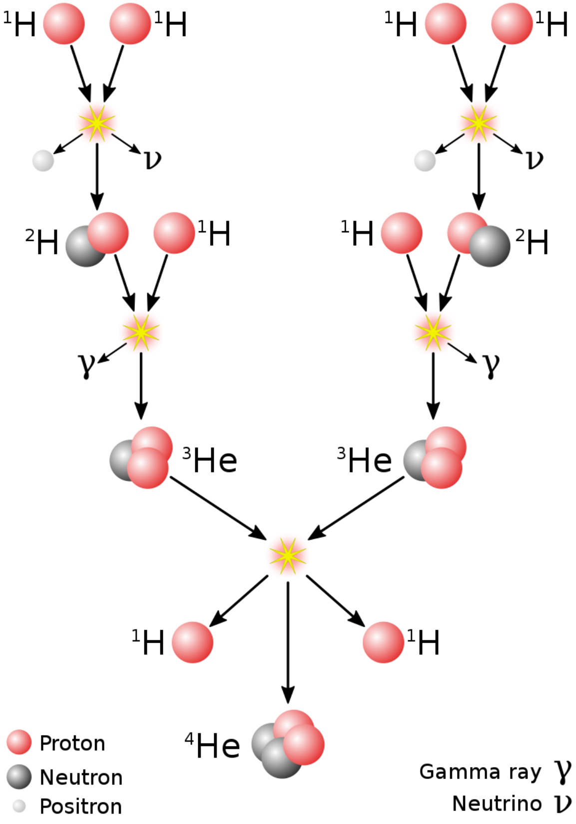
https://i.stack.imgur.com/wXQD5.jpg

Santarius, J. F., “Fusion Space Propulsion – A Shorter Time Frame Than You Think”, JANNAF, Monterey, 5-8 December 2005.

