Introduction to Nuclear Fusion as An Energy Source

Po-Yu Chang

Institute of Space and Plasma Sciences, National Cheng Kung University

Lecture 10

2026 spring semester

Tuesday 9:00-12:00

Materials:

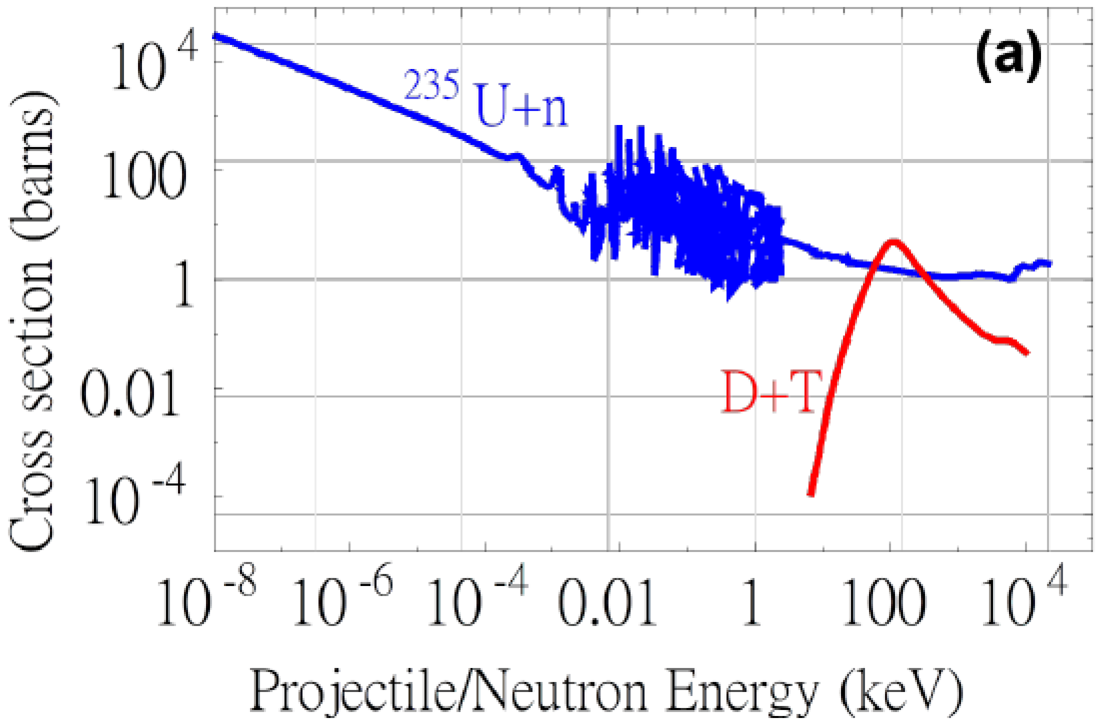
https://capst.ncku.edu.tw/PGS/index.php/teaching/

Online courses:

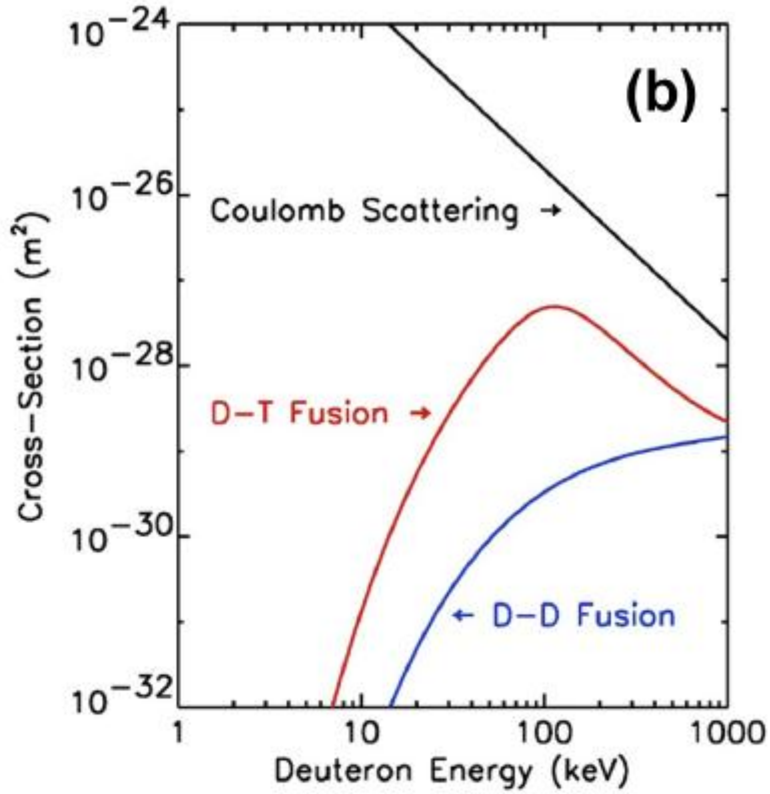
https://reurl.cc/GaLmxA (Google meet)

2026/5/14 updated

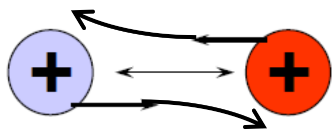
1. [5 points in total] Shown in the following figure (a) is the comparison between cross sections of ^{235}U+n fission and D+T fusion. Please explain why nuclear fission is much easier to achieve than nuclear fusion.

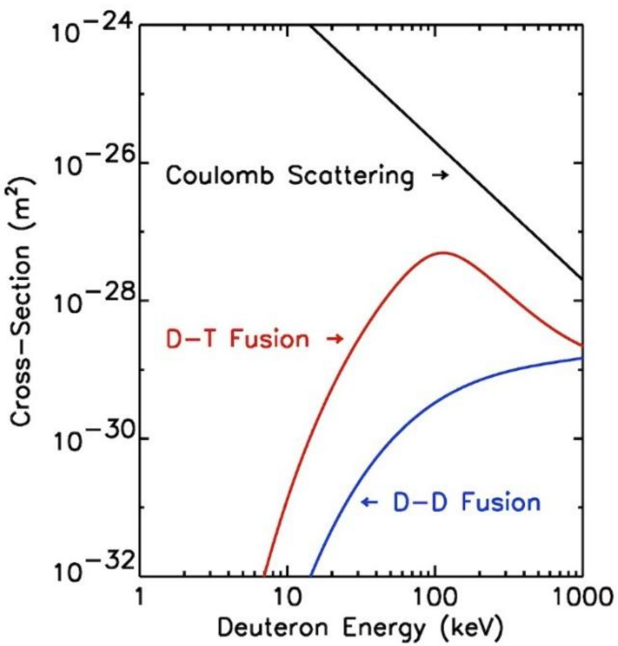


2. [10 points in total] Shown in the above figure (b) is the comparison between cross sections of coulomb scattering, D-T fusion, and D-D fusion. Please explain why "beam-target" fusion is not viable for energy generation.

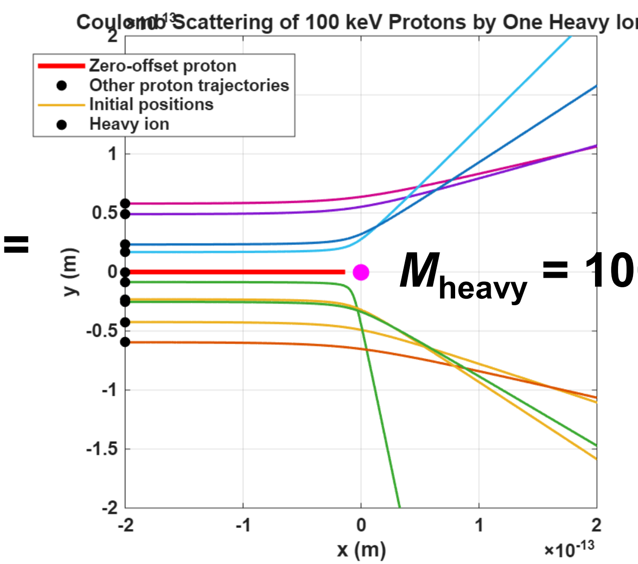


Coulomb scattering is more likely to happen than nuclear fusion





$E_{incoming}$ = 100 keV



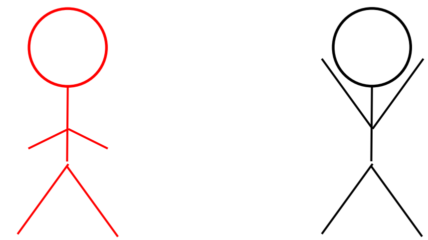
$M_{heavy} = 1000\ M_p$

- **Beam particles lose their energy before making a fusion reaction, i.e., they only thermalize the fuel. Therefore, beam fusion does NOT work!**

https://www6.lehigh.edu/~eus204/lab/PCL_fusion.php#x1-10096
https://www.16pic.com/photo/pic_3815585.html

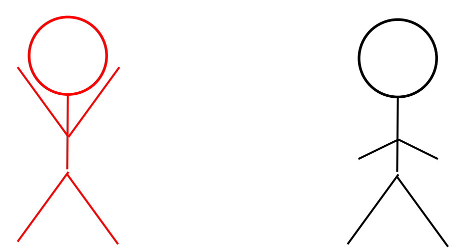
Who wins the game? Who wins the money?

- **One game:**



	Alice	Bob
Winning probability	0.00001%	99.99999%
Winning money	$100	$0
Who win after one game?	X	O
Who win the money after one game?	X	X
Who wins most the games over one night?	X	O
Who wins the money after one night?	O	X

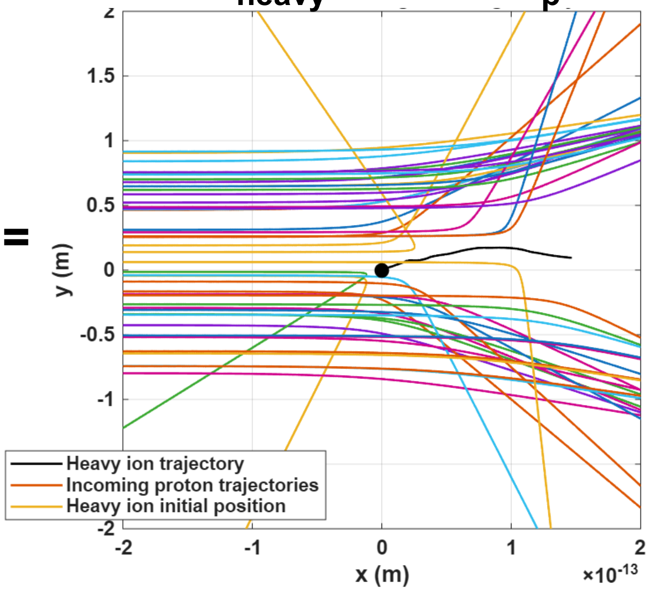
- **One night:**



Hot plasma needs to be confined so that particles have chance to collide with each other

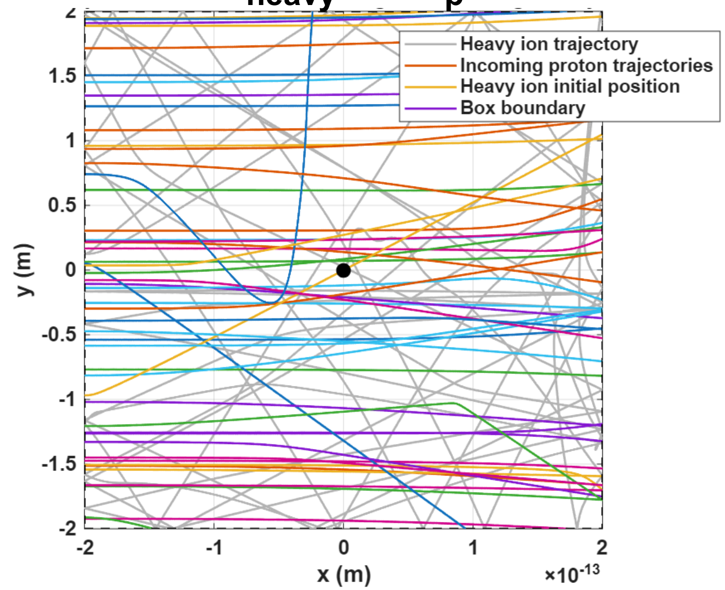
$M_{heavy} = 1000\ M_p$

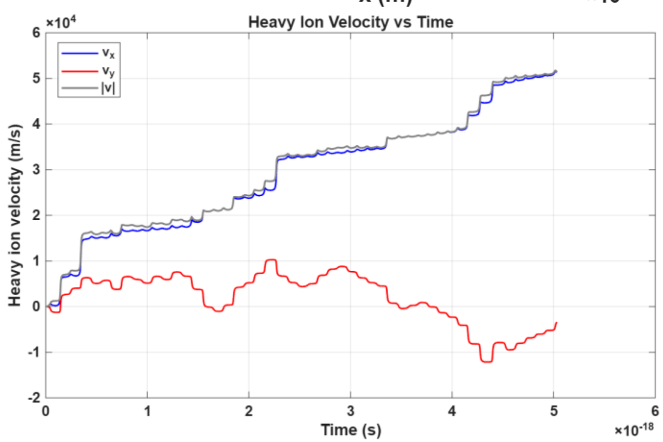
$E_{incoming}$ = 100 keV

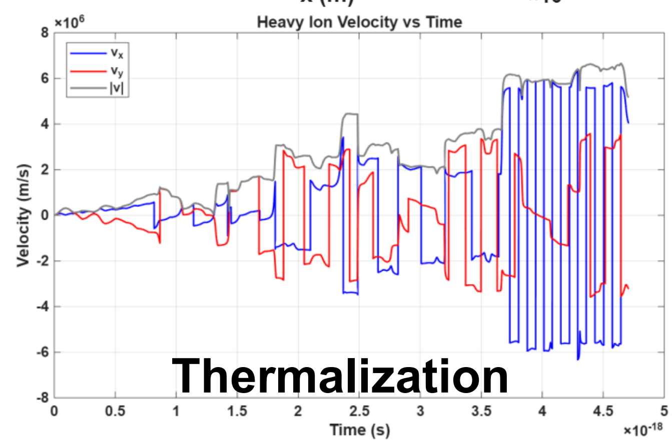


$M_{heavy} = M_p$

$E_{incoming}$ = 100 keV

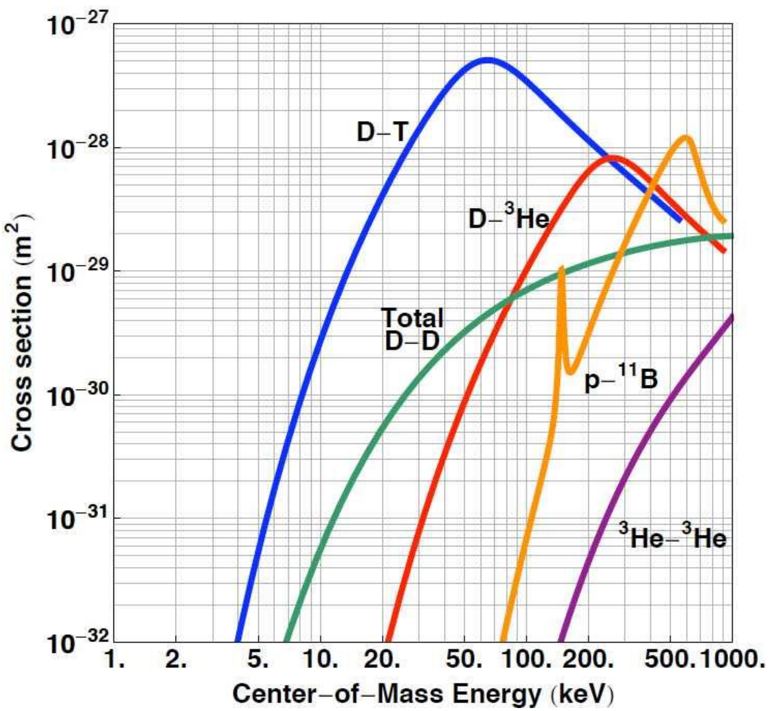


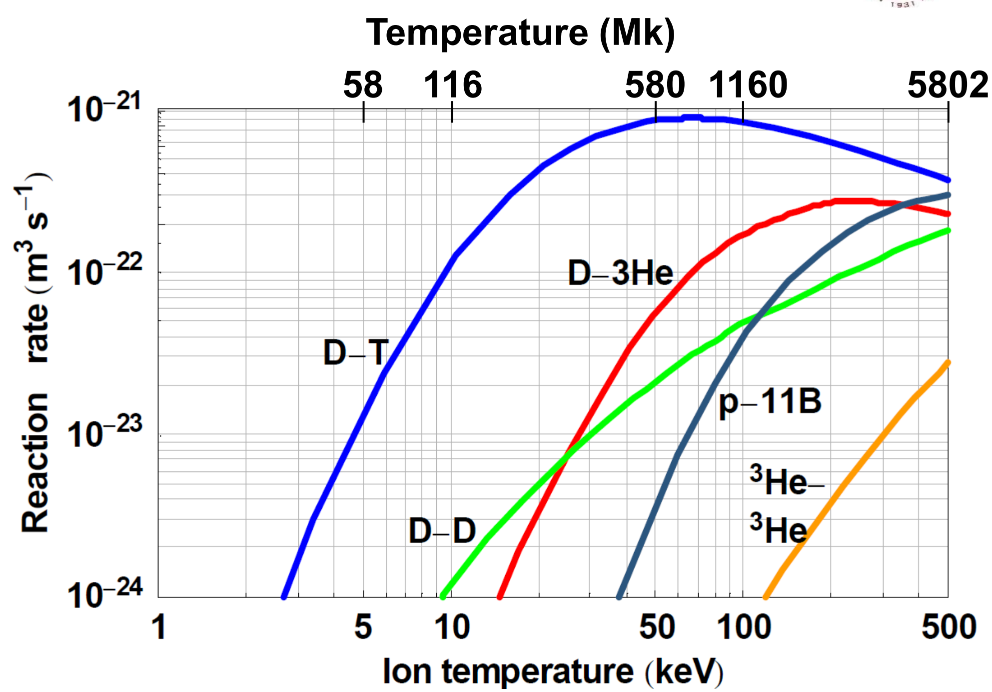




- **Beam particles just thermalize the fuel. Beam-target fusion does NOT work!**

Fusion doesn't come easy, a "hot plasma" at 100M ⁰C is needed





$D + D \rightarrow T + p$

$\quad\;\; \rightarrow He^3 + n$

$D + T \rightarrow He^4 + n$

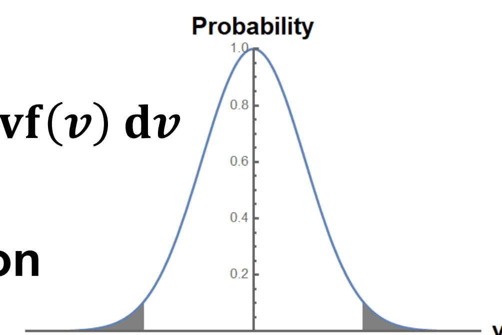
$D + He^3 \rightarrow He^4 + p$

$p + B^{11} \rightarrow 3He^3$

- **Reaction rate:**

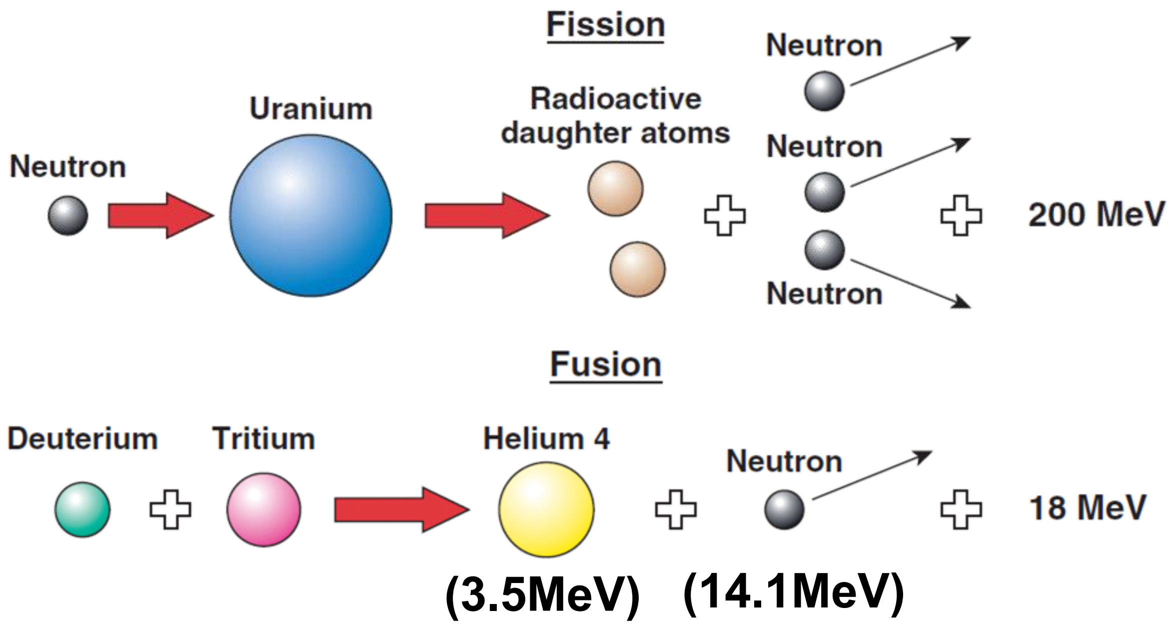
$$<\sigma v> = \int \sigma(v) v f(v)\, dv$$

Cross section



https://i.stack.imgur.com/wXQD5.jpg
Santarius, J. F., "Fusion Space Propulsion – A Shorter Time Frame Than You Think", JANNAF, Monterey, 5-8 December 2005.

3. [5 points in total] When a neutron collides with an ^{235}U, the nuclear fission reaction releases energy of $\sim$ 200 MeV. On the other hand, in a deuterium-tritium (D-T) fusion reaction, only 17.6 MeV of energy is released. Why does the D-T fusion reaction provide more energy than the neutron-^{235}U fission reaction for the same amount of fuel in mass?



- **Fission of ^{235}U:** $$\frac{Q}{A} = \frac{200\ \mathrm{MeV}}{(235+1)\mathrm{amu}} = 0.85\ \mathrm{MeV/amu}$$
- **Fusion of D+T:** $$\frac{Q}{A} = \frac{17.6\ \mathrm{MeV}}{(2+3)\mathrm{amu}} = 3.5\ \mathrm{MeV/amu}$$

4. [5 points in total] To have a self-sustain system such that the plasma can be heated by itself and remains hot, the heating from the alpha particles generated in the D-T fusion reactions ($D + T \rightarrow {}^4He + n$) needs to be higher than the energy losses, which are mostly by Bremmstrahlung radiations at high temperature. As a result, an ignition condition, called the Lawson criterion, where $P\tau > 10$ atm $-$ s, needs to be achieved. Please explain the physical meaning, NOT the math, of the criterion.

- **Steady state 0-D power balance:**

  $$S_\alpha + S_h = S_B + S_k$$

  S_α: α particle heating

  S_h: external heating

  S_B: Bremsstrahlung radiation

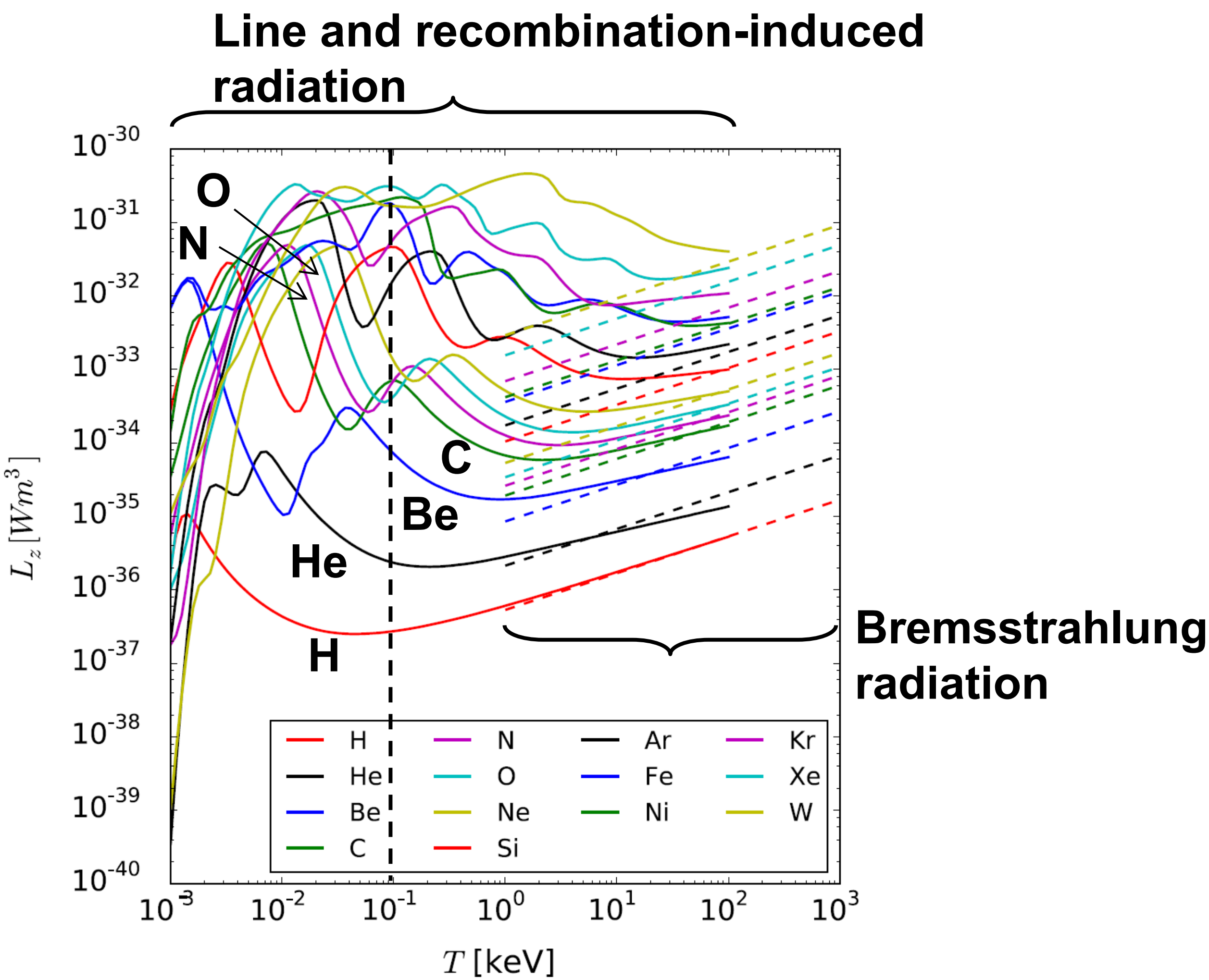
  S_k: heat conduction lost

Ignition condition: Pτ > 10 atm-s = 10 Gbar - ns

- **P: pressure, or called energy density**
- **τ is confinement time**

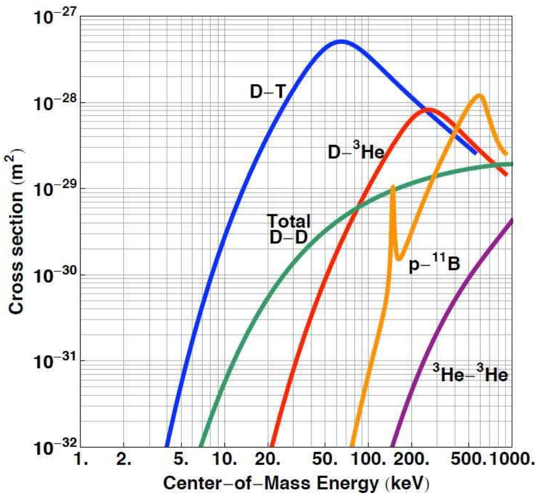
5. [10 points in total] The magnetic field of Formosa Integrated Research Spherical Tokamak (FIRST) is $B_{\mathrm{T}} = 0.1$ T at $R = 45$ cm. The small radius of FIRST is $a = 32$ cm. Hydrogen gas will be used in FIRST. To simplify the problem, assume the cross section of FIRST on the poloidal plane is a perfect circle with the small radius $a = 32$ cm. When the temperature of protons achieves 100 eV, is the field of 0.1 T is enough to confine 100-eV protons?

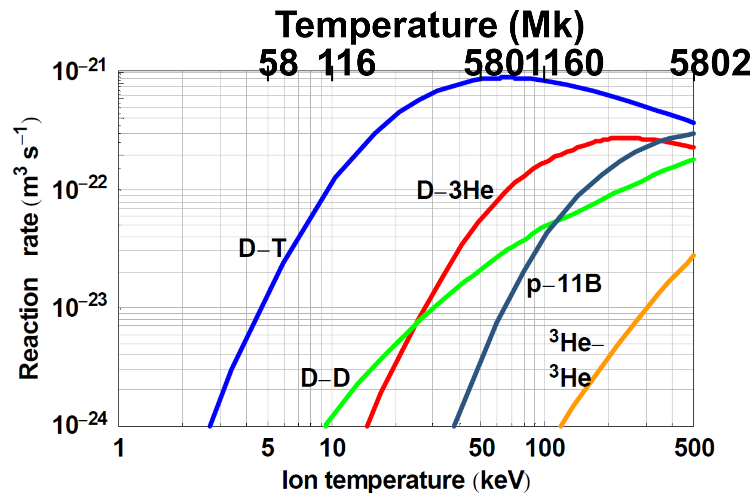
6. [10 points in total] Shown in the following is the loss function from line, recombination-induced, and bremsstrahlung radiation for different atoms at different temperatures. Please explain **(a)** what a "radiation barrier" at 100 eV is [7 points], and **(b)** why a temperature of 100 eV was picked as the milestone for the Formosa Integrated Research Spherical Tokamak (FIRST)? [3 points]



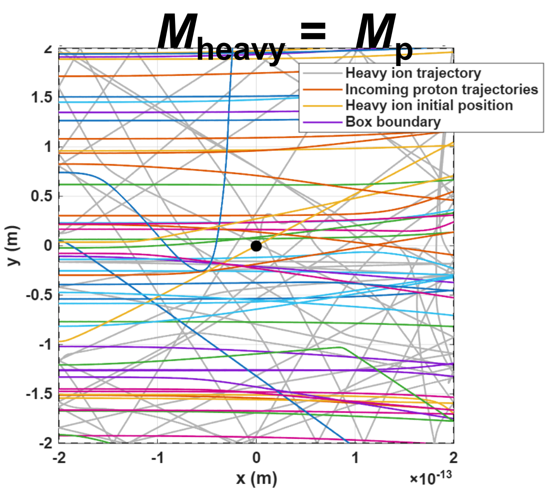
H. Lux, etc., Fusion Eng. Des. **42**, 101 (2015)

7. [5 points in total] Shown in the following figure (a) are the cross sections of different fusion reactions for different center-of-mass energy (in keV). On the other hand, the following figure (b) are the reaction rates of different fusion reactions at different temperature (in keV). Please explain what's the difference between "cross sections" and "reaction rates"?





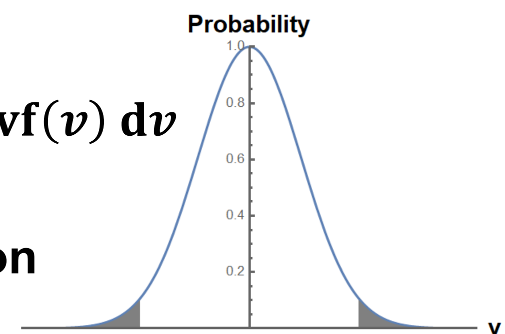
$E_{incoming}$ = 100 keV



- **Reaction rate:**

$$< \sigma v >= \int \sigma(v) v f(v)\, dv$$

Cross section

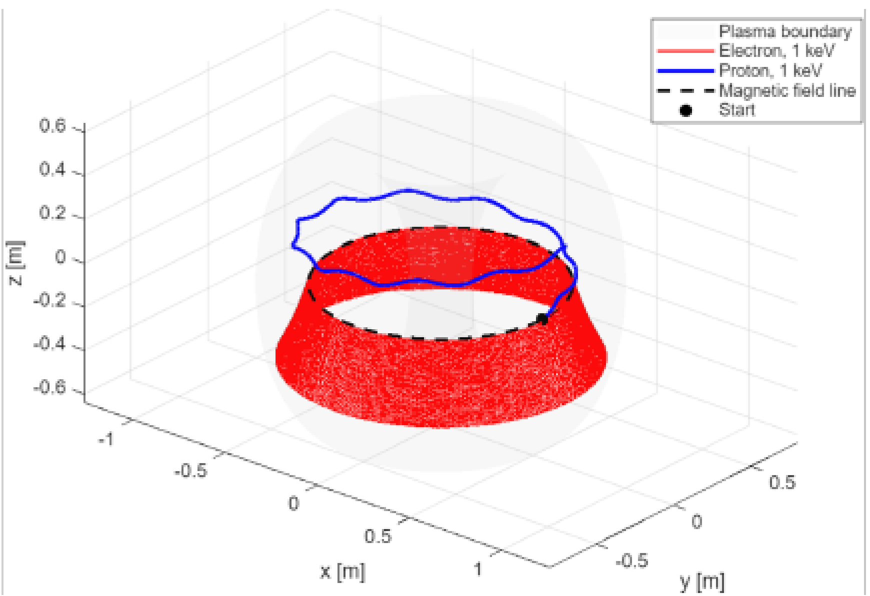


https://i.stack.imgur.com/wXQD5.jpg

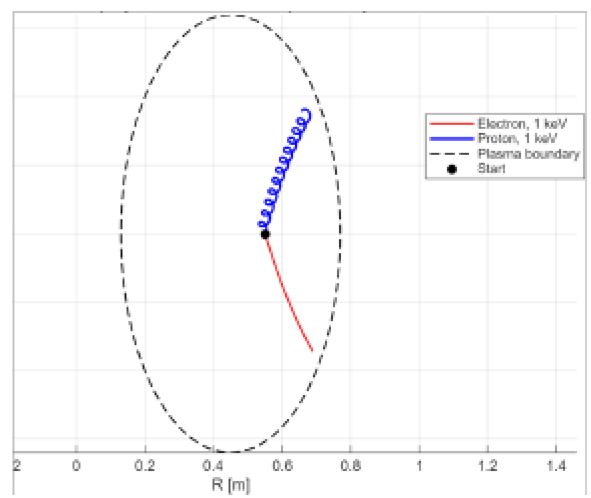
Santarius, J. F., "Fusion Space Propulsion – A Shorter Time Frame Than You Think", JANNAF, Monterey, 5-8 December 2005.

8. [10 points in total] In homework 1, we plotted trajectories of electrons and protons with only toroidal magnetic field and a negative electric field to mimic consequence of the charge separation. Following are the 3D trajectories of electrons and protons in figure (a), and the poloidal-plane projection of trajectories in figure (b). Red curves are for electrons while blue curves are for protons in both figures. Please explain what happen and why particles can not be confined with only toroidal fields.

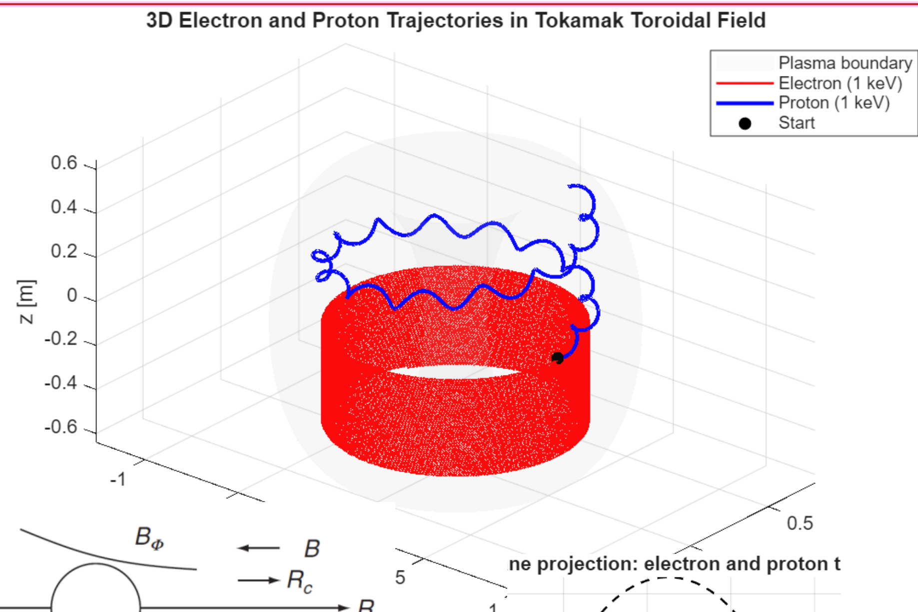
(a) Trajectories of electrons and Protons with only Toroidal fields

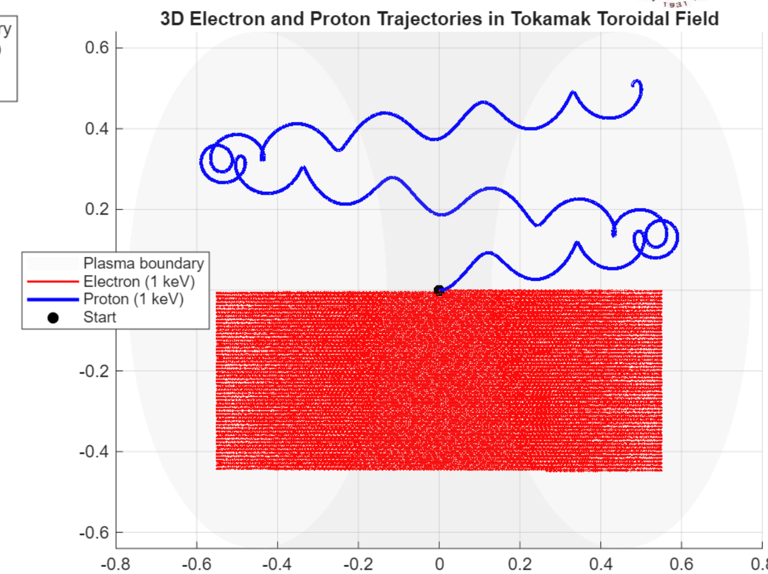


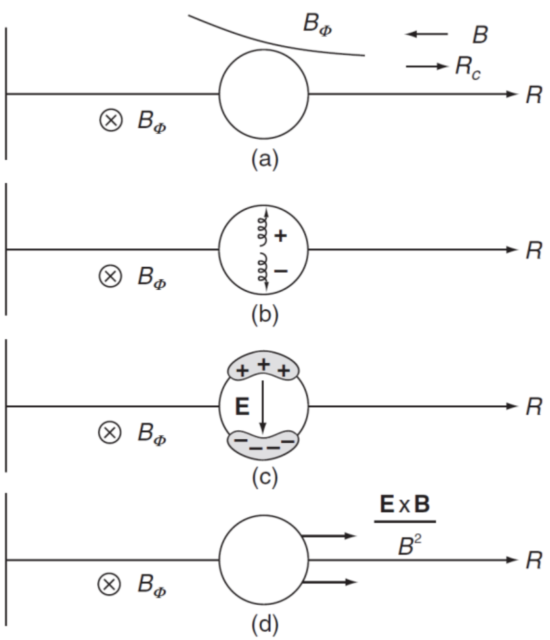
(b) Poloidal-plane projection of trajectories

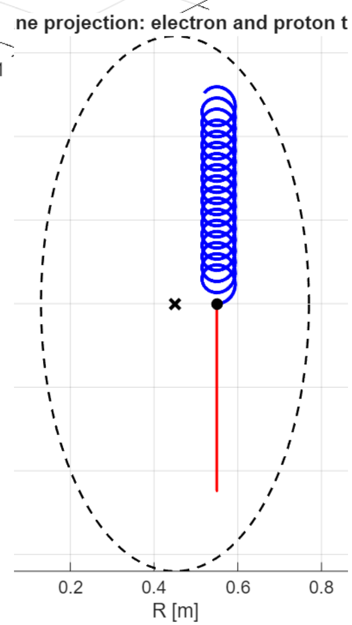


4. [10 points in total] A spherical tokamak with a major radius $R = 45$ cm, a minor radius $a = 32$ cm, and an elongation of $\kappa = 2$. The toroidal field $B_T = 0.1$ T at 45 cm. The plasma current $I_p = 0$. An **electron** with an energy of 1 keV. Initial position at $R = 55$ cm. Please compare the result with condition 3.

