Introduction to Nuclear Fusion as An Energy Source

Po-Yu Chang

Institute of Space and Plasma Sciences, National Cheng Kung University

Lecture 12

2024 spring semester

Wednesday 9:10-12:00

Materials:

https://capst.ncku.edu.tw/PGS/index.php/teaching/

Online courses:

https://nckucc.webex.com/nckucc/j.php?MTID=ma76b50f97b1c6d72db61de9eaa9f0b27

2024/5/29 updated

Note!

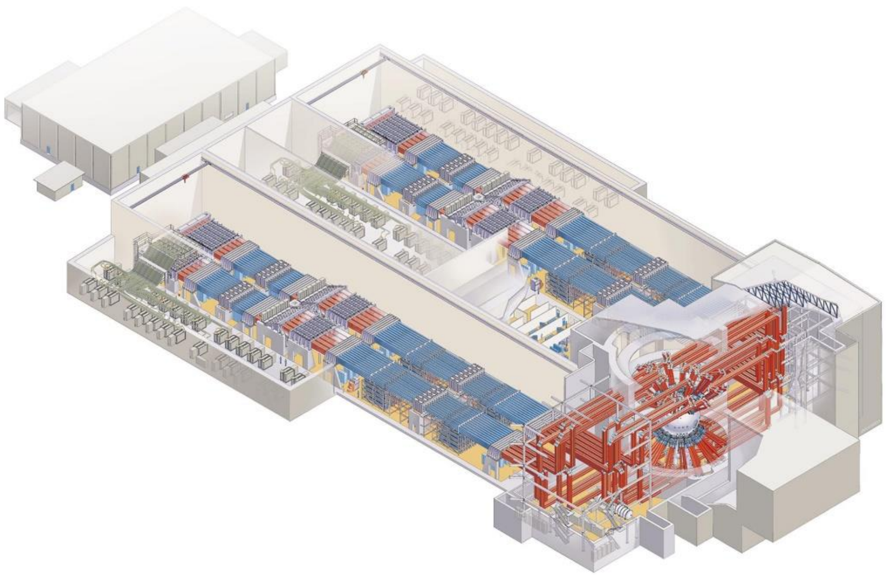
- **Final exam 6/12 (One double-sided A4 cheating sheet is allowed.)**
- **Last class 6/19**

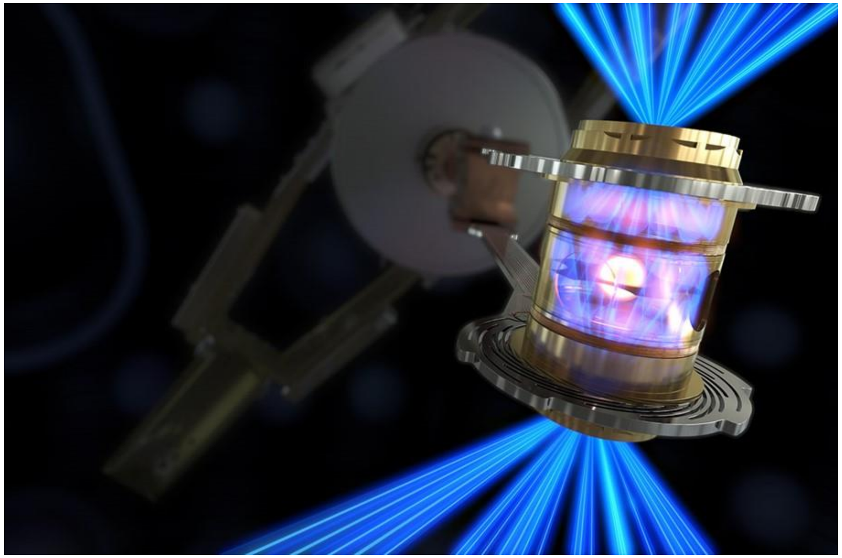
Course Outline

- Inertial confinement fusion (ICF)
  - Plasma frequency and critical density
  - Direct- and indirect- drive
  - Laser generated pressure (Inverse bremsstrahlung and Ablation pressure)
  - Burning fraction, why compressing a capsule?
  - Implosion dynamics
  - Shock (Compression with different adiabat)
  - Laser pulse shape
  - Rocket model, shell velocity
  - Laser-plasma interaction (Stimulated Raman Scattering, SRS; Stimulated Brillouin Scattering, SBS; Two-plasmon decay)
  - Instabilities (Rayleigh-taylor instability, Kelvin-Helmholtz instability, Richtmeyer-Meshkov instability)

Significant breakthrough was achieved in ICF recently

- **Inertial confinement fusion (ICF)**



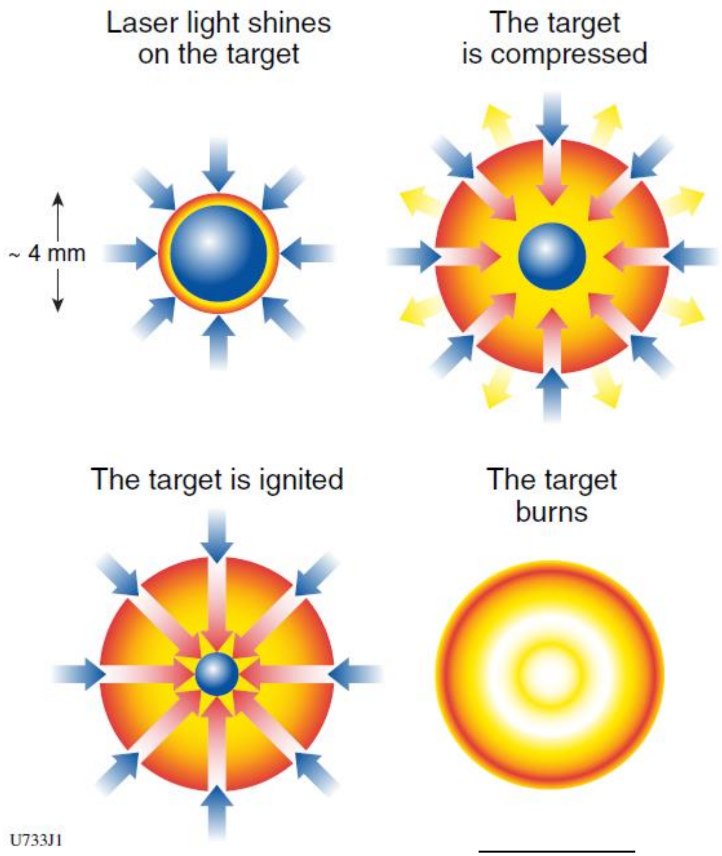


- **National Ignition Facility (NIF) demonstrated a gain grater than 1 for the first time on 2022/12/5. The yield of 3.15 MJ from the 2.05-MJ input laser energy, i.e., Q=1.5.**

https://zh.wikipedia.org/wiki/國家點火設施
https://www.science.org/content/article/historic-explosion-long-sought-fusion-breakthrough

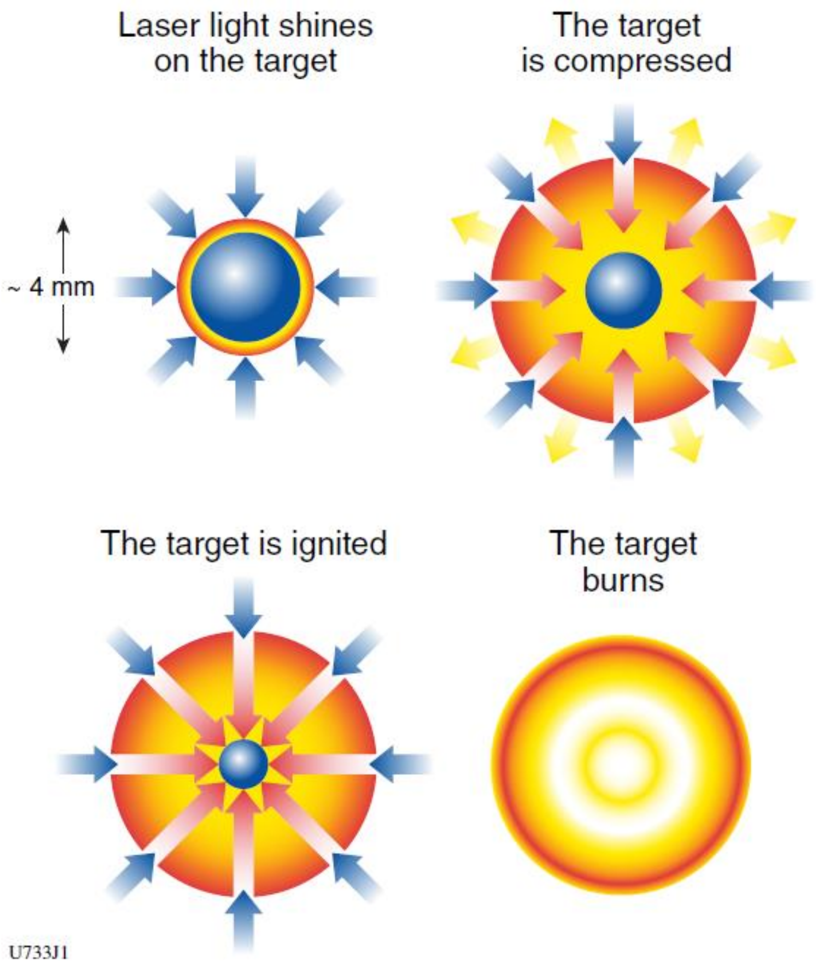
Don’t confine it!

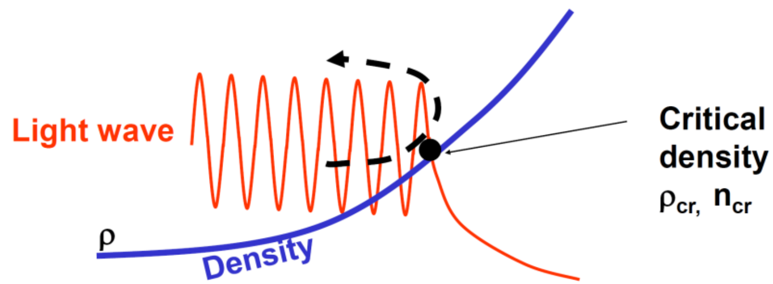
- **Solution 2: Inertial confinement fusion (ICF). Or you can say it is confined by its own inertia: P~Gigabar, τ~nsec, T~10 keV (10^8 ^{0}C)**

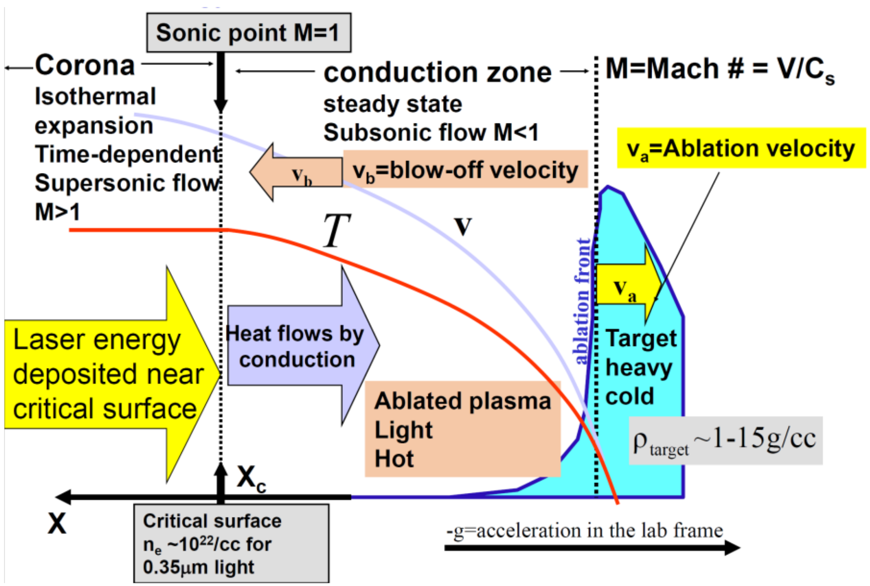


Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester

Compression happens when outer layer of the target is heated by laser and ablated outward

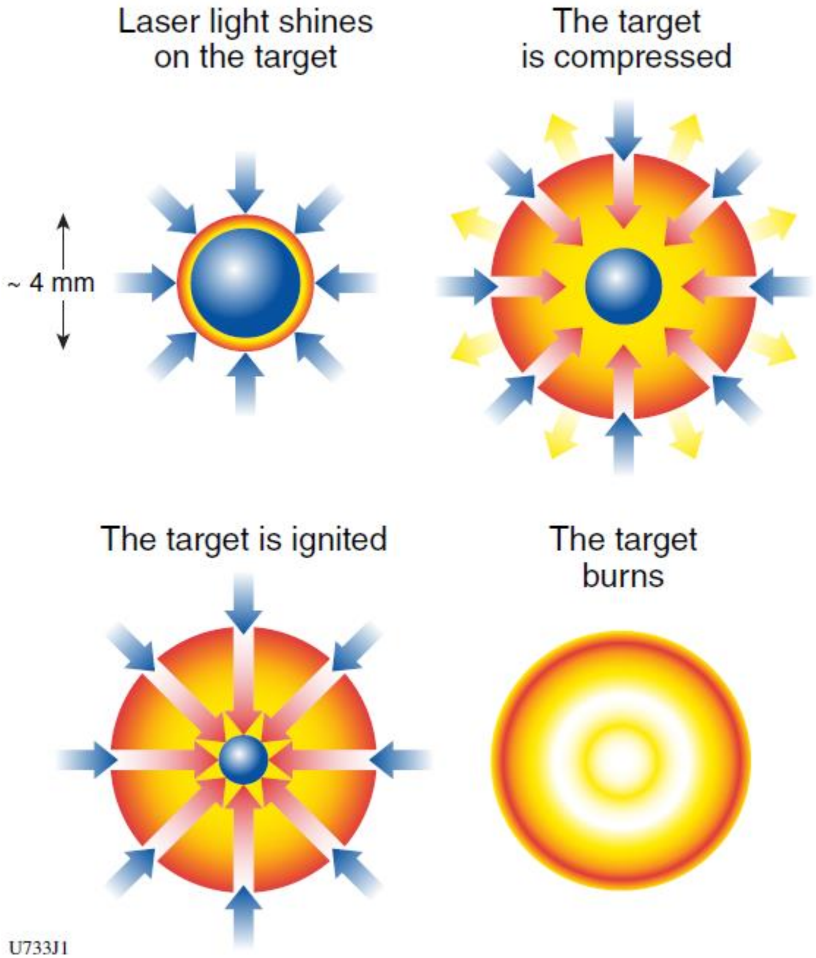




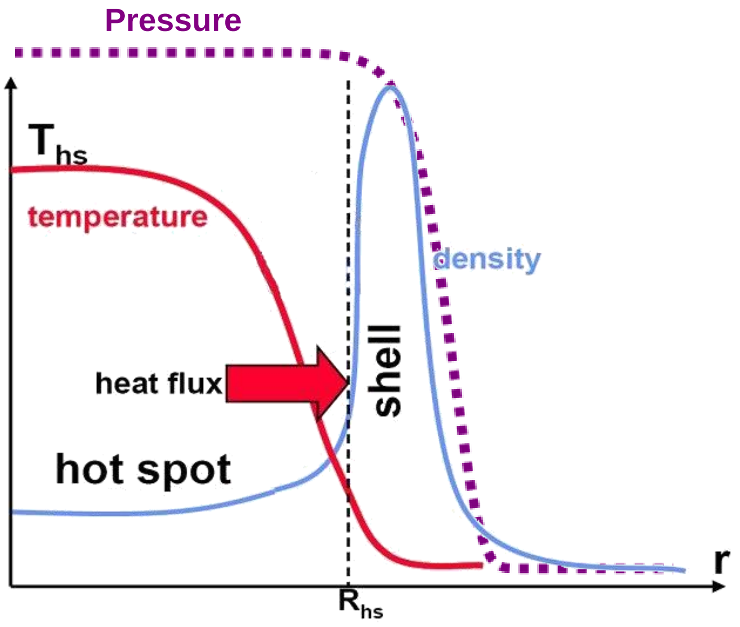


Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester
R. Betti, HEDSA HEDP Summer School, 2015

Plasma is confined by its own inertia in inertial confinement fusion (ICF)



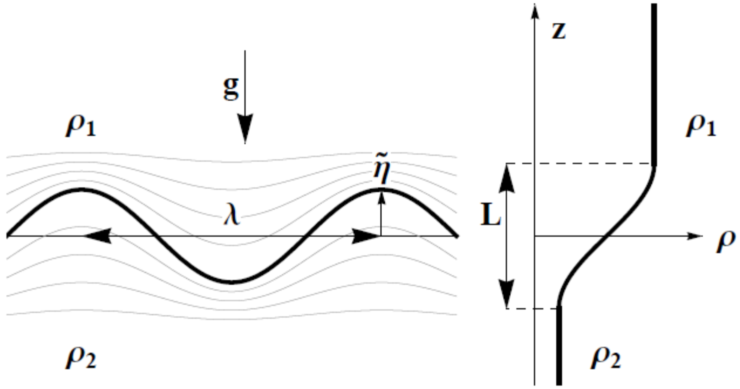
Spatial profile at stagnation



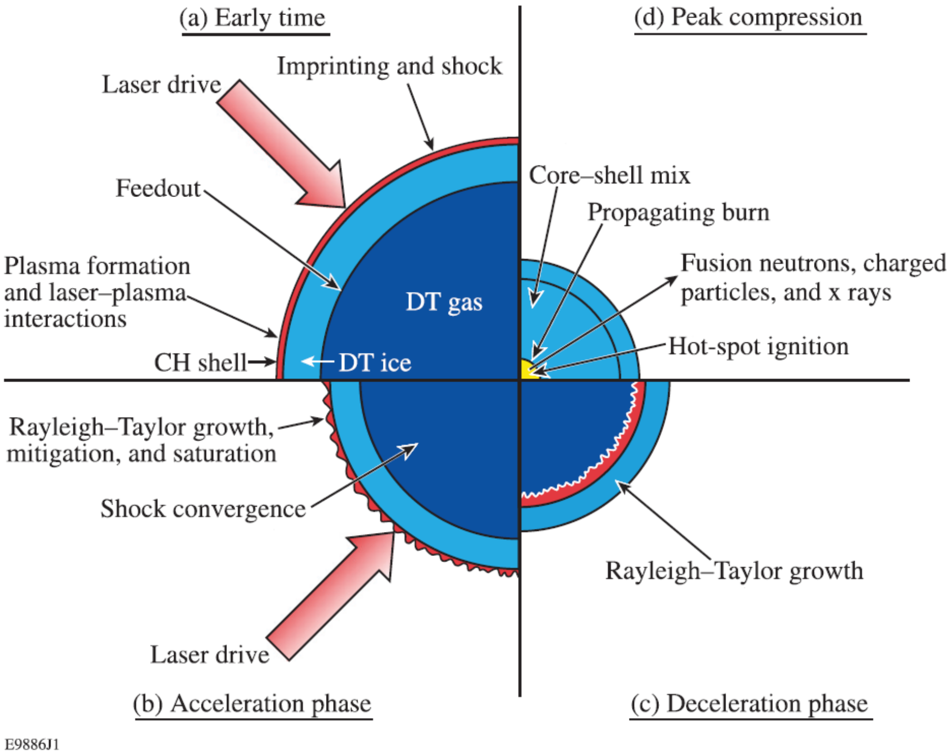
Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester

A ball can not be compressed uniformly by being squeezed between several fingers

- **Rayleigh-Taylor instability**

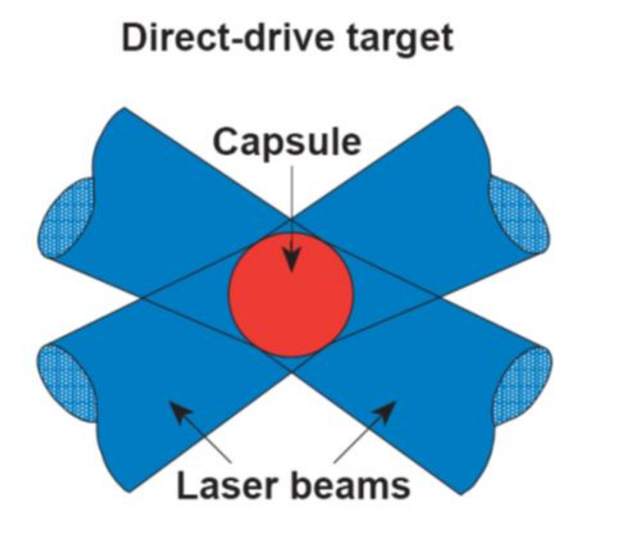


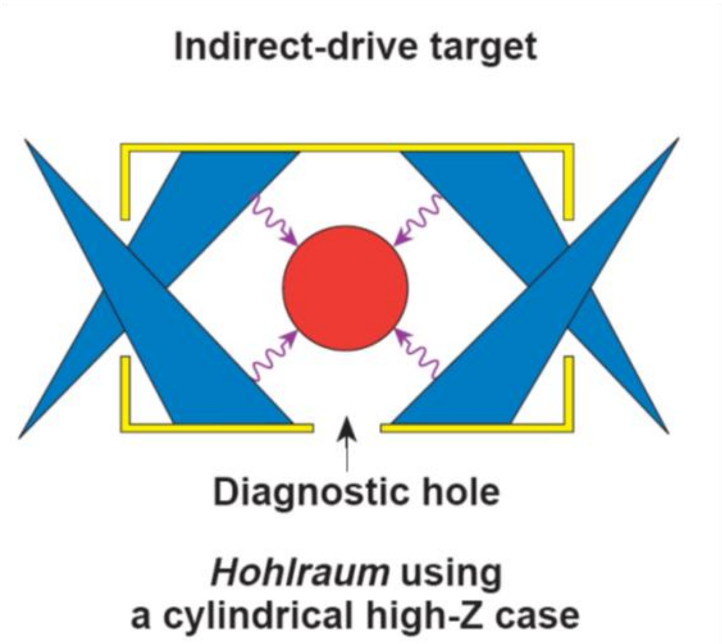
- **Stages of a target implosion**



P.-Y. Chang, PhD Thesis, U of Rochester (2013)
R. S. Craxton, etc., *Phys. Plasmas* **22**, 110501 (2015)

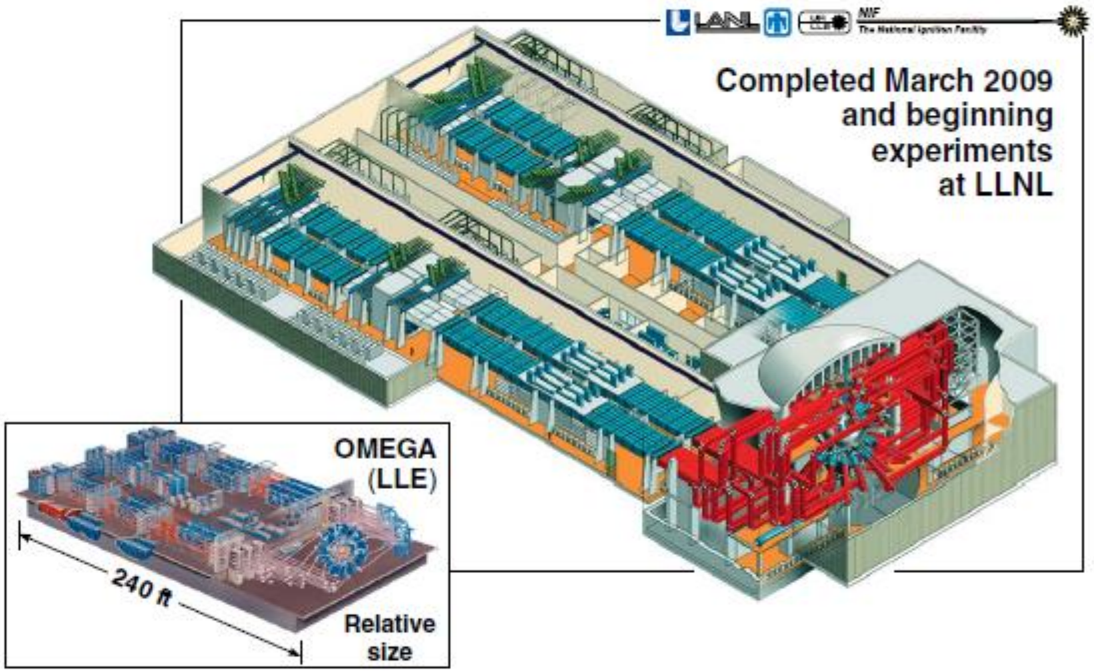
A spherical capsule can be imploded through directly or indirectly laser illumination





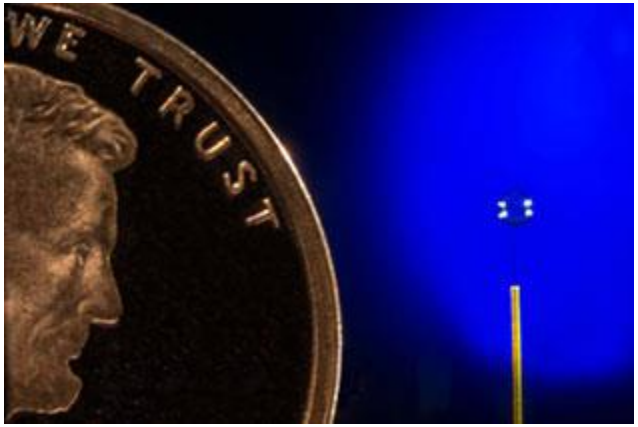
*R. Betti, HEDSA HEDP Summer School, 2015

The 1.8-MJ National Ignition Facility (NIF) will demonstrate ICF ignition and modest energy gain

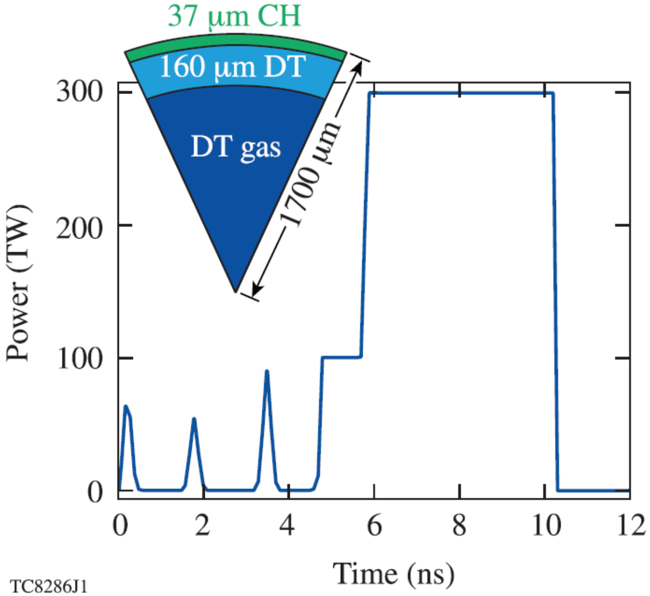


OMEGA experiments are integral to an ignition demonstration on the NIF.

Targets used in ICF



- **Triple-point temperature : 19.79 K**

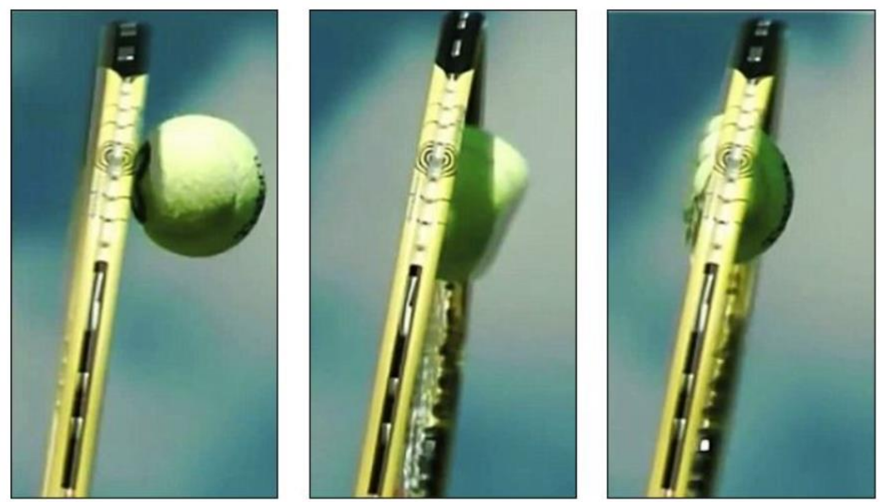


http://www.lle.rochester.ed
https://en.wikipedia.org/wiki/Inertial_confinement_fusion
R. S. Craxton, etc., *Phys. Plasmas* **22**, 110501 (2015)

Softer material can be compressed to higher density

- **Compression of a baseball**

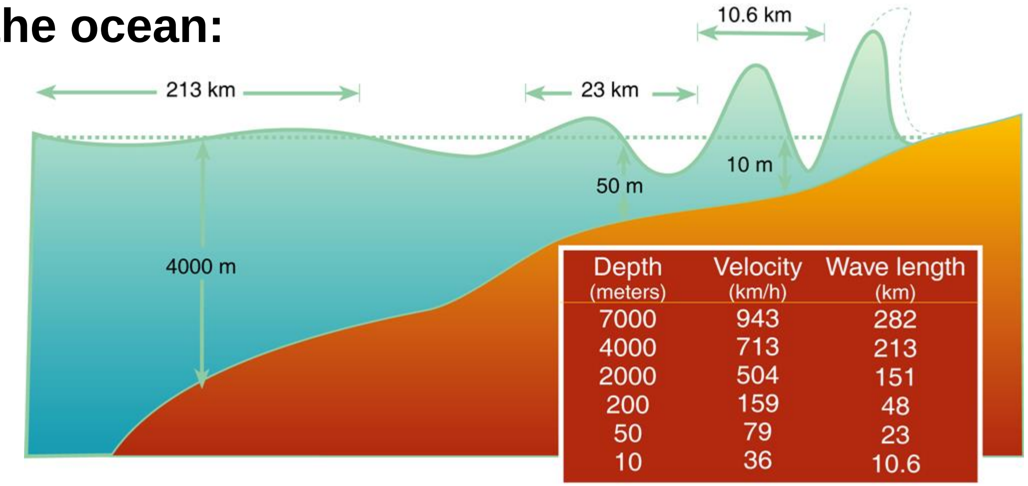
- **Compression of a tennis ball**



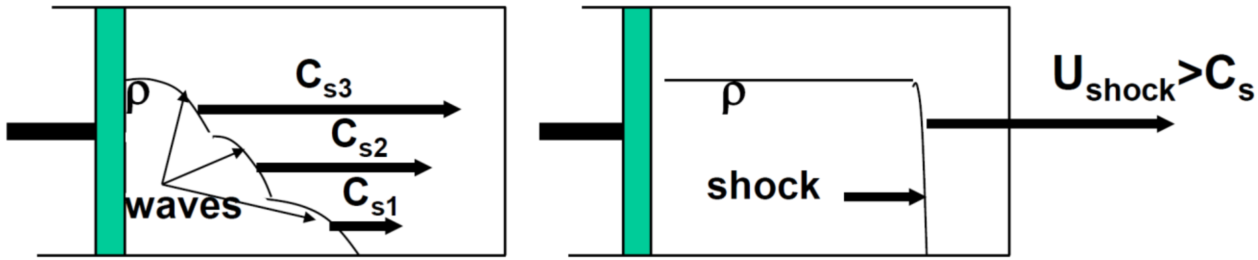
https://www.youtube.com/watch?v=uxlIdMoAwbY
https://newsghana.com.gh/wimbledon-slow-motion-video-of-how-a-tennis-ball-turns-to-goo-after-serve/

A shock is formed due to the increasing sound speed of a compressed gas/plasma

- **Wave in the ocean:**



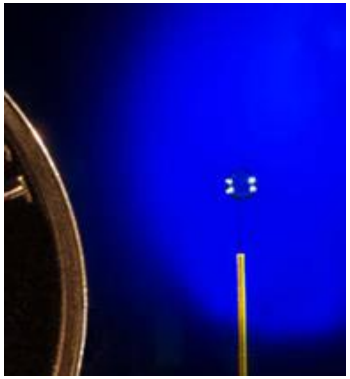
- **Acoustic/compression wave driven by a piston:**



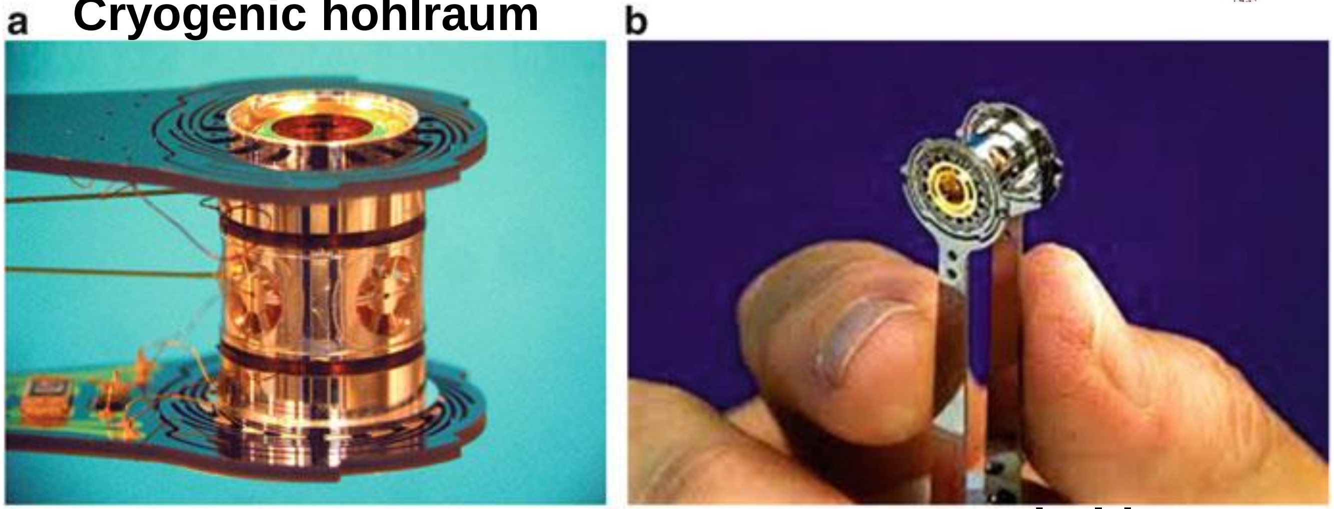
$$C_s \sim \sqrt{\frac{p}{\rho}} \sim \sqrt{\frac{\alpha\rho^{5/3}}{\rho}} \sim \sqrt{\alpha}\rho^{1/3}$$

http://neamtic.ioc-unesco.org/tsunami-info/the-cause-of-tsunamis
*R. Betti, HEDSA HEDP Summer School, 2015

Targets used in ICF

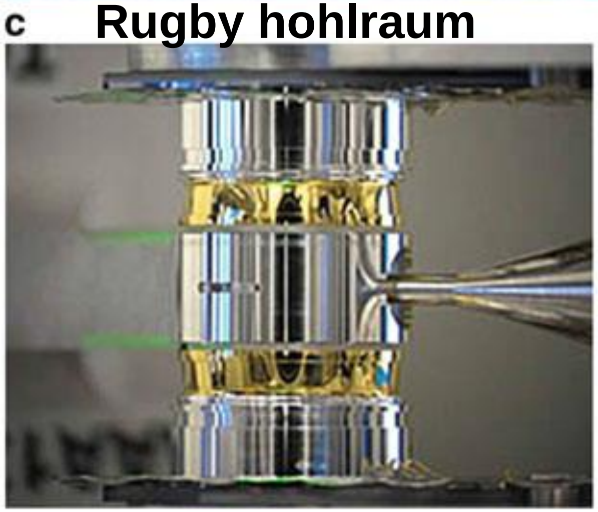


Cryogenic hohlraum

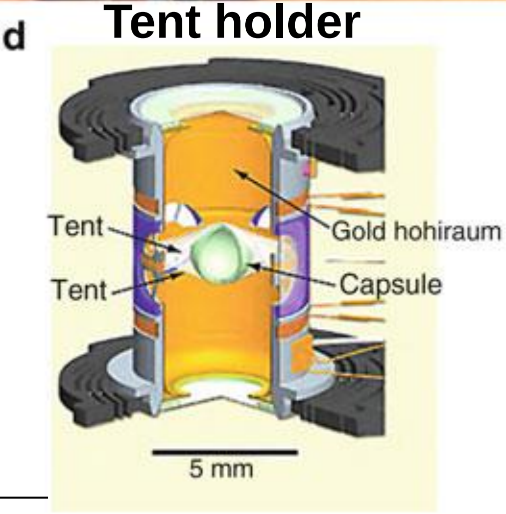


Cryogenic shroud

Rugby hohlraum

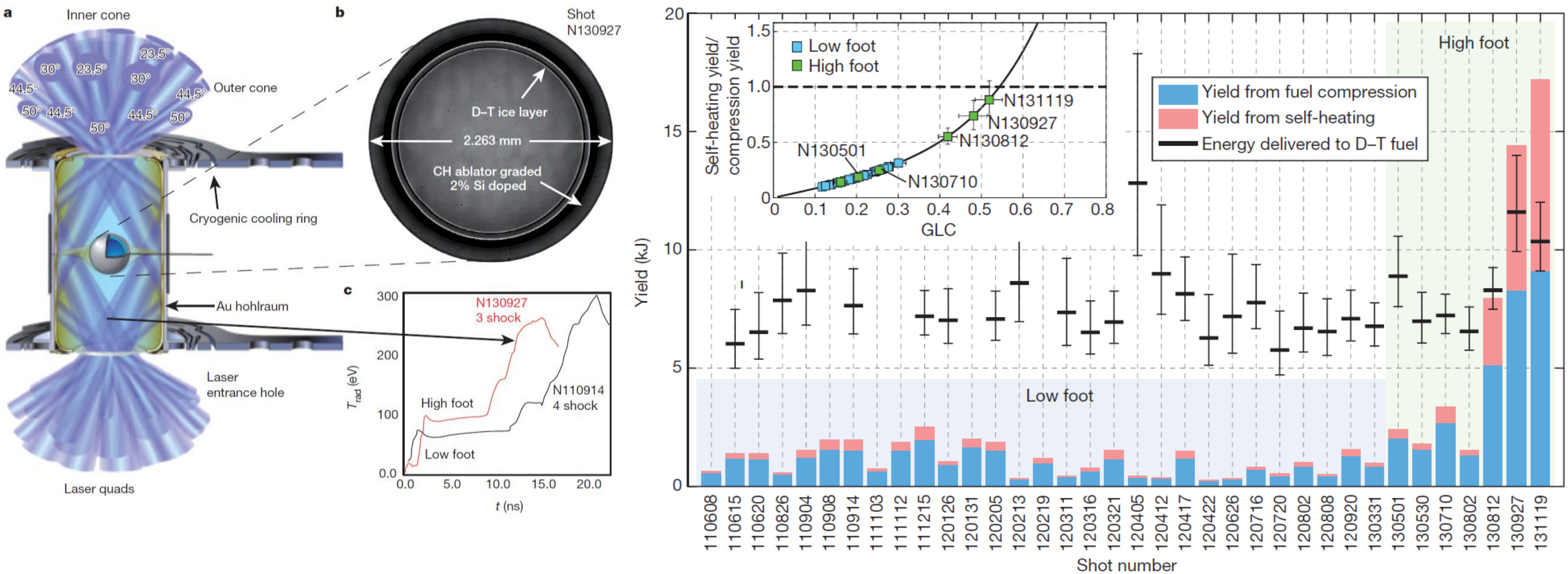


Tent holder



https://www.lle.rochester.edu/index.php/2014/11/10/next-generation-cryo-target/
Introduction to Plasma Physics and Controlled Fusion 3rd Edition, by Francis F. Chen
https://www.llnl.gov/news/nif-shot-lights-way-new-fusion-ignition-phase