Fusion in the sun provides the energy

- **Proton-proton chain in sun or smaller**

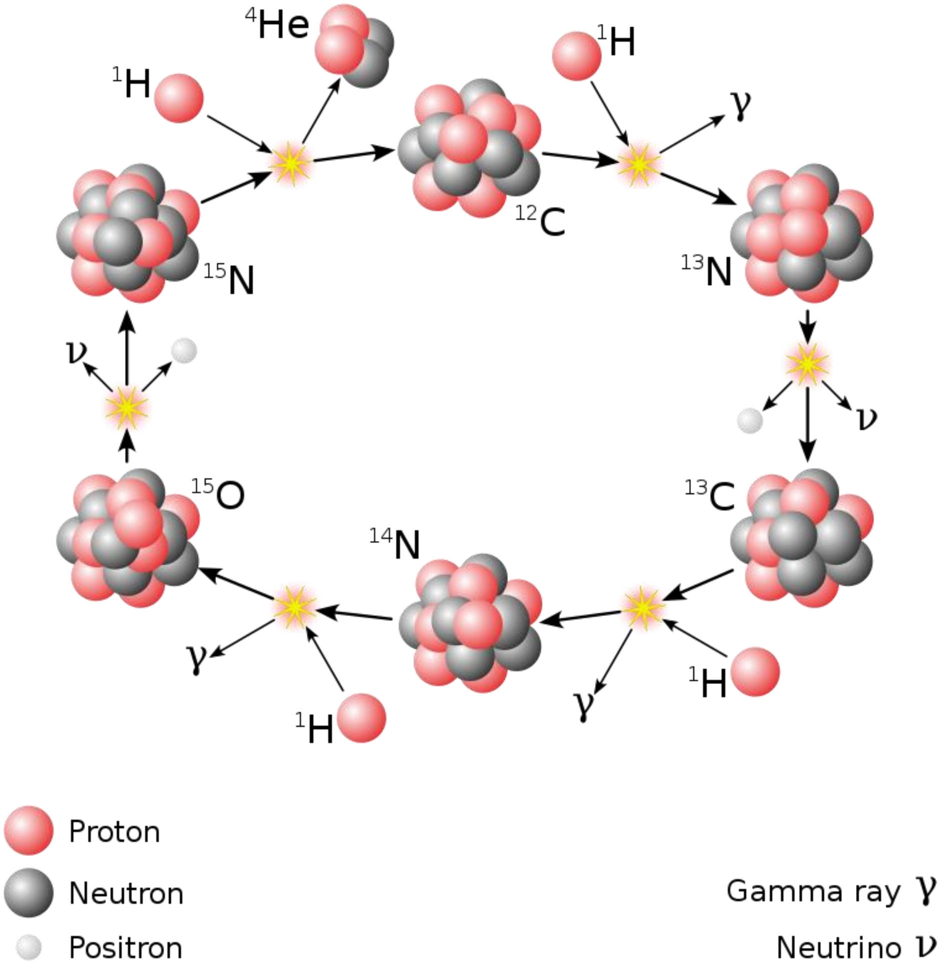




- **Particles are confined by the gravity.**

https://en.wikipedia.org/wiki/Sun
https://en.wikipedia.org/wiki/Nuclear_fusion

In heavy sun, the fusion reaction is the CNO cycle



https://en.wikipedia.org/wiki/Nuclear_fusion

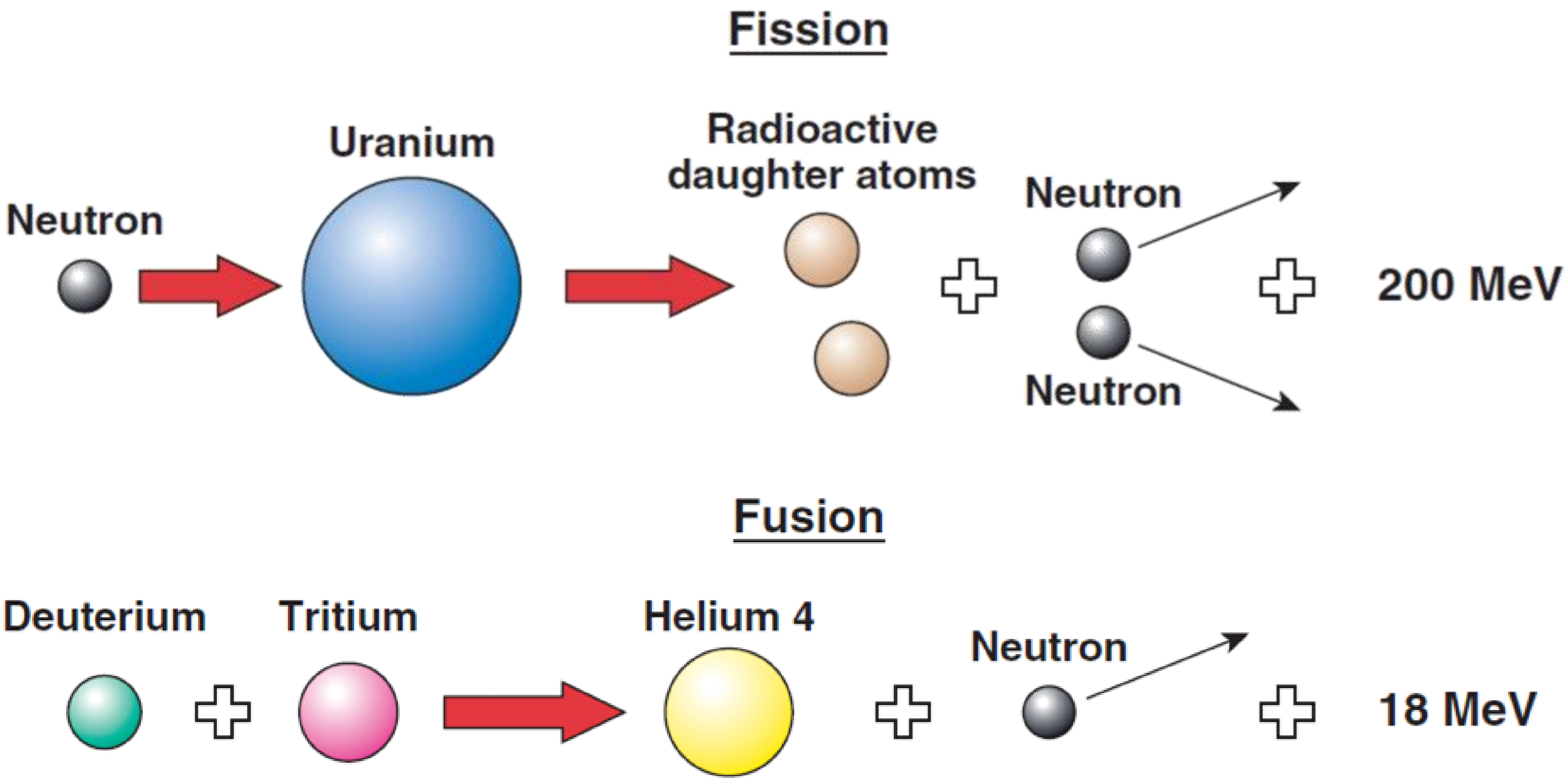
The cross section of proton-proton chain is much smaller than D T fusion