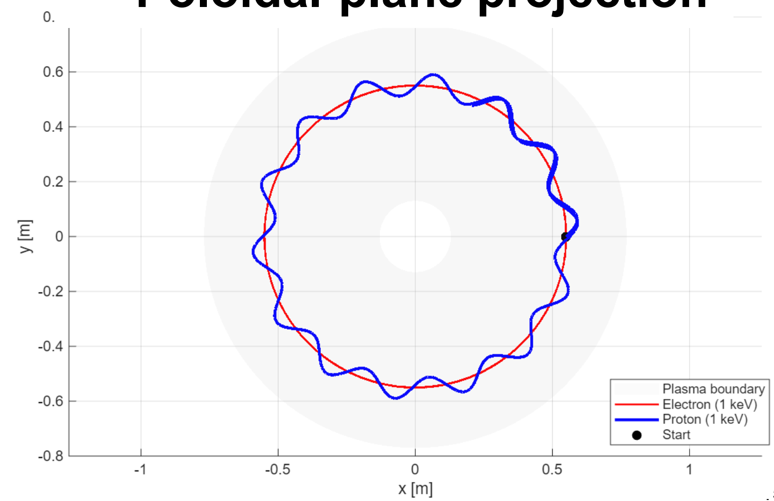




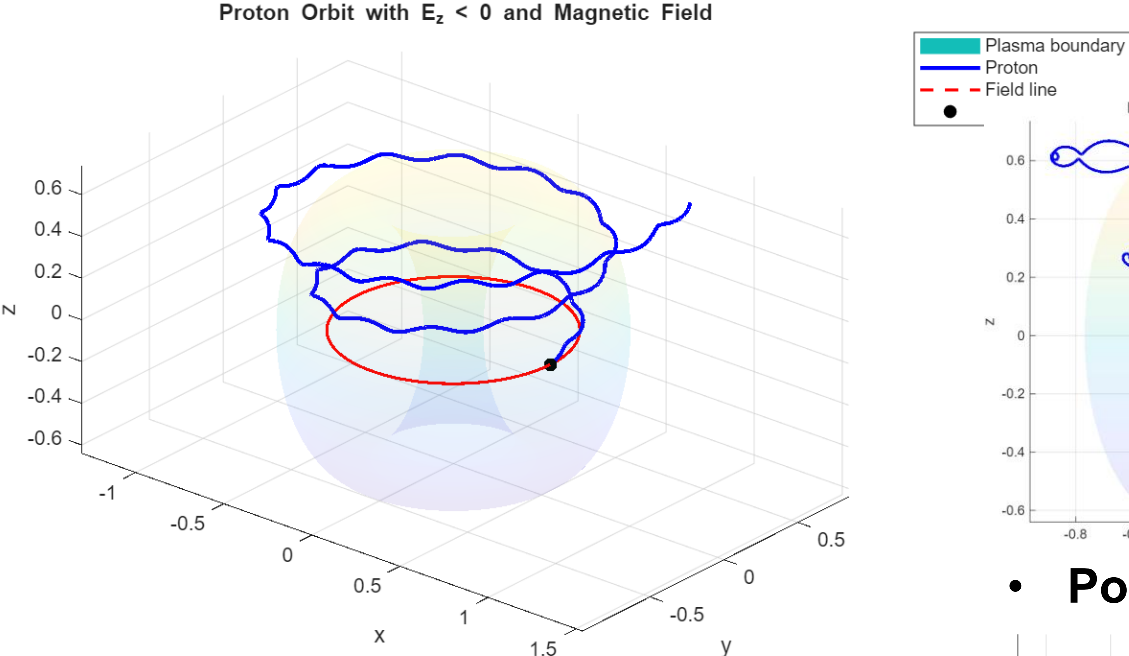


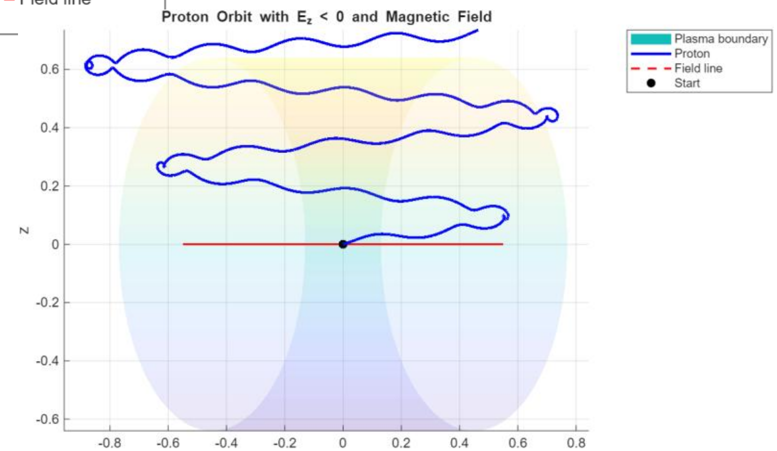


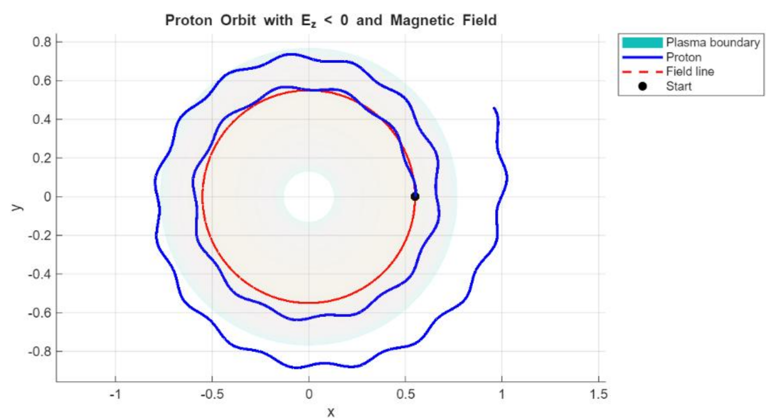
- **Poloidal-plane projection**



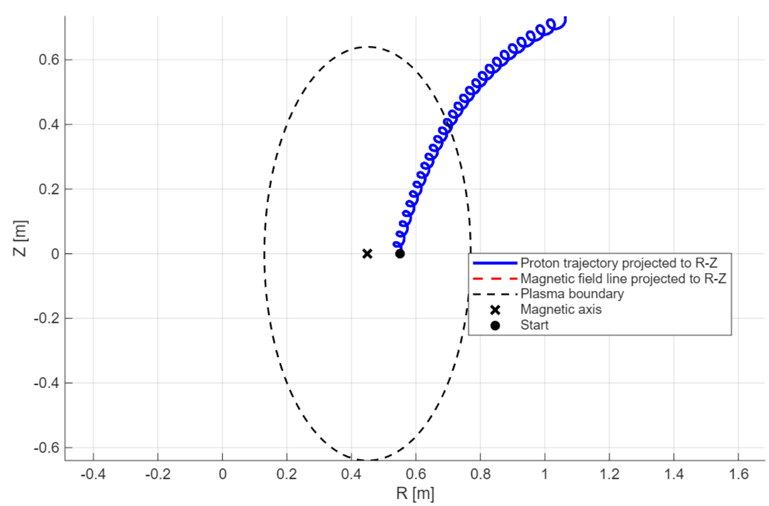
5. [10 points in total] A spherical tokamak with a major radius $R = 45$ cm, a minor radius $a = 32$ cm, and an elongation of $\kappa = 2$. The toroidal field $B_T = 0.1$ T at 45 cm. The plasma current $I_p = 0$. A electric field $E_z = -1000$ V/m in $\hat{z}$. A **proton** with an energy of 1 keV. Initial position at $R = 55$ cm.



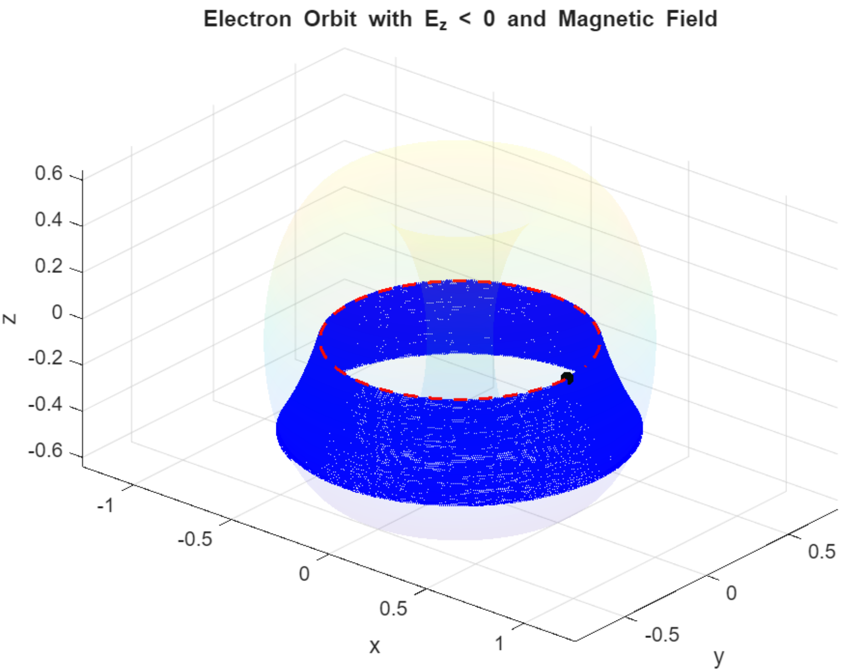


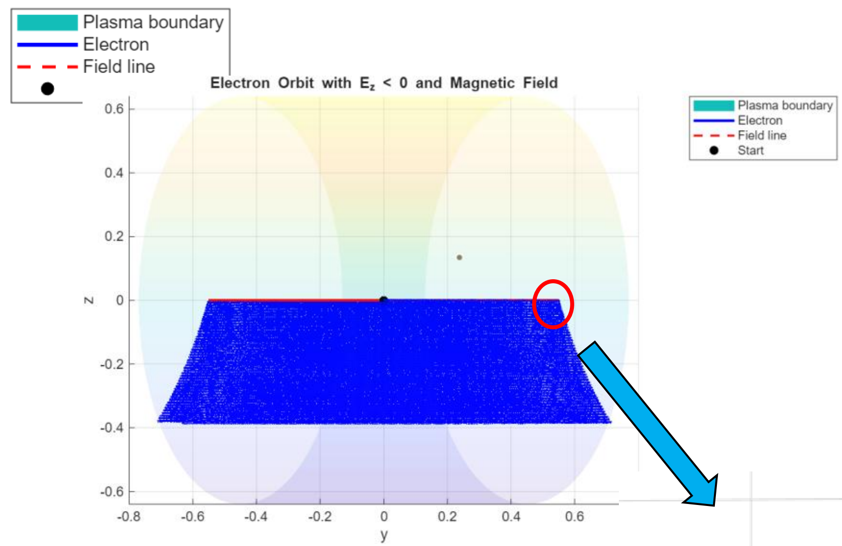


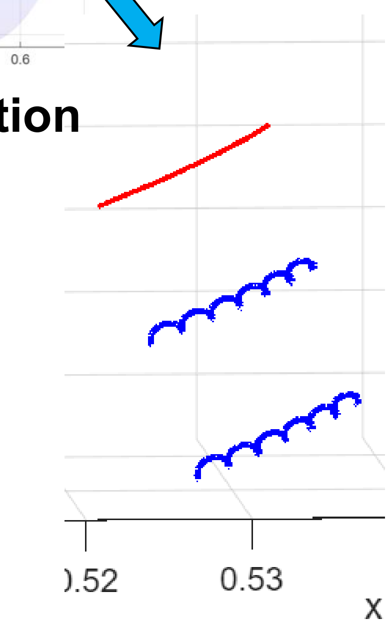
- **Poloidal-plane projection**



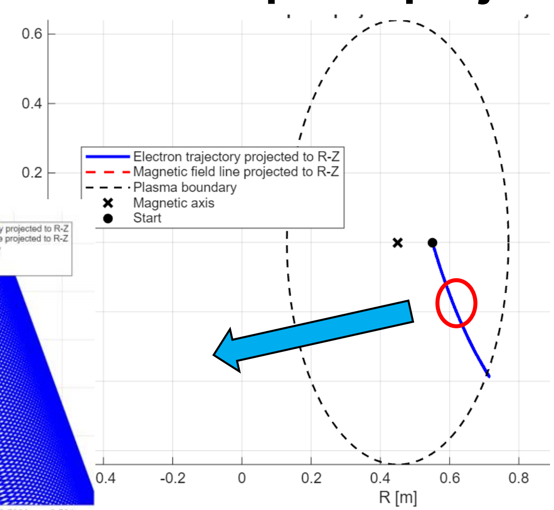
6. [10 points in total] A spherical tokamak with a major radius $R = 45$ cm, a minor radius $a = 32$ cm, and an elongation of $\kappa = 2$. The toroidal field $B_{\mathrm{T}} = 0.1$ T at 45 cm. The plasma current $I_{\mathrm{p}} = 0$. A electric field $E_z = -1000$ V/m in $\hat{z}$. An **electron** with an energy of 1 keV. Initial position at $R = 55$ cm. Please compare the result with condition 5.

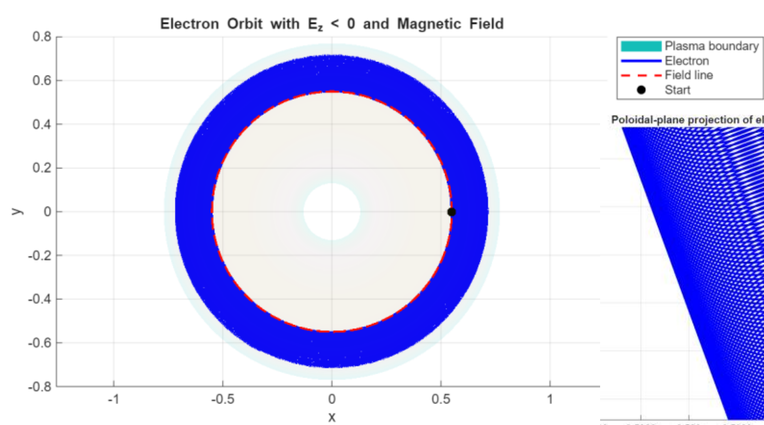


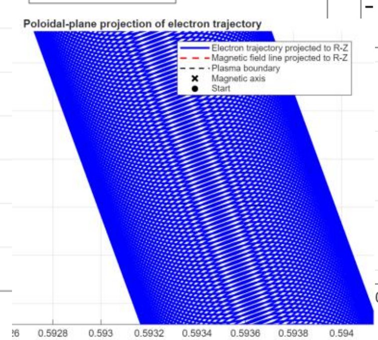




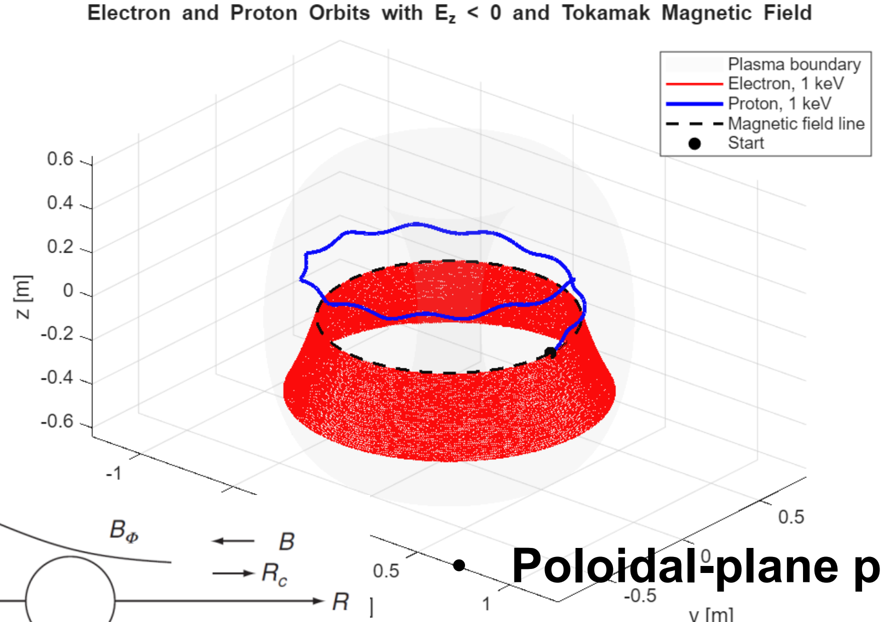
- **Poloidal-plane projection**

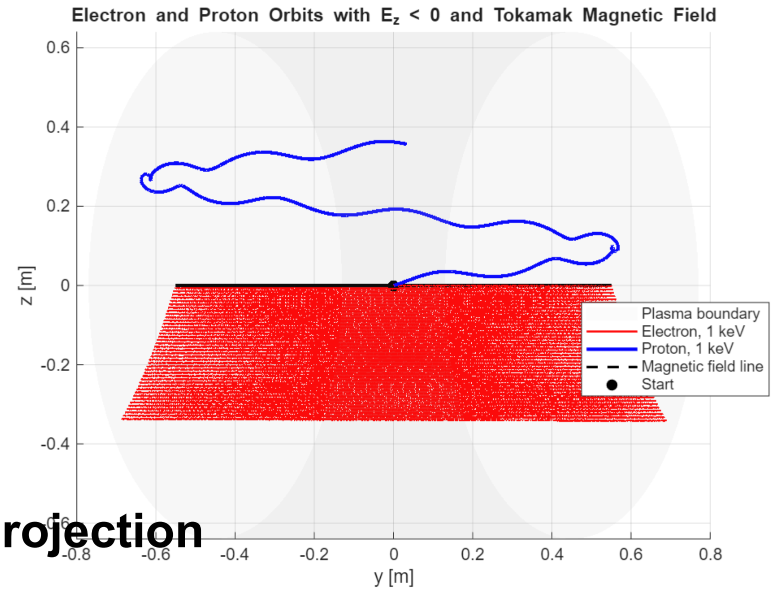




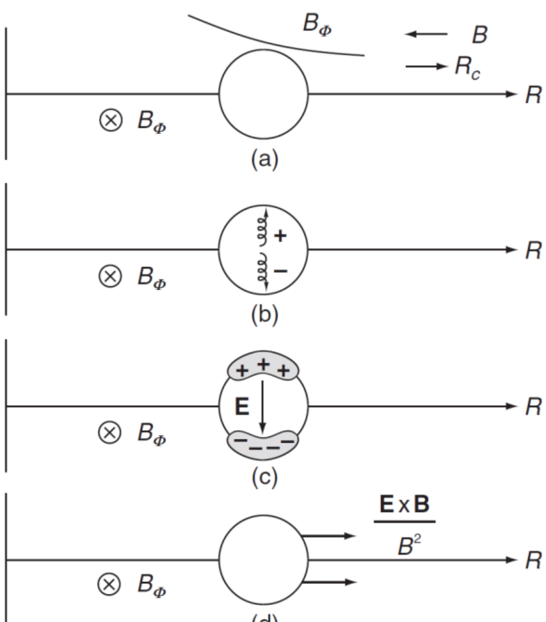


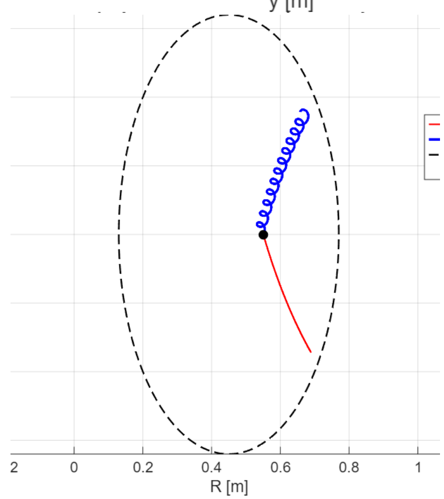
6. [10 points in total] A spherical tokamak with a major radius $R = 45$ cm, a minor radius $a = 32$ cm, and an elongation of $\kappa = 2$. The toroidal field $B_T = 0.1$ T at 45 cm. The plasma current $I_p = 0$. A electric field $E_z = -1000$ V/m in $\hat{z}$. An **electron** with an energy of 1 keV. Initial position at $R = 55$ cm. Please compare the result with condition 5.

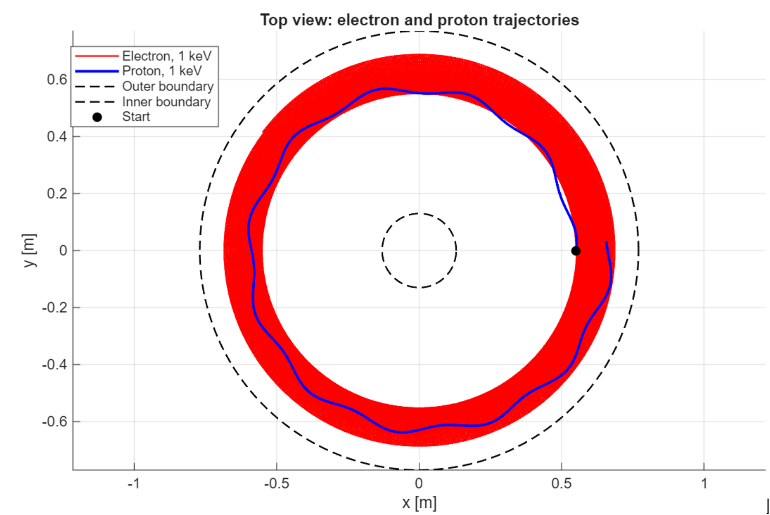




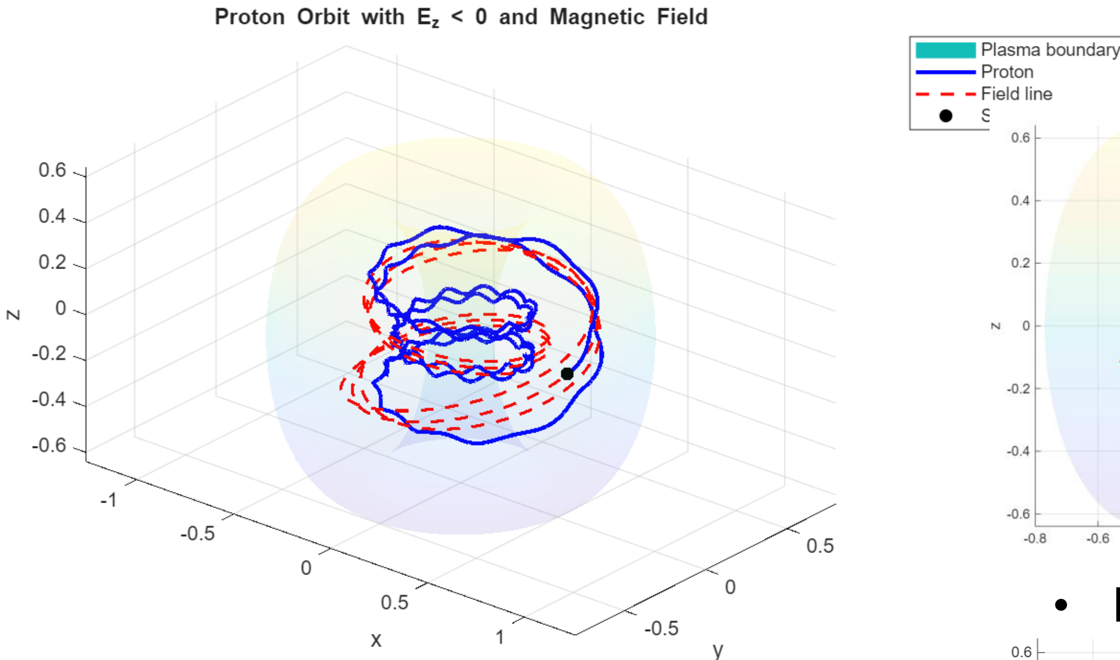
- **Poloidal-plane projection**

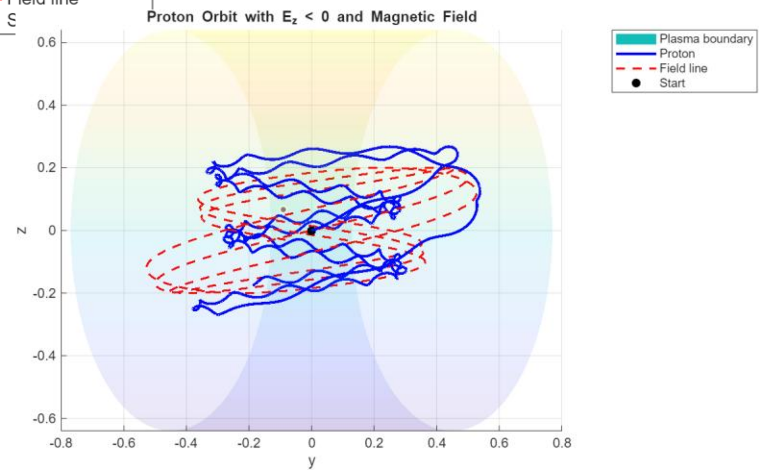


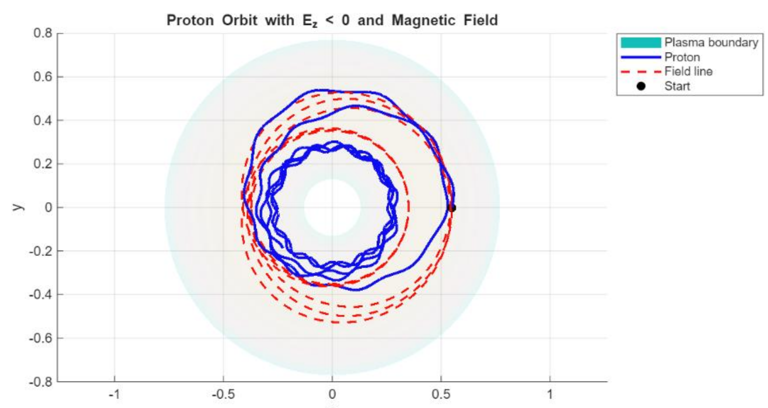




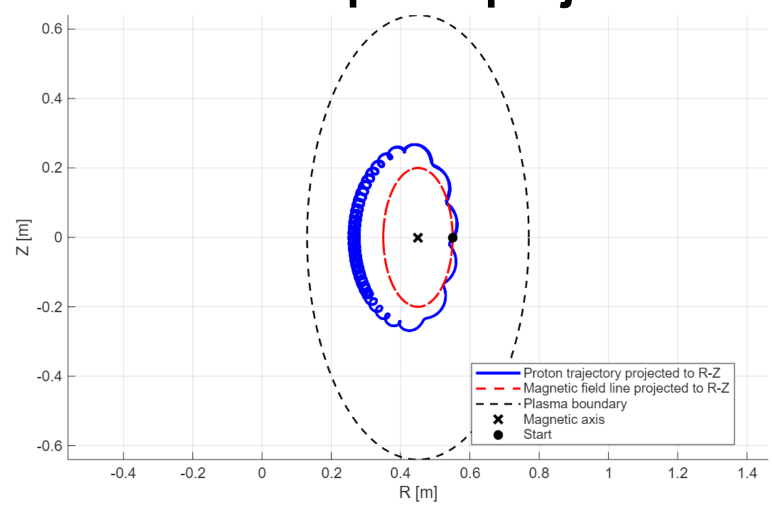
7. [10 points in total] A spherical tokamak with a major radius $R = 45$ cm, a minor radius $a = 32$ cm, and an elongation of $\kappa = 2$. The toroidal field $B_{\mathrm{T}} = 0.1$ T at 45 cm. The plasma current $I_{\mathrm{p}} = 100$ kA. A **proton** with an energy of 1 keV. Initial position at $R = 55$ cm.



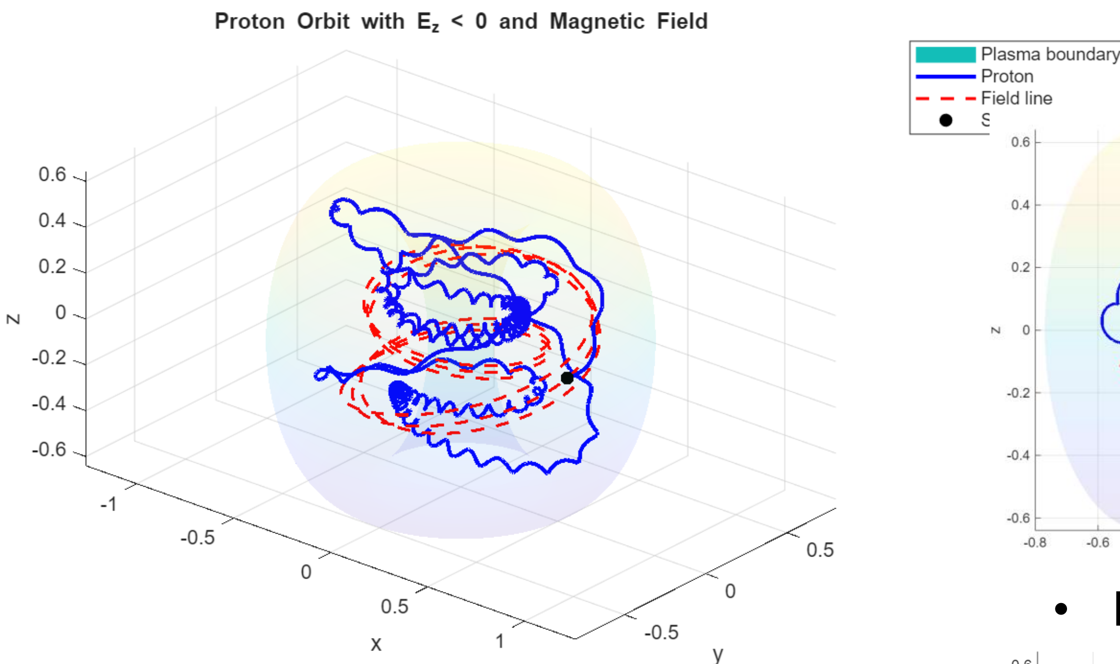


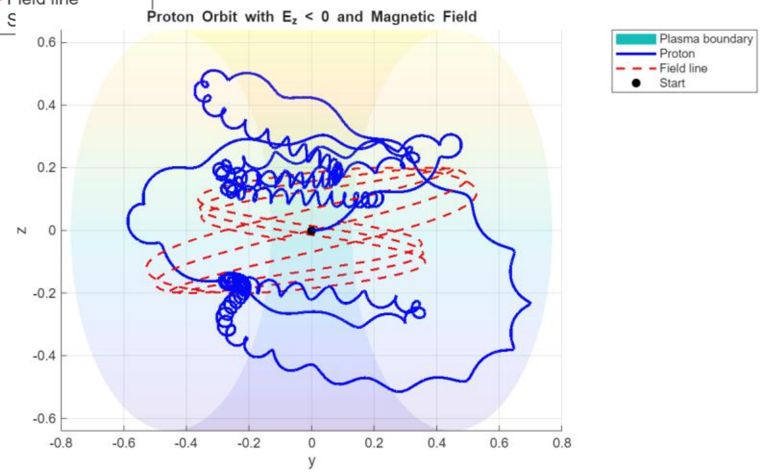


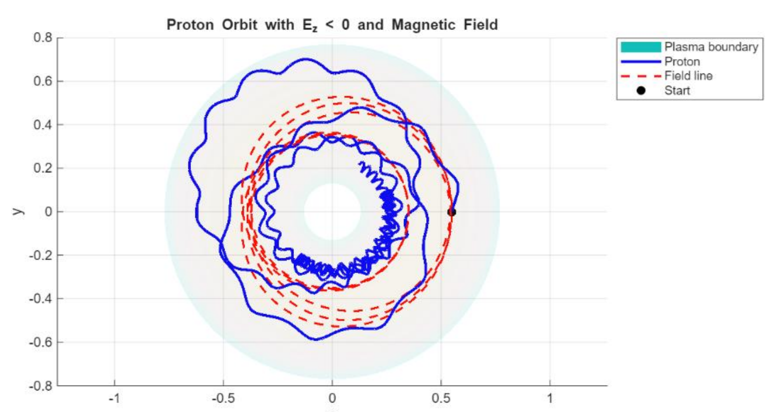
- **Poloidal-plane projection**



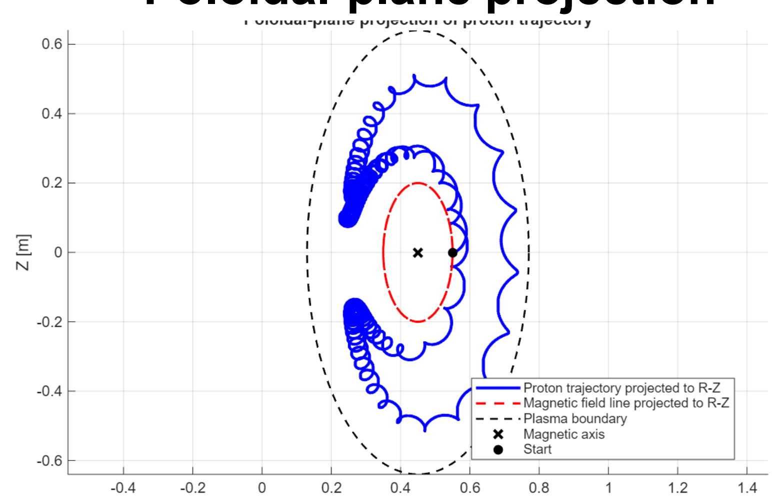
7. [10 points in total] A spherical tokamak with a major radius $R = 45$ cm, a minor radius $a = 32$ cm, and an elongation of $\kappa = 2$. The toroidal field $B_\mathrm{T} = 0.1$ T at 45 cm. The plasma current $I_\mathrm{p} = 100$ kA. A **proton** with an energy of 1 keV. Initial position at $R = 55$ cm.



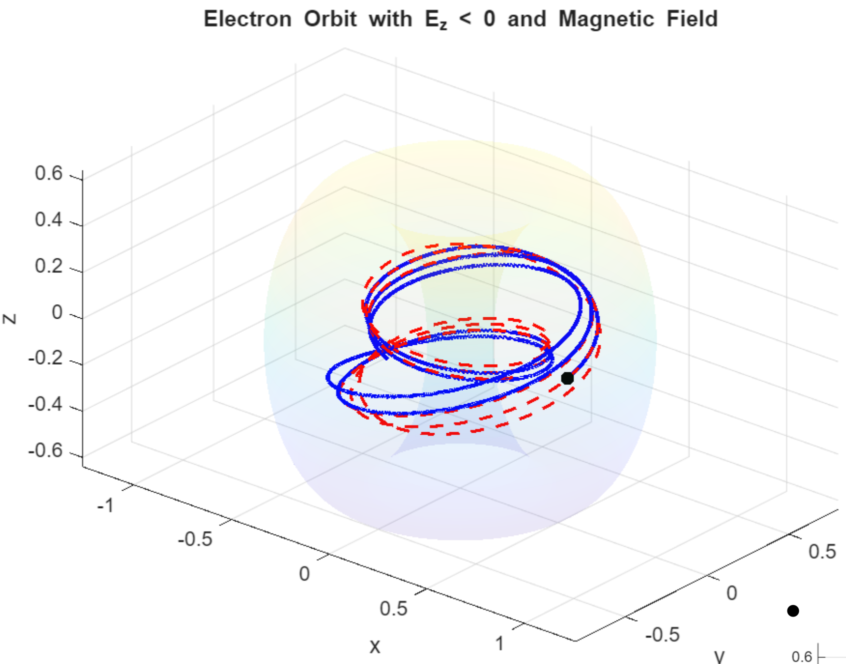


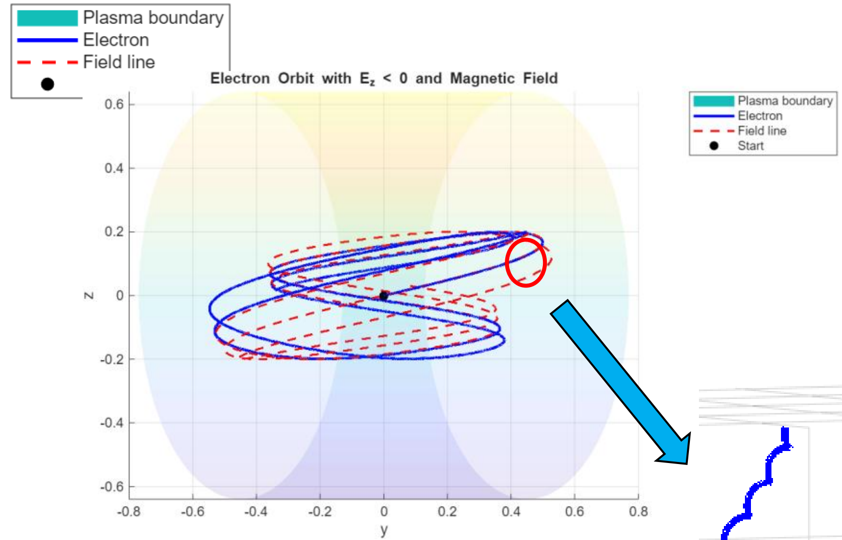


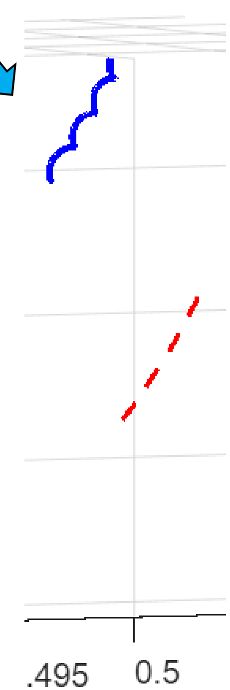
- **Poloidal-plane projection**

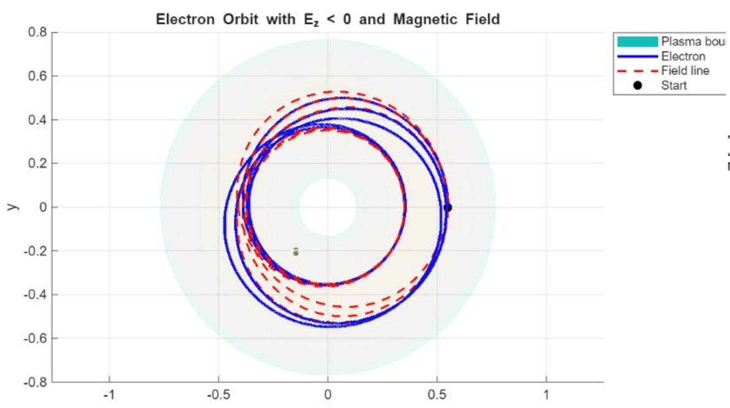


8. [10 points in total] A spherical tokamak with a major radius $R = 45$ cm, a minor radius $a = 32$ cm, and an elongation of $\kappa = 2$. The toroidal field $B_{\mathrm{T}} = 0.1$ T at 45 cm. The plasma current $I_{\mathrm{p}} = 100$ kA. An **electron** with an energy of 1 keV. Initial position at $R = 55$ cm. Please compare the result with condition 7.

