Introduction to Nuclear Fusion as An Energy Source

Po-Yu Chang

Institute of Space and Plasma Sciences, National Cheng Kung University

Lecture 11

2025 spring semester

Tuesday 9:00-12:00

Materials:

https://capst.ncku.edu.tw/PGS/index.php/teaching/

Online courses:

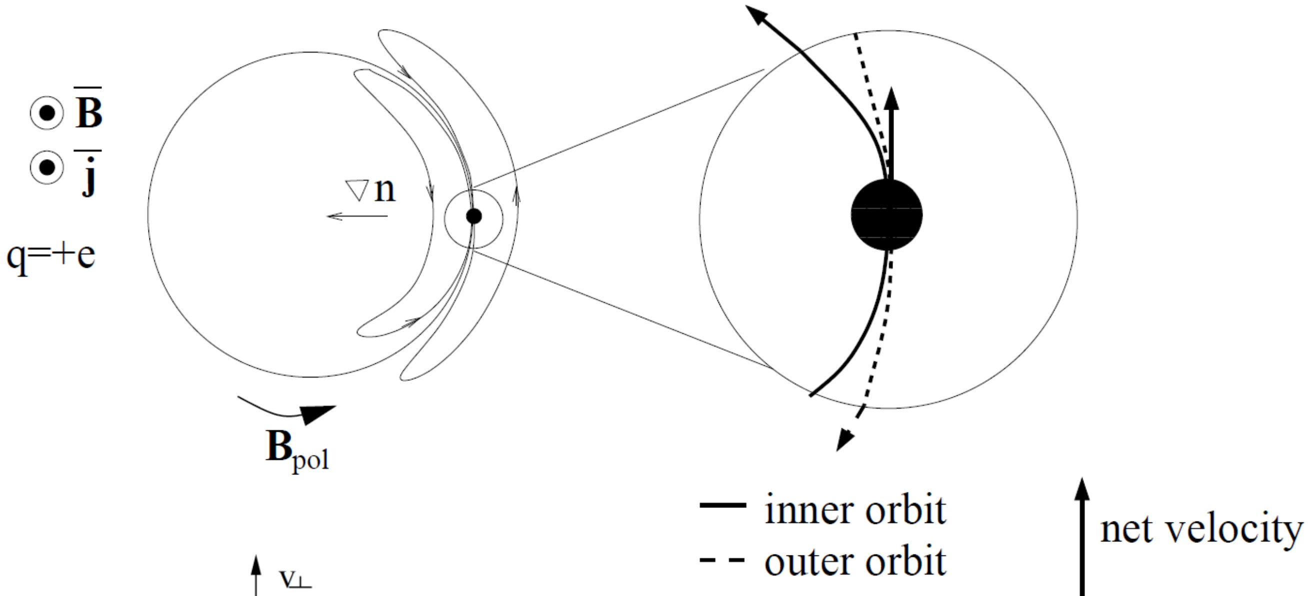
https://nckucc.webex.com/nckucc/j.php?MTID=mf1a33a5dab5eb71de9da4380ae888592

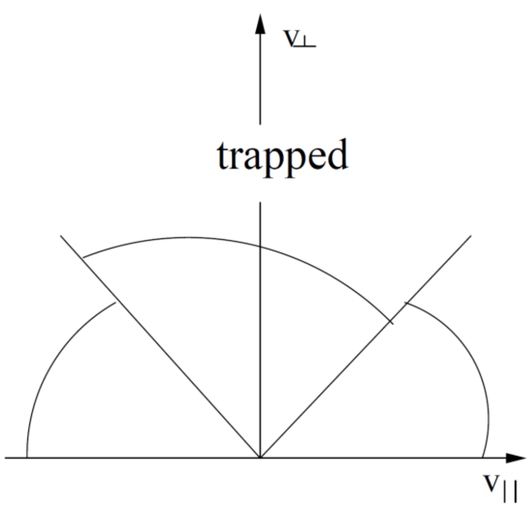
2025/5/20 updated

Note!

- **Final exam 6/ 3 (One double-sided A4 cheating sheet is allowed.)**
- **Contents after the midterm, i.e, after (included) Lecture 8**

A banana current is generated when there is a pressure gradient in the plasma

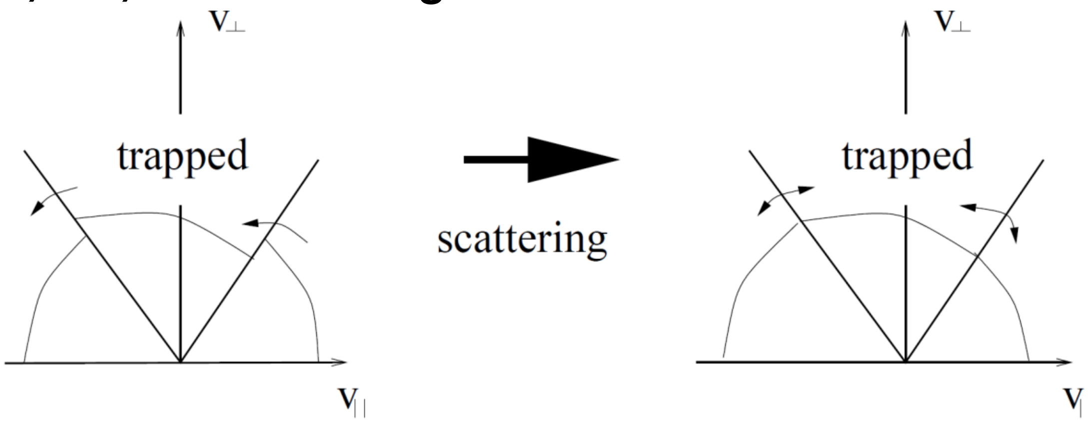


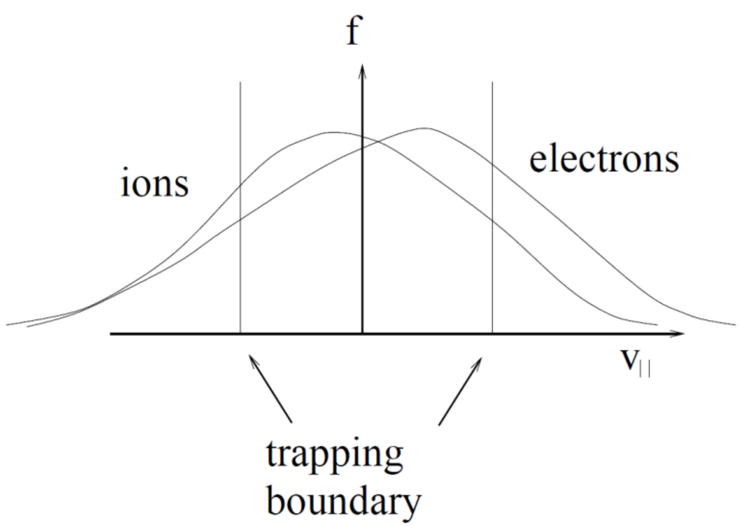


Samuli Saarelma, Helsinki University of Technology, PhD Thesis 2015

Bootstrap current is generated when passing particles are scattered by the trapped particles

- **Scattering smooths the velocity distribution and shifts it in the parallel direction, i.e., a current is generated. It is called the bootstrap current.**



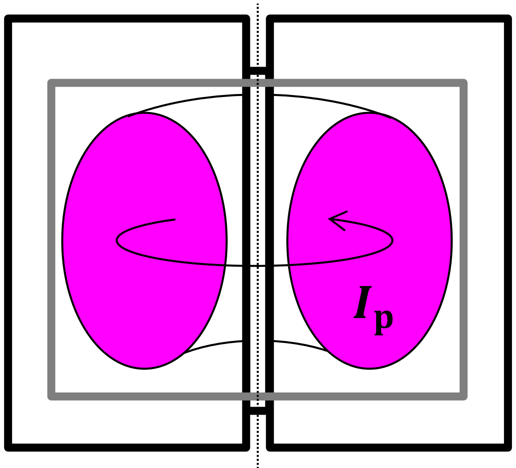


$$\boldsymbol{j = -enu_{||\mathrm{e}} + enu_{||\mathrm{i}} = 4\epsilon^{3/2}\frac{1}{B_{\mathrm{p}}}T\frac{dn}{dr}}$$

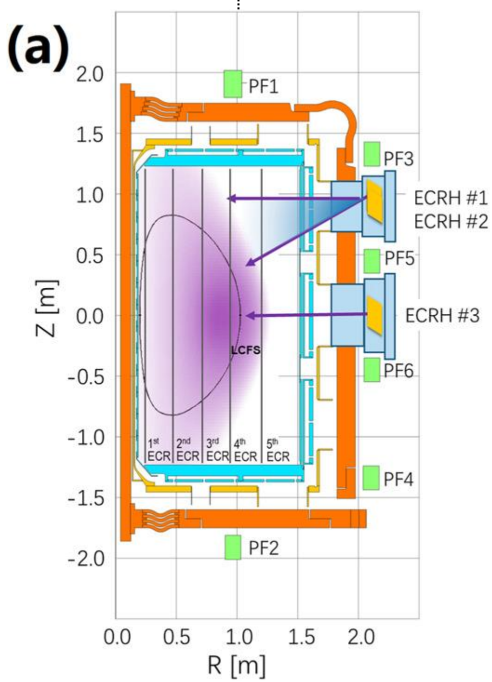
- **The bootstrap current is vital for steady-state operation.**

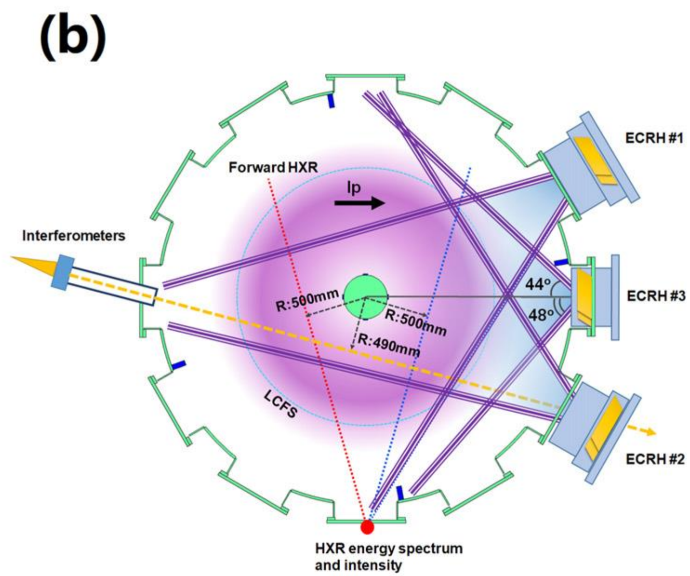
Samuli Saarelma, Helsinki University of Technology, PhD Thesis 2015

Momentum exchange may be needed to drive plasma current



$$\vec{j}_p = \Sigma q n \vec{v} = -e n_e \vec{v}_e + e n_i \vec{v}_i$$

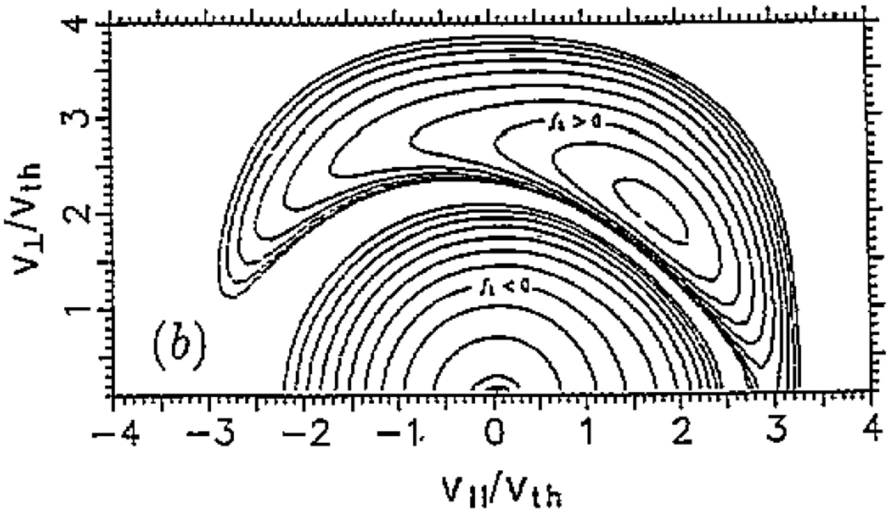




Yuejiang Shi et al, Nucl. Fusion **62**, 086047 (2022)

The collisional re-distribution of the ECRH-driven anisotropy in $E_\perp$ causes some parallel momentum to flow from e^- to ions

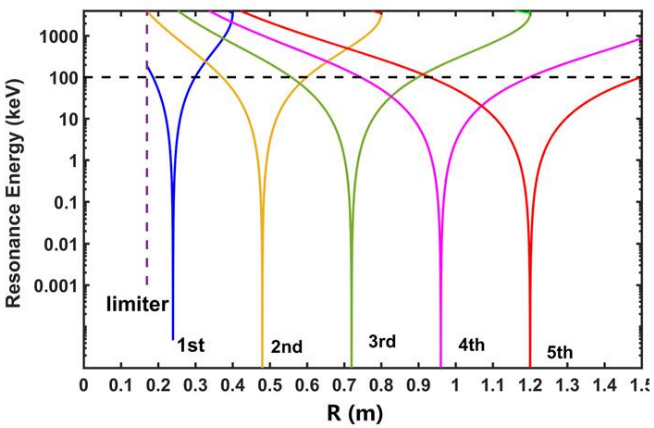
- **Coulomb collisions are more efficient at lower energies.**

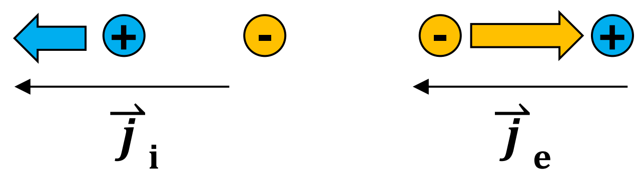


Velocity: $v_2 > v_1$

Collisions: $\nu_2 < \nu_1$

- **Electron cyclotron current drive:**





$$\vec{j}_p = -e n_e \vec{v}_e + e n_i \vec{v}_i$$

$$\vec{P} = n_e m_e \vec{v}_e + n_i m_i \vec{v}_i \approx 0$$

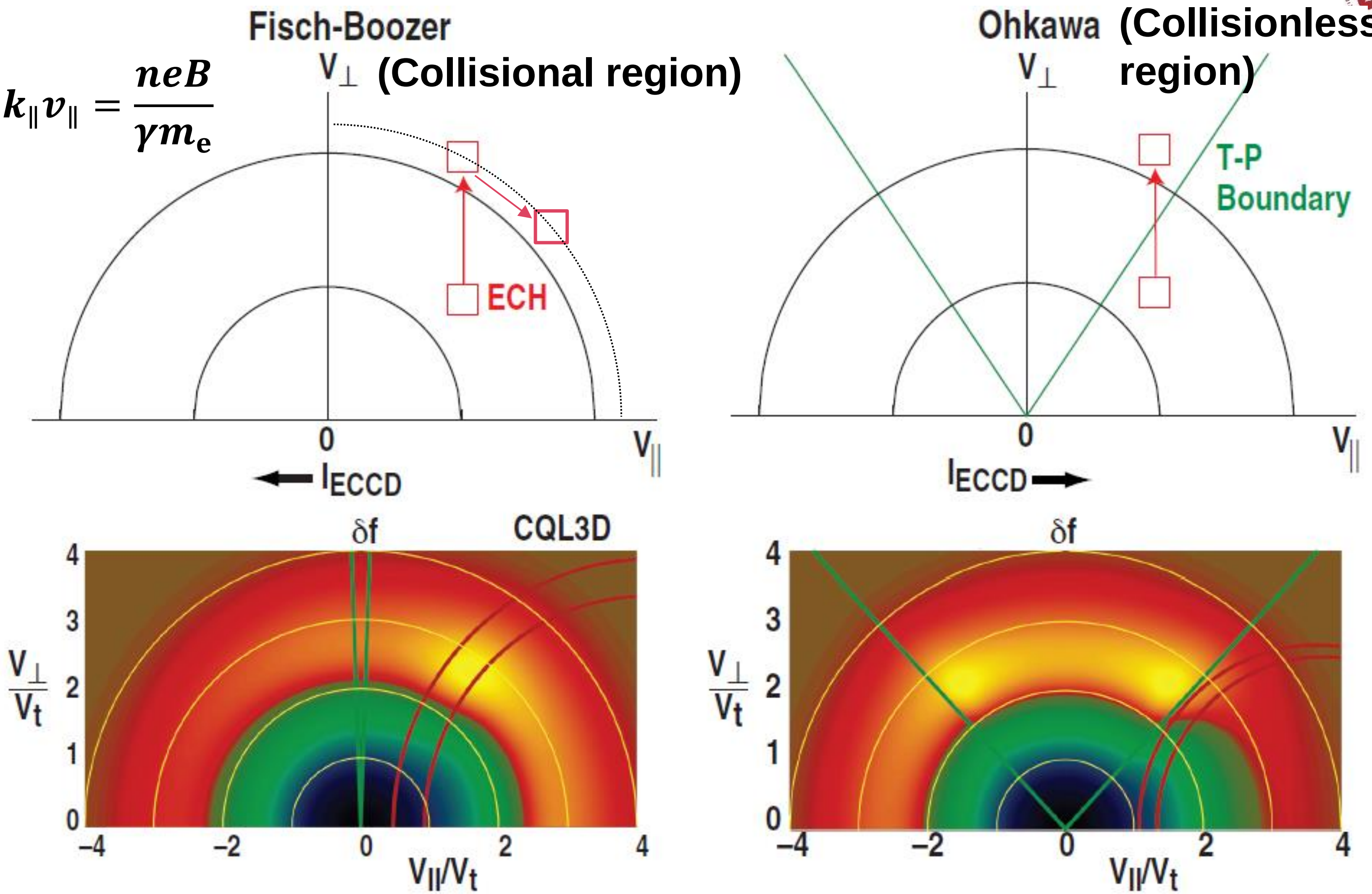
V. Erckmann and U. Gasparion, Plasma Phys. Control. Fusion **36**, 1869 (1994)
Yuejiang Shi et al, Nucl. Fusion **62**, 086047 (2022)

Passing electrons can be trapped if the $v_\perp$ is increased by heating

$$\omega - k_\parallel v_\parallel = \frac{neB}{\gamma m_e}$$

(Collisional region)

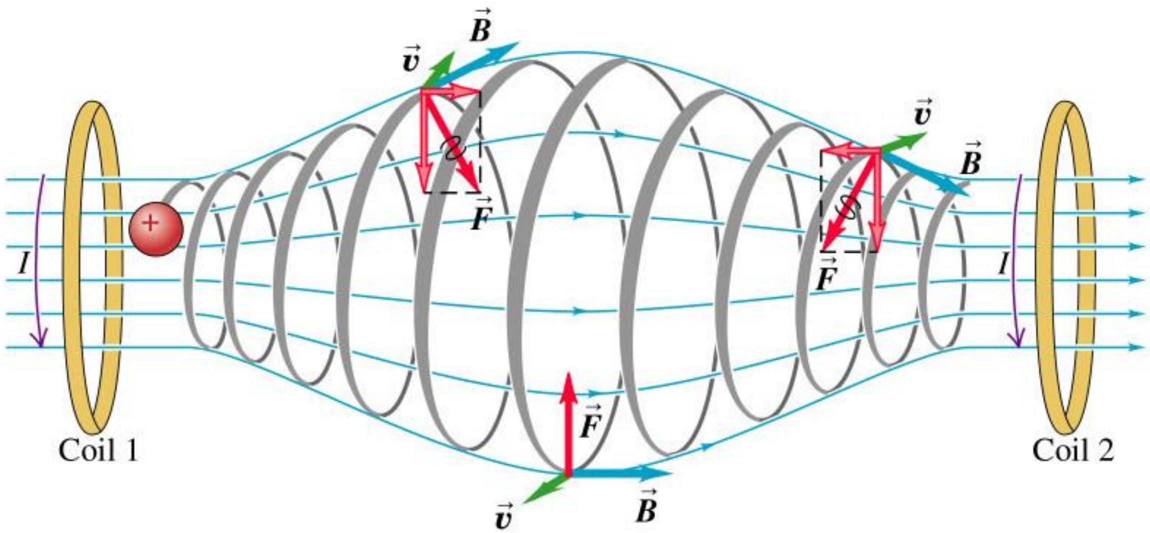
(Collisionless region)

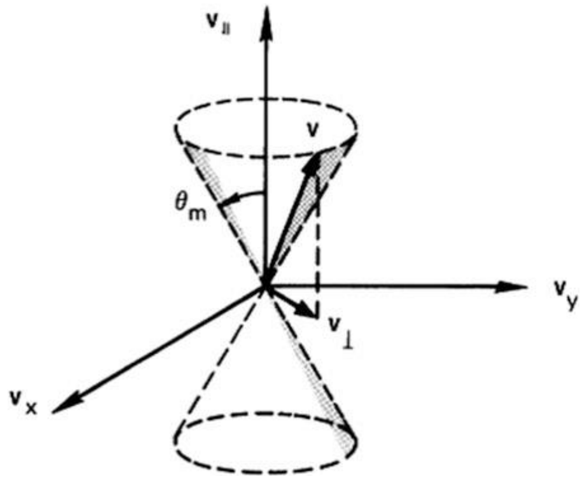


Lecture note "Physics and Applications of Electron Cyclotron Heating and Current Drive" by R. Prater from GA at 6th ITER International School 2012, Institute for Plasma Research, Gandhinagar Ahmedabad, India, December 2-6, 2012

Charged particles can be partially confined by a magnetic mirror machine

- **Charged particles with small $v_{||}$ eventually stop and are reflected while those with large $v_{||}$ escape.**





$$v'_\perp{}^2 = v_{\perp o}{}^2 + v_{||o}{}^2 \equiv v_o{}^2$$

$$\frac{B_o}{B'} = \frac{v_{\perp o}{}^2}{v'_\perp{}^2} = \frac{v_{\perp o}{}^2}{v_o{}^2} \equiv \sin^2\theta$$

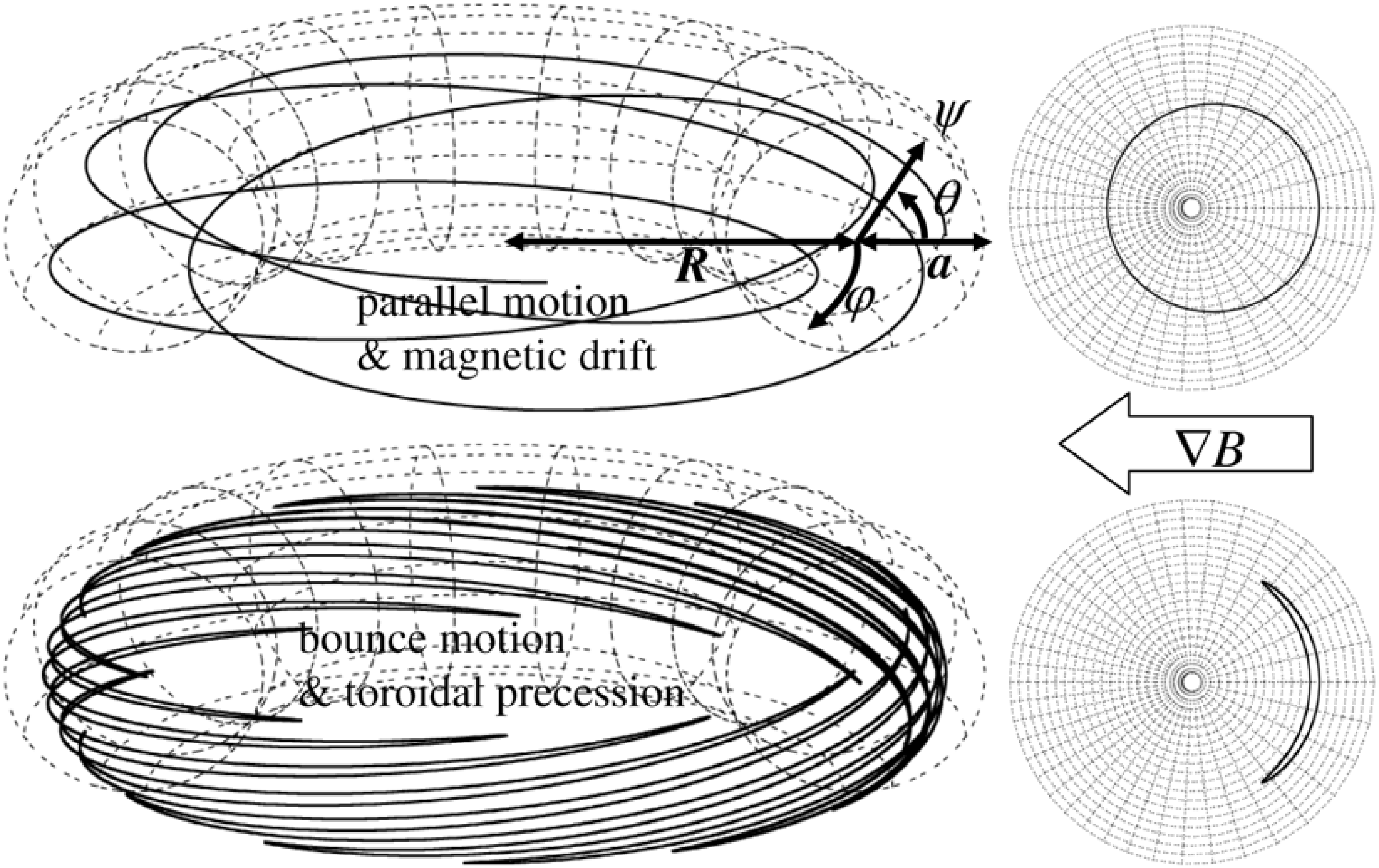
$$\frac{1}{2}mv^2 = \frac{1}{2}mv_{||}{}^2 + \frac{1}{2}mv_\perp{}^2 \qquad \text{Invarient: } \mu \equiv \frac{1}{2}\frac{mv_\perp{}^2}{B}$$

$$\frac{B_o}{B_m} \equiv \frac{1}{R_m} = \sin^2\theta_m$$

- **Large $v_{||}$ may occur from collisions between particles.**
- **Those confined charged particle are eventually lost due to collisions.**

https://i.stack.imgur.com/GlzGZ.jpg

The trajectories of charged particles follow the toroidal field lines



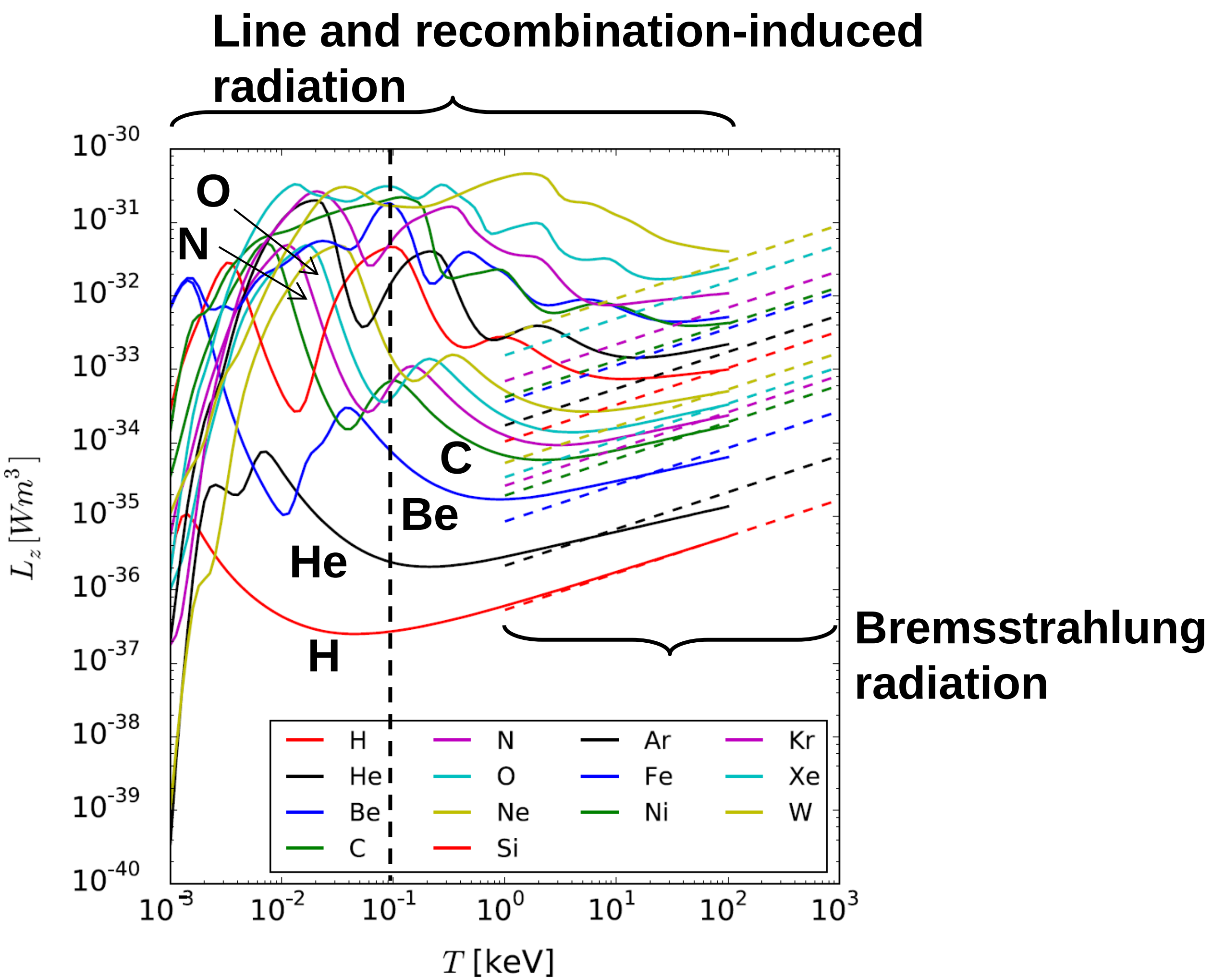
X. Garbet et al, Nucl. Fusion **50**, 043002 (2010)

Comparison of Fisch-Boozer Mechanism and Ohkawa Mechanism

Aspect	Fisch–Boozer Mechanism[1]	Ohkawa Mechanism[2]
Physical Process	Asymmetric heating of passing electrons with subsequent collisional momentum transfer	Selective de-trapping of barely trapped electrons into passing orbits (collisionless mechanism)
Requires collisions?	Yes (collisional mechanism)	No (collisionless pitch-angle scattering)
Key Particle Population	Passing electrons	Trapped (or barely trapped) electrons
Wave absorption location	Depends on Doppler-shifted resonance; typically near magnetic axis or mid-radius	Usually near edge where barely trapped particles are abundant

1 N. J. Fisch and A. H. Boozer, Phys. Rev. Lett. 45, 720 (1980).
2 T. Ohkawa, “Steady state operation of tokamaks by rf heating,” General Atomics Report No. GA-A13847 (1976).

Temperature of 100 eV is the threshold of radiation barrier by impurities



H. Lux, etc., Fusion Eng. Des. **42**, 101 (2015)

Reference for MCF

- Jeffrey P. Freidberg, Ideal Magnetohydrodynamics
- John Wesson, Tokamaks
- Tokamak Physics by 陳騮 院士

Course Outline

- Inertial confinement fusion (ICF)
  - Plasma frequency and critical density
  - Direct- and indirect- drive
  - Laser generated pressure (Inverse bremsstrahlung and Ablation pressure)
  - Burning fraction, why compressing a capsule?
  - Implosion dynamics
  - Shock (Compression with different adiabat)
  - Laser pulse shape
  - Rocket model, shell velocity
  - Laser-plasma interaction (Stimulated Raman Scattering, SRS; Stimulated Brillouin Scattering, SBS; Two-plasmon decay)
  - Instabilities (Rayleigh-taylor instability, Kelvin-Helmholtz instability, Richtmeyer-Meshkov instability)

Under what conditions the plasma keeps itself hot?