Reaction	$\sigma_{10\ \mathrm{keV}}$ (barn)	$\sigma_{100\ \mathrm{keV}}$ (barn)	σ_{max} (barn)	ε_{max} (keV)
$D+T\rightarrow\alpha+n$	$2.72x10^{-2}$	3.43	5.0	64
$D+T\rightarrow T+p$	$2.81x10^{-4}$	$3.3x10^{-2}$	0.06	1250
$D+T\rightarrow {}^3He+n$	$2.78x10^{-4}$	$3.7x10^{-2}$	0.11	1750
$T+T\rightarrow\alpha+2n$	$7.90x10^{-4}$	$3.4x10^{-2}$	0.16	1000
$D+{}^3He\rightarrow\alpha+p$	$2.2x10^{-7}$	0.1	0.9	250
$p+{}^6Li\rightarrow\alpha+{}^3He$	$6x10^{-10}$	$7x10^{-3}$	0.22	1500
$p+{}^{11}B\rightarrow 3\alpha$	$(4.6x10^{-17})$	$3x10^{-4}$	1.2	550
$p+p\rightarrow D+e^++v$	$(3.6x10^{-26})$	$(4.4x10^{-25})$		
$p+{}^{12}C\rightarrow{}^{13}N+\gamma$	$(1.9x10^{-26})$	$2.0x10^{-10}$	1.0x10.4	400
${}^{12}C+{}^{12}C$ (all branches)		$(5.0x10^{-103})$		

- “( )” are theoretical values while others are measured values.

The Physics of Inertial Fusion, by Stefano Atzeni and Jürgen Meyer-Ter-Vehn

Nuclear fusion and fission release energy through energetic neutrons



(3.5MeV) (14.1MeV)

Nuclear fusion provides more energy per atomic mass unit (amu) than nuclear fission

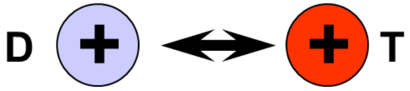
- **Fusion of D+T:** $$\frac{Q}{A}=\frac{17.6\mathbf{MeV}}{(3+2)\mathbf{amu}}=3.5\frac{\mathbf{MeV}}{\mathbf{amu}}$$
- **Fission of ^{235}U+n:** $$\frac{Q}{A}=\frac{200\mathbf{MeV}}{(235+1)\mathbf{amu}}=0.85\frac{\mathbf{MeV}}{\mathbf{amu}}$$

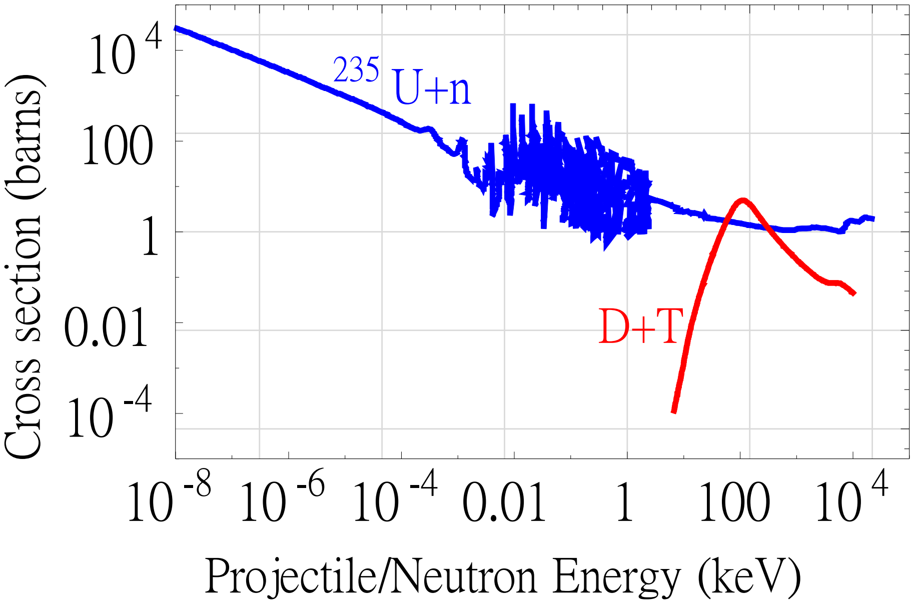
	Half-life (years)
U235	7.04×10^8
U238	4.47×10^9
…	
Tritium	12.3