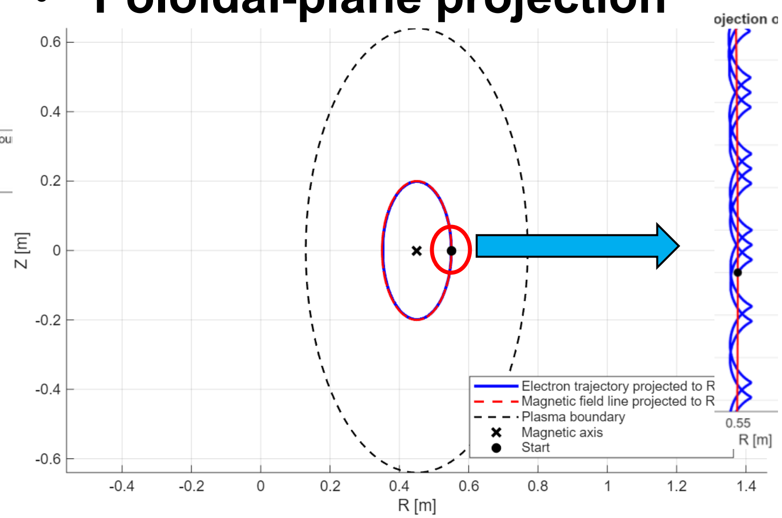




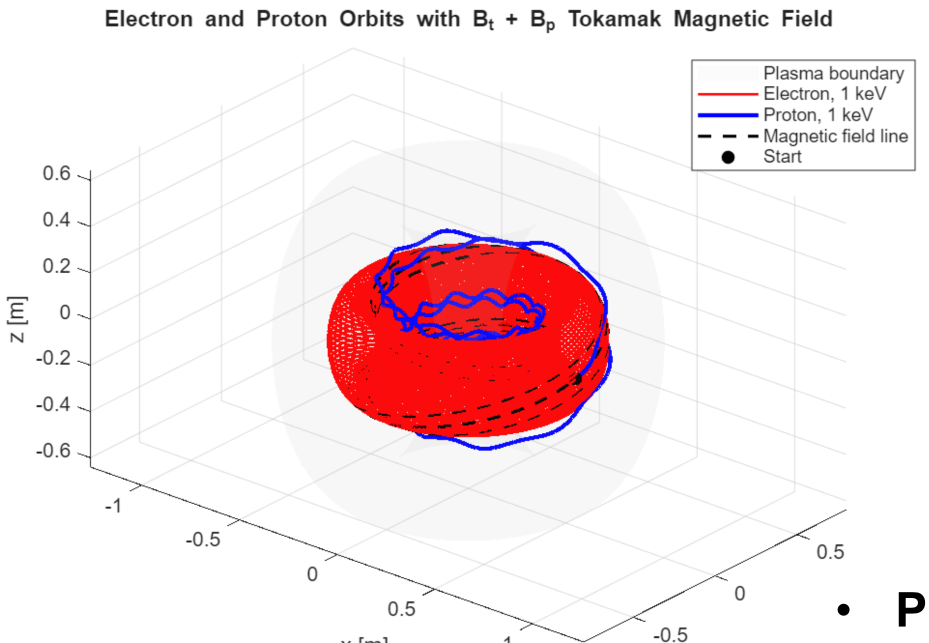


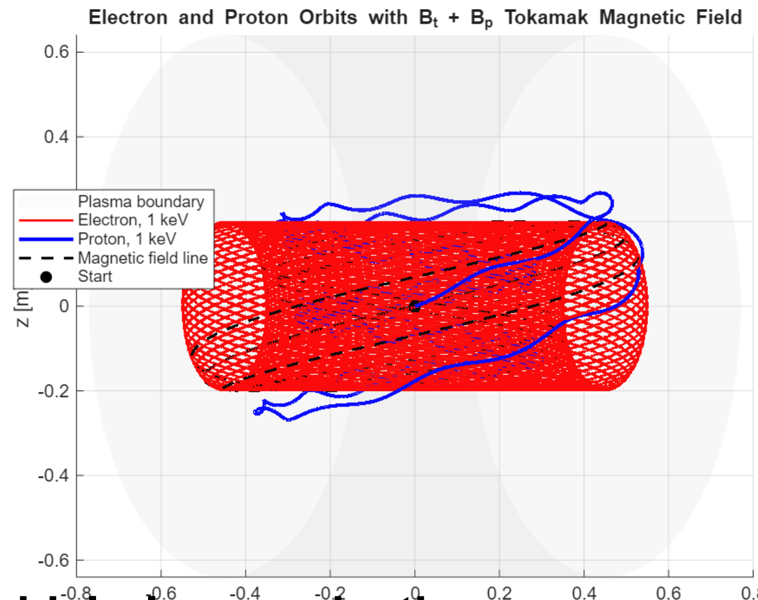


- **Poloidal-plane projection**

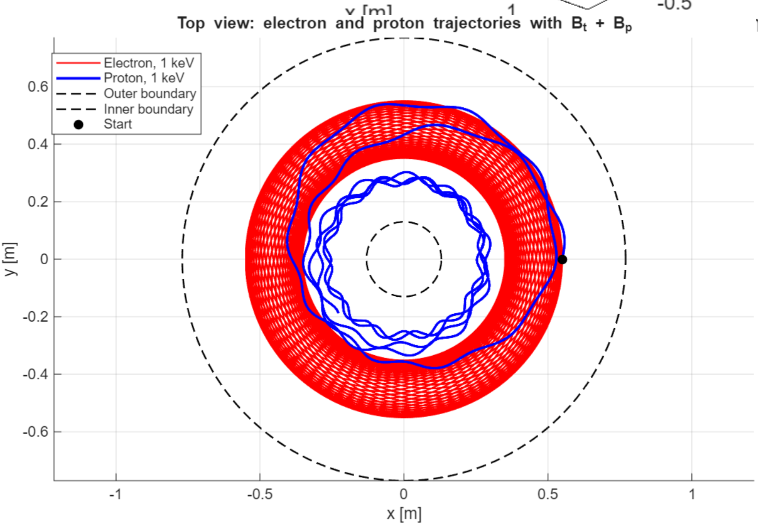


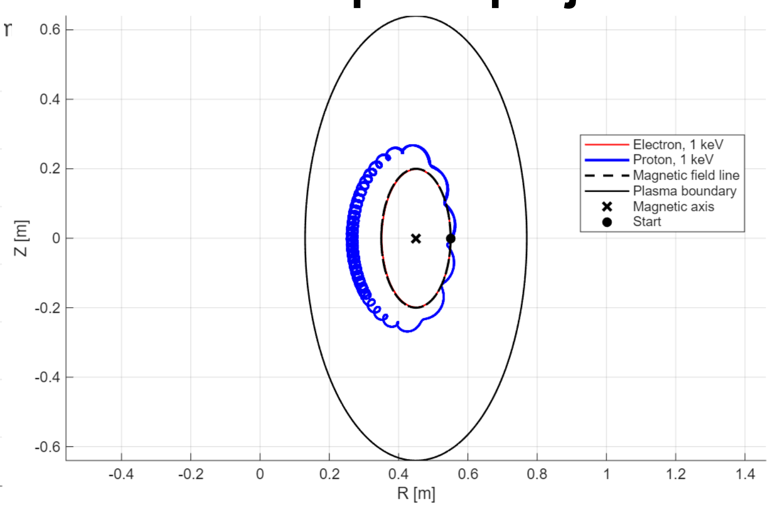
8. [10 points in total] A spherical tokamak with a major radius $R = 45$ cm, a minor radius $a = 32$ cm, and an elongation of $\kappa = 2$. The toroidal field $B_\mathrm{T} = 0.1$ T at 45 cm. The plasma current $I_\mathrm{p} = 100$ kA. An **electron** with an energy of 1 keV. Initial position at $R = 55$ cm. Please compare the result with condition 7.





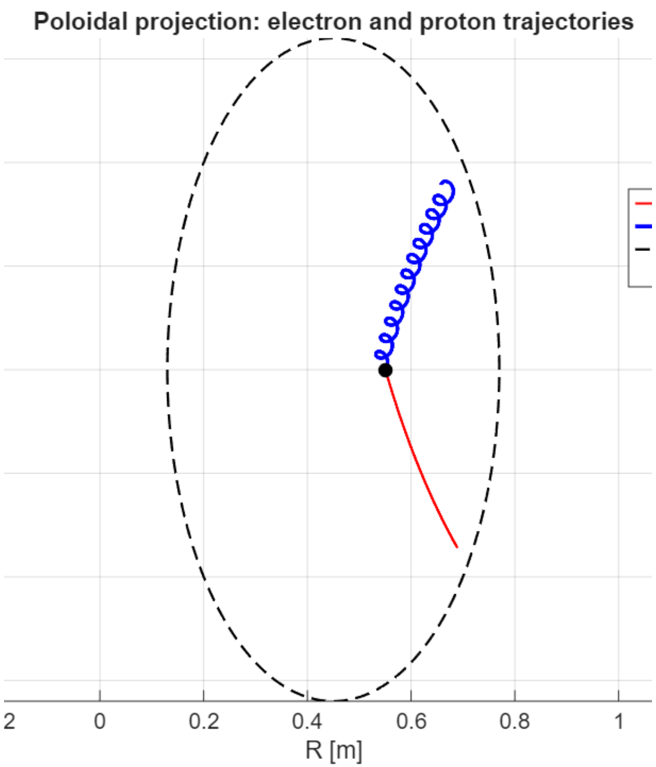
- **Poloidal-plane projection**



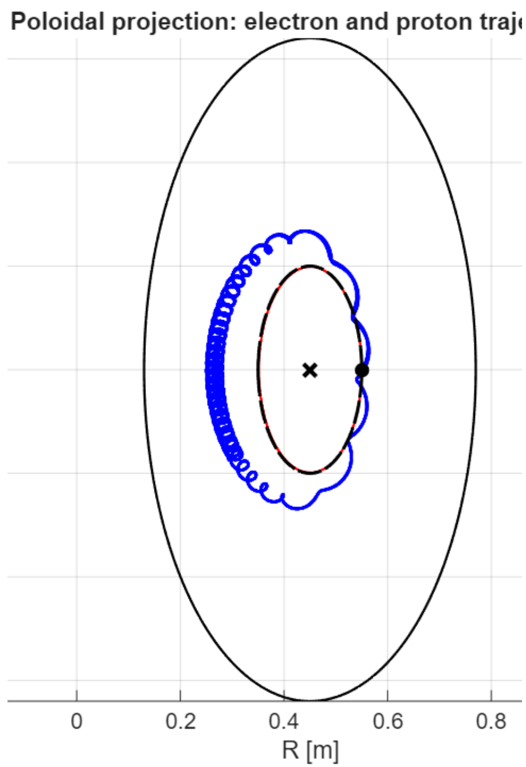


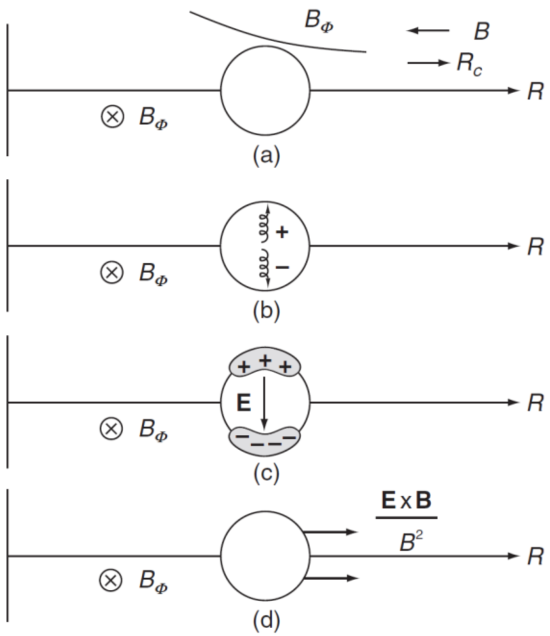
Plasma current plays an important role for confinement

- **W/O plasma current**

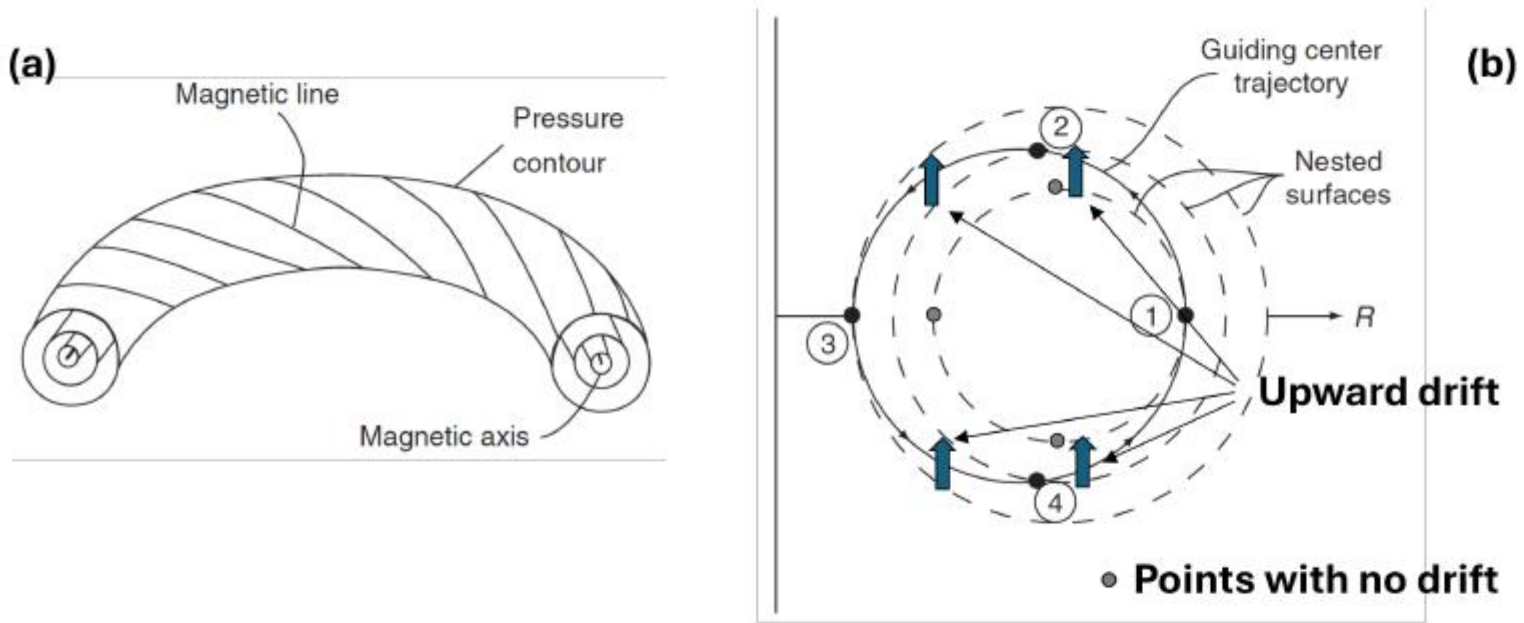


- **W/ plasma current**

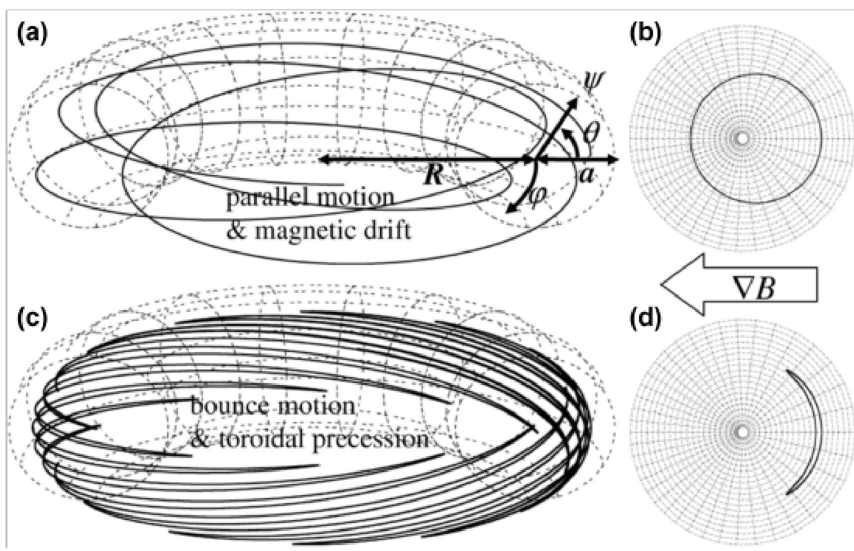




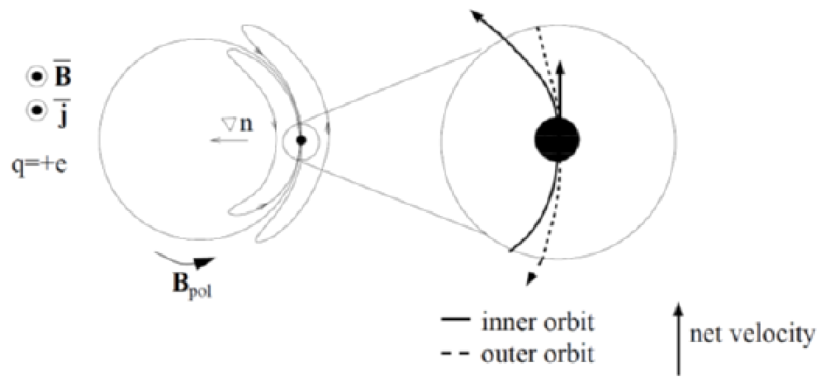
9. [10 points in total] To confine the plasma in a Tokamak, magnetic fields are helical as shown in the following figure (a). Please use the following figure (b) to explain why the plasma can be confined.



10. [25 points in total] Shown in the following figures are the particle trajectories in a Tokamak. Figure (a) is the trajectory of the passing particle, while figure (b) is the projection of the trajectory of the passing particle on the poloidal plane. Figure (c) is the trajectory of the bouncing particle, while figure (d) is the projection of the trajectory of the bouncing particle on the poloidal plane, which is called the banana orbit. Please answer the following questions: **(a)** When a charged particle follows a field line, why does it experience a nonuniform toroidal magnetic field? [5 points] **(b)** Since the toroidal magnetic field is not a constant along the particle's trajectory, the "mirror effect" occurs. Why are there two categories of particle trajectories? What determines if particles are bouncing or passing? [10 points] **(c)** If there is no drift, what does Fig. (b) look like? [5 points] **(d)** If there is no drift, what does Fig. (d) look like? [5 points]



11. [10 points in total] Shown in the following figure are two adjacent banana orbits. A bootstrap current can be generated when the number of particles in two adjacent banana orbits is different. Please explain **(a)** why banana orbits are essential for generating bootstrap currents. [5 points] and **(b)** why a bootstrap current can be generated when a pressure gradient across two adjacent banana orbits occurs. [5 points]

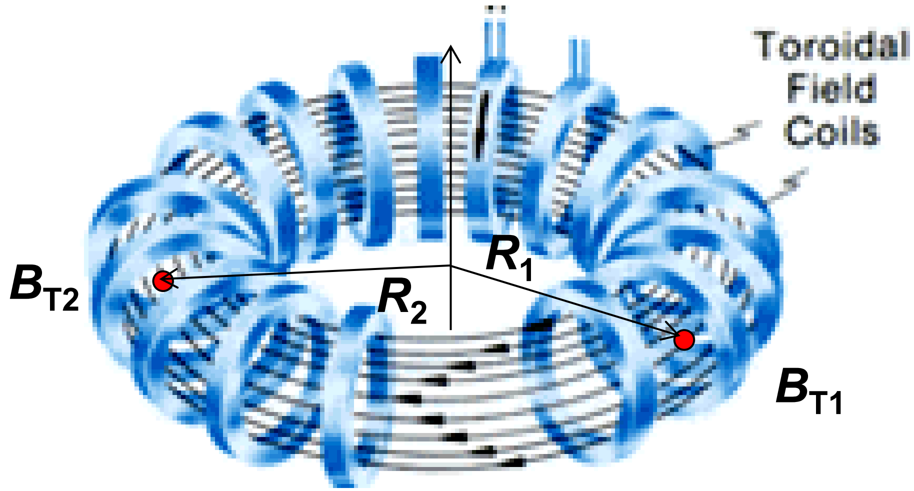


With poloidal fields, charged particles see nonuniform toroidal magnetic field

- **W/o poloidal field**

$$R_1 = R_2$$

$$B_{T1} = B_{T2}$$



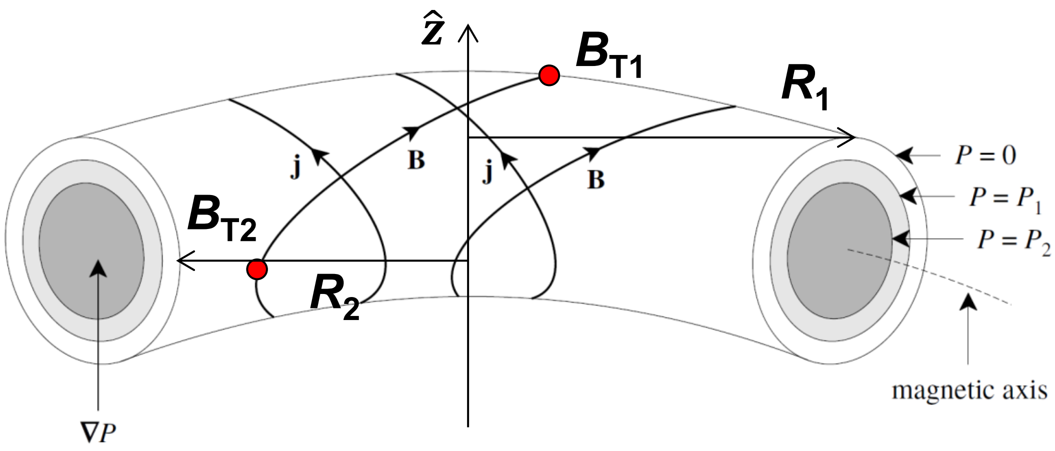
$$B_T \propto \frac{1}{R}$$

$$B_T \gg B_p$$

- **W/ poloidal field**

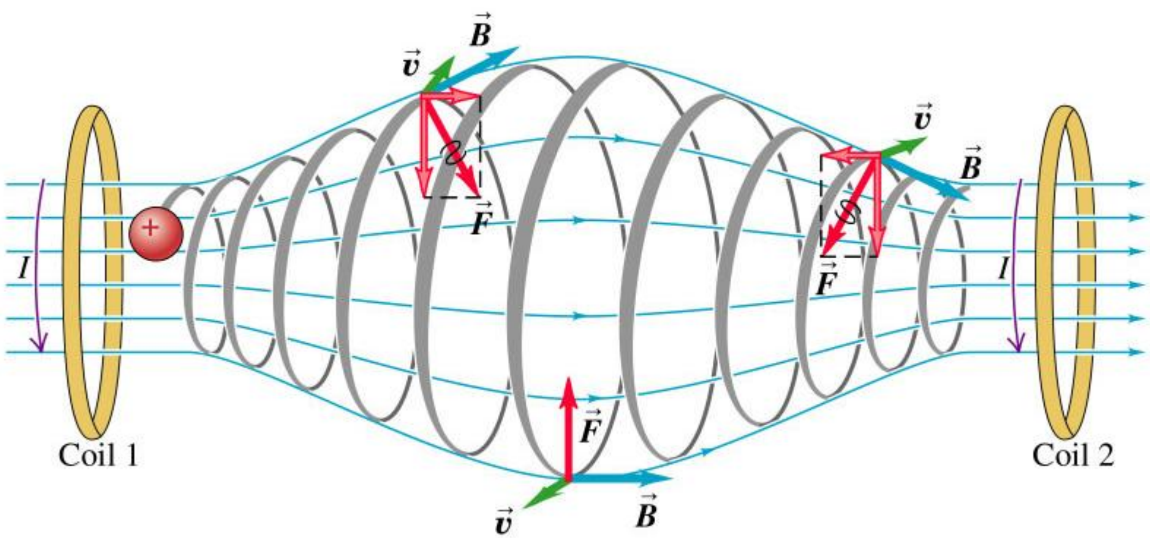
$$R_1 > R_2$$

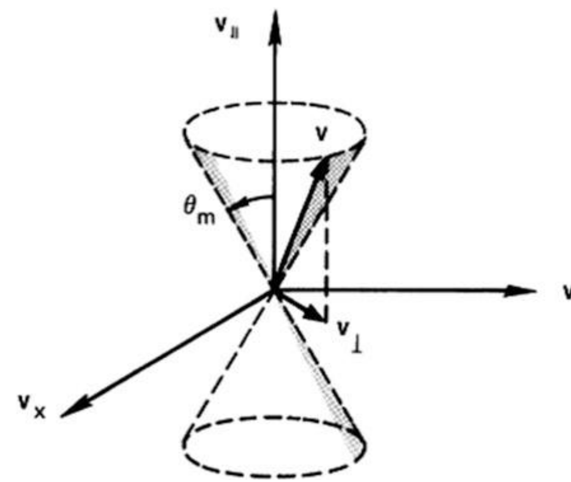
$$B_{T1} < B_{T2}$$



Charged particles can be partially confined by a magnetic mirror machine

- **Charged particles with small $v_{||}$ eventually stop and are reflected while those with large $v_{||}$ escape.**





$$v'_\perp{}^2 = v_{\perp o}{}^2 + v_{||o}{}^2 \equiv v_o{}^2$$

$$\frac{B_o}{B'} = \frac{v_{\perp o}{}^2}{v'_\perp{}^2} = \frac{v_{\perp o}{}^2}{v_o{}^2} \equiv \sin^2\theta$$

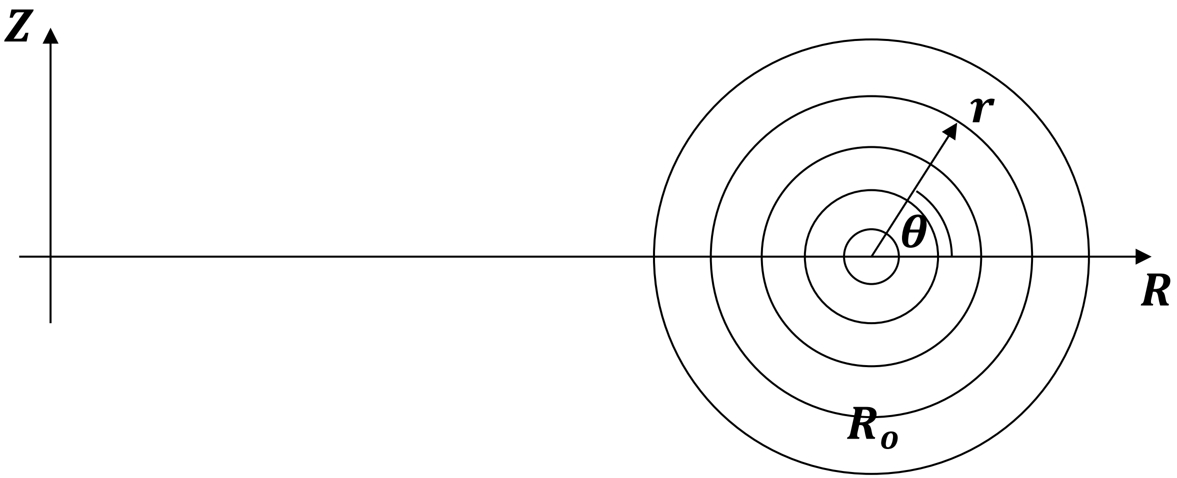
$$\frac{1}{2}mv^2 = \frac{1}{2}mv_{||}{}^2 + \frac{1}{2}mv_\perp{}^2 \qquad \text{Invarient: } \mu \equiv \frac{1}{2}\frac{mv_\perp{}^2}{B}$$

$$\frac{B_o}{B_m} \equiv \frac{1}{R_m} = \sin^2\theta_m$$

- **Large $v_{||}$ may occur from collisions between particles.**
- **Those confined charged particle are eventually lost due to collisions.**

https://i.stack.imgur.com/GlzGZ.jpg

Parallel velocity changes when particles follow field the field line



$R \gg r$

$B_\mathrm{T} \gg B_\mathrm{p}$

$R = R_\mathrm{o} + r\cos\theta = R_\mathrm{o}(1+\epsilon\cos\theta)$ **Inverse aspect ratio:** $\epsilon \equiv \dfrac{r}{R_\mathrm{o}}$

$$B \simeq \frac{B_\mathrm{o}}{1+\epsilon\cos\theta} \simeq B_\mathrm{o}(1-\epsilon\cos\theta)$$

Invarient: $\mu \equiv \dfrac{1}{2}\dfrac{m{v_\perp}^2}{B}$ $\quad \dfrac{{v_\perp}^2}{B_o(1-\epsilon cos\theta)} = \dfrac{{v_{\perp 0}}^2}{B_o(1-\epsilon)}$ $\quad {v_\perp}^2 = \dfrac{{v_{\perp 0}}^2(1-\epsilon cos\theta)}{1-\epsilon}$

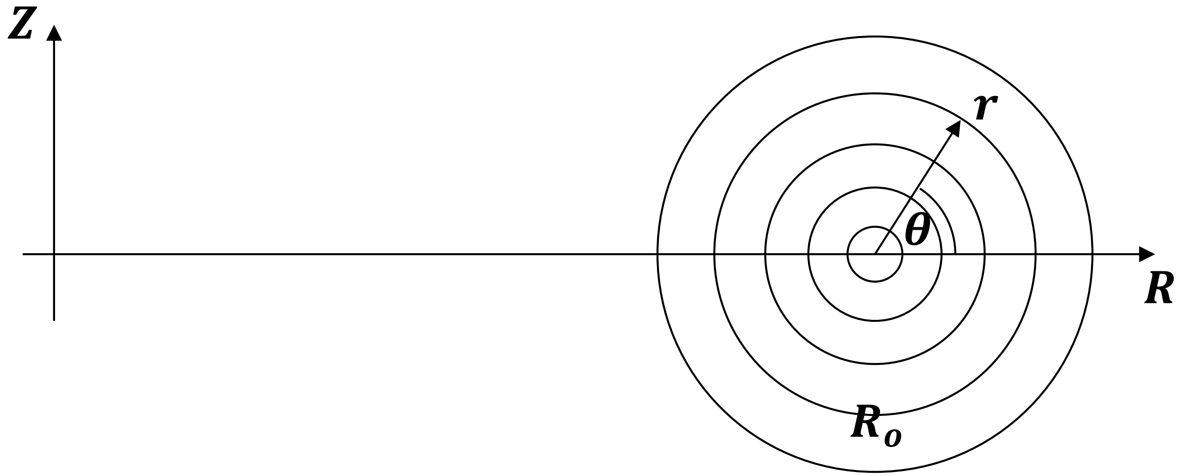
$$v^2 = {v_\perp}^2 + {v_{||}}^2 = {v_{\perp o}}^2 + {v_{||o}}^2$$

$$v_{||}{}^2 = v^2\left(1 - \frac{{v_\perp}^2}{v^2}\right)$$

$$v_{||}{}^2 = v^2\left(1 - \frac{{v_{\perp 0}}^2}{v^2}\frac{(1-\epsilon\cos\theta)}{1-\epsilon}\right)$$

$$\approx v^2\left(1 - \frac{{v_{\perp 0}}^2}{v^2}\left(1 + 2\epsilon\sin^2\left(\frac{\theta}{2}\right)\right)\right)$$

Particles may be trapped by nonuniform magnetic field



$R \gg r$

$B_{\mathrm{T}} \gg B_{\mathrm{p}}$

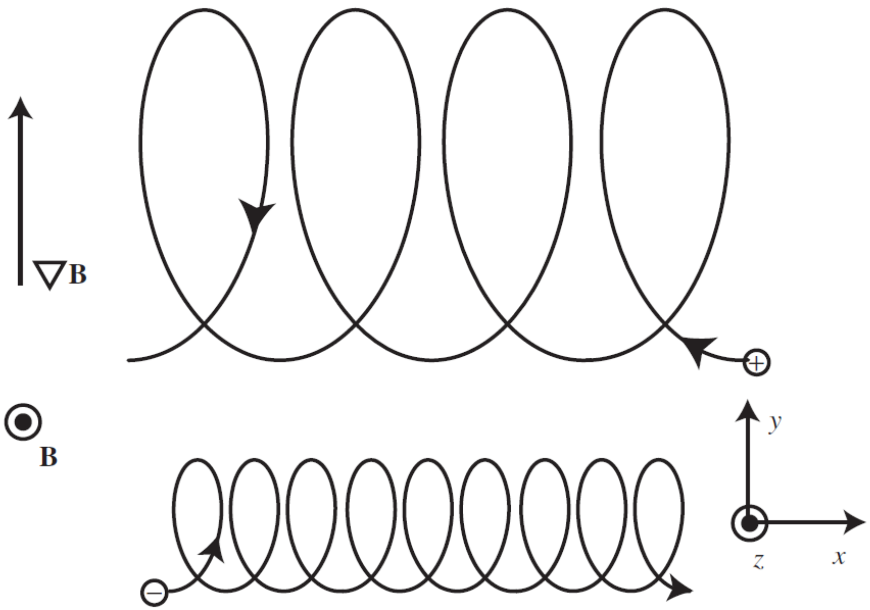
$$\epsilon \equiv \frac{r}{R_{\mathrm{o}}}$$

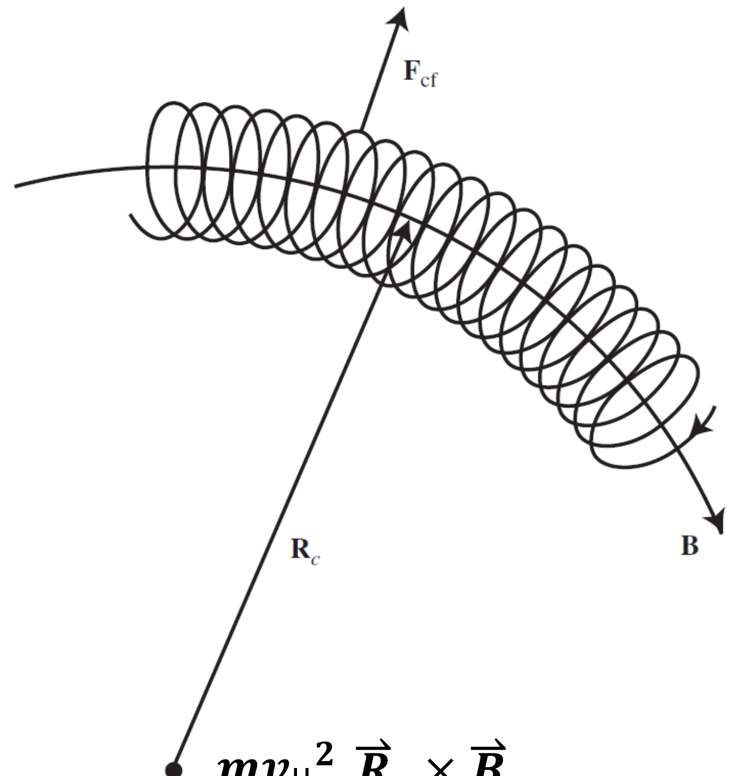
$$v_{||}^2 \approx v^2\left(1 - \frac{{v_{\perp 0}}^2}{v^2}\left(1 + 2\epsilon\sin^2\left(\frac{\theta}{2}\right)\right)\right)$$

- For $v_{||}^2 \geq 0$, particles are passing.
- For $v_{||}^2 \leq 0$, particles are trapped.