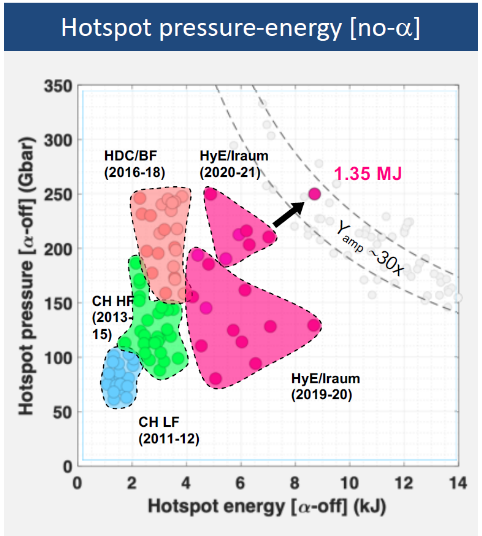
Nature letter "Fuel gain exceeding unity in an inertially confined fusion implosion"

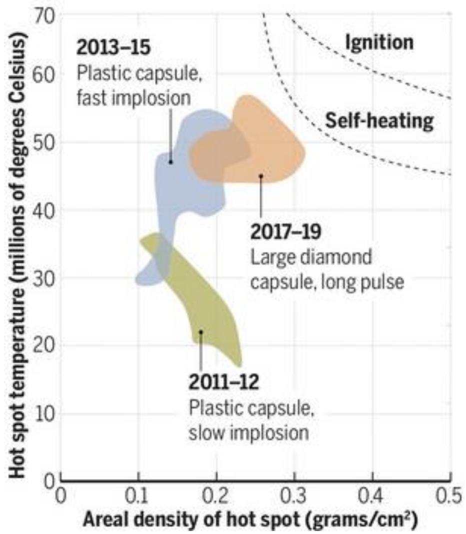


- **Fuel gain exceeding unity was demonstrated for the first time.**

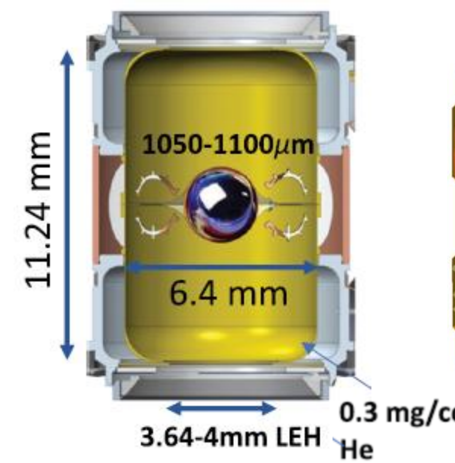
Nature 506, p343, 2014

The hot spot has entered the burning plasma regime

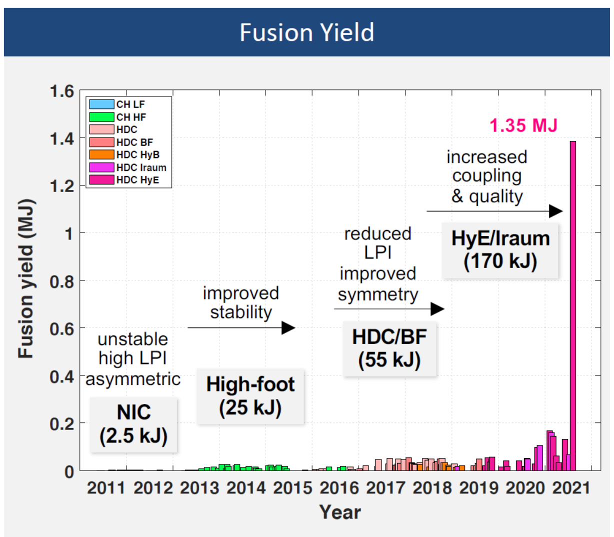




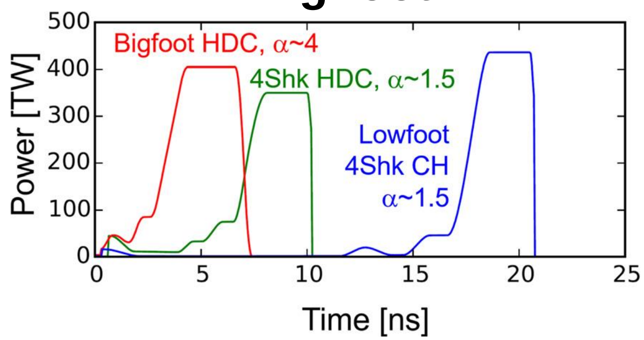
- **Hybrid-E**

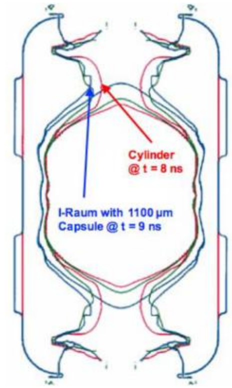


- **I-raum**



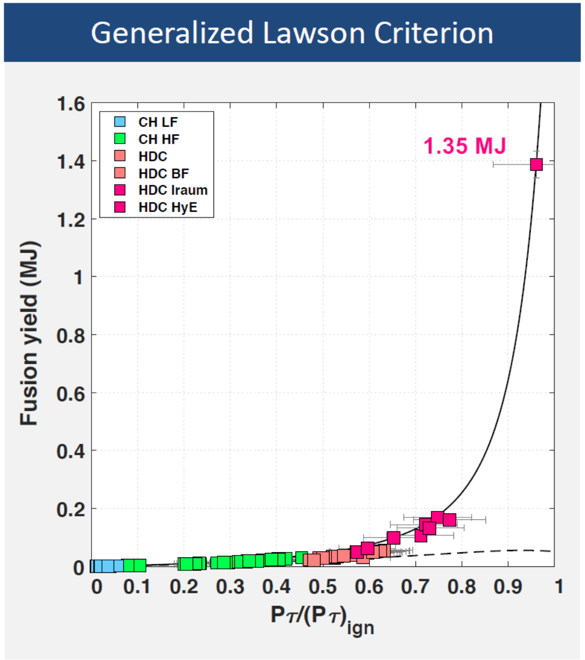
- **Big foot**

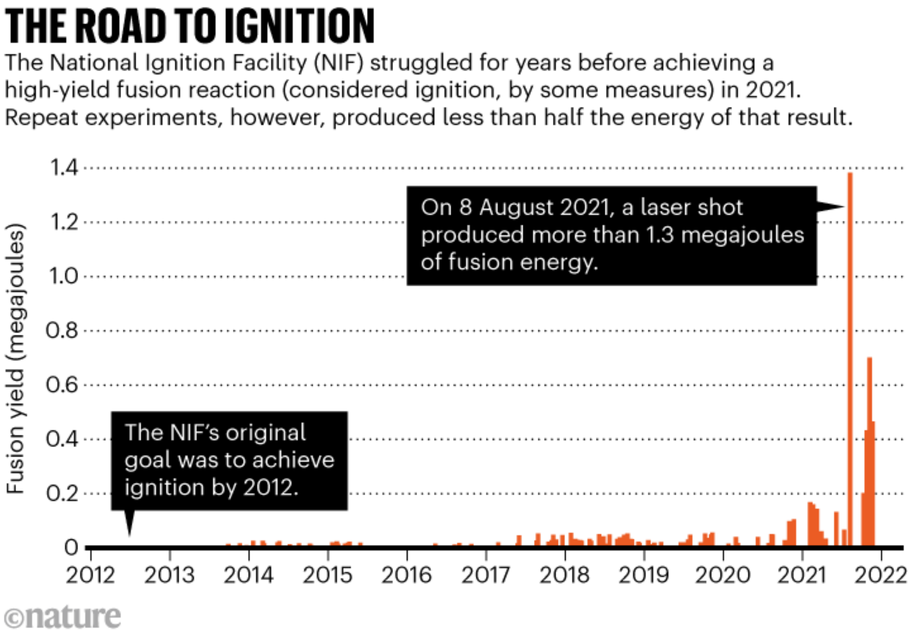




T. Ma, ARPA-E workshop, April 26, 2022
Science 370, p1019, 2020
D. T. Casey, etc., Phys. Plasmas, 25, 056308 (2018)
A. L. Kritcher, etc., Phys. Plasmas, 28, 072706 (2021)
H. F. Robey, etc., Phys. Plasmas, 25, 012711 (2018)

National Ignition Facility (NIF) achieved a yield of more than 1.3 MJ from ~1.9 MJ of laser energy in 2021 (Q~0.7)



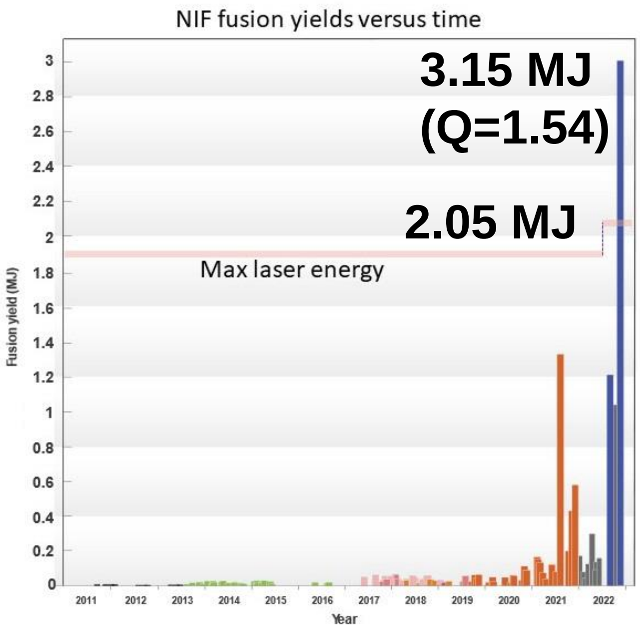


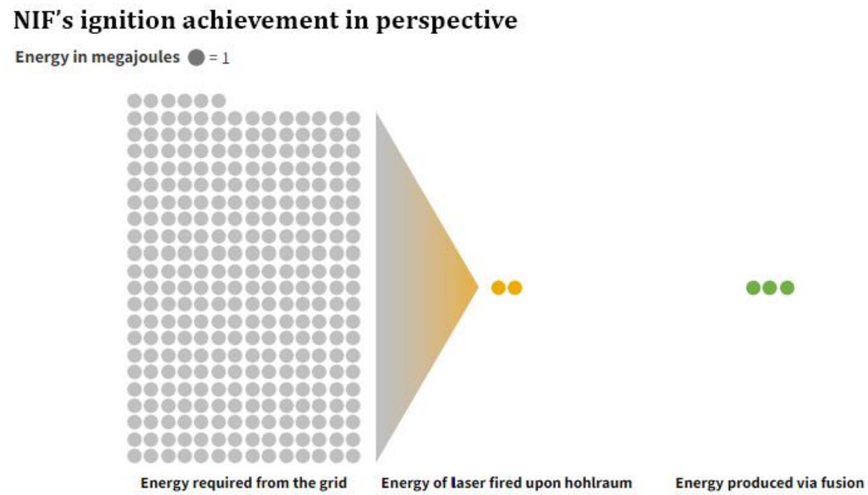
- **National Ignition Facility (NIF) achieved a yield of more than 1.3 MJ (Q~0.7). This advancement puts researchers at the threshold of fusion ignition.**

- **Laser-fusion facility heads back to the drawing board.**

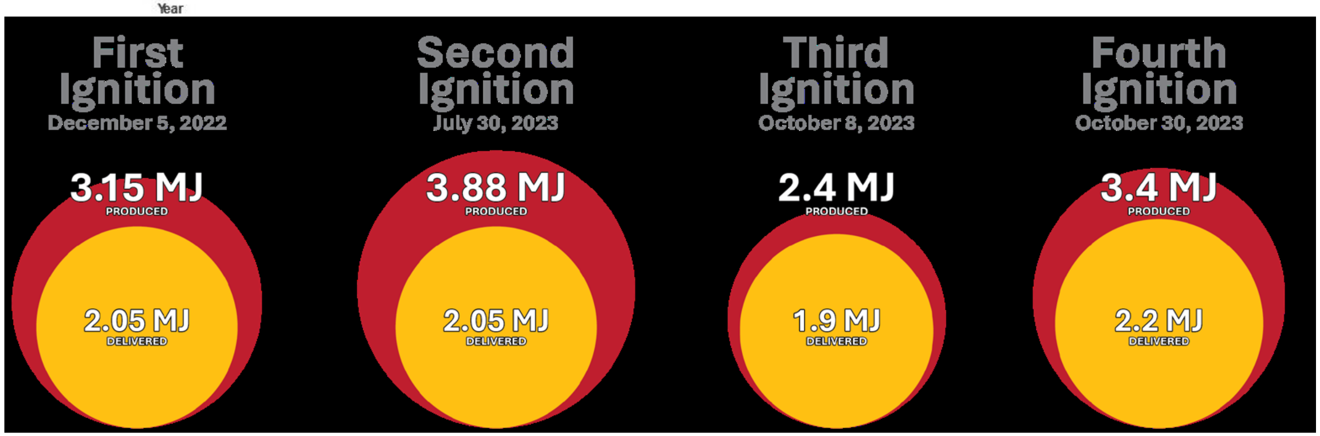
T. Ma, ARPA-E workshop, April 26, 2022
J. Tollefson, Nature (News) 608, 20 (2022)

“Ignition” (target yield larger than one) was achieved in NIF on 2022/12/5





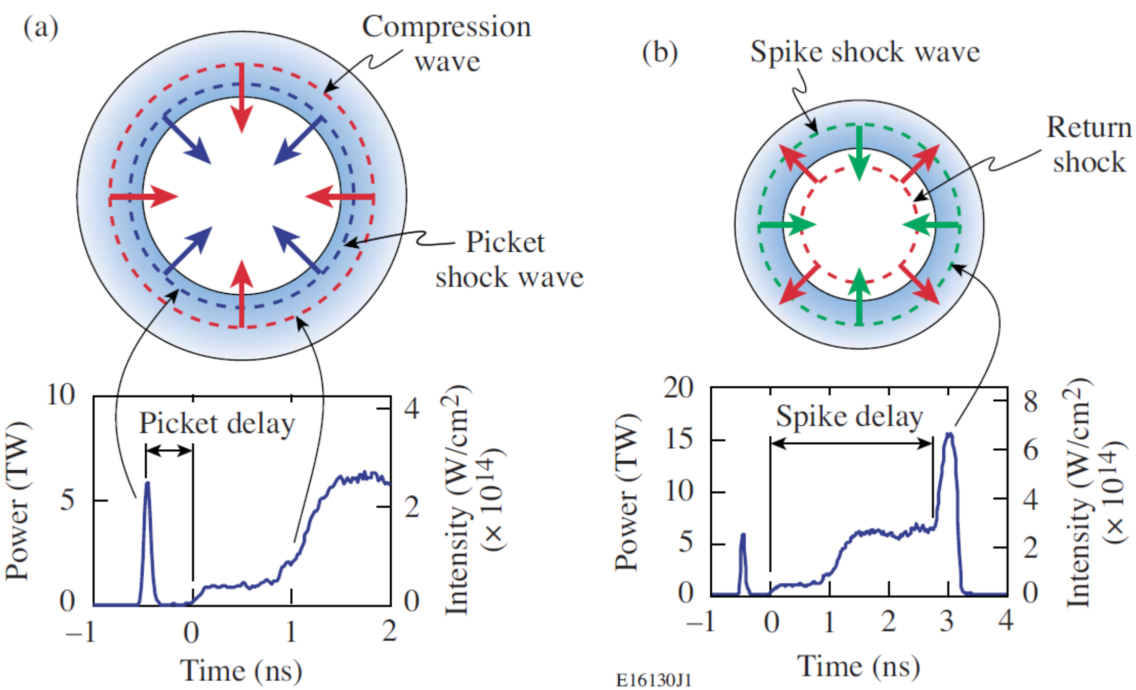
300 MJ 2.05 MJ 3.15 MJ



https://physicstoday.scitation.org/do/10.1063/PT.6.2.20221213a/full/
The age of ignition: anniversary edition, LLNL-BR-857901

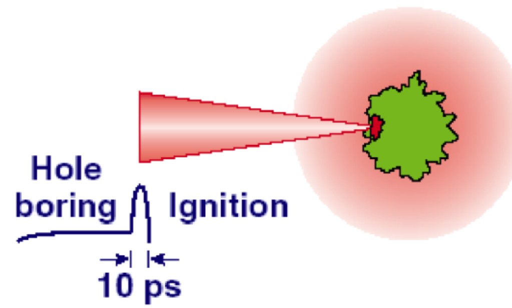
External “spark” can be used for ignition

- **Shock ignition**

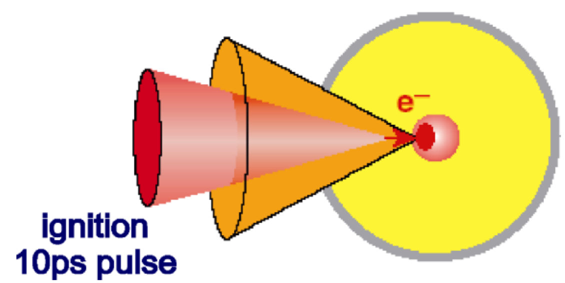


- **Fast ignition**

a) channeling FI concept



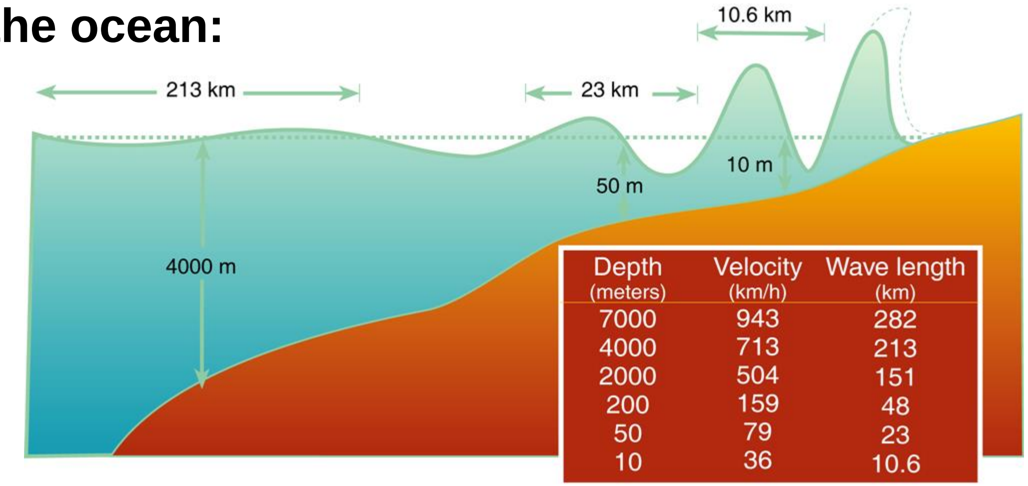
b) cone-in-shell FI concept



J. Badziak, Bull.Polish Acad. Sci. Tech. Sci.Phys. Plasmas 15, 056306 (2008)

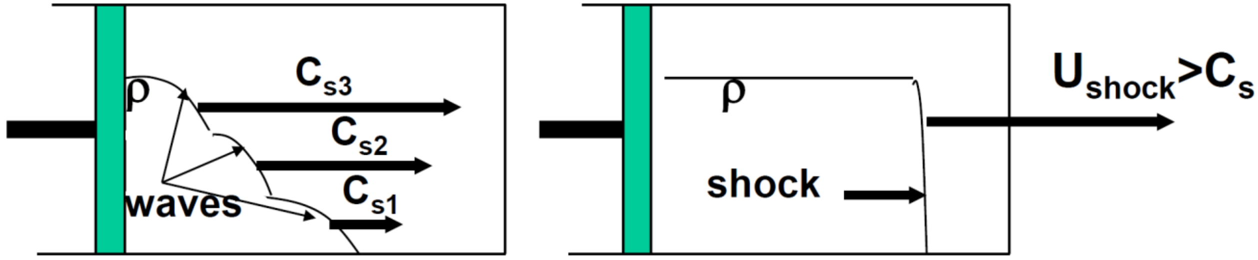
A shock is formed due to the increasing sound speed of a compressed gas/plasma