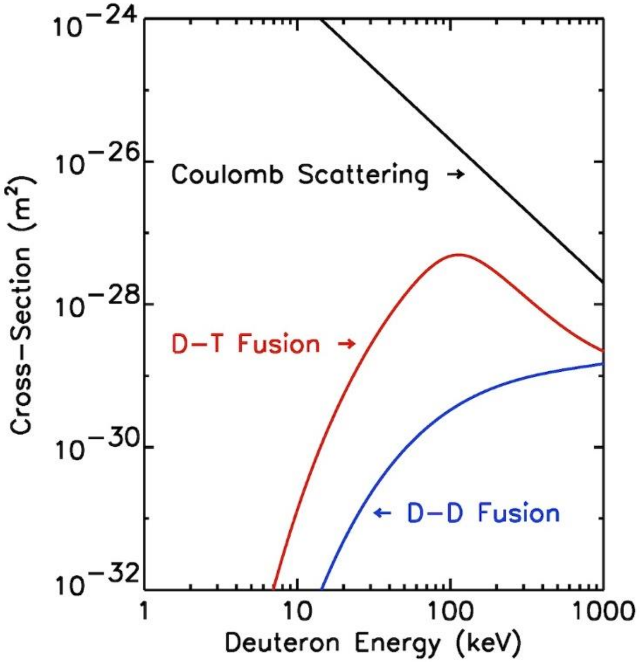
$n + Li^6 \rightarrow He^4 + T$

Fusion is much harder than fission

- **Fission:** $n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3n + 177\ \mathrm{MeV}$
- **Fusion:** $D + T \rightarrow He^4\,(3.5\ \mathrm{MeV}) + n\,(14.1\ \mathrm{MeV})$



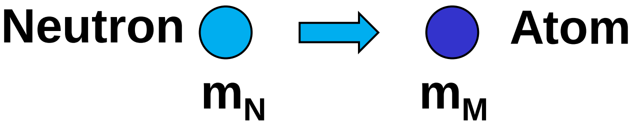




https://www6.lehigh.edu/~eus204/lab/PCL_fusion.php#x1-10096

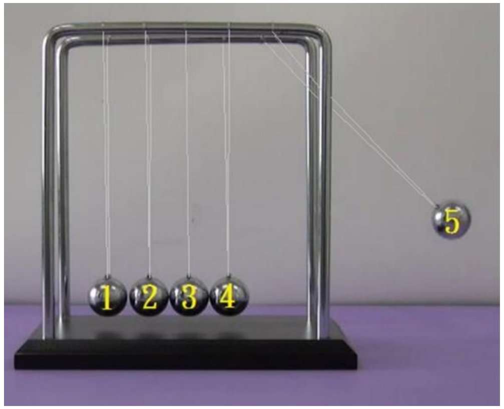
Fast neutrons are slowed down due to the collisions

- **A moderator is used to slow down fast neutrons but not to absorb neutrons.**
- **For m_M~m_N, the energy decrement is higher. Therefore, H slows down neutron most efficiently.**
- **However, H + n → D, i.e., H absorbs neutrons.**



Neutron **Atom**

m_N **m_M**



- **The best option is the D in the heavy water (D_2O).**

	Energy decrement	Neutron scattering cross section (σ_s) (Barns)	Neutron absorption cross section (σ_a) (Barns)
H	1	49 (H_2O)	0.66 (H_2O)
D	0.7261	10.6 (D_2O)	0.0013 (D_2O)
C	0.1589	4.7 (Graphite)	0.0035 (Graphite)