Charge particles drift across magnetic field lines when the magnetic field is not uniform or curved

- **Gradient-B drift**
- **Curvature drift**





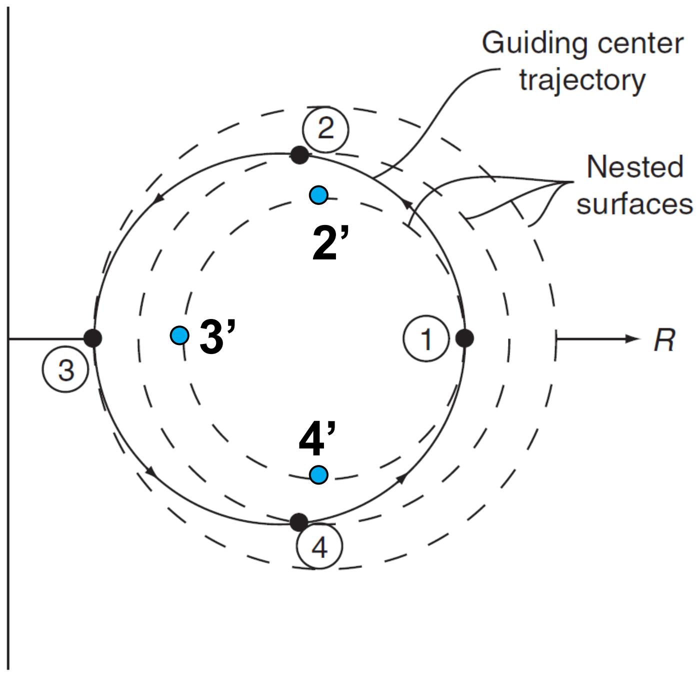
$$\vec{v}_{\nabla} = \frac{m v_{\perp}^{2}}{2q} \frac{\vec{B} \times \nabla B}{B^{3}}$$

$$\vec{v}_{R} = \frac{m v_{||}^{2}}{2q} \frac{\vec{R}_{c} \times \vec{B}}{R_{c} B^{2}}$$

$$\vec{v}_{\text{total}} = \vec{v}_{R} + \vec{v}_{\nabla} = \frac{\vec{B} \times \nabla B}{\omega_{c} B^{2}} \left(v_{||}^{2} + \frac{1}{2} v_{\perp}^{2} \right) = \frac{m}{q} \frac{\vec{R}_{c} \times \vec{B}}{R_{c}^{2} B^{2}} \left(v_{||}^{2} + \frac{1}{2} v_{\perp}^{2} \right)$$

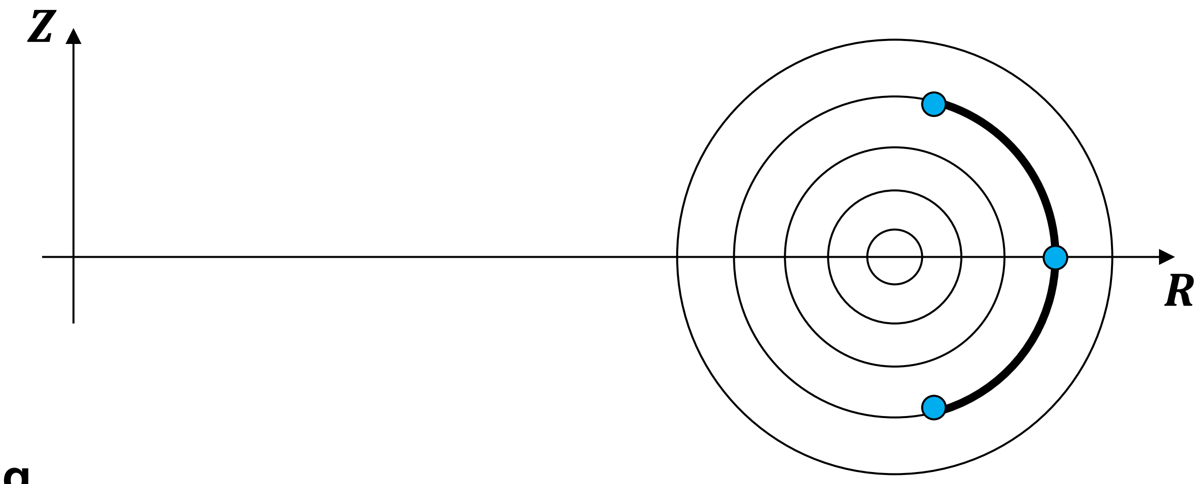
For passing particles, they drift back to the original position with a "semicircle" orbit

$$\vec{v}_{\text{total}} = \vec{v}_{\text{R}} + \vec{v}_{\nabla} = \frac{\vec{B} \times \nabla B}{\omega_c B^2}\left(v_{||}{}^2 + \frac{1}{2}v_{\perp}{}^2\right) = \frac{m}{q}\frac{\vec{R}_c \times \vec{B}}{R_c{}^2 B^2}\left(v_{||}{}^2 + \frac{1}{2}v_{\perp}{}^2\right)$$

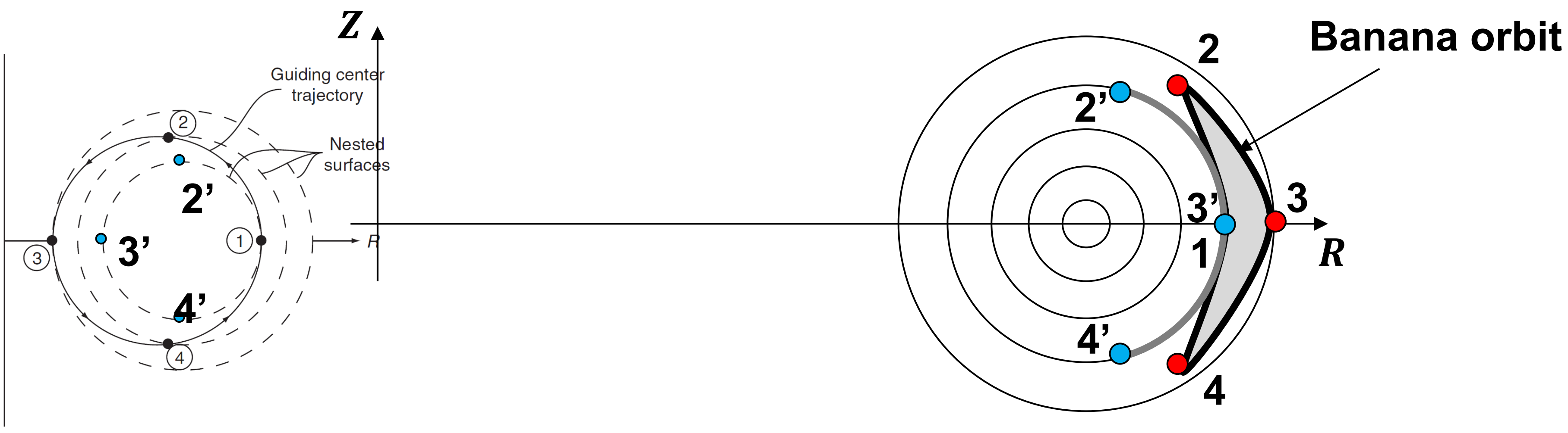


For trapped particles, they drift back to the original position with a banana orbit

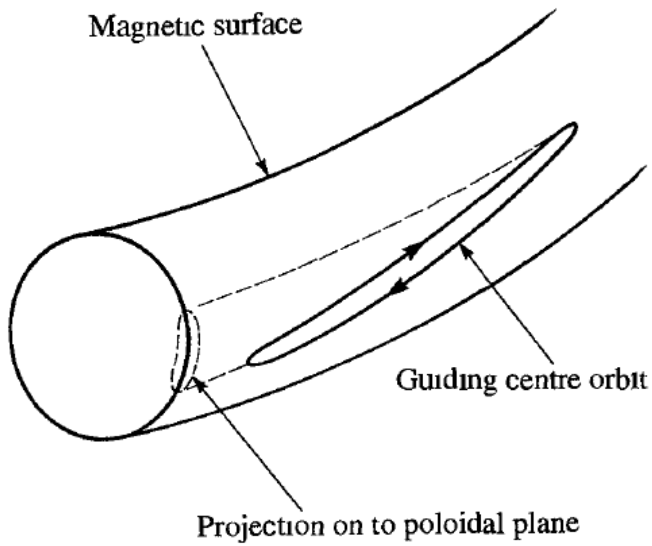
- W/o drifting

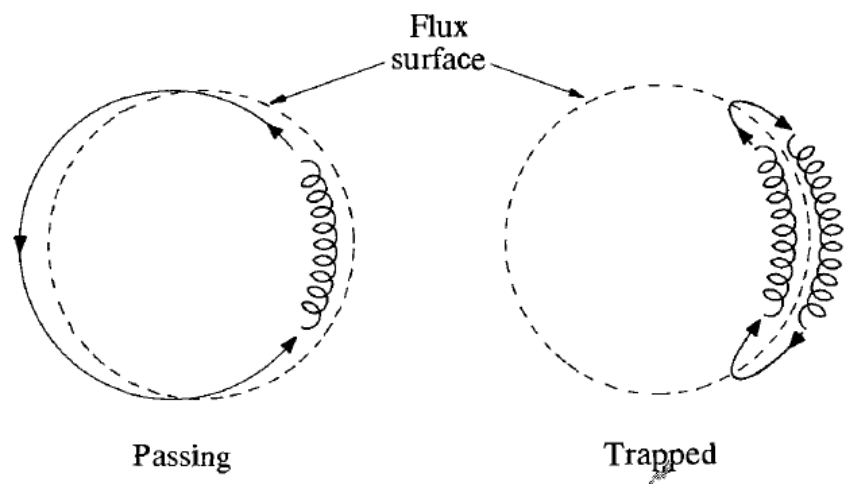


- W/ drifting



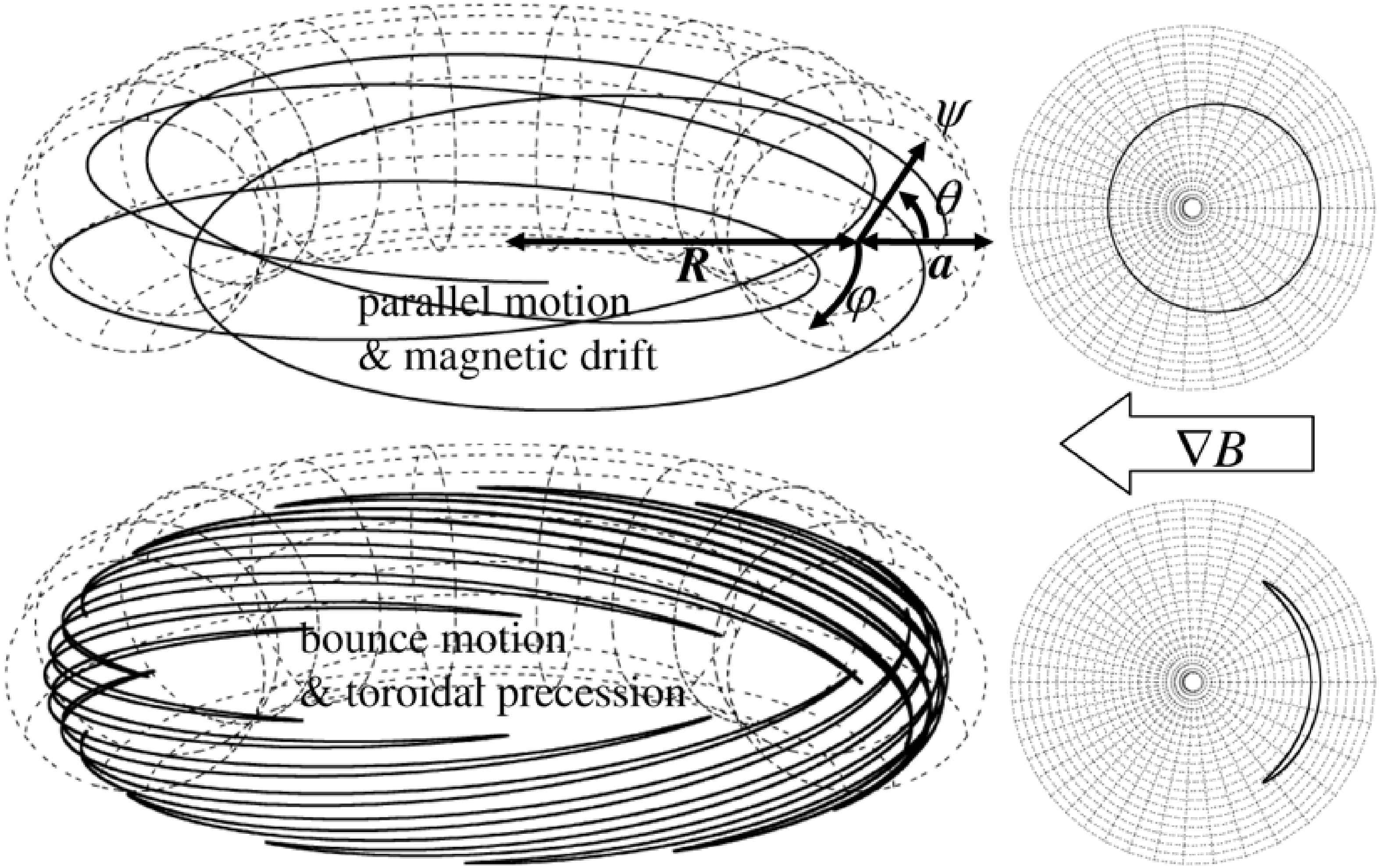
Trajectories of charged particles





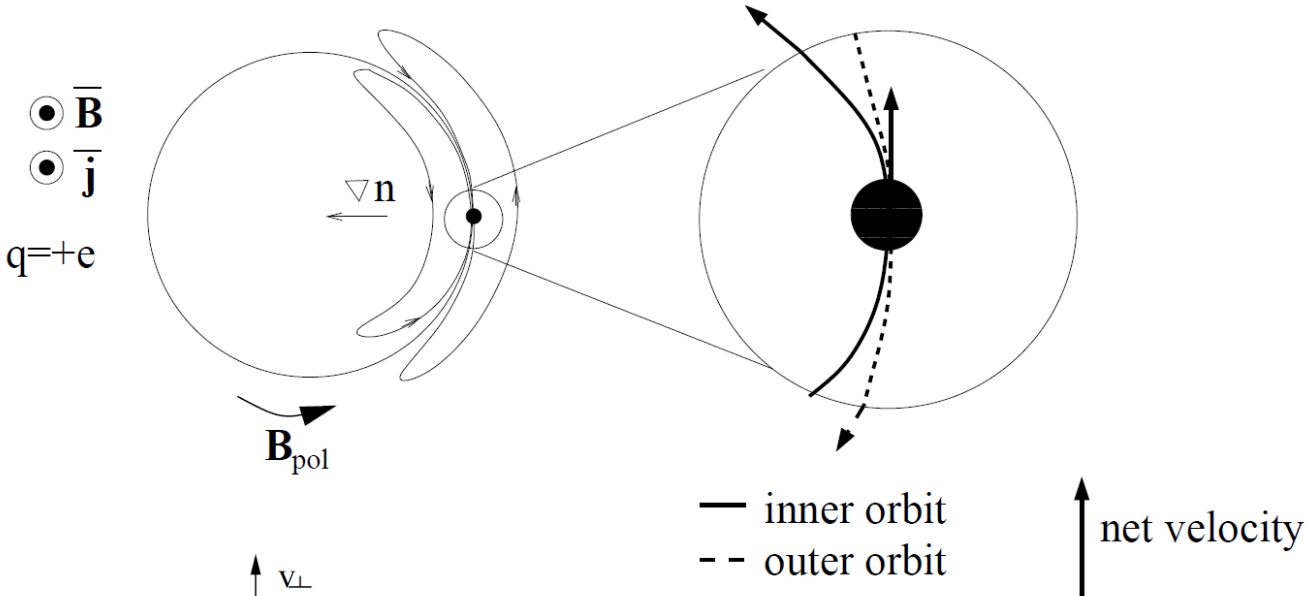
John Wesson, Tokamaks 3rd edition

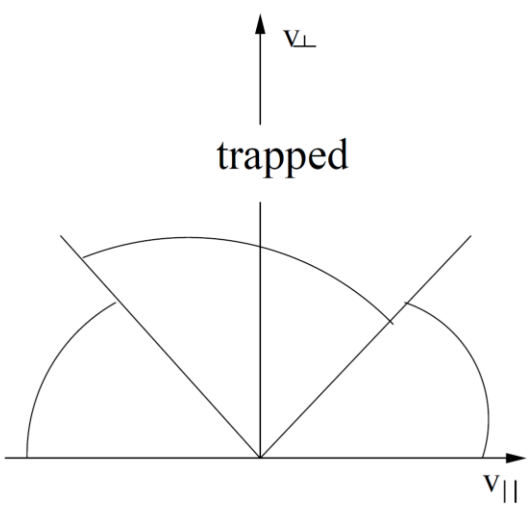
The trajectories of charged particles follow the toroidal field lines



X. Garbet et al, Nucl. Fusion **50**, 043002 (2010)

A banana current is generated when there is a pressure gradient in the plasma

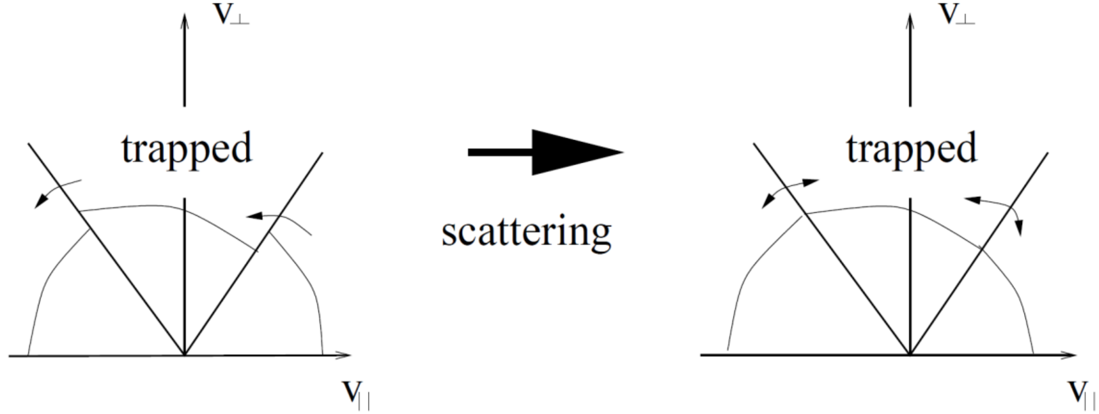


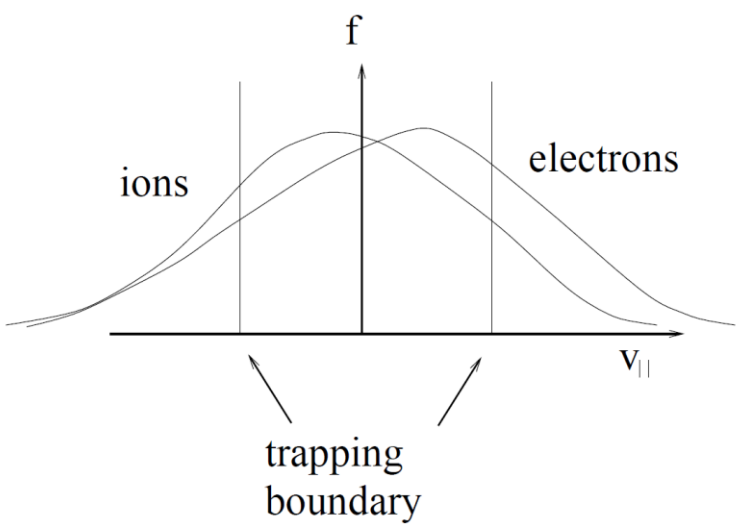


Samuli Saarelma, Helsinki University of Technology, PhD Thesis 2015

Bootstrap current is generated when passing particles are scattered by the trapped particles

- **Scattering smooths the velocity distribution and shifts it in the parallel direction, i.e., a current is generated. It is called the bootstrap current.**





$$j = -enu_{||e} + enu_{||i} = 4\epsilon^{3/2}\frac{1}{B_p}T\frac{dn}{dr}$$

- **The bootstrap current is vital for steady-state operation.**

Samuli Saarelma, Helsinki University of Technology, PhD Thesis 2015

When forces are balances, the system is in the equilibrium state, or called "Magnetohydrostatics"

- **Equilibrium state:**

$$\rho_{\mathrm{m}}\left[\frac{\partial \vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\,\vec{v}\right]=\vec{j}\times\vec{B}-\nabla p\equiv 0$$

$$\vec{j}\times\vec{B}=\nabla p$$

$$\nabla\times\vec{B}=\mu_{\mathrm{o}}\,\vec{j}$$

$$\vec{j}\times\vec{B}=\frac{1}{\mu_{\mathrm{o}}}(\vec{\nabla}\times\vec{B})\times\vec{B}=\frac{1}{\mu_{\mathrm{o}}}\left[(\vec{B}\cdot\vec{\nabla})\,\vec{B}-\frac{1}{2}\,\vec{\nabla}\,B^2\right]=\nabla p$$

$$\nabla\left(P+\frac{B^2}{2\mu_{\mathrm{o}}}\right)=\frac{1}{\mu_o}(\vec{B}\cdot\vec{\nabla})\,\vec{B}$$

Magnetic pressure **Magnetic tension** ← **Forces caused by curvature of the field lines**

$$\vec{j}\perp\nabla p \qquad \vec{B}\perp\nabla p \quad \Rightarrow \quad \vec{j}\cdot\nabla p=0 \qquad \vec{B}\cdot\nabla p=0$$

- **The surfaces with p = constant are both magnetic surfaces (i.e., they are made up of magnetic field lines) and current surfaces (i.e., they are made of current flow lines).**

Plasma condition can be obtained by solving Grad-Shafranov equation

$$\Delta^*\psi = -\mu_o R^2 \frac{dp}{d\psi} - \frac{1}{2}\frac{dF^2}{d\psi} \qquad \text{where } \Delta^*\psi = R^2\nabla\cdot\left(\frac{\nabla\psi}{R^2}\right) \qquad \mu_o j_\phi = -\frac{1}{R}\Delta^*\psi$$

$$j_\phi = R\frac{dp}{d\psi} + \frac{1}{2\mu_o R}\frac{dF^2}{d\psi}$$

- **The usual strategy to solve the Grad-Shafranov equation:**
  1. **Specify two free functions, the plasma pressure $p = p(\psi)$ and the toroidal field function $F = F(\psi)$.**
  2. **Solve the equation with specified boundary conditions to determine the flux function $\psi(R, z)$.**
  3. **Calculation the magnetic field using the following equations:**

     $$B_R = -\frac{1}{R}\frac{\partial\psi}{\partial z} \qquad B_\phi = \frac{F(\psi)}{R} \qquad B_z = \frac{1}{R}\frac{\partial\psi}{\partial R}$$

  4. **The pressure profile can then be obtained from $p = p\big(\psi(R, z)\big)$.**

Application of solving Grad-Shafranov equation for designing a tokamak

- **Given I_{plasma}, $p(\psi)$, $I(\psi)$, I_{coils}, free boundary of plasma, perfect conductor as the chamber.**
- **Given I_{plasma}, $p(\psi)$, $I(\psi)$, I_{coils}, free boundary of plasma, insulator chamber.**
- **Given I_{plasma}, $p(\psi)$, $I(\psi)$, I_{coils}, free boundary of plasma, chamber with eddy current.**
- **Given I_{plasma}, $p(\psi)$, $I(\psi)$, fixed boundary of plasma. Then, use I_{coils}, free boundary of plasma and match the plasma shape calculated in the fixed boundary condition.**

$$\Delta^*\psi = -\mu_o R^2 \frac{dp}{d\psi} - \frac{1}{2}\frac{dF^2}{d\psi} \qquad \text{where } \Delta^*\psi = R^2 \nabla \cdot \left(\frac{\nabla\psi}{R^2}\right)$$

$$j_\phi = R\frac{dp}{d\psi} + \frac{1}{2\mu_o R}\frac{dF^2}{d\psi} \qquad I_P = -2\pi F(\psi)$$

$$\mu_o \vec{j} = \left(\frac{\nabla F}{R}\right) \times \hat{\phi} + \left(-\frac{1}{R}\Delta^*\psi\right)\hat{\phi} \qquad \vec{B} = \left(\frac{\nabla\psi}{R}\right) \times \hat{\phi} + \frac{F(\psi)}{R}\hat{\phi}$$

Application of solving Grad-Shafranov equation for reconstructing a tokamak equilibrium state

- **Measure**
  - **boundary conditions, including ψ, B, etc., on the wall (using flux loop and B-dot probe).**
  - **Pressure.**
  - **Plasma current (using Rogowski coil).**
- **Reconstruct $\psi(r,z)$, j, $p(\psi)$, $I(\psi)$, etc.**

$$\Delta^*\psi = -\mu_o R^2 \frac{dp}{d\psi} - \frac{1}{2}\frac{dF^2}{d\psi} \qquad \text{where } \Delta^*\psi = R^2\nabla\cdot\left(\frac{\nabla\psi}{R^2}\right)$$

$$I_P = -2\pi F(\psi)$$

$$\mu_o\vec{j} = \left(\frac{\nabla F}{R}\right)\times\hat{\phi} + \left(-\frac{1}{R}\Delta^*\psi\right)\hat{\phi} \qquad \vec{B} = \left(\frac{\nabla\psi}{R}\right)\times\hat{\phi} + \frac{F(\psi)}{R}\hat{\phi}$$

Example of the analytical solution of the Grad-Shafranov equation

$$\Delta^*\psi = -\mu_o R^2 \frac{dp}{d\psi} - \frac{1}{2}\frac{dF^2}{d\psi}$$

- For $\mu_o \frac{dp}{d\psi} = -C_2$ $\quad \frac{1}{2}\frac{dF^2}{d\psi} = C_1$

$$\psi(R,z) = -\frac{C_1}{2}z^2 + \frac{C_2}{8}R^4 + C_3 + C_4R^2 + C_5\left(R^4 - 4R^2z^2\right)$$

$$B_R(R,z) = -\frac{1}{R}\left(-C_1 z - 8C_5R^2 z\right) \qquad B_z(R,z) = \frac{1}{R}\left(\frac{C_2}{2}R^3 + 2C_4R + C_5\left(4R^3 - 8Rz^2\right)\right)$$

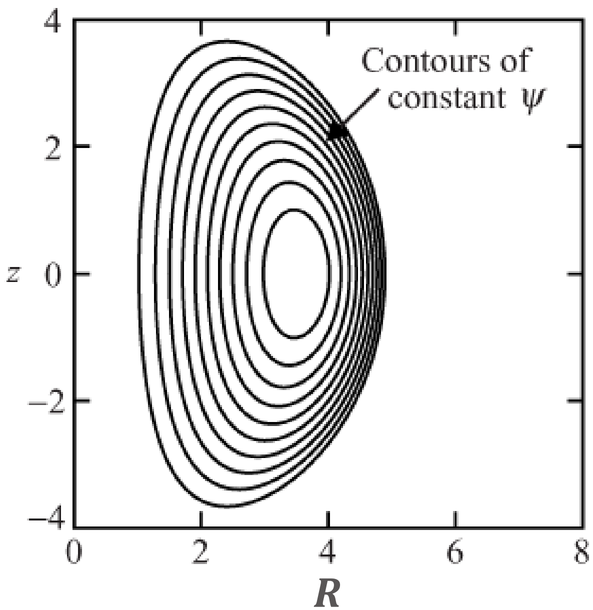
$C_1 = 1$

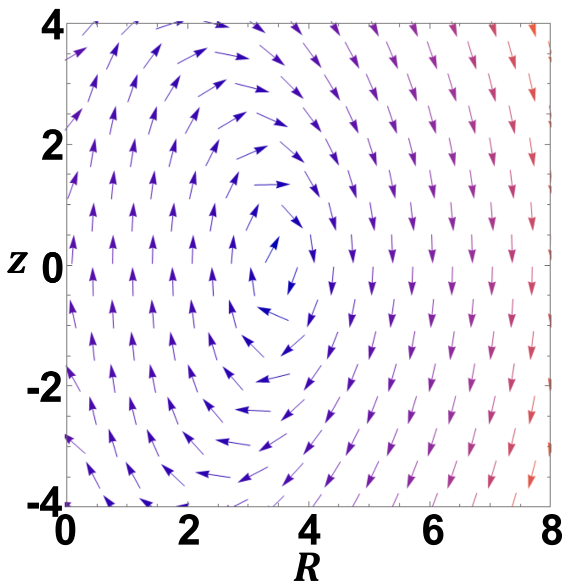
$C_2 = -8$

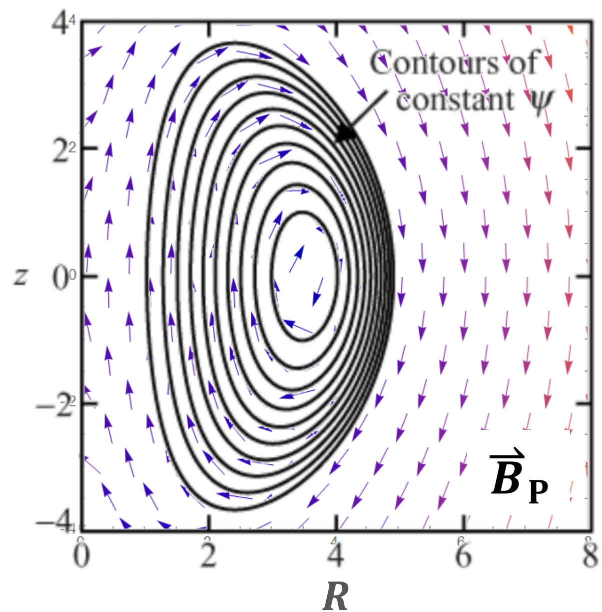
$C_3 = -20$

$C_4 = 20$

$C_5 = 0.2$



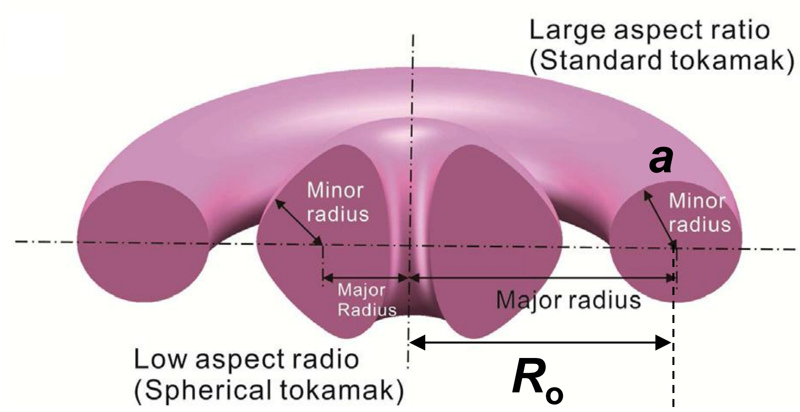




Examples: Large Aspect Ratio Tokamak

$$\Delta^*\psi = -\mu_o R^2 \frac{dp}{d\psi} - \frac{1}{2}\frac{dF^2}{d\psi}$$

$$j_\phi = R\frac{dp}{d\psi} + \frac{1}{2\mu_o R}\frac{dF^2}{d\psi}$$



- **Assuming:**
  - $R_o >> a$
  - $j_\Phi = j_o = constant$
  - $F(\psi) = B_o R_o = constant$

$$\Delta^*\psi = R^2\nabla\cdot\left(\frac{\nabla\psi}{R^2}\right) = \frac{\partial^2\psi}{\partial z^2} + R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right)$$

$$\Rightarrow \frac{dp}{d\psi} = \text{const} \qquad \mu_o j_\phi = -\frac{1}{R}\Delta^*\psi \qquad \Rightarrow \quad -\mu_o R j_o = \frac{\partial^2\psi}{\partial z^2} + R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right)$$

$$\psi(R,Z) = \frac{1}{2}A\left(R^2 - {R_o}^2\right) + \frac{1}{2}BZ^2$$

$$\Delta^*\psi = A + B = -\mu_o R j_o$$

$$\text{Let } A = B \equiv -\frac{\mu_o j_o}{2}$$

$$\psi(R,Z) = -\frac{\mu_o j_o}{4}\left[\left(R^2 - {R_o}^2\right) + Z^2\right]$$

Equilibrium reconstruction

- **Magnetic Diagnostics (Essential for the Grad-Shafranov equation)**
  - **Magnetic flux loops: Measure poloidal flux ψ at different locations.**
  - **Poloidal field (PF) probes: Measure B_p.**
  - **Toroidal field (TF) coils: Provide information on B_ϕ.**
  - **Rogowski coils: Measure total plasma current I_p.**
  - **Hall probes: Measure local magnetic fields.**
- **Interferometry & Thomson Scattering - Provide electron density profile $n_e(r)$.**
- **X-ray & Spectroscopy - Measure soft X-ray emission, used to infer internal plasma profiles.**
- **Motional Stark Effect (MSE) & Polarimetry - Measure the pitch angle of the magnetic field, used to infer the safety factor profile $q(\psi)$.**
- **Soft X-ray and ECE Imaging - Can give additional constraints on plasma shape and profiles.**

Equilibrium reconstruction – conti.

$$\Delta^{*}\psi = -\mu_o R^2 \frac{dp}{d\psi} - \frac{1}{2}\frac{dF^2}{d\psi} \qquad j_\phi = R\frac{dp}{d\psi} + \frac{1}{2\mu_o R}\frac{dF^2}{d\psi}$$

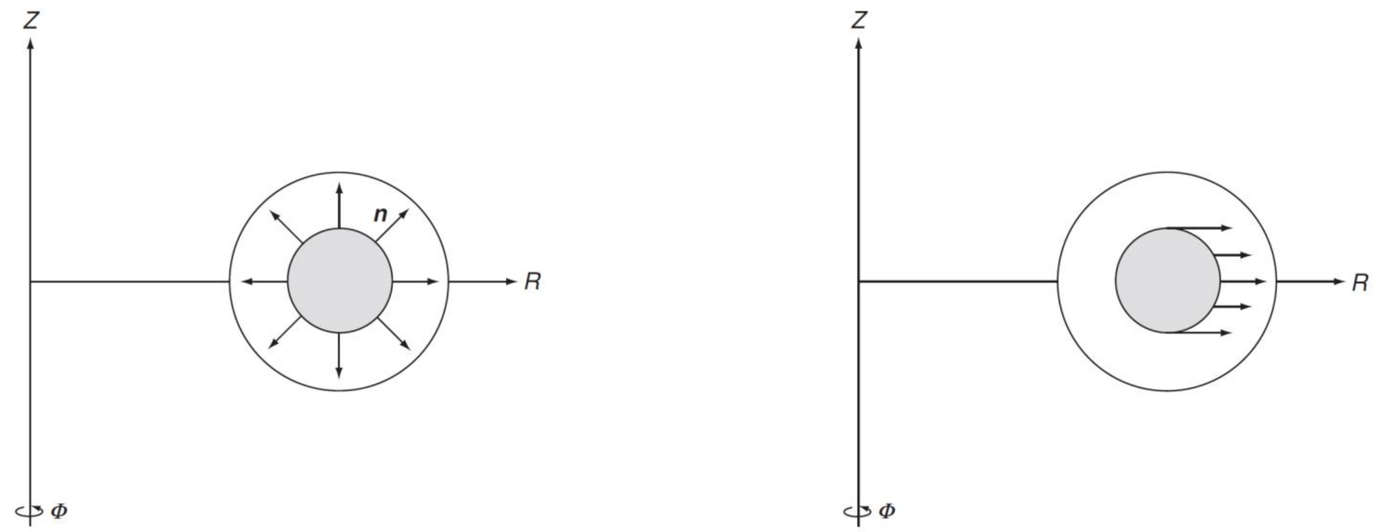
$j_\phi(\psi)$ $p(\psi)$ $F(\psi)$ **<= They are constrained with each other. When two are given, the third can be calculated.**

- **Adjustable functions:** $F(\psi)$ **and** $p(\psi)$ **are written in analytical forms.**
- **Solve Grad-Shafranov for** $\psi(R,Z)$
- **Compute derived quantities (flux loops, magnetic probes, pitch angles)**
- **Compare to diagnostics**
- **Adjust profile parameters**
- **Repeat until residuals are minimized**
- **Calculate** $j_\phi(\psi)$

Magnetically confined toroidal equilibrium

1. Radial pressure balance in the poloidal plan needs to be provided so that the pressure contours form closed nested surfaces. Both toroidal and poloidal fields can readily accomplish this task.
2. The radially outward expansion force inherent in all toroidal geometries needs to be balanced without sacrificing stability.

- Forces associated with toroidal force balance are usually than those corresponding to radial pressure balance. However, they are more difficult to compensate.