- Steady state 0-D power balance:

$$S_\alpha + S_h = S_B + S_k$$

S_α: α particle heating

S_h: external heating

S_B: Bremsstrahlung radiation

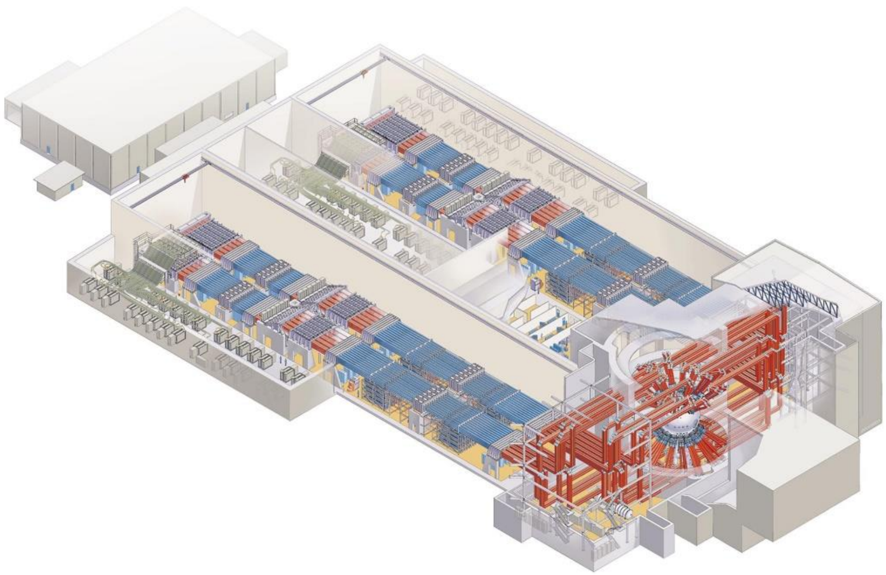
S_k: heat conduction lost

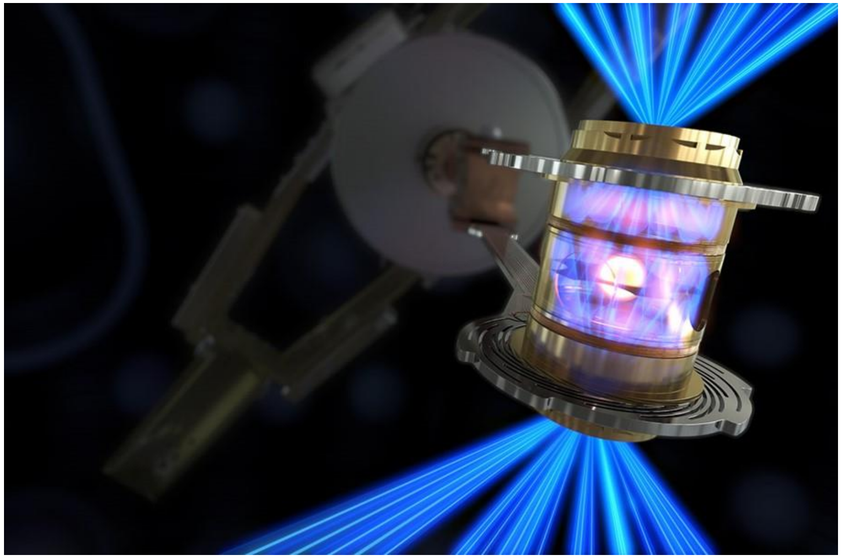
Ignition condition: $P\tau$ > 10 atm-s = 10 Gbar - ns

- P: pressure, or called energy density
- τ is confinement time

Significant breakthrough was achieved in ICF recently

- **Inertial confinement fusion (ICF)**



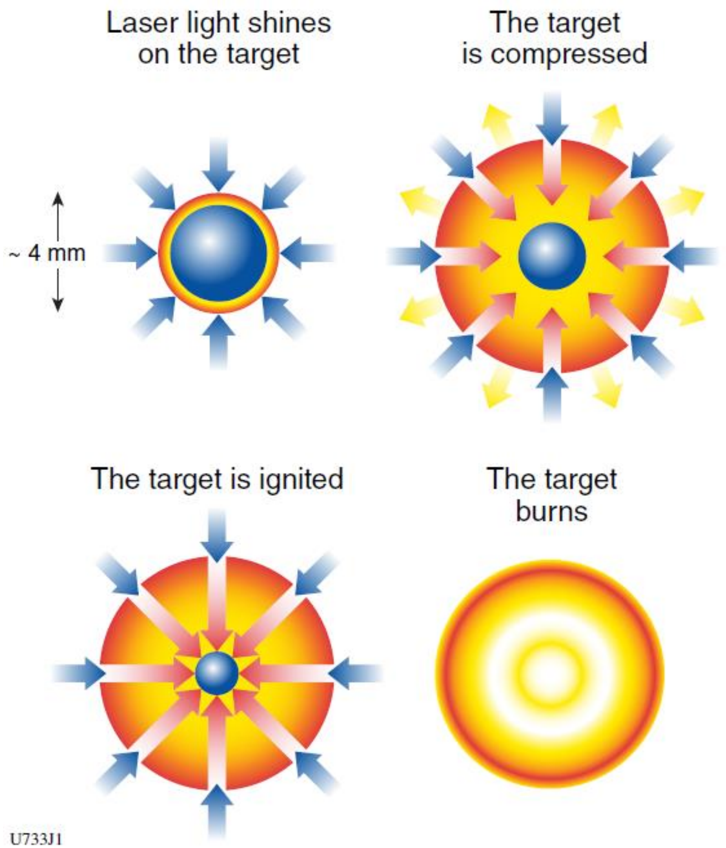


- **National Ignition Facility (NIF) demonstrated a gain grater than 1 for the first time on 2022/12/5. The yield of 3.15 MJ from the 2.05-MJ input laser energy, i.e., Q=1.5.**

https://zh.wikipedia.org/wiki/國家點火設施
https://www.science.org/content/article/historic-explosion-long-sought-fusion-breakthrough

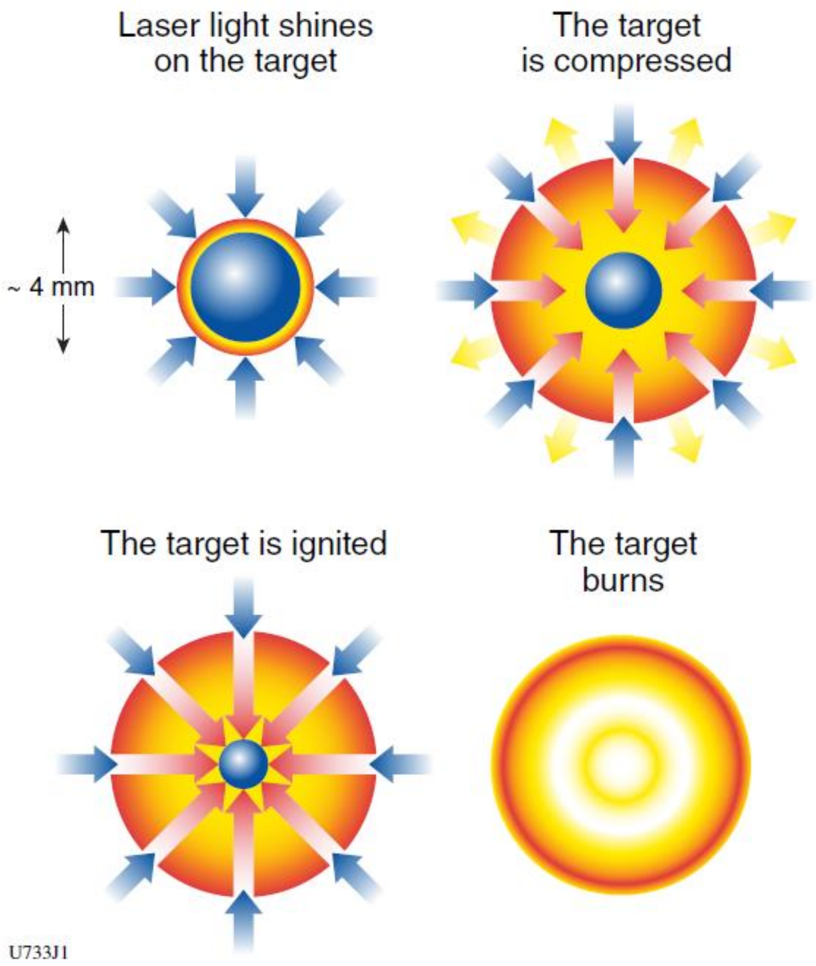
Don’t confine it!

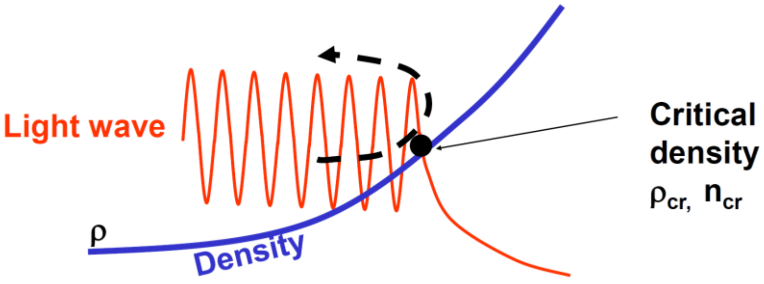
- **Solution 2: Inertial confinement fusion (ICF). Or you can say it is confined by its own inertia: P~Gigabar, τ~nsec, T~10 keV (10^8 ^{0}C)**

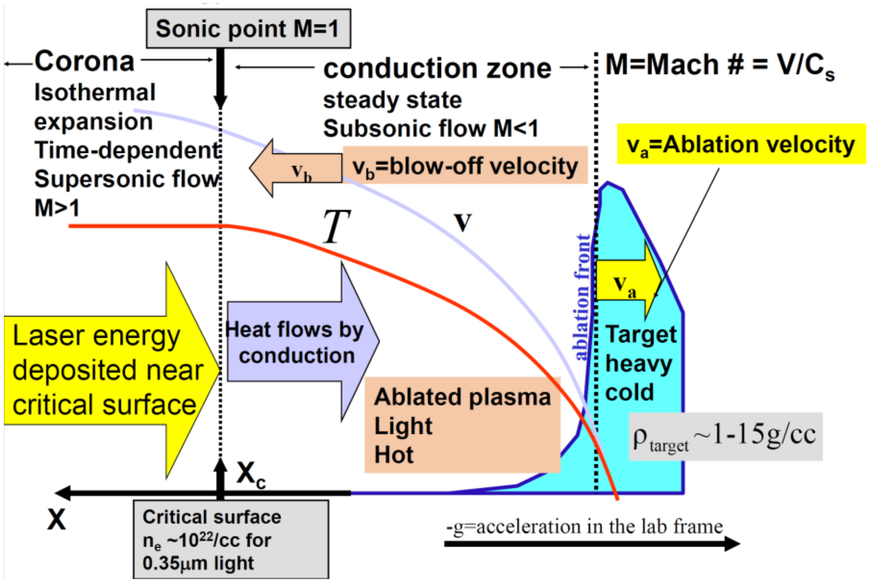


Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester

Compression happens when outer layer of the target is heated by laser and ablated outward

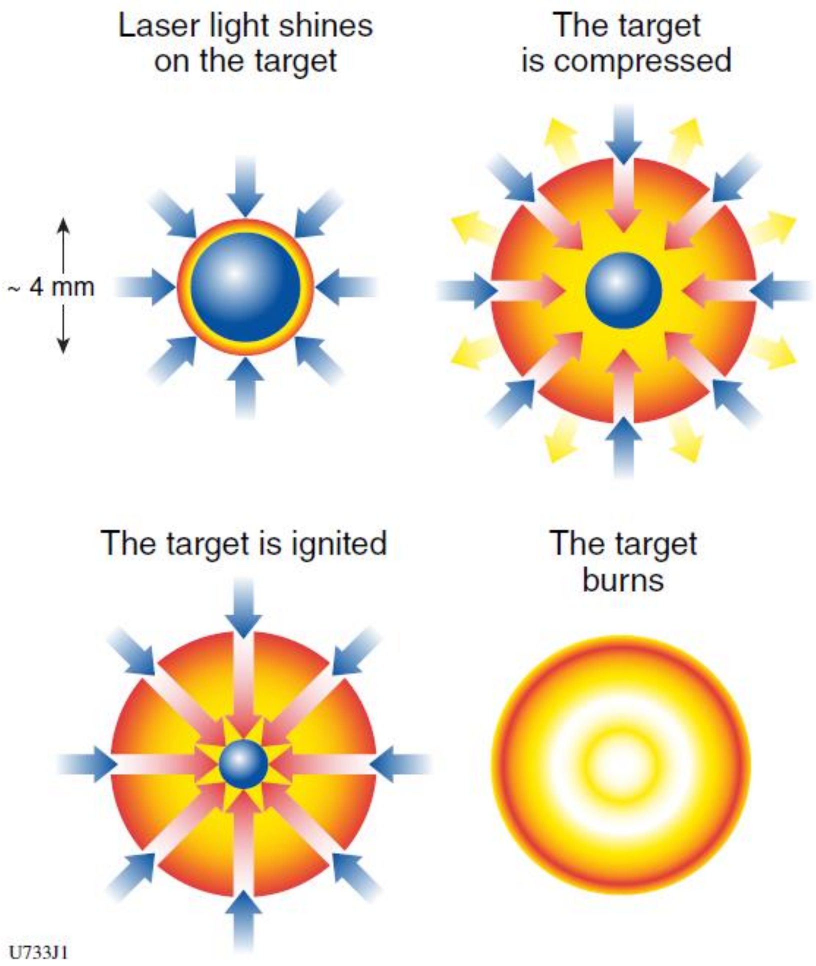




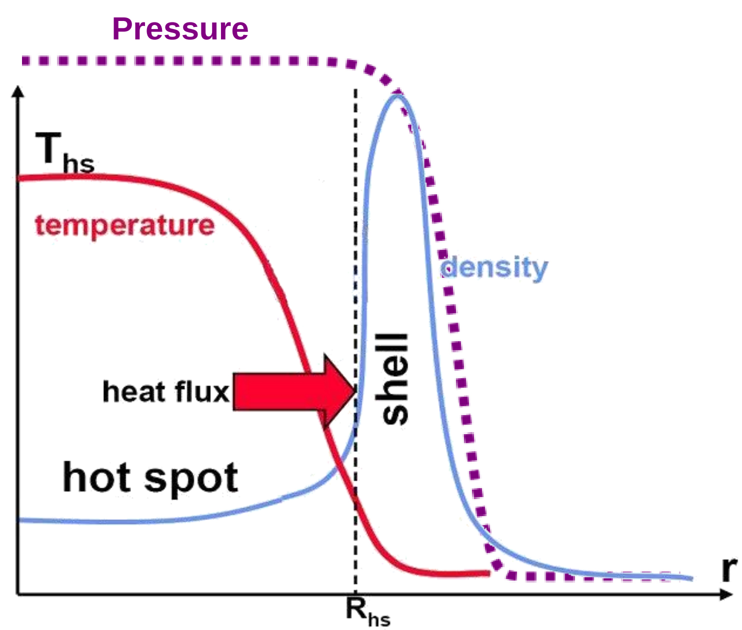


Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester
R. Betti, HEDSA HEDP Summer School, 2015

Plasma is confined by its own inertia in inertial confinement fusion (ICF)



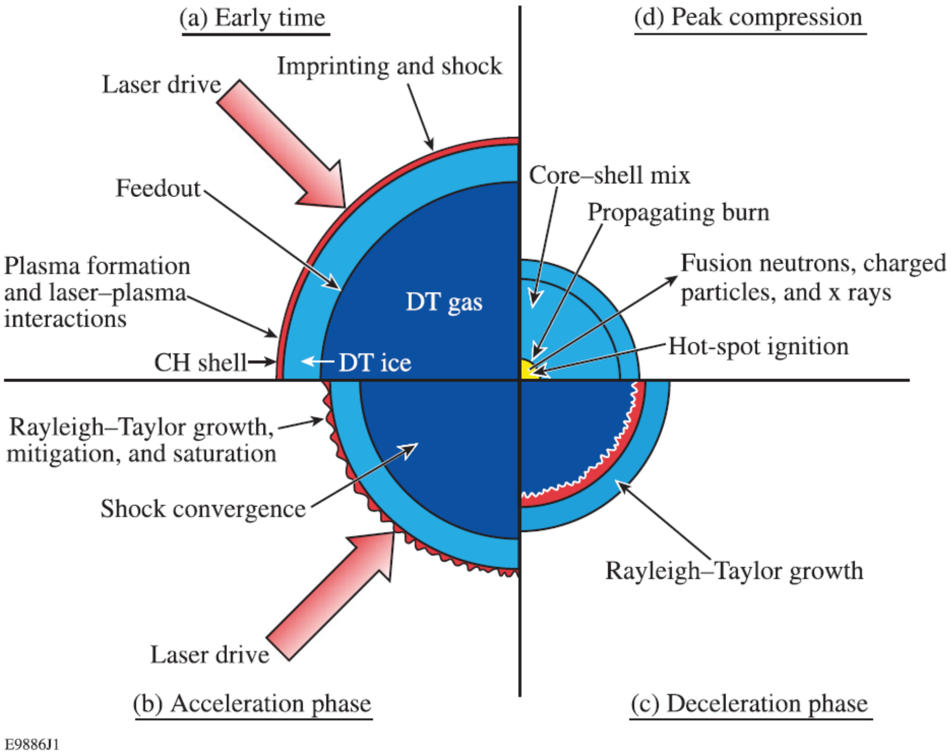
Spatial profile at stagnation



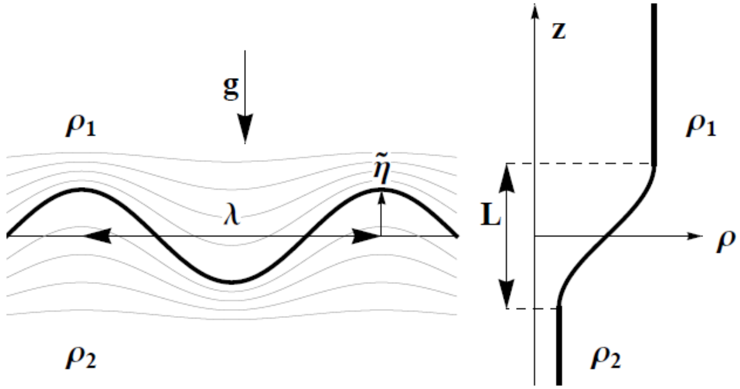
Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester

A ball can not be compressed uniformly by being squeezed between several fingers

- **Stages of a target implosion**

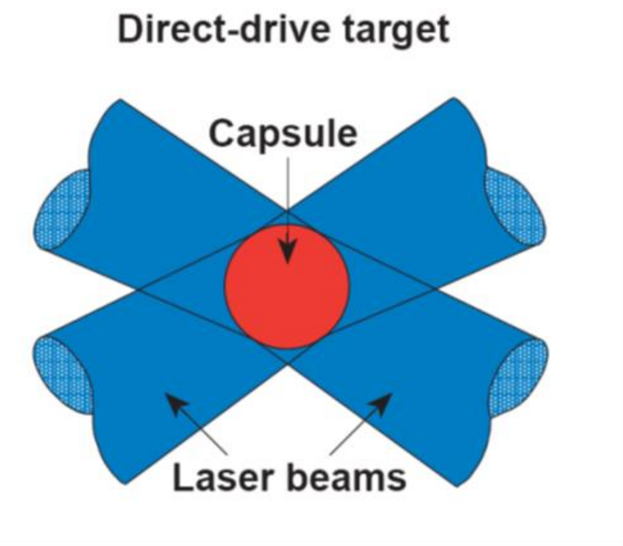


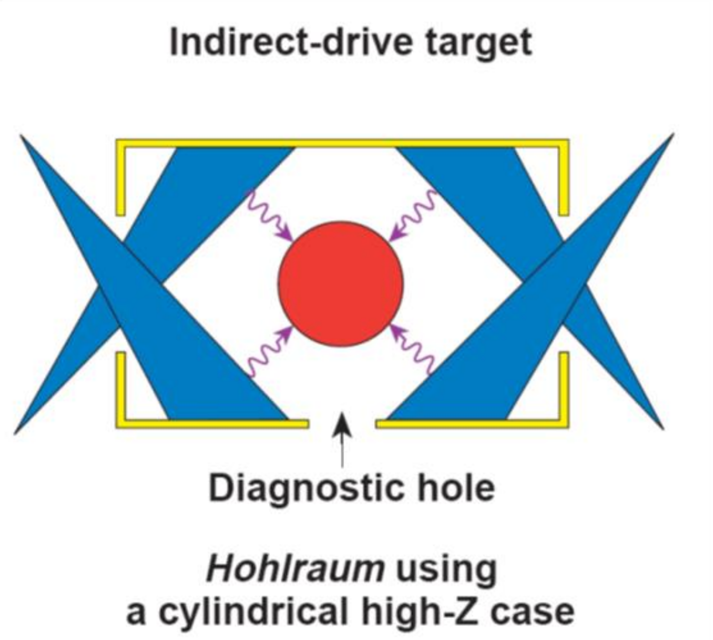
- **Rayleigh-Taylor instability**



P.-Y. Chang, PhD Thesis, U of Rochester (2013)
R. S. Craxton, etc., *Phys. Plasmas* **22**, 110501 (2015)

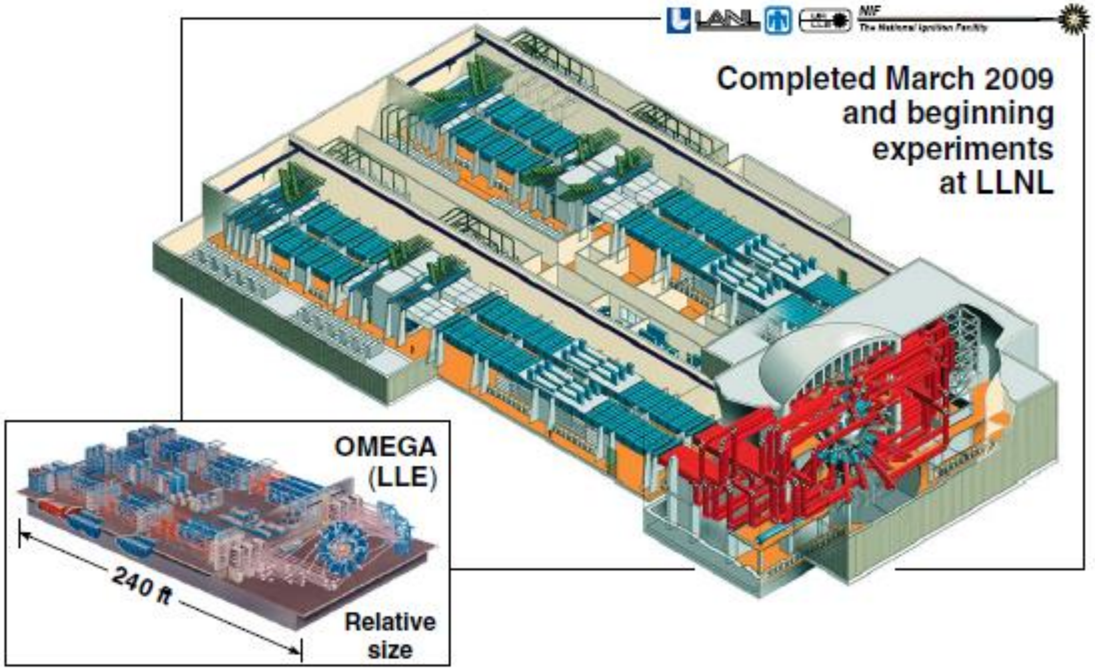
A spherical capsule can be imploded through directly or indirectly laser illumination





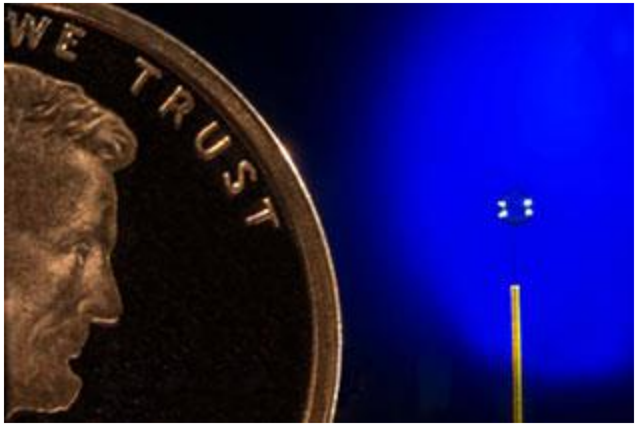
*R. Betti, HEDSA HEDP Summer School, 2015

The 1.8-MJ National Ignition Facility (NIF) will demonstrate ICF ignition and modest energy gain

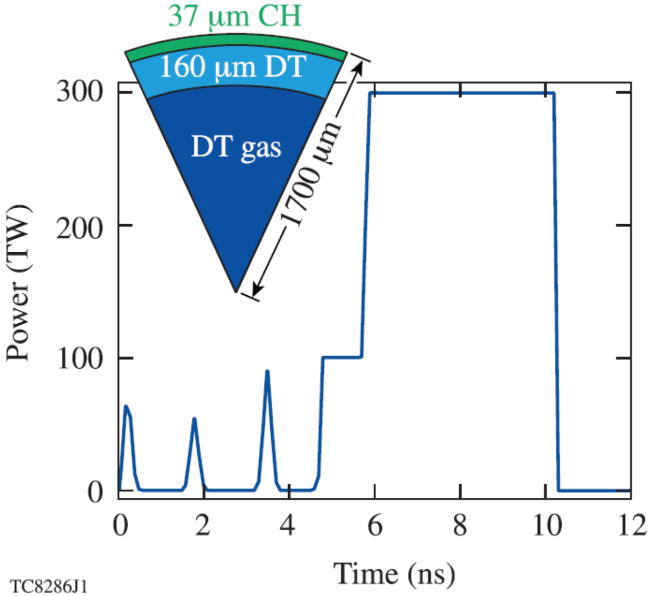


OMEGA experiments are integral to an ignition demonstration on the NIF.

Targets used in ICF



- **Triple-point temperature : 19.79 K**

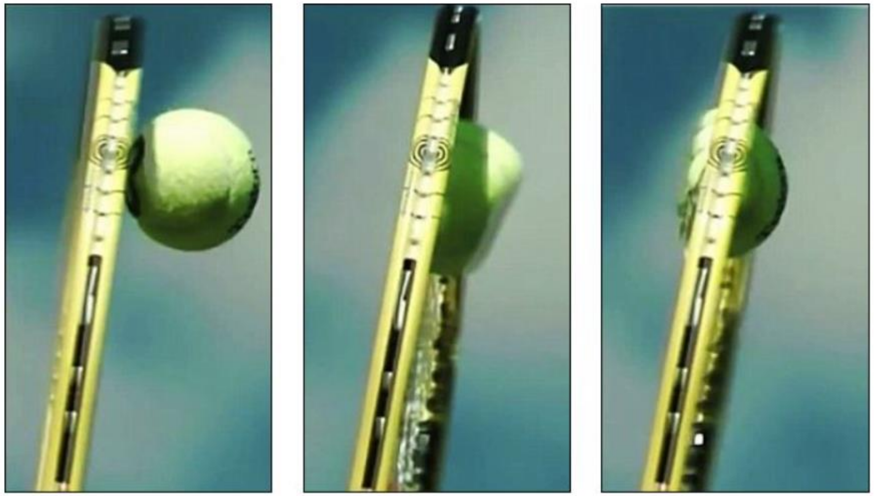


http://www.lle.rochester.ed
https://en.wikipedia.org/wiki/Inertial_confinement_fusion
R. S. Craxton, etc., *Phys. Plasmas* **22**, 110501 (2015)

Softer material can be compressed to higher density

- **Compression of a baseball**

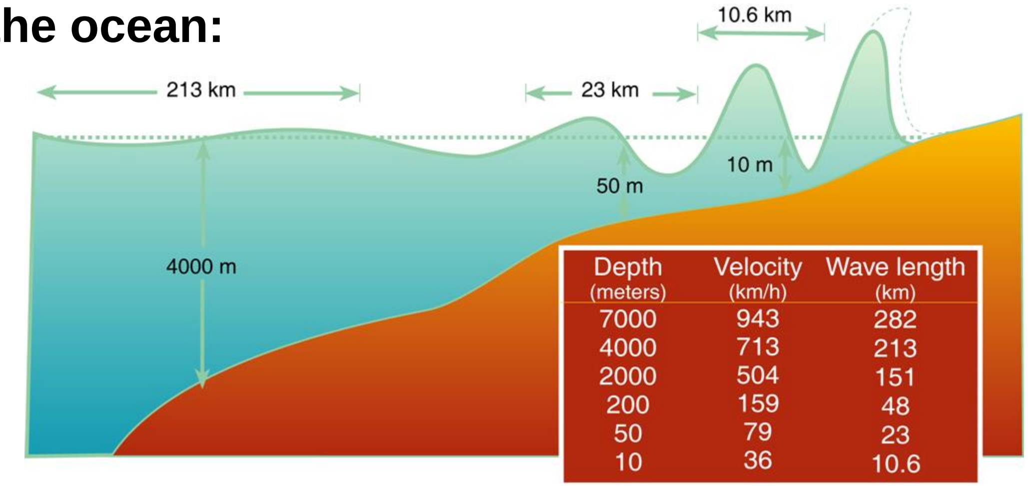
- **Compression of a tennis ball**



https://www.youtube.com/watch?v=uxlIdMoAwbY
https://newsghana.com.gh/wimbledon-slow-motion-video-of-how-a-tennis-ball-turns-to-goo-after-serve/

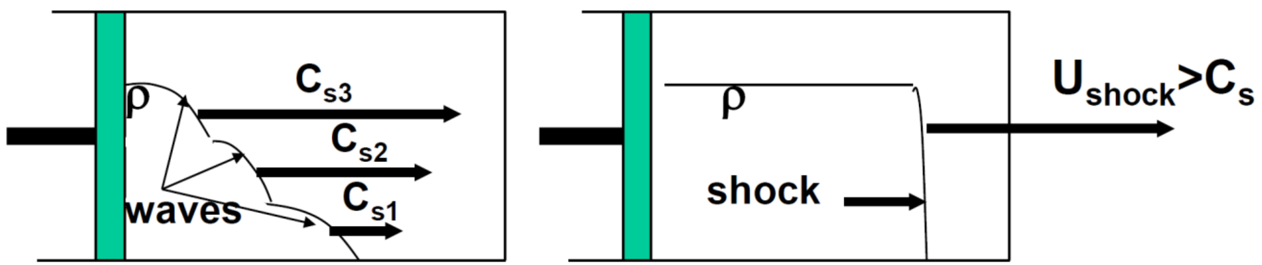
A shock is formed due to the increasing sound speed of a compressed gas/plasma

- **Wave in the ocean:**



Depth (meters)	Velocity (km/h)	Wave length (km)
7000	943	282
4000	713	213
2000	504	151
200	159	48
50	79	23
10	36	10.6

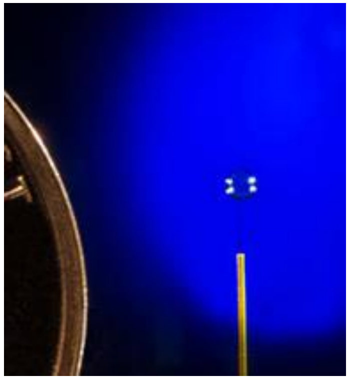
- **Acoustic/compression wave driven by a piston:**



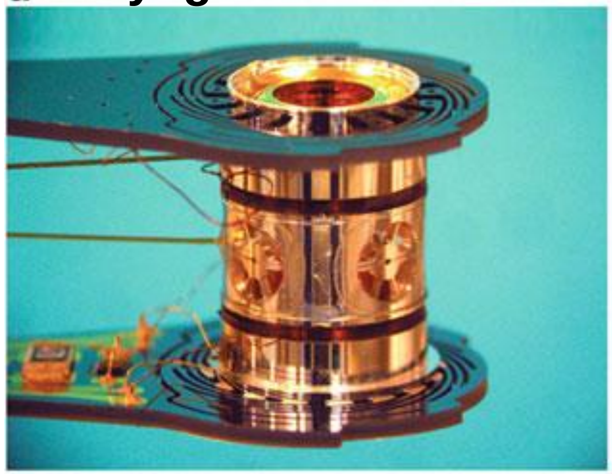
$$C_s \sim \sqrt{\frac{p}{\rho}} \sim \sqrt{\frac{\alpha \rho^{5/3}}{\rho}} \sim \sqrt{\alpha}\rho^{1/3}$$

http://neamtic.ioc-unesco.org/tsunami-info/the-cause-of-tsunamis
*R. Betti, HEDSA HEDP Summer School, 2015

Targets used in ICF



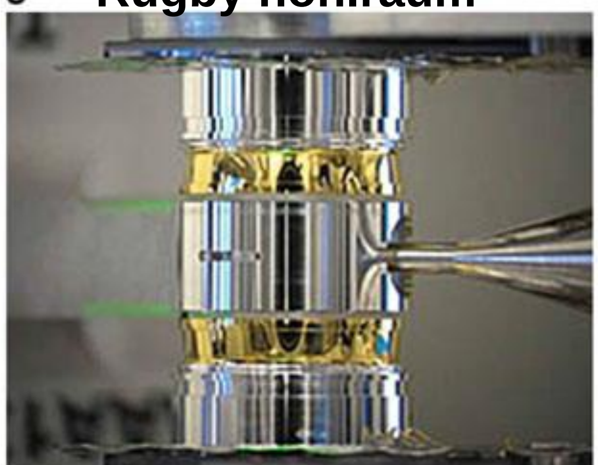
a Cryogenic hohlraum



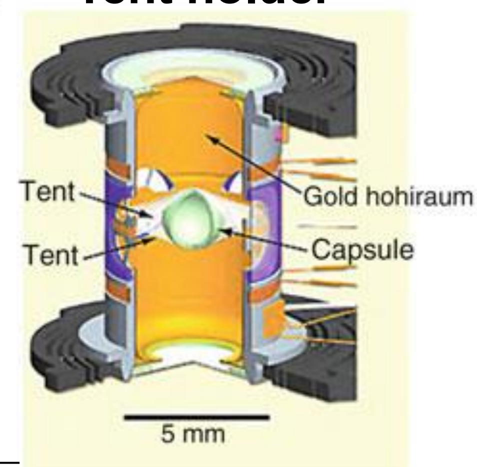
b

Cryogenic shroud

c Rugby hohlraum

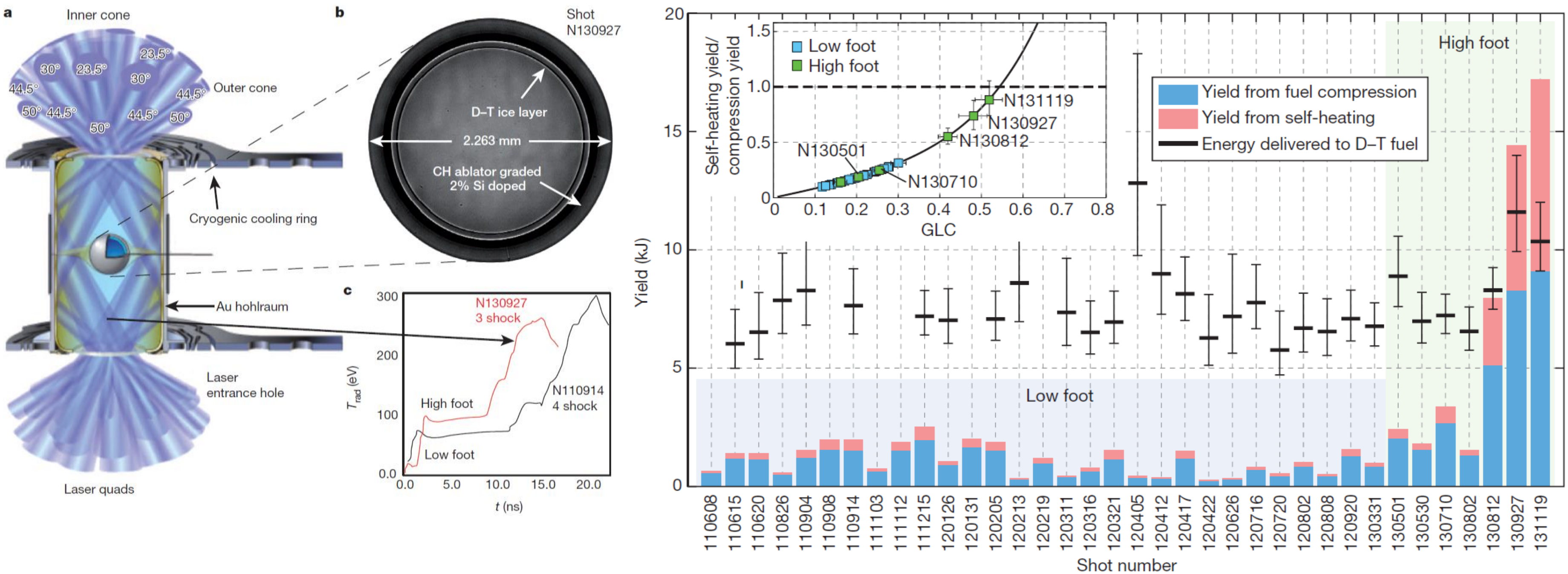


d Tent holder



https://www.lle.rochester.edu/index.php/2014/11/10/next-generation-cryo-target/
Introduction to Plasma Physics and Controlled Fusion 3rd Edition, by Francis F. Chen
https://www.llnl.gov/news/nif-shot-lights-way-new-fusion-ignition-phase