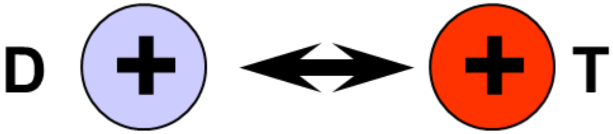
https://en.wikipedia.org/wiki/Neutron_moderator#cite_note-Weston-4
https://energyeducation.ca/encyclopedia/Neutron_moderator#cite_note-3

Comparison between nuclear fission and nuclear fusion

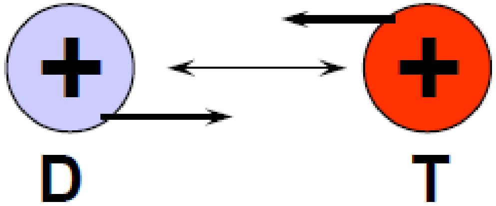
	Nuclear Fission	Nuclear Fusion
Chain reaction	**Yes**	**No**
Melt down	**Possible**	**Impossible**
Nuclear waste	**High radiative**	**Low radiative / None**

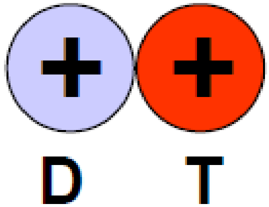
A “hot plasma” at 100M ⁰C is needed

- Probability for fusion reactions to occur is low at low temperatures due to the coulomb repulsion force.



- If the ions are sufficiently hot, i.e., large random velocity, they can collide by overcoming coulomb repulsion



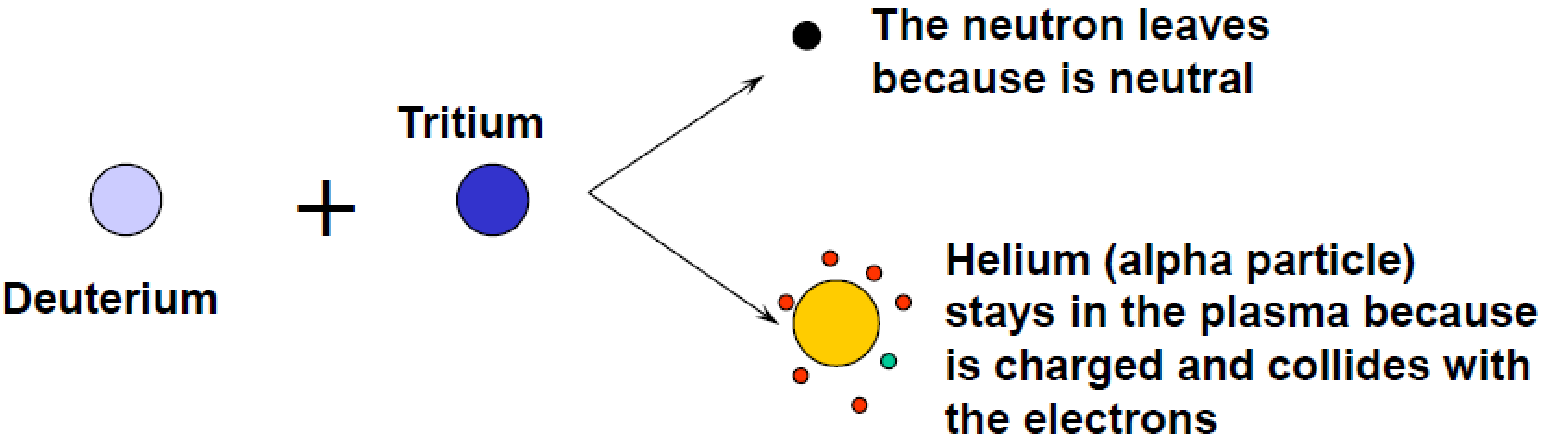


---

*R. Betti, HEDSA HEDP Summer School, 2015

It takes a lot of energy or power to keep the plasma at 100M ^{0}C

- Let the plasma do it itself!



- The α-particles heat the plasma.

*R. Betti, HEDSA HEDP Summer School, 2015

Under what conditions the plasma keeps itself hot?

- **Steady state 0-D power balance:**

$S_\alpha+S_h=S_B+S_k$

S_α: α particle heating

S_h: external heating

S_B: Bremsstrahlung radiation

S_k: heat conduction lost

Ignition condition: $P\tau > 10$ atm-s = 10 Gbar - ns

- **P: pressure, or called energy density**
- **τ is confinement time**