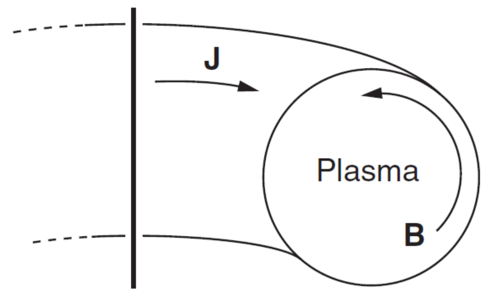
Toroidal configuration with a purely poloidal magnetic field

- **Hoop force:**

$$\psi_1 = \psi_2 \equiv \psi$$

$$S_1 < S_2 \quad B_1 > B_2$$

$$\vec{F}_{\mathrm{H,R}} \propto \hat{e}_{\mathrm{R}} \frac{{B_1}^2 S_1 - {B_2}^2 S_2}{2\mu_o} > 0$$

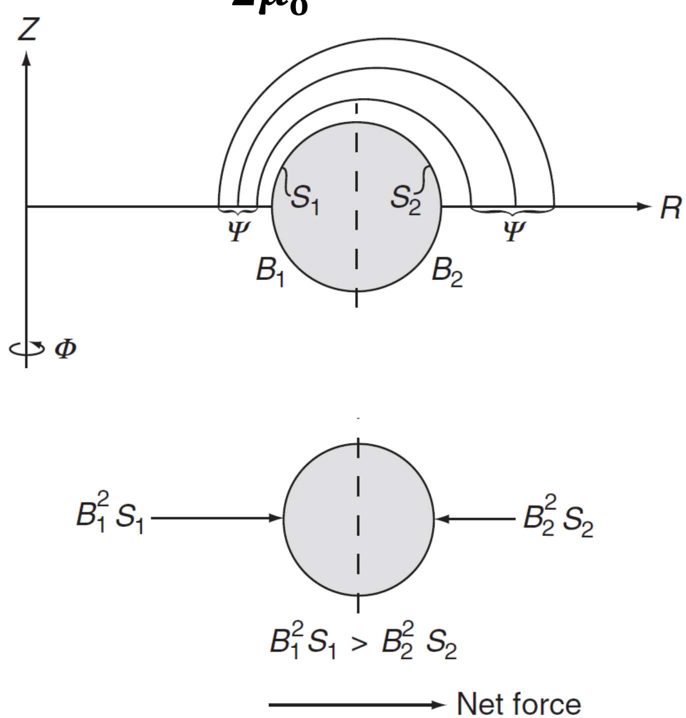


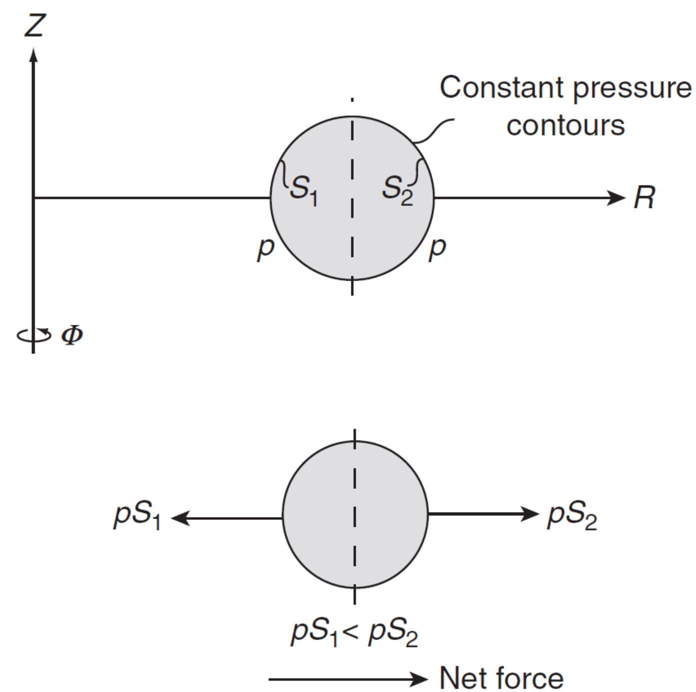
- **Tire tube force**

$$p_1 = p_2 \equiv p$$

$$S_1 < S_2$$

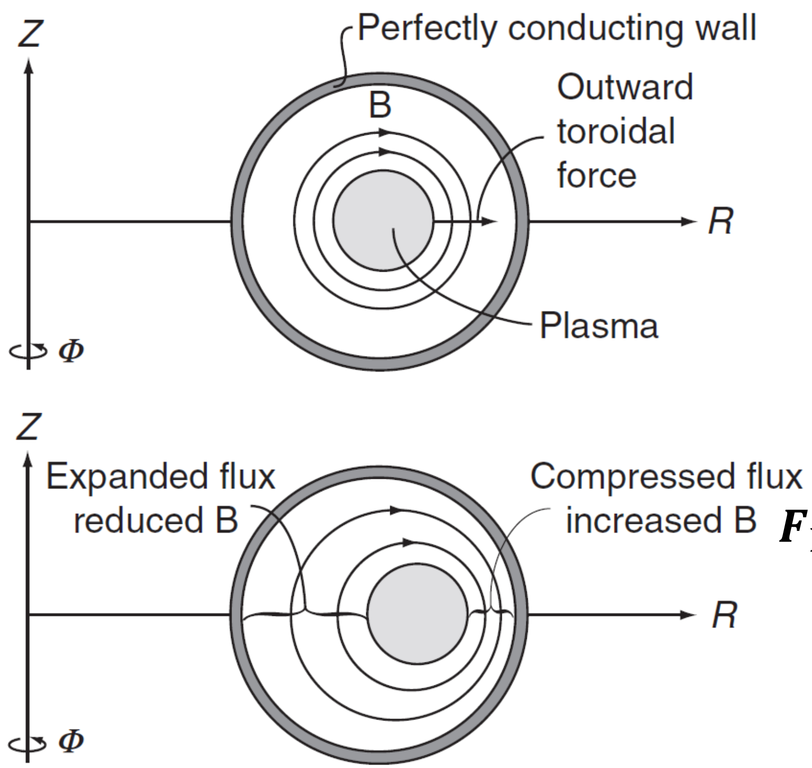
$$\vec{F}_{\mathrm{T,R}} \propto -\hat{e}_{\mathrm{R}}(pS_1 - \mathrm{p}S_2) > 0$$

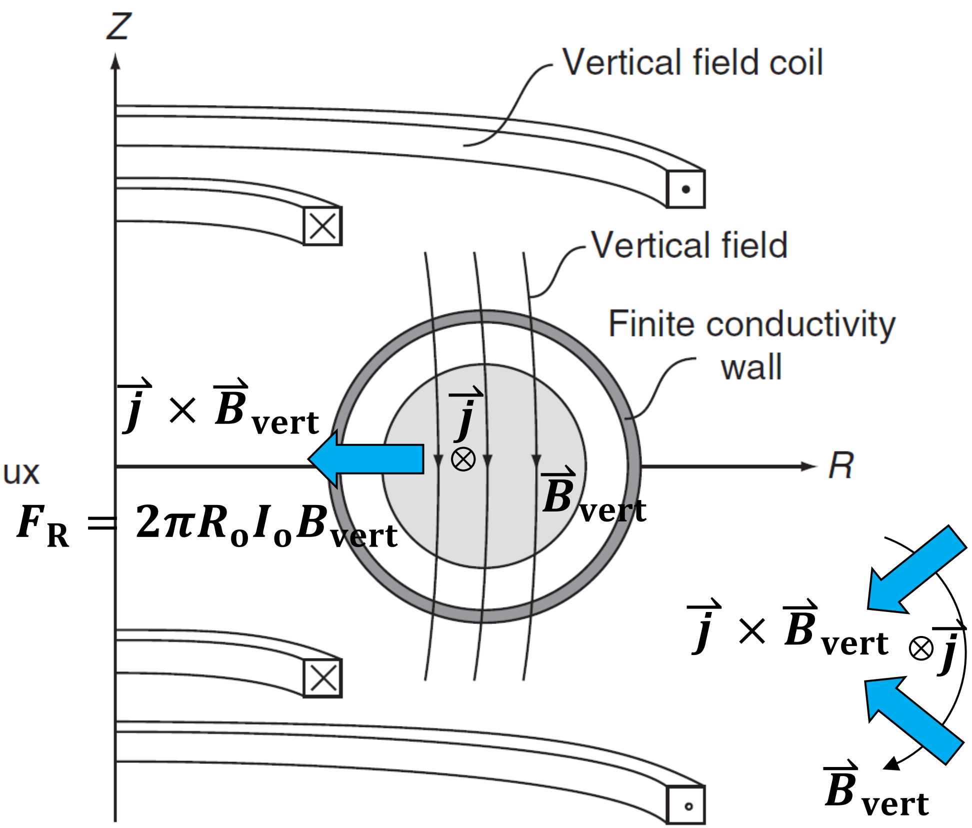




The outward force can be compensated by either a perfectly conducting shell or externally applied vertical field

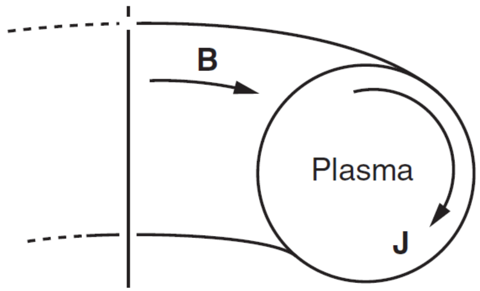
- **Perfectly conducting shell**
- **Externally applied vertical field**



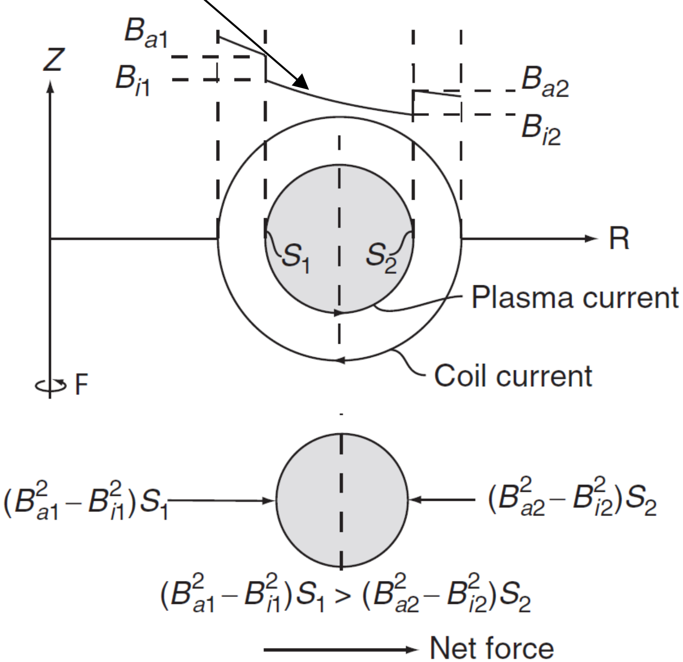


- **With a finite conductivity wall, flux can only remain compressed for about a skin time.**
- **This configuration develops disastrous MHD instabilities (z pinch).**

Toroidal configuration with a purely toroidal magnetic field, stable but NOT balanced

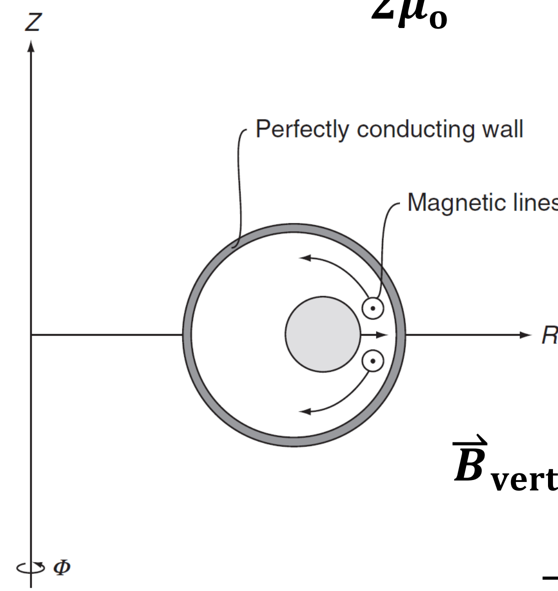


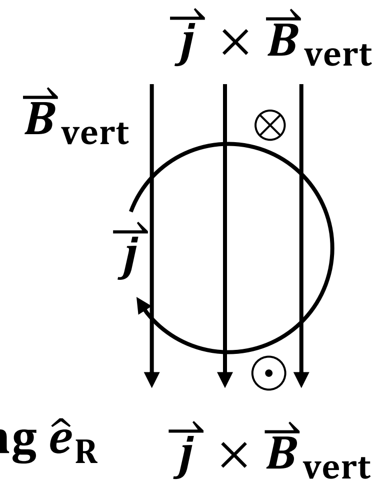
Plasma is diamagnetic



$$\vec{B} = B_\phi \hat{e}_\phi \qquad B_\phi = B_o \frac{R_o}{R} \qquad B_o = \frac{\mu_o I_c}{2\pi R_o}$$

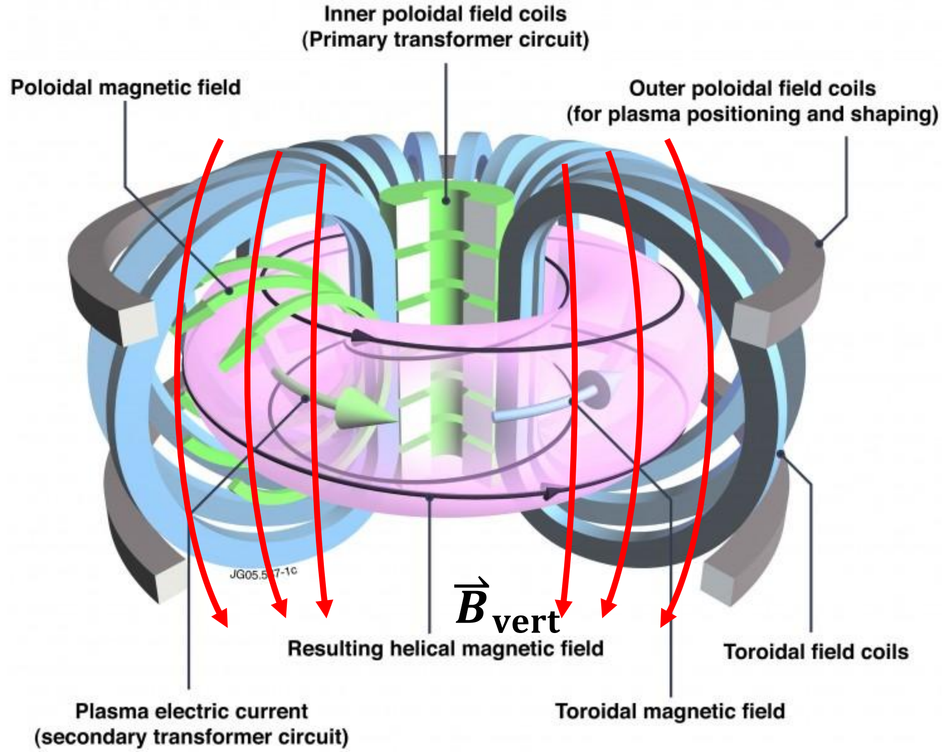
$$\vec{F}_R \propto \hat{e}_R \frac{({B_{a1}}^2 - {B_{i1}}^2)S_1 - ({B_{a2}}^2 - {B_{i2}}^2)S_2}{2\mu_o} > 0$$





$\vec{j} \times \vec{B}_{vert}$
is not pointed along $\hat{e}_R$

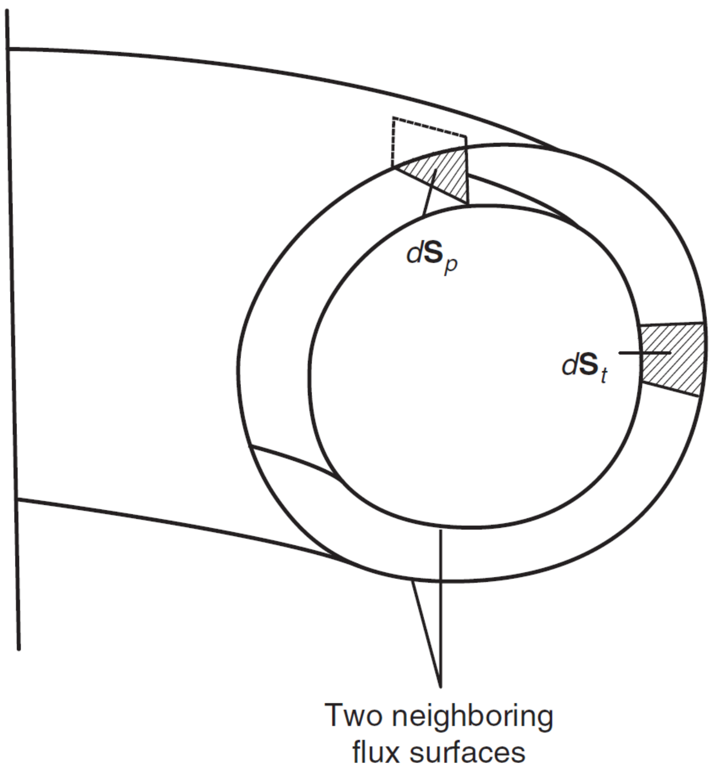
Coils in a tokamak



- **Toroidal field coils (in poloidal direction) – generate toroidal field for confinement.**
- **Poloidal field coils – generate vertical field for plasma positioning and shaping.**
- **Central solenoid – for breakdown and generating plasma current (in toroidal direction) and thus generating poloidal field for confinement.**

https://www.euro-fusion.org/2011/09/tokamak-principle-2/

Fluxes and currents



- **Poloidal flux:** $\psi_p = \int \vec{B} \cdot d\vec{S}_p$

  $\psi_p = \psi_p(p)$

- **Toroidal flux:** $\psi_t = \int \vec{B} \cdot d\vec{S}_t$

- **Poloidal current:** $I_p = \int \vec{j} \cdot d\vec{S}_p$

- **Toroidal current:** $I_t = \int \vec{j} \cdot d\vec{S}_t$

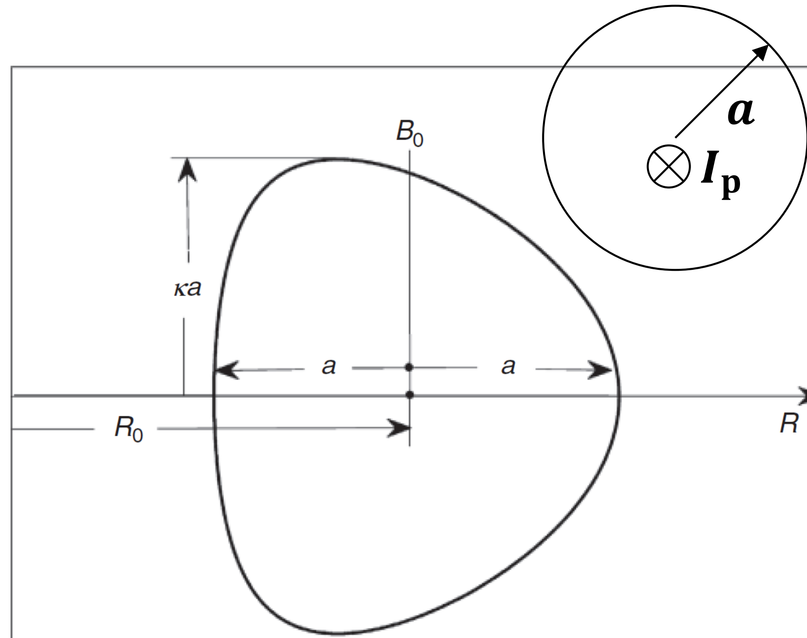
Normalized plasma pressure, β

$$\beta = \frac{\text{plasma pressure}}{\text{magnetic pressure}} = \frac{2\mu_o\langle p\rangle}{B^2}$$

- **Plasma pressure:** $\langle p\rangle = \frac{1}{V_p}\int p\, d\vec{r}$
- **Magnetic pressure:** $P_B = \frac{B^2}{2\mu_o}$

$$B^2 = {B_t}^2 + {B_p}^2 = {B_o}^2 + \left(\frac{\mu_o I_p}{2\pi a}\right)^2 \frac{2}{1+\kappa^2}$$

$${B_t}^2 = {B_o}^2 \qquad B_o = B\ @R = R_o$$



- **Elongation:** κ

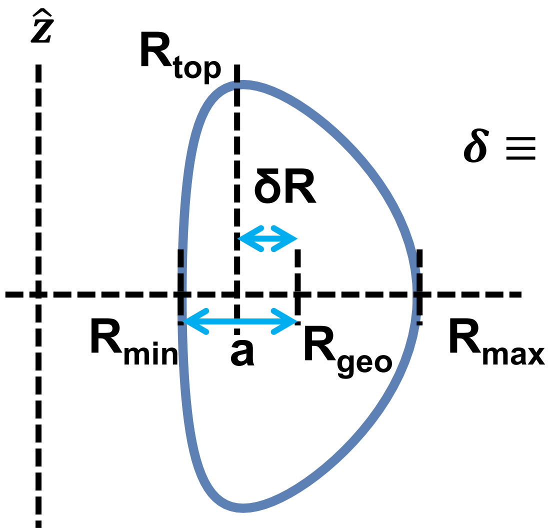
$${B_p}^2 = \left(\frac{\mu_o I_p}{2\pi a}\right)^2 = \left(\frac{\mu_o I_p}{C_p}\right)^2$$

$$C_p \approx 2\pi a\sqrt{\frac{1+\kappa^2}{2}}$$

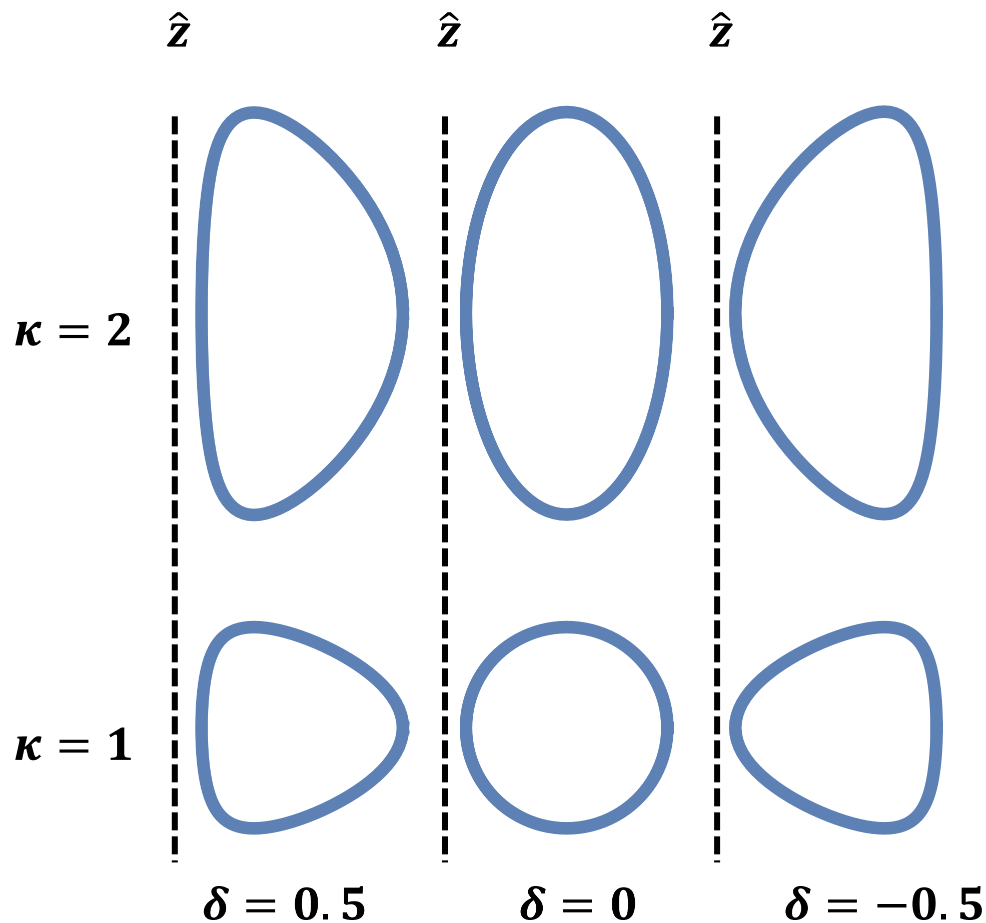
$$\beta_t = \frac{2\mu_o\langle p\rangle}{{B_o}^2} \qquad \beta_p = \frac{4\pi^2 a^2(1+\kappa^2)p}{\mu_o {I_p}^2}$$

$$\frac{1}{\beta} = \frac{1}{\beta_t} + \frac{1}{\beta_p}$$

Different poloidal shapes



$$\delta \equiv \frac{\delta R}{a}$$



$$r = R + a\mathrm{Cos}\big(\theta + \delta \mathrm{Sin}(\theta)\big)$$

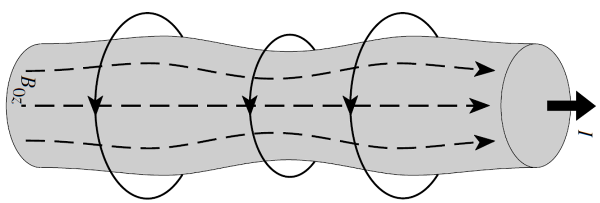
$$z = a\kappa \mathrm{Sin}(\theta)$$

- **Aspect ratio:** $\frac{R}{a}$
- **Elongation:** κ
- **Triangularity:** δ

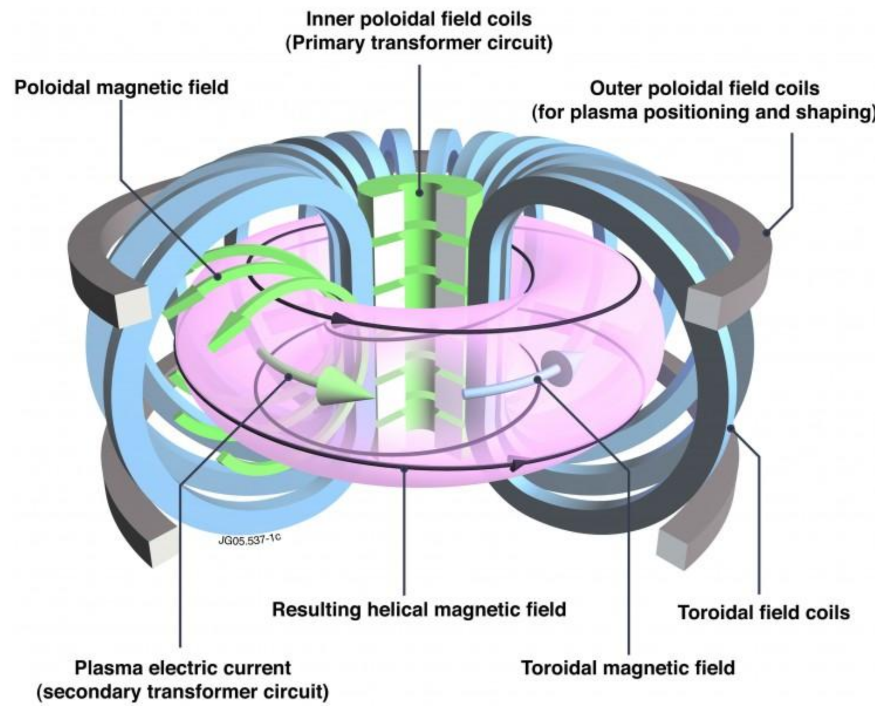
Safety factor

- **Kink Safety Factor:**

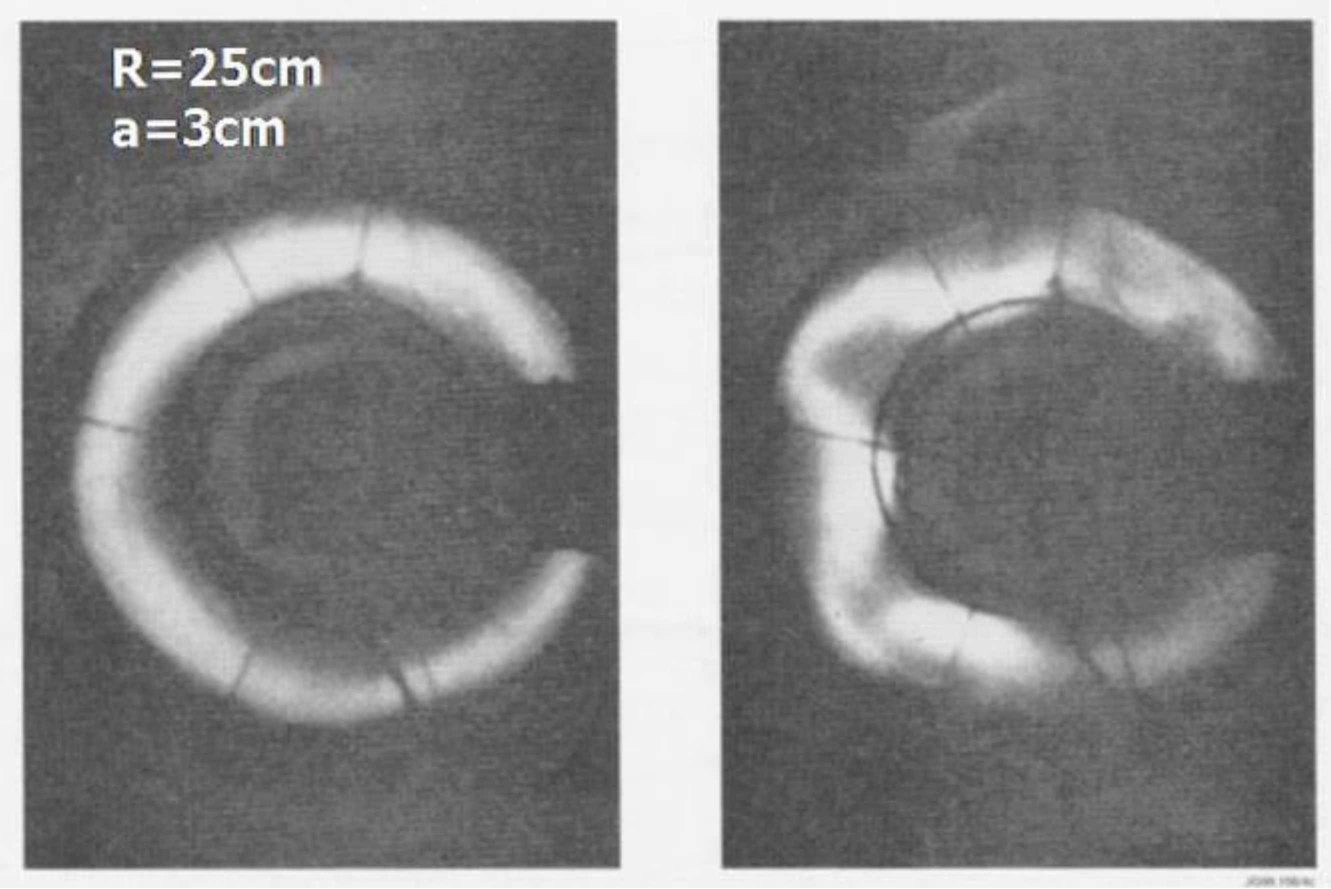
$$q^*(r) = \frac{aB_o}{R_oB_p} = \frac{2\pi a^2 \kappa B_o}{\mu_o R_o I_o}$$



$$q(r) = \frac{rB_z(r)}{R_oB_\theta(r)}$$

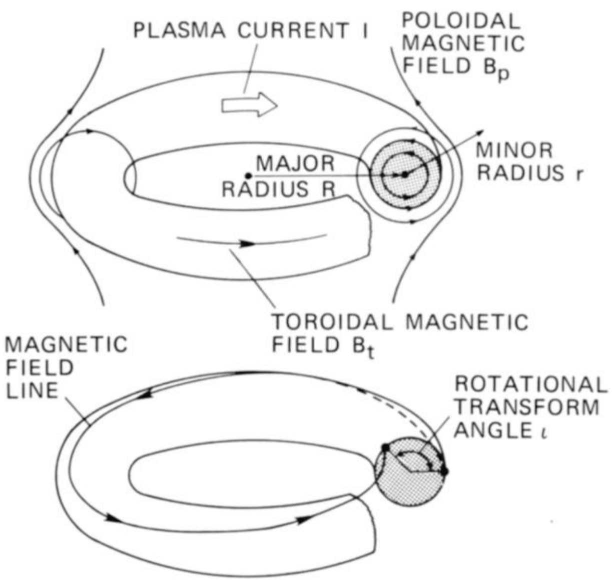


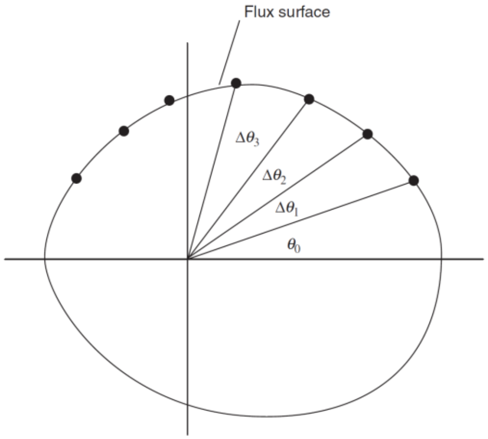
Kink instability in action in a 3 by 25-cm pyrex tube at Aldermaston



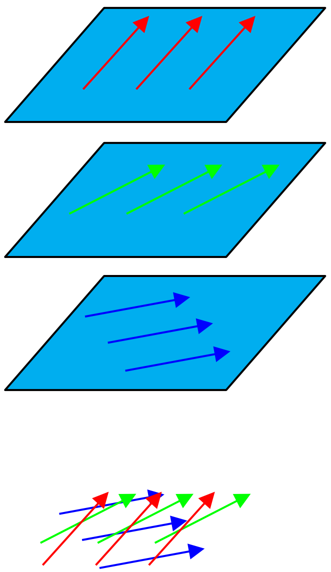
https://en.wikipedia.org/wiki/Kink_instability
R A Bingham et al 2026 Plasma Phys. Control. Fusion 68 030201

Safety factor





- **Shear:**



- **Rotational transform:** $\iota \equiv \lim_{N\to\infty} \frac{1}{N}\sum_{1}^{N} \Delta\theta_{\mathrm{n}}$
- **MHD safety factor:** $q(V) \equiv \frac{2\pi}{\iota(V)} = \frac{d\psi_t/dV}{d\psi_p/dV}$

$\psi_{\mathrm{t}} = \psi_{\mathrm{t}}(V) \quad \psi_{\mathrm{p}} = \psi_{\mathrm{p}}(V)$

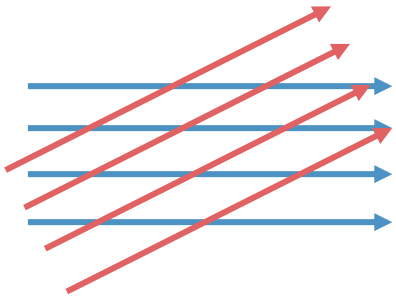
- **Shear:** $s(V) \equiv 2\frac{V}{q}\frac{dq}{dV}$

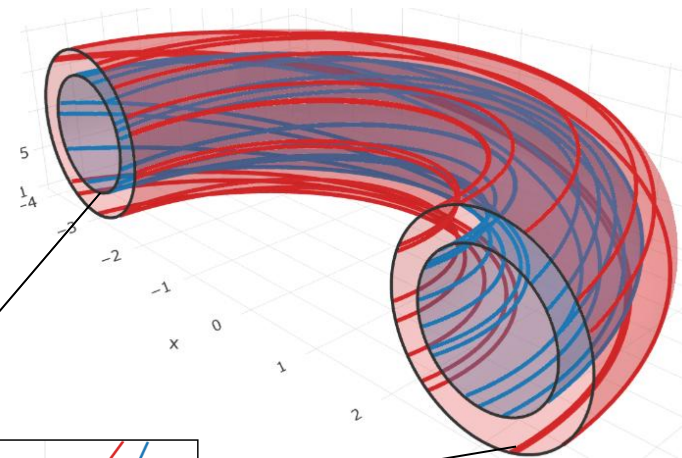
$\iota(V) \equiv 2\pi\left(\frac{d\psi_p/dV}{d\psi_t/dV}\right)$

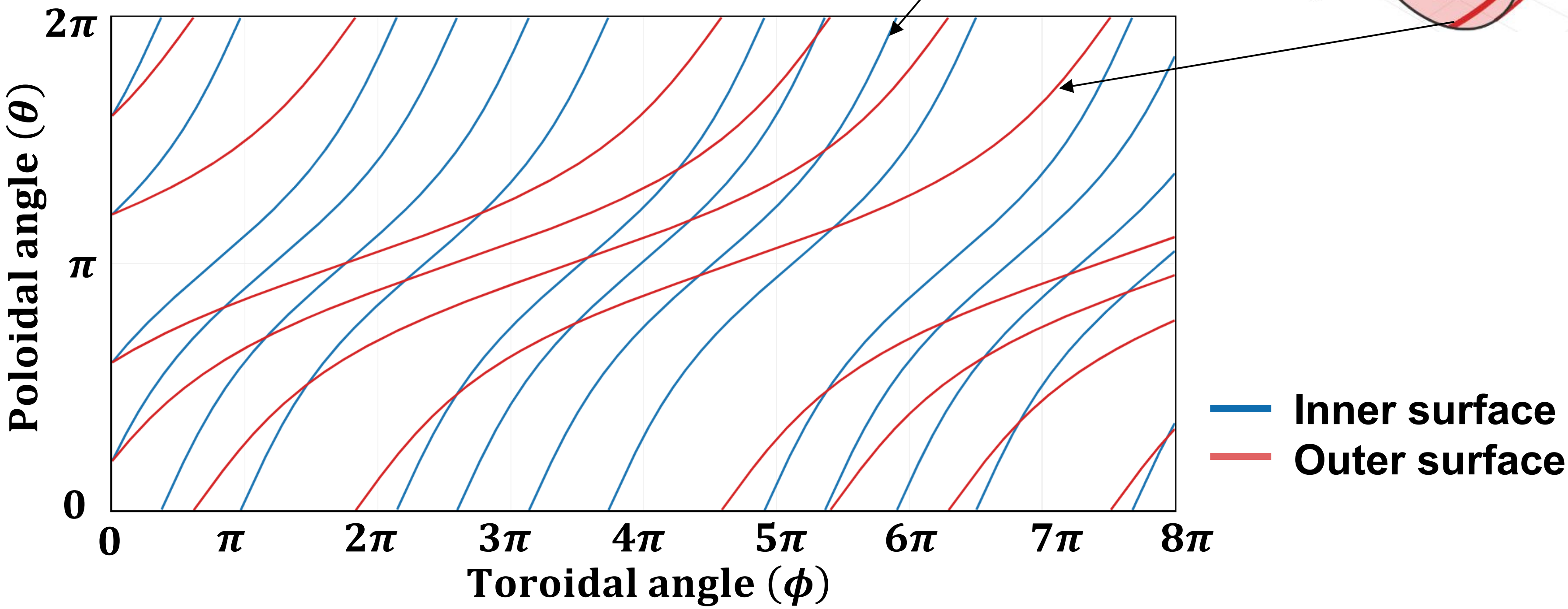
C. C. Baker, *et al*, Nuclear Technology/Fusion, 1, 5 (1981)

Global magnetic shear

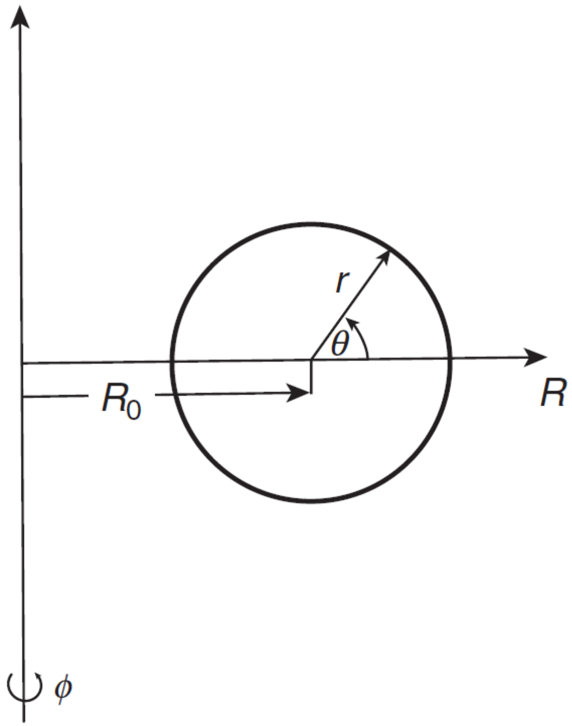
- **Shear**







Volume of the constant flux



$$R = R_o + r\cos\theta$$

$$Z = r\sin\theta$$

$$\psi = \psi(r, \theta)$$

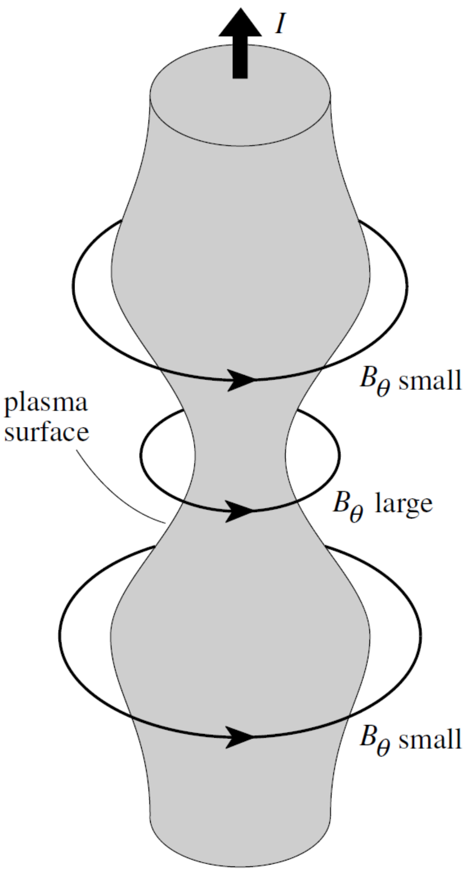
$$r = \hat{r}(\theta, \psi)$$

$$V(\psi) = \int_0^{2\pi}\int_0^{2\pi}\int_0^{\hat{r}} R r\,dr\,d\theta\,d\phi$$

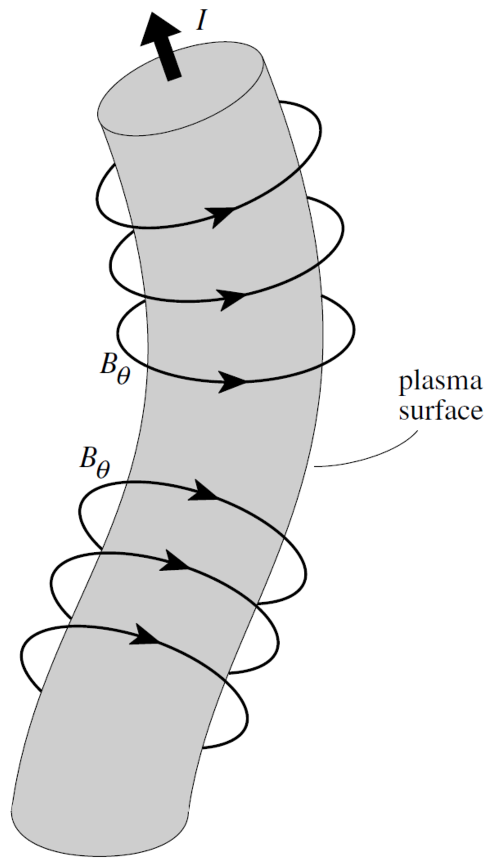
$$V(\psi) = \pi R_o \int_0^{2\pi} d\theta\,\hat{r}\left[1 + \frac{2}{3}\left(\frac{\hat{r}}{R_o}\right)\cos\theta\right]$$