- **Wave in the ocean:**



Depth (meters)	Velocity (km/h)	Wave length (km)
7000	943	282
4000	713	213
2000	504	151
200	159	48
50	79	23
10	36	10.6

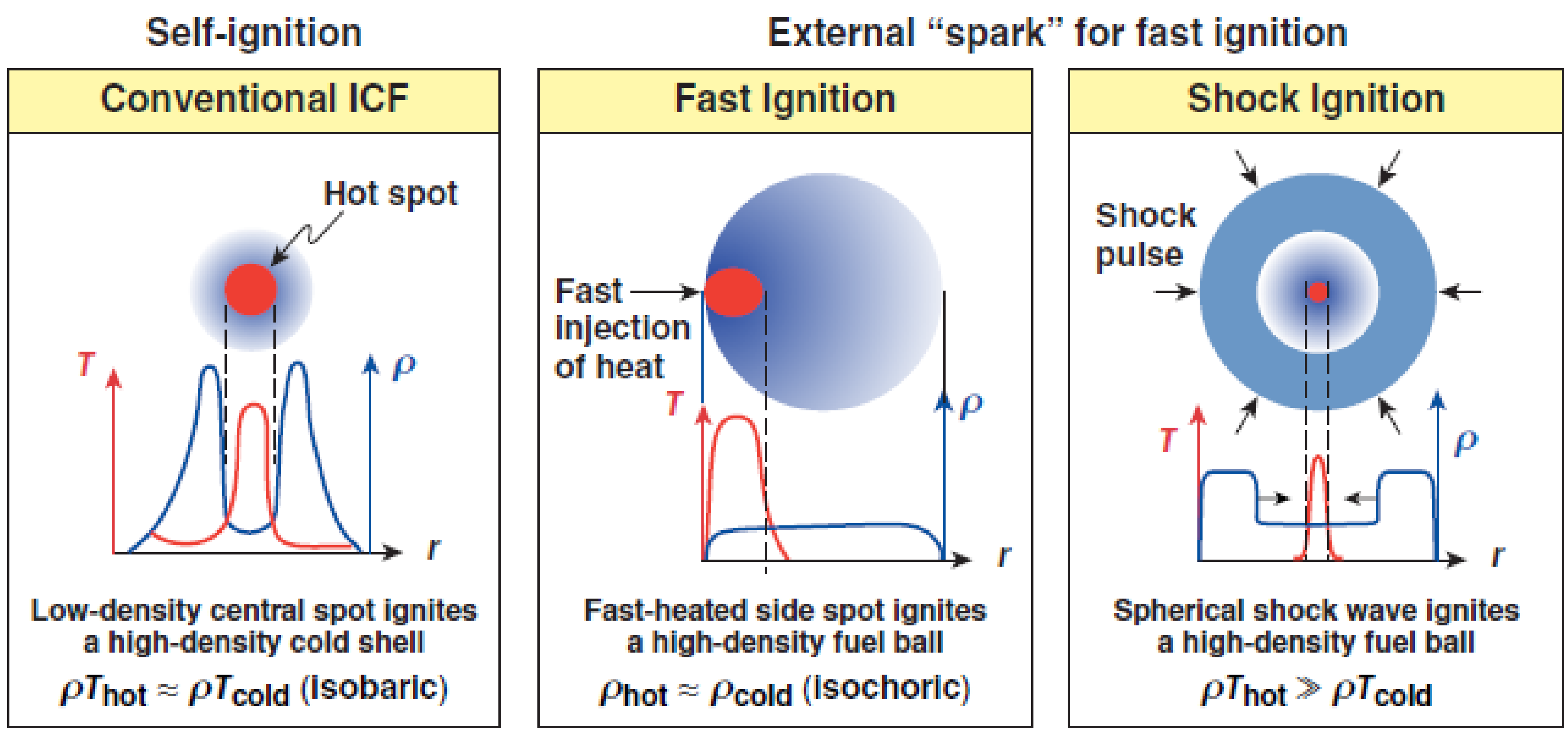
- **Acoustic/compression wave driven by a piston:**



$$C_s \sim \sqrt{\frac{p}{\rho}} \sim \sqrt{\frac{\alpha\rho^{5/3}}{\rho}} \sim \sqrt{\alpha}\rho^{1/3}$$

http://neamtic.ioc-unesco.org/tsunami-info/the-cause-of-tsunamis
*R. Betti, HEDSA HEDP Summer School, 2015

Ignition can happen by itself or being triggered externally

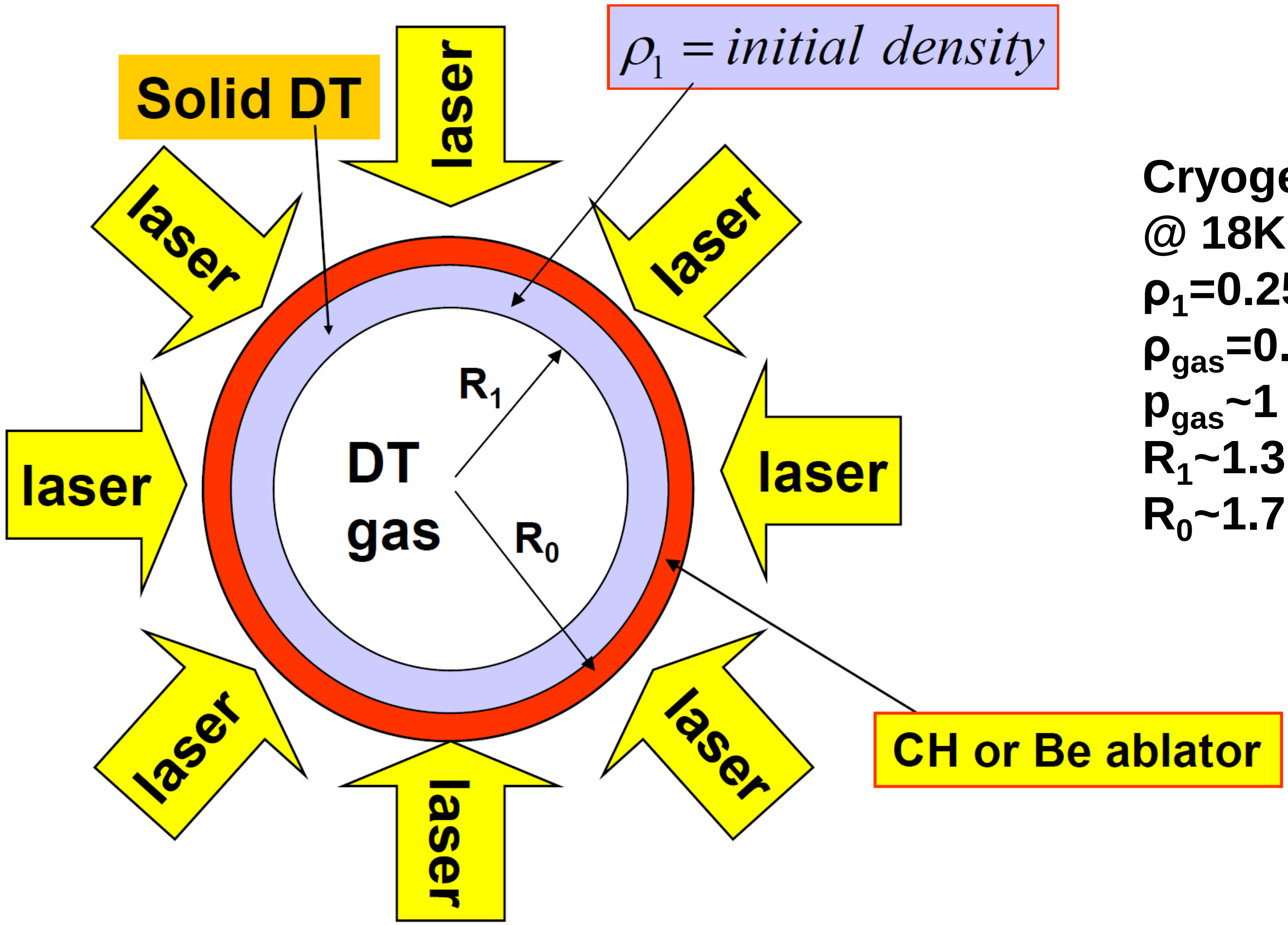


P. B. Radha, Fusion Energy Conference,2018

Reference for ICF

- Riccardo Betti, University of Rochester, HEDSA HEDP summer school, San Diego, CA, August 16-21, 2015
- ICF lectures for course PHY558/ME533
- The physics of inertial fusion, by S. Atzeni, J. Meyer-Ter-Vehn

Laser-driven imploding capsules are mm-size shells with hundreds of μm thick layers of cryogenic solid DT



Cryogenic solid DT ice @ 18K
ρ_1=0.25 g/cc
ρ_{gas}=0.3-0.6 mg/cc
p_{gas}~1 atm
R_1~1.3 mm
R_0~1.7 mm

Conservation equations of gas-dynamics and ideal gas EOS are used for DT plasma

Mass conservation: $\partial_t \rho + \partial_x(\rho\, \vec{v}) = 0$

Momentum conservation: $\partial_t(\rho\, \vec{v}) + \partial_x\left(p + \rho v^2\right) = \vec{F}$

Energy conservation: $\partial_t \epsilon + \partial_x(\vec{v}\,(\epsilon + p) - \kappa \partial_x T) = \text{source} + \text{sinks}$

Ideal gas EOS: $p = (n_e T_e + n_i T_i) = 2nT = \frac{2}{m_i}\rho_i T = \frac{\rho T}{A}$

Total energy per unit volume: $\epsilon = \frac{3}{2}p + \rho\frac{v^2}{2}$

Mass density: $\rho = n_i m_i$

Plasma thermal conductivity: κ

The plasma thermal conductivity is written in a power law of T

$$n\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\kappa\frac{\partial T}{\partial x}\right) \rightarrow n\frac{T}{t} \sim \frac{\kappa T}{x^2} \Rightarrow \kappa \sim n\frac{x^2}{t}$$

$$x \Rightarrow \lambda_{\mathrm{mfp}} \sim v_{\mathrm{th}}\tau_{\mathrm{coll}} = \frac{v_{\mathrm{th}}}{\nu_{\mathrm{coll}}} \qquad t \Rightarrow \tau_{\mathrm{coll}} = \frac{1}{\nu_{\mathrm{coll}}} \qquad \Rightarrow \kappa \sim n\frac{v_{\mathrm{th}}^2}{\nu_{\mathrm{coll}}}$$

$$v_{\mathrm{th}}^2 \sim \frac{T}{m_{\mathrm{e}}} \qquad \nu_{\mathrm{coll}} \sim \frac{n}{T^{3/2}} \qquad \Rightarrow \kappa \sim T^{5/2}$$

v_{th}: thermal velocity
ν_{coll}: collision frequency
τ_{coll}: collision time

Plasma thermal conductivity

$$\kappa \approx \kappa_0 T^{5/2}$$

Sound speed in an ideal DT gas/plasma

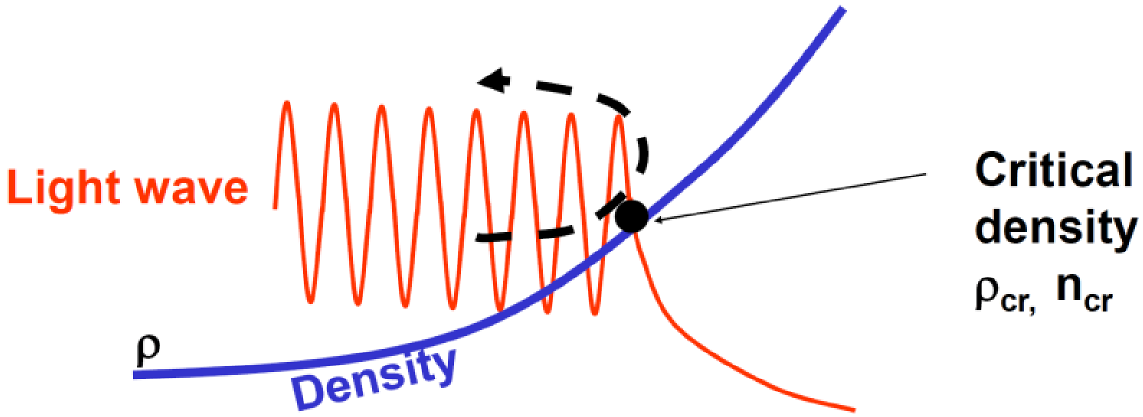
- **Adiabatic sound speed when the entropy is conserved along the fluid motion**

$$C_{\mathrm{s}}^{\mathrm{adiabatic}} = C_{\mathrm{s}}\,(\text{constant entropy}) = \sqrt{\frac{5}{3}\frac{p}{\rho}} = \sqrt{\frac{10}{3}\frac{T}{m_{\mathrm{i}}}}$$

- **Isothermal sound speed when the temperature is constant along the fluid motion**

$$C_{\mathrm{s}}^{\mathrm{isothermal}} = C_{\mathrm{s}}\,(\text{constant temperature}) = \sqrt{\frac{p}{\rho}} = \sqrt{\frac{2T}{m_{\mathrm{i}}}}$$

The laser light cannot propagate past a critical density

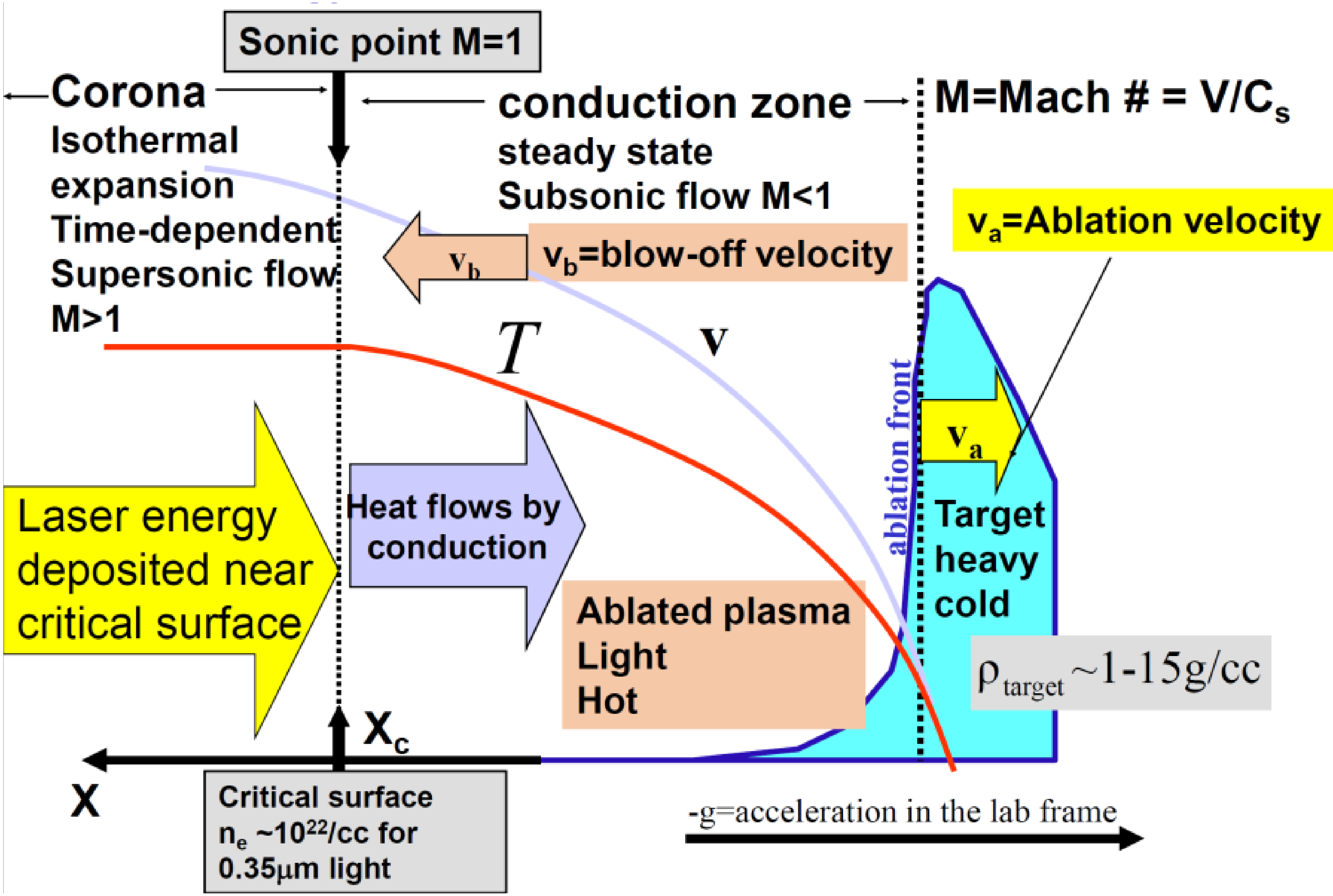


- **Critical density is given by plasma frequency=laser frequency**

$$\omega_{\mathrm{L}} = \frac{2\pi c}{\lambda_{\mathrm{L}}} \qquad \omega_{\mathrm{pe}} = \sqrt{\frac{4\pi n_{\mathrm{e}} e^2}{m_{\mathrm{e}}}}$$

$$\omega_{\mathrm{L}}^2 = \omega_{\mathrm{pe}}^2 \qquad n_{\mathrm{e}}^{\mathrm{cr}} = \frac{1.1 \times 10^{21}}{\lambda_{\mathrm{L},\mu\mathrm{m}}^2}\ \mathrm{cm}^{-3}$$

The laser generates a pressure by depositing energy at the critical surface



Consider the stead state equations of motion in the conduction zone

$$\partial_t \rho + \partial_x(\rho\,\vec{v}) = 0$$

$$\partial_t(\rho\,\vec{v}) + \partial_x\left(p + \rho v^2\right) = \vec{F}$$

$$\partial_t \epsilon + \partial_x(\vec{v}\,(\epsilon + p) - \kappa \partial_x T) = \text{source} + \text{sinks}$$

$$p = (n_e T_e + n_i T_i) = 2nT = \frac{2}{m_i}\rho_i T = \frac{\rho T}{A}$$

$$\frac{d}{dt} = 0 \;\Rightarrow$$

$$\frac{d}{dx}(\rho v) = 0$$

$$\frac{d}{dx}\left(p + \rho v^2\right) = 0$$

$$\frac{d}{dx}\left(v\left(\frac{5}{2}p + \frac{\rho v^2}{2}\right) - \kappa\frac{dT}{dx}\right) = 0$$

$$\kappa = \kappa_o T^{5/2}$$

$$p = \frac{\rho T}{A} \qquad A = \frac{m_i}{1+z}$$

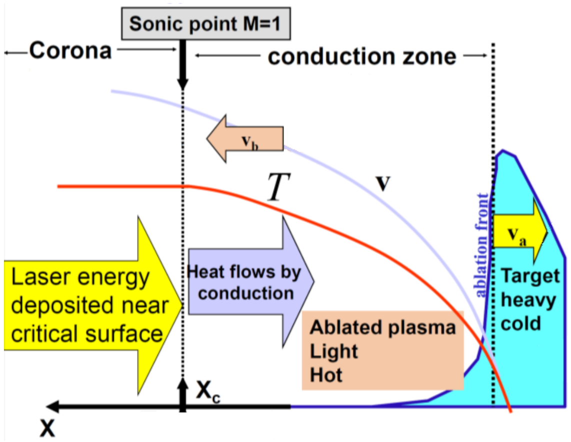
- **Integrate with space:**

$$\rho v = \rho_c v_c$$

$$p + \rho v^2 = p_c + \rho_c {v_c}^2 \qquad \frac{\rho T}{A} + \rho v^2 = \frac{\rho_c T_c}{A} + \rho_c {v_c}^2$$

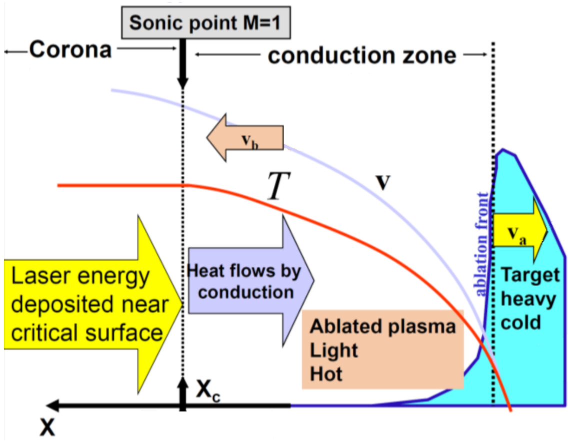
$$\rho v\left(\frac{T}{Av} + v\right) = \rho_c v_c\left(\frac{T_c}{A v_c} + v_c\right)$$

$$\frac{T}{Av} + v = \frac{T_c}{A v_c} + v_c \qquad v\left(\frac{1}{M^2} + 1\right) = v_c\left(\frac{1}{{M_c}^2} + 1\right)$$



Consider the stead state equations of motion in the conduction zone

$$\frac{T}{Av}+v=\frac{T_c}{Av_c}+v_c \qquad v^2-v\left(\frac{T_c}{Av_c}+v_c\right)+\frac{T}{A}=0$$



$$v=\frac{1}{2}\left(\frac{T_c}{Av_c}+v_c\pm\sqrt{\left(\frac{T_c}{Av_c}+v_c\right)^2-\frac{4T}{A}}\right)$$

- Near the target where $T \ll T_c$, one expect that $v \ll v_c$. Therefore,

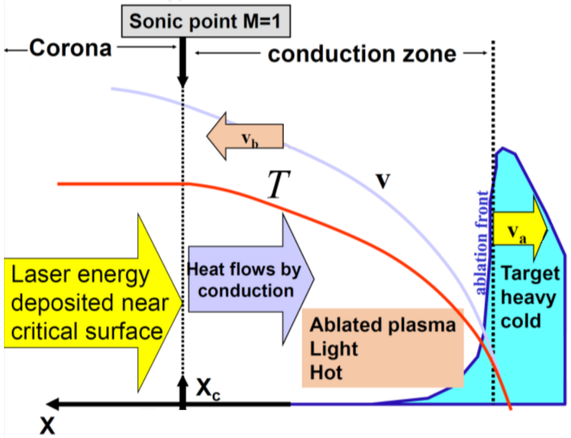
$$v=\frac{1}{2}\left(\frac{T_c}{Av_c}+v_c-\sqrt{\left(\frac{T_c}{Av_c}+v_c\right)^2-\frac{4T}{A}}\right)$$

- At $T = T_c$, $v = v_c$:

$$v_c=\frac{1}{2}\left(\frac{T_c}{Av_c}+v_c-\left|\frac{T_c}{Av_c}-v_c\right|\right)$$

Consider the stead state equations of motion in the conduction zone

- If $\frac{T_c}{Av_c} - v_c \le 0$ $\quad v_c = \frac{T_c}{Av_c}$ $\quad M_c = 1$
- If $\frac{T_c}{Av_c} - v_c \ge 0$ $\quad v_c = v_c$ $\quad M_c \le 1$
- Pick M_c = 1, i.e., the flow is sonic at the critical surface.



- Integrate the energy equation in the conduction zone:

Assuming $M_c \ll 1$, i.e., $p \gg \rho v^2$

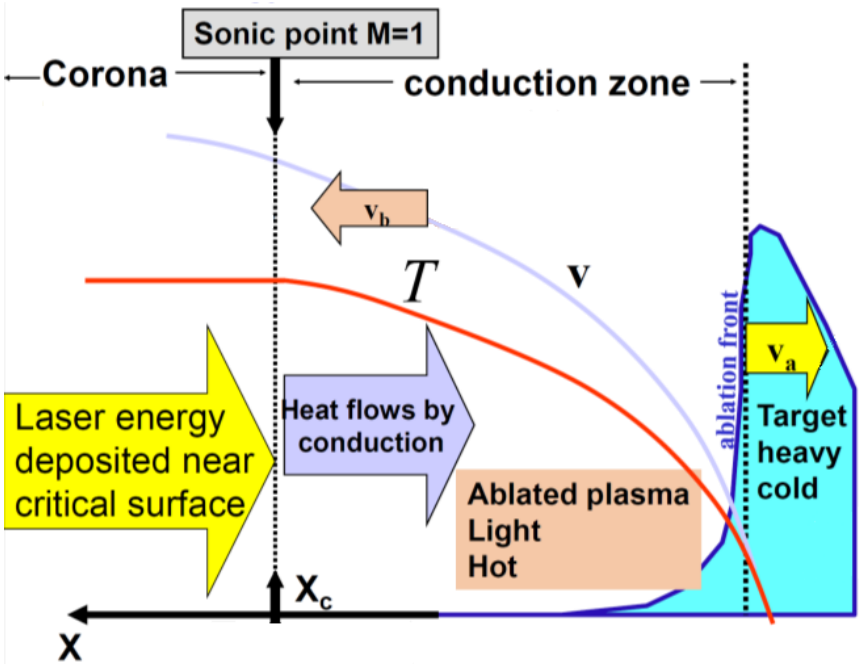
$$\frac{5}{2}pv - \kappa\frac{dT}{dx} = \frac{5}{2}p_o v_o - \left(\kappa\frac{dT}{dx}\right)_o = \frac{5}{2}\frac{\rho_o T_o}{A}v_o - \left(\kappa_o T^{5/2}\frac{dT}{dx}\right)_o \rightarrow 0$$

$$\frac{5}{2}\frac{\rho T v}{A} - \kappa_o T^{5/2}\frac{dT}{dx} = 0$$

$$\frac{5}{2}\frac{\rho_c v_c T}{A} - \kappa_o T^{5/2}\frac{dT}{dx} = 0$$

$$T = T_c\left(1 + \frac{25}{4A}\frac{\rho_c v_c}{k_o {T_c}^{5/2}}(x - x_c)\right)^{2/5}$$

Consider the stead state equations of motion in the conduction zone



$$T = T_c\left(1 + \frac{25}{4A}\frac{\rho_c v_c}{k_o {T_c}^{5/2}}(x - x_c)\right)^{2/5}$$

$$v = \frac{1}{2}\left(\frac{T_c}{Av_c} + v_c - \sqrt{\left(\frac{T_c}{Av_c} + v_c\right)^2 - \frac{4T}{A}}\right)$$

$$\rho = \frac{\rho_c v_c}{v}$$

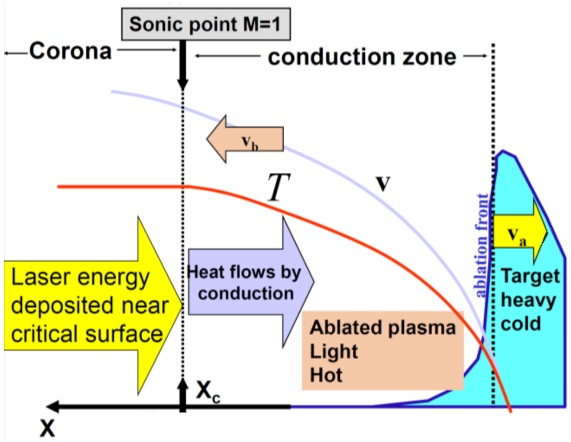
The plasma keeps expanding in the corona zone so that no steady state can be found

- For x > x_c:

$$\partial_t \rho + \partial_x(\rho v) = 0$$

$$\rho(\partial_t v + v\partial_x v) + \partial_x p = 0$$

$$\partial_t\left(\frac{3p}{2} + \frac{\rho v^2}{2}\right) + \partial_x\left(v\left(\frac{5p}{2} + \frac{\rho v^2}{2}\right) - \kappa\frac{\partial T}{\partial x}\right) = 0$$



- The temperature in the corona is high.

$$\kappa = \kappa_o T^{5/2} \;\Rightarrow \text{very large} \Rightarrow \frac{\partial T}{\partial x} = 0 \Rightarrow \; T = T_c = \text{constant} \qquad p = \frac{\rho T_c}{A}$$

$$\rho(\partial_t v + v\partial_x v) + \frac{T_c}{A}\partial_x \rho = 0$$

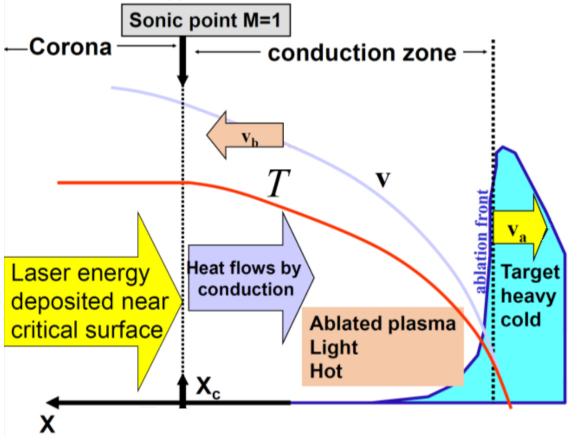
- Self-similar solutions depending on $\xi = \dfrac{z}{t}$ $\qquad z \equiv x - x_c$

$$\partial_t \rightarrow -\frac{\xi}{t}\partial_\xi \qquad \partial x \rightarrow \frac{1}{t}\partial_\xi$$

The plasma keeps expanding in the corona zone so that no steady state can be found

$$\partial_t\rho + \partial_x(\rho v) = 0$$

$$\xi = \frac{z}{t} \qquad z \equiv x - x_c$$



$$\rho(\partial_t v + v\partial_x v) + \frac{T_c}{A}\partial_x\rho = 0$$

$$\partial_t \to -\frac{\xi}{t}\partial_\xi$$

$$-\xi\partial_\xi\rho + \partial_\xi(\rho v) = 0 \qquad \frac{\partial_\xi\rho}{\rho} = \frac{\partial_\xi v}{\xi - v}$$

$$\partial x \to \frac{1}{t}\partial_\xi$$

$$\rho\left(-\frac{\xi}{t}\partial_\xi v + \frac{v}{t}\partial_\xi v\right) + \frac{T_c}{A}\frac{1}{t}\partial_\xi\rho = 0 \qquad (\xi - v)\partial_\xi v = \frac{T_c}{A}\frac{\partial_\xi\rho}{\rho}$$

$$(\xi - v)\partial_\xi v = \frac{T_c}{A}\frac{\partial_\xi v}{\xi - v} \qquad (\xi - v)^2 = \frac{T_c}{A} \qquad v = \xi \pm \sqrt{\frac{T_c}{A}}$$

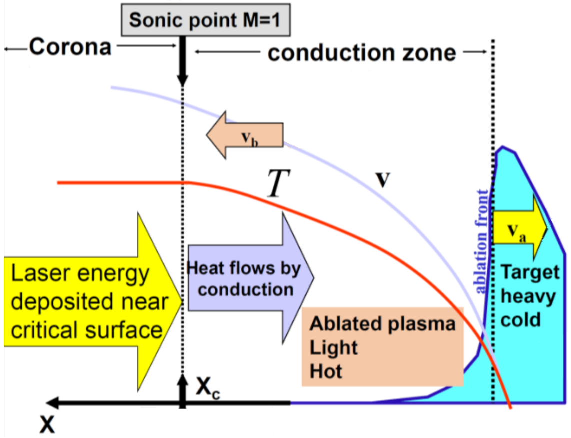
- **At z=0 ($x=x_c$),** $v = v_c = \sqrt{\frac{T_c}{A}}$ ➡ $v = \xi + \sqrt{\frac{T_c}{A}}$

The plasma keeps expanding in the corona zone so that no steady state can be found

$$\frac{\partial_\xi \rho}{\rho} = \frac{\partial_\xi v}{\xi - v}$$

$$(\xi - v)\partial_\xi v = \frac{T_c}{A}\frac{\partial_\xi \rho}{\rho}$$

$$v = \xi + \sqrt{\frac{T_c}{A}}$$



$$\partial_\xi \ln\rho = -\frac{\partial_\xi v}{\sqrt{T_c/A}} \qquad \ln\rho = -\frac{\xi}{\sqrt{T_c/A}} + \text{constant}$$

$$\rho = \rho_c e^{-\frac{\xi}{\sqrt{T_c/A}}} \qquad \rho = \rho_c \text{ at } \xi = 0$$

$$v = \frac{x - x_c}{t} + \sqrt{\frac{T_c}{A}}$$

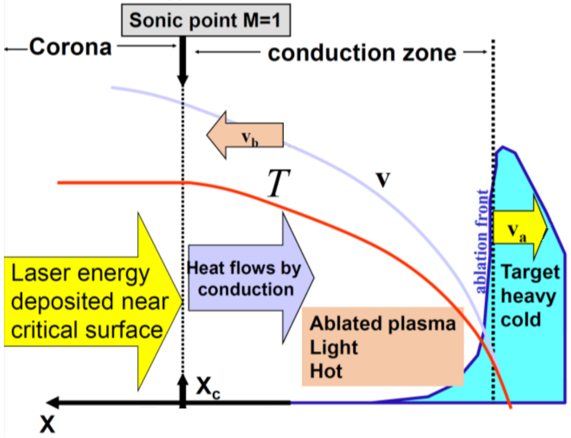
$$\rho = \rho_c exp\left(-\frac{x - x_c}{t\sqrt{T_c/A}}\right)$$

- **Laser energy is absorbed at the critical surface:**

$$\frac{\partial}{\partial t}\left(\frac{3p}{2} + \frac{\rho v^2}{2}\right) + \frac{\partial}{\partial x}\left(v\left(\frac{5p}{2} + \frac{\rho v^2}{2}\right) - \kappa\frac{\partial T}{\partial x}\right) = I\delta(x)$$

The plasma keeps expanding in the corona zone so that no steady state can be found

- **Laser energy is absorbed at the critical surface:**



$$\frac{\partial}{\partial t}\left(\frac{3p}{2}+\frac{\rho v^2}{2}\right)+\frac{\partial}{\partial x}\left(v\left(\frac{5p}{2}+\frac{\rho v^2}{2}\right)-\kappa\frac{\partial T}{\partial x}\right)=I\delta(x)$$

- **The jump conditions are**

$$\left[-\kappa\frac{\partial T}{\partial x}\right]_{x_c^-}^{x_c^+}=I=-\kappa^+\left(\frac{\partial T}{\partial x}\right)^+ +\kappa^-\left(\frac{\partial T}{\partial x}\right)^-$$

$$\kappa^-\left(\frac{\partial T}{\partial x}\right)^- \simeq \frac{5}{2}\frac{\rho_c v_c T_c}{A}+\frac{1}{2}\rho_c v_c^{\,3}=3\frac{\rho_c v_c T_c}{A}=3\rho_c\left(\frac{T_c}{A}\right)^{3/2}$$

$$\kappa^+\left(\frac{\partial T}{\partial x}\right)^+ =? \qquad \left(\frac{\partial T}{\partial x}\right)^+\to 0 \qquad \kappa^+\to\infty$$

The plasma keeps expanding in the corona zone so that no steady state can be found

- **Total energy in the corona:**

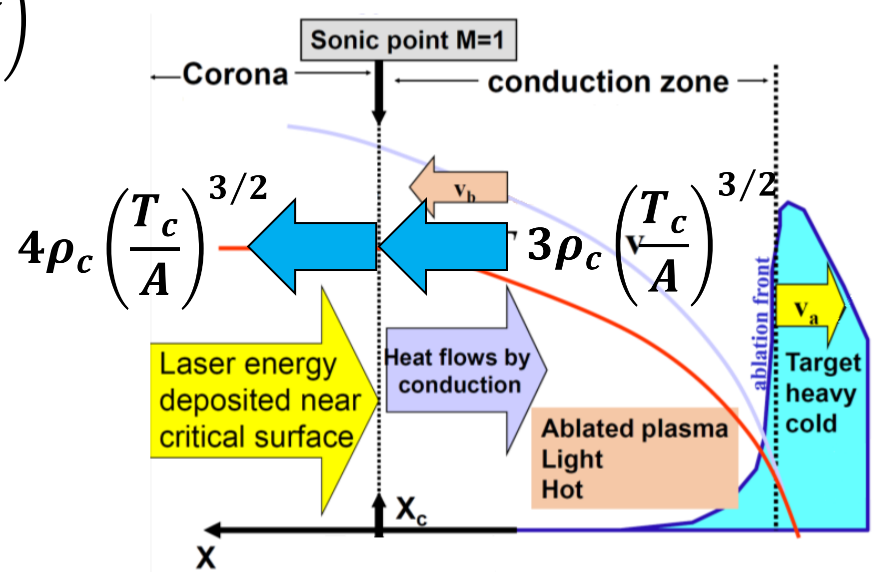
$$\epsilon = \int_{x_c}^{\infty} dx \left(\frac{3}{2} p + \frac{1}{2} \rho v^2 \right) = \int_{0}^{\infty} dz \left(\frac{3}{2} \rho \frac{T_c}{A} + \frac{1}{2} \rho v^2 \right)$$

$$= t \int_{0}^{\infty} d\xi \rho_c e^{-\frac{\xi}{\sqrt{T_c/A}}} \left(\frac{3}{2} \frac{T_c}{A} + \frac{1}{2} \xi^2 + \xi \sqrt{\frac{T_c}{A}} + \frac{1}{2} \frac{T_c}{A} \right)$$

$$= t \left(\frac{T_c}{A} \right)^{3/2} \rho_c \int_{0}^{\infty} d\zeta e^{-\zeta} \left(2 + \frac{1}{2} \zeta^2 + \zeta \right)$$

$$= 4 \rho_c \left(\frac{T_c}{A} \right)^{3/2} t$$

$$\frac{d\epsilon}{dt} = 4 \rho_c \left(\frac{T_c}{A} \right)^{3/2}$$



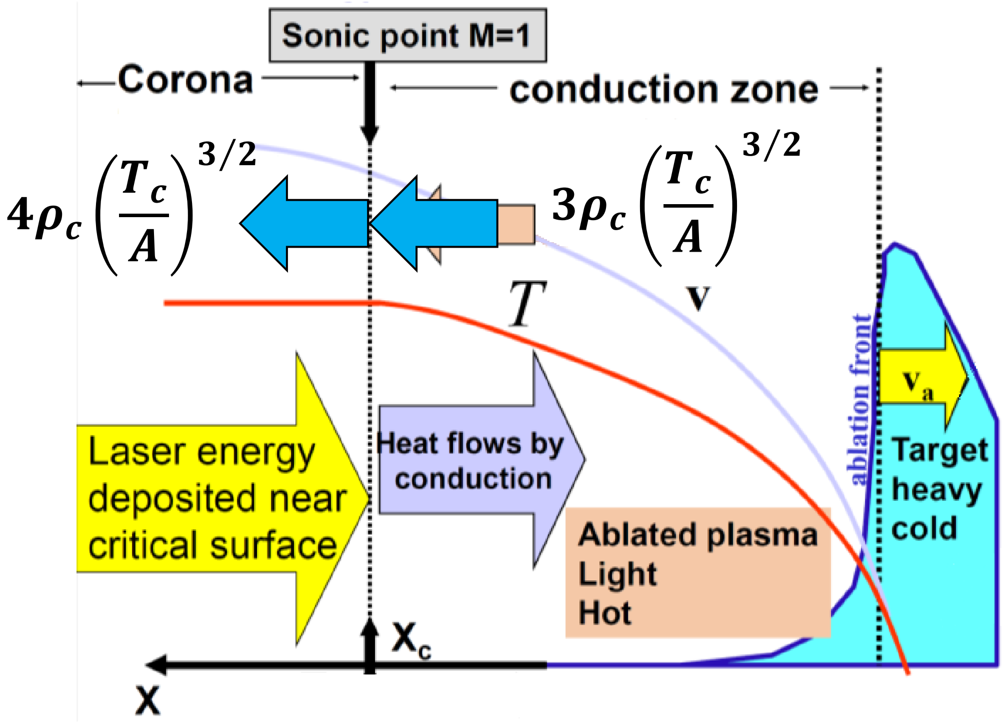
The plasma keeps expanding in the corona zone so that no steady state can be found

$$\kappa^- \left(\frac{\partial T}{\partial x}\right)^- = 3\rho_c \left(\frac{T_c}{A}\right)^{3/2}$$

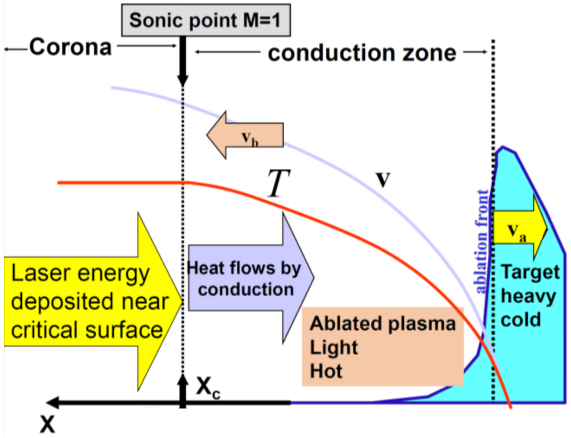
$$-\kappa^+ \left(\frac{\partial T}{\partial x}\right)^+ = \rho_c \left(\frac{T_c}{A}\right)^{3/2}$$

$$I = -\kappa^+ \left(\frac{\partial T}{\partial x}\right)^+ + \kappa^- \left(\frac{\partial T}{\partial x}\right)^-$$

$$= 4\rho_c \left(\frac{T_c}{A}\right)^{3/2}$$



The plasma keeps expanding in the corona zone so that no steady state can be found



- **Total ablation pressure (static + dynamic):**

$$P_A = \frac{\rho_c T_c}{A} + \rho_c {v_c}^2 = 2\frac{\rho_c T_c}{A} \sim \rho_c \frac{I^{2/3}}{{\rho_c}^{2/3}} \sim {\rho_c}^{1/3} I^{2/3}$$

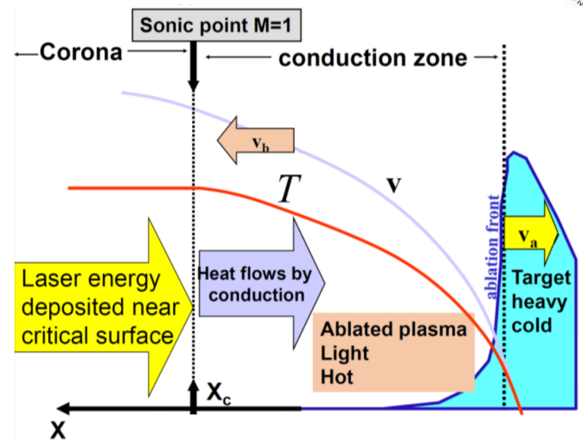
$$v_c = \sqrt{\frac{T_c}{A}} \qquad I = 4\rho_c \left(\frac{T_c}{A}\right)^{3/2}$$

- **Temperature at critical surface:** $T_c \sim \left(\frac{I}{\rho_c}\right)^{2/3}$

- **Velocity at critical surface:** $v_c \sim \left(\frac{I}{\rho_c}\right)^{1/3}$

- **Ablation rate:** $\rho_c v_c \sim {\rho_c}^{2/3} I^{1/3}$

Pressure generated by a laser is obtained using energy conservation equation



- **Energy conservation equation:**

$$\partial_t \varepsilon + \partial_x \left[\vec{v}\left(\varepsilon + p\right) - \kappa \partial_x T\right] = \underbrace{I\delta\left(x - x_{\mathrm{cr}}\right)}_{\mathrm{w/cm^2 s} \cdot 1/cm}$$

- **Since the temperature gradients are small in the corona, the heat flux is small**

$$\kappa \partial_{\mathrm{x}} T\left(x \geq x_{\mathrm{cr}}\right) << \kappa \partial_{\mathrm{x}} T\left(x \leq x_{\mathrm{cr}}\right) \qquad \left(\kappa \partial_{\mathrm{x}} T\left(x \geq x_{\mathrm{cr}}\right) \approx \frac{1}{3} \kappa \partial_{\mathrm{x}} T\left(x \leq x_{\mathrm{cr}}\right)\right)$$

- **Integrate around critical surface x_c**

$$\begin{aligned} \int_{x_{\mathrm{cr}}^-}^{x_{\mathrm{cr}}^+} \left\{\partial_t \varepsilon + \partial_x \left[\vec{v}\left(\varepsilon + p\right) - \kappa \partial_x T\right]\right\} dx &= \int_{x_{\mathrm{cr}}^-}^{x_{\mathrm{cr}}^+} \left\{I\delta\left(x - x_{\mathrm{cr}}\right)\right\} dx \\ \partial_t \varepsilon x\Big|_{x_{\mathrm{cr}}^-}^{x_{\mathrm{cr}}^+} + \left[v\left(\varepsilon + p\right)\right]_{x_{\mathrm{cr}}^-}^{x_{\mathrm{cr}}^+} - \left[\kappa \partial_x T\right]_{x_{\mathrm{cr}}^-}^{x_{\mathrm{cr}}^+} &= I \end{aligned}$$

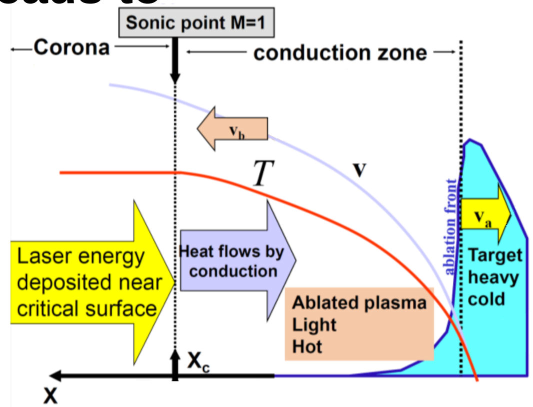
$$-\left[\kappa \partial_x T\right]_{x_{\mathrm{cr}}^-}^{x_{\mathrm{cr}}^+} = I$$

Laser produced ablation pressure

$$\partial_t \varepsilon + \partial_x \left[\vec{v}\left(\varepsilon + p\right) - \kappa \partial_x T\right] = I \delta\left(x - x_{\mathrm{cr}}\right) \qquad -\left[\kappa \partial_x T\right]_{x_{\mathrm{cr}}^-}^{x_{\mathrm{cr}}^+} = I$$

- **Solving at steady state in the conduction zone ($x<x_c$) leads to**

$$v\left(\varepsilon + p\right) \sim \kappa \partial_x T \qquad \text{for } x \leq x_{\mathrm{cr}}^-$$



- **At the sonic point (i.e., critical surface)** $C_{\mathrm{s}} \sim \sqrt{p/\rho}$

$$I = \left[v\left(\varepsilon + p\right)\right]_{x_{\mathrm{cr}}^-} = C_{\mathrm{s}}\left(\frac{5}{2}p_{\mathrm{cr}} + \rho_{\mathrm{cr}}\frac{C_{\mathrm{s}}^2}{2}\right) \sim \frac{p_{\mathrm{cr}}^{3/2}}{\rho_{\mathrm{cr}}^{1/2}}$$

- **The total pressure (static+dynamic) is the ablation pressure**

$$p_{\mathrm{A}} = \left[p + \rho v^2\right]_{x=x_{\mathrm{cr}}} = 2p_{\mathrm{cr}} \sim \left(I\rho_{\mathrm{cr}}^{1/2}\right)^{2/3} \sim \left(\frac{I}{\lambda_{\mathrm{L}}}\right)^{2/3} \qquad n_{\mathrm{cr,e}} = \frac{1.1\times 10^{21}}{{\lambda_{\mathrm{L,\mu m}}}^2}\,\mathrm{cm}^{-3}$$

- **The laser-produced total (ablation) pressure on target:**

$$p_{\mathrm{A}}(\mathrm{Mbar}) \approx 83\left(\frac{I_{15}}{\lambda_{\mathrm{L,\mu m}}/0.35}\right)^{2/3}$$

I_{15}: laser intensity in 10^{15} w/cm^2
$\lambda_{\mathrm{L,\mu m}}$: laser wavelength in μm

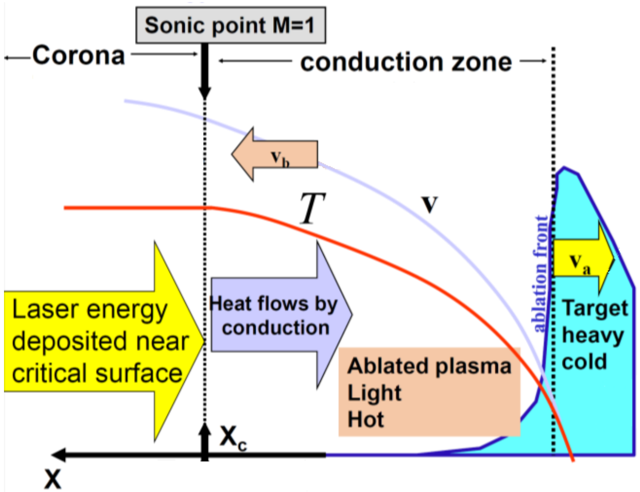
Mass ablation rate induced by the laser

- **At steady state, the mass flow across the critical surface must equal the mass flow off the shell (i.e., the mass ablation rate   )**

$$\dot{m}_{\mathrm{a}} = \rho v = \rho_{\mathrm{cr}} v_{\mathrm{cr}} = \rho_{\mathrm{cr}} C_{\mathrm{s}}^{\mathrm{cr}} = \rho_{\mathrm{cr}} \sqrt{\frac{p_{\mathrm{cr}}}{\rho_{\mathrm{cr}}}} = \sqrt{\rho_{\mathrm{cr}} p_{\mathrm{cr}}}$$

$$\rho_{\mathrm{cr}} \sim \frac{1}{\lambda_{\mathrm{L}}^2} \qquad p_{\mathrm{cr}} \sim \left(\frac{I}{\lambda}\right)^{2/3} \qquad \Rightarrow \dot{m}_{\mathrm{a}} = \frac{I^{1/3}}{\lambda_{\mathrm{L}}^{4/3}}$$

$$\dot{m}_{\mathbf{a}} = 3.3 \times 10^5 \frac{I_{15}{}^{1/3}}{\lambda_{\mathrm{L}}{}^{4/3}} \ \mathrm{g/cm^2\,s}$$



Entropy of an ideal gas/plasma

- **The entropy S is a property of a gas just like p, T, and ρ**

$$S = c_{\rm v} \ln \left[\frac{p}{\rho^{5/3}} {\rm const} \right] = c_{\rm v} \ln \alpha \qquad \alpha = {\rm const} \frac{p}{\rho^{5/3}}$$

- **α is called the "adiabat"**
- **The entropy/adiabat S/α changes through dissipation or heat sources/sinks**

$$\rho \left(\frac{\partial S}{\partial t} + \vec{u} \cdot \nabla S \right) = \frac{DS}{Dt} = \mu \frac{|\nabla \vec{u}|^2}{T} + \frac{\nabla \cdot \kappa \nabla T}{T} + {\rm sources/sinks}$$

- **In an ideal gas (no dissipation) and without sources and sinks, the entropy/adiabat is a constant of motion of each fluid element**

$$\frac{DS}{dt} = 0 \Rightarrow S \ , \ \alpha = {\rm const} \Rightarrow p \sim \alpha \rho^{5/3}$$

It is easier to compress a low adiabat (entropy) gas

- **Smaller α -> less work to compress from low to high density**

$$W_{1\rightarrow 2} = -\int pdV \sim -\int_{\rho_1}^{\rho_2} \alpha \rho^{5/3} d\left(\frac{M}{\rho}\right) \sim \alpha M \left(\rho_2^{2/3} - \rho_1^{2/3}\right)$$

- **Smaller α -> higher density for the same pressure**

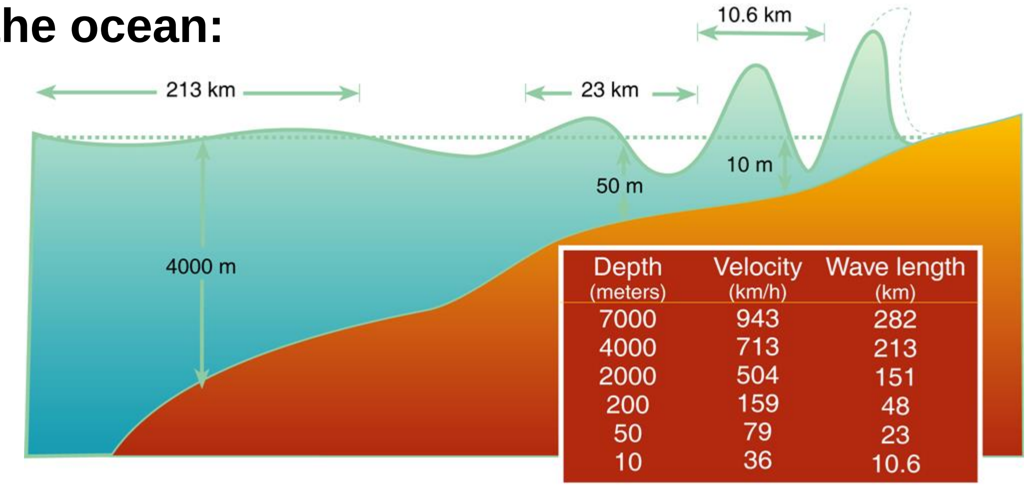
$$\alpha \sim \frac{p}{\rho^{5/3}} \Rightarrow \rho \sim \left(\frac{p}{\alpha}\right)^{3/5}$$

- **In HEDP, the constant in adiabat definition comes from the normalization of the pressure against the Fermi pressure.**
- **When thermal effects are negligible at very high densities, the pressure is proportional to $\rho^{5/3}$ due to the quantum mechanical effects (degenerate electron gas) just like isentropic flow**

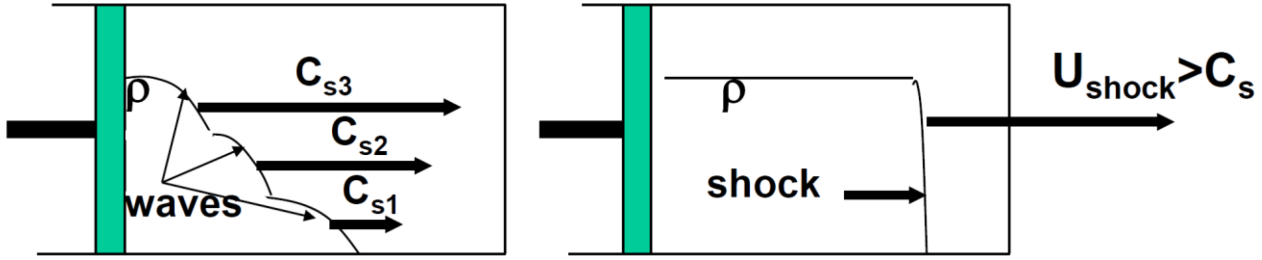
$$\alpha \equiv \frac{p}{p_{\mathrm{F}}} \quad \Rightarrow \alpha_{\mathrm{DT}} = \frac{p_{\mathrm{Mbar}}}{2.2\rho_{\mathrm{g/cc}}^{5/3}}$$

A shock is formed due to the increasing sound speed of a compressed gas/plasma

- **Wave in the ocean:**



- **Acoustic/compression wave driven by a piston:**



$$C_s \sim \sqrt{\frac{p}{\rho}} \sim \sqrt{\frac{\alpha\rho^{5/3}}{\rho}} \sim \sqrt{\alpha}\rho^{1/3}$$

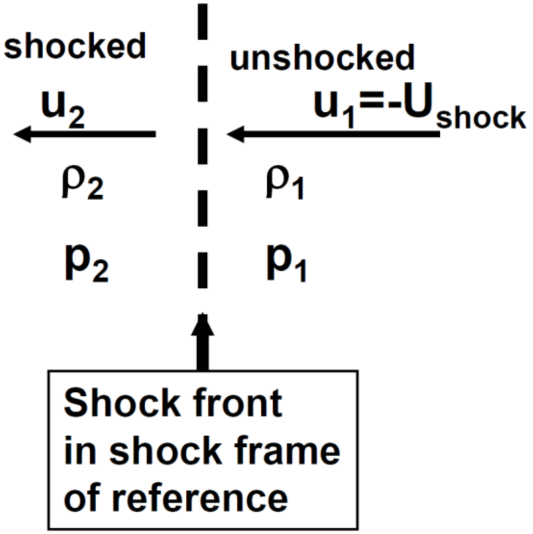
http://neamtic.ioc-unesco.org/tsunami-info/the-cause-of-tsunamis

Rankine-Hugoniot conditions are obtained using conservation of mass, momentum and energy across the shock front

$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$u_1 \left(\varepsilon_1 + p_1\right) = u_2 \left(\varepsilon_2 + p_2\right)$$



- **Ideal gas/plasma:**

$$\varepsilon = \frac{3}{2} p + \rho \frac{u^2}{2}$$

- **With assigned ρ_1, p_1, and p_2, ρ_2, u_2, and u_1=-U_{shock} can be obtained using Rankine-Hugoniont conditions**

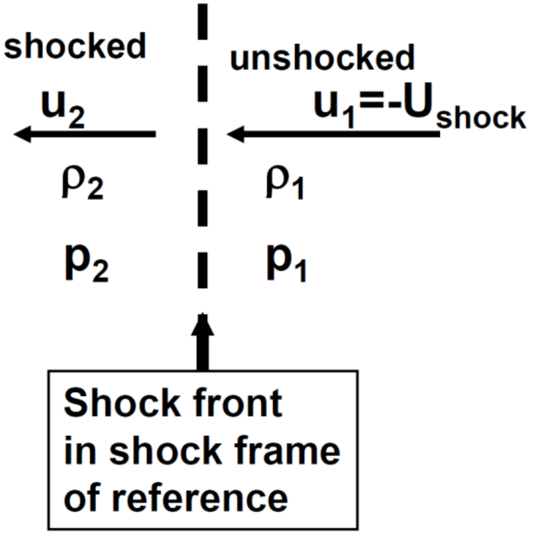
For a strong shock where $p_2 >> p_1$, the R-H conditions are simplified

$$\frac{\rho_2}{\rho_1} \approx 4$$

$$U_{\text{shock}} = -u_1 \approx \sqrt{\frac{4p_2}{3\rho_1}}$$

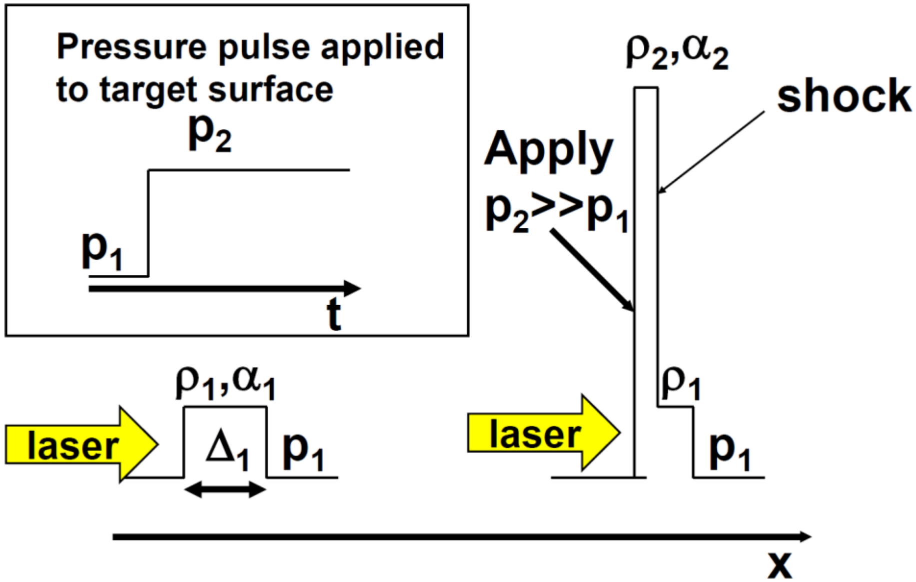
$$u_2 \approx \sqrt{\frac{p_2}{12\rho_1}}$$

$$\frac{\alpha_2}{\alpha_1} = \frac{p_2/\rho_2^{5/3}}{p_1/\rho_1^{5/3}} \approx \frac{1}{4^{5/4}} \frac{p_2}{p_1} >> 1$$



- **The adiabat increases through the shock.**

In an ideal gas/plasma, the adiabat α only raises when a shock is present



- **Post-shock density**

$$\rho_2 \approx 4\rho_1$$

- **Adiabat set by the shock for DT:**

$$\alpha_2 \approx \frac{p_{2,\mathrm{Mbar}}}{2.2\left(4\rho_{1,\mathrm{g/cc}}\right)^{5/3}}$$

- **Time required for the shock to reach the rear target surface (shock break-out time, t_{sb})**

$$t_{\mathrm{sb}} = \frac{\Delta_1}{u_{\mathrm{shock}}} = \Delta_1\sqrt{\frac{3\rho_1}{4p_2}} \propto \sqrt{\frac{1}{\alpha_2\rho_1^{2/3}}}$$

Higher laser intensity leads to higher adiabat

- **For a cryogenic solid DT target at 18 k:**

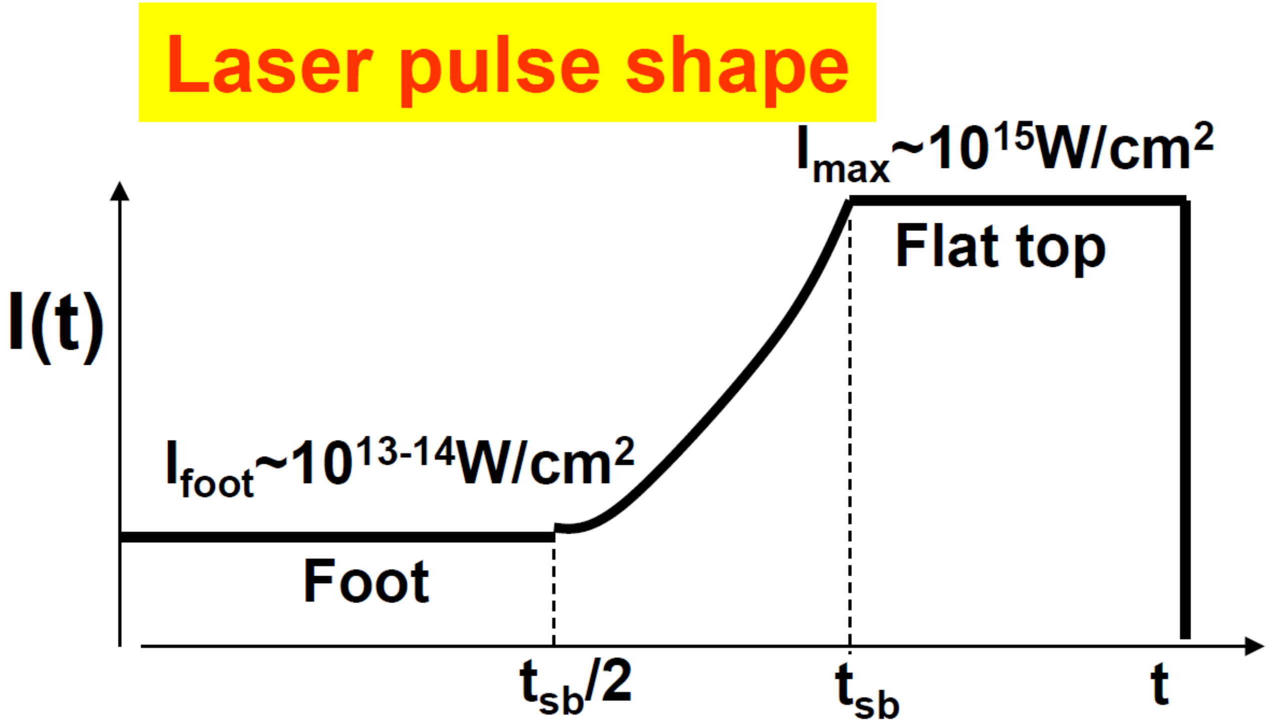
$$\rho_1 = 0.25\ \mathrm{g/cc} \qquad \alpha = \frac{p_{\mathrm{Mbar}}}{2.2} \qquad p \approx 83 \left(\frac{I_{15}}{\lambda_{\mu\mathrm{m}}/0.35} \right)^{2/3}$$

$$I \approx 4.3 \times 10^{12}\ \mathrm{w/cm^2} \quad \Rightarrow \quad p = 2.2\ \mathrm{Mbar} \quad \Rightarrow \quad \alpha = 1$$

$$I \approx 1.2 \times 10^{13}\ \mathrm{w/cm^2} \quad \Rightarrow \quad p = 4.4\ \mathrm{Mbar} \quad \Rightarrow \quad \alpha = 2$$

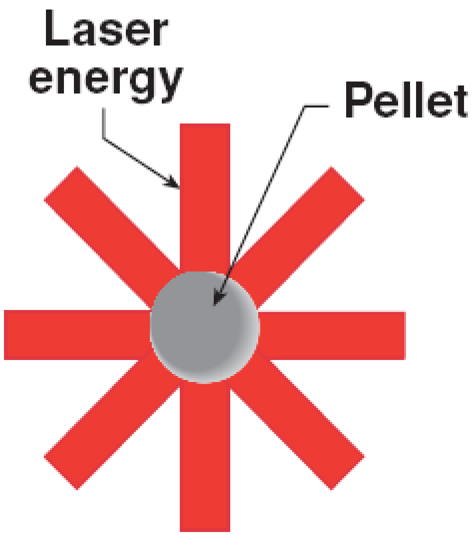
$$I \approx 2.2 \times 10^{13}\ \mathrm{w/cm^2} \quad \Rightarrow \quad p = 6.6\ \mathrm{Mbar} \quad \Rightarrow \quad \alpha = 3$$

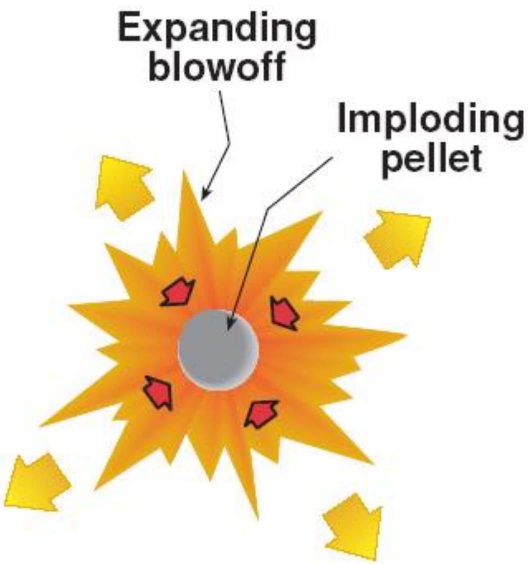
The pressure must be "slowly" increased after the first shock to avoid raising the adiabat



- After the foot of the laser pulse, the laser intensity must be raised starting at about $0.5t_{sb}$ and reach its peak at about t_{sb}
- Reaching I_{max} at t_{sb} prevents a rarefaction/decompression wave to propagate back from the rear target surface and decompress the target.

Most of the absorbed laser energy goes into the kinetic and thermal energy of the expanding blow-off plasma





- **The rocket model:**

Shell Newton's law

$$M\frac{du}{dt} = -4\pi R^2 p_{\mathrm{a}}$$

Shell mass decreases due to ablation

$$\frac{dM}{dt} = -4\pi R^2 \dot{m}_{\mathrm{a}}$$

p_a =ablation rate x exhaust velocity

$$p_{\mathrm{a}} = \dot{m}_{\mathrm{a}} u_{\mathrm{exhaust}}$$

Shell velocity can be obtained by integrating the rocket equations

$$M\frac{du}{dt} = -4\pi R^2 p_\mathrm{a} \qquad \frac{dM}{dt} = -4\pi R^2 \dot{m}_\mathrm{a} \qquad p_\mathrm{a} = \dot{m}_\mathrm{a} u_\mathrm{exhaust}$$

$$\begin{aligned}
M\frac{du}{dt} &= -4\pi R^2 p_\mathrm{a} = -4\pi R^2 \dot{m}_\mathrm{a} u_\mathrm{exhaust} \\
&= -4\pi R^2 u_\mathrm{exhaust} \frac{1}{-4\pi R^2}\frac{dM}{dt} \\
&= u_\mathrm{exhaust}\frac{dM}{dt} \\
\int du &= u_\mathrm{exhaust}\int \frac{dM}{M} \\
u_\mathrm{shell} &= u_\mathrm{exhaust}\ln\left(\frac{M_\mathrm{initial}}{M_\mathrm{final}}\right)
\end{aligned}$$

$$E_\mathrm{kin}^\mathrm{shell} = \frac{M_\mathrm{final}}{2} u_\mathrm{shell}^2 = \frac{M_\mathrm{final}}{2}\left[u_\mathrm{exhaust}\ln\left(\frac{M_\mathrm{initial}}{M_\mathrm{final}}\right)\right]^2$$

$$E_\mathrm{exhaust} = (M_\mathrm{initial} - M_\mathrm{final})\left(\frac{u_\mathrm{exhaust}^2}{2} + \frac{3}{2}\frac{p_\mathrm{ex}}{\rho_\mathrm{ex}}\right)$$

(dynamic + static)

$M_\mathrm{exhaust} = M_\mathrm{initial} - M_\mathrm{final}$

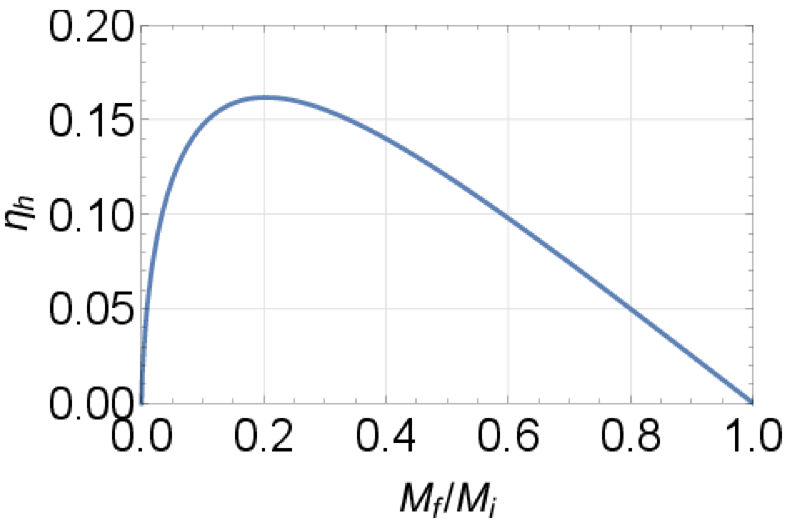
Maximum hydro efficiency is about 15%

$$E_{\rm kin}^{\rm shell} = \frac{M_{\rm final}}{2} u_{\rm shell}^2 = \frac{M_{\rm final}}{2} \left[u_{\rm exhaust} \ln \left(\frac{M_{\rm initial}}{M_{\rm final}} \right) \right]^2$$

$$E_{\rm exhaust} = (M_{\rm initial} - M_{\rm final}) \left(\frac{u_{\rm exhaust}^2}{2} + \frac{3}{2} \frac{p_{\rm ex}}{\rho_{\rm ex}} \right)$$

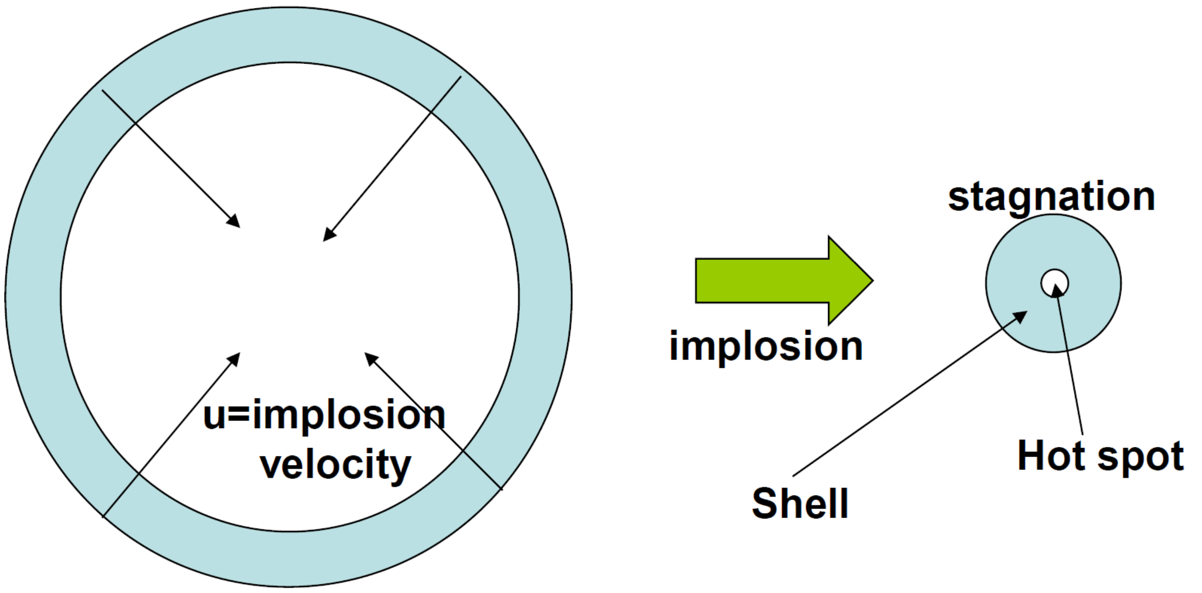
$$\text{Take } u_{\rm exhaust}^2 \approx C_{\rm s}^2 \approx \frac{p_{\rm ex}}{\rho_{\rm ex}}$$

$$\eta_{\rm h} = \frac{E_{\rm kin}^{\rm shell}}{E_{\rm exhaust}} = \frac{M_{\rm f}/M_{\rm i} \left[\ln \left(M_{\rm f}/M_{\rm i} \right) \right]^2}{4 \left(1 - M_{\rm f}/M_{\rm i} \right)}$$



One dimensional implosion hydrodynamics

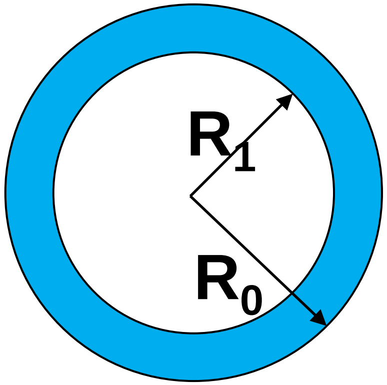
- What are the stagnation values of the relevant hydrodynamic properties?



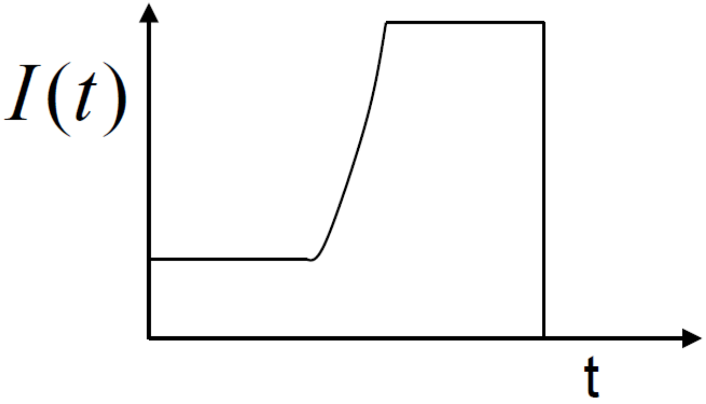
ρ?
ρR?
P?

What variables can be controlled?

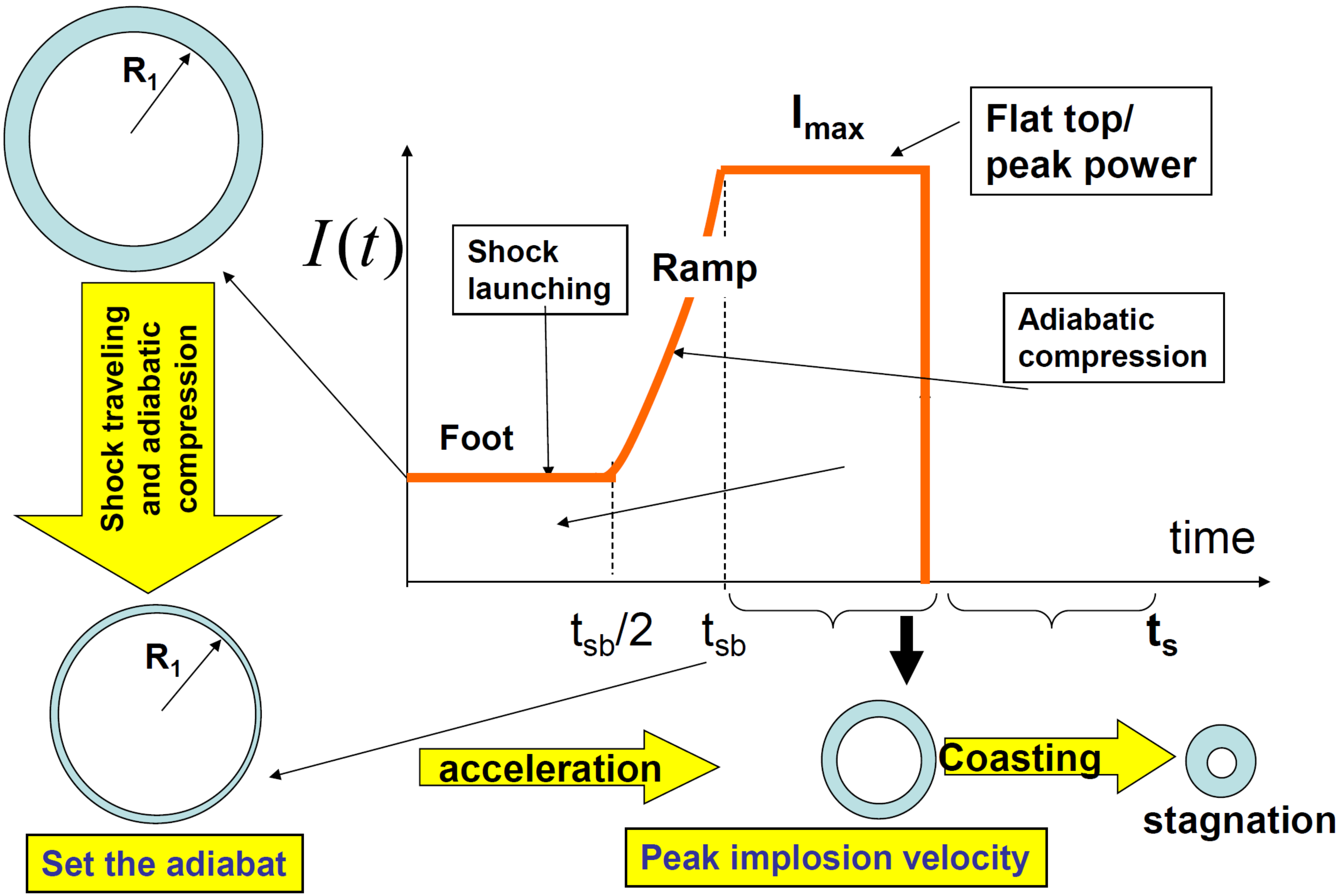
- Shell outer radius R_0 at time t=0
- Shell inner radius R_1 at time t=0
- The total laser energy on target
- Adiabat α through shocks
- Applied pressure p(t) through the pulse shape I(t)



$$\alpha \sim \frac{p}{\rho^{5/3}} \qquad p \sim I^{2/3}$$

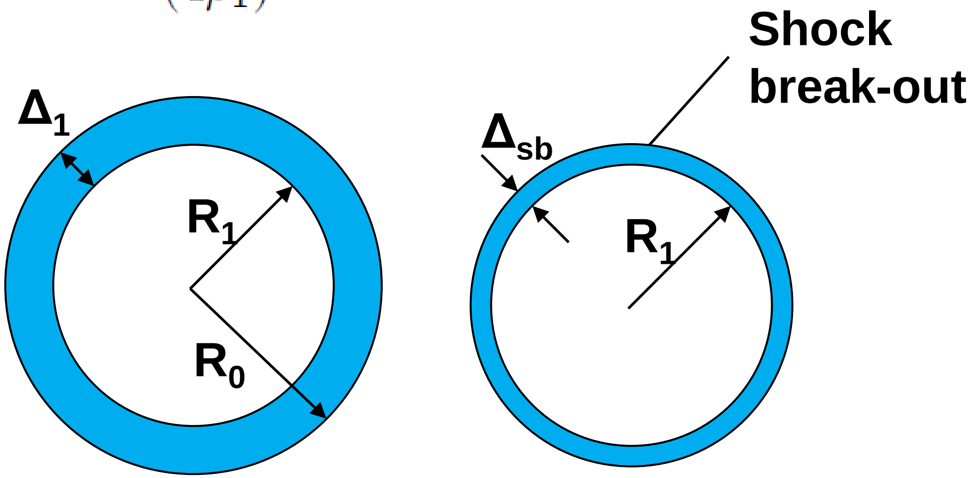


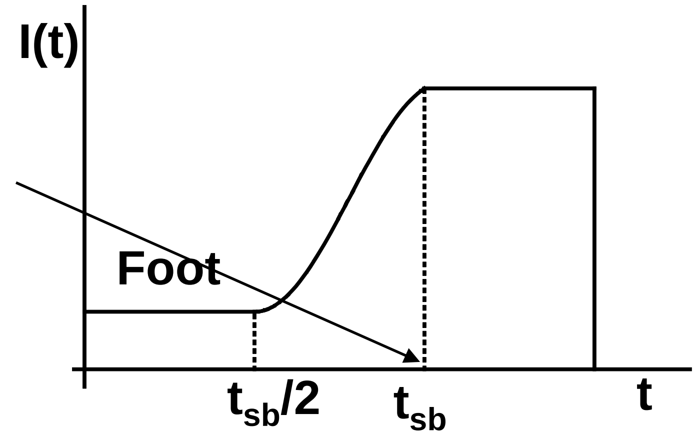
There are three stages in the laser pulse: foot, ramp, and flat top



The adiabat is set by the shock launched by the foot of the laser pulse

$$\alpha \sim \frac{p_{\text{foot}}}{(4\rho_1)^{5/3}} \qquad \rho_1 = \text{initial density}$$





$$m_{sb} \sim 4\pi R_1^{\,2}\Delta_1\rho_1 = 4\pi R_1^{\,2}\Delta_{sb}\rho_{sb}$$

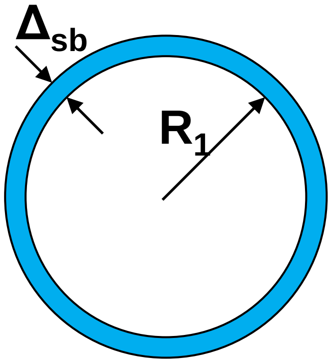
$$\Delta_1\rho_1 = \Delta_{sb}\rho_{sb}$$

$$\rho_{sb} \sim \left(\frac{p_{\max}}{\alpha}\right)^{5/3} = 4\rho_1\left(\frac{p_{\max}}{p_{\text{foot}}}\right)^{5/3}$$

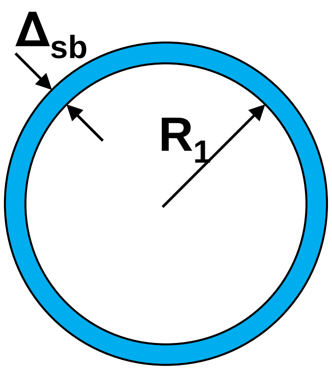
$$\Delta_{sb} = \Delta_1\frac{\rho_1}{\rho_{sb}} = \Delta_1\frac{\rho_1}{4\rho_1\left(\frac{p_{max}}{p_{foot}}\right)^{3/5}} = \frac{\Delta_1}{4}\left(\frac{p_{foot}}{p_{max}}\right)^{3/5}$$

Density and thickness at shock break out time are expressed in laser intensity

- **Use** $p \sim I^{2/3}$
- **Shell density** $\rho_{\rm sb} \sim 4\rho_1 \left(\frac{p_{\rm max}}{p_{\rm foot}}\right)^{3/5} = 4\rho_1 \left(\frac{I_{\rm max}}{I_{\rm foot}}\right)^{2/5}$
- **Shell thickness** $\Delta_{\rm sb} \sim \frac{\Delta_1}{4} \left(\frac{p_{\rm foot}}{p_{\rm max}}\right)^{3/5} = \frac{\Delta_1}{4} \left(\frac{I_{\rm foot}}{I_{\rm max}}\right)^{2/5}$
- **Shell radius** $R \approx R_1$



The aspect ratio is maximum at shock break out



$$\textbf{Aspect ratio} \equiv \frac{\boldsymbol{R}}{\boldsymbol{\Delta}}$$

$$A_1 = \frac{R_1}{\Delta_1} = \text{initial aspect ratio}$$

$$A_{\rm sb} = IFAR = \frac{R_1}{\Delta_{\rm sb}} = 4A_1 \left(\frac{I_{\rm max}}{I_{\rm foot}}\right)^{2/5}$$

$$A_{\rm sb} = A_{\rm max}$$

IFAR ≡ Maximum In-Flight-Aspect-Ratio = aspect ratio at shock break-out

The IFAR scales with the Mach number

- **The shell kinetic energy = the work done on the shell**

$$M u_{\rm max}^2 \sim \int_R^{R_1} p r^2 dr \sim p\left(R_1^3 - R^3\right) \approx p R_1^3$$

$$R_1^{\ 3} = \frac{M u_{max}^{\ \ 2}}{p}$$

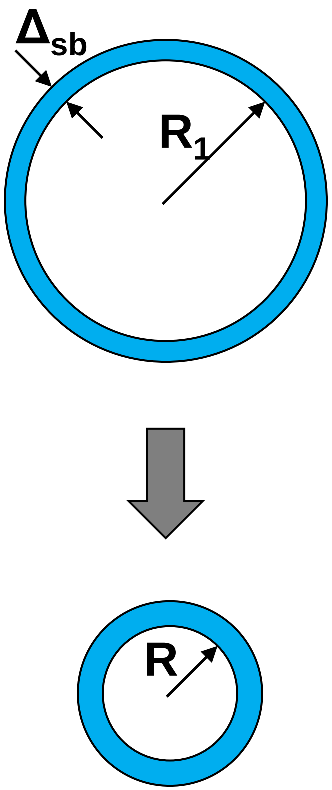
$$M \sim \rho_{\rm sb} \Delta_{\rm sb} R_1^2 \qquad \Delta_{sb} \sim \frac{M}{\rho_{sb} R_1^{\ 2}} \qquad R_1 >> R$$

$$IFAR = \frac{R_1}{\Delta_{sb}} = \frac{R_1}{\frac{M}{\rho_{sb} R_1^{\ 2}}} = \frac{\rho_{sb} R_1^{\ 3}}{M} = \frac{\rho_{sb}}{M} \frac{M u_{max}^{\ \ 2}}{p}$$

$$= \frac{u_{max}^{\ \ 2}}{p/\rho_{sb}} \sim Mach_{max}^{\ \ 2}$$

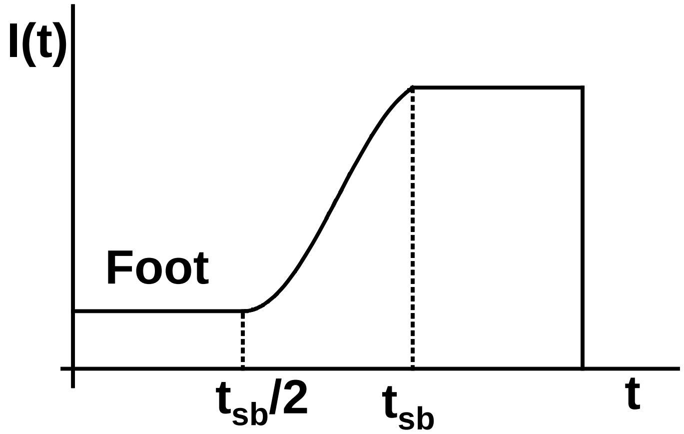
$$\rho \sim (p/\alpha)^{3/5} \qquad p \sim I^{2/3}$$

$$IFAR \sim \frac{u_{max}^{\ \ 2}}{\alpha^{3/5} I^{4/15}}$$



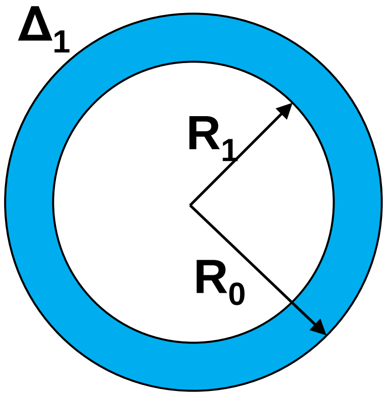
The final implosion velocity can be found using IFAR

$$u_{\max}^2 \sim IFAR \times \alpha^{3/5} I^{4/15}$$



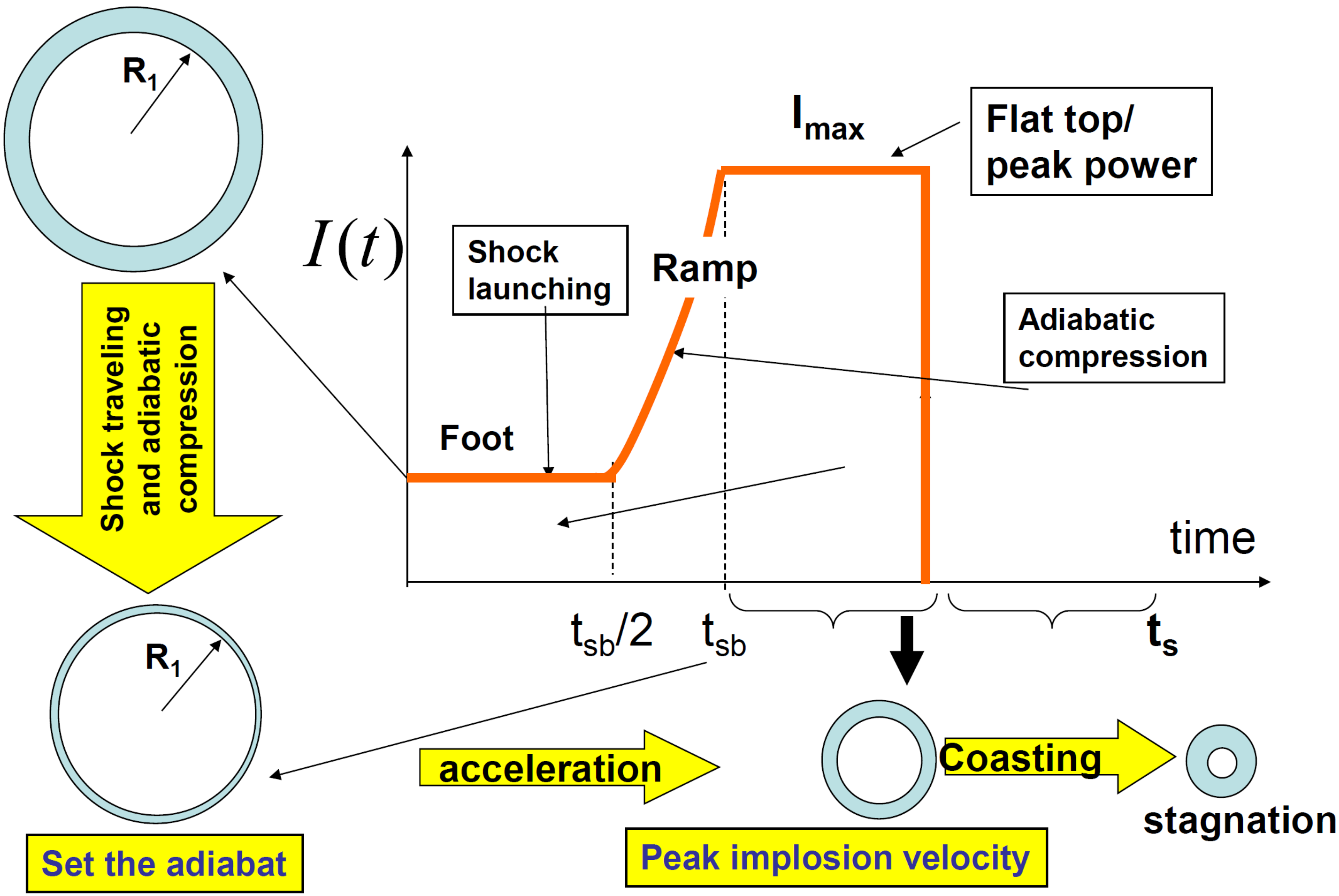
$$IFAR = 4A_1 \left(\frac{I_{\max}}{I_{\text{foot}}}\right)^{2/5}$$

$$A_1 = \frac{R_1}{\Delta_1}$$



$$u_{\max,\text{cm}/s} \approx 10^7 \sqrt{0.7 A_1 \alpha^{3/5} I_{15,\max}{}^{4/15} \left(\frac{I_{\max}}{I_{\text{foot}}}\right)^{2/5}}$$

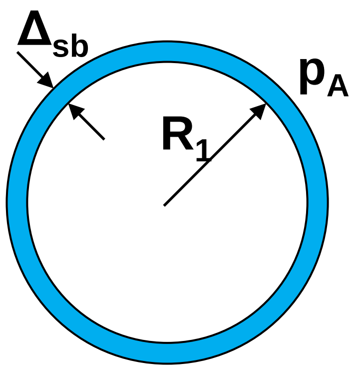
There are three stages in the laser pulse: foot, ramp, and flat top



A simple implosion theory can be derived in the limit of infinite initial aspect ratio

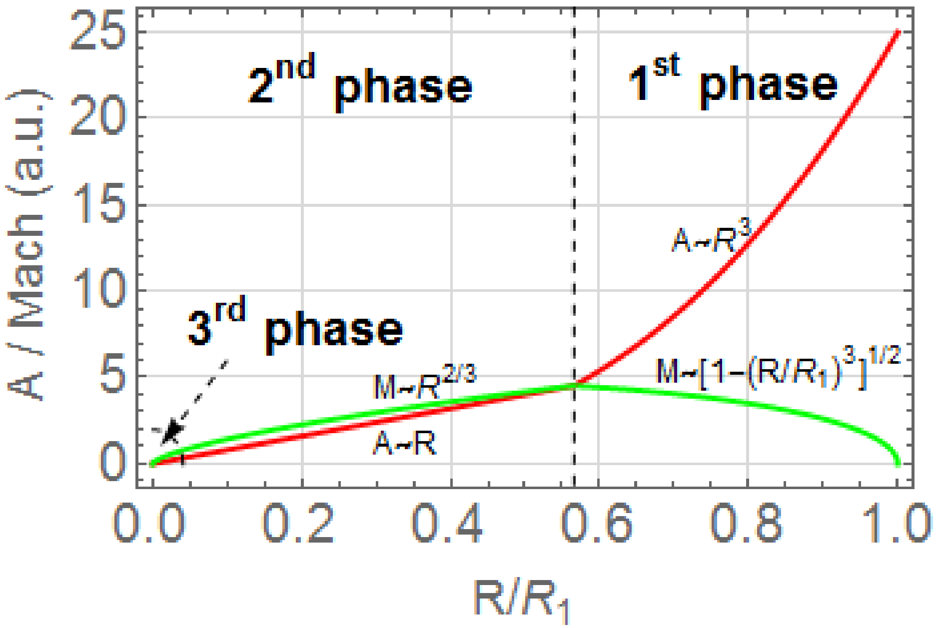
- **Start from a high aspect ratio shell (thin shell) at the beginning of the acceleration phase**
  - **Constant ablated pressure**
  - **The adiabat is set and kept fixed by the first and the only shock**

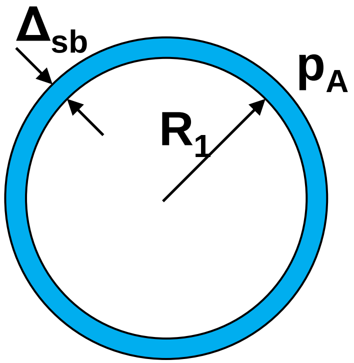
$$IFAR = A_{\rm sb} = \frac{R_1}{\Delta_{\rm sb}} >> 1$$



Basko and Meyer-ter-vehn, Phys. Rev. Lett. *88*, 244502-1, 2002

The implosion are divided in 3 phases after the shock break out





- 1st phase: acceleration
- 2nd phase: coasting
- 3rd phase: stagnation