Introduction to Nuclear Fusion as An Energy Source



Institute of Space and Plasma Sciences, National Cheng Kung University

Lecture 14

2024 spring semester

Wednesday 9:10-12:00

Materials:

https://capst.ncku.edu.tw/PGS/index.php/teaching/

Online courses:

https://nckucc.webex.com/nckucc/j.php?MTID=ma76b50f97b1c6d72db61de 9eaa9f0b27

2024/6/19 updated 1

Course Outline

Water Bank

- Inertial confinement fusion (ICF)
 - Plasma frequency and critical density
 - Direct- and indirect- drive
 - Laser generated pressure (Inverse bremsstrahlung and Ablation pressure)
 - Burning fraction, why compressing a capsule?
 - Implosion dynamics
 - Shock (Compression with different adiabat)
 - Laser pulse shape
 - Rocket model, shell velocity
 - Laser-plasma interaction (Stimulated Raman Scattering, SRS; Stimulated Brillouin Scattering, SBS; Two-plasmon decay)
 - Instabilities (Rayleigh-taylor instability, Kelvin-Helmholtz instability, Richtmeyer-Meshkov instability)



- Riccardo Betti, University of Rochester, HEDSA HEDP summer school, San Diego, CA, August 16-21, 2015
- ICF lectures for course PHY558/ME533
- The physics of inertial fusion, by S. Atzeni, J. Meyer-Ter-Vehn

There are three stages in the laser pulse: foot, ramp, and flat top



The adiabat is set by the shock launched by the foot of the laser pulse



Density and thickness at shock break out time are expressed in laser intensity

• Use $p \sim I^{2/3}$

• Shell density
$$\rho_{sb} \sim \rho_1 \left(\frac{p_{max}}{p_{foot}}\right)^{5/3} = 4\rho_1 \left(\frac{I_{max}}{I_{foot}}\right)^{2/5}$$

• Shell thickness $\Delta_{sb} \sim \frac{\Delta_1}{4} \left(\frac{p_{foot}}{p_{max}}\right)^{3/5} = \frac{\Delta_1}{4} \left(\frac{I_{foot}}{I_{max}}\right)^{2/5}$

• Shell radius $R \approx R_1$



The aspect ratio is maximum at shock break out





Aspect ratio
$$\equiv \frac{R}{\Delta}$$

 $A_1 = \frac{R_1}{\Delta_1} = \text{initial aspect ratio}$
 $A_{\text{sb}} = IFAR = \frac{R_1}{\Delta_{\text{sb}}} = 4A_1 \left(\frac{I_{\text{max}}}{I_{\text{foot}}}\right)^{2/5}$

 $A_{\rm sb} = A_{\rm max}$

IFAR = Maximum In-Flight-Aspect-Ratio = aspect ratio at shock break-out

The IFAR scales with the Mach number

• The shell kinetic energy = the work done on the shell

$$Mu_{max}^{2} \sim -\int_{R_{1}}^{R} pr^{2} dr \sim p(R_{1}^{3} - R^{3}) \approx pR_{1}^{3} \qquad R_{1}^{3} = \frac{Mu_{max}^{2}}{p}$$

$$M \sim \rho_{sb} \Delta_{sb} R_{1}^{2} \qquad \Delta_{sb} \sim \frac{M}{\rho_{sb} R_{1}^{2}} \qquad R_{1} >> R$$

$$IFAR = \frac{R_{1}}{\Delta_{sb}} = \frac{R_{1}}{\frac{M}{\rho_{sb} R_{1}^{2}}} = \frac{\rho_{sb} R_{1}^{3}}{M} = \frac{\rho_{sb}}{M} \frac{Mu_{max}^{2}}{p}$$

$$= \frac{u_{max}^{2}}{p/\rho_{sb}} \sim Mach_{max}^{2}$$

$$\alpha \sim \frac{p}{\rho^{5/3}} \qquad p \sim I^{2/3} \qquad IFAR \sim \frac{u_{max}^{2}}{\alpha^{3/5} I^{4/15}}$$

The final implosion velocity can be found using IFAR

$$u_{\max}^{2} \sim IFAR \times \alpha^{3/5} I^{4/15}$$

$$IFAR = 4A_{1} \left(\frac{I_{\max}}{I_{foot}}\right)^{2/5}$$

$$A_{1} = \frac{R_{1}}{A_{1}}$$

$$u_{\max, \operatorname{cm}/s} \approx 10^{7} \sqrt{0.7A_{1}\alpha^{3/5} I_{15,\max}^{4/15} \left(\frac{I_{\max}}{I_{foot}}\right)^{2/5}}$$

There are three stages in the laser pulse: foot, ramp, and flat top



A simple implosion theory can be derived in the limit of infinite initial aspect ratio

- Start from a high aspect ratio shell (thin shell) at the beginning of the acceleration phase
 - Constant ablated pressure
 - The adiabat is set and kept fixed by the first and the only shock

$$IFAR = A_{\rm sb} = \frac{R_1}{\Delta_{\rm sb}} \gg 1$$



The implosion are divided in 3 phases after the shock break out



- 1st phase: acceleration
- 2nd phase: coasting
- 3rd phase: stagnation

Summary of phase 1 (acceleration phase)



Summary of phase 2 (coasting phase)



$$Mach_2 = Mach_{max} \simeq A_2 = \sqrt{A_{sb}}$$

$$\Delta \simeq \text{constant} = \Delta_2 \sim \frac{R_1}{A_{\text{sb}}^{2/3}} \qquad \bar{\rho} \simeq \rho_2 \left(\frac{R_2}{R}\right)^2 \sim \rho_{\text{sb}} \left(\frac{R_2}{R}\right)^2$$

How about the 3rd phase where A~1?







- 1st phase: acceleration
- 2nd phase: coasting
- 3rd phase: stagnation

The thin shell model breaks down when A~1



- When A~1 => Δ~R, the "void" inside the shell closes and a "return shock" propagating outward is generated due to the collision of the shell with itself
- The density is compressed by a factor no more than 4 even if the strong shock is generated

 $ho_{st} \sim 4
ho_3 \sim
ho_3$ where ho_3 is the density right before the void closure

The stagnated density scales with square of the maximum Mach number



$$\rho_{3} \sim \rho_{2} \left(\frac{R_{2}}{R_{3}}\right)^{2} \sim \rho_{sb} \left(\frac{R_{2}}{R_{3}}\right)^{2} \qquad \bar{\rho} \simeq \rho_{2} \left(\frac{R_{2}}{R}\right)^{2}$$

$$A = A_{3} \sim 1 \Rightarrow \frac{R_{3}}{\Delta_{3}} \sim \frac{R_{3}}{\Delta_{2}} \sim 1 \Rightarrow R_{3} \sim \Delta_{2}$$

$$\rho_{st} \sim \rho_{3} \sim \rho_{sb} \left(\frac{R_{2}}{\Delta_{2}}\right)^{2} \sim \rho_{sb} A_{2}^{2} \sim \rho_{sb} Mach_{2}^{2} \sim \rho_{sb} Mach_{max}^{2}$$

$$\frac{\rho_{st}}{\rho_{st}} \sim Mach_{max}^{2} \qquad \text{Density compression scaling law.}$$

 $\rho_{\rm sb}$

The stagnated pressure scales to the 4th power of the maximum Mach number



Conservation of energy at stagnation:

$$p_{st}R_{st}^{3} \sim mu_{max}^{2} \qquad R_{st} \sim R_{3} \sim \Delta_{3} \sim \Delta_{2} \implies p_{st}\Delta_{2}^{3} \sim \underline{mu_{max}}^{2} \sim \underline{\rho_{2}R_{2}^{2}\Delta_{2}}u_{max}^{2}$$

$$\Rightarrow p_{st} \sim \rho_{2} \left(\frac{R_{2}}{\Delta_{2}}\right)^{2} u_{max}^{2} = \rho_{2}A_{2}^{2}u_{max}^{2} \sim p_{2}Mach_{2}^{2}\frac{u_{max}^{2}}{p_{2}/\rho_{2}} \sim p_{A}Mach_{2}^{4} \sim p_{A}Mach_{max}^{4}$$

$$Mach_{2} = Mach_{max} \simeq A_{2} = \sqrt{A_{sb}}$$

$$\alpha_{st} \sim \frac{p_{st}}{\rho_{st}^{5/3}} \sim \frac{p_{A}^{4}Mach_{max}^{4}}{\rho_{sb}^{5/3}Mach_{max}^{10/3}} = \alpha_{sb}Mach_{max}^{2/3}$$

$$\frac{\alpha_{st}}{\alpha_{sb}} \sim Mach_{max}^{2/3}$$

$$\frac{\rho_{st}}{\rho_{sb}} \sim Mach_{max}^{2}$$

Scaling of the areal density of the compressed core

Amplification of areal density

$$\rho_{\rm st}R_{\rm st} \sim \rho_{\rm st}^{2/3} \left(\underline{\rho_{\rm st}R_{\rm st}^3}\right)^{1/3} \sim \rho_{\rm sb}^{2/3} Mach_{\rm max}^{4/3} \underline{Mass^{1/3}}$$

$$\sim \frac{\rho_{\rm sb}^{2/3}}{\rho_1^{2/3}} Mach_{\rm max}^{4/3} \rho_1^{2/3} (\rho_1 R_1^2 \Delta_1)^{1/3} \qquad A_1 = \frac{R_1}{\Delta_1}$$

$$\rho_{\rm st}R_{\rm st} \sim (\rho_1 \Delta_1) Mach_{\rm max}^{4/3} A_1^{2/3} \left(\frac{\rho_{\rm sb}}{\rho_1}\right)^{2/3}$$

$$\frac{\rho_{\rm sb}}{\rho_1} = 4 \left(\frac{I_{\rm max}}{I_{\rm foot}}\right)^{2/5}$$

 $\frac{\rho_{\rm st}}{Mach_{\rm max}}$

 $\rho_{\rm sb}$

$$(\rho R)_{\rm st} \sim (\rho_1 \Delta_1) IFAR^{2/3} A_1^{2/3} \left(\frac{I_{\rm max}}{I_{\rm foot}}\right)^{4/15}$$
$$E_{\rm las} = 4\pi R_1^2 I_{\rm max} t_{\rm imp} \approx 4\pi R_1^2 I_{\rm max} \frac{R_1}{u_{\rm max}}$$
$$E_{\rm las} \approx \frac{4\pi R_1^3 I_{\rm max}}{u_{\rm max}}$$

 $u_{\rm max}$

Summary

$$A_{\rm sb} = IFAR = 4A_1 \left(\frac{I_{\rm max}}{I_{\rm foot}}\right)^{2/5} \qquad u_{\rm max,cm/s} \approx 10^7 \sqrt{0.7A_1 \alpha^{3/5} I_{15,\rm max}^{4/15} \left(\frac{I_{\rm max}}{I_{\rm foot}}\right)^{2/5}}$$

$$\rho_{\rm st} \sim \rho_{\rm sb} Mach_{\rm max}^2 \sim \rho_1 IFAR \left(\frac{I_{\rm max}}{I_{\rm foot}}\right)^{2/5}$$

$$p_{\rm st} \sim p_A Mach_{\rm max}^4 \sim p_A IFAR^2$$

$$\alpha_{\rm st} \sim \alpha_{\rm sb} Mach_{\rm max}^{2/3} \sim \alpha_{\rm sb} IFAR^{1/3}$$
$$(\rho R)_{\rm st} \sim (\rho_1 \Delta_1) IFAR^{2/3} A_1^{2/3} \left(\frac{I_{\rm max}}{I_{\rm foot}}\right)^{4/15}$$

Calculation of the burn-up fraction



$$4\pi \int_{0}^{R_{f}} r^{2} dr \left(\frac{\partial n_{i}}{\partial t} = -\nabla \cdot (n_{i}v) - \frac{n_{i}^{2}}{2} \langle \sigma v \rangle \right) \qquad \rho$$

$$4\pi \int_{0}^{R_{f}} r^{2} \frac{\partial n_{i}}{\partial t} dr = 4\pi \frac{d}{dt} \int_{0}^{R_{f}} r^{2} n_{i} dr - 4\pi \dot{R}_{f} R_{f}^{2} n_{i}$$

$$= -n_{i}v 4\pi R_{f}^{2} - \frac{n_{i}^{2}}{2} \langle \sigma v \rangle V_{f}$$
(neglect)

$$\frac{d}{dt}\left(\int_{a(t)}^{b(t)} f(x,t)dx\right) = f(b(t),t)\frac{db(t)}{dt} - f(a(t),t)\frac{da(t)}{dt} + \int \frac{\partial}{\partial t}f(x,t)dx$$

~

(Leibniz integral rule)

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$$N_{f} \equiv \frac{4\pi}{3} R_{f}^{3} n_{i} \equiv V_{f} n_{i}$$

$$\frac{dN_{f}}{dt} - 3N_{f} \frac{\dot{R}_{f}}{R_{f}} = -\frac{N_{f}^{2}}{V_{f}} \frac{\langle \sigma v \rangle}{2}$$

$$\frac{d_{t}N_{f}}{N_{f}^{2}} - \frac{3\dot{R}_{f}}{N_{f}R_{f}} = -\frac{\langle \sigma v \rangle}{2V_{f}}$$

$$R_{f}^{3} \frac{d}{dt} \left(\frac{1}{N_{f}}\right) + 3R_{f}^{2} \frac{\dot{R}_{f}}{N_{f}} = \frac{d}{dt} \left(\frac{R_{f}^{3}}{N_{f}}\right) = \frac{\langle \sigma v \rangle}{2V_{f}} R_{f}^{3}$$

$$\frac{d}{dt} \left(\frac{R_{f}^{3}}{N_{f}}\right) = \frac{\langle \sigma v \rangle}{2V_{f}} R_{f}^{3} \qquad \frac{R_{f}^{3}}{N_{f}} = \int_{0}^{t} \frac{\langle \sigma v \rangle}{2V_{f}} R_{f}^{3} dt + \frac{R_{0}^{3}}{N_{0}}$$

$$R_{f} = R_{0} - C_{s}t \qquad dt = -\frac{dR_{f}}{C_{s}} \qquad V_{f} = \frac{4\pi}{3} R_{f}^{3}$$

$$\frac{R_{f}^{3}}{N_{f}} = -\int_{R_{0}}^{R_{f}} \frac{\langle \sigma v \rangle}{2 \times 4\pi/3} \frac{dR_{f}}{C_{s}} + \frac{R_{0}^{3}}{N_{0}} \qquad \int_{V_{f}}^{V_{f}} = \frac{V_{0}}{N_{0}} \left[1 + \frac{\langle \sigma v \rangle}{2C_{s}} n_{0}R_{0} \left(1 - \frac{R_{f}}{R_{0}}\right)\right]$$

$$\frac{R_{f}^{3}}{N_{f}} = \frac{\langle \sigma v \rangle}{2C_{s}} \frac{3}{4\pi} (R_{0} - R_{f}) + \frac{R_{0}^{3}}{N_{0}} \qquad \xi \equiv \frac{\langle \sigma v \rangle}{2C_{s}} n_{0}R_{0}$$

$$\frac{V_{f}}{R_{f}} = \frac{\langle \sigma v \rangle}{2C_{s}} R_{0} \left(1 - \frac{R_{f}}{R_{0}}\right) + \frac{V_{0}}{N_{0}} \qquad \frac{V_{f}}{N_{f}} = \frac{1}{n_{0}} \left[1 + \xi \left(1 - \frac{R_{f}}{R_{0}}\right)\right]$$

$$\begin{split} \frac{V_f}{N_f} &= \frac{1}{n_0} \left[1 + \xi \left(1 - \frac{R_f}{R_0} \right) \right] & n_i = \frac{N_f}{V_f} \\ \text{#Burned ions} &= \int_0^t \frac{\langle \sigma v \rangle}{2} n_i^2 V_f dt = \int_0^t \frac{\langle \sigma v \rangle}{2} \frac{N_f^2}{V_f} dt = -\int_{R_0}^{R_f} \frac{\langle \sigma v \rangle}{2} \left(\frac{N_f}{V_f} \right)^2 V_f \frac{dR_f}{C_s} \\ &= \int_{R_f}^{R_0} \frac{\langle \sigma v \rangle}{2} \frac{n_0^2}{\left[1 + \xi \left(1 - \frac{R_f}{R_0} \right) \right]^2} \left(\frac{R_f}{R_0} \right)^3 V_0 R_0 \frac{dR_f/R_0}{C_s} V_{f,0} = \frac{4\pi}{3} R_{f,0}^3 \\ &= \int \frac{\langle \sigma v \rangle}{2} \frac{n_0^2}{\left[1 + \xi (1 - x) \right]^2} x^3 V_0 \frac{R_0}{C_s} dx = N_0 \xi \int_0^1 \frac{x^3 dx}{\left[1 + \xi (1 - x) \right]^2} \\ &= N_0 \frac{\xi [6 + \xi (9 + 2\xi)] - 6(1 + \xi)^2 Ln[1 + \xi]}{2\xi^3} & \xi \equiv \frac{\langle \sigma v \rangle}{2C_s} n_0 R_0 \end{split}$$

 $\Theta(\xi) = \frac{\xi[6 + \xi(9 + 2\xi)] - 6(1 + \xi)^2 \text{Ln}[1 + \xi]}{2\xi^3}$

#Burn-up Fraction

$$C_{s} = \sqrt{\frac{T_{e} + T_{i}}{m_{i}}} = \sqrt{\frac{2T}{m_{i}}} \qquad \rho = n_{0}m_{i} \qquad m_{i} = \frac{m_{D} + m_{T}}{2} = 2.5 \times 1.67 \times 10^{-27} \text{kg}$$

$$\xi = \frac{\langle \sigma v \rangle}{\sqrt{T}} (\rho R_{0}) \frac{1}{2\sqrt{2m_{i}}} = \frac{\langle \sigma v \rangle_{m^{3}/s}}{\sqrt{T_{keV} \times 1.6 \times 10^{-16}}} \frac{(\rho R_{0})_{g/cm^{2}} \times 10}{2\sqrt{5 \times 1.67 \times 10^{-27}}}$$

$$\xi \simeq \frac{1.25 \times 10^{-22}}{\sqrt{16 \times 10^{-16}}} \frac{10(\rho R_{0})_{g/cm^{2}}}{2\sqrt{5 \times 1.67 \times 10^{-27}}} = 0.54(\rho R_{0})_{g/cm^{2}}$$

$$\frac{\langle \sigma v \rangle}{\sqrt{T_{keV}}} = 1.25 \times 10^{-22} \qquad (@ T = 40 \text{ keV})$$

H.-S. Bosch and G.M. HaleNucl. Fusion **32** (1992) 611

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Smallest areal density (ρR)



Energy gain

Fusion energy
$$= \frac{M_0}{2m_i} \epsilon_f \theta$$

 $\epsilon_f = 17.6 \text{MeV}$
Energy gain $= \frac{\text{Fusion Energy}}{\text{Input Energy}}$
 \cdot Input energy: the sphere is heated to the temperature T
Thermal energy in sphere: $\frac{3}{2}(n_{i0}T_i + n_{e0}T_e)V_0$
 $n_{i0} = n_{e0} \equiv n_0$ $T_e = T_i \Rightarrow 3n_0 \text{TV}_0 = 3\frac{M_0}{m_i}T$
Set heating efficiency: $\eta = \frac{\text{Thermal Energy}}{\text{Input Energy}}$
 $Gain = \frac{\frac{M_0}{2m_i}\epsilon_f\theta}{3\frac{M_0}{m_i}T/\eta} = \eta \frac{M_0}{2m_i} \frac{\epsilon_f \theta}{3\frac{M_0}{m_i}T} = \frac{\eta}{6}\frac{\epsilon_f}{T}\theta$ $Gain = \eta 293\left(\frac{10}{T_{\text{keV}}}\right)\theta$

The power to heat the plasma is enormous



• Consider the small T limit:

$$\Theta(\xi) \approx \frac{\xi}{4+\xi} \qquad \xi \equiv \frac{\langle \sigma v \rangle}{2C_s} n_0 R_0 = \frac{\langle \sigma v \rangle}{\sqrt{T}} (\rho R_0) \frac{1}{2\sqrt{2m_i}}$$

 $\langle \sigma v \rangle {\sim} T^4$ for T
ightarrow 0 , then $\xi {\sim} T^{7/2}$ and $Gain {\sim} T^{5/2}
ightarrow 0$

Required input power:

$$P_{w} = \frac{E_{\text{input}}}{\tau_{\text{input}}} \quad \tau_{\text{input}} \ll \tau_{\text{burn}} = \frac{R}{C_{s}} \quad \text{(Heat out before it runs away)}$$

$$P_{w} = \frac{E_{\text{input}}}{\mu R/C_{s}} = \frac{E_{\text{thermal}}}{\eta \mu R/C_{s}} = 3 \frac{M_{0}}{m_{i}} T \frac{1}{R} \frac{C_{s}}{\eta \mu} \qquad \tau_{\text{input}} = \mu \frac{R}{C_{s}} \quad \text{Ex: } \mu \sim 0.1$$

$$\frac{P_{w}}{M_{0}} = \frac{3}{m_{i}} \frac{T}{R} \frac{C_{s}}{\eta \mu} = \frac{3}{m_{i}} \frac{T}{R} \sqrt{\frac{2T}{m_{i}}} \frac{1}{\eta \mu} \qquad \frac{P_{w}}{M_{0}} = 10^{18} \left(\frac{T_{\text{keV}}}{10}\right)^{3/2} \frac{0.1}{\mu} \frac{1}{R_{\text{cm}}} \frac{1}{\eta} \quad \text{Watts/g}$$

A clever way is needed to ignite a target

• For T = 10 keV

$$\xi \approx 0.18(\rho R)$$
 $Gain|_{10keV} \approx 293\eta \frac{0.18\rho R}{4+0.18\rho R} \approx 293\eta \frac{\rho R_{g/cm^2}}{22+\rho R_{g/cm^2}}$

For T=40 keV

$$\xi \approx 0.54(\rho R)$$
 $Gain|_{40keV} \approx 73\eta \frac{\rho R_{g/cm^2}}{7 + \rho R_{g/cm^2}}$

 $ho R_{g/cm^2} \gtrsim rac{22}{2.93\eta - 1}$ For Gains $\gtrsim 100$ - T = 10 keV10 keV $ho R \gtrsim 22 g/\mathrm{cm}^2 \quad \eta > 1$ 40 keV $ho R_{g/cm^2} \gtrsim rac{7}{0.73\eta - 1}$ - T = 40 keV-40 $ho R \gtrsim 7 g/\mathrm{cm}^2 \quad \eta > 1$ 0.5 1.0 1.5 2.0 0.0 How do we get $\eta > 1$? η

Requirement to ignite a target



• For *T*=10 keV and $\rho R \gtrsim$ 22 g/cm²

$$\rho R = \frac{4\pi}{3} \frac{\rho R^3}{4\pi R^2/3} = \frac{M_0}{\frac{4\pi}{3}R^2} = \frac{3}{4\pi} \frac{M_0}{R^2} \gtrsim 22 \ g/\text{cm}^2$$
$$\frac{M_0}{R^2} \gtrsim 92 \ g/\text{cm}^2$$
$$P_w \Big|_{10keV} = 10^{18} \left(\frac{T_{keV}}{10}\right)^{3/2} \frac{0.1}{\mu} \frac{M_0}{R_{cm}} \frac{1}{\eta} = 10^{18} \frac{0.1}{\mu} \frac{1}{\eta} 92R_{cm} \quad \text{Watts}$$
$$P_w \Big|_{10keV} \approx 10^{20} \frac{0.1}{\mu} \frac{R_{cm}}{\eta} \text{Watts}$$
$$\cdot \text{ For } T=40 \text{keV} \quad \bullet \text{ Needed:}$$

$$ho R \gtrsim 7 \implies rac{M_0}{R^2} \gtrsim 30 \, g/\mathrm{cm}^2$$

$$P_w\Big|_{40keV} \approx 2.4 \times 10^{20} \frac{0.1}{\mu} \frac{R_{\rm cm}}{\eta}$$
 Watts

Needed:
$$R_{\rm cm} \ll 1$$

 $\eta \gg 1$
 $\mu \gg 0.1$

Requirements to ignite a target



$$P_w\Big|_{10keV} \approx 10^{20} \frac{0.1}{\mu} \frac{R_{cm}}{\eta}$$
 Watts

- $R_{\rm cm} \ll 1$: sphere size in the order of 100's um
- $\eta \gg 1$: input energy amplification
- $\mu \gg 0.1$: energy delivery time decoupled from burn time. Need longer energy delivery time. Need to bring down power to ~10¹⁵ W

Math....#!@%\$\$#&^%\$#



$$P_{w} = 10^{18} \frac{M_{0,g}}{\eta} \left(\frac{T_{\text{keV}}}{10}\right)^{3/2} \frac{0.1}{\mu} \frac{1}{R_{\text{cm}}} \text{ Watts}/g$$

$$\tau_{\text{input}} = \mu \frac{R}{C_{s}} \text{ Ex: } \mu \sim 0.1 \quad \eta = \frac{\text{Thermal Energy}}{\text{Input Energy}}$$

$$Gain = 293\eta \left(\frac{10}{T_{\text{keV}}}\right) \Theta(\xi) \qquad \Theta(\xi) \approx \frac{\xi}{4+\xi} \qquad \xi = \frac{\langle \sigma v \rangle}{2m_{i}C_{s}} (\rho R_{0})$$

$$G_{\text{max}} \equiv 293\eta \left(\frac{10}{T_{\text{keV}}}\right) \qquad G = G_{\text{max}} \frac{\xi}{4+\xi} \Longrightarrow \xi = \frac{4G}{G_{\text{max}}-G}$$

$$P_{w} = \frac{10^{18}}{\eta} \left(\frac{T_{\text{kev}}}{10}\right)^{3/2} \frac{0.1}{\mu} \frac{4\pi}{3} \frac{\rho R_{0}^{3}}{R_{0}} = \frac{10^{18}}{\eta} \left(\frac{T_{\text{kev}}}{10}\right)^{3/2} \frac{0.1}{\mu} \frac{4\pi}{3} (\rho R_{0}) R_{0}$$

More math...!#\$%%^&*&^(*&%)(#%!@\$#%%^*&*%(

$$P_{w} = \frac{10^{18}}{\eta} \left(\frac{T_{kev}}{10}\right)^{3/2} \frac{0.1}{\mu} \frac{4\pi}{3} \frac{\rho R_{0}^{3}}{R_{0}} = \frac{10^{18}}{\eta} \left(\frac{T_{kev}}{10}\right)^{3/2} \frac{0.1}{\mu} \frac{4\pi}{3} (\rho R_{0}) R_{0}$$

$$= \frac{10^{18}}{\eta} \left(\frac{T_{kev}}{10}\right)^{3/2} \frac{0.1}{\mu} \frac{4\pi}{3} R_{0} \frac{2m_{i}C_{s}}{\langle \sigma v \rangle \langle \sigma v$$

Need to lower the power by 5 orders of magnitude



$$P_w \approx \frac{7 \times 10^{19}}{\eta} \frac{0.1}{\mu} R_{0,\text{cm}} \frac{G}{G_{\text{max}}}$$
 Watts

- μ ↑
- $\eta \uparrow$: require the fuel ignition from a "spark." Ignite only a small portion of the DT plasma, i.e., $M_h << M_0$
- $R_0 \downarrow$: smaller system size

:

$$P_{w} = P_{w}(M_{0})\frac{M_{h}}{M_{0}}$$

$$P_{w}^{\min} = \frac{7 \times 10^{15}}{\eta_{h}} \left(\frac{M_{h}/M_{0}}{0.01}\right) \left(\frac{R_{0,\mu m}}{100}\right) \left(\frac{0.1}{\mu}\right) \left(\frac{G}{G_{\max}}\right) \text{Watts}$$
Effective increase in η



$$P_w^{\min} = \frac{7 \times 10^{15}}{\eta_h} \left(\frac{M_h/M_0}{0.01}\right) \left(\frac{R_{0,\mu m}}{100}\right) \left(\frac{0.1}{\mu}\right) \left(\frac{G}{G_{\max}}\right) \text{Watts}$$

- For the case of using a huge laser, ex: 1MJ.
- The ignition requires temperatures $T \gtrsim 5 \text{keV}$, then the energy required for ignition is

$$E_{\text{ign}} \approx 3 \frac{M_h}{m_i} \frac{T}{\eta_h}$$
$$M_h \approx \frac{m_i}{3} \frac{\eta_h E_{\text{ign}}}{T}$$
$$M_{h,\mu g} \approx 17 \left(\frac{5}{T_{\text{keV}}}\right) E_{\text{igm,MJ}} \left(\frac{\eta_h}{0.01}\right) \qquad M_h \approx 20 \mu g$$
Target design using an 1MJ laser - continue

- For "inefficient" heating mechanism ($\eta_h \approx 1\%$), the mass that can be heated to $T \approx 5$ keV is in the order of $M_h \approx 20 \ \mu g$.
- If $M_{\rm h}/M_0 \approx 0.01$, then $M_0 \approx 2$ mg.
- Assuming that the burned-up fraction $\Theta \approx \frac{\rho R}{7 + \rho R}$ for $\Theta \approx 30\% \rightarrow \rho R \approx 3 g/cm^2$ $M_0 = \frac{4\pi}{3}\rho R^3 = \frac{4\pi}{3}R^2(\rho R)$ $R = \sqrt{\frac{4\pi}{3}\frac{M_0}{\rho R}} = 126\sqrt{\frac{M_{0,mg}}{2}}\sqrt{\frac{3}{\rho R}}\mu m$ $\rho = \frac{3M_0}{4\pi R^3} = 240\sqrt{\frac{M_{0,mg}}{2}}\left(\frac{126}{R_{um}}\right)^3 g/cm^3 \leftrightarrow 1000$ $\rho_{\rm DT} = 0.25 g/cm^3$
- DT must be compressed ~1000 times
- The initial radius of a 2 mg sphere of *DT* is $R_{init} \simeq 2.6$ mm while the final radius $R_{final} \simeq 100 \ \mu m$, the convergence ratios of 30 ~ 40 are required.

Requirements of the density and size of the ignition mass



 $M_h \approx 20 \mu g$

$$R_h \simeq \sqrt{\frac{3}{4\pi} \frac{M_h}{\rho_h R_h}} \approx 40 \mu \mathrm{m}$$

$$\rho_h \approx \frac{(\rho_h R_h)}{R_h} = \frac{0.3}{40 * 10^{-4}} = 75 \, g/\mathrm{cm}^3$$

Summary



Possible fuel assembly for 1MJ ICF driver



There are alternative

TRAPPING Fusion fire

When a superhot, ionized plasma is trapped in a magnetic field, it will fight to escape. Reactors are designed to keep it confined for long enough for the nuclei to fuse and produce energy.

A CHOICE OF FUELS

Many light isotopes will fuse to release energy. A deuterium-tritium mix ignites at the lowest temperature, roughly 100 million kelvin, but produces neutrons that make the reactor radioactive. Other fuels avoid that, but ignite at much higher temperatures.



Magnetic field coils

Liquid metal vortex

http://www.nextbigfuture.com/2016/05/nuclear-fusion-comany-tri-alpha-energy.html



Commonwealth Fusion Systems, a MIT spin-out company, is building a high-magnetic field tokamak





- Fusion power $\propto B^4$.
- The fusion gain Q > 2 is expected for SPARC tokamak.

Merging compression is used to heat the tokamak at the start-up process in ST40 Tokamak at Tokamak Energy Ltd



• High temperature superconductors are used.

• B_T ~ 3 T



 Merging compression



M. Gryaznevich, etc., Fusion Eng. Design, **123**,177 (2017) https://www.tokamakenergy.co.uk/ P. F. Buxton, etc., Fusion Eng. Design, **123**, 551 (2017)

Reconnection





https://www.youtube.com/watch?v=7sS3Lpzh0Zw

Merging compression is used to heat the plasma



http://www.100milliondegrees.com/merging-compression/ P. F. Buxton, etc., Fusion Eng. Design, **123**, 551 (2017)

A strong magnetic field reduces the heat flux



• Typical hot spot conditions: $R_{hs} \sim 40 \ \mu m, \rho \sim 20 \ g/cm^3, T \sim 5 \ keV:$ $B > 10 \ MG$ is needed for $\chi > 1$

Magnetic-flux compression can be used to provide the needed magnetic field.

Principle of frozen magnetic flux in a good conductor is used to compress fields



 $\Phi = \pi r_0^2 B_0 = \pi r^2 B$

M. Hohenberger, P.-Y. Chang, *et al.*, Phys. Plasmas <u>19</u>, 056306 (2012). ₄₆

Plasma can be pinched by parallel propagating plasmas





https://en.wikipedia.org/wiki/Pinch_(plasma_physics) 47

Plasma can be heated via pinches



Sandia's Z machine is the world's most powerful and efficient laboratory radiation source





- Stored energy: 20 MJ
- Marx charge voltage: 85 kV
- Peak electrical power: 85 TW
- Peak current: 26 MA
- Rise time: 100 ns
- Peak X-ray emissions: 350 TW
- Peak X-ray output: 2.7 MJ

Z machine





Z machine







- Stored energy: 20 MJ
- Peak electrical power: 85 TW
- Peak current: 26 MA
- Rise time: 100 ns
- Peak X-ray output: 2.7 MJ

Z machine discharge





Before and after shots

• Before shots



SAND2017-0900PE_The sandia z machine - an overview of the world's most powerful pulsed power facility.pdf

• After shots



Promising results were shown in MagLIF concept conducted at the Sandia National Laboratories



The stagnation plasma reached fusion-relevant temperatures with a 70 km/s implosion velocity

S. A. Slutz et al Phys. Plasmas 17 056303 (2010)

M. R. Gomez et al Phys. Rev. Lett. 113 155003 (2014) 54

MagLIF target





Neutron yield increased by 100x with preheat and external magnetic field.





M. R. Gomez *et al* Phys. Rev. Lett. 113 155003 (2014) 56

Sheared flow stabilizes MHD instabilities



 $\frac{dV_Z}{dr} \neq 0$

- M. G. Haines, etc., Phys. Plasmas 7, 1672 (2000) U. Shumlak, etc., Physical Rev. Lett. 75, 3285 (1995)
- U. Shumlak, etc., ALPHA Annual Review Meeting 2017

A z-pinch plasma can be stabilized by sheared flows



https://www.zapenergyinc.com/about A. D. Stepanov, etc., Phys. Plasmas 27, 112503 (2020)

Elevated electron temperature coincident with observed fusion reactions in a sheared-flow-stabilized z pinch



Fusion reactor concept by ZAP energy



https://www.zapenergyinc.com/about E. G. Forbes, etc., Fusion Sci. Tech. 75, 599 (2019)

Spherical torus (ST) and compact torus (CT)

Spherical torus (ST)



- Compact torus (CT)
 - Spheromak



• Field reversed configuration (FRC)



Zhe Gao, Matter Radiat. Extremes **1**, 153 (2016) https://en.wikipedia.org/wiki/Field-reversed_configuration

Field reverse configuration is used in Tri-alpha energy



Field reverse configuration is used in Tri-alpha energy



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NBI for Tri-Alpha Energy Technologies





Neutral beams are injected in to the chamber for spinning the FRC





https://tae.com/media/ https://zh.wikihow.com/%E5%9C%A8%E6%89%8B%E6%8C%87%E4%B8%8A%E8%BD%AC%E7%AF%AE%E7%90%83

FRC sustain longer with neutral beam injection





General fusion is a design ready to be migrated to a power plant



A spherical tokamak is first generated



Plasma injector for the spherical tokamak





A spherical tokamak is generated in a liquid metal vortex





The spherical tokamak is compressed by the pressure provided by the sournding hydraulic pistons



BBC: General Fusion to build its Fusion Demonstration Plant in the UK, at the UKAEA Culham Campus



By Matt McGrath Environment correspondent

🕑 17 June





A company backed by Amazon's Jeff Bezos is set to build a large-scale nuclear fusion demonstration plant in Oxfordshire.

Canada's General Fusion is one of the leading private firms aiming to turn the
Helion energy is compressing the two merging FRCs





Two FRCs are accelerated toward each other





Two FRCs merge with each other



ectricity Recapture

plasma expands, it pushes back on the magnetic y Faraday's law, the change in field induces t, which is directly recaptured as electricity. This usion electricity is used to power homes and unities, efficiently and affordably.

site uses cookies. Read more about our privacy policy & terms of use.

The merged FRC is compressed electrically to high temperature





e uses cookies. Read more about our privacy policy & terms of use.

Similar concept will be studied in our laboratory. •

Projectile Fusion is being established at First Light Fusion Ltd, UK





• I_{peak}=14 MA w/ T_{rise}~2us.





 High pressure is generated by the colliding shock.

https://firstlightfusion.com/ B. Tully and N. Hawker, Phys. Rev. **E93**, 053105 (2016) 77

A gas gun is used to eject the projectile





https://www.youtube.com/watch?v=JN7lyxC11n0 https://www.youtube.com/watch?v=aW4eufacf-8

Many groups aim to achieve ignition in the MCF regime in the near future

ITER – 2025 First Plasma
2035 D-T Exps
2050 DEMO



https://www.iter.org https://www.tokamakenergy.co.uk/ https://www.psfc.mit.edu/sparc

- Tokamak energy, UK
 - 2025 Gain
 - 2030 to power grid



 Commonwealth Fusion Systems, USA – 2025 Gain



Fusion is blooming



We are closed to ignition!



A. J. Webster, Phys. Educ. **38**, 135 (2003) R. Betti, etc., Phys. Plasmas, **17**, 058102 (2010)

Many groups aim to achieve ignition in the MCF regime in the near future

ITER – 2025 First Plasma
2035 D-T Exps
2050 DEMO



https://www.iter.org https://www.tokamakenergy.co.uk/ https://www.psfc.mit.edu/sparc

- Tokamak energy, UK
 - 2025 Gain
 - 2030 to power grid



 Commonwealth Fusion Systems, USA – 2025 Gain



A new design using a spherical chamber can tolerate several potential shapes and sides calculated by the theory group