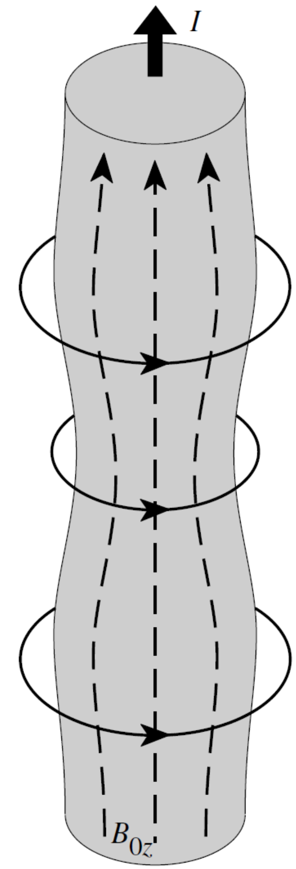
A cylindrical plasma column is stable when the safety factor is greater than unity

- **Sausage instability (m=0)**



- **Kink instability**





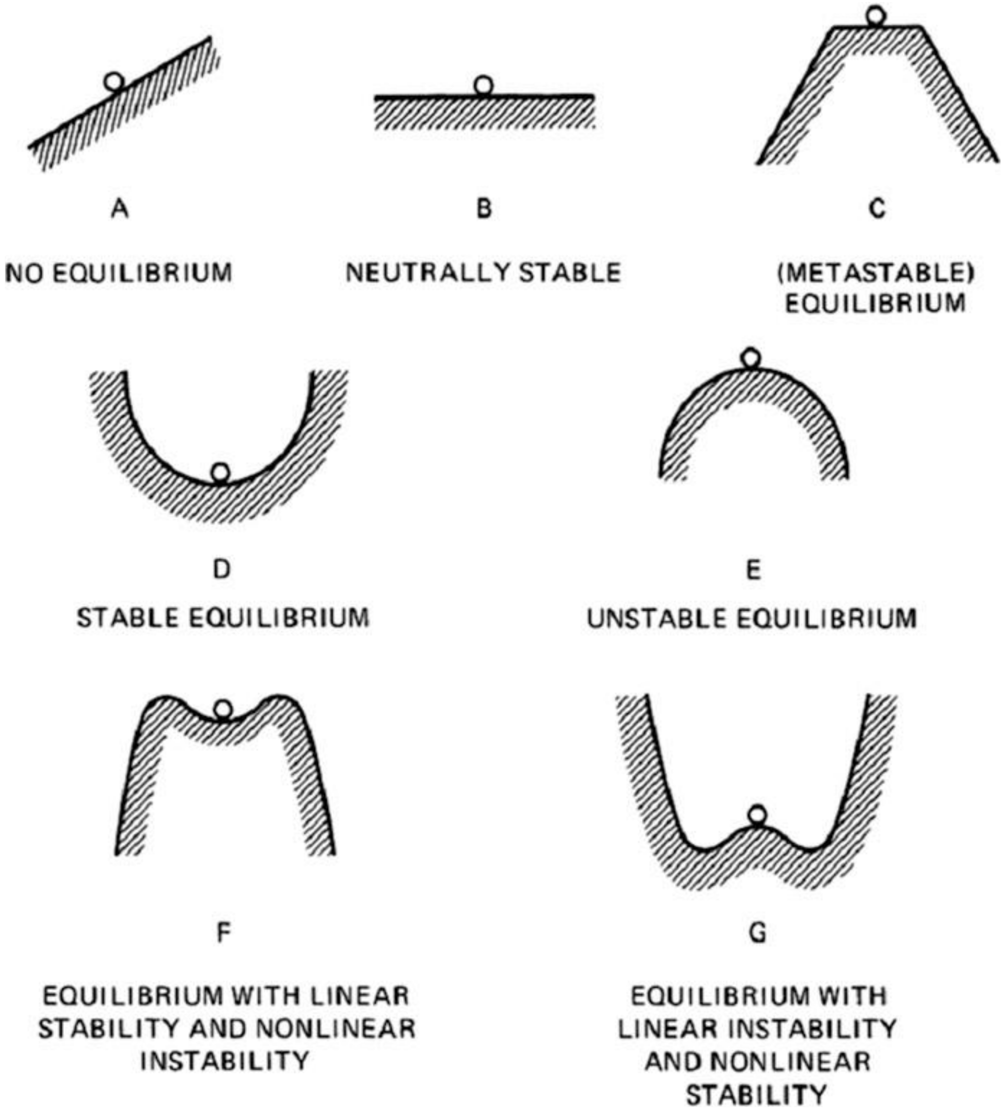
- **MHD Safety factor:** $q(r) = \dfrac{rB_z(r)}{R_o B_\theta(r)}$ **Kruskal–Shafranov limit**

Magnetic well

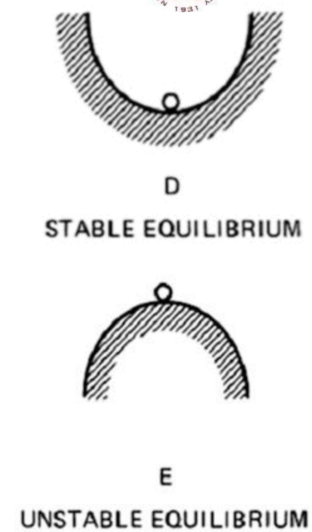
$$\widehat{W} = 2\frac{V}{\langle B^2\rangle}\frac{d}{dV}\left\langle\frac{B^2}{2}\right\rangle$$

$$= 2\frac{V}{\langle B^2\rangle}\frac{d}{dV}\left\langle\mu_o p + \frac{B^2}{2}\right\rangle$$

- **A magnetic well is a quantity that measures plasma stability against short perpendicular wavelength modes driven by the plasma pressure gradient.**



Variational formulation for checking stabilization



- **Equilibrium state:** $\vec{j}_o \times \vec{B}_o = \nabla p_o$
- **Momentum eq:** $\rho_m \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} \right] = \vec{j} \times \vec{B} - \nabla \cdot \overleftrightarrow{P}$

$$\rho_m = \rho_o + \tilde{\rho}_1 \qquad p = p_o + \tilde{p}_1 \qquad \vec{v} = \vec{v}_o + \tilde{\vec{v}}_1 = \tilde{\vec{v}}_1 \equiv \frac{\partial \vec{\xi}}{\partial t}$$

$$\vec{j} = \vec{j}_o + \tilde{\vec{j}}_1 \qquad \vec{B} = \vec{B}_o + \tilde{\vec{B}}_1 \equiv \vec{B}_o + \vec{Q}$$

$$\rho_m \frac{\partial^2 \vec{\xi}}{\partial t^2} = \vec{F}(\vec{\xi}) \qquad \vec{F}(\vec{\xi}) = \vec{j}_o \times \vec{Q} + \tilde{\vec{j}}_1 \times \vec{B}_o - \nabla \tilde{p}_1$$

$$\vec{F}(\vec{\xi}) = \frac{1}{\mu_o}(\nabla \times \vec{B}_o) \times \vec{Q} + \frac{1}{\mu_o}(\nabla \times \vec{Q}) \times \vec{B}_o + \nabla(\vec{\xi} \cdot \nabla p + \gamma p \nabla \cdot \vec{\xi})$$

- **The change in potential energy associated with the perturbation:**

$$\delta W = -\frac{1}{2} \int \vec{\xi}^* \cdot \vec{F}(\vec{\xi}) d\vec{r}$$

- **Stable requirement:** $\delta W \geq 0$

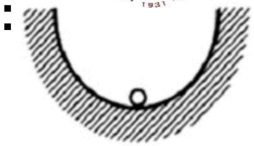
$$\delta W_F = \frac{1}{2} \int d\vec{r} \left[\frac{|\vec{Q}_\perp|^2}{\mu_o} + \frac{B^2}{\mu_o} |\nabla \cdot \vec{\xi}_\perp + 2\vec{\xi}_\perp \cdot \vec{\kappa}|^2 + \gamma p |\nabla \cdot \vec{\xi}|^2 - 2(\vec{\xi}_\perp \cdot \nabla p)(\vec{\kappa} \cdot \vec{\xi}_\perp^*) - J_{||}(\vec{\xi}_\perp^* \times \vec{b}) \cdot \vec{Q}_\perp \right]$$

Variational formulation for checking stabilization

- **The change in potential energy associated with the perturbation:**

$$\delta W = -\frac{1}{2}\int \vec{\xi}^{*}\cdot\vec{F}\left(\vec{\xi}\right)d\,\vec{r}$$

- **Stable requirement:** $\delta W \geq 0$



D

STABLE EQUILIBRIUM

E

UNSTABLE EQUILIBRIUM

$$\delta W_F = \frac{1}{2}\int d\,\vec{r}\left[\frac{\left|\vec{Q}_\perp\right|^2}{\mu_o} + \frac{B^2}{\mu_o}\left|\nabla\cdot\vec{\xi}_\perp + 2\,\vec{\xi}_\perp\cdot\vec{\kappa}\right|^2 + \gamma p\left|\nabla\cdot\vec{\xi}\right|^2 \right.$$
$$\left. -2\left(\vec{\xi}_\perp\cdot\nabla p\right)\left(\vec{\kappa}\cdot\vec{\xi}_\perp^{\,*}\right) - J_{||}\left(\vec{\xi}_\perp^{\,*}\times\vec{b}\right)\cdot\vec{Q}_\perp\right]$$

- **Stabilization terms:**

$\dfrac{\left|\vec{Q}_\perp\right|^2}{\mu_o}$: **For bending magnetic field lines, (shear Alfvén wave).**

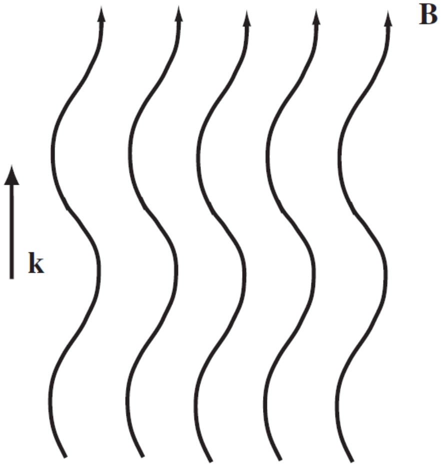
$\dfrac{B^2}{\mu_o}\left(\left|\nabla\cdot\vec{\xi}_\perp + 2\,\vec{\xi}_\perp\cdot\vec{\kappa}\right|\right)^2$: **For compressing the magnetic field, (compressional Alfvén wave).**

$\gamma p\left|\nabla\cdot\vec{\xi}\right|^2$: **For compressing the plasma, (sound wave).**

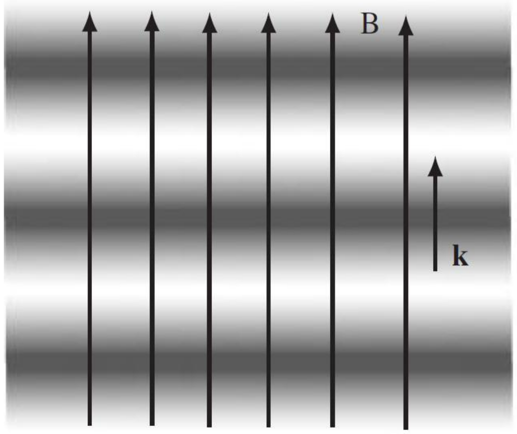
- **Destabilization terms:** $-2\left(\vec{\xi}_\perp\cdot\nabla p\right)\left(\vec{\kappa}\cdot\vec{\xi}_\perp^{\,*}\right) - J_{||}\left(\vec{\xi}_\perp^{\,*}\times\vec{b}\right)\cdot\vec{Q}_\perp$

Alfvén waves

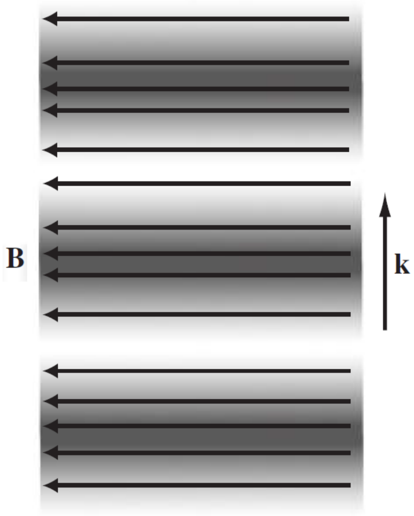
- **Shear Alfvén wave:**



- **Longitudinal sound wave (the slow magnetosonic wave)**

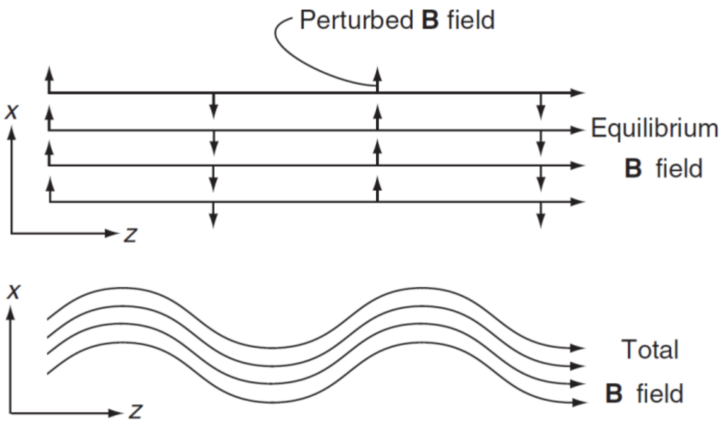


- **The fast magnetosonic wave (Compressional Alfvén wave):**

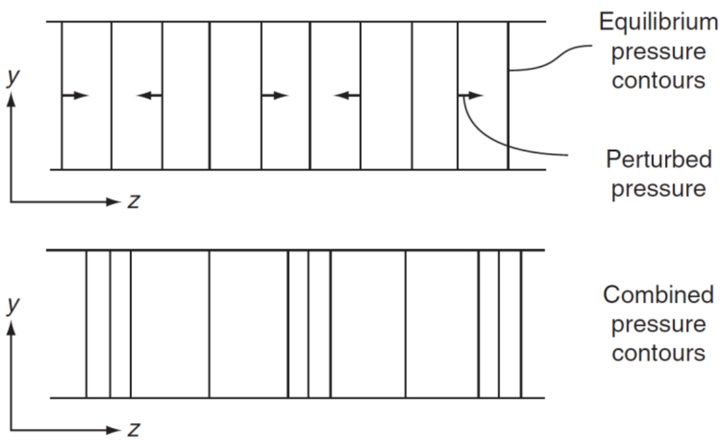


Alfvén waves

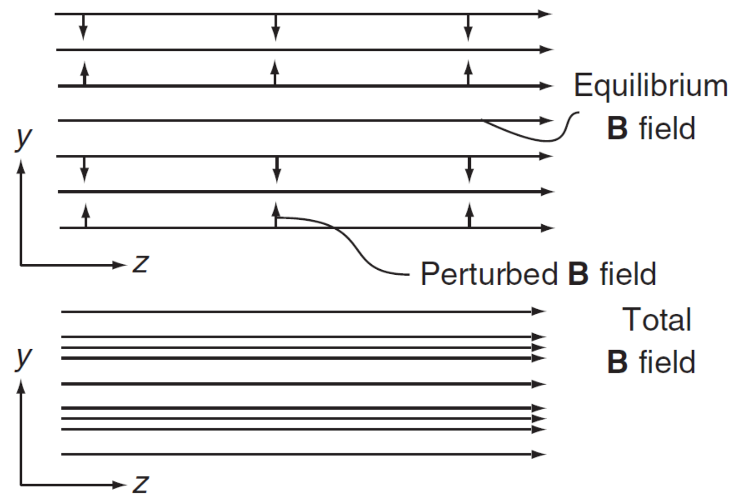
- **Shear Alfvén wave:**



- **The slow magnetosonic wave (Shear + Compressional Alfvén wave):**



- **The fast magnetosonic wave (Compressional Alfvén wave):**



Classification of MHD instabilities

- **Locations:**
  - **Internal/Fixed boundary modes: mode structure does not require any motion of the plasma-vacuum interface away from its equilibrium position.**
  - **External/Free-boundary modes: the plasma-vacuum interface moves from its equilibrium position during an unstable MHD perturbation.**
- **Dominant destabilizing term**
  - **Pressure-driven modes: the dominant destabilizing term is the one proportional to ∇p.**
  - **Current-driven modes: the dominant destabilizing term is the one proportional to $J_{||}$.**

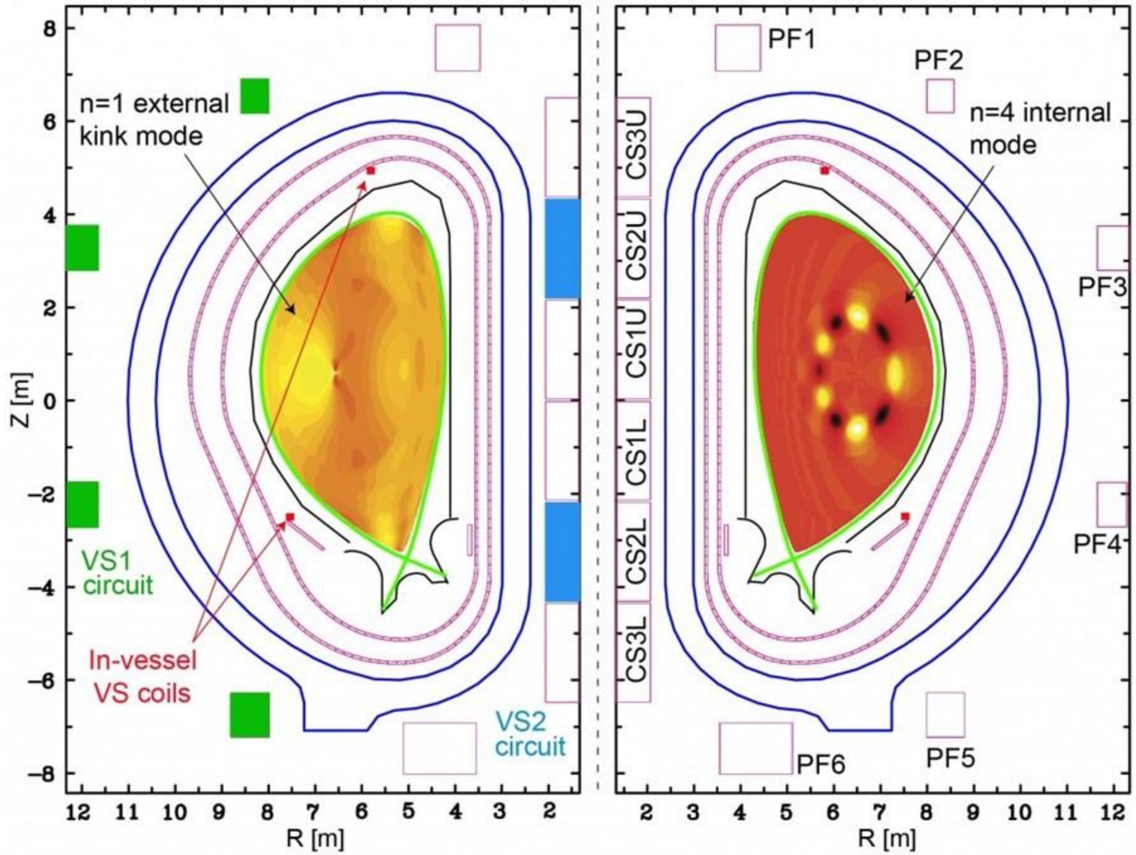
$$\delta W_F = \frac{1}{2}\int d\vec{r}\left[\frac{\left|\vec{Q}_\perp\right|^2}{\mu_o} + \frac{B^2}{\mu_o}\left|\nabla\cdot\vec{\xi}_\perp + 2\vec{\xi}_\perp\cdot\vec{\kappa}\right|^2 + \gamma p\left|\nabla\cdot\vec{\xi}\right|^2 - 2(\vec{\xi}_\perp\cdot\nabla p)\left(\vec{\kappa}\cdot\vec{\xi}_\perp^{\,*}\right) - J_{||}\left(\vec{\xi}_\perp^{\,*}\times\vec{b}\right)\cdot\vec{Q}_\perp\right]$$

Classification of MHD instabilities

- **Locations:**
  - **Internal/Fixed boundary modes: mode structure does not require any motion of the plasma-vacuum interface away from its equilibrium position.**
  - **External/Free-boundary modes: the plasma-vacuum interface moves from its equilibrium position during an unstable MHD perturbation.**
- **Dominant destabilizing term**
  - **Current-driven modes, e.g., kink instability, sausage instability.**
  - **Pressure-driven modes, e.g., interchange mode, ballooning mode.**

External mode vs internal mode

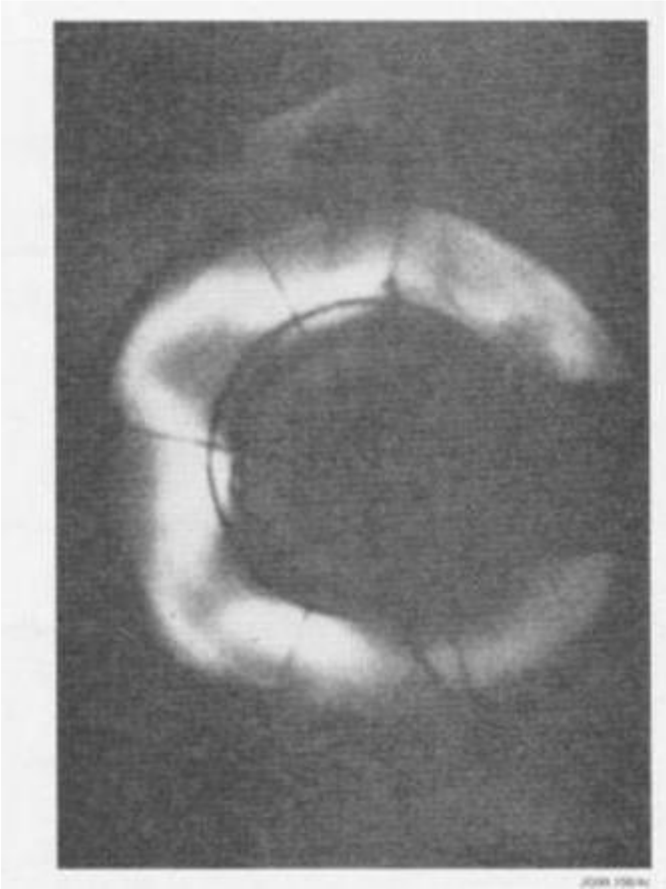
- **Predicted behaviors of the plasma in ITER**



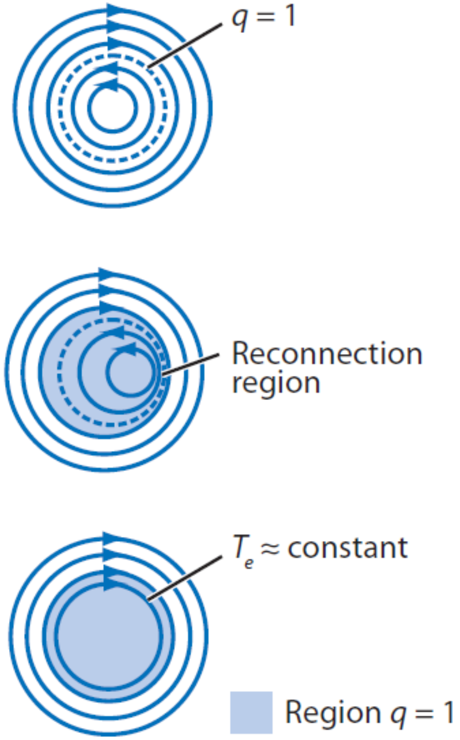
https://www.iter.org/newsline/-/3401

External vs internal kink instability

- **External kink instability**



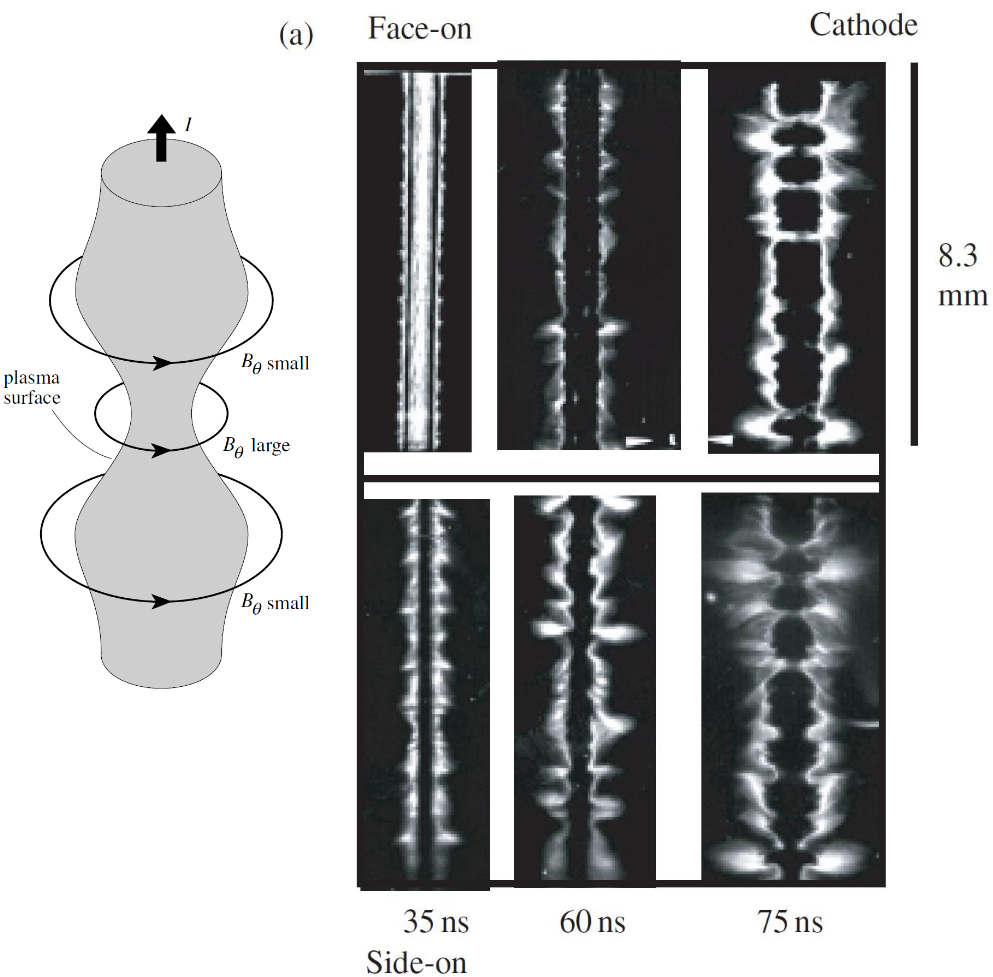
- **Internal kink instability**



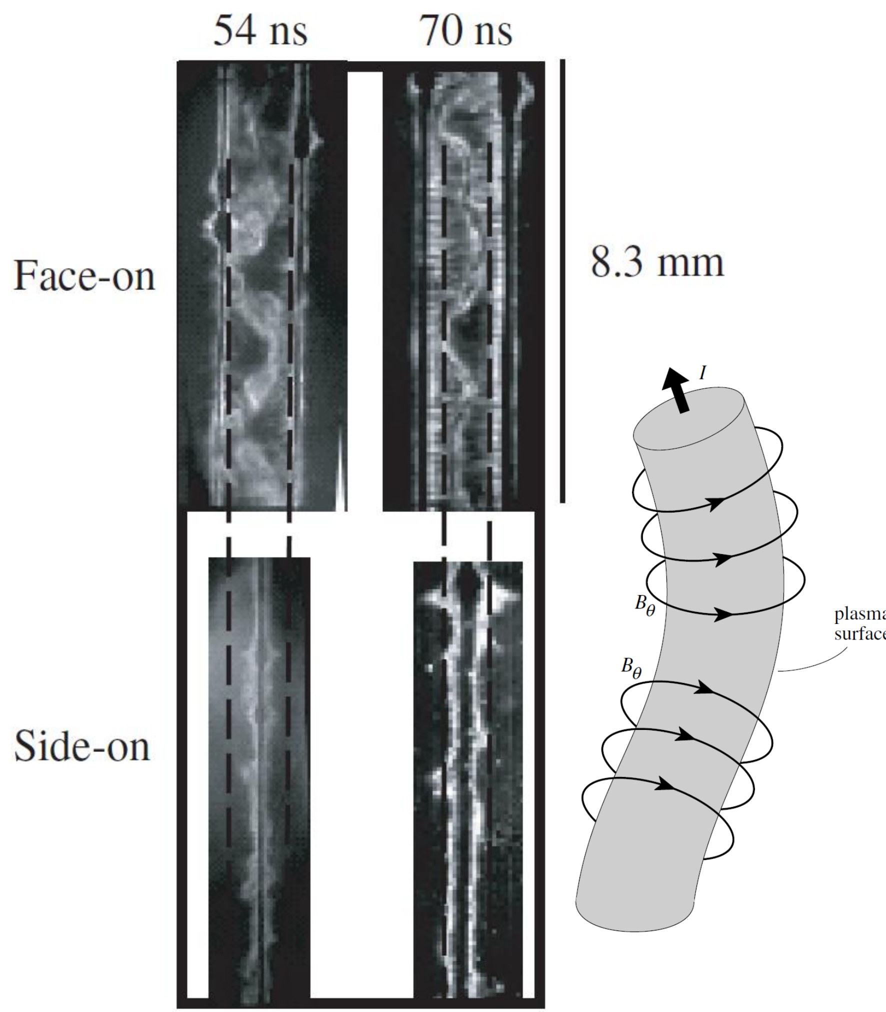
https://en.wikipedia.org/wiki/Kink_instability
R A Bingham et al 2026 Plasma Phys. Control. Fusion 68 030201
E. G. Zweibel and M. Yamada, Annu. Rev. Astron. Astrophys., 47, 291 (2009)

Current-driven instability

- **Sausage instability (m=0)**

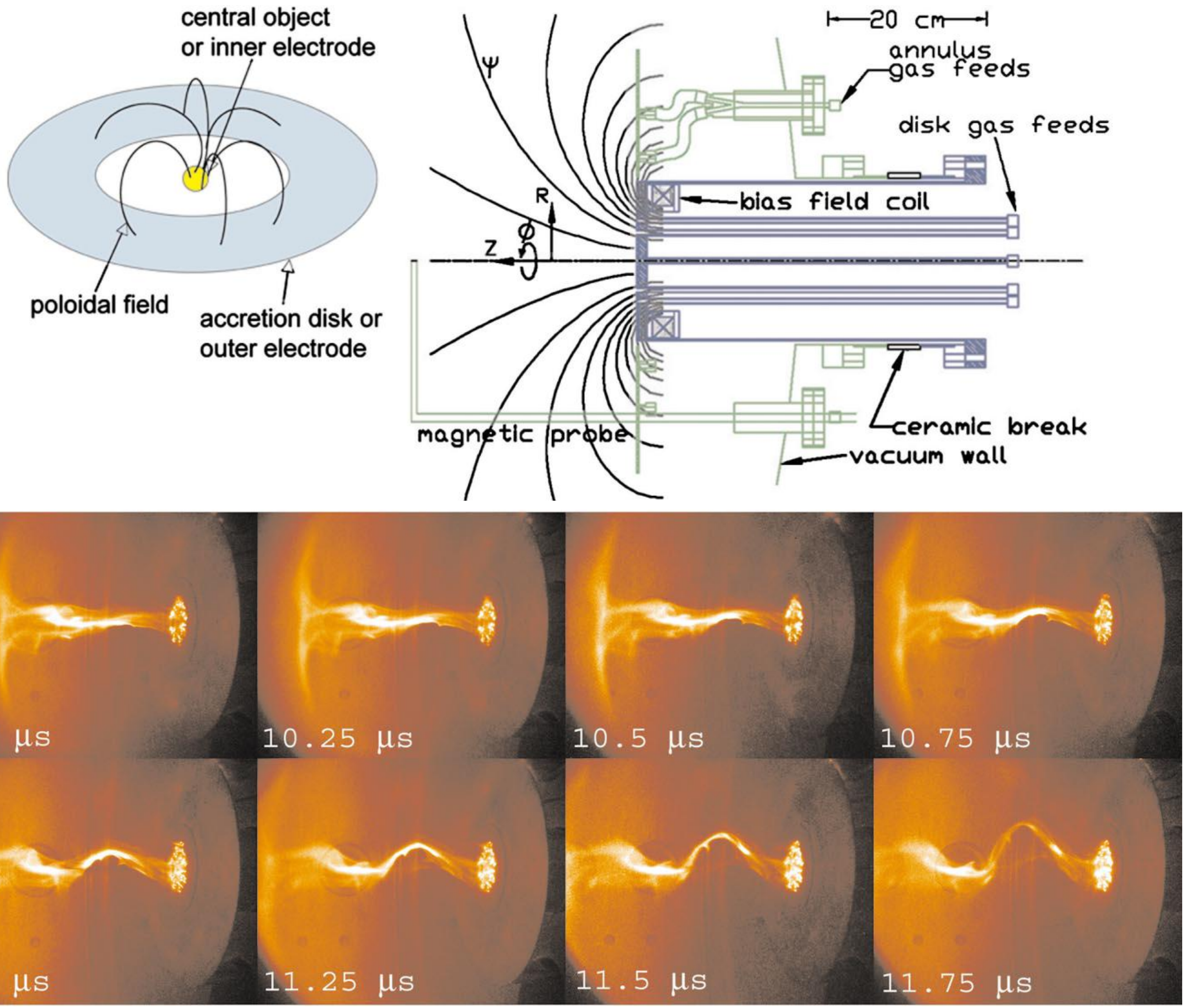


- **Kink instability (m=1)**



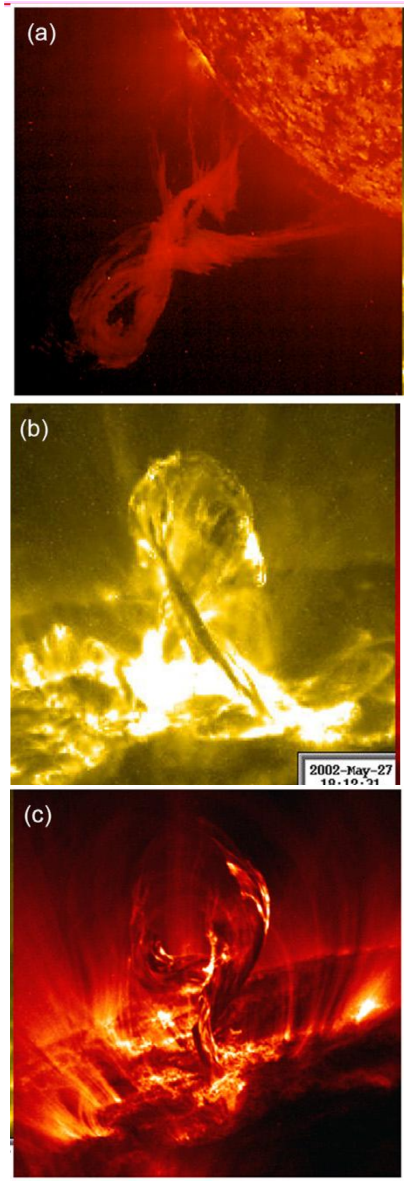
F. N. Beg, et. Al., Plasma Phys. Control. Fusion **46**, 1 (2004)

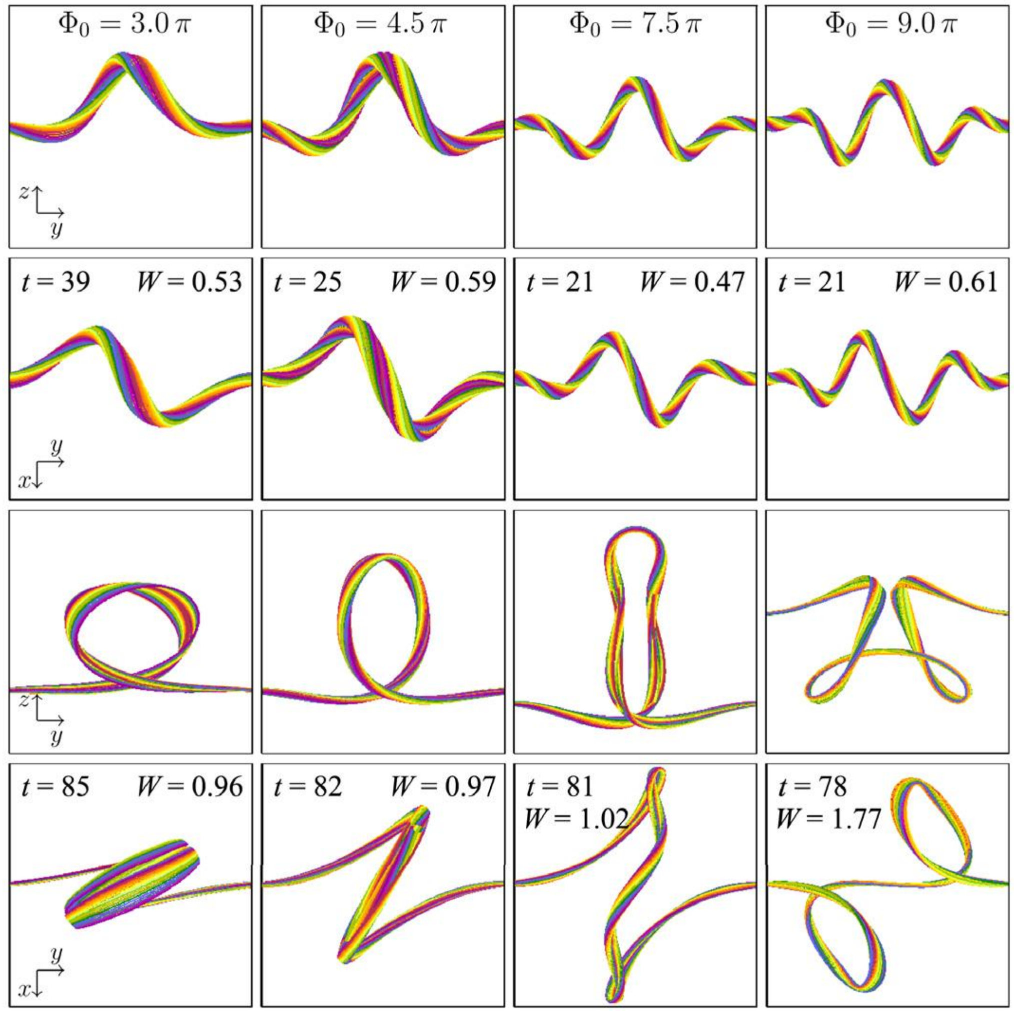
Kink instabilities in the lab



S. C. Hsu, P. M. Bellan, Phys. Plasma **12**, 032103 (2005)

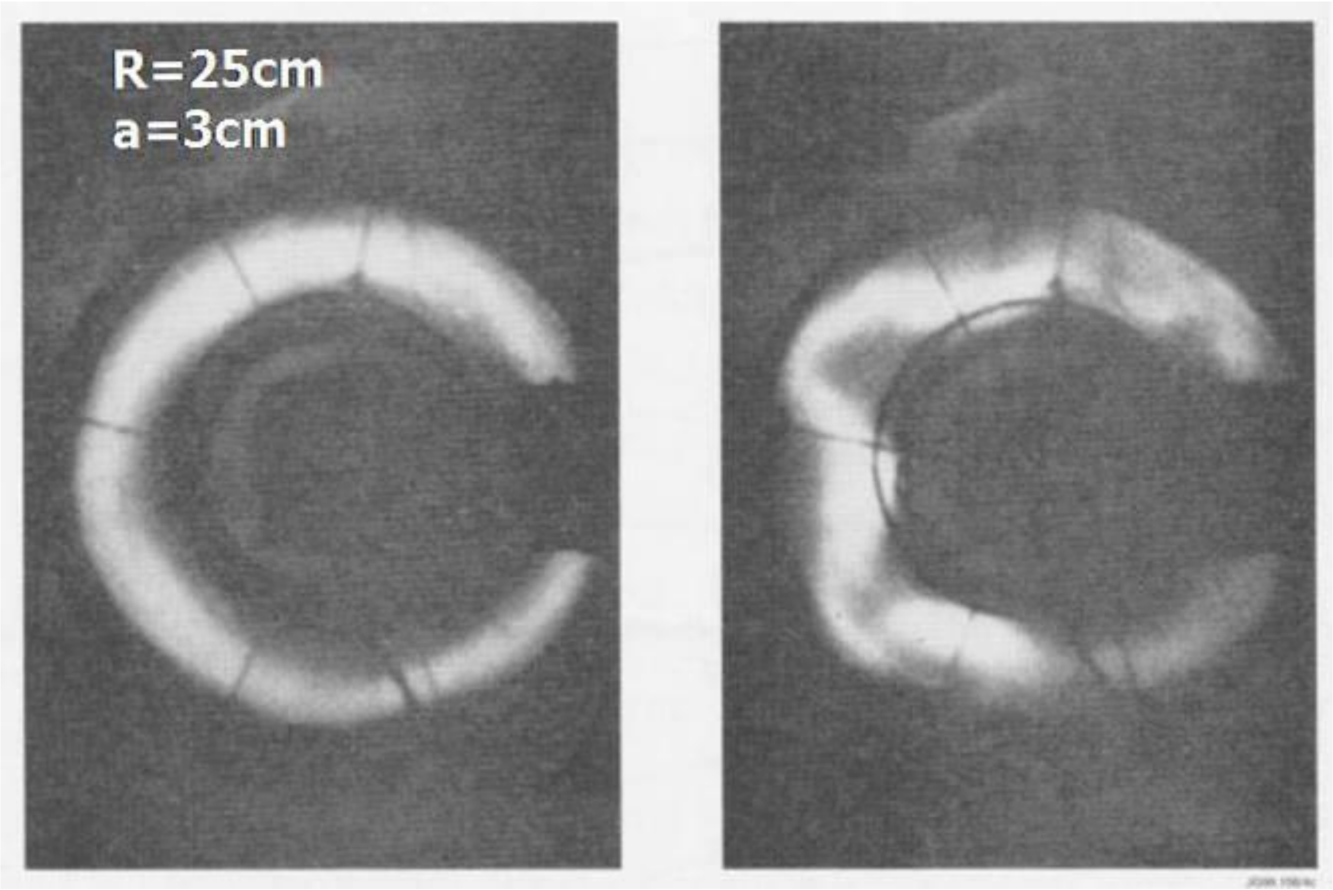
Kink instabilities in space





T Török *et al*, Plasma Phys. Control. Fusion **56**, 064012 (2014)