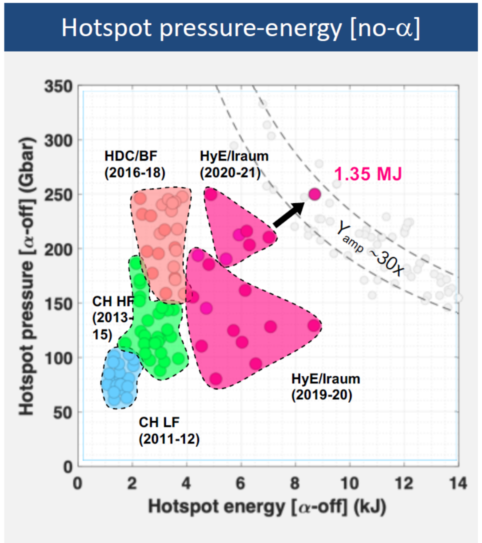
Nature letter “Fuel gain exceeding unity in an inertially confined fusion implosion”

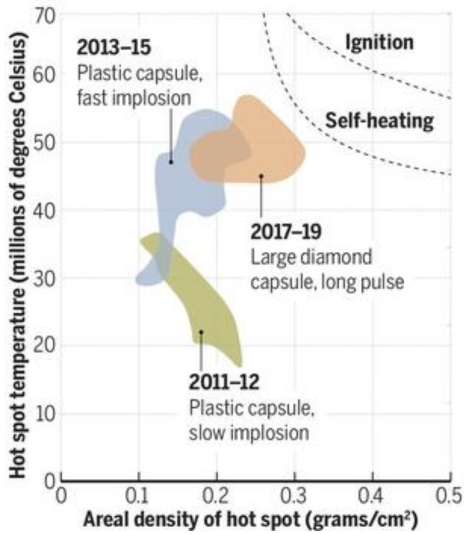


- **Fuel gain exceeding unity was demonstrated for the first time.**

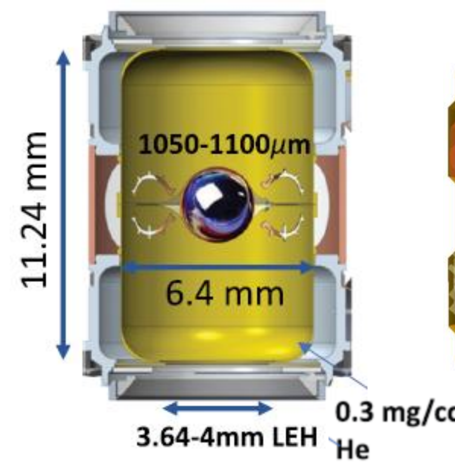
Nature 506, p343, 2014

The hot spot has entered the burning plasma regime

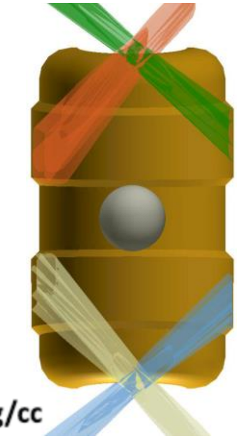


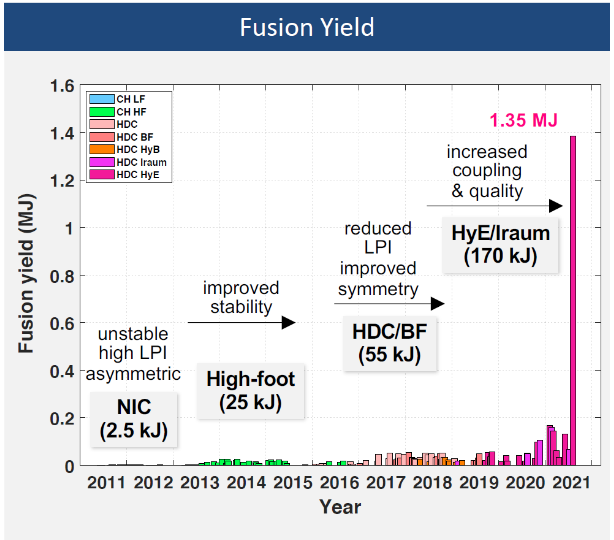


- **Hybrid-E**

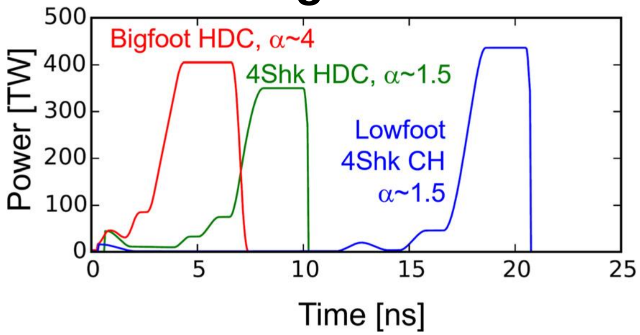


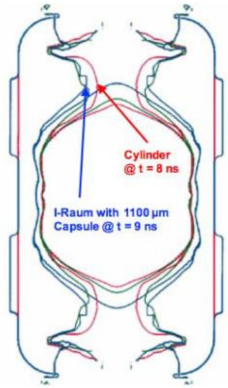
- **I-raum**





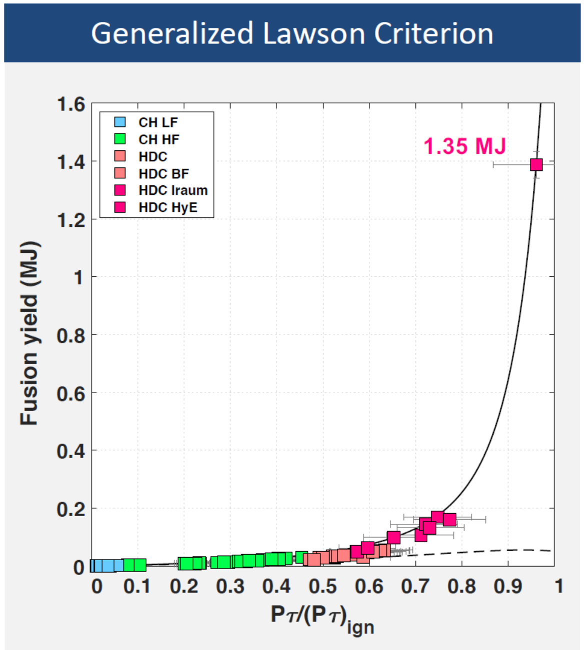
- **Big foot**

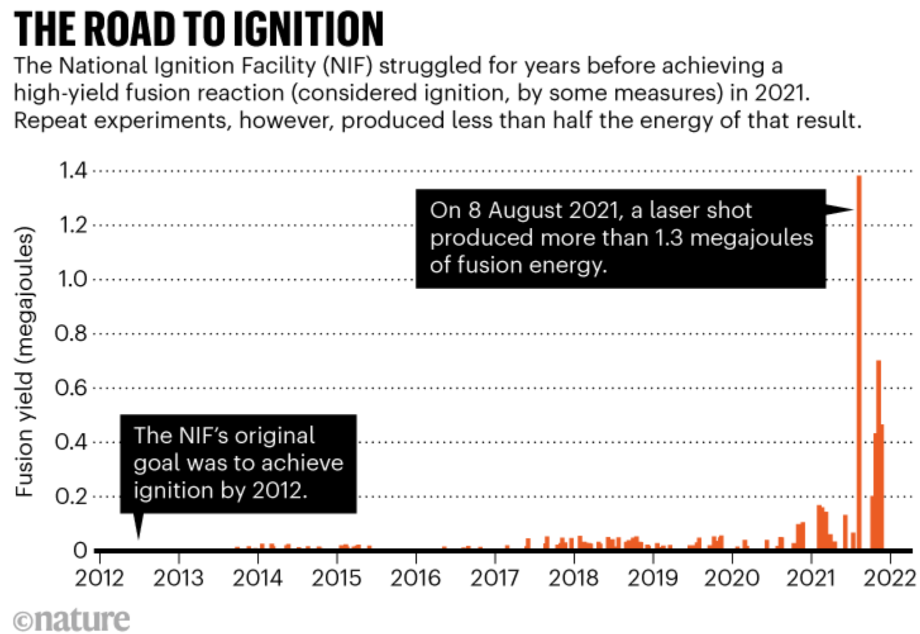




T. Ma, ARPA-E workshop, April 26, 2022
Science 370, p1019, 2020
D. T. Casey, etc., Phys. Plasmas, 25, 056308 (2018)
A. L. Kritcher, etc., Phys. Plasmas, 28, 072706 (2021)
H. F. Robey, etc., Phys. Plasmas, 25, 012711 (2018)

National Ignition Facility (NIF) achieved a yield of more than 1.3 MJ from ~1.9 MJ of laser energy in 2021 (Q~0.7)

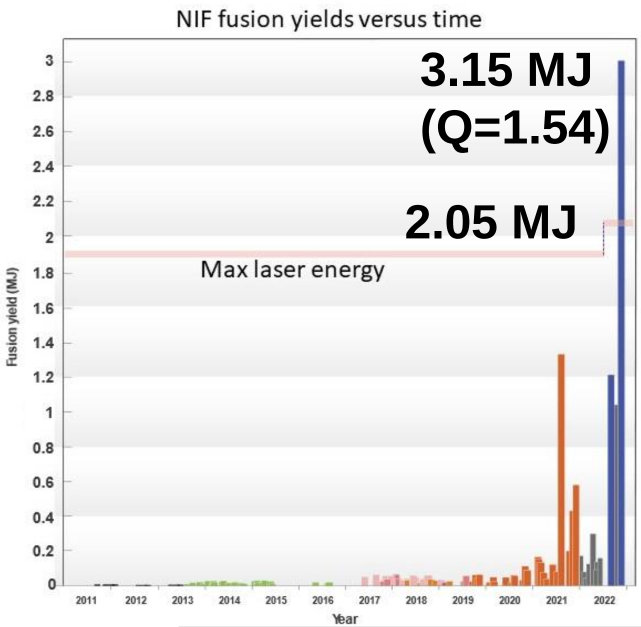


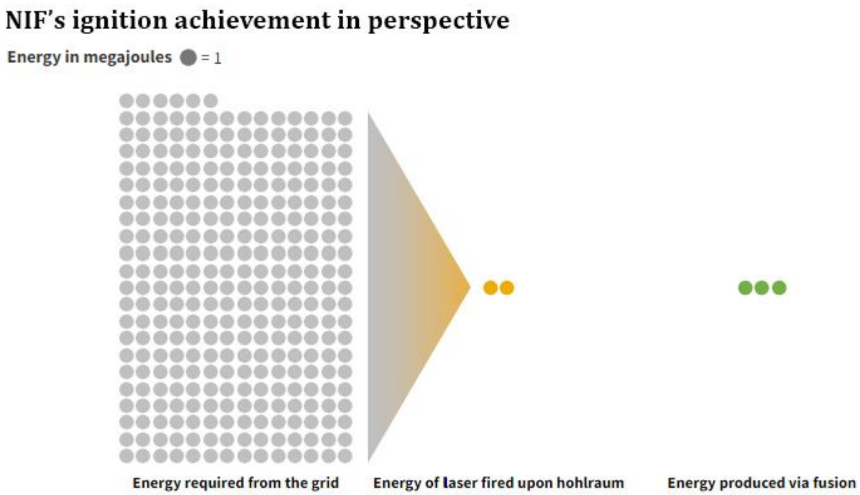


- **National Ignition Facility (NIF) achieved a yield of more than 1.3 MJ (Q~0.7). This advancement puts researchers at the threshold of fusion ignition.**
- **Laser-fusion facility heads back to the drawing board.**

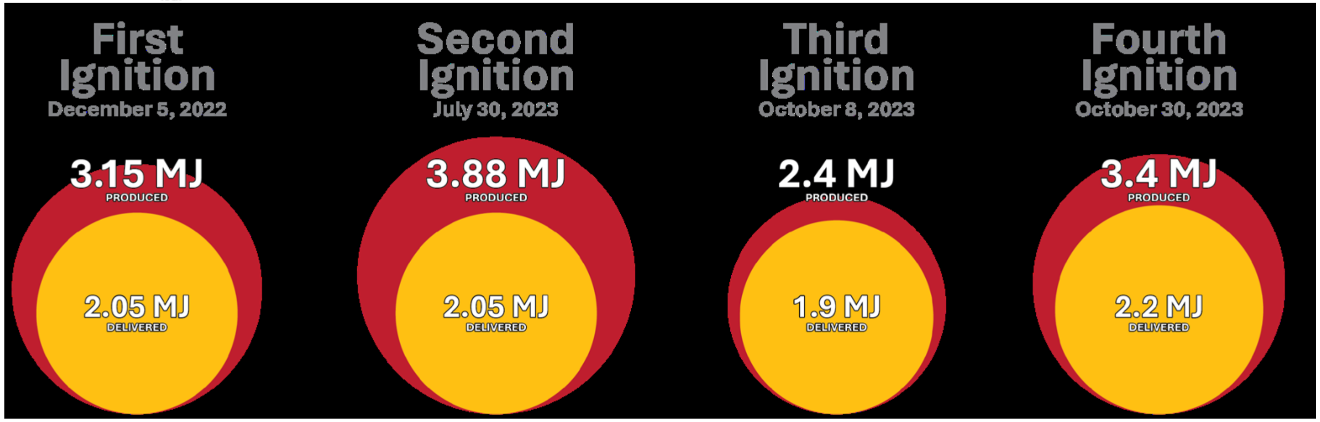
T. Ma, ARPA-E workshop, April 26, 2022
J. Tollefson, Nature (News) 608, 20 (2022)

"Ignition" (target yield larger than one) was achieved in NIF on 2022/12/5



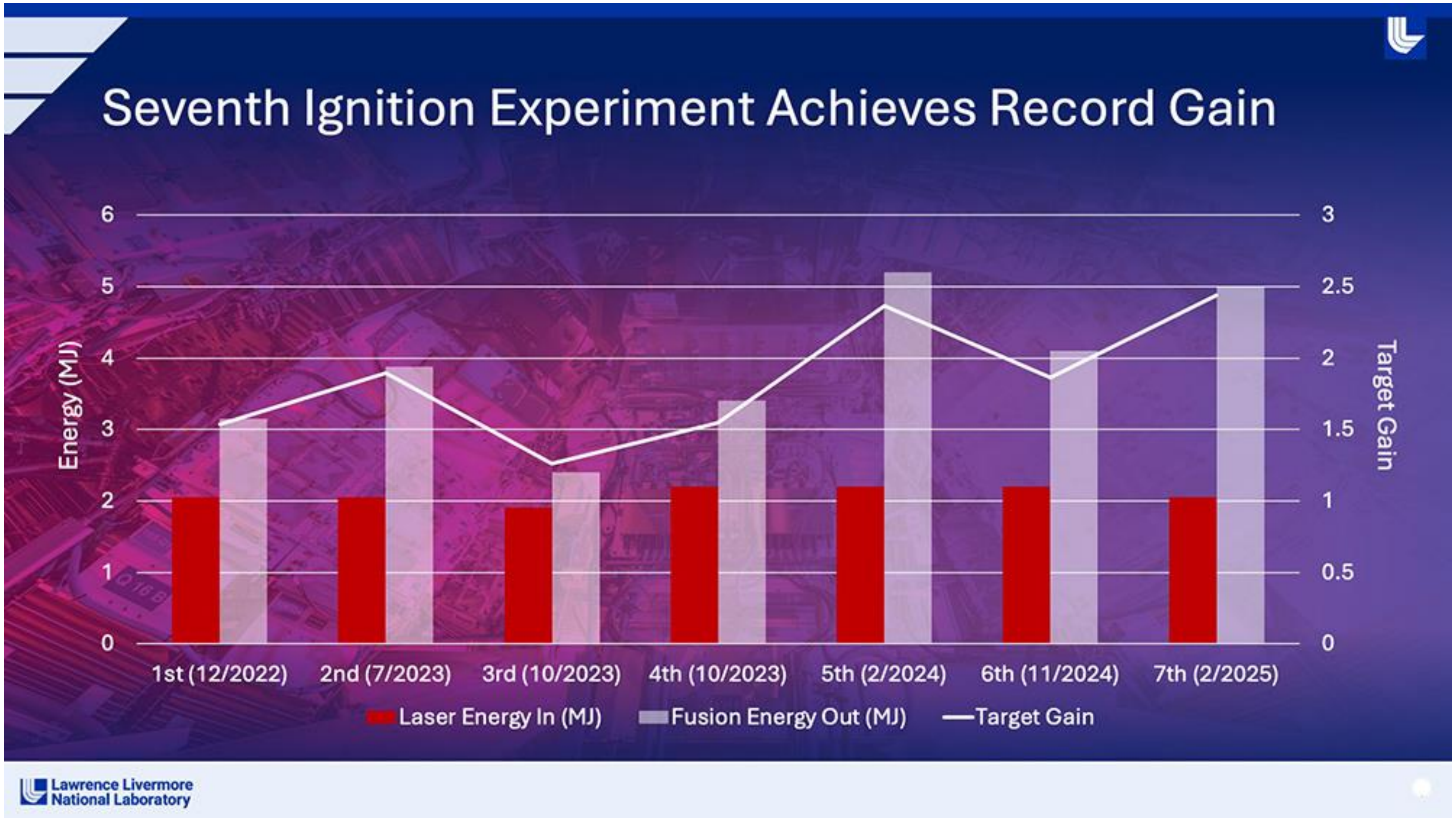


300 MJ | 2.05 MJ | 3.15 MJ



https://physicstoday.scitation.org/do/10.1063/PT.6.2.20221213a/full/
The age of ignition: anniversary edition, LLNL-BR-857901

A gain over 2.5 has been achieved

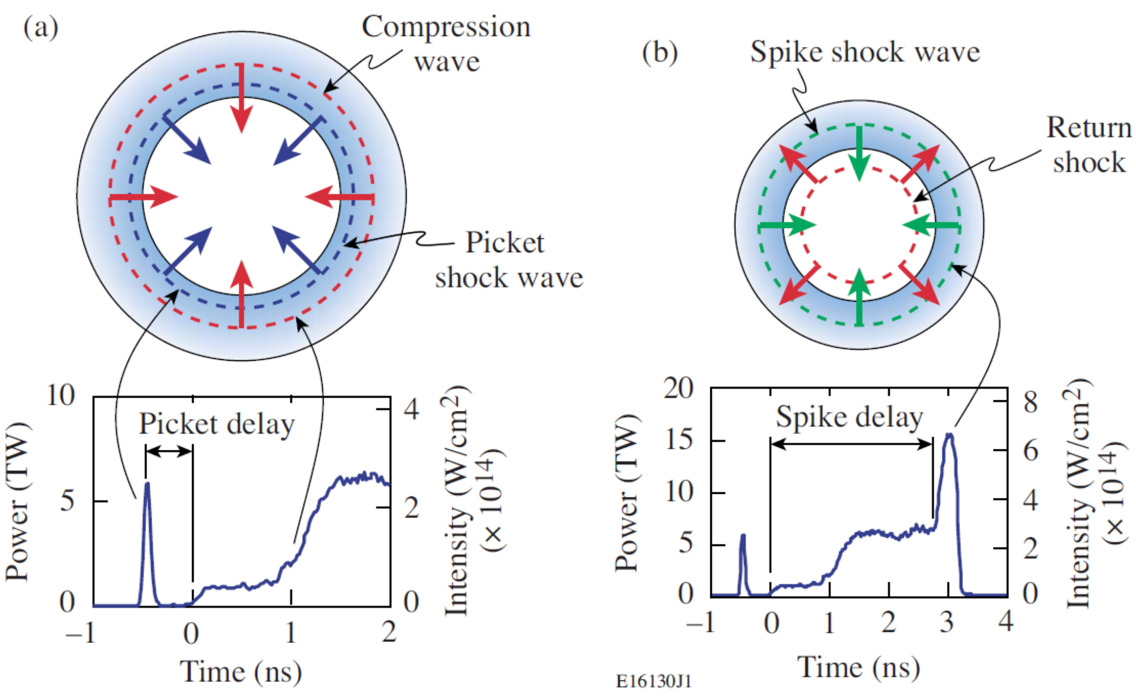


- **In recent attempts, the team at NIF increased the yield of the experiment, first to 5.2 megajoules and then again to 8.6 megajoules, according to a source with knowledge of the experiment.**

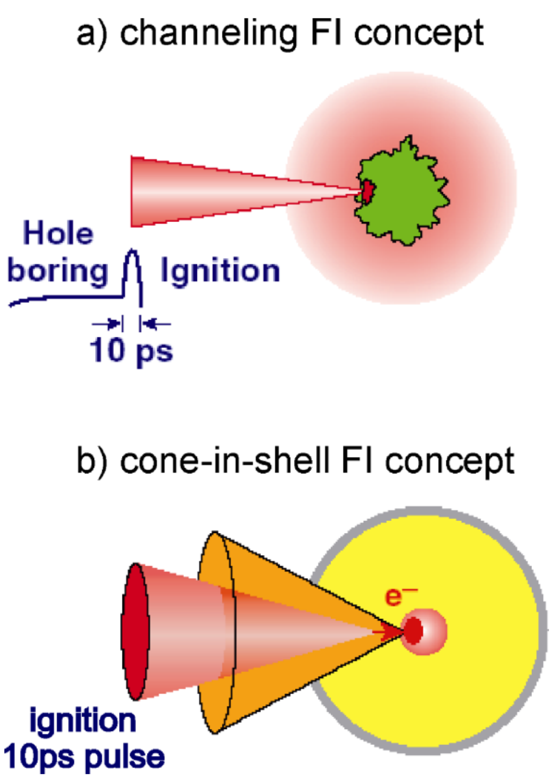
https://lasers.llnl.gov/science/achieving-fusion-ignition
https://techcrunch.com/2025/05/17/laser-powered-fusion-experiment-more-than-doubles-its-power-output/
https://www.notebookcheck.net/Laser-powered-fusion-NIF-hits-record-8-6-MJ-output.1019076.0.html

External “spark” can be used for ignition

- **Shock ignition**



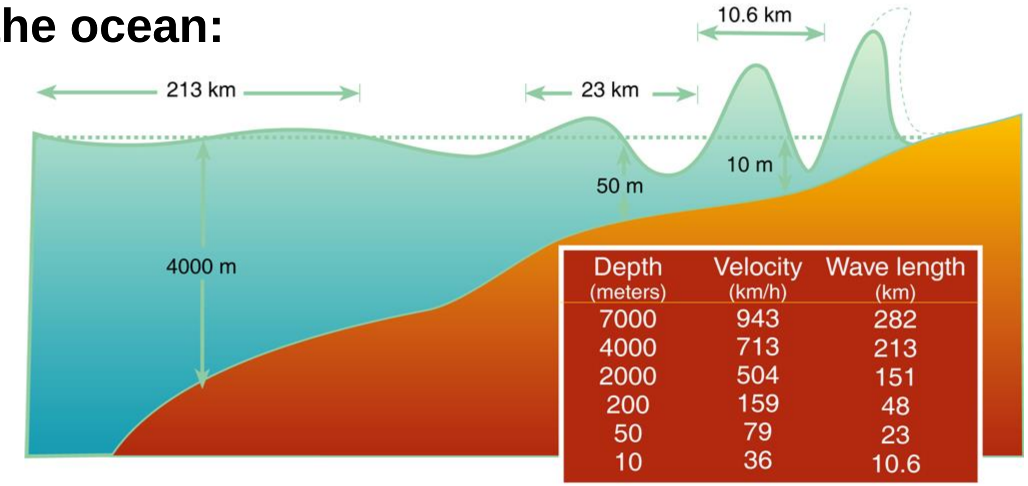
- **Fast ignition**



J. Badziak, Bull.Polish Acad. Sci. Tech. Sci.Phys. Plasmas 15, 056306 (2008)

A shock is formed due to the increasing sound speed of a compressed gas/plasma

- **Wave in the ocean:**



Depth (meters)	Velocity (km/h)	Wave length (km)
7000	943	282
4000	713	213
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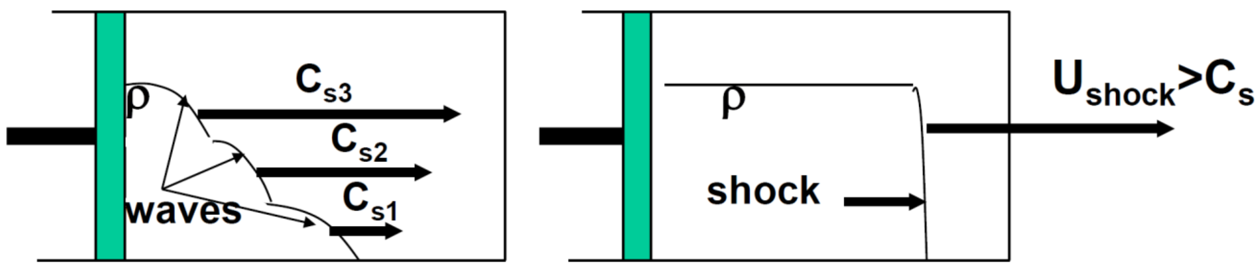
200	159	48
50	79	23
10	36	10.6

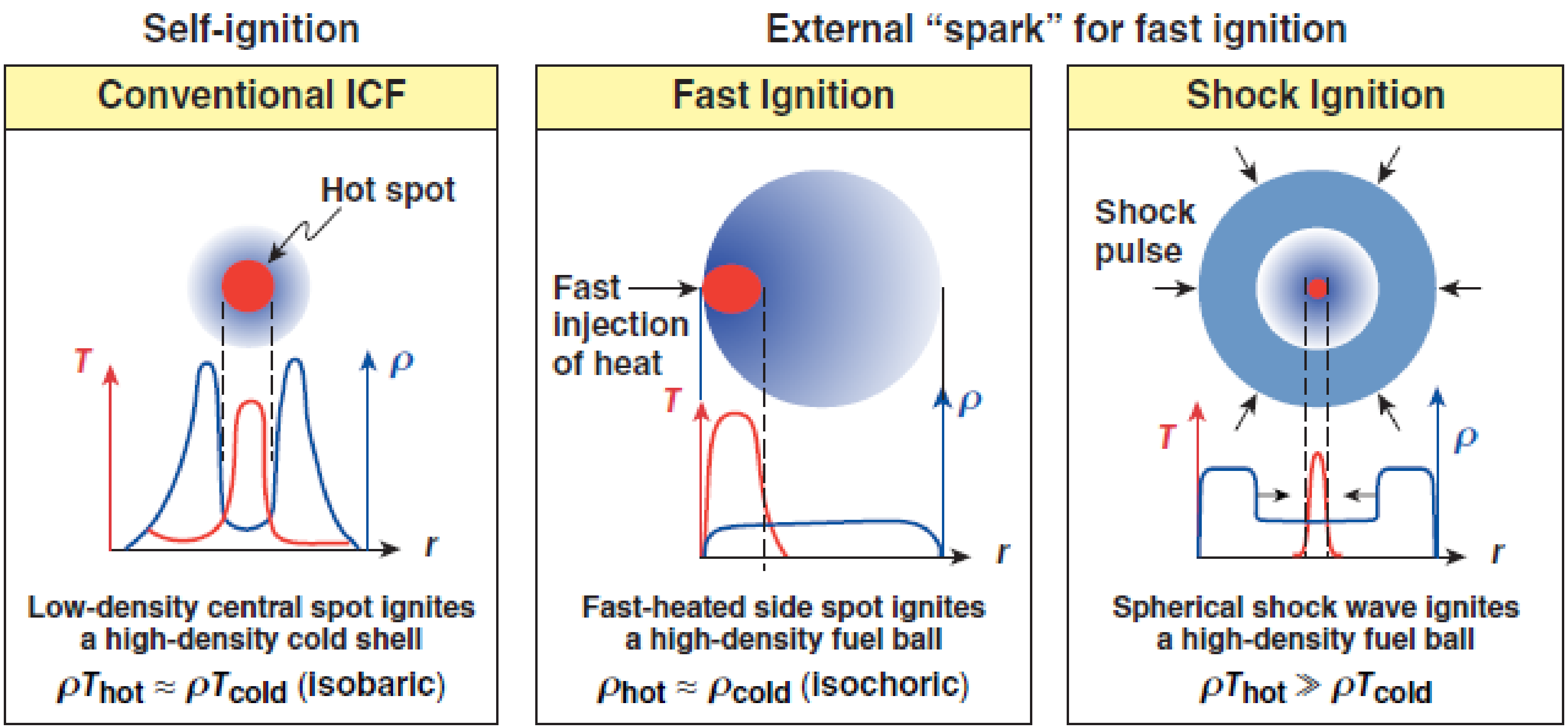
- **Acoustic/compression wave driven by a piston:**



$$C_s \sim \sqrt{\frac{p}{\rho}} \sim \sqrt{\frac{\alpha\rho^{5/3}}{\rho}} \sim \sqrt{\alpha}\rho^{1/3}$$

http://neamtic.ioc-unesco.org/tsunami-info/the-cause-of-tsunamis
*R. Betti, HEDSA HEDP Summer School, 2015

Ignition can happen by itself or being triggered externally

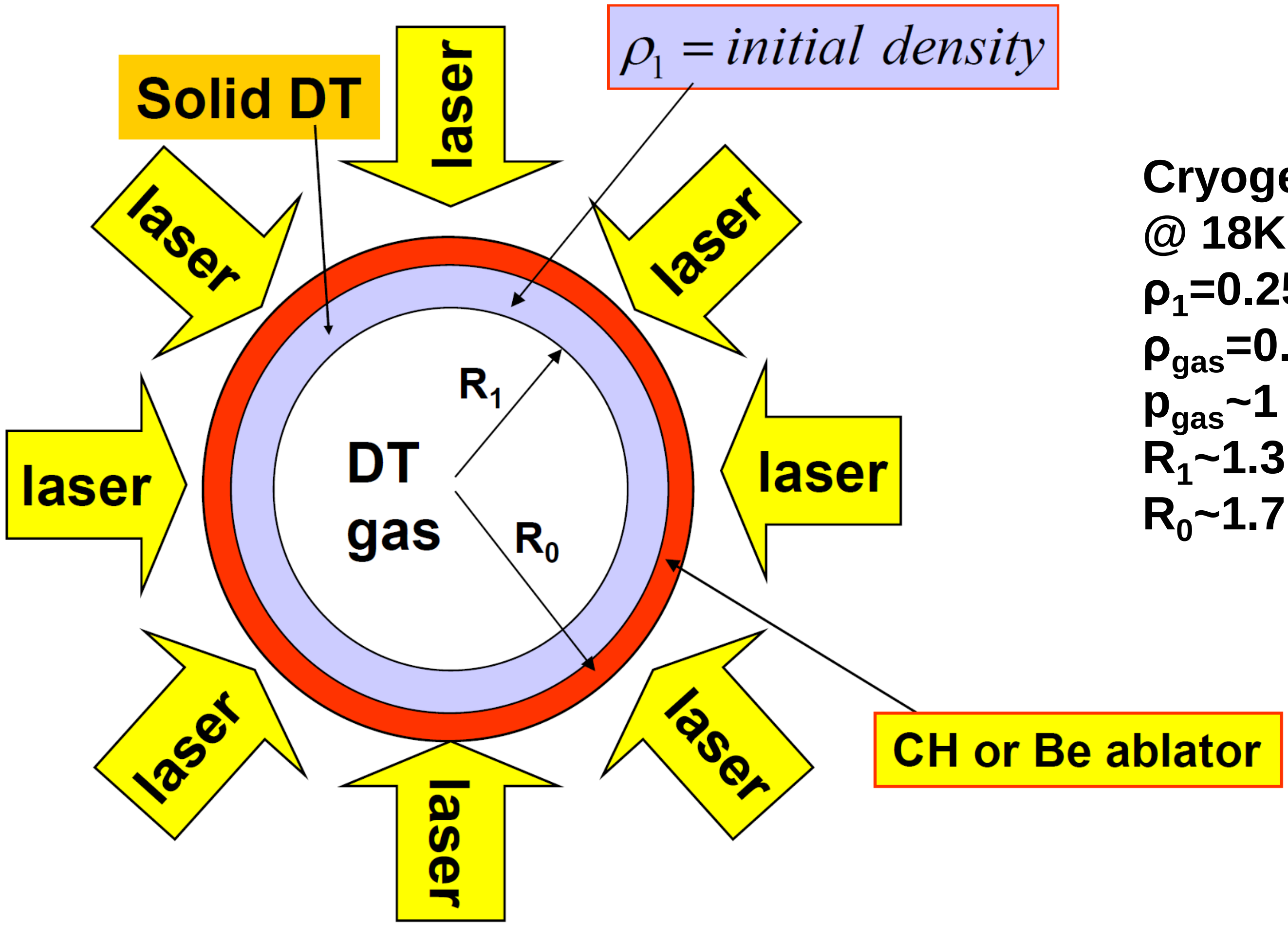


P. B. Radha, Fusion Energy Conference,2018

Reference for ICF

- Riccardo Betti, University of Rochester, HEDSA HEDP summer school, San Diego, CA, August 16-21, 2015
- ICF lectures for course PHY558/ME533
- The physics of inertial fusion, by S. Atzeni, J. Meyer-Ter-Vehn

Laser-driven imploding capsules are mm-size shells with hundreds of μm thick layers of cryogenic solid DT



Cryogenic solid DT ice @ 18K
ρ_1=0.25 g/cc
ρ_{gas}=0.3-0.6 mg/cc
p_{gas}~1 atm
R_1~1.3 mm
R_0~1.7 mm

Conservation equations of gas-dynamics and ideal gas EOS are used for DT plasma

Mass conservation: $\partial_t \rho + \partial_x(\rho\,\vec{v}) = 0$

Momentum conservation: $\partial_t(\rho\,\vec{v}) + \partial_x\left(p + \rho v^2\right) = \vec{F}$

Energy conservation: $\partial_t \epsilon + \partial_x[\vec{v}\,(\epsilon + p) - \kappa\partial_x T] = \mathrm{source} + \mathrm{sinks}$

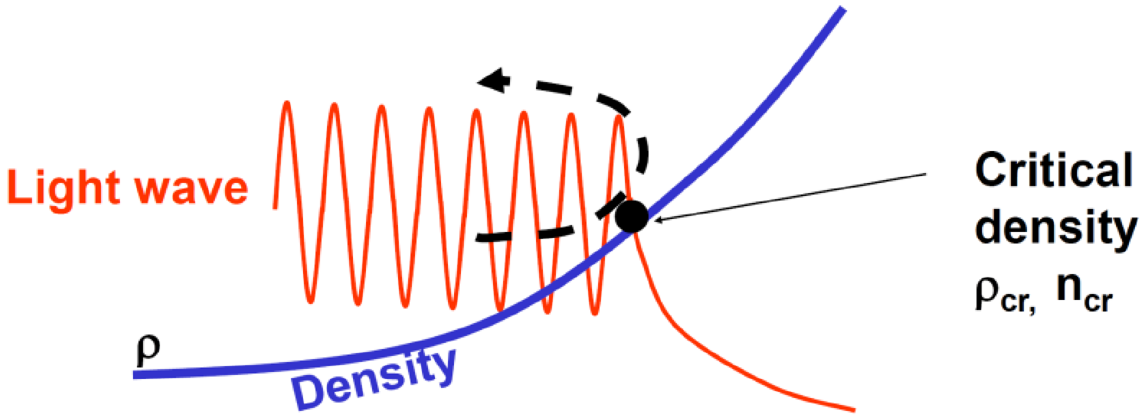
Ideal gas EOS: $p = (n_e T_e + n_i T_i) = 2nT = \frac{2}{m_i}\rho_i T = \frac{\rho T}{A}$

Total energy per unit volume: $\epsilon = \frac{3}{2}p + \rho\frac{v^2}{2}$

Mass density: $\rho = n_i m_i$

Plasma thermal conductivity: κ

The laser light cannot propagate past a critical density

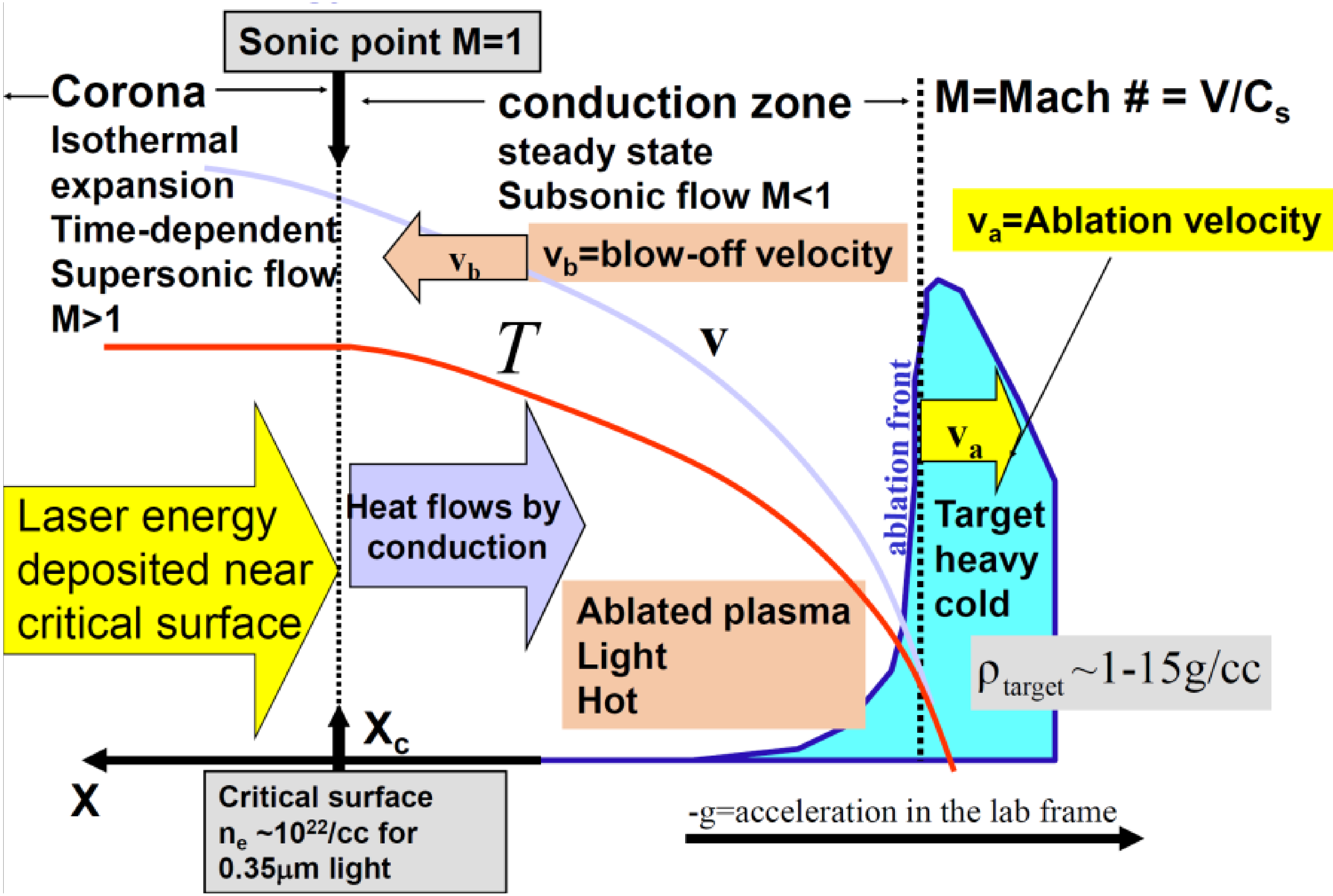


- **Critical density is given by plasma frequency=laser frequency**

$$\omega_{\mathrm{L}} = \frac{2\pi c}{\lambda_{\mathrm{L}}} \qquad \omega_{\mathrm{pe}} = \sqrt{\frac{4\pi n_{\mathrm{e}} e^2}{m_{\mathrm{e}}}}$$

$$\omega_{\mathrm{L}}^2 = \omega_{\mathrm{pe}}^2 \qquad \boxed{n_{\mathrm{e}}^{\mathrm{cr}} = \frac{1.1 \times 10^{21}}{\lambda_{\mathrm{L},\mu\mathrm{m}}^2}\ \mathrm{cm}^{-3}}$$

The laser generates a pressure by depositing energy at the critical surface



The plasma thermal conductivity is written in a power law of T

$$n\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\kappa\frac{\partial T}{\partial x}\right) \rightarrow n\frac{T}{t} \sim \frac{\kappa T}{x^2} \Rightarrow \kappa \sim n\frac{x^2}{t}$$

$$x \Rightarrow \lambda_{\mathrm{mfp}} \sim v_{\mathrm{th}}\tau_{\mathrm{coll}} = \frac{v_{\mathrm{th}}}{\nu_{\mathrm{coll}}} \qquad t \Rightarrow \tau_{\mathrm{coll}} = \frac{1}{\nu_{\mathrm{coll}}} \qquad \Rightarrow \kappa \sim n\frac{v_{\mathrm{th}}^2}{\nu_{\mathrm{coll}}}$$

$$v_{\mathrm{th}}^2 \sim \frac{T}{m_{\mathrm{e}}} \qquad \nu_{\mathrm{coll}} \sim \frac{n}{T^{3/2}} \qquad \Rightarrow \kappa \sim T^{5/2}$$

v_{th}: thermal velocity
v_{coll}: collision frequency
τ_{coll}: collision time

Plasma thermal conductivity

$$\boxed{\kappa \approx \kappa_o T^{5/2}}$$

Sound speed in an ideal DT gas/plasma

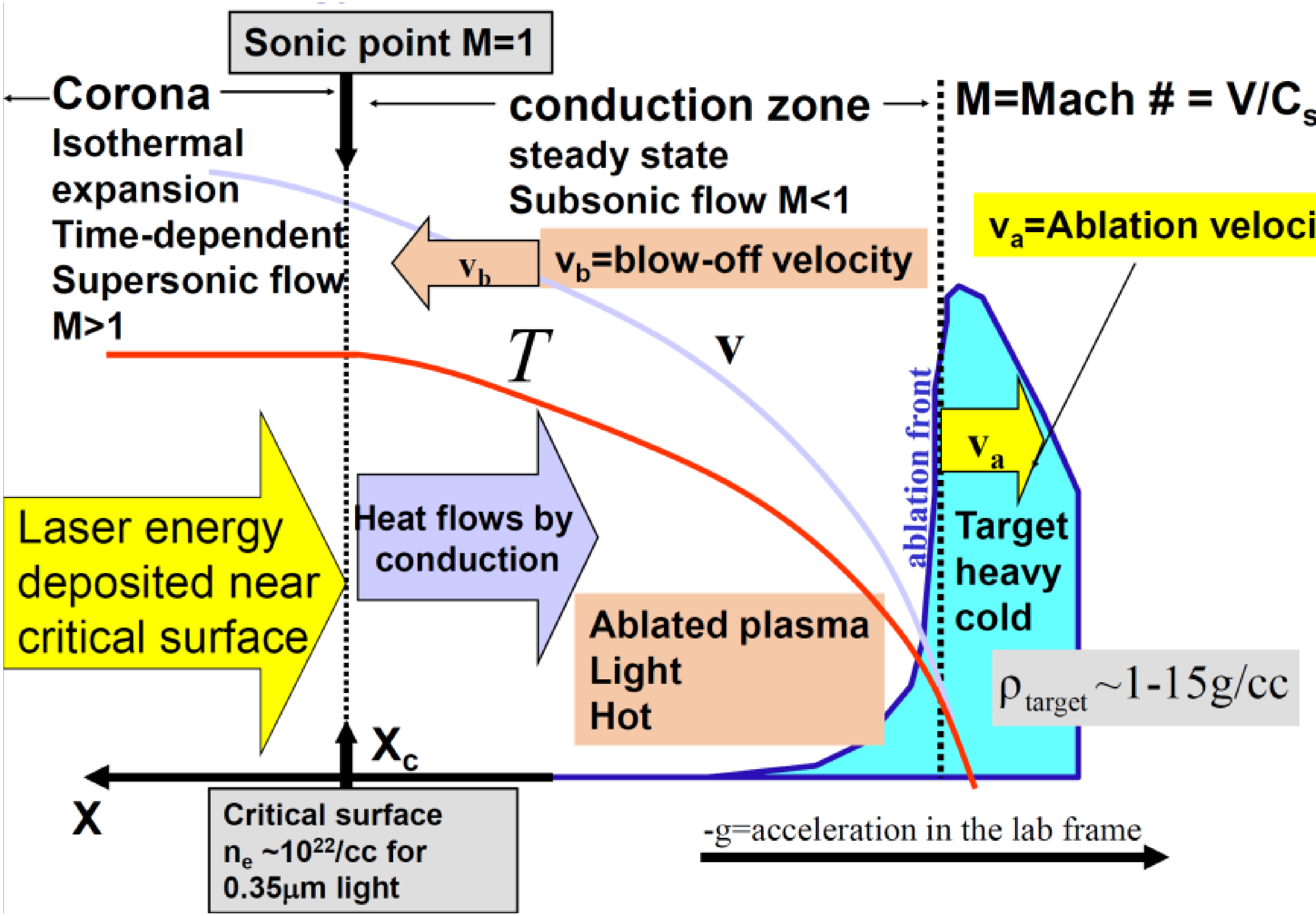
- **Adiabatic sound speed when the entropy is conserved along the fluid motion**

$$C_{\mathrm{s}}^{\mathrm{adiabatic}} = C_{\mathrm{s}}\,(\text{constant entropy}) = \sqrt{\frac{5}{3}\frac{p}{\rho}} = \sqrt{\frac{10}{3}\frac{T}{m_{\mathrm{i}}}}$$

- **Isothermal sound speed when the temperature is constant along the fluid motion**

$$C_{\mathrm{s}}^{\mathrm{isothermal}} = C_{\mathrm{s}}\,(\text{constant temperature}) = \sqrt{\frac{p}{\rho}} = \sqrt{\frac{2T}{m_{\mathrm{i}}}}$$

The laser generates a pressure by depositing energy at the critical surface



Consider the stead state equations of motion in the conduction zone

$$\partial_t\rho + \partial_x(\rho\,\vec{v}) = 0$$

$$\partial_t(\rho\,\vec{v}) + \partial_x\left(p + \rho v^2\right) = \vec{F}$$

$$\partial_t\epsilon + \partial_x[\vec{v}\,(\epsilon + p) - \kappa\partial_x T] = \text{source} + \text{sinks}$$

$$p = (n_e T_e + n_i T_i) = 2nT = \frac{2}{m_i}\rho_i T = \frac{\rho T}{A}$$

$$\frac{d}{dt} = 0$$

$$\frac{d}{dx}(\rho v) = 0$$

$$\frac{d}{dx}\left(p + \rho v^2\right) = 0$$

$$\frac{d}{dx}\left[v\left(\frac{5}{2}p + \frac{\rho v^2}{2}\right) - \kappa\frac{dT}{dx}\right] = 0$$

$$\kappa = \kappa_o T^{5/2}$$

$$p = \frac{\rho T}{A} \qquad A = \frac{m_i}{1+z}$$

$$c_S = \sqrt{\frac{p}{\rho}} = \sqrt{\frac{T}{A}}$$

- **Integrate with space:**

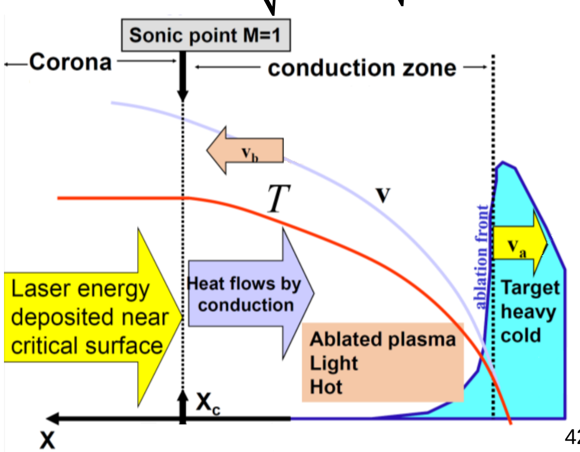
$$\rho v = \rho_c v_c$$

$$p + \rho v^2 = p_c + \rho_c {v_c}^2$$

$$\frac{\rho T}{A} + \rho v^2 = \frac{\rho_c T_c}{A} + \rho_c {v_c}^2$$

$$\rho v\left(\frac{T}{Av} + v\right) = \rho_c v_c\left(\frac{T_c}{Av_c} + v_c\right)$$

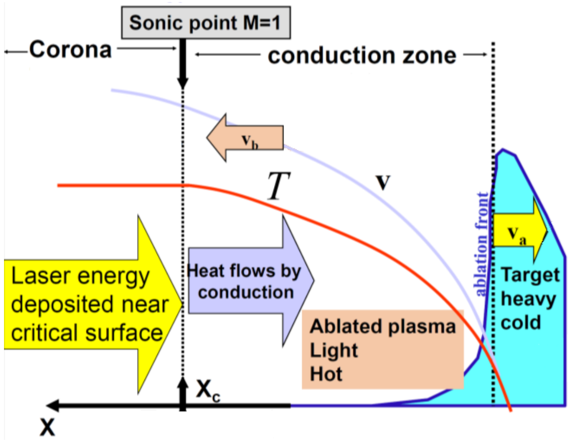
$$\frac{T}{Av} + v = \frac{T_c}{Av_c} + v_c \Rightarrow v\left(\frac{1}{M^2} + 1\right) = v_c\left(\frac{1}{{M_c}^2} + 1\right)$$



Consider the stead state equations of motion in the conduction zone

$$\frac{T}{Av} + v = \frac{T_c}{Av_c} + v_c \qquad v^2 - v\left(\frac{T_c}{Av_c} + v_c\right) + \frac{T}{A} = 0$$

$$v = \frac{1}{2}\left[\frac{T_c}{Av_c} + v_c \pm \sqrt{\left(\frac{T_c}{Av_c} + v_c\right)^2 - \frac{4T}{A}}\right]$$



- Near the target where $T << T_c$, one expect that $v << v_c$. Therefore,

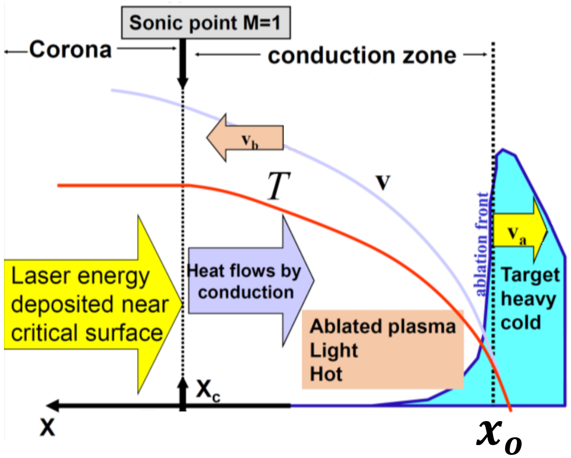
$$v = \frac{1}{2}\left[\frac{T_c}{Av_c} + v_c - \sqrt{\left(\frac{T_c}{Av_c} + v_c\right)^2 - \frac{4T}{A}}\right]$$

- At $T = T_c$, $v = v_c$:

$$v_c = \frac{1}{2}\left[\frac{T_c}{Av_c} + v_c - \left|\frac{T_c}{Av_c} - v_c\right|\right]$$

Consider the stead state equations of motion in the conduction zone

- If $\dfrac{T_c}{Av_c} - v_c \le 0$ $\qquad v_c = \dfrac{T_c}{Av_c}$ $\qquad M_c = 1$
- If $\dfrac{T_c}{Av_c} - v_c \ge 0$ $\qquad v_c = v_c$ $\qquad M_c \le 1$
- Pick M_c = 1, i.e., the flow is sonic at the critical surface.



- Integrate the energy equation in the conduction zone:

$$\frac{d}{dx}\left[v\left(\frac{5}{2}p + \cancel{\frac{\rho v^2}{2}}\right) - \kappa\frac{dT}{dx}\right] = 0$$

Assuming $M_c \ll 1$, i.e., $p \gg \rho v^2$

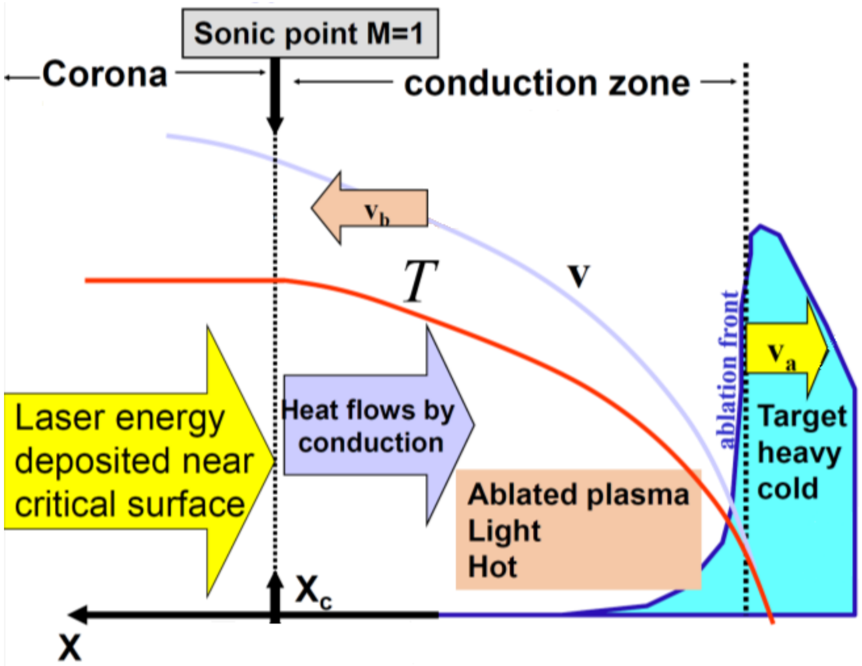
$$\frac{5}{2}pv - \kappa\frac{dT}{dx} = \frac{5}{2}p_o v_o - \left(\kappa\frac{dT}{dx}\right)_o = \frac{5}{2}\frac{\rho_o T_o}{A}v_o - \left(\kappa_o T^{5/2}\frac{dT}{dx}\right)_o \to 0$$

$$\frac{5}{2}\frac{\rho T v}{A} - \kappa_o T^{5/2}\frac{dT}{dx} = 0$$

$$\frac{5}{2}\frac{\rho_c v_c T}{A} - \kappa_o T^{5/2}\frac{dT}{dx} = 0$$

$$T = T_c\left[1 + \frac{25}{4A}\frac{\rho_c v_c}{k_o {T_c}^{5/2}}(x - x_c)\right]^{2/5}$$

Consider the stead state equations of motion in the conduction zone

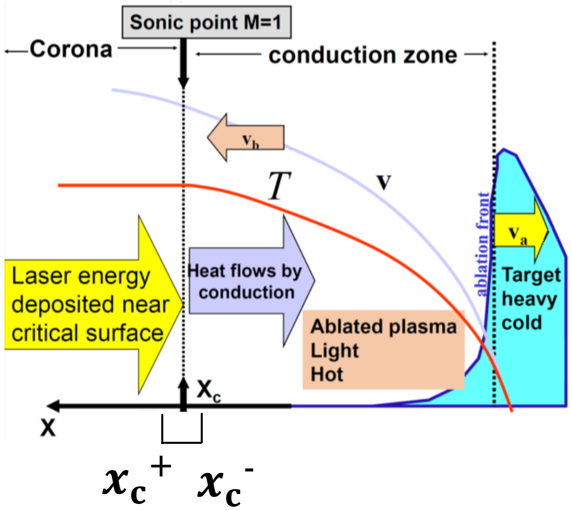


$$T = T_c \left[1 + \frac{25}{4A} \frac{\rho_c v_c}{k_o {T_c}^{5/2}} (x - x_c) \right]^{2/5}$$

$$v = \frac{1}{2} \left[\frac{T_c}{A v_c} + v_c - \sqrt{\left(\frac{T_c}{A v_c} + v_c \right)^2 - \frac{4T}{A}} \right]$$

$$\rho = \frac{\rho_c v_c}{v}$$

Pressure generated by a laser is obtained using energy conservation equation



- **Energy conservation equation:**

$$\partial_t \epsilon + \partial_x[\vec{v}\,(\epsilon + p) - \kappa\partial_x T] = \text{source} + \text{sinks}$$
$$= \underbrace{I\delta(x - x_c)}_{\mathrm{w/cm^2 s \cdot 1}/cm}$$

- **Since the temperature gradients are small in the corona, the heat flux is small**

$$\kappa\partial_{\mathrm{x}} T\,(x \geq x_{\mathrm{cr}}) << \kappa\partial_{\mathrm{x}} T\,(x \leq x_{\mathrm{cr}}) \qquad \left(\kappa\partial_{\mathrm{x}} T\,(x \geq x_{\mathrm{cr}}) \approx \frac{1}{3}\kappa\partial_{\mathrm{x}} T\,(x \leq x_{\mathrm{cr}})\right)$$

- **Integrate around critical surface x_c**

$$\int_{x_{\mathrm{cr}}^-}^{x_{\mathrm{cr}}^+} \{\partial_t \varepsilon + \partial_x [\vec{v}\,(\varepsilon + p) - \kappa\partial_x T]\}\, dx = \int_{x_{\mathrm{cr}}^-}^{x_{\mathrm{cr}}^+} \{I\delta\,(x - x_{\mathrm{cr}})\}\, dx$$

$$\partial_t \varepsilon x|_{x_{\mathrm{cr}}^-}^{x_{\mathrm{cr}}^+} + [v\,(\varepsilon + p)]_{x_{\mathrm{cr}}^-}^{x_{\mathrm{cr}}^+} - [\kappa\partial_x T]_{x_{\mathrm{cr}}^-}^{x_{\mathrm{cr}}^+} = I$$

$$-[\kappa\partial_x T]_{x_{\mathrm{cr}}^-}^{x_{\mathrm{cr}}^+} = I \Rightarrow I = -\kappa^+\left(\frac{\partial T}{\partial x}\right)^+ + \kappa^-\left(\frac{\partial T}{\partial x}\right)^- \approx \kappa^-\left(\frac{\partial T}{\partial x}\right)^-$$

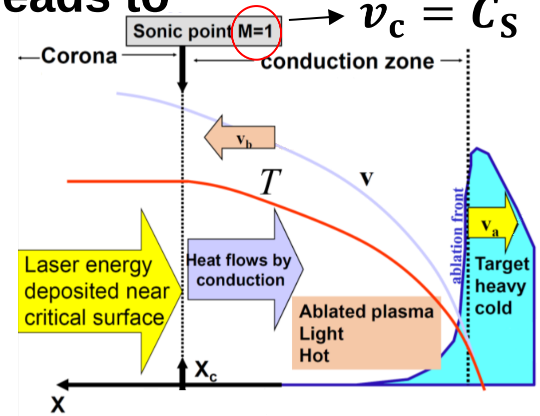
Laser produced ablation pressure

$$\partial_t \epsilon + \partial_x[\vec{v}(\epsilon + p) - \kappa \partial_x T] = \text{source} + \text{sinks} = I\delta(x - x_c) \qquad I \approx \kappa^{-}\left(\frac{\partial T}{\partial x}\right)^{-}$$

- **Solving at steady state in the conduction zone ($x<x_c$) leads to**

$$v\,(\varepsilon + p) \sim \kappa \partial_x T \qquad \text{for } x \le x_{\rm cr}^{-}$$

$$v_c = C_s$$



- **At the sonic point (i.e., critical surface)** $C_{\rm s} \sim \sqrt{p/\rho}$

$$I = [v\,(\varepsilon + p)]_{x_{\rm cr}^{-}} = C_{\rm s}\left(\frac{5}{2}p_{\rm cr} + \rho_{\rm cr}\frac{C_{\rm s}^2}{2}\right) \sim \frac{p_{\rm cr}^{3/2}}{\rho_{\rm cr}^{1/2}}$$

- **The total pressure (static+dynamic) is the ablation pressure**

$$\epsilon = \frac{3}{2}p + \rho\frac{v^2}{2}$$

$$p_{\rm A} = \left[p + \rho v^2\right]_{x = x_{\rm cr}} = 2p_{\rm cr} \sim \left(I\rho_{\rm cr}^{1/2}\right)^{2/3} \sim \left(\frac{I}{\lambda_{\rm L}}\right)^{2/3}$$

$$n_{\rm e}^{\rm cr} = \frac{1.1 \times 10^{21}}{\lambda_{\rm L,\mu m}^2}\,\text{cm}^{-3}$$

- **The laser-produced total (ablation) pressure on target:**

$$\boxed{p_{\rm A}(\text{Mbar}) \approx 83\left(\frac{I_{15}}{\lambda_{\rm L,\mu m}/0.35}\right)^{2/3}}$$

I_{15}: laser intensity in 10^{15}w/cm^2
$\lambda_{\rm L,\mu m}$: laser wavelength in μm

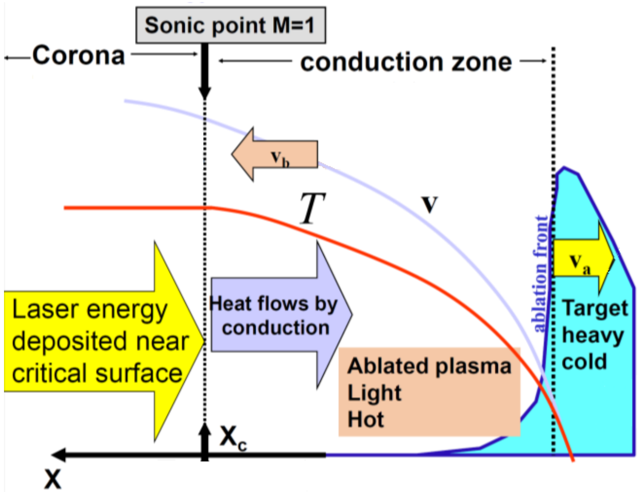
Mass ablation rate induced by the laser

- At steady state, the mass flow across the critical surface must equal the mass flow off the shell (i.e., the mass ablation rate)

$$\dot{m}_{\mathrm{a}} = \rho v = \rho_{\mathrm{cr}} v_{\mathrm{cr}} = \rho_{\mathrm{cr}} C_{\mathrm{s}}^{\mathrm{cr}} = \rho_{\mathrm{cr}} \sqrt{\frac{p_{\mathrm{cr}}}{\rho_{\mathrm{cr}}}} = \sqrt{\rho_{\mathrm{cr}} p_{\mathrm{cr}}}$$

$$\rho_{\mathrm{cr}} \sim \frac{1}{\lambda_{\mathrm{L}}^2} \qquad p_{\mathrm{cr}} \sim \left(\frac{I}{\lambda}\right)^{2/3} \qquad \Rightarrow \dot{m}_{\mathrm{a}} = \frac{I^{1/3}}{\lambda_{\mathrm{L}}^{4/3}}$$

$$\boxed{\dot{m}_{\mathrm{a}} = 3.3 \times 10^5 \frac{I_{15}^{1/3}}{\lambda_{\mathrm{L},\mu\mathrm{m}}^{4/3}} \ \mathrm{g/cm^2 s}}$$



Entropy of an ideal gas/plasma

- **The entropy S is a property of a gas just like p, T, and ρ**

$$S = c_{\mathrm{v}} \ln \left[\frac{p}{\rho^{5/3}} \mathrm{const} \right] = c_{\mathrm{v}} \ln \alpha \qquad \alpha = \mathrm{const} \frac{p}{\rho^{5/3}}$$

- **α is called the "adiabat"**
- **The entropy/adiabat S/α changes through dissipation or heat sources/sinks**

$$\rho \left(\frac{\partial S}{\partial t} + \vec{u} \cdot \nabla S \right) = \frac{DS}{Dt} = \mu \frac{|\nabla \vec{u}|^2}{T} + \frac{\nabla \cdot \kappa \nabla T}{T} + \mathrm{sources/sinks}$$

- **In an ideal gas (no dissipation) and without sources and sinks, the entropy/adiabat is a constant of motion of each fluid element**

$$\frac{DS}{dt} = 0 \Rightarrow S\ ,\ \alpha = \mathrm{const} \Rightarrow p \sim \alpha \rho^{5/3}$$

It is easier to compress a low adiabat (entropy) gas

- **Smaller α -> less work to compress from low to high density**

$$W_{1\to 2} = -\int p dV \sim -\int_{\rho_1}^{\rho_2} \alpha \rho^{5/3} d\left(\frac{M}{\rho}\right) \sim \alpha M \left(\rho_2^{2/3} - \rho_1^{2/3}\right)$$

- **Smaller α -> higher density for the same pressure**

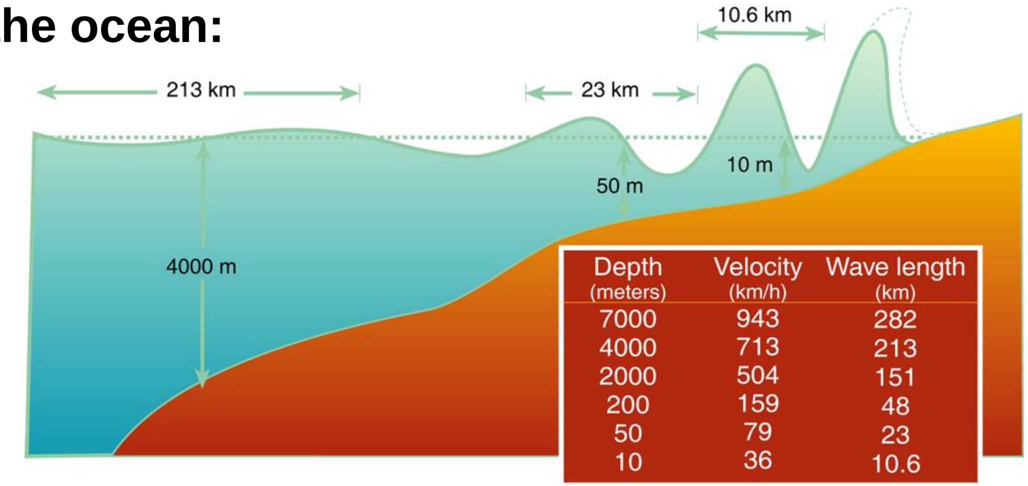
$$\alpha \sim \frac{p}{\rho^{5/3}} \Rightarrow \rho \sim \left(\frac{p}{\alpha}\right)^{3/5}$$

- **In HEDP, the constant in adiabat definition comes from the normalization of the pressure against the Fermi pressure.**
- **When thermal effects are negligible at very high densities, the pressure is proportional to ρ5/3 due to the quantum mechanical effects (degenerate electron gas) just like isentropic flow**

$$\alpha \equiv \frac{p}{p_F} \quad \Rightarrow \alpha_{DT} = \frac{p_{\mathrm{Mbar}}}{2.2\rho_{\mathrm{g/cc}}^{5/3}}$$

A shock is formed due to the increasing sound speed of a compressed gas/plasma

- **Wave in the ocean:**



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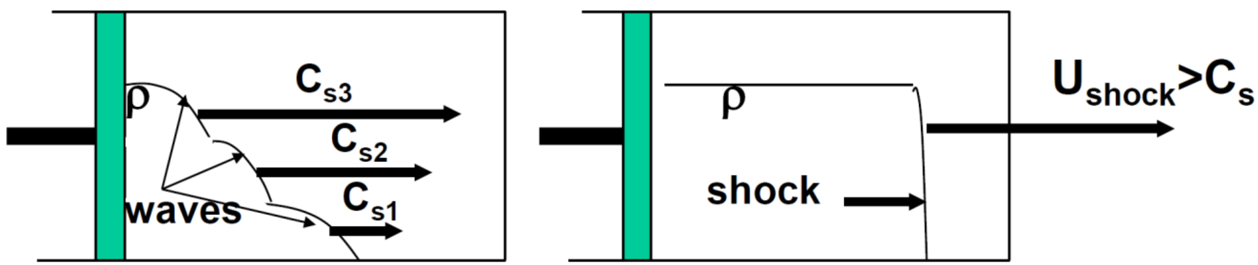


$$C_s \sim \sqrt{\frac{p}{\rho}} \sim \sqrt{\frac{\alpha \rho^{5/3}}{\rho}} \sim \sqrt{\alpha}\rho^{1/3}$$

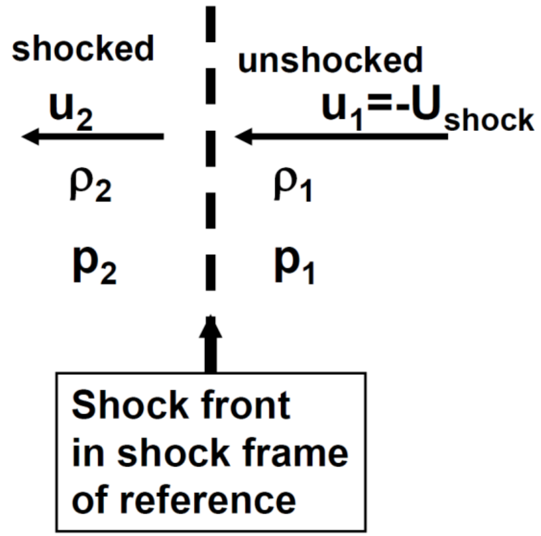
http://neamtic.ioc-unesco.org/tsunami-info/the-cause-of-tsunamis

Rankine-Hugoniot conditions are obtained using conservation of mass, momentum and energy across the shock front

$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$u_1\left(\varepsilon_1 + p_1\right) = u_2\left(\varepsilon_2 + p_2\right)$$



- **Ideal gas/plasma:**

$$\varepsilon = \frac{3}{2}p + \rho\frac{u^2}{2}$$

- **With assigned ρ_1, p_1, and p_2, ρ_2, u_2, and u_1=-U_{shock} can be obtained using Rankine-Hugoniont conditions**

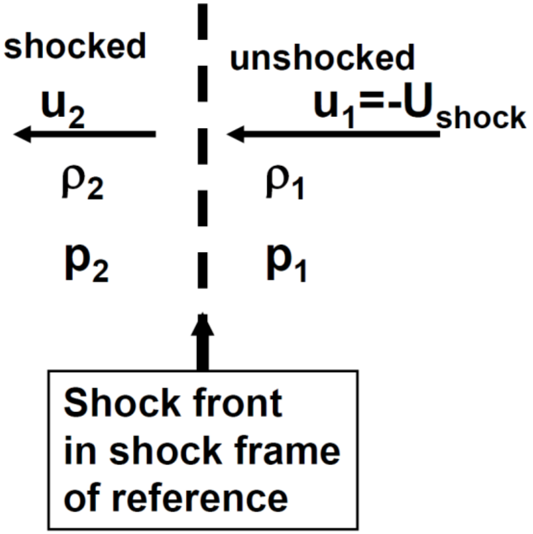
For a strong shock where $p_2 >> p_1$, the R-H conditions are simplified

$$\frac{\rho_2}{\rho_1} \approx 4$$

$$U_{\text{shock}} = -u_1 \approx \sqrt{\frac{4p_2}{3\rho_1}}$$

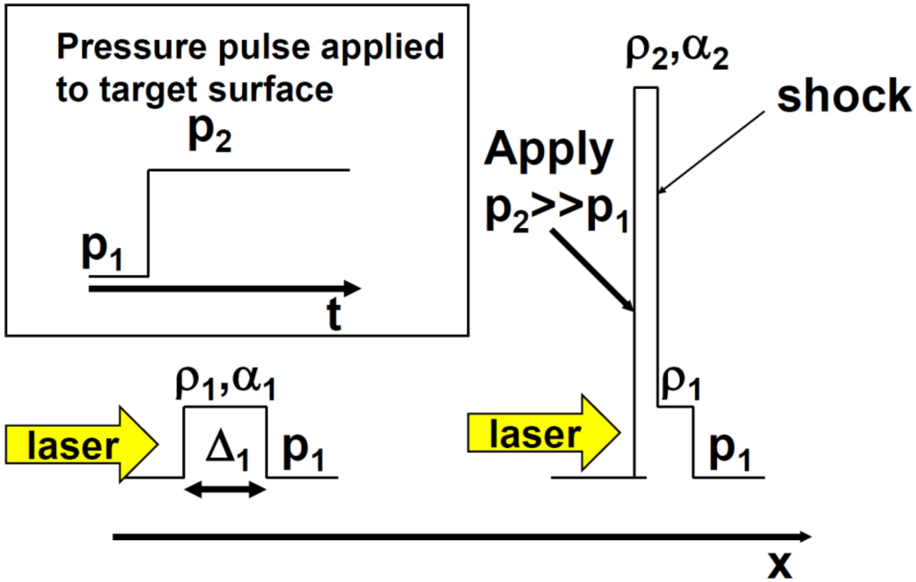
$$u_2 \approx \sqrt{\frac{p_2}{12\rho_1}}$$

$$\frac{\alpha_2}{\alpha_1} = \frac{p_2/\rho_2^{5/3}}{p_1/\rho_1^{5/3}} \approx \frac{1}{4^{5/4}}\frac{p_2}{p_1} >> 1$$



- **The adiabat increases through the shock.**

In an ideal gas/plasma, the adiabat α only raises when a shock is present



- **Post-shock density**

$$\rho_2 \approx 4\rho_1$$

- **Adiabat set by the shock for DT:**

$$\alpha_2 \approx \frac{p_{2,\mathrm{Mbar}}}{2.2\left(4\rho_{1,\mathrm{g/cc}}\right)^{5/3}}$$

- **Time required for the shock to reach the rear target surface (shock break-out time, t_{sb})**

$$t_{\mathrm{sb}} = \frac{\Delta_1}{u_{\mathrm{shock}}} = \Delta_1\sqrt{\frac{3\rho_1}{4p_2}} \propto \sqrt{\frac{1}{\alpha_2\rho_1^{2/3}}}$$

$$U_{\mathrm{shock}} = -u_1 \approx \sqrt{\frac{4p_2}{3\rho_1}}$$

Higher laser intensity leads to higher adiabat

- **For a cryogenic solid DT target at 18 k:**

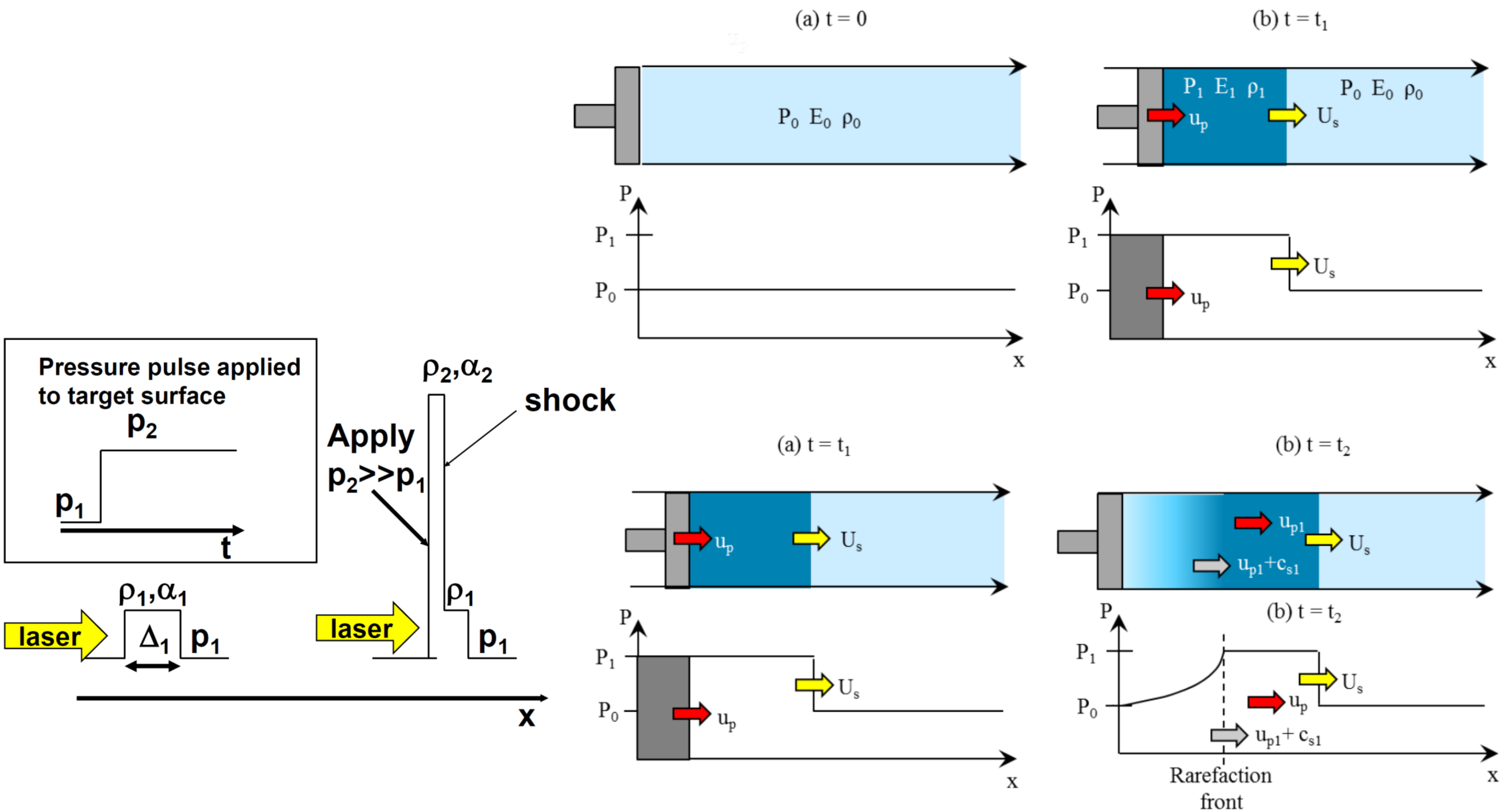
$$\rho_1 = 0.25\ \mathrm{g/cc} \qquad \alpha = \frac{p_{\mathrm{Mbar}}}{2.2} \qquad p \approx 83\left(\frac{I_{15}}{\lambda_{\mu\mathrm{m}}/0.35}\right)^{2/3}$$

$$I \approx 4.3\times 10^{12}\ \mathrm{w/cm^2} \quad \Rightarrow \quad p = 2.2\ \mathrm{Mbar} \quad \Rightarrow \quad \alpha = 1$$

$$I \approx 1.2\times 10^{13}\ \mathrm{w/cm^2} \quad \Rightarrow \quad p = 4.4\ \mathrm{Mbar} \quad \Rightarrow \quad \alpha = 2$$

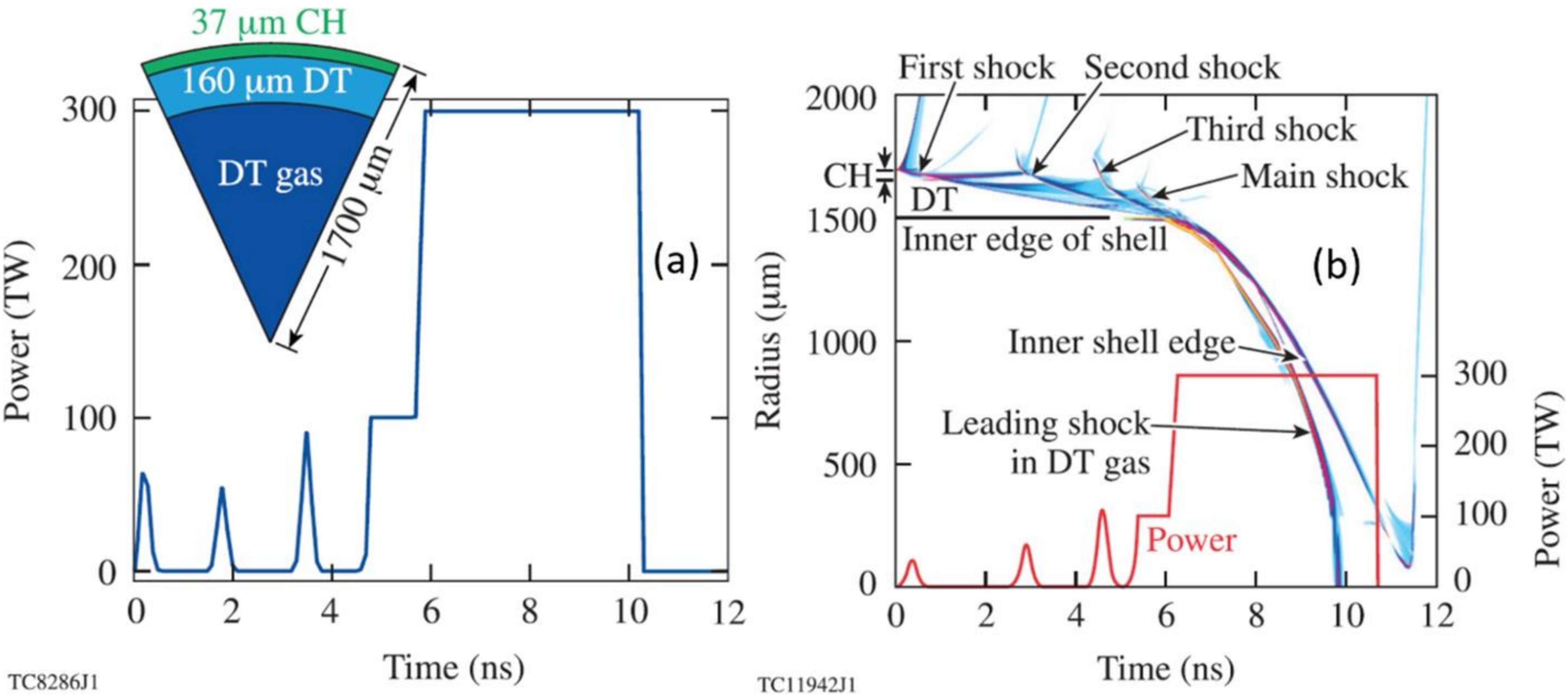
$$I \approx 2.2\times 10^{13}\ \mathrm{w/cm^2} \quad \Rightarrow \quad p = 6.6\ \mathrm{Mbar} \quad \Rightarrow \quad \alpha = 3$$

A shock or a rarefaction wave may be formed depending on the driving force from the piston



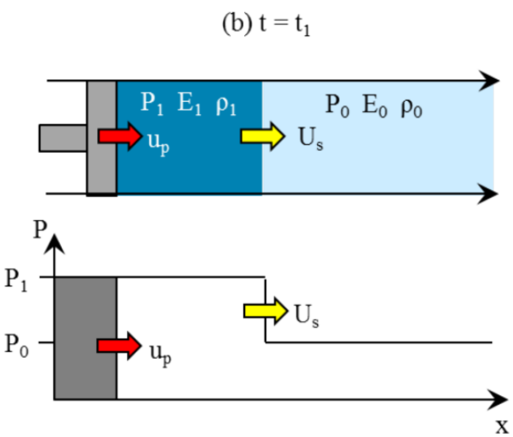
Michelle Colleen Gregor, PhD Thesis, U of Rochester, 2017

Multiple shock can be generated with multiple pickets

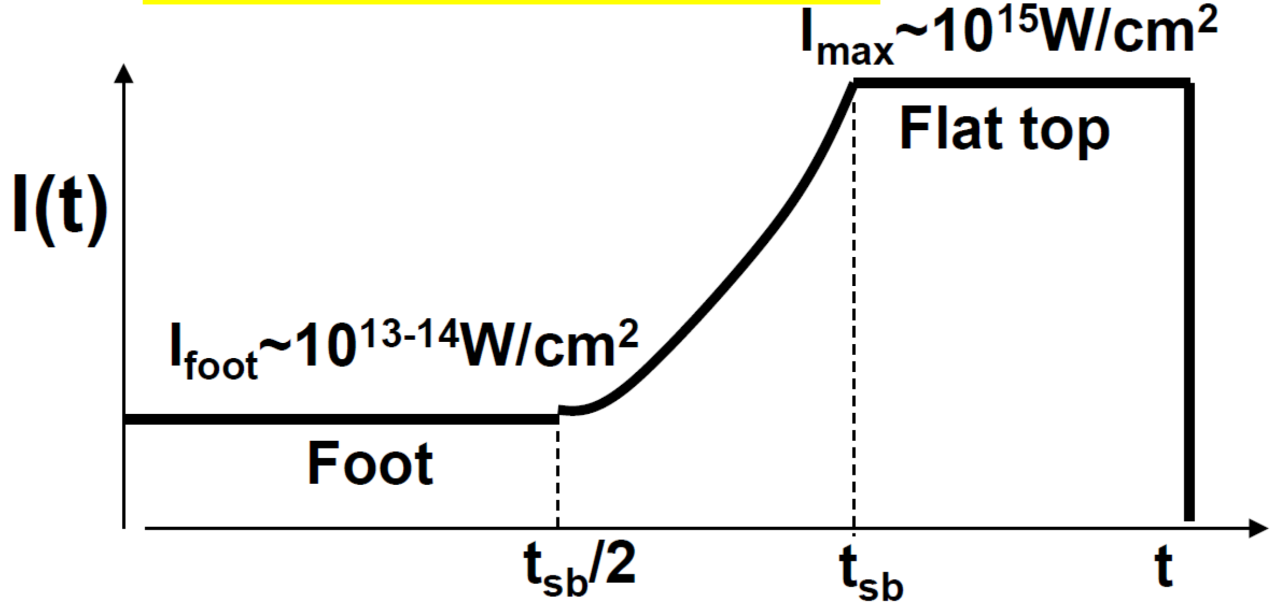


T. Ditmire, etc., J. Fusion Energy **42**, 27 (2023)
R. S. Craxton, etc., Phys. Plasmas 22, 110501 (2015)

The pressure must be “slowly” increased after the first shock to avoid raising the adiabat

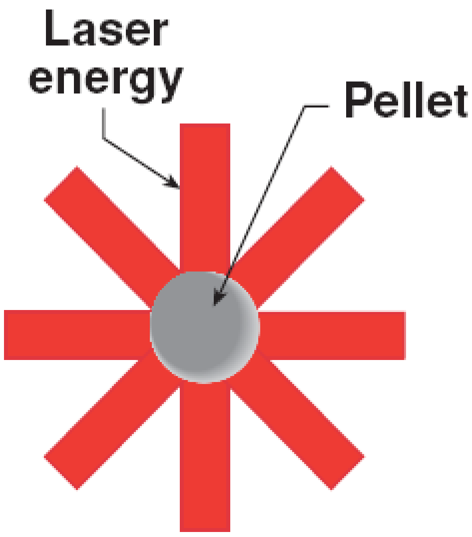


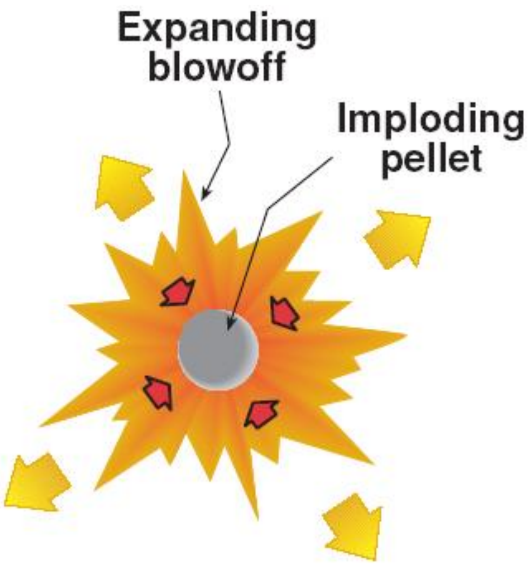
Laser pulse shape



- **After the foot of the laser pulse, the laser intensity must be raised starting at about $0.5t_{sb}$ and reach its peak at about t_{sb}**
- **Reaching I_{max} at t_{sb} prevents a rarefaction/decompression wave to propagate back from the rear target surface and decompress the target.**

Most of the absorbed laser energy goes into the kinetic and thermal energy of the expanding blow-off plasma





- **The rocket model:**

Shell Newton's law

$$M\frac{du}{dt} = -4\pi R^2 p_\mathrm{a}$$

Shell mass decreases due to ablation

$$\frac{dM}{dt} = -4\pi R^2 \dot{m}_\mathrm{a}$$

p_a =ablation rate x exhaust velocity

$$p_\mathrm{a} = \dot{m}_\mathrm{a} u_\mathrm{exhaust}$$

Shell velocity can be obtained by integrating the rocket equations

$$M\frac{du}{dt} = -4\pi R^2 p_{\rm a} \qquad \frac{dM}{dt} = -4\pi R^2 \dot{m}_{\rm a} \qquad p_{\rm a} = \dot{m}_{\rm a} u_{\rm exhaust}$$

$$\begin{aligned}
M\frac{du}{dt} &= -4\pi R^2 p_{\rm a} = -4\pi R^2 \dot{m}_{\rm a} u_{\rm exhaust} \\
&= -4\pi R^2 u_{\rm exhaust}\frac{1}{-4\pi R^2}\frac{dM}{dt} \\
&= u_{\rm exhaust}\frac{dM}{dt} \\
\int du &= u_{\rm exhaust}\int\frac{dM}{M} \\
u_{\rm shell} &= u_{\rm exhaust}\ln\left(\frac{M_{\rm initial}}{M_{\rm final}}\right)
\end{aligned}$$

$$E_{\rm kin}^{\rm shell} = \frac{M_{\rm final}}{2}u_{\rm shell}^2 = \frac{M_{\rm final}}{2}\left[u_{\rm exhaust}\ln\left(\frac{M_{\rm initial}}{M_{\rm final}}\right)\right]^2$$

$$E_{\rm exhaust} = (M_{\rm initial} - M_{\rm final})\left(\frac{u_{\rm exhaust}^2}{2} + \frac{3}{2}\frac{p_{\rm ex}}{\rho_{\rm ex}}\right)$$

(dynamic + static)

$\boldsymbol{M_{\rm exhaust} = M_{\rm initial} - M_{\rm final}}$

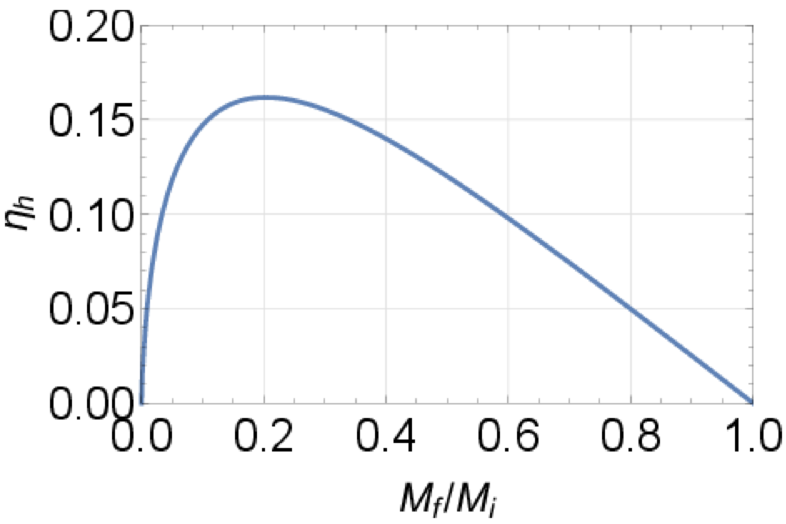
Maximum hydro efficiency is about 15%

$$E_{\mathrm{kin}}^{\mathrm{shell}} = \frac{M_{\mathrm{final}}}{2} u_{\mathrm{shell}}^2 = \frac{M_{\mathrm{final}}}{2} \left[u_{\mathrm{exhaust}} \ln \left(\frac{M_{\mathrm{initial}}}{M_{\mathrm{final}}} \right) \right]^2$$

$$E_{\mathrm{exhaust}} = (M_{\mathrm{initial}} - M_{\mathrm{final}}) \left(\frac{u_{\mathrm{exhaust}}^2}{2} + \frac{3}{2} \frac{p_{\mathrm{ex}}}{\rho_{\mathrm{ex}}} \right)$$

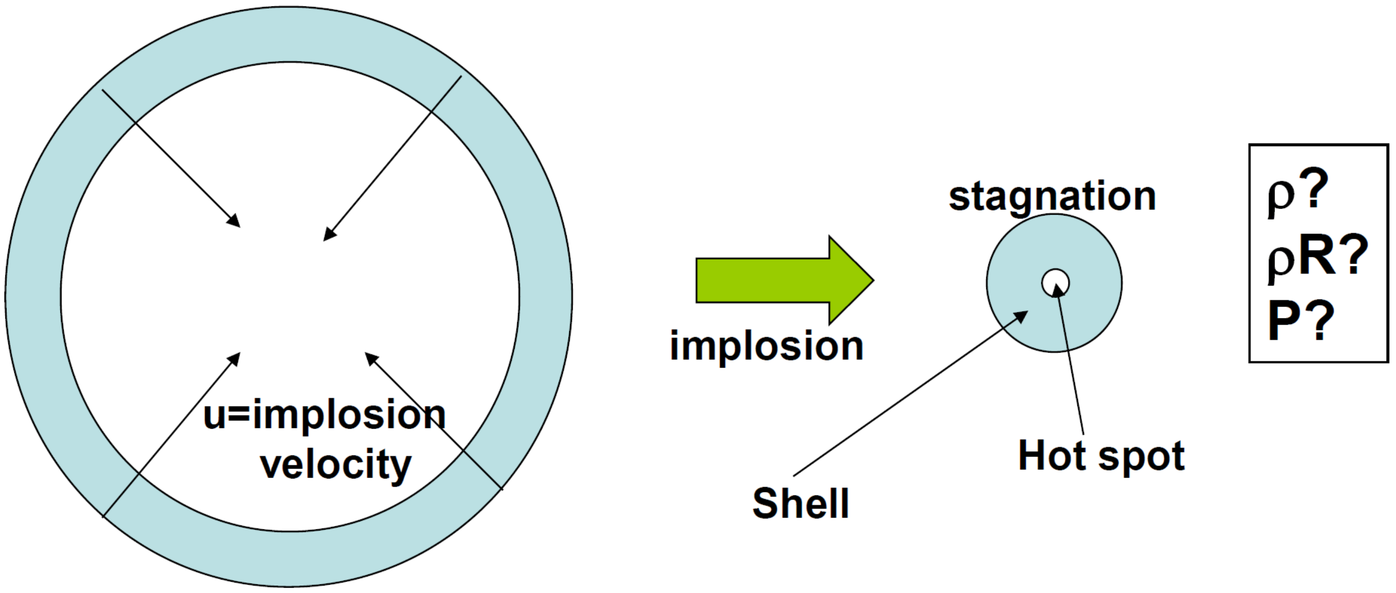
Take $u_{\mathrm{exhaust}}^2 \approx C_{\mathrm{s}}^2 \approx \dfrac{p_{\mathrm{ex}}}{\rho_{\mathrm{ex}}}$

$$\eta_{\mathrm{h}} = \frac{E_{\mathrm{kin}}^{\mathrm{shell}}}{E_{\mathrm{exhaust}}} = \frac{M_{\mathrm{f}}/M_{\mathrm{i}} \left[\ln \left(M_{\mathrm{f}}/M_{\mathrm{i}} \right) \right]^2}{4 \left(1 - M_{\mathrm{f}}/M_{\mathrm{i}} \right)}$$



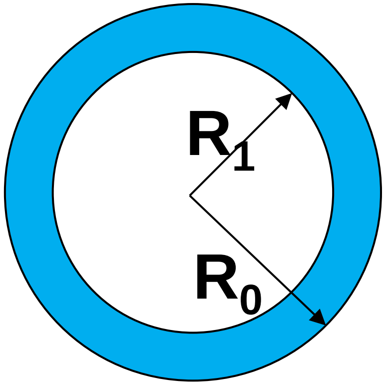
One dimensional implosion hydrodynamics

- **What are the stagnation values of the relevant hydrodynamic properties?**

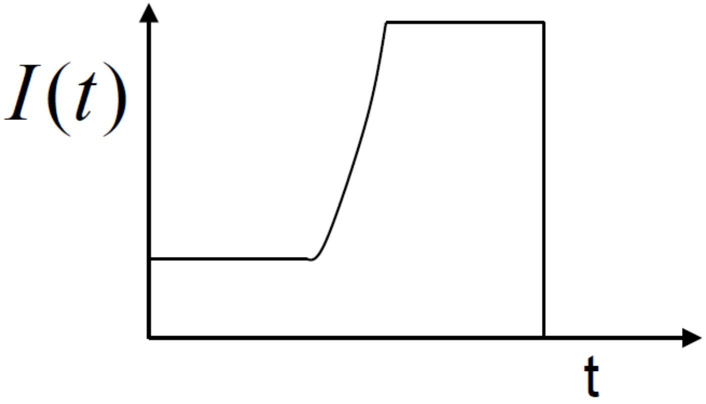


What variables can be controlled?

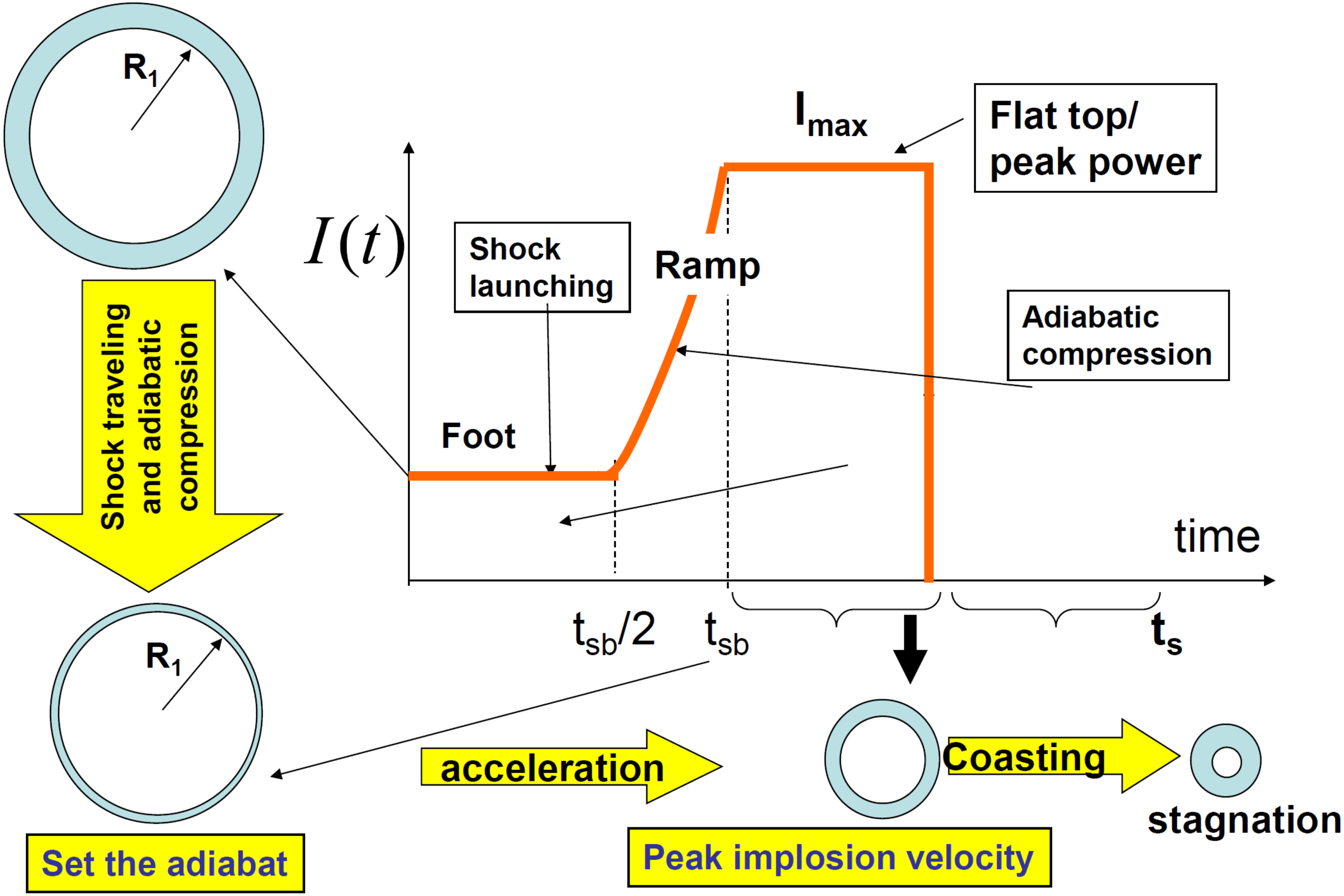
- Shell outer radius R_0 at time t=0
- Shell inner radius R_1 at time t=0
- The total laser energy on target
- Adiabat α through shocks
- Applied pressure p(t) through the pulse shape I(t)



$$\alpha \sim \frac{p}{\rho^{5/3}} \qquad p \sim I^{2/3}$$

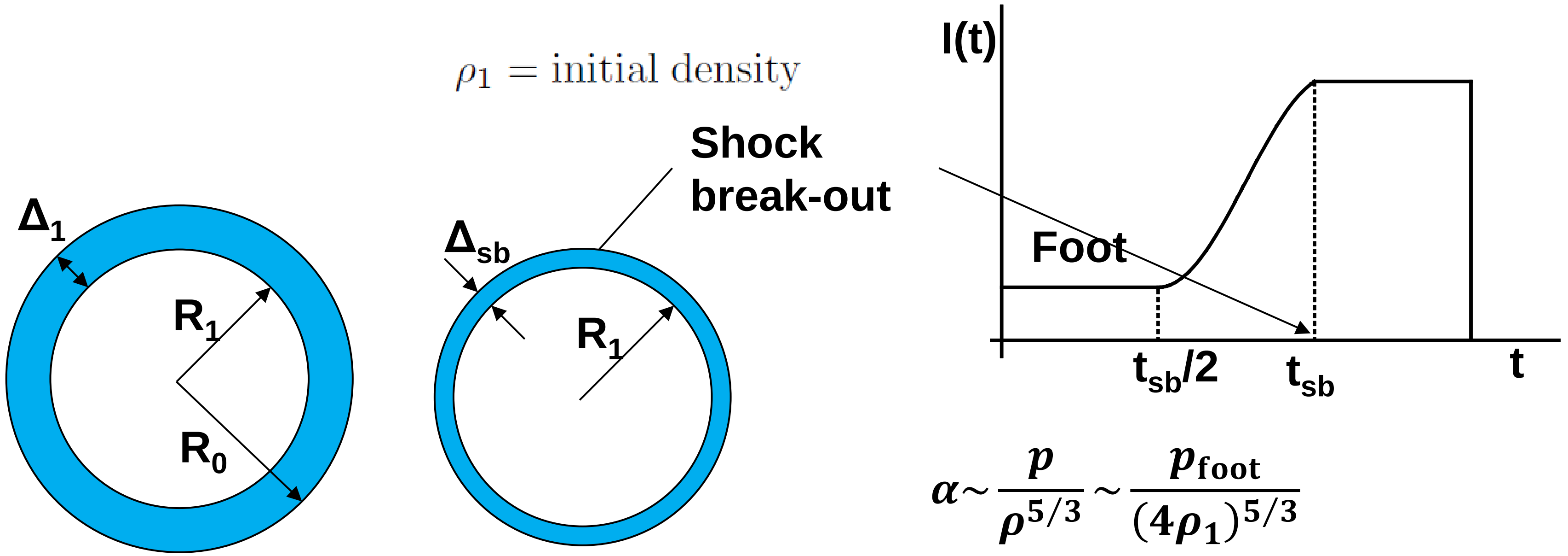


There are three stages in the laser pulse: foot, ramp, and flat top



The adiabat is set by the shock launched by the foot of the laser pulse

$\rho_1 = \text{initial density}$



$$\alpha \sim \frac{p}{\rho^{5/3}} \sim \frac{p_{\text{foot}}}{(4\rho_1)^{5/3}}$$

$$\rho_{sb} \sim \left(\frac{p_{\max}}{\alpha}\right)^{5/3} \doteq 4\rho_1 \left(\frac{p_{\max}}{p_{\text{foot}}}\right)^{5/3}$$

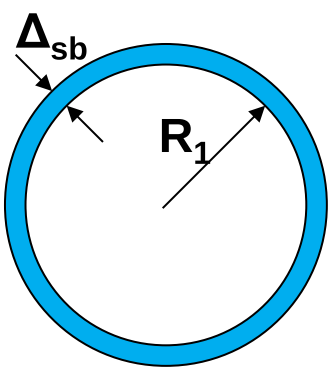
$$m_{sb} \sim 4\pi {R_1}^2 \Delta_1 \rho_1 = 4\pi {R_1}^2 \Delta_{sb} \rho_{sb}$$

$$\Delta_1 \rho_1 = \Delta_{sb} \rho_{sb}$$

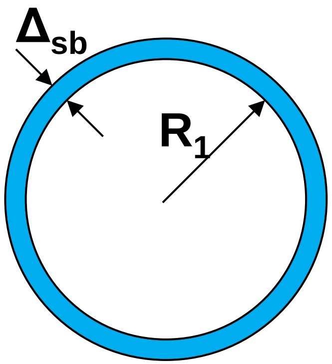
$$\Delta_{sb} = \Delta_1 \frac{\rho_1}{\rho_{sb}} \sim \Delta_1 \frac{\rho_1}{4\rho_1 \left(\frac{p_{\max}}{p_{\text{foot}}}\right)^{3/5}} = \frac{\Delta_1}{4} \left(\frac{p_{\text{foot}}}{p_{\max}}\right)^{3/5}$$

Density and thickness at shock break out time are expressed in laser intensity

- Use $p \sim I^{2/3}$
- Shell density $\rho_{\mathrm{sb}} \sim \rho_1 \left(\frac{p_{\mathrm{max}}}{p_{\mathrm{foot}}}\right)^{5/3} = 4\rho_1 \left(\frac{I_{\mathrm{max}}}{I_{\mathrm{foot}}}\right)^{2/5}$
- Shell thickness $\Delta_{\mathrm{sb}} \sim \frac{\Delta_1}{4} \left(\frac{p_{\mathrm{foot}}}{p_{\mathrm{max}}}\right)^{3/5} = \frac{\Delta_1}{4} \left(\frac{I_{\mathrm{foot}}}{I_{\mathrm{max}}}\right)^{2/5}$
- Shell radius $R \approx R_1$



The aspect ratio is maximum at shock break out



$$\text{Aspect ratio} \equiv \frac{R}{\Delta}$$

$$A_1 = \frac{R_1}{\Delta_1} = \text{initial aspect ratio}$$

$$A_{\rm sb} = IFAR = \frac{R_1}{\Delta_{\rm sb}} = 4A_1\left(\frac{I_{\rm max}}{I_{\rm foot}}\right)^{2/5}$$

$$A_{\rm sb} = A_{\rm max}$$

IFAR ≡ Maximum In-Flight-Aspect-Ratio = aspect ratio at shock break-out

The IFAR scales with the Mach number

- The shell kinetic energy = the work done on the shell

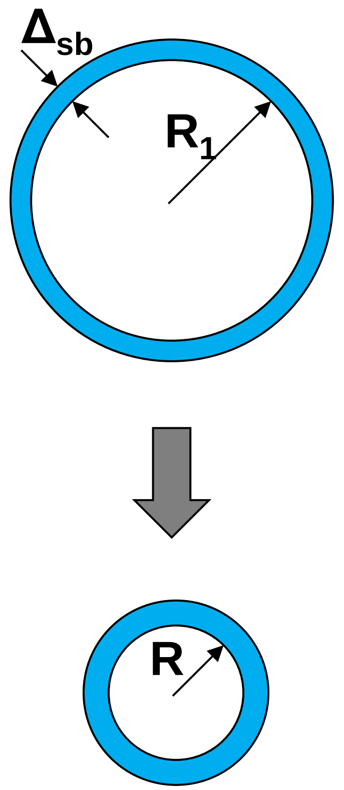
$$Mu_{max}{}^2 \sim -\int_{R_1}^{R} pr^2 dr \sim p\left(R_1{}^3 - R^3\right) \approx pR_1{}^3 \qquad R_1{}^3 = \frac{Mu_{max}{}^2}{p}$$

$$M \sim \rho_{sb}\Delta_{sb}R_1{}^2 \qquad \Delta_{sb} \sim \frac{M}{\rho_{sb}R_1{}^2} \qquad R_1 >> R$$

$$IFAR = \frac{R_1}{\Delta_{sb}} = \frac{R_1}{\dfrac{M}{\rho_{sb}R_1{}^2}} = \frac{\rho_{sb}R_1{}^3}{M} = \frac{\rho_{sb}}{M}\frac{Mu_{max}{}^2}{p}$$

$$= \frac{u_{max}{}^2}{p/\rho_{sb}} \sim Mach_{max}{}^2$$

$$\alpha \sim \frac{p}{\rho^{5/3}} \qquad p \sim I^{2/3} \qquad \boxed{IFAR \sim \frac{u_{max}{}^2}{\alpha^{3/5} I^{4/15}}}$$



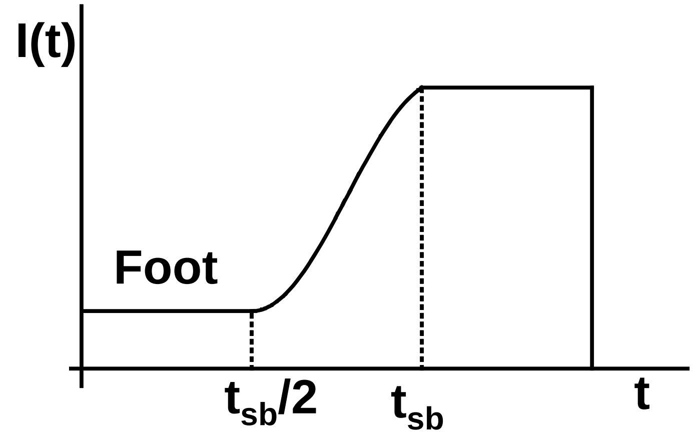
The final implosion velocity can be found using IFAR

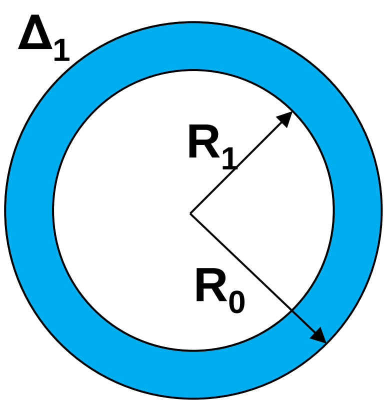
$$u_{\max}{}^{2} \sim IFAR \times \alpha^{3/5} I^{4/15}$$

$$IFAR = 4A_1 \left(\frac{I_{\max}}{I_{\text{foot}}}\right)^{2/5}$$

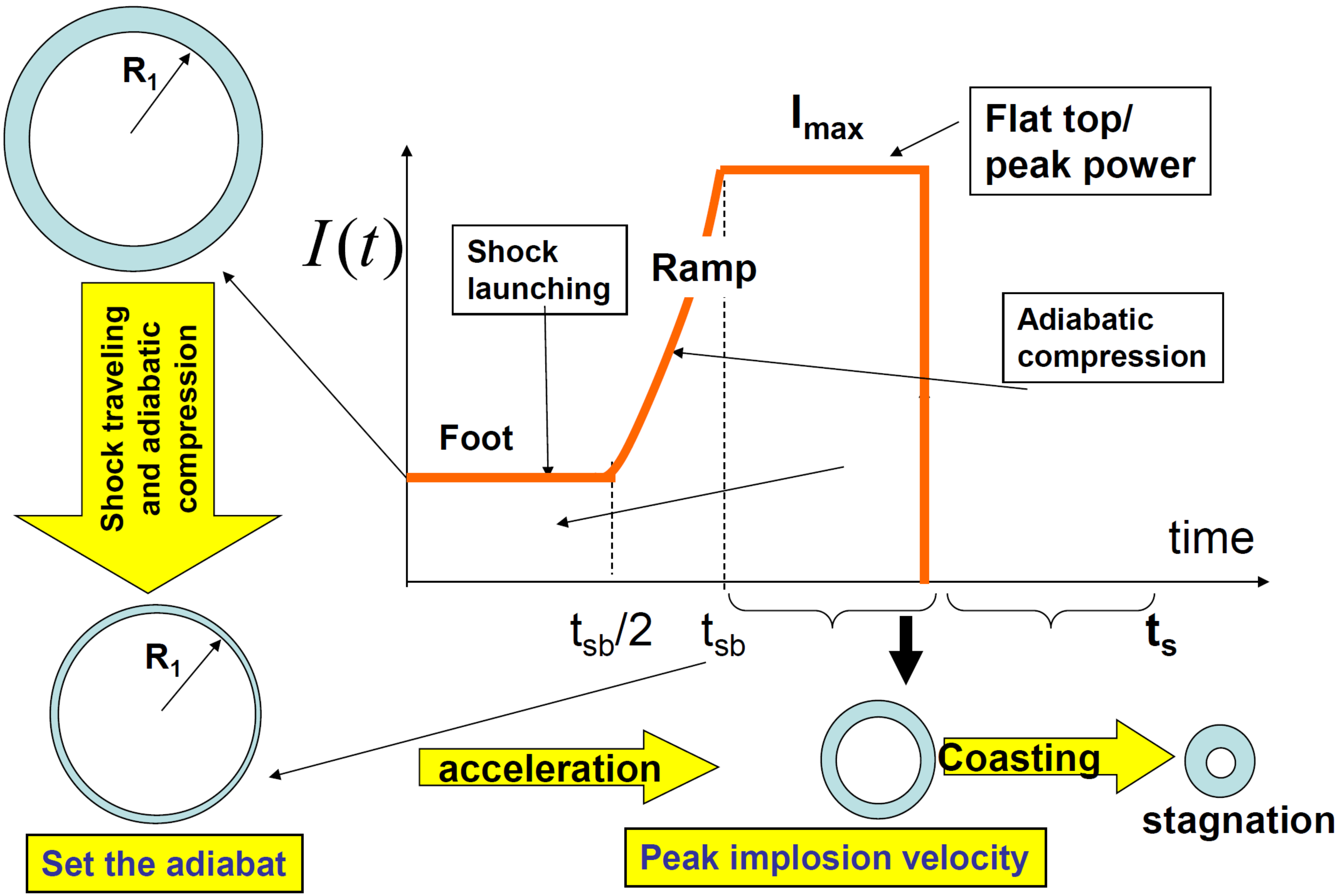
$$A_1 = \frac{R_1}{\Delta_1}$$

$$\boxed{u_{\max,\text{cm}/s} \approx 10^7 \sqrt{0.7 A_1 \alpha^{3/5} I_{15,\max}{}^{4/15} \left(\frac{I_{\max}}{I_{\text{foot}}}\right)^{2/5}}}$$





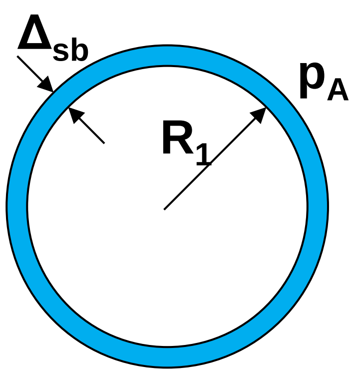
There are three stages in the laser pulse: foot, ramp, and flat top



A simple implosion theory can be derived in the limit of infinite initial aspect ratio

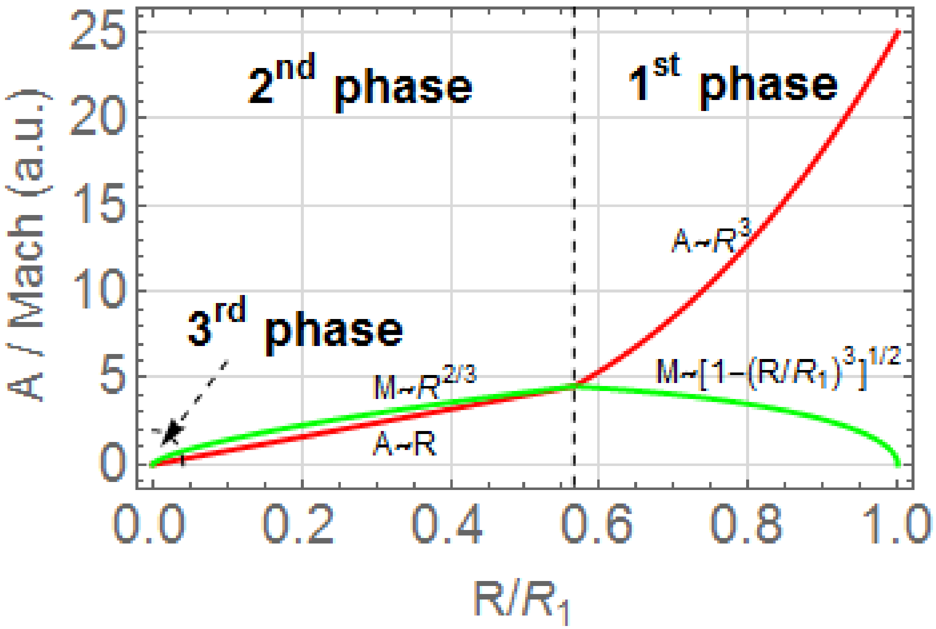
- **Start from a high aspect ratio shell (thin shell) at the beginning of the acceleration phase**
  - **Constant ablated pressure**
  - **The adiabat is set and kept fixed by the first and the only shock**

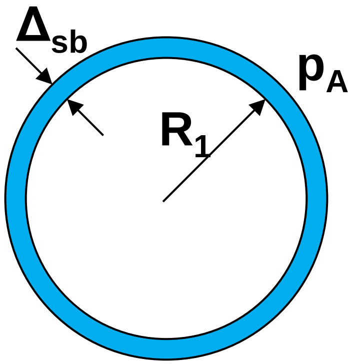
$$IFAR = A_{\mathbf{sb}} = \frac{R_1}{\Delta_{\mathbf{sb}}} \gg 1$$



Basko and Meyer-ter-vehn, Phys. Rev. Lett. *88*, 244502-1, 2002

The implosion are divided in 3 phases after the shock break out





- 1st phase: acceleration
- 2nd phase: coasting
- 3rd phase: stagnation

The shell density is constant

- **Shell expansion/contraction:** $$t_{\rm ex} \sim \frac{\Delta}{C_{\rm s}}$$
- **Implosion time:** $$t_{\rm i} \sim \frac{R}{u_{\rm i}}$$

$$\frac{t_{\rm i}}{t_{\rm ex}} \sim \frac{R}{\Delta}\frac{C_{\rm s}}{u_{\rm i}} = \frac{A}{Mach} \qquad A = \frac{R}{\Delta} \qquad Mach = \frac{u}{C_{\rm s}}$$

- **In the acceleration phase** $A \sim Mach^2$

$$\frac{t_{\rm i}}{t_{\rm ex}} \sim \frac{A}{Mach} \sim Mach \sim \sqrt{A} >> 1 \ \Rightarrow\ \rho \approx \text{const}$$

- **From mass conservation:**

$$M \sim 4\pi R^2 \Delta\rho \ \Rightarrow\ \Delta \sim R^{-2} \qquad A = \frac{R}{\Delta} \sim R^3 \ \Rightarrow\ A = A_{\rm sb}\left(\frac{R}{R_1}\right)^3$$

The shell density is constant

- **Shell expansion/contraction:** $t_{\mathrm{ex}} \sim \frac{\Delta}{C_s}$
- **Implosion time:** $t_{\mathrm{i}} \sim \frac{R}{u_{\mathrm{i}}}$

$$\frac{t_{\mathrm{i}}}{t_{\mathrm{ex}}} \sim \frac{R}{\Delta}\frac{C_s}{u_{\mathrm{i}}} = \frac{A}{Mach} \qquad A = \frac{R}{\Delta} \qquad Mach = \frac{u}{C_s}$$

- **In the acceleration phase** $A \sim Mach^2$ $\quad IFAR \sim {Mach_{max}}^2$ (p29)

$$\frac{t_{\mathrm{i}}}{t_{\mathrm{ex}}} \sim \frac{A}{Mach} \sim Mach \sim \sqrt{A} \gg 1 \quad \Rightarrow \rho \approx \mathrm{const}$$ **(Thin shell)**

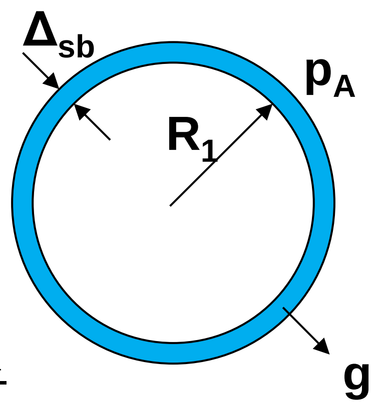
(implosion time >> expansion/contraction time)

- **From mass conservation:**

$$M \sim 4\pi R^2 \Delta\rho \quad \Rightarrow \quad \Delta \sim R^{-2} \qquad\qquad A = \frac{R}{\Delta} \sim R^3 \Rightarrow \quad A = A_{\mathrm{sb}}\left(\frac{R}{R_1}\right)^3$$

The shell density is constant

- **Shell expansion/contraction:** $t_{\mathrm{ex}} \sim \dfrac{\Delta_{\mathrm{sb}}}{C_{\mathrm{s}}}$
- **Implosion time:** $t_{\mathrm{imp}} \sim \dfrac{R_1}{u_{\mathrm{i}}}$

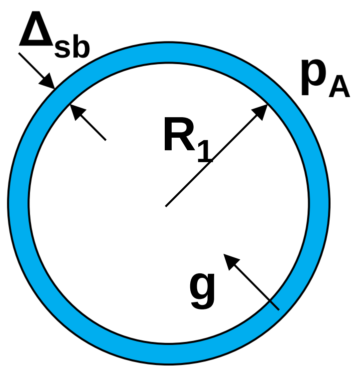


$$\frac{t_{imp}}{t_{ex}} \sim \frac{R_1}{\Delta_{sb}} \frac{C_s}{u_i} = \frac{A_{sb}}{Mach} \gg 1 \qquad A_{sb} = \frac{R_1}{\Delta_{sb}} \qquad Mach = \frac{u_{\mathrm{i}}}{C_{\mathrm{s}}}$$

- **The pressure and the density are constant throughout the whole shell.**

The shell density is constant

- **In the shell frame of reference:**



$$\rho(\partial_t \vec{w} + \vec{w}\cdot\nabla\,\vec{w}) = -\nabla p + \rho g\hat{r}$$

Neglect the first two term (check later) $\Rightarrow \frac{dp}{dr} = -\rho\ddot{R}$

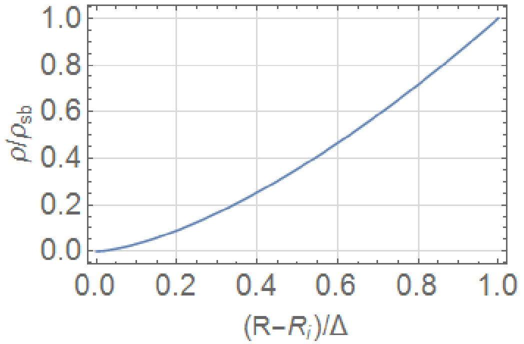
Use $p = \alpha_0\rho^{5/3}$ **and integrate along r:**

$$\alpha_0\frac{d\rho^{5/3}}{dr} = -\rho\ddot{R} \quad\Rightarrow\quad \alpha_0\frac{d\rho^{5/3}}{\rho} = -\ddot{R}dr \quad\Rightarrow\quad \alpha_0\frac{5}{3}\int_{@R_i}^{@R}\frac{\rho^{2/3}}{\rho}d\rho = -\ddot{R}(t)\int_{R_i}^{R}dr$$

$$\rho = \rho_{sb}\left(\frac{R-R_i}{\Delta}\right)^{3/2}$$

$$\text{where } \Delta = -\frac{5}{2}\frac{\alpha_0{\rho_{sb}}^{2/3}}{\ddot{R}} = -\frac{3}{2}\frac{5}{3}\frac{p_A}{\rho_{sb}}\frac{1}{\ddot{R}}$$

$$= -\frac{3}{2}\frac{C_s}{\ddot{R}(t)}$$



The requirement of the 1st phase is obtained using mass conservation

- **Mass conservation:**

$$m = \int_{R_i}^{R_i+\Delta} \rho r^2 dr = \rho_{\mathrm{sb}} \int_{R_i}^{R_i+\Delta} \left(\frac{r-R_i}{\Delta}\right)^{3/2} r^2 dr \qquad \rho = \rho_{\mathrm{sb}} \left(\frac{R-R_i}{\Delta}\right)^{3/2}$$

$$\simeq \rho_{sb} {R_i}^2 \Delta \int_{R_i}^{R_i+\Delta} \left(\frac{r-R_i}{\Delta}\right)^{3/2} d\left(\frac{r-R_i}{\Delta}\right) = \frac{2}{5}\rho_{sb} {R_i}^2 \Delta \sim \frac{2}{5}\rho_{sb} R^2 \Delta$$

$$\Delta = \frac{5}{2}\frac{m}{\rho_{\mathrm{sb}} R^2} \Rightarrow \dot{\Delta} = \frac{5}{2}\frac{m}{\rho_{\mathrm{sb}}}(-2)\frac{\dot{R}}{R^3} = -2\frac{\dot{R}}{R}\Delta = -2\frac{v}{A} \qquad \dot{\Delta} = -2\frac{v}{A} \qquad t_{\mathrm{imp}} \sim \frac{R_1}{u_{\mathrm{i}}}$$

$$\rho(\partial_{\mathrm{t}}\vec{w} + \vec{w}\cdot\nabla\vec{w}) = -\nabla p + \rho g \hat{r} \qquad \vec{w} \sim \dot{\Delta} \qquad \partial_t \sim 1/t_{\mathrm{imp}} \qquad \nabla \sim 1/\Delta$$

$$\Rightarrow \rho\left(\frac{\dot{\Delta}}{t_{\mathrm{imp}}} + \frac{\dot{\Delta}^2}{\Delta}\right) \sim -\frac{p}{\Delta} + \rho\ddot{R} \qquad \rho\frac{\dot{\Delta}}{t_{\mathrm{imp}}} \sim \rho\frac{v}{A t_{\mathrm{imp}}} \sim \rho\frac{v^2}{AR} \qquad \rho\frac{\dot{\Delta}^2}{\Delta} \sim \rho\frac{v^2}{A^2\Delta} \sim \rho\frac{v^2}{AR}$$

$$\rho\frac{\dot{\Delta}}{t_{\mathrm{imp}}} \Big/ \frac{p}{\Delta} \sim \rho\frac{v^2}{AR}\frac{\Delta}{p} \sim \frac{v^2}{{C_s}^2}\frac{1}{A^2} = \frac{Mach^2}{A^2}$$

- $Mach \ll A$ **is the requirement for the 1st phase**

Aspect ratio and Mach number are functions of radius

$$A = \frac{R}{\Delta} = R^3\left(\frac{2}{5}\frac{\rho_{sb}}{m}\right) \propto R^3 \Rightarrow$$

$$\boxed{A = A_{sb}\left(\frac{R}{R_1}\right)^3 = \mathrm{IFAR}\left(\frac{R}{R_1}\right)^3}$$

$$\Delta = -\frac{3}{2}\frac{C_s{}^2}{\ddot{R}} \quad \text{(p36)} \Rightarrow \quad \ddot{R} = -\frac{3}{2}\frac{C_s{}^2}{\Delta} = -\frac{3}{2}\left(\frac{2}{5}\frac{\rho_{sb}R^2}{m}\right)\left(\frac{5}{3}\frac{p_A}{\rho_{sb}}\right)$$

$$\dot{R}\frac{d\dot{R}}{\mathrm{dt}} = -\frac{p_A}{m}R^2\dot{R} \qquad \frac{1}{2}\int d\dot{R}^2 = -\frac{p_A}{m}\int R^2\mathrm{dR} \qquad \dot{R}^2 = \frac{2}{3}\frac{p_A R_1{}^3}{m}\left[1-\left(\frac{R}{R_1}\right)^3\right]$$

$$Mach^2 = \frac{\dot{R}^2}{C_s{}^2} = \frac{2}{3}\frac{p_A R_1{}^3}{m}\frac{3}{5}\frac{\rho_{sb}}{p_A}\left[1-\left(\frac{R}{R_1}\right)^3\right] = \frac{2}{5}\frac{R_1{}^3\rho_{sb}}{m}\left[1-\left(\frac{R}{R_1}\right)^3\right]$$

$$\boxed{Mach = Mach_{max}\left[1-\left(\frac{R}{R_1}\right)^3\right]^{1/2}}$$

$$Mach_{\max}{}^2 = \frac{2}{5}\frac{R_1{}^3\rho_{sb}}{m} = \frac{2}{5}\frac{5}{2}\frac{R_1{}^3\rho_{sb}}{\rho_{sb}R_1{}^2\Delta_{sb}} = \frac{R_1}{\Delta_{sb}} = A_{sb}$$

The model breaks down when A~Mach

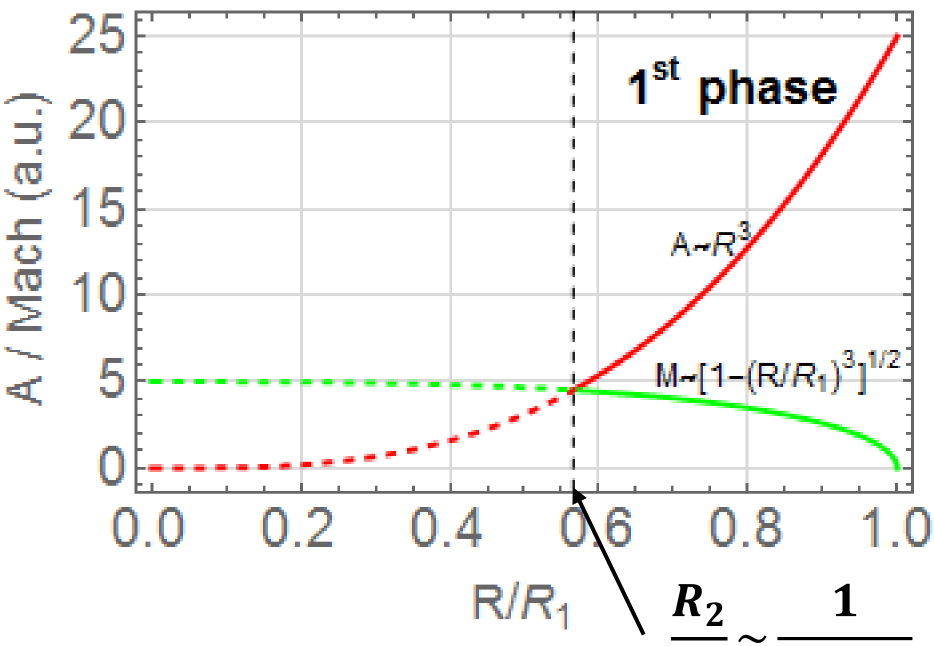
$\boldsymbol{A \sim \mathrm{Mach}}$ $$A_{\mathrm{sb}}\left(\frac{R}{R_1}\right)^3 \sim \mathrm{Mach}_{\max}\left[1-\left(\frac{R}{R_1}\right)^3\right]^{1/2} = \sqrt{A_{\mathrm{sb}}}\left[1-\left(\frac{R}{R_1}\right)^3\right]^{1/2}$$

$$A_{\mathrm{sb}}\left(\frac{R}{R_1}\right)^6 \sim 1-\left(\frac{R}{R_1}\right)^3 \quad \Rightarrow \quad A_{\mathrm{sb}}\left(\frac{R}{R_1}\right)^6 + \left(\frac{R}{R_1}\right)^3 - 1 \sim 0$$

$$\left(\frac{R}{R_1}\right)^3 \sim \frac{-1 \pm \sqrt{1+4A_{\mathrm{sb}}}}{2A_{\mathrm{sb}}} \sim \frac{-1 \pm 2\sqrt{A_{\mathrm{sb}}}}{2A_{\mathrm{sb}}} \sim \frac{1}{\sqrt{A_{\mathrm{sb}}}} \qquad \because \sqrt{A_{\mathrm{sb}}} >> 1$$

$$\frac{R}{R_1} \sim \frac{1}{{A_{\mathrm{sb}}}^{1/6}} \ll 1 \qquad A = A_{\mathrm{sb}}\left(\frac{R}{R_1}\right)^3 \sim \sqrt{A_{\mathrm{sb}}} \gg 1$$

Summary of phase 1 (acceleration phase)



$$\frac{R_2}{R_1} \sim \frac{1}{{A_{sb}}^{1/6}}$$

$$\boxed{\frac{1}{{A_{sb}}^{1/6}} < \frac{R}{R_1} \leq 1}$$

$$\boxed{A = A_{sb}\left(\frac{R}{R_1}\right)^3 = \mathrm{IFAR}\left(\frac{R}{R_1}\right)^3}$$

$$\boxed{Mach = Mach_{max}\left[1 - \left(\frac{R}{R_1}\right)^3\right]^{1/2}}$$

$$Mach_2 \simeq \mathrm{Mach_{max}}\left(1 - \frac{1}{\sqrt{A_{sb}}}\right)^{1/2} \simeq \mathrm{Mach_{max}} = \sqrt{A_{sb}} \qquad A_2 \sim \sqrt{A_{sb}}$$

The 2nd phase starts when R < R_2

- A decreases as R decreases. Eventually, A < Mach
- A >> 1 is required for thin shell model
- Assuming that the laser is off (coasting phase) when $R/R_1 \sim {A_{sb}}^{1/6}$

$$\rho(\partial_t \vec{w} + \vec{w} \cdot \nabla \vec{w}) = -\nabla p \qquad t_{imp2} \sim \frac{R_2}{u_i}$$

$$\Rightarrow \frac{\dot{\Delta}}{t_{imp2}} + \frac{\dot{\Delta}^2}{\Delta} \sim -\frac{p/\rho}{\Delta} \qquad \frac{\dot{\Delta}}{t_{imp2}} = \frac{\dot{\Delta}}{\Delta}\frac{u_i}{R_2/\Delta} = \frac{\dot{\Delta}}{\Delta}\frac{u_i}{A}$$

$$\underbrace{\frac{\dot{\Delta} u_i}{A}}_{(1)} + \underbrace{\dot{\Delta}^2}_{(2)} \sim \underbrace{{C_s}^2}_{(3)}$$

- There are two cases:
  - Case 1: (3) << (1) and/or (2)
  - Case 2: (3) ~ (1) and/or (2)

The shell thickness does not change in the 2nd phase (coasting phase)

$$\underbrace{\frac{\dot{\Delta} u_i}{A}}_{(1)} + \underbrace{\dot{\Delta}^2}_{(2)} \sim \underbrace{C_s{}^2}_{(3)}$$

- **Case 1: (3) << (1) and/or (2)**

$$\dot{\Delta}\left(\frac{u_i}{A} + \dot{\Delta}\right) \sim 0 \qquad \dot{\Delta} \sim 0 \quad \text{or} \quad \Delta \equiv \Delta_2 = \text{constant}$$

- **Case 2: (3) ~ (1) and/or (2) and** $A << Mach$
  - **(3) ~ (1)** $\quad \dfrac{\dot{\Delta} u_i}{A} \sim C_s{}^2 \Rightarrow \dot{\Delta} \sim \dfrac{C_s A}{u_i / C_s} = \dfrac{C_s A}{\text{Mach}}$

$$\frac{\delta\Delta}{\Delta} \sim \frac{\dot{\Delta} t_{\text{imp2}}}{\Delta} \sim \frac{1}{\Delta}\frac{C_s A}{\text{Mach}}\frac{R_2}{u_i} \sim \frac{A^2}{\text{Mach}^2} << 1$$

  - **(3) ~ (2)** $\quad \dot{\Delta}^2 \sim C_s{}^2 \qquad \dfrac{\delta\Delta}{\Delta} \sim \dfrac{\dot{\Delta} t_{\text{imp2}}}{\Delta} \sim \dfrac{C_s}{\Delta}\dfrac{R_2}{u_i} \sim \dfrac{A}{\text{Mach}} << 1$

Change of shell thickness is small!

$$\frac{R_2}{R_1} \sim \frac{1}{A_{\text{sb}}{}^{1/6}}$$

$$A_2 \sim \sqrt{A_{\text{sb}}}$$

$$\boxed{\Delta \equiv \Delta_2 = \text{constant} = \Delta_2 \frac{R_2}{R_2} = \frac{R_2}{A_2}\frac{R_1}{R_1} = \frac{1}{A_{sb}{}^{1/6}}\frac{R_1}{\sqrt{A_{sb}}} \sim \frac{R_1}{A_{sb}{}^{2/3}}}$$

To verify that A << Mach

- **Comparison of A and Mach:**

$$A \approx \frac{R}{h_2}\frac{R_2}{R_2} = A_2\left(\frac{R}{R_2}\right) \qquad Mach \sim \frac{u_i}{C_s} \sim \frac{u_i}{\sqrt{p/\rho}} \sim \frac{u_i}{\sqrt{\alpha\rho^{2/3}}} = \frac{u_i}{\alpha^{1/2}\rho^{1/3}}$$

$$m \sim \bar{\rho}R^2\Delta \simeq \bar{\rho}R^2\Delta_2 \qquad \Rightarrow\ \bar{\rho} \simeq \frac{m}{R^2\Delta_2}$$

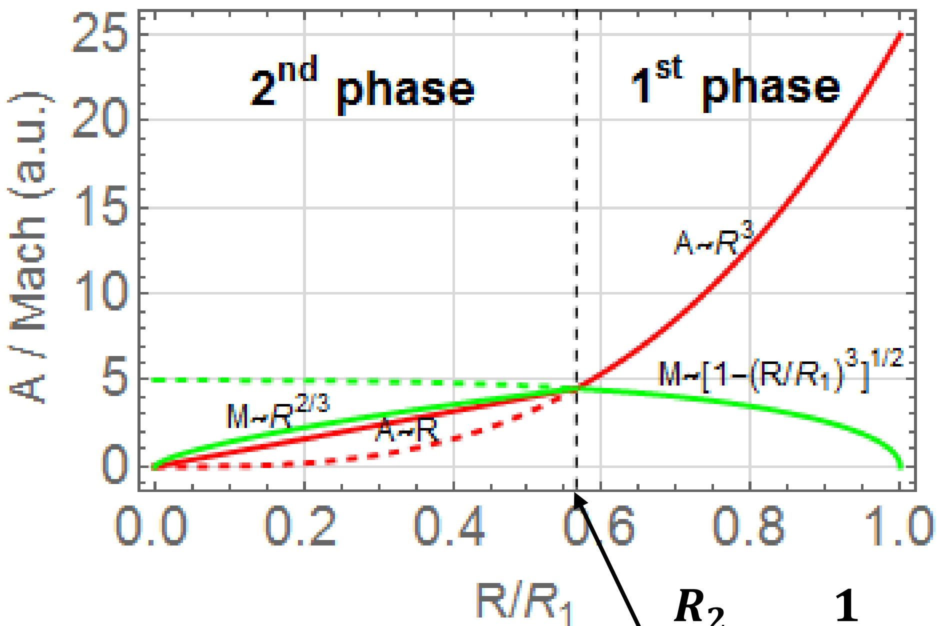
$$Mach \sim \frac{u_i}{\alpha^{1/2}}\left(\frac{R^2\Delta_2}{m}\right)^{1/3} = \frac{u_i}{\alpha^{1/2}}\left(\frac{\Delta_2 {R_2}^2}{m}\right)^{1/3}\left(\frac{R}{R_2}\right)^{2/3} = \mathrm{Mach}_2\left(\frac{R}{R_2}\right)^{2/3}$$

$$\text{where } \mathrm{Mach}_2 = \mathrm{Mach}(R = R_2) = \frac{u_i}{\alpha^{1/2}}\left(\frac{{R_2}^2\Delta_2}{m}\right)^{1/3} \sim A_2 \sim \sqrt{A_{\mathrm{sb}}}$$

$$\frac{A}{\mathrm{Mach}} \sim \frac{A_2\left(\frac{R}{R_2}\right)}{\mathrm{Mach}_2\left(\frac{R}{R_2}\right)^{2/3}} \sim \left(\frac{R}{R_2}\right)^{1/3} \ll 1$$

- **Requirement for thin shell model:** $A \gg 1 \Rightarrow A_2\left(\frac{R}{R_2}\right) \gg 1 \Rightarrow \frac{R}{R_2} \gg \frac{1}{A_2} \sim \frac{1}{\sqrt{A_{\mathrm{sb}}}}$

Summary of phase 2 (coasting phase)



$$\frac{R_2}{R_1} \sim \frac{1}{A_{\mathrm{sb}}{}^{1/6}}$$

$$1 < A < \sqrt{A_{\mathrm{sb}}} \qquad A < \mathrm{Mach}$$

$$\boxed{\frac{1}{\sqrt{A_{\mathrm{sb}}}} \sim \frac{1}{A_2} < \frac{R}{R_2} < 1}$$

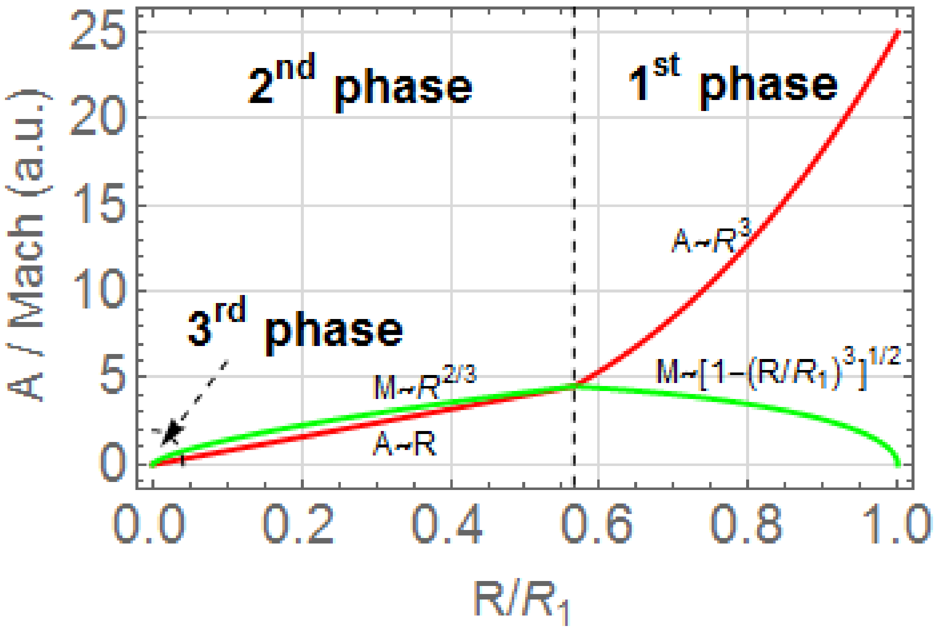
$$\boxed{A = A_2\left(\frac{R}{R_2}\right) \sim \sqrt{A_{\mathrm{sb}}}\left(\frac{R}{R_2}\right)}$$

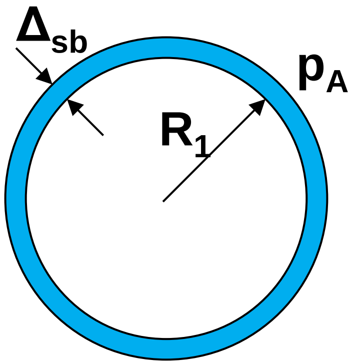
$$\boxed{Mach \sim \mathrm{Mach}_2\left(\frac{R}{R_2}\right)^{2/3} \sim \sqrt{A_{\mathrm{sb}}}\left(\frac{R}{R_2}\right)^{2/3}}$$

$$Mach_2 = \mathrm{Mach}_{\mathrm{max}} \simeq A_2 = \sqrt{A_{\mathrm{sb}}}$$

$$\varDelta \simeq \mathrm{constant} = \varDelta_2 \sim \frac{R_1}{A_{\mathrm{sb}}{}^{2/3}} \qquad \bar{\rho} \simeq \rho_2\left(\frac{R_2}{R}\right)^2 \sim \rho_{\mathrm{sb}}\left(\frac{R_2}{R}\right)^2$$

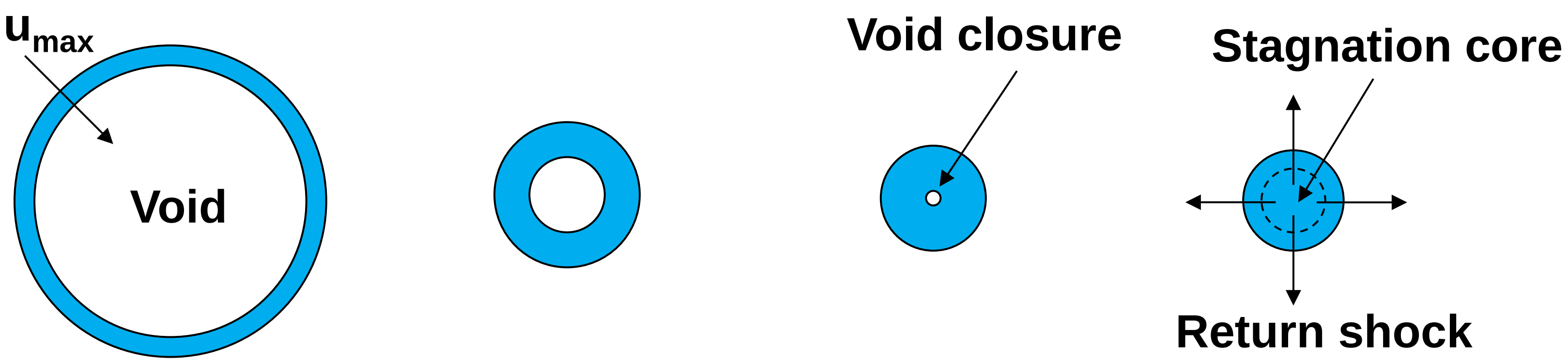
How about the 3rd phase where A~1 ?





- 1st phase: acceleration
- 2nd phase: coasting
- 3rd phase: stagnation

The thin shell model breaks down when A~1



- When A~1 => Δ~R, the “void” inside the shell closes and a “return shock” propagating outward is generated due to the collision of the shell with itself
- The density is compressed by a factor no more than 4 even if the strong shock is generated

$\rho_{st} \sim \rho_3$ where ρ_3 is the density right before the void closure

The stagnated density scales with square of the maximum Mach number

$$\rho_3 \sim \rho_2 \left(\frac{R_2}{R_3}\right)^2 \sim \rho_{\mathrm{sb}} \left(\frac{R_2}{R_3}\right)^2 \qquad \bar{\rho} \simeq \rho_2 \left(\frac{R_2}{R}\right)^2 \ \ \text{(p40)}$$

$$A = A_3 \sim 1 \Rightarrow \frac{R_3}{\Delta_3} \sim \frac{R_3}{\Delta_2} \sim 1 \Rightarrow R_3 \sim \Delta_2$$

$$\rho_{\mathrm{st}} \sim \rho_3 \sim \rho_{\mathrm{sb}} \left(\frac{R_2}{\Delta_2}\right)^2 \sim \rho_{\mathrm{sb}} {A_2}^2 \sim \rho_{\mathrm{sb}} {Mach_2}^2 \sim \rho_{\mathrm{sb}} {Mach_{\max}}^2$$

$$\boxed{\frac{\rho_{\mathrm{st}}}{\rho_{\mathrm{sb}}} \sim {Mach_{\max}}^2} \longleftarrow$$

Density compression scaling law.

The stagnated pressure scales to the 4th power of the maximum Mach number

- **Conservation of energy at stagnation:**

$$p_{\mathrm{st}} R_{\mathrm{st}}^{3} \sim m u_{\max}^{2} \qquad R_{\mathrm{st}} \sim R_3 \sim \Delta_3 \sim \Delta_2 \quad \Rightarrow \quad p_{\mathrm{st}} \Delta_2^{3} \sim \underline{m} u_{\max}^{2} \sim \underline{\rho_2 R_2^{2} \Delta_2} u_{\max}^{2}$$

$$\Rightarrow p_{\mathrm{st}} \sim \rho_2 \left(\frac{R_2}{\Delta_2}\right)^2 u_{\max}^{2} = \rho_2 A_2^{2} u_{\max}^{2} \sim p_2 Mach_2^{2} \frac{u_{\max}^{2}}{p_2/\rho_2} \sim p_A Mach_2^{4} \sim p_A Mach_{\max}^{4}$$

$$\boxed{\frac{p_{\mathrm{st}}}{p_{\mathrm{A}}} \sim Mach_{\max}^{4}}$$

$$Mach_2 = Mach_{\max} \simeq A_2 = \sqrt{A_{\mathrm{sb}}}$$

$$\alpha_{\mathrm{st}} \sim \frac{p_{\mathrm{st}}}{\rho_{\mathrm{st}}^{5/3}} \sim \frac{p_A Mach_{\max}^{4}}{\rho_{\mathrm{sb}}^{5/3} Mach_{\max}^{10/3}} = \alpha_{\mathrm{sb}} Mach_{\max}^{2/3}$$

$$\boxed{\frac{\alpha_{\mathrm{st}}}{\alpha_{\mathrm{sb}}} \sim Mach_{\max}^{2/3}}$$

$$\frac{\rho_{\mathrm{st}}}{\rho_{\mathrm{sb}}} \sim Mach_{\max}^{2}$$

Scaling of the areal density of the compressed core

$$\rho_{st}R_{st} \sim \rho_{st}\Delta_2 \sim \left(\frac{p_{st}}{\alpha_{st}}\right)^{3/5} \frac{\Delta_2}{R_2}\frac{R_2}{R_1}R_1 \sim \left(\frac{p_A\,\mathrm{Mach_{max}}^4}{\alpha_{sb}\,\mathrm{Mach_{max}}^{2/3}}\right)^{3/5} \frac{1}{A_2}\frac{1}{A_{sb}^{1/6}}R_1$$

$A_2 \sim \mathrm{Mach_{max}}$ $\qquad$ $A_{sb} \sim \mathrm{Mach_{max}}^2$

$$\rho_{st}R_{st} \sim \left(\frac{p_A}{\alpha_{sb}}\right)^{3/5} \mathrm{Mach_{max}}^2 \frac{1}{\mathrm{Mach_{max}}}\frac{1}{\mathrm{Mach_{max}}^{1/3}}R_1$$

$$\sim \left(\frac{p_A}{\alpha_{sb}}\right)^{3/5} \mathrm{Mach_{max}}^{2/3} R_1 \sim \left(\frac{p_A}{\alpha_{sb}}\right)^{3/5} \frac{u_{max}^{2/3}}{(p_A/\rho_{sb})^{1/3}}\frac{p_0^{1/3}R_1}{p_0^{1/3}}$$

$$\sim \left(\frac{p_A}{\alpha_{sb}}\right)^{3/5} \frac{u_{max}^{2/3}}{(p_A^{2/5}\alpha_{sb}^{3/5})^{1/3}}\frac{(p_A R_1^3)^{1/3}}{p_A^{1/3}} \sim \frac{p_A^{2/15}}{\alpha_{sb}^{4/5}}u_{max}^{2/3}E_k^{1/3}$$

$$E_k \sim p_A R_1^3$$

$$E_k \sim E_{las} \;\Rightarrow\; \boxed{\rho_{st}R_{st} \sim \frac{p_A^{2/15}u_{max}^{2/3}E_{las}^{1/3}}{\alpha_{sb}^{4/5}}}$$

Amplification of areal density

$$\rho_{\mathrm{st}} R_{\mathrm{st}} \sim \rho_{\mathrm{st}}{}^{2/3} \left(\rho_{\mathrm{st}} R_{\mathrm{st}}{}^{3}\right)^{1/3} \sim \rho_{\mathrm{sb}}{}^{2/3} Mach_{\max}{}^{4/3} Mass^{1/3}$$

$$\sim \frac{\rho_{\mathrm{sb}}{}^{2/3}}{\rho_1{}^{2/3}} Mach_{\max}{}^{4/3} \rho_1{}^{2/3} \left(\rho_1 R_1{}^{2} \Delta_1\right)^{1/3}$$

$$\rho_{\mathrm{st}} R_{\mathrm{st}} \sim (\rho_1 \Delta_1) Mach_{\max}{}^{4/3} A_1{}^{2/3} \left(\frac{\rho_{\mathrm{sb}}}{\rho_1}\right)^{2/3}$$

$$\frac{\rho_{\mathrm{sb}}}{\rho_1} = 4 \left(\frac{I_{\max}}{I_{\mathrm{foot}}}\right)^{2/5}$$

$$\boxed{(\rho R)_{\mathrm{st}} \sim (\rho_1 \Delta_1) \mathrm{IFAR}^{2/3} A_1{}^{2/3} \left(\frac{I_{\max}}{I_{\mathrm{foot}}}\right)^{4/15}}$$

$$E_{\mathrm{las}} = 4\pi R_1{}^{2} I_{\max} t_{\mathrm{imp}} \approx 4\pi R_1{}^{2} I_{\max} \frac{R_1}{u_{\max}}$$

$$E_{\mathrm{las}} \approx \frac{4\pi R_1{}^{3} I_{\max}}{u_{\max}}$$

Summary

$$A_{\mathrm{sb}} = \mathrm{IFAR} = 4A_1\left(\frac{I_{\mathrm{max}}}{I_{\mathrm{foot}}}\right)^{2/5} \qquad u_{\mathrm{max,cm}/s} \approx 10^7\sqrt{0.7A_1\alpha^{3/5}{I_{15,\mathrm{max}}}^{4/15}\left(\frac{I_{\mathrm{max}}}{I_{\mathrm{foot}}}\right)^{2/5}}$$

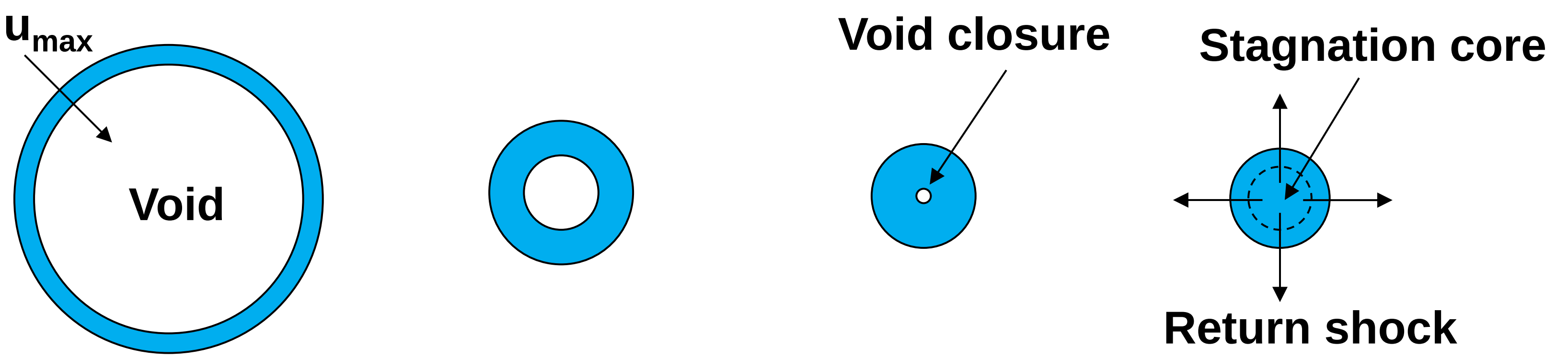
$$\rho_{\mathrm{st}} \sim \rho_{\mathrm{sb}}{\mathrm{Mach}_{\mathrm{max}}}^2 \sim \rho_1 \mathrm{IFAR}\left(\frac{I_{\mathrm{max}}}{I_{\mathrm{foot}}}\right)^{2/5}$$

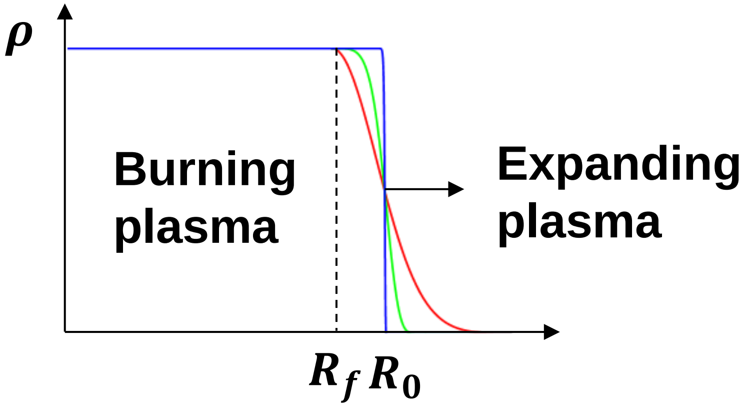
$$p_{\mathrm{st}} \sim p_A{\mathrm{Mach}_{\mathrm{max}}}^4 \sim p_A \mathrm{IFAR}^2$$

$$\alpha_{\mathrm{st}} \sim \alpha_{\mathrm{sb}}{\mathrm{Mach}_{\mathrm{max}}}^{2/3} \sim \alpha_{\mathrm{sb}} \mathrm{IFAR}^{1/3}$$

$$(\rho R)_{\mathrm{st}} \sim (\rho_1 \Delta_1)\mathrm{IFAR}^{2/3}{A_1}^{2/3}\left(\frac{I_{\mathrm{max}}}{I_{\mathrm{foot}}}\right)^{4/15}$$

Calculation of the burn-up fraction



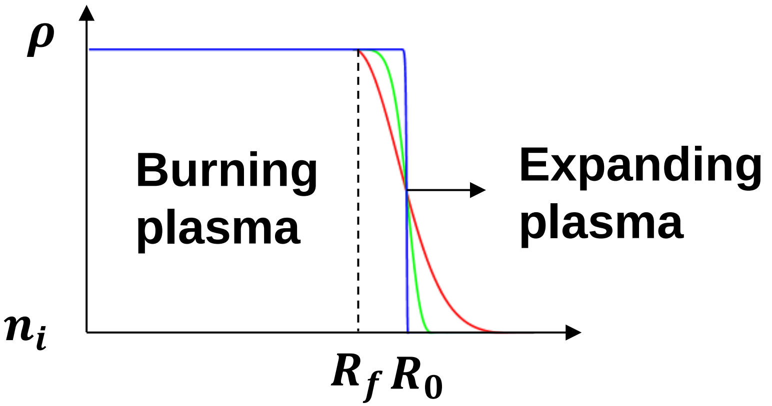


$$R_f = R_0 - C_s t$$

$$\frac{\partial n_i}{\partial t} = -\nabla \cdot (n_i v) - \frac{{n_i}^2}{4} < \sigma v > \times 2$$

Calculation of the burn-up fraction - continue

$$4\pi\int_0^{R_f} r^2 dr\left(\frac{\partial n_i}{\partial t} = -\nabla\cdot(n_i v) - \frac{{n_i}^2}{2}\langle\sigma v\rangle\right)$$



$$4\pi\int_0^{R_f} r^2\frac{\partial n_i}{\partial t}dr = 4\pi\partial_t\int_0^{R_f} r^2 n_i dr - 4\pi\dot{R}_f {R_f}^2 n_i$$

$$= -n_i v 4\pi {R_f}^2 - \frac{{n_i}^2}{2}\langle\sigma v\rangle V_f$$

(neglect)

(Leibniz integral rule)

$$N_f \equiv \frac{4\pi}{3}{R_f}^3 n_i \equiv V_f n_i$$

$$d_t N_f - 3N_f\frac{\dot{R}_f}{R_f} = -\frac{{N_f}^2}{V_f}\frac{\langle\sigma v\rangle}{2}$$

$$\frac{d_t N_f}{{N_f}^2} - \frac{3\dot{R}_f}{N_f R_f} = -\frac{\langle\sigma v\rangle}{2V_f}$$

$$d_t\frac{1}{N_f} + \frac{3\dot{R}_f}{N_f R_f} = \frac{\langle\sigma v\rangle}{2V_f}$$

$$ {R_f}^3 d_t\frac{1}{N_f} + 3{R_f}^2\frac{\dot{R}_f}{N_f}$$

$$= \frac{d}{\mathrm{dt}}\left(\frac{{R_f}^3}{N_f}\right) = \frac{\langle\sigma v\rangle}{2V_f}{R_f}^3$$

Calculation of the burn-up fraction - continue

$$\frac{d}{\mathrm{dt}}\left(\frac{{R_f}^3}{N_f}\right)=\frac{\langle\sigma v\rangle}{2V_f}{R_f}^3 \qquad \frac{{R_f}^3}{N_f}=\int_0^t\frac{\langle\sigma v\rangle}{2V_f}{R_f}^3dt+\frac{{R_0}^3}{N_0}$$

$$R_f=R_0-C_st \qquad dt=-\frac{\mathrm{dR}_f}{C_s} \qquad V_f=\frac{4\pi}{3}{R_f}^3$$

$$\frac{{R_f}^3}{N_f}=-\int_{R_0}^{R_f}\frac{\langle\sigma v\rangle}{2\times 4\,\pi/3}\frac{dR_f}{C_s}+\frac{{R_0}^3}{N_0} \qquad n_0=\frac{N_0}{V_0}$$

$$\frac{{R_f}^3}{N_f}=-\int_{R_0}^{R_f}\frac{\langle\sigma v\rangle}{2C_s}\frac{3}{4\pi}dR_f+\frac{{R_0}^3}{N_0} \qquad \frac{V_f}{N_f}=\frac{V_0}{N_0}\left[1+\frac{\langle\sigma v\rangle}{2C_s}n_0R_0\left(1-\frac{R_f}{R_0}\right)\right]$$

$$\frac{{R_f}^3}{N_f}=\frac{\langle\sigma v\rangle}{2C_s}\frac{3}{4\pi}\left(R_0-R_f\right)+\frac{{R_0}^3}{N_0} \qquad \boxed{\xi\equiv\frac{\langle\sigma v\rangle}{2C_s}n_0R_0}$$

$$\frac{V_f}{N_f}=\frac{\langle\sigma v\rangle}{2C_s}\mathrm{R}_0\left(1-\frac{R_f}{R_0}\right)+\frac{V_0}{N_0} \qquad \boxed{\frac{V_f}{N_f}=\frac{1}{n_0}\left[1+\xi\left(1-\frac{R_f}{R_0}\right)\right]}$$

Calculation of the burn-up fraction - continue

$$\frac{V_f}{N_f} = \frac{1}{n_0}\left[1 + \xi\left(1 - \frac{R_f}{R_0}\right)\right] \qquad n_i = \frac{N_f}{V_f}$$

#Burned ions $= \int_0^t \frac{\langle\sigma v\rangle}{2} n_i^2 V_f dt = \int_0^t \frac{\langle\sigma v\rangle}{2} \frac{N_f{}^2}{V_f} dt = -\int_{R_0}^{R_f} \frac{\langle\sigma v\rangle}{2} \left(\frac{N_f}{V_f}\right)^2 V_f \frac{dR_f}{C_s}$

$$= \int_{R_f}^{R_0} \frac{\langle\sigma v\rangle}{2} \frac{n_0{}^2}{\left[1 + \xi\left(1 - \frac{R_f}{R_0}\right)\right]^2} \left(\frac{R_f}{R_0}\right)^3 V_0 R_0 \frac{dR_f/R_0}{C_s}$$

$$= \int \frac{\langle\sigma v\rangle}{2} \frac{n_0{}^2}{[1 + \xi(1 - x)]^2} x^3 V_0 \frac{R_0}{C_s} dx = N_0 \xi \int_0^1 \frac{x^3 dx}{[1 + \xi(1 - x)]^2}$$

$$= N_0 \frac{\xi[6 + \xi(9 + 2\xi)] - 6(1 + \xi)^2 Ln[1 + \xi]}{2\xi^3}$$

#Burn-up Fraction

$$\boxed{\Theta(\xi) = \frac{\xi[6 + \xi(9 + 2\xi)] - 6(1 + \xi)^2 \mathrm{Ln}[1 + \xi]}{2\xi^3}}$$

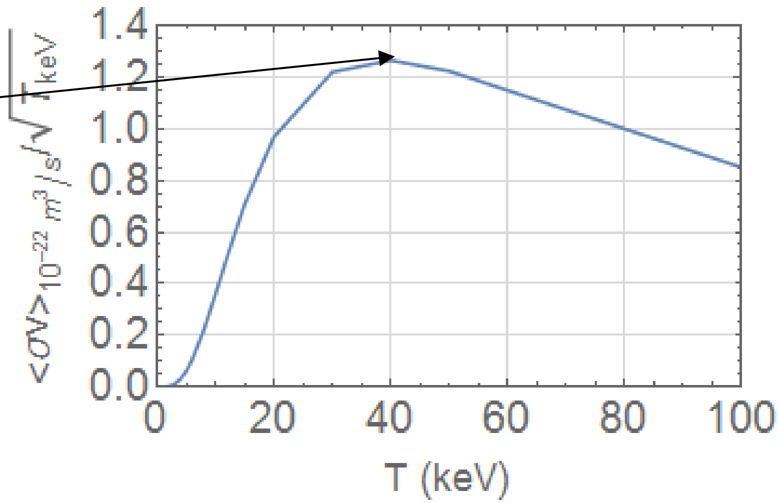
Calculation of the burn-up fraction - continue

$$C_s = \sqrt{\frac{T_e + T_i}{m_i}} = \sqrt{\frac{2T}{m_i}} \qquad \rho = n_0 m_i \qquad m_i = \frac{m_D + m_T}{2} = 2.5 \times 1.67 \times 10^{-27}\mathrm{kg}$$

$$\xi = \frac{\langle\sigma v\rangle}{\sqrt{T}}(\rho R_0)\frac{1}{2\sqrt{2m_i}} = \frac{\langle\sigma v\rangle_{m^3/s}}{\sqrt{T_{keV} \times 1.6 \times 10^{-16}}}\frac{(\rho R_0)_{g/cm^2} \times 10}{2\sqrt{5 \times 1.67 \times 10^{-27}}}$$

$$\xi \simeq \frac{1.25 \times 10^{-22}}{\sqrt{1.6 \times 10^{-16}}}\frac{10(\rho R_0)_{g/cm^2}}{2\sqrt{5 \times 1.67 \times 10^{-27}}} = 0.54(\rho R_0)_{g/cm^2}$$

$$\left.\frac{\langle\sigma v\rangle}{\sqrt{T_{keV}}}\right]_{\max} = 1.25 \times 10^{-22} \quad @\ T = 40\ \mathrm{keV}$$



H.-S. Bosch and G.M. HaleNucl. Fusion ***32*** (1992) 611

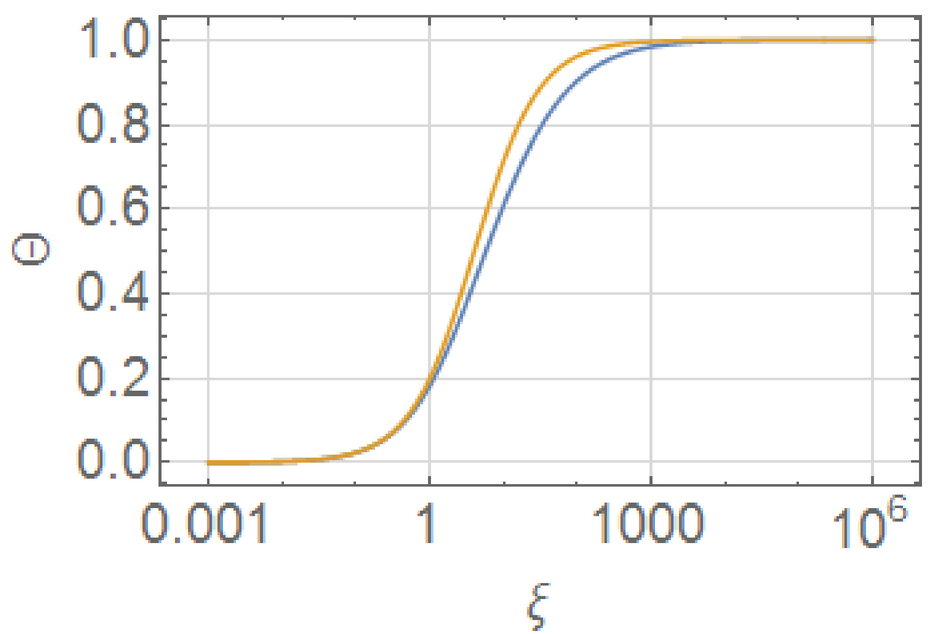
Smallest areal density (ρR)

#Burned-up Fraction $$\Theta(\xi) = \frac{\xi[6 + \xi(9 + 2\xi)] - 6(1 + \xi)^2 \mathrm{Ln}[1 + \xi]}{2\xi^3}$$

$$\lim_{\xi \to 0} \Theta(\xi) = \frac{\xi}{4} \qquad \lim_{\xi \to \infty} \Theta(\xi) = 1 \qquad \Theta(\xi) \approx \frac{\xi}{4 + \xi} \qquad \xi \simeq 0.54(\rho R_0)_{g/cm^2}$$

$$\Theta(\xi) \approx \frac{0.54\rho\mathrm{R}}{4 + 0.54\rho\mathrm{R}}$$

$$\boxed{\Theta(\xi) \approx \frac{(\rho\mathrm{R})_{g/\mathrm{cm}^2}}{7 + (\rho\mathrm{R})_{g/\mathrm{cm}^2}}}$$



Large *ρR* is needed to have high burn-up fraction.

For energy applications:

$$\boxed{\Theta \gtrsim 0.3} \qquad \boxed{\rho R \geq 3\ g/\mathrm{cm}^2}$$

Energy gain

Mass = M_0
Temp = T
DT
Volume = V_0

Fusion energy $= \dfrac{M_0}{2m_i}\epsilon_f \Theta$

$\epsilon_f = 17.6\text{MeV}$

Energy gain $= \dfrac{\text{Fusion Energy}}{\text{Input Energy}}$

- Input energy: the sphere is heated to the temperature T

Thermal energy in sphere: $\dfrac{3}{2}(n_{i0}T_i + n_{e0}T_e)V_0$

$n_{i0} = n_{e0} \equiv n_0 \qquad T_e = T_i \Longrightarrow 3n_0\text{TV}_0 = 3\dfrac{M_0}{m_i}T$

Set heating efficiency: $\eta = \dfrac{\text{Thermal Energy}}{\text{Input Energy}}$

$$\text{Gain} = \frac{\frac{M_0}{2m_i}\epsilon_f\Theta}{3\frac{M_0}{m_i}T/\eta} = \eta\frac{M_0}{2m_i}\frac{\epsilon_f\Theta}{3\frac{M_0}{m_i}T} = \frac{\eta}{6}\frac{\epsilon_f}{T}\Theta$$

$$\boxed{\text{Gain} = \eta\, 293\left(\frac{10}{T_{\text{keV}}}\right)\Theta}$$

The power to heat the plasma is enormous

- **Consider the small T limit:**

$$\boldsymbol{\Theta}(\xi) \approx \frac{\xi}{4+\xi} \qquad \xi \equiv \frac{\langle\sigma v\rangle}{2C_s} n_0 R_0 = \frac{\langle\sigma v\rangle}{\sqrt{T}} (\rho R_0) \frac{1}{2\sqrt{2m_i}}$$

$$\langle\sigma v\rangle \sim T^4 \ \text{ for } \ T \to 0 \ \text{, then} \quad \xi \sim T^{7/2} \quad \text{and} \quad Gain \sim T^{5/2} \to 0$$

- **Required input power:**

$$P_w = \frac{E_{\text{input}}}{\tau_{\text{input}}} \qquad \tau_{\text{input}} \ll \tau_{\text{burn}} = \frac{R}{C_s}$$ **(Heat out before it runs away)**

$$P_w = \frac{E_{\text{input}}}{\mu R/C_s} = \frac{E_{\text{thermal}}}{\eta\mu R/C_s} = 3\frac{M_0}{m_i}\frac{T}{R}\frac{C_s}{\eta\mu} \qquad \tau_{\text{input}} = \mu\frac{R}{C_s} \quad \text{Ex: } \mu \sim 0.1$$

$$\frac{P_w}{M_0} = \frac{3}{m_i}\frac{T}{R}\frac{C_s}{\eta\mu} = \frac{3}{m_i}\frac{T}{R}\sqrt{\frac{2T}{m_i}}\frac{1}{\eta\mu}$$

$$\boxed{\frac{P_w}{M_0} = 10^{18}\left(\frac{T_{\text{keV}}}{10}\right)^{3/2} \frac{0.1}{\mu}\frac{1}{R_{\text{cm}}}\frac{1}{\eta}\ \text{Watts}/g}$$

A clever way is needed to ignite a target

- **For T = 10 keV**

$$\xi \approx 0.18(\rho R) \qquad Gain|_{10\text{keV}} \approx 293\eta \frac{0.18\rho R}{4 + 0.18\rho R} \approx 293\eta \frac{\rho R_{g/\text{cm}^2}}{22 + \rho R_{g/\text{cm}^2}}$$

- **For T=40 keV**

$$\xi \approx 0.54(\rho R) \qquad Gain|_{40\text{keV}} \approx 73\eta \frac{\rho R_{g/\text{cm}^2}}{7 + \rho R_{g/\text{cm}^2}}$$

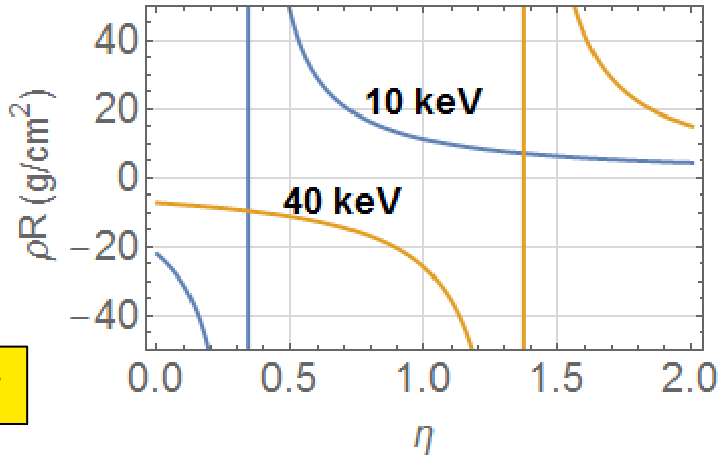
- **For Gains $\gtrsim$ 100**
  - **T = 10 keV**

    $$\rho R \gtrsim 22\, g/\text{cm}^2 \quad \eta > 1$$

  - **T = 40 keV**

    $$\eta > 1$$

How do we get $\eta > 1$?



Requirement to ignite a target

- **For T=10 keV and ρR ≳ 22 g/cm²**

$$\rho R = \frac{4\pi}{3}\frac{\rho R^3}{4\pi R^2/3} = \frac{M_0}{\frac{4\pi}{3}R^2} = \frac{3}{4\pi}\frac{M_0}{R^2} \gtrsim 22\, g/\mathrm{cm}^2$$

$$\frac{M_0}{R^2} \gtrsim 92\, g/\mathrm{cm}^2$$

$$P_w\Big|_{10keV} = 10^{18}\left(\frac{T_{\mathrm{keV}}}{10}\right)^{3/2}\frac{0.1}{\mu}\frac{M_0}{R_{\mathrm{cm}}}\frac{1}{\eta} = 10^{18}\frac{0.1}{\mu}\frac{1}{\eta}92R_{\mathrm{cm}} \quad \text{Watts}$$

$$P_w\Big|_{10keV} \approx 10^{20}\frac{0.1}{\mu}\frac{R_{\mathrm{cm}}}{\eta}\text{Watts}$$

- **For T=40keV**

$$\rho R \gtrsim 7 \;\Rightarrow\; \frac{M_0}{R^2} \gtrsim 30\, g/\mathrm{cm}^2$$

$$P_w\Big|_{40keV} \approx 2.4\times 10^{20}\frac{0.1}{\mu}\frac{R_{\mathrm{cm}}}{\eta}\text{Watts}$$

- **Needed:**

  $R_{\mathrm{cm}} \ll 1$

  $\eta \gg 1$

  $\mu \gg 0.1$

Requirements to ignite a target

$$P_w\Big|_{10keV} \approx 10^{20}\,\frac{0.1}{\mu}\,\frac{R_{\mathrm{cm}}}{\eta}\,\mathrm{Watts}$$

- $R_{\mathrm{cm}} \ll 1$ **: sphere size in the order of 100's um**
- $\eta \gg 1$ **: input energy amplification**
- $\mu \gg 0.1$ **: energy delivery time decoupled from burn time. Need longer energy delivery time. Need to bring down power to ~10^{15} W**

Math….#!@%$$#&^%$#

$$P_w = 10^{18}\frac{M_{0,g}}{\eta}\left(\frac{T_{\mathrm{keV}}}{10}\right)^{3/2}\frac{0.1}{\mu}\frac{1}{R_{\mathrm{cm}}}\ \mathrm{Watts}/g$$

$$\tau_{\mathrm{input}} = \mu\frac{R}{C_s}\quad \text{Ex: } \mu\sim 0.1 \qquad \eta = \frac{\text{Thermal Energy}}{\text{Input Energy}}$$

$$\text{Gain} = 293\eta\left(\frac{10}{T_{\mathrm{keV}}}\right)\Theta(\xi) \qquad \Theta(\xi) \approx \frac{\xi}{4+\xi} \qquad \xi = \frac{\langle\sigma v\rangle}{2m_i C_s}(\rho \mathrm{R_0})$$

$$G_{\max} \equiv 293\eta\left(\frac{10}{T_{\mathrm{keV}}}\right) \qquad G = G_{\max}\frac{\xi}{4+\xi} \Longrightarrow \xi = \frac{4G}{G_{\max}-G}$$

$$P_w = \frac{10^{18}}{\eta}\left(\frac{T_{\mathrm{kev}}}{10}\right)^{3/2}\frac{0.1}{\mu}\frac{4\pi}{3}\frac{\rho {R_0}^3}{R_0} = \frac{10^{18}}{\eta}\left(\frac{T_{\mathrm{kev}}}{10}\right)^{3/2}\frac{0.1}{\mu}\frac{4\pi}{3}(\rho R_0)R_0$$

More math…!#$%%^&*&^(*&%)(#%!@$#%%^*&*%(

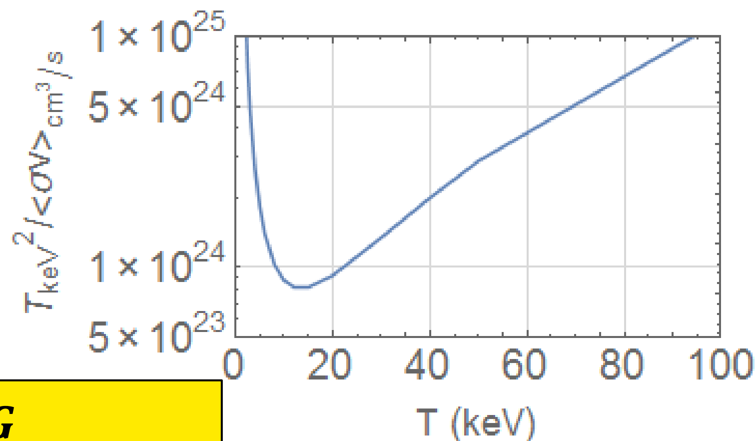
$$P_w = \frac{10^{18}}{\eta}\left(\frac{T_{\mathrm{kev}}}{10}\right)^{3/2}\frac{0.1}{\mu}\frac{4\pi}{3}\frac{\rho {R_0}^3}{R_0} = \frac{10^{18}}{\eta}\left(\frac{T_{\mathrm{kev}}}{10}\right)^{3/2}\frac{0.1}{\mu}\frac{4\pi}{3}(\rho R_0)R_0$$

$$= \frac{10^{18}}{\eta}\left(\frac{T_{\mathrm{kev}}}{10}\right)^{3/2}\frac{0.1}{\mu}\frac{4\pi}{3}R_0\frac{2m_i C_s}{<\sigma v>}\xi \quad \text{where} \quad \xi \equiv \frac{\langle\sigma v\rangle}{2m_i C_s}(\rho R)$$

$$= \frac{10^{18}}{\eta}\left(\frac{T_{\mathrm{kev}}}{10}\right)^{3/2}\frac{0.1}{\mu}\frac{32\pi}{3}R_{0,\mathrm{cm}}\frac{\sqrt{\mathrm{T}m_i}}{<\sigma v>}\frac{G}{G_{\max}-G} \quad \text{where} \quad C_s = \sqrt{\frac{2T}{m_i}}$$

$$P_w = \frac{10^{18}}{\eta}\frac{{T_{\mathrm{kev}}}^2}{\langle\sigma v\rangle_{\mathrm{cm}^2/s}}\frac{0.1}{\mu}R_{0,\mathrm{cm}}\frac{G}{G_{\max}-G} \quad \text{Watts}$$

$$\left.\frac{{T_{kev}}^2}{\langle\sigma v\rangle_{cm^2/s}}\right|_{min} = 8\times 10^{23} \quad \text{for} \quad T = 14\mathrm{keV}$$



$$\frac{G}{G_{\max}-G} \approx \frac{G}{G_{\max}}$$

$$\boxed{P_w \approx \frac{7\times 10^{19}}{\eta}\frac{0.1}{\mu}R_{0,\mathrm{cm}}\frac{G}{G_{\max}}\text{Watts}}$$

Need to lower the power by 5 orders of magnitude

$$P_w \approx \frac{7\times 10^{19}}{\eta}\frac{0.1}{\mu}R_{0,\mathrm{cm}}\frac{G}{G_{\max}}\,\mathrm{Watts}$$

- μ ↑ :
- η ↑ : require the fuel ignition from a "spark." Ignite only a small portion of the DT plasma, i.e., $M_h << M_0$
- R_0 ↓ : smaller system size

$$P_w = P_w(M_0)\frac{M_h}{M_0}$$

$$P_w{}^{\min} = \frac{7\times 10^{15}}{\eta_h}\left(\frac{M_h/M_0}{0.01}\right)\left(\frac{R_{0,\mu\mathrm{m}}}{100}\right)\left(\frac{0.1}{\mu}\right)\left(\frac{G}{G_{\max}}\right)\mathrm{Watts}$$

Effective increase in η

Lawson's Ignition Criterion

- **Approximate the power losses:**

$$\text{Plasma energy:}\quad \epsilon = \frac{3}{2}\mathrm{PV} = \frac{3}{2}(2\mathrm{nT})V = 3\mathrm{nTV}$$

$$P_{\text{losses}}{}^{\text{total}} = P_{\text{losses}} + P_{\text{brem}} = \frac{3\mathrm{nTV}}{\tau_E} + C_b n^2\sqrt{T}V$$

Confinement time

- **Plasma energy balance:**

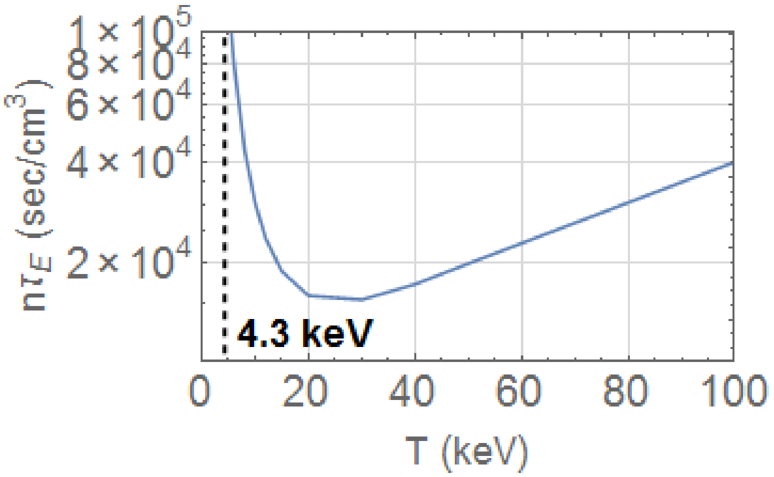
$$\frac{d\epsilon}{\mathrm{dt}} = V\left\{\langle\sigma v\rangle\frac{n^2}{4}\epsilon_\alpha - C_b n^2\sqrt{T} - \frac{3\mathrm{nT}}{\tau_E}\right\} > 0$$

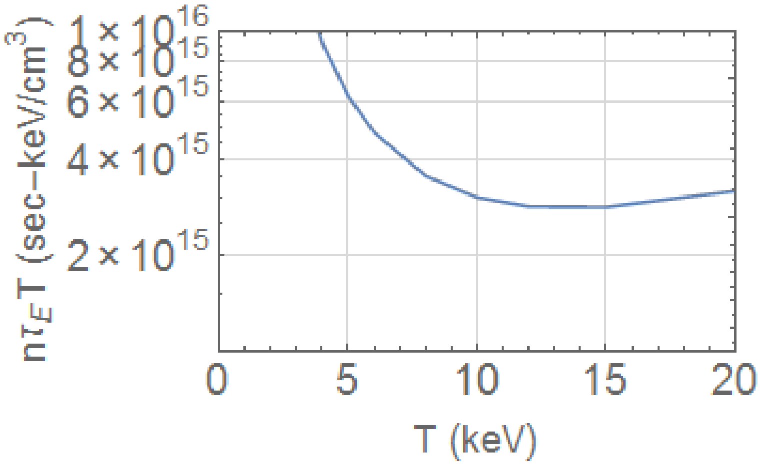
$$n^2\left[\frac{1}{4}\langle\sigma v\rangle\epsilon_\alpha - C_b\sqrt{T}\right] > \frac{3\mathrm{nT}}{\tau_E}$$

$$\boxed{n\tau_E > \frac{3T}{\frac{1}{4}\langle\sigma v\rangle\epsilon_\alpha - C_b\sqrt{T}}}$$

Temperature needs to be greater than ~5 keV to ignite

$$n\tau_E > \frac{3T}{\frac{1}{4}\langle\sigma v\rangle\epsilon_\alpha - C_b\sqrt{T}}$$





$$n\tau_E > 2 \times 10^4\ \mathrm{sec/cm^3}$$

$$nT\tau_E > 3.5 \times 10^{15}\ \mathrm{keV - sec/cm^3}$$

Target design using an 1MJ laser

$$P_w^{\min} = \frac{7\times 10^{15}}{\eta_h}\left(\frac{M_h/M_0}{0.01}\right)\left(\frac{R_{0,\mu\mathrm{m}}}{100}\right)\left(\frac{0.1}{\mu}\right)\left(\frac{G}{G_{\max}}\right)\mathrm{Watts}$$

- For the case of using a huge laser, ex: 1MJ.
- The ignition requires temperatures $T \gtrsim 5\mathrm{keV}$, then the energy required for ignition is

$$E_{\mathrm{ign}} \approx 3\frac{M_h}{m_i}\frac{T}{\eta_h}$$

$$M_h \approx \frac{m_i}{3}\frac{\eta_h E_{\mathrm{ign}}}{T}$$

$$M_{h,\mu\mathrm{g}} \approx 17\left(\frac{5}{T_{\mathrm{keV}}}\right)E_{\mathrm{igm,MJ}}\left(\frac{\eta_h}{0.01}\right) \qquad M_h \approx 20\mu\mathrm{g}$$

Target design using an 1MJ laser - continue

- **For "inefficient" heating mechanism $(\eta_h \approx 1\%)$, the mass that can be heated to $T \approx 5$ keV is in the order of $M_h \approx 20$ μg.**
- **If $M_h/M_0 \approx 0.01$, then $M_0 \approx 2$ mg .**
- **Assuming that the burned-up fraction** $\Theta \approx \dfrac{\rho R}{7 + \rho R}$

for $\Theta \approx 30\% \rightarrow \rho R \approx 3\ g/cm^2$

$$M_0 = \frac{4\pi}{3}\rho R^3 = \frac{4\pi}{3} R^2(\rho R) \qquad R = \sqrt{\frac{4\pi}{3}\frac{M_0}{\rho R}} = 126\sqrt{\frac{M_{0,\mathrm{mg}}}{2}}\sqrt{\frac{3}{\rho R}}\,\mu\mathrm{m}$$

$$\rho = \frac{3M_0}{4\pi R^3} = 240\sqrt{\frac{M_{0,\mathrm{mg}}}{2}}\left(\frac{126}{R_{\mu\mathrm{m}}}\right)^3\ g/\mathrm{cm}^3 \qquad \rho_{\mathrm{DT}} = 0.25\ g/\mathrm{cm}^3$$

- **DT must be compressed ~1000 times**
- **The initial radius of a 2 mg sphere of *DT* is $R_{init} \simeq 2.6$ mm while the final radius $R_{final} \simeq 100\ \mu m$, the convergence ratios of 30 ~ 40 are required.**

Requirements of the density and size of the ignition mass

$$M_h \approx 20\mu\text{g}$$

$$\rho_h R_h \approx 0.3\ g/\text{cm}^2 \longleftarrow$$ **To stop 3.5 MeV α particles**

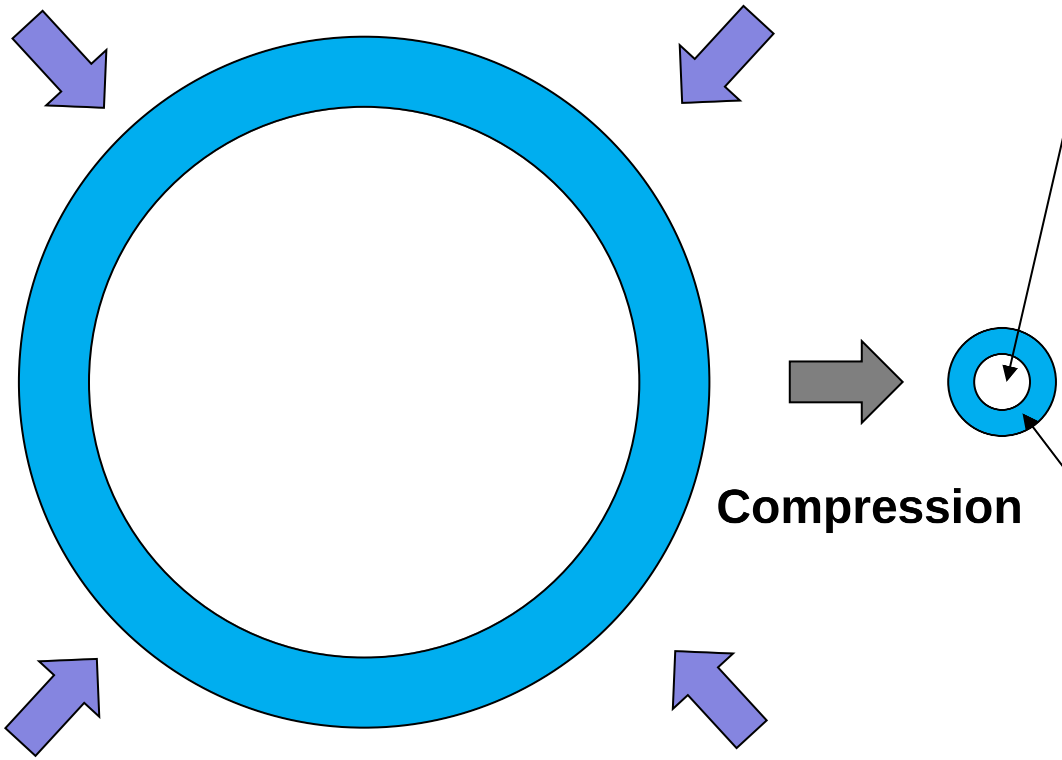
$$R_h \simeq \sqrt{\frac{3}{4\pi}\frac{M_h}{\rho_h R_h}} \approx 40\mu\text{m}$$

$$\rho_h \approx \frac{(\rho_h R_h)}{R_h} = \frac{0.3}{40 * 10^{-4}} = 75\ g/\text{cm}^3$$

Summary

- **Possible fuel assembly for 1MJ ICF driver**

$E_{\text{laser}} = 1\text{MJ}$ $\quad \eta_h \approx 1\%$



Hot spot

$M_h \approx 20\mu\text{g}$ $\quad (\rho R)_h \approx 0.3\, g/\text{cm}^2$

$\rho_h \approx 70\, g/\text{cm}^3$ $\quad R_h \approx 40\mu\text{m}$

$$\frac{M_h}{M_0} \sim 0.01$$

Dense fuel

$M_0 \approx 2\text{mg}$ $\quad (\rho R) \approx 3\, g/\text{cm}^2$

$\rho \approx 250\, g/\text{cm}^3$ $\quad R \approx 120\mu\text{m}$

$\rho_{\text{DT}} = 0.25\, g/\text{cm}^3$ $\quad R_{\text{init}} \simeq 2.6\text{mm}$

Convergence ratio ~20

Backup

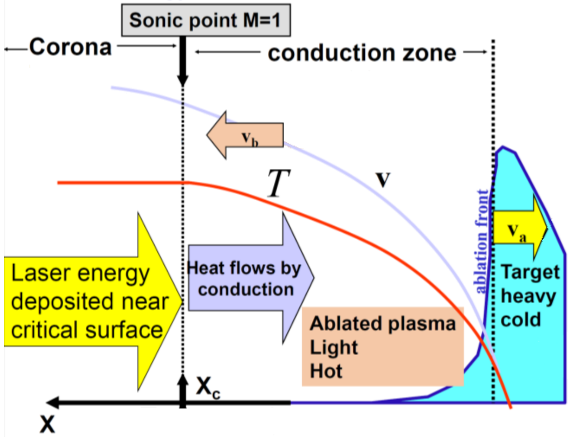
The plasma keeps expanding in the corona zone so that no steady state can be found

- For $x > x_c$:

$$\partial_t \rho + \partial_x(\rho v) = 0$$

$$\rho(\partial_t v + v\partial_x v) + \partial_x p = 0$$

$$\partial_t \left(\frac{3p}{2} + \frac{\rho v^2}{2}\right) + \partial_x \left[v\left(\frac{5p}{2} + \frac{\rho v^2}{2}\right) - \kappa \frac{\partial T}{\partial x}\right] = 0$$



- The temperature in the corona is high.

$$\kappa = \kappa_o T^{5/2} \Rightarrow \text{very large} \Rightarrow \frac{\partial T}{\partial x} = 0 \Rightarrow T = T_c = \text{constant} \qquad p = \frac{\rho T_c}{A}$$

$$\rho(\partial_t v + v\partial_x v) + \frac{T_c}{A}\partial_x \rho = 0$$

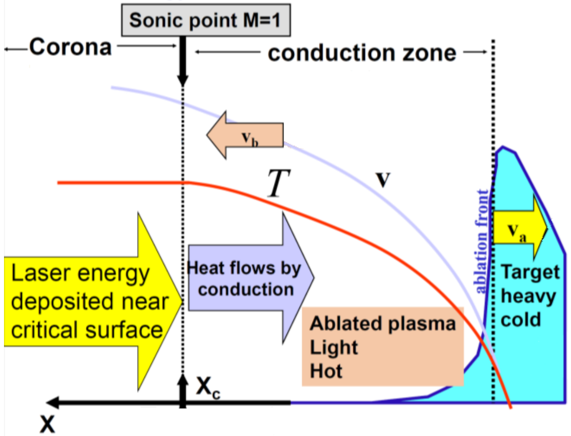
- Self-similar solutions depending on $\xi = \dfrac{z}{t}$ $\qquad z \equiv x - x_c$

$$\partial_t = \frac{\partial \xi}{\partial t}\frac{\partial}{\partial \xi} = -\frac{z}{t^2}\frac{\partial}{\partial \xi} \to -\frac{\xi}{t}\partial_\xi \qquad \partial_x = \frac{\partial \xi}{\partial x}\frac{\partial}{\partial \xi} = \frac{1}{t}\frac{\partial}{\partial \xi} \to \frac{1}{t}\partial_\xi$$

The plasma keeps expanding in the corona zone so that no steady state can be found

$$\partial_t \rho + \partial_x(\rho v) = 0$$

$$\xi = \frac{z}{t} \qquad z \equiv x - x_c$$



$$\rho(\partial_t v + v\partial_x v) + \frac{T_c}{A}\partial_x \rho = 0$$

$$\partial_t \rightarrow -\frac{\xi}{t}\partial_\xi$$

$$-\xi\partial_\xi \rho + \partial_\xi(\rho v) = 0 \qquad \frac{\partial_\xi \rho}{\rho} = \frac{\partial_\xi v}{\xi - v}$$

$$\partial x \rightarrow \frac{1}{t}\partial_\xi$$

$$\rho\left(-\frac{\xi}{t}\partial_\xi v + \frac{v}{t}\partial_\xi v\right) + \frac{T_c}{A}\frac{1}{t}\partial_\xi \rho = 0 \qquad (\xi - v)\partial_\xi v = \frac{T_c}{A}\frac{\partial_\xi \rho}{\rho}$$

$$(\xi - v)\partial_\xi v = \frac{T_c}{A}\frac{\partial_\xi v}{\xi - v} \qquad (\xi - v)^2 = \frac{T_c}{A} \qquad v = \xi \pm \sqrt{\frac{T_c}{A}}$$

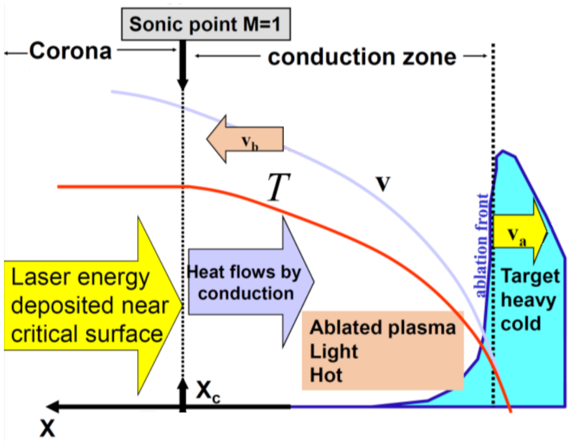
- **At z=0 (x=x_c),** $v = v_c = \sqrt{\frac{T_c}{A}}$ ➡ $v = \xi + \sqrt{\frac{T_c}{A}}$

The plasma keeps expanding in the corona zone so that no steady state can be found

$$\frac{\partial_\xi \rho}{\rho} = \frac{\partial_\xi v}{\xi - v}$$

$$(\xi - v)\partial_\xi v = \frac{T_c}{A}\frac{\partial_\xi \rho}{\rho}$$

$$v = \xi + \sqrt{\frac{T_c}{A}}$$



$$\partial_\xi \ln\rho = -\frac{\partial_\xi v}{\sqrt{T_c/A}}$$

$$\ln\rho = -\frac{\xi}{\sqrt{T_c/A}} + \text{constant}$$

$$\rho = \rho_c e^{-\frac{\xi}{\sqrt{T_c/A}}}$$

$$\rho = \rho_c \text{ at } \xi = 0$$

$$v = \frac{x - x_c}{t} + \sqrt{\frac{T_c}{A}}$$

$$\rho = \rho_c \exp\left[-\frac{x - x_c}{t\sqrt{T_c/A}}\right]$$

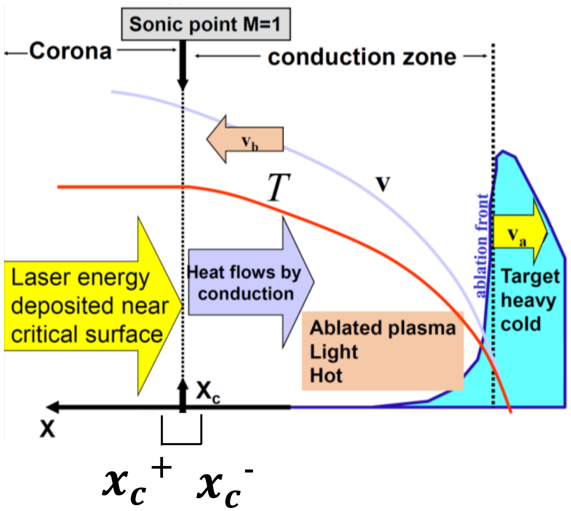
- **Laser energy is absorbed at the critical surface:**

$$\frac{\partial}{\partial t}\left(\frac{3p}{2} + \frac{\rho v^2}{2}\right) + \frac{\partial}{\partial x}\left[v\left(\frac{5p}{2} + \frac{\rho v^2}{2}\right) - \kappa\frac{\partial T}{\partial x}\right] = I\delta(x)$$

The plasma keeps expanding in the corona zone so that no steady state can be found

- **Laser energy is absorbed at the critical surface:**

$$\frac{\partial}{\partial t}\left(\frac{3p}{2}+\frac{\rho v^2}{2}\right)+\frac{\partial}{\partial x}\left[v\left(\frac{5p}{2}+\frac{\rho v^2}{2}\right)-\kappa\frac{\partial T}{\partial x}\right]=I\delta(x)$$



- **The jump conditions are**

$$\left[-\kappa\frac{\partial T}{\partial x}\right]_{x_c^-}^{x_c^+}=I=-\kappa^+\left(\frac{\partial T}{\partial x}\right)^+ +\kappa^-\left(\frac{\partial T}{\partial x}\right)^-$$

$$T=T_c\left[1+\frac{25}{4A}\frac{\rho_c v_c}{k_o {T_c}^{5/2}}(x-x_c)\right]^{2/5}$$

$$\kappa^-\left(\frac{\partial T}{\partial x}\right)^-\simeq\frac{5}{2}\frac{\rho_c v_c T_c}{A}+\frac{1}{2}\rho_c {v_c}^3=3\frac{\rho_c v_c T_c}{A}=3\rho_c\left(\frac{T_c}{A}\right)^{3/2}$$

$$\kappa^+\left(\frac{\partial T}{\partial x}\right)^+=? \qquad \left(\frac{\partial T}{\partial x}\right)^+\to 0 \qquad \kappa^+\to\infty$$

The plasma keeps expanding in the corona zone so that no steady state can be found

- **Total energy in the corona:**

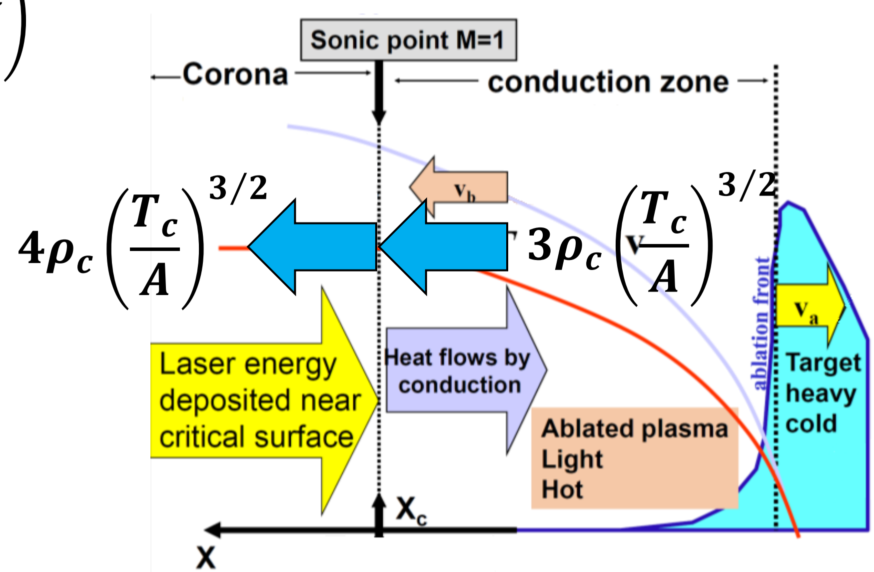
$$\epsilon = \int_{x_c}^{\infty} dx \left(\frac{3}{2} p + \frac{1}{2} \rho v^2 \right) = \int_0^{\infty} dz \left(\frac{3}{2} \rho \frac{T_c}{A} + \frac{1}{2} \rho v^2 \right)$$

$$= t \int_0^{\infty} d\xi \rho_c e^{-\frac{\xi}{\sqrt{T_c/A}}} \left(\frac{3}{2} \frac{T_c}{A} + \frac{1}{2} \xi^2 + \xi \sqrt{\frac{T_c}{A}} + \frac{1}{2} \frac{T_c}{A} \right)$$

$$= t \left(\frac{T_c}{A} \right)^{3/2} \rho_c \int_0^{\infty} d\zeta e^{-\zeta} \left(2 + \frac{1}{2} \zeta^2 + \zeta \right)$$

$$= 4 \rho_c \left(\frac{T_c}{A} \right)^{3/2} t$$

$$\frac{d\epsilon}{dt} = 4 \rho_c \left(\frac{T_c}{A} \right)^{3/2}$$



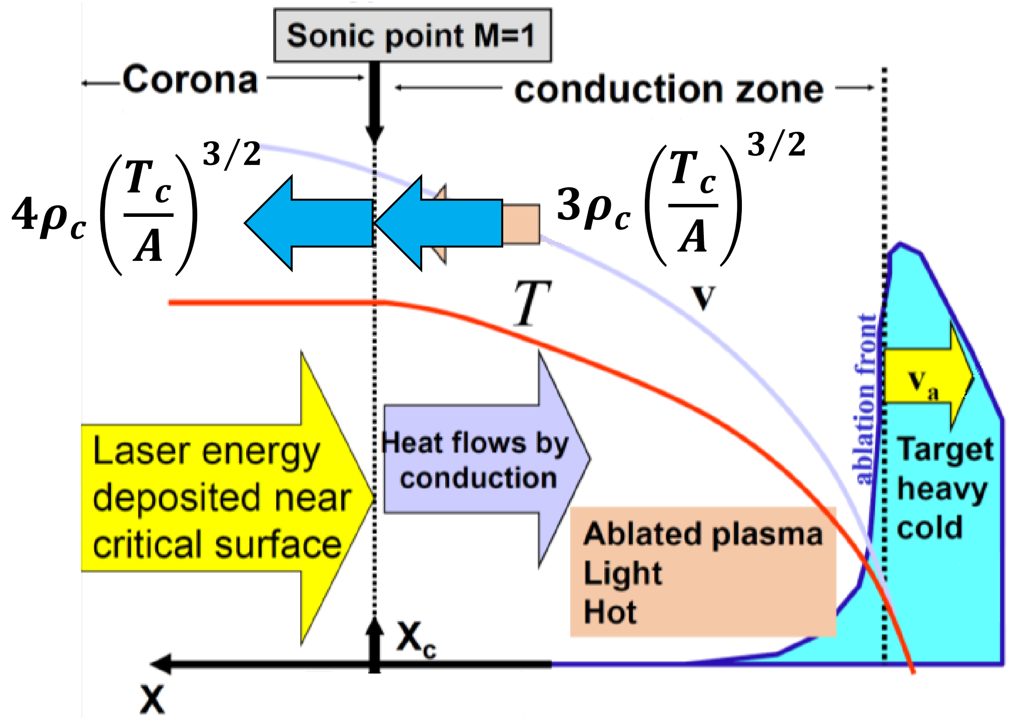
The plasma keeps expanding in the corona zone so that no steady state can be found

$$\kappa^-\left(\frac{\partial T}{\partial x}\right)^- = 3\rho_c\left(\frac{T_c}{A}\right)^{3/2}$$

$$-\kappa^+\left(\frac{\partial T}{\partial x}\right)^+ = \rho_c\left(\frac{T_c}{A}\right)^{3/2}$$

$$I = -\kappa^+\left(\frac{\partial T}{\partial x}\right)^+ + \kappa^-\left(\frac{\partial T}{\partial x}\right)^-$$

$$= 4\rho_c\left(\frac{T_c}{A}\right)^{3/2}$$

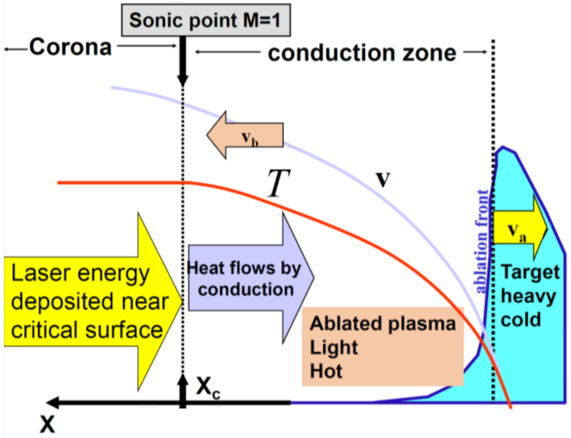


The plasma keeps expanding in the corona zone so that no steady state can be found

- **Total ablation pressure (static + dynamic):**

$$P_A = \frac{\rho_c T_c}{A} + \rho_c {v_c}^2 = 2\frac{\rho_c T_c}{A} \sim \rho_c \frac{I^{2/3}}{{\rho_c}^{2/3}} \sim {\rho_c}^{1/3} I^{2/3}$$

$$v_c = \sqrt{\frac{T_c}{A}} \qquad I = 4\rho_c \left(\frac{T_c}{A}\right)^{3/2}$$



- **Temperature at critical surface:** $T_c \sim \left(\frac{I}{\rho_c}\right)^{2/3}$

- **Velocity at critical surface:** $v_c \sim \left(\frac{I}{\rho_c}\right)^{1/3}$

- **Ablation rate:** $\rho_c v_c \sim {\rho_c}^{2/3} I^{1/3}$