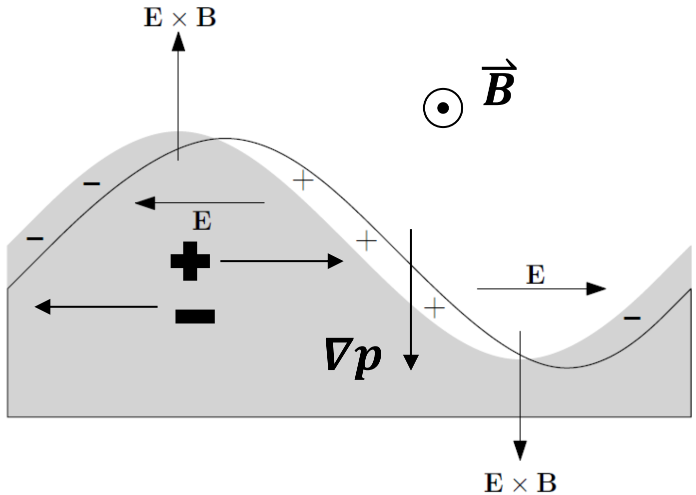
Kink instability in Tokamak

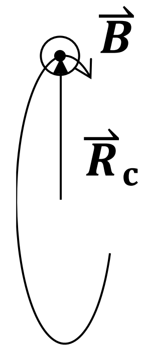


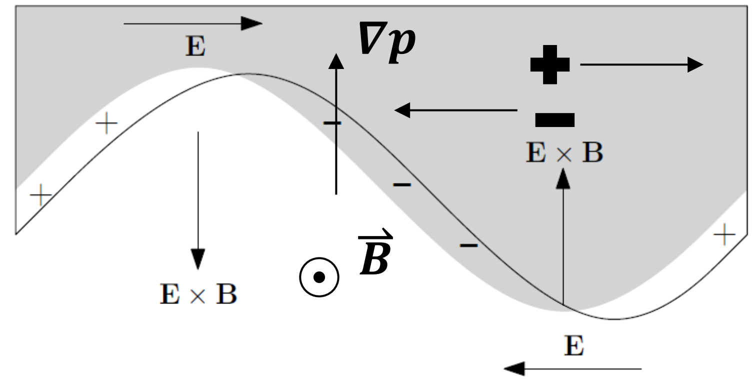
https://en.wikipedia.org/wiki/Kink_instability

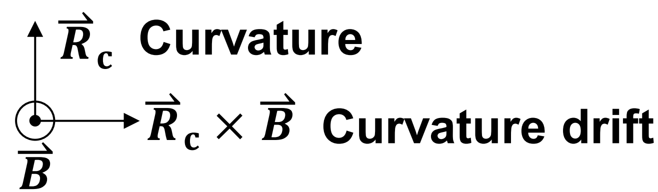
Pressure driven instability – interchange perturbations

- **Unstable: bad curvature** $\vec{R}_c \cdot \nabla p < 0$
- **stable: good curvature** $\vec{R}_c \cdot \nabla p > 0$

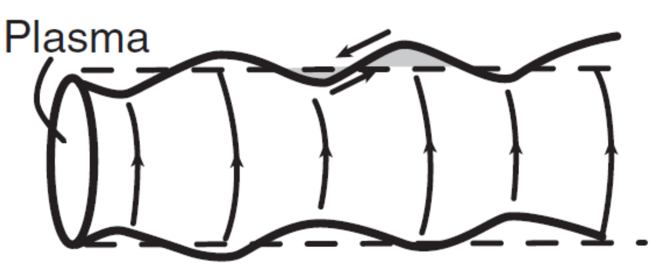








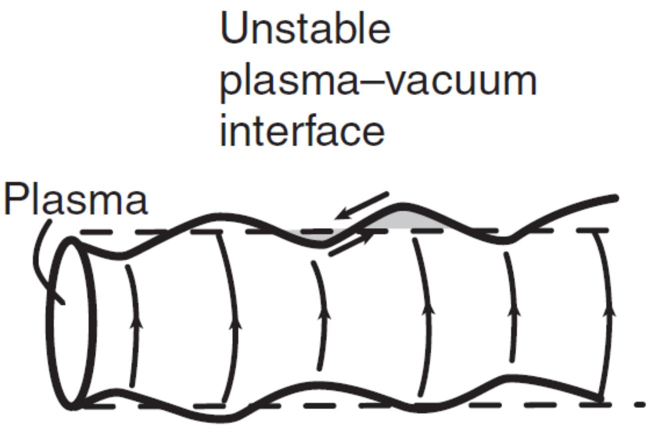
Unstable
plasma–vacuum
interface



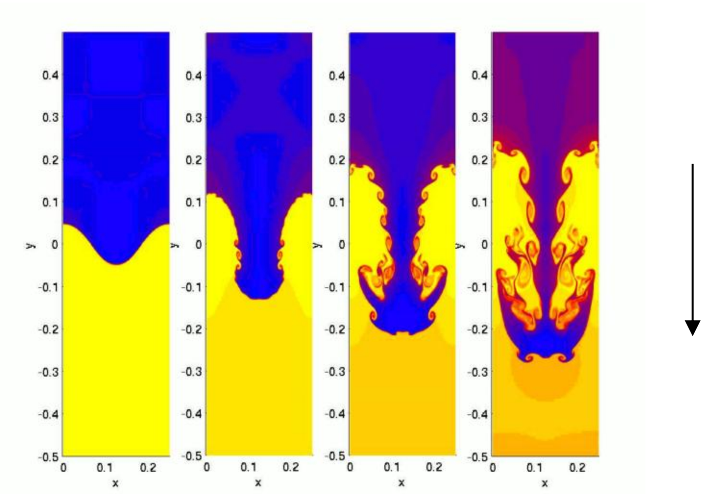
Stable
plasma–vacuum
interface

Ben Dudson, Lecture note of Magnetic Confinement Fusion 2014

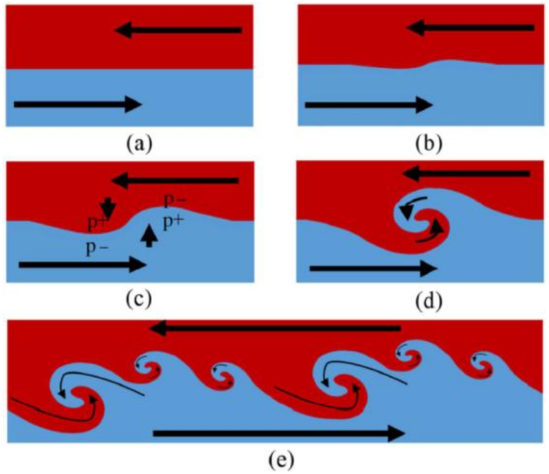
Rayleigh-Taylor instability



- **Rayleigh-Taylor instability**

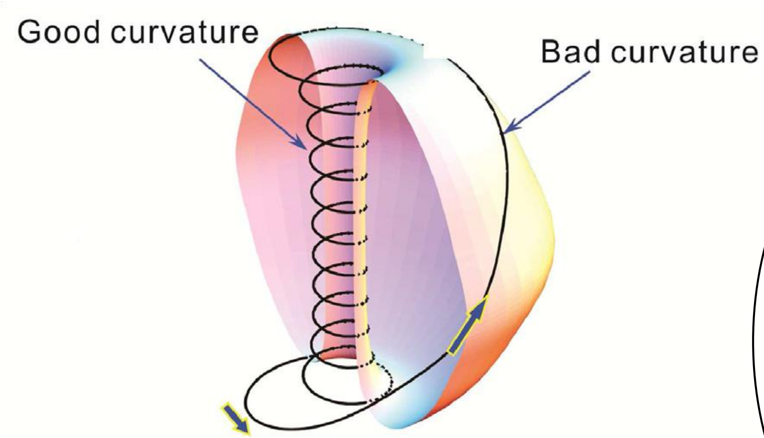


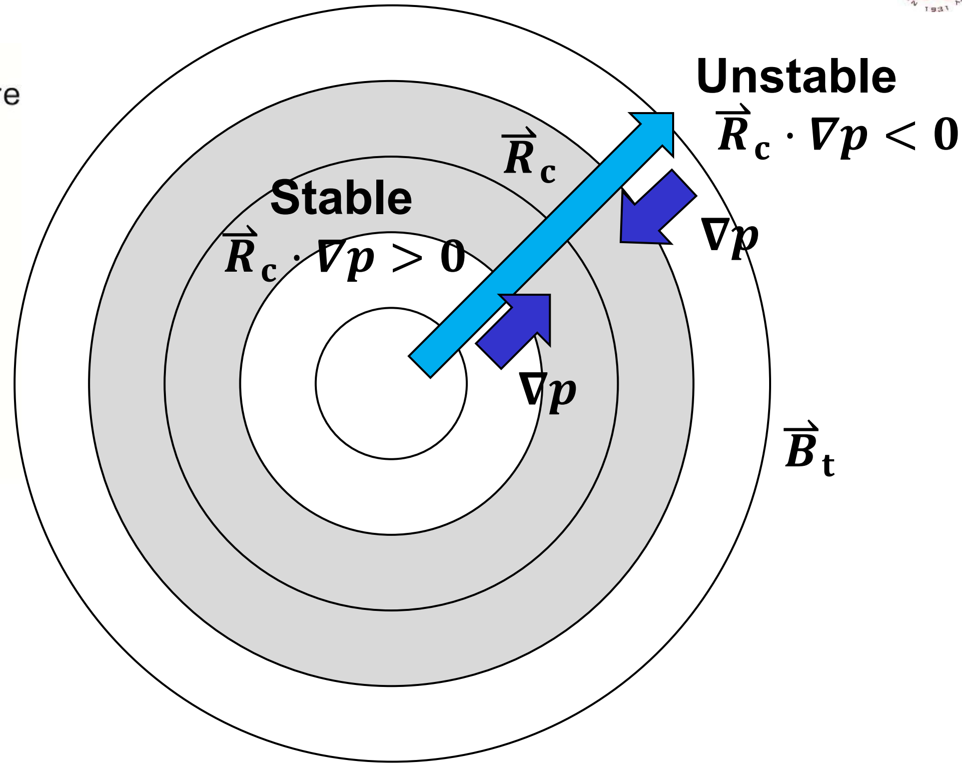
- **Kelvin-Helmholtz instability**



https://en.wikipedia.org/wiki/Rayleigh%E2%80%93Taylor_instability
https://en.wikipedia.org/wiki/Kelvin%E2%80%93Helmholtz_instability
Xie Lei et al, Energy Report **7**, 2262 (2021)

Pressure driven instability – interchange perturbations



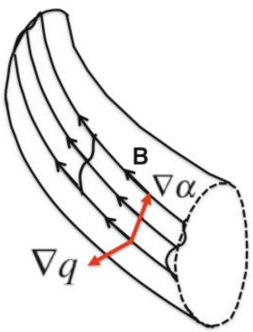


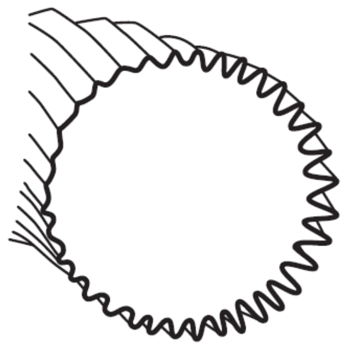
- **Suydam criterion for cylindrical plasmas:** $$\mu_o \frac{2r^2}{{B_\theta}^2}\frac{1}{s^2}\vec{R}_c \cdot \nabla p > -\frac{1}{4}$$ **Shear:** $s = \frac{r}{q}\frac{dq}{dr}$

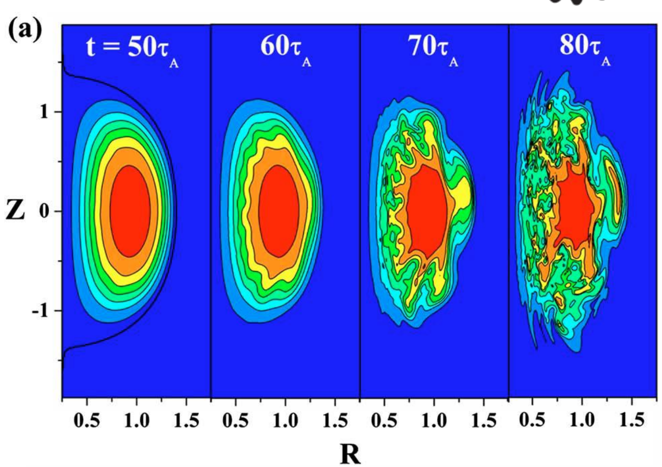
- **Mercier criterion for tokamak:** $$D = -\mu_o \frac{2r}{B^2}\frac{1}{s^2}\frac{dp}{dr}\left(1-q^2\right) < \frac{1}{4}$$

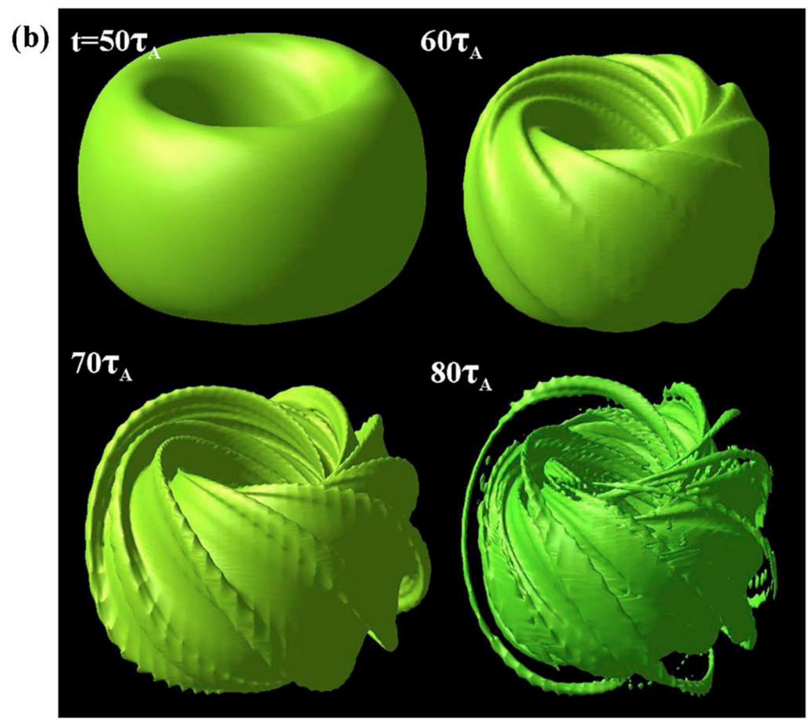
Ballooning mode – show wavelength mode

$$\delta W_F = \frac{1}{2}\int d\,\vec{r}\left[\frac{\left|\vec{Q}_\perp\right|^2}{\mu_o} + \frac{B^2}{\mu_o}\left|\nabla\cdot\vec{\xi}_\perp + 2\,\vec{\xi}_\perp\cdot\vec{\kappa}\right|^2 + \gamma p\left|\nabla\cdot\vec{\xi}\right|^2 \right.$$
$$\left. -2\left(\vec{\xi}_\perp\cdot\nabla p\right)\left(\vec{\kappa}\cdot\vec{\xi}_\perp^{\,*}\right) - J_{||}\left(\vec{\xi}_\perp^{\,*}\times\vec{b}\right)\cdot\vec{Q}_\perp\right]$$



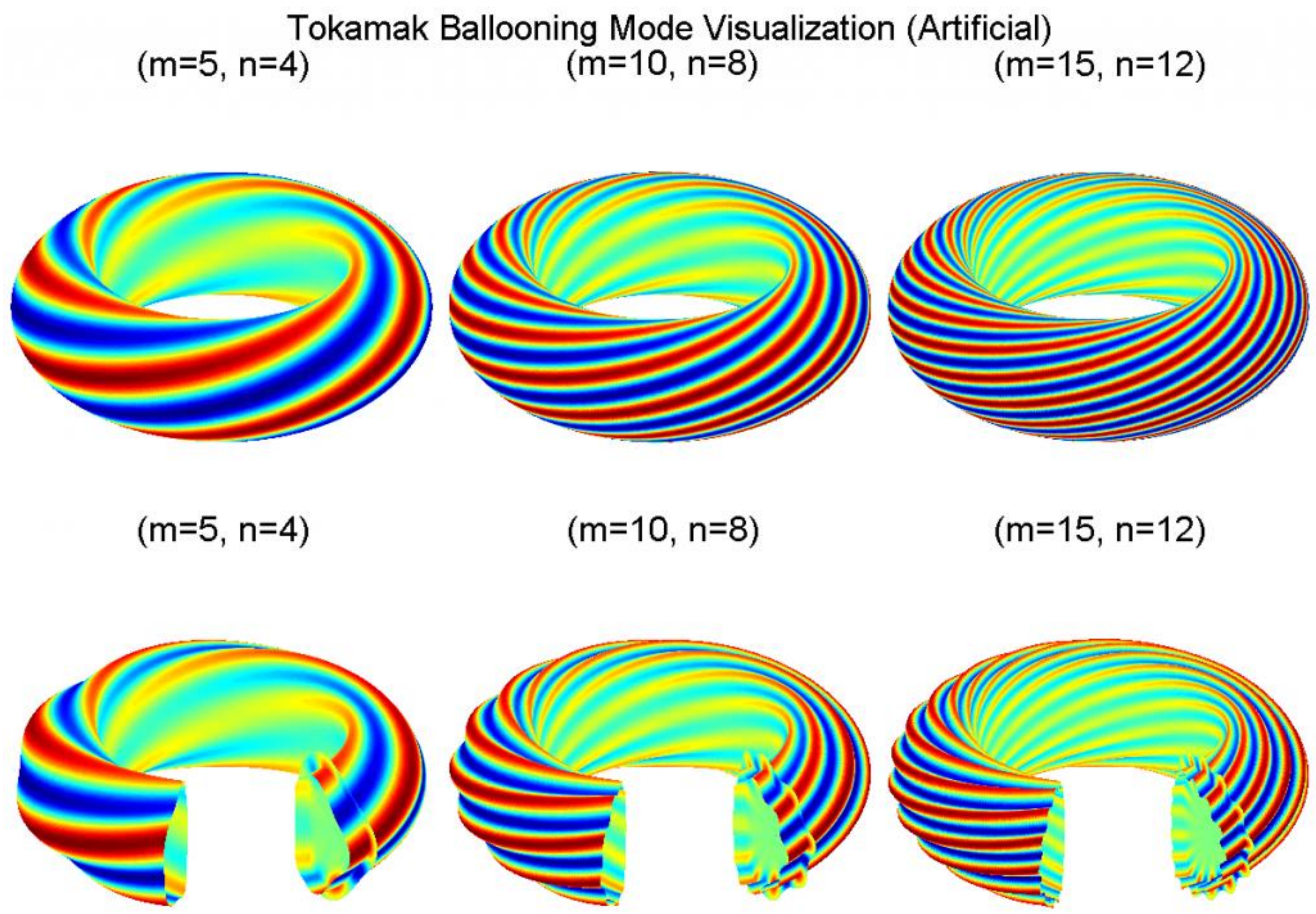






R. Khan et al, Phys. Plasmas **14**, 062302 (2007)

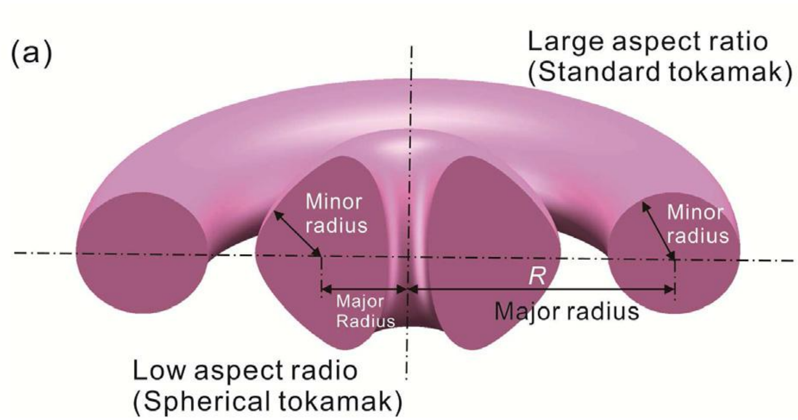
Ballooning mode – show wavelength mode

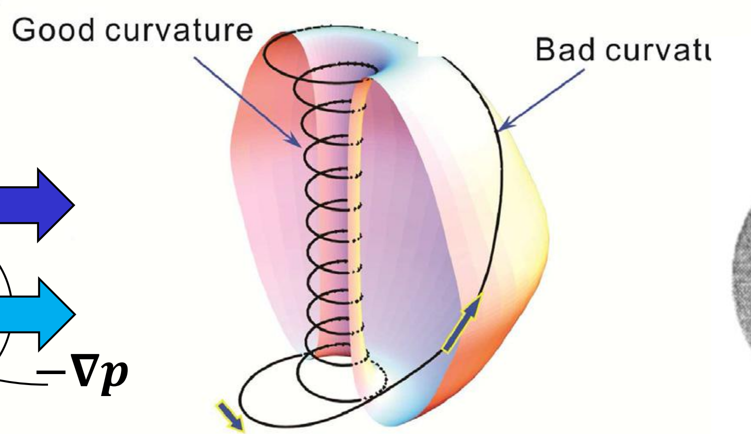


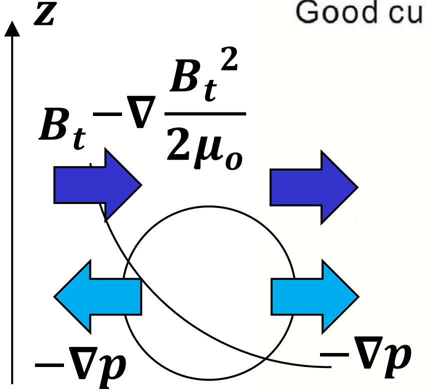
https://blog.sciencenet.cn/home.php?mod=space&uid=222053&do=album&picid=308476#pic_block

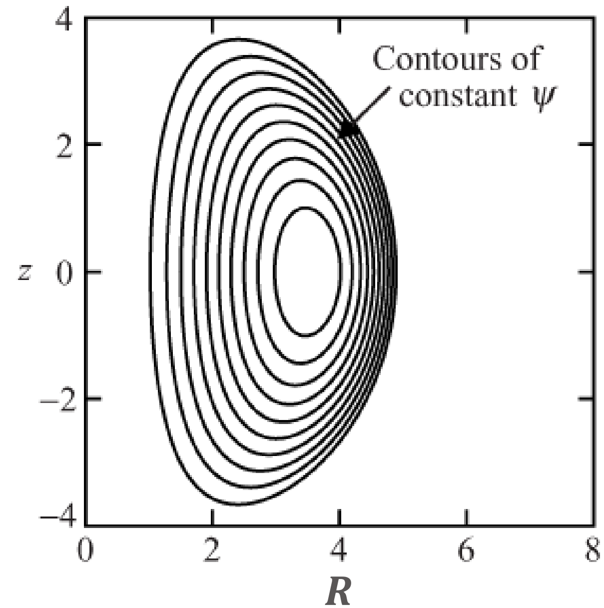
The Spherical tokamak

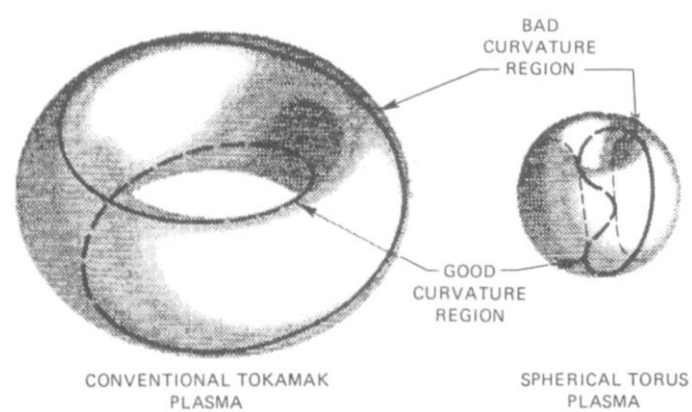
- **Aspect ratio R_o/a ~ 1.6**









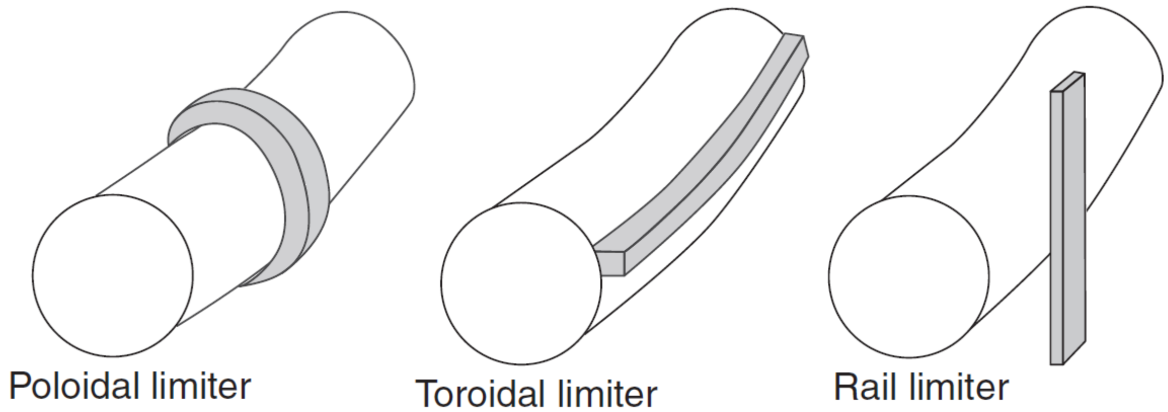


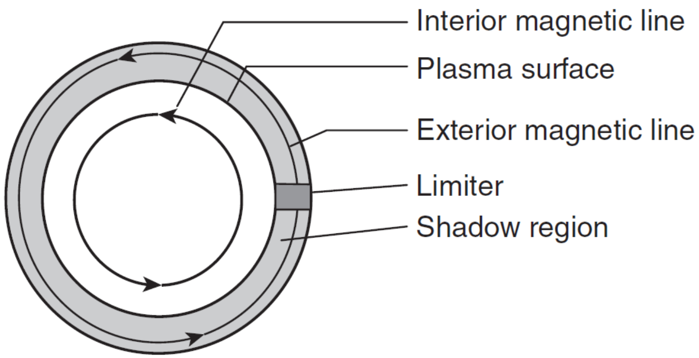
Zhe Gao, Matter and Radia. Extrem., **1**, 153 (2016)
Y-k. M.Peng and D. J. Strickler, Nucl. Fusion **26**, 769 (1986)

The Spherical tokamak

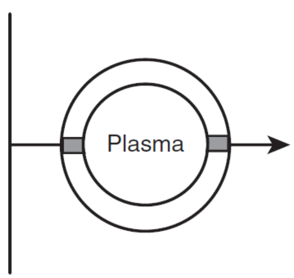
- Aspect ratio $R_o/a \sim 1.6$
- Advantages:
  - Higher β_t limit.
  - A compact design almost spherical in appearance.
- Challenges:
  - Minimum space is given in the center of the torus to accommodate the toroidal field coils.
  - With a very compact design the technology associated with the construction and maintenance of the device may be more difficult than for a "normal" tokamak.
  - Large currents will have to be driven noninductively, a costly and physically difficult requirement.

Limiter protects the vacuum chamber from plasma bombardment and defines the edge of the plasma

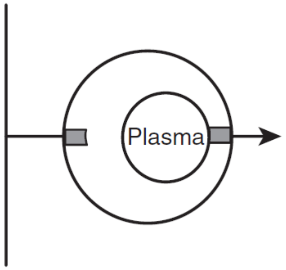




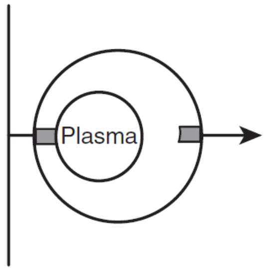
- **Vertical field is correct.**



- **Vertical field is too small.**

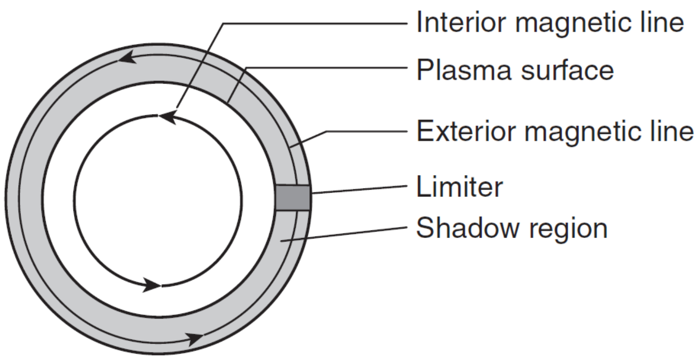


- **Vertical field is too large.**

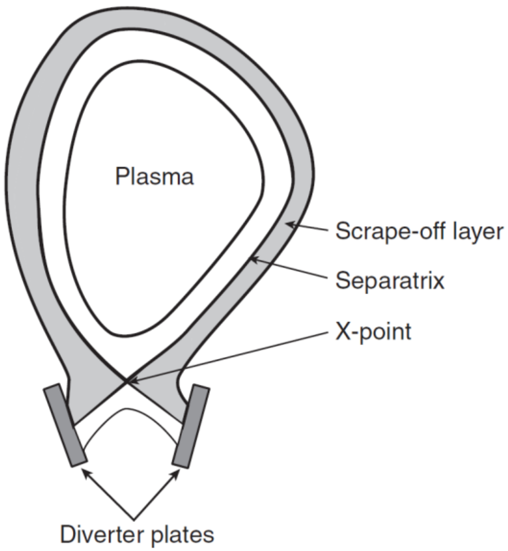


Limiter protects the vacuum chamber from plasma bombardment and defines the edge of the plasma

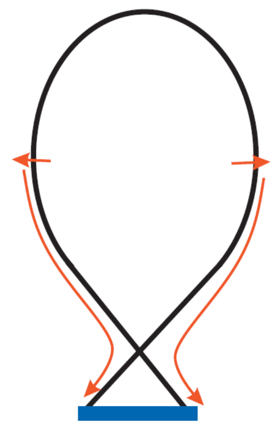
- A mechanical limiter is a robust piece of material, often made of tungsten, molybdenum, or graphite placed inside the vacuum chamber.
- Some of the particles of the limiter surface may escape. Neutral particles can penetrate some distance into the plasma before being ionized.
- The high-z impurities can lead to significant additional energy loss in the plasma through radiation.
- In ignition experiments and fusion reactors, the bombardment is more intense and extends over longer periods of time. In addition, if the impurity level is too high, it may not be possible to achieve a high enough temperature to ignite.



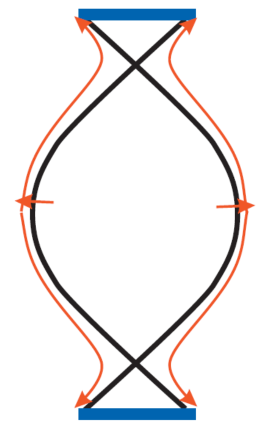
The magnetic divertor – guide a narrower layer of magnetic lines away from the edge of the plasma



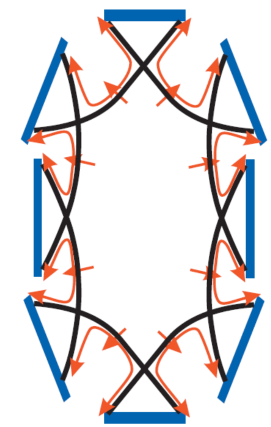
- **Single-null poloidal-field divertors for tokamak**



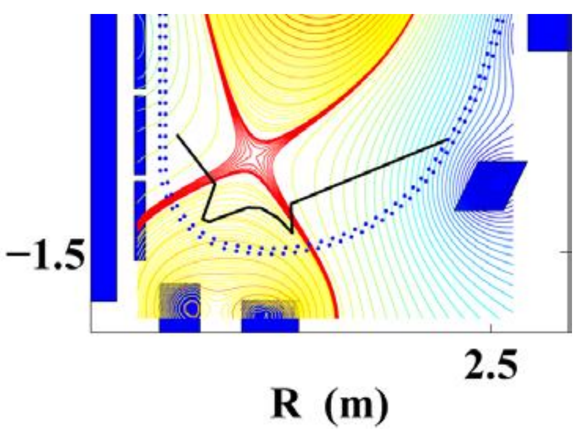
- **Double-null poloidal-field divertors for tokamak**



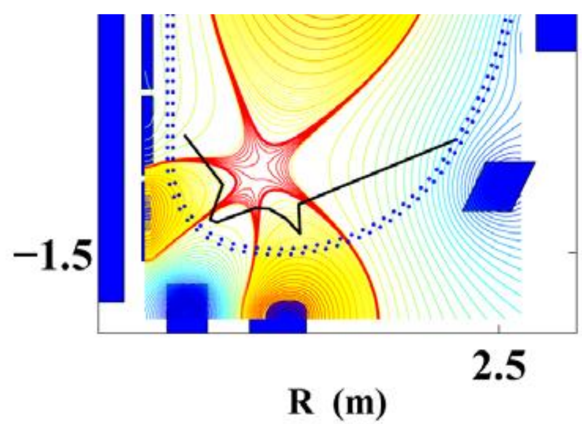
- **Island divertor for stellerators**



- **Standard**



- **Snowflake**



Y. Feng, Nucl. Fusion, **46**, 807 (2006)
L Xue *et al*, Plasma Phys. Control. Fusion **58**, 055005 (2016)

Pros and cons of a divertor

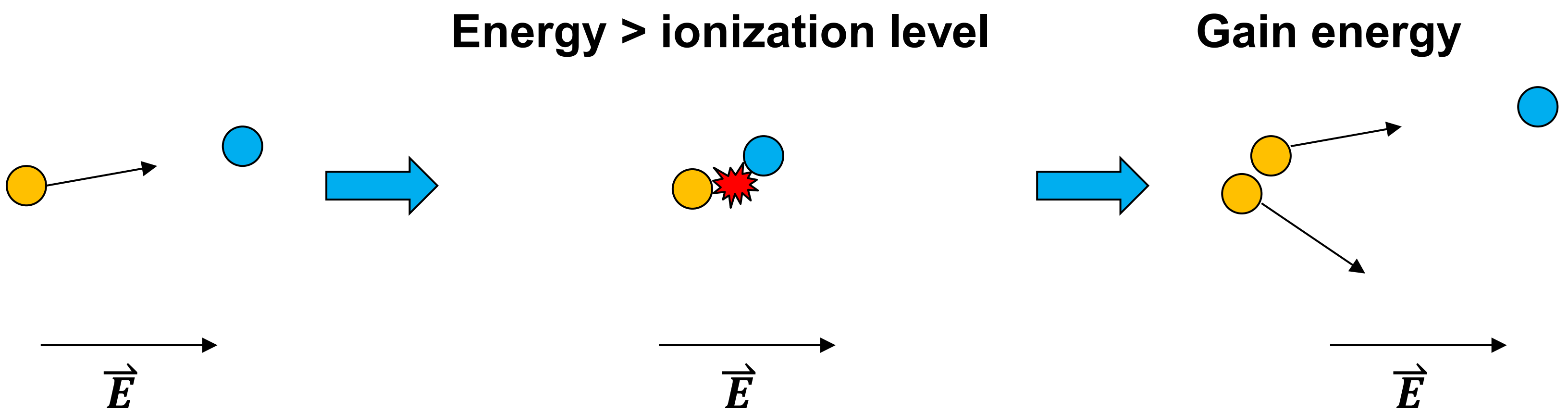
- **Advantages:**
  - The collector plate is remote from the plasma. There is space available to spread out the magnetic lines.
  - A lower intensity of particles and energy bombard the collector plate leading to a longer replacement time.
  - It is more difficult for impurities to migrate into the plasma.
  - There are longer distance distances to travel and if a neutral particle becomes ionized before or during the time it crosses the divertor layer on its way toward the plasma, its parallel motion then carries it back to the collector plate.
  - The larger divertor chamber provides more access to pump out impurities.
  - The plasma edge is not in direct contact with a solid material such as a limiter.
- **Disadvantages: larger and more complex system and more expensive.**

Course Outline

- Magnetic confinement fusion (MCF)
  - Gyro motion, MHD
  - 1D equilibrium (z pinch, theta pinch)
  - Drift: ExB drift, grad B drift, and curvature B drift
  - Tokamak, Stellarator (toroidal field, poloidal field)
  - Magnetic flux surface
  - 2D axisymmetric equilibrium of a torus plasma: Grad-Shafranov equation.
  - Stability (Kink instability, sausage instability, Safety factor Q)
  - Central-solenoid (CS) start-up (discharge) and current drive
  - CS-free current drive: electron cyclotron current drive, bootstrap current.
  - Auxiliary Heating: ECRH, Ohmic heating, Neutral beam injection.

Collisions play an important role in ionization process

- At the microscopic level, breakdown requires the presence of sufficiently energy charge particles that have acquired enough energy from the applied electric field between two energy-dissipating collisions to ionize the material and to create more charge particles.



In most cases, electrons dominate the breakdown process since its mobility is much larger than that of ions

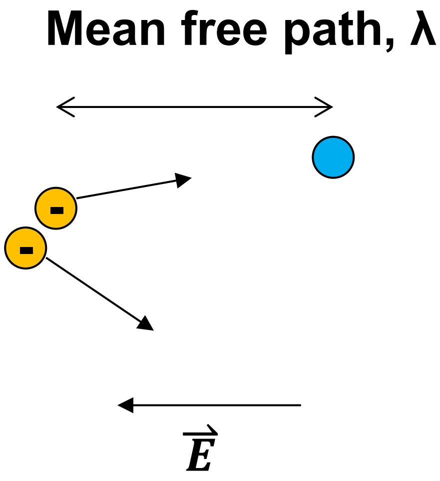
$$E_{\mathrm{k}} = \frac{1}{2} m v^2 \qquad v = \sqrt{\frac{2E_{\mathrm{k}}}{m}} \qquad E_{\mathrm{k}} \sim kT$$

Collision time: $$t = \frac{s}{\sqrt{\frac{2E_{\mathrm{k}}}{m}}} \sim \frac{n^{-1/3}}{\sqrt{T}}\sqrt{m} \qquad n = \frac{\#/}{V} \sim \frac{\#//}{S^3} \qquad s \sim n^{-1/3}$$

$$\frac{m_{\mathrm{i}}}{m_{\mathrm{e}}} \sim 2000 \times \text{Atomic mass} \qquad \frac{t_{\mathrm{i}}}{t_{\mathrm{e}}} \sim 45 \times \sqrt{A}$$

Mean free path is important in ionization process

- For an electron to acquire enough energy between collisions, its <u>mean free path</u> in the material must be sufficiently long.



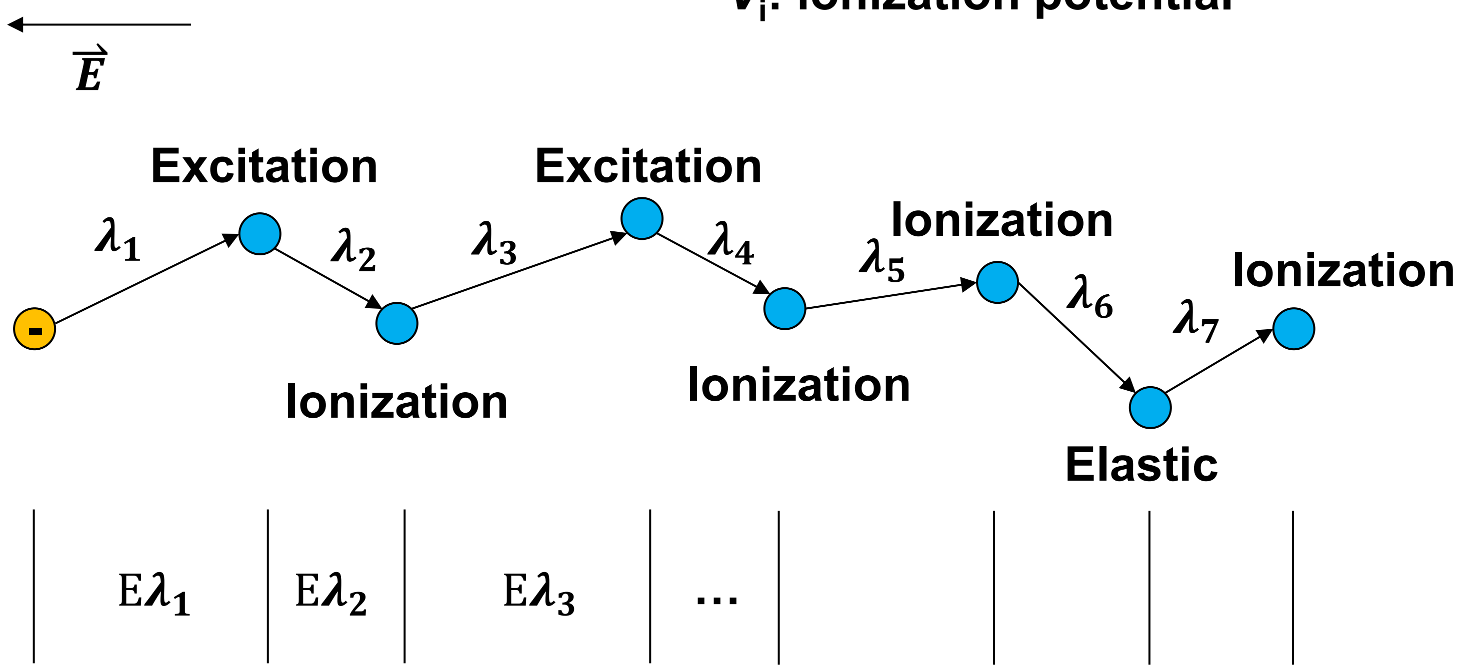
$$E_{\mathrm{k}} = e \times E \times \lambda = \mathrm{eV}$$

Electron impact ionization is the most important process in a breakdown of gases

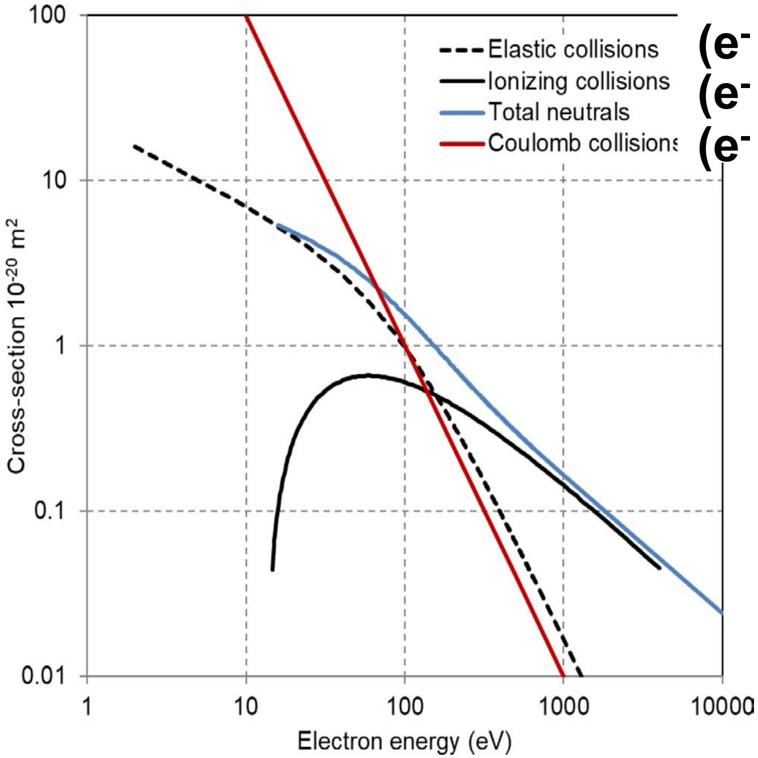
- **Electron impact ionization:** $A + e^- \rightarrow A^+ + e^- + e^-$
  - **The most important process in the breakdown of gases but is not sufficient alone to result in the breakdown.**

$$eE\lambda_{e,i} \geq eV_i$$

V_i: ionization potential



Collision cross-sections of elastic, ionizing collisions between e^- and H_2 and coulomb collisions



(e^- vs H_2)
($e^- + H_2 \rightarrow 2e^- + H^+ + H$)
(e^- vs H^+)

$$\sigma_{\mathrm{elastic,m^2}} = \frac{1.75\times 10^{-16}}{\left(W_{\mathrm{e,eV}}{}^{1.5} + 750\right)\sqrt{W_{\mathrm{e,eV}}}}$$

$$\sigma_{\mathrm{ionizing,m^2}} \sim 3\times 10^{-20}\left(ln\epsilon - 0.69 + \frac{0.66}{\epsilon}\right)\epsilon^{-1}$$

$$\epsilon = \frac{W_{\mathrm{e}}}{E_{\mathrm{ion}}} \quad E_{\mathrm{ion}} \sim 15\ \mathrm{eV\ for\ H_2}$$

$$\sigma_{\mathrm{coulomb}} = \frac{e^4}{4\pi\epsilon_{\mathrm{o}}{}^2 m_{\mathrm{e}}{}^2 v_{\mathrm{e}}{}^2}\ln\Lambda$$

$$= \frac{e^4}{16\pi\epsilon_{\mathrm{o}}{}^2 W_{\mathrm{e}}{}^2}\ln\Lambda \sim 10^{-16} W_{\mathrm{e}}{}^{-2}$$

$$\mathrm{for}\ \ln\Lambda \sim 13-15$$

$$\nu = n v_E \sigma$$

$$\lambda = \frac{1}{n\sigma}$$

P. C. de Vries and Y. Gribov, Nucl. Fusion, **59**, 096043 (2019)

Townsend avalanche process for Tokamak breakdown

- **The first Townsend coefficient α: the number of ionizing collisions made on the average by an electron as it travels 1 m along the electric field:**

$$\alpha \sim \frac{1}{\lambda_i} = \frac{\nu_{ei}}{\bar{v}_e} = \frac{n_0 \langle \sigma v_e \rangle_{ne}}{\bar{v}_e} = \frac{p}{T} \frac{\langle \sigma v \rangle_{ne}}{\bar{v}_e} \equiv Ap \qquad A \equiv \frac{1}{T} \frac{\langle \sigma v \rangle_{ne}}{\bar{v}_e}$$

- **Number of primary electrons with energy higher than the ionization potential:**

$$dn_e = -n_e \frac{dx_i}{\lambda_i} \quad \Rightarrow \quad \frac{n_e(x_i)}{n_{e0}} = \exp\left(-\frac{x_i}{\lambda_i}\right)$$

$$\alpha \equiv \frac{\#/\text{ ionization collisions}}{\text{per electron}} \times (\#/\text{electron with E} > \text{ionization potential})$$

$$= \frac{1}{\lambda_i} \frac{n_e(x_i)}{n_{e0}} = \frac{1}{\lambda_i} \exp\left(-\frac{x_i}{\lambda_i}\right)$$

$$\alpha = Ap \exp(-Apx_i)$$

A **= 3.83 m^{-1}Pa^{-1} = 1060 m^{-1}Torr^{-1}**

B **= 96.6 Vm^{-1}Pa^{-1} = 35000 m^{-1}Torr^{-1}**

$$\alpha = Ap\exp\left(-\frac{AV^*}{E/p}\right) \equiv Ap\exp\left(-\frac{B}{E/p}\right) \qquad x_i \approx \frac{V^*}{E} \text{ where } V^* > V_i$$

- **The parameters *A* and *B* must be experimentally determined.**

Paschen-like curve for minimum breakdown voltage

$$\alpha \sim \frac{1}{\lambda_{\mathrm{i}}} = Ap \qquad \alpha = Ap\exp\left(-\frac{B}{E/p}\right)$$

A **= 3.83 m^{-1}Pa^{-1} = 1060 m^{-1}Torr^{-1}**

B **= 96.6 Vm^{-1}Pa^{-1} = 35000 m^{-1}Torr^{-1}**

- **For *p*=1 mPa,** $\lambda_{\mathrm{i}} \sim 262\ \mathrm{m}$ **, for ITER,** $2\pi r_{\mathrm{o}} \sim 38\ \mathrm{m}$ $\quad \dfrac{\lambda_{\mathrm{i}}}{2\pi r_{\mathrm{o}}} \sim 7$

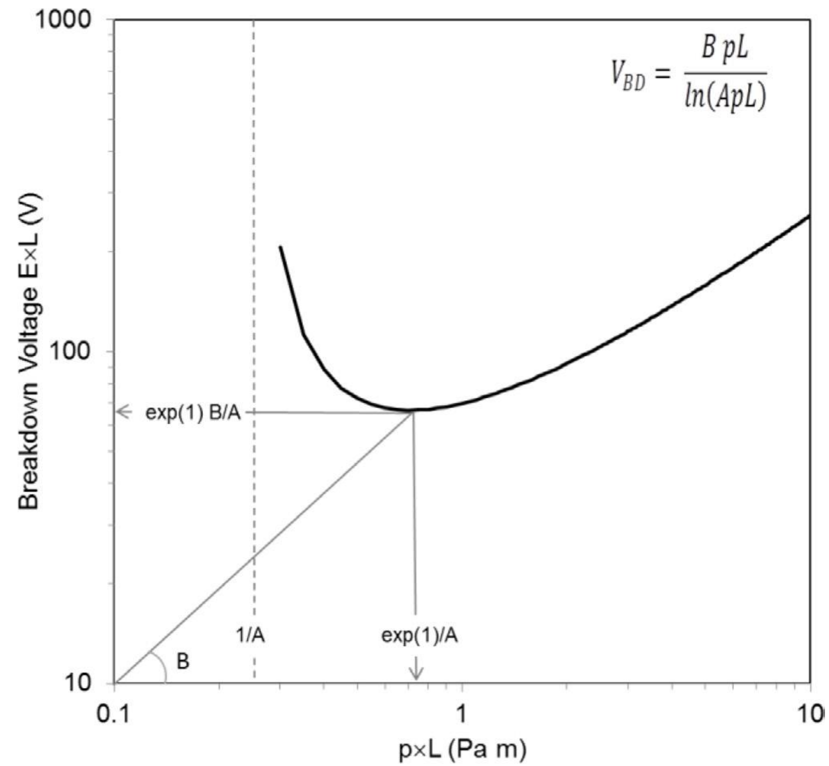
- **For breakdown to happen:**

$$\alpha L > 1 \qquad \alpha L = ApL\exp\left(-\frac{BpL}{V_{\mathrm{BD}}}\right) > 1$$

$$\exp\left(-\frac{BpL}{V_{BD}}\right) > \frac{1}{ApL}$$

$$-\frac{BpL}{V_{\mathrm{BD}}} > -\ln(ApL)$$

$$V_{\mathrm{BD}} > \frac{BpL}{\ln(ApL)} \qquad E_{\mathrm{BD}} > \frac{Bp}{\ln(ApL)}$$



P. C. de Vries and Y. Gribov, Nucl. Fusion, **59**, 096043 (2019)

Perpendicular stray-field (B_z) needs to be as small as possible

- **For *p*=1 mPa,** $\lambda_{\mathrm{i}} \sim 262\ \mathrm{m}$ **, for ITER,** $2\pi r_{\mathrm{o}} \sim 38\ \mathrm{m}$ $\quad \dfrac{\lambda_{\mathrm{i}}}{2\pi r_{\mathrm{o}}} \sim 7$

  $\dfrac{B_z}{B_T} \sim 10^{-3} \qquad \lambda_i \times \dfrac{B_z}{B_T} = 0.26\ \mathrm{m}$

- **For ITER,**

  $$E \sim E_{\mathrm{loop}} = 0.3\ \mathrm{V/m}$$

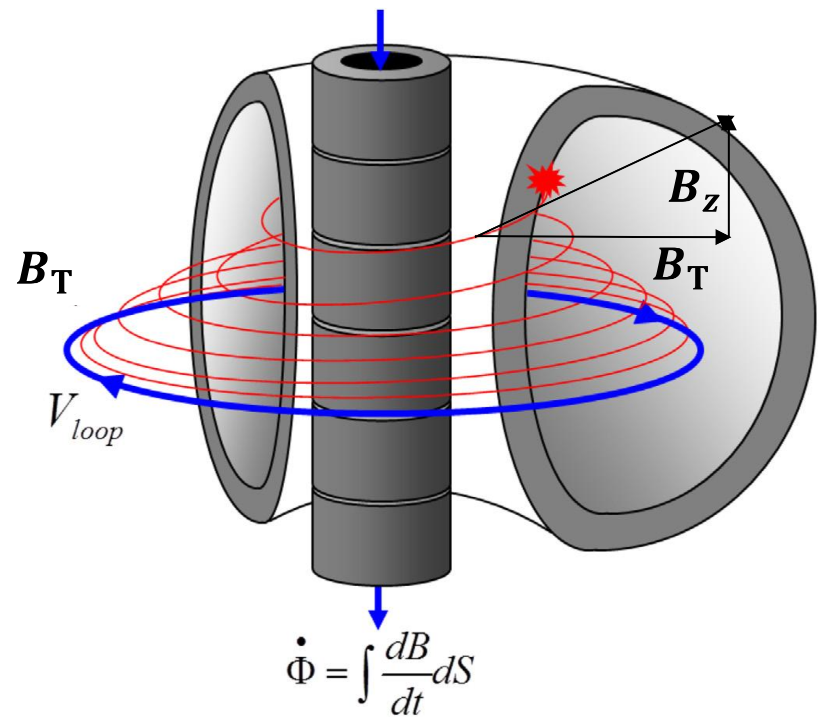
  $$p = 1\ \mathrm{mPa} \qquad L_{\mathrm{BD}} = 357\ \mathrm{m}$$

- **Required loop field:**

  $$E_{\mathrm{BD}} > \frac{Bp}{\ln(ApL)}$$

  $$E_{\mathrm{BD}} > \frac{1.25 \times 10^4 P_{\mathrm{Torr}}}{\ln(510 P L_{\mathrm{c}})}$$

  $$L_{\mathrm{c}} = 0.25 a_{\mathrm{eff}} \left(\frac{B_z}{B_T}\right)$$



- **W/ preionization:** $E_{\mathrm{T}} \dfrac{B_{\mathrm{T}}}{B_z} \geq 100\ \mathrm{V/m}$
- **Purely Ohmic discharges:** $E_{\mathrm{T}} \dfrac{B_{\mathrm{T}}}{B_z} \geq 1000\ \mathrm{V/m}$

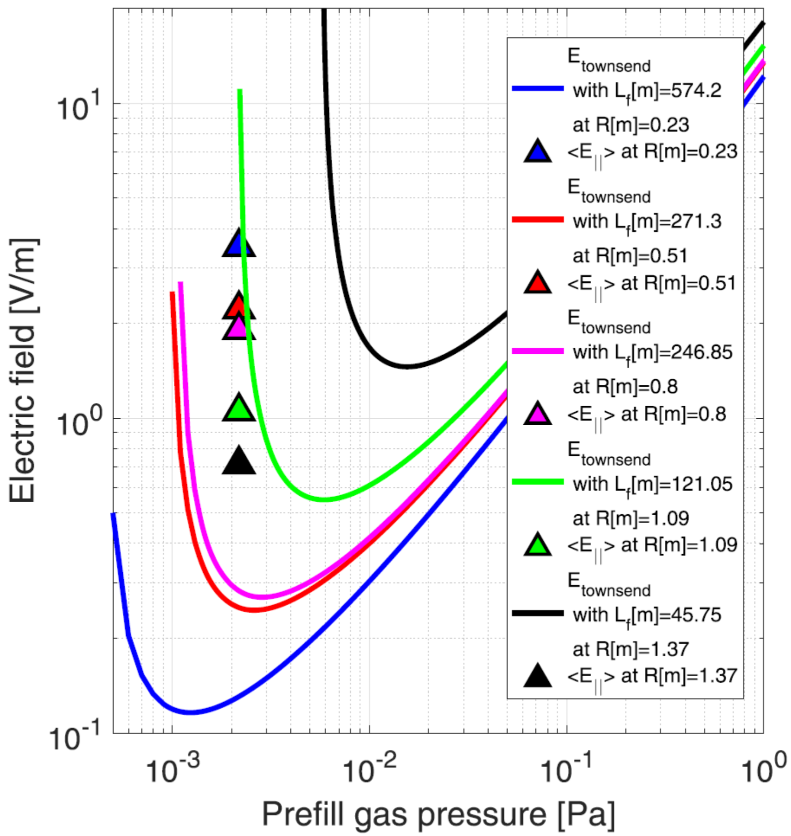
P. C. de Vries and Y. Gribov, Nucl. Fusion, **59**, 096043 (2019)
S. J. Doyle et al, Fusion Eng. Des. **171**, 112706 (2021)

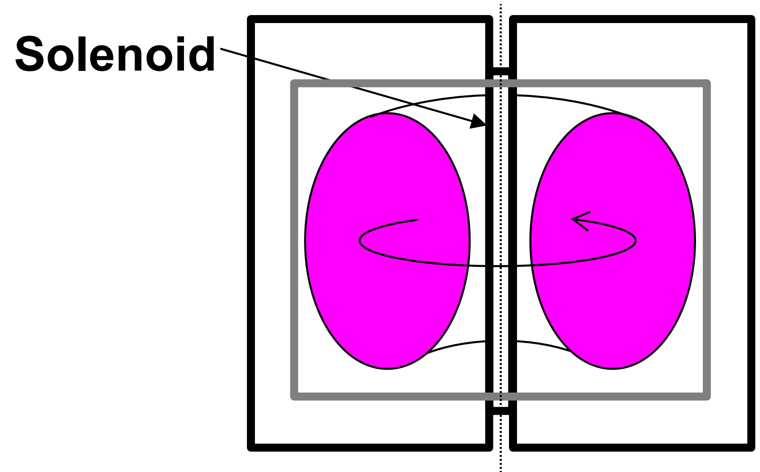
Examples or required loop electric fields

$$E_{\mathrm{BD}} > \frac{1.25 \times 10^4 P_{\mathrm{Torr}}}{\ln(510 P L_c)}$$

$$L_c = 0.25 a_{\mathrm{eff}} \left(\frac{B_z}{B_T}\right)$$

- W/ preionization: $E_{\mathrm{T}} \frac{B_{\mathrm{T}}}{B_{\mathrm{z}}} \geq 100\,\mathrm{V/m}$
- Purely Ohmic discharges: $E_{\mathrm{T}} \frac{B_{\mathrm{T}}}{B_{\mathrm{z}}} \geq 1000\,\mathrm{V/m}$





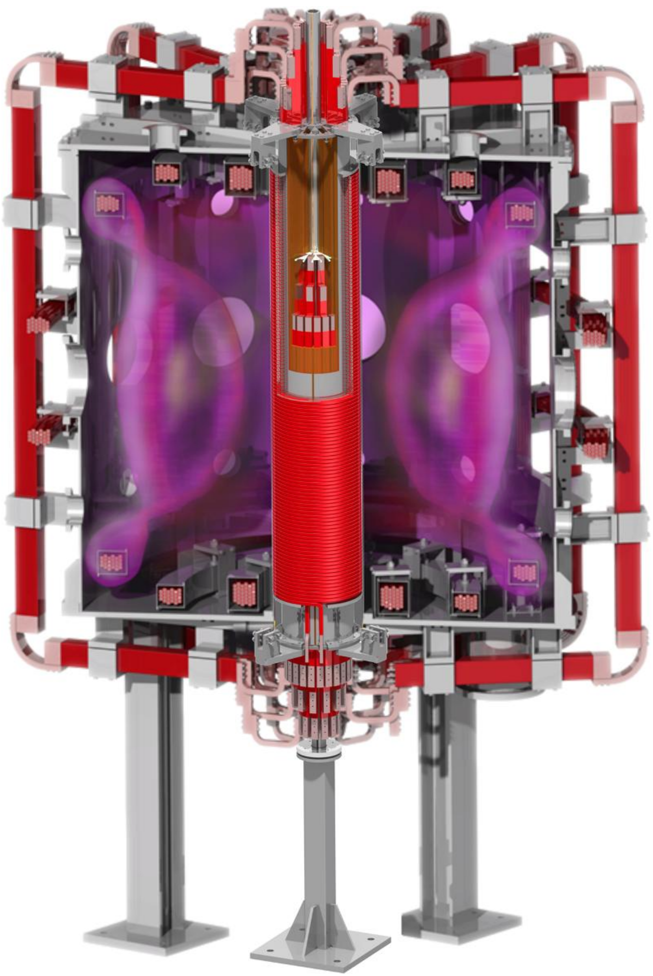
$$V_{\phi} = -\frac{\partial \phi}{\partial t} \equiv M \frac{\partial I_{CS}}{\partial t}$$

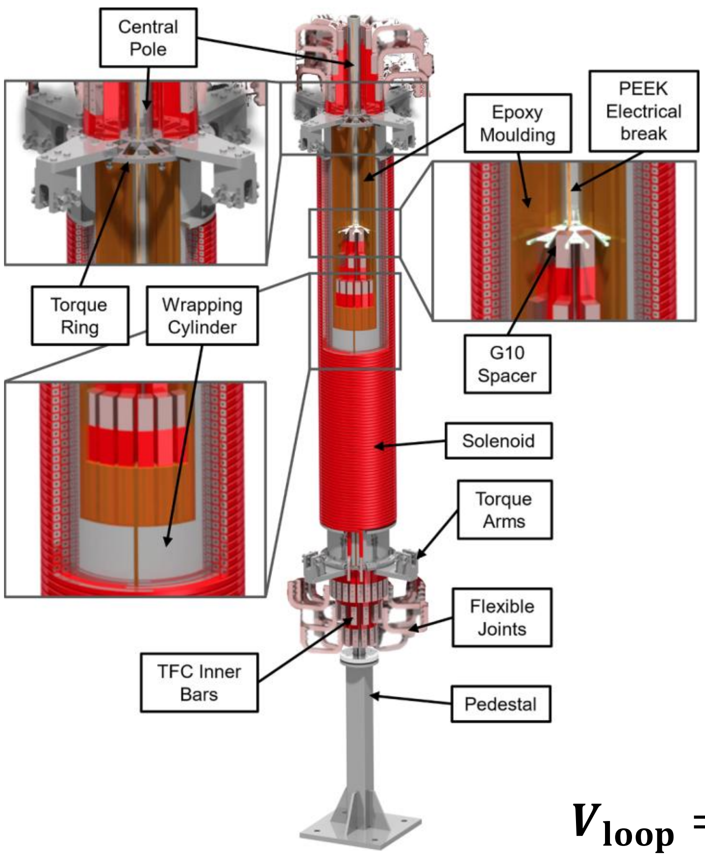
$$E_{\phi} = -\frac{1}{2\pi r} \frac{\partial \phi}{\partial t}$$

H.-T. Kim, etc., Nucl. Fusion **62**, 126012 (2022)
S. J. Doyle et al, Fusion Eng. Des. **171**, 112706 (2021)

Central solenoid can be used to provide the required loop voltage for breakdown

- **SMART:**

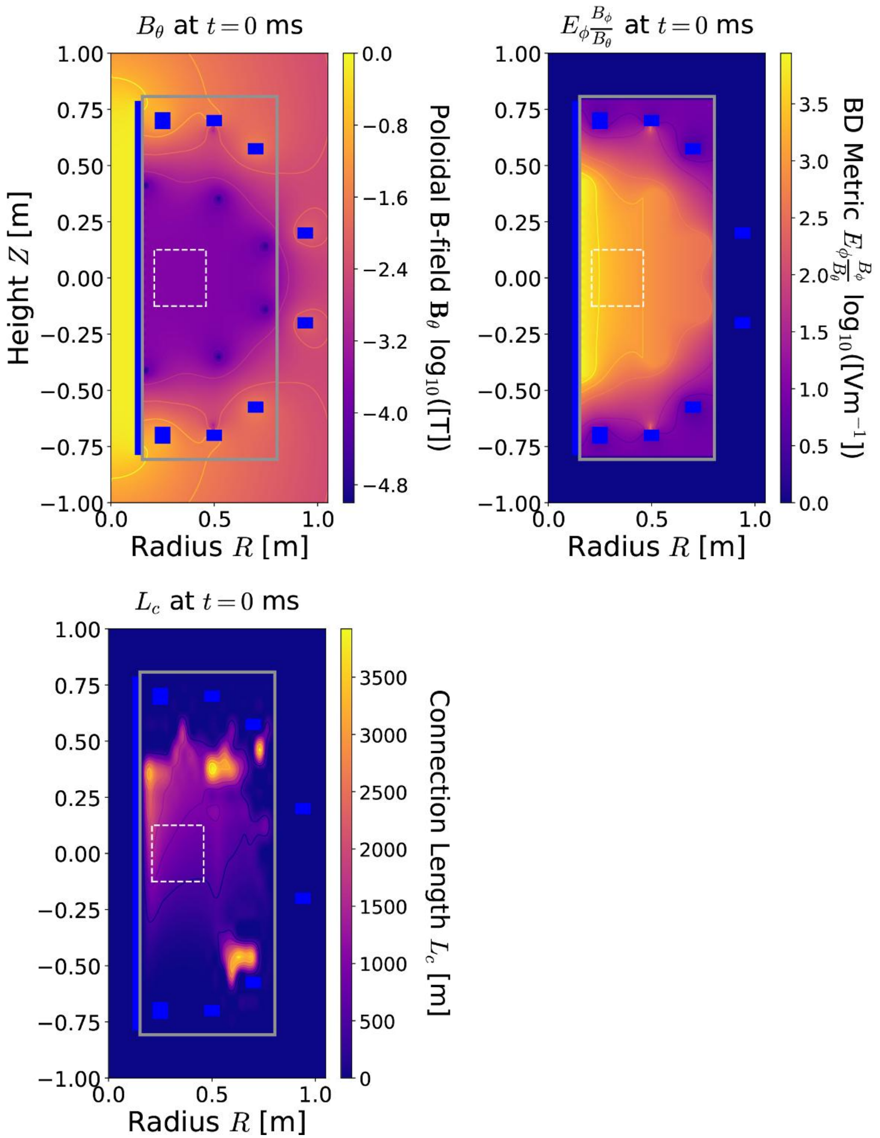




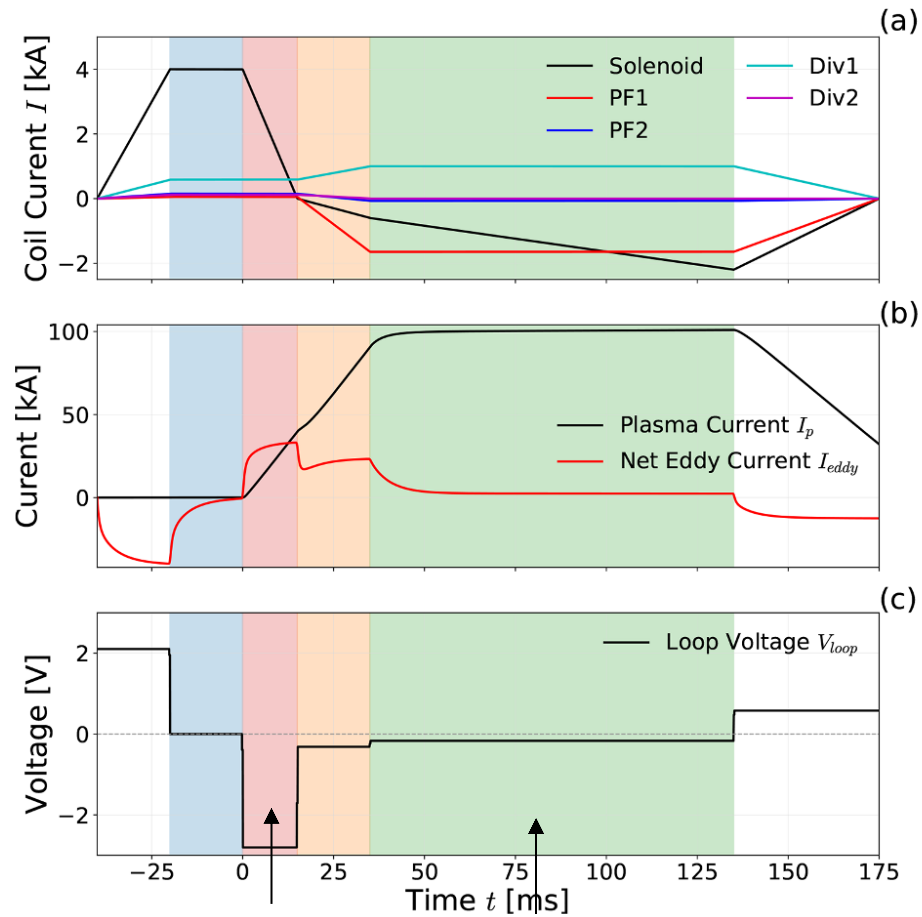
$$V_{\text{loop}} = \frac{A_{\text{sol}}\mu N_{\text{sol}}}{L_{\text{sol}}}\frac{dI_{\text{sol}}}{dt}$$

J. Segado-Fernandez et al, Fusion Eng. Des. **193**, 113832 (2023)

Poloidal coils are used to reduce the stray field during breakdown



- **Currents of SMART**

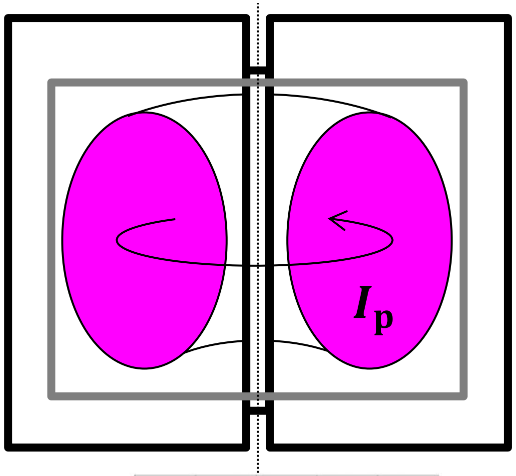


Breakdown

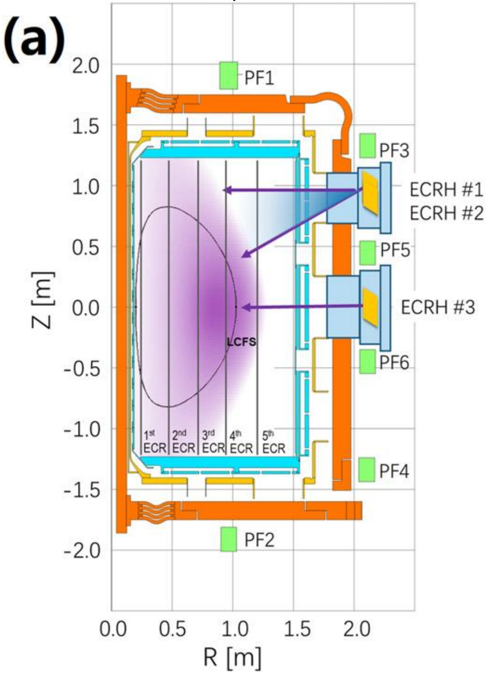
Plasma current is driven

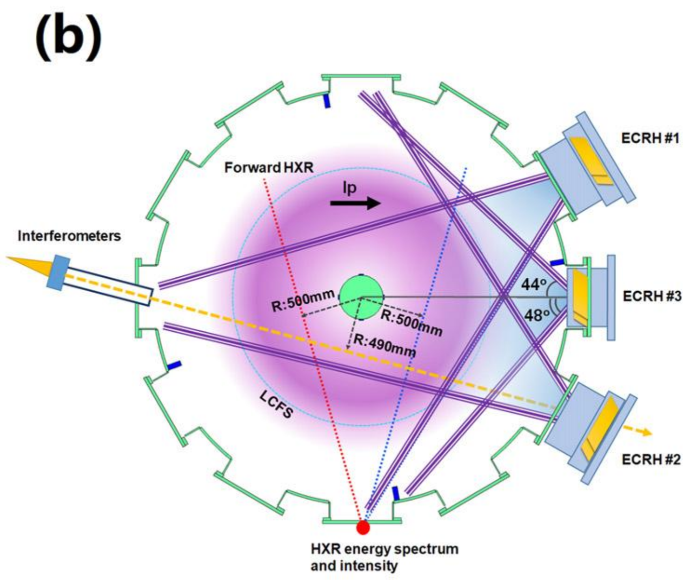
S. J. Doyle, etc., Fusion Eng. Des. **171**, 112706 (2021)

Momentum exchange may be needed to drive plasma current



$$\vec{j}_{\mathrm{p}} = \Sigma q n \vec{v} = -e n_{\mathrm{e}} \vec{v}_{\mathrm{e}} + e n_{\mathrm{i}} \vec{v}_{\mathrm{i}}$$





Yuejiang Shi et al, Nucl. Fusion **62**, 086047 (2022)

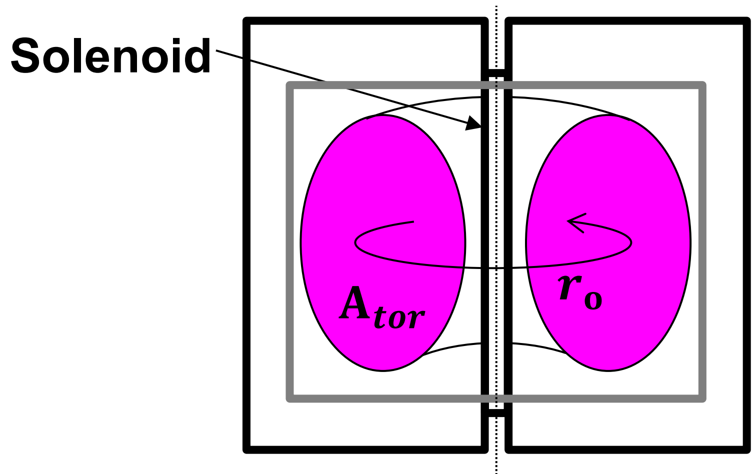
Solenoid can be used to drive the plasma current

$$L_{\mathrm{tor}}\frac{dI}{dt}+IR=V_{\mathrm{loop}}=M\frac{dI_{\mathrm{sol}}}{dt}$$

$$L_{\mathrm{tor}}=\mu_{\mathrm{o}}r_{\mathrm{o}}\left[\ln\left(\frac{8r_{\mathrm{o}}}{a}\right)-1.5\right]$$

$$R_{\mathrm{spitzer}}=\eta_{\mathrm{spiter}}\frac{2\pi r_{\mathrm{o}}}{A_{\mathrm{tor}}}$$

$$\eta_{\mathrm{spiter}}=5.2\times10^{-3}Z\ln\Lambda\,T_{e,(\mathrm{eV})}^{-3/2}$$



Current is initially driven at the surface and then diffuses into the plasma

- **Simplified Ohm's law:** $\vec{E} + \vec{v} \times \vec{B} = \eta \vec{j}$
- **Assuming a stationary plasma:** $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = \nabla \times (\eta \vec{j})$
- **Assuming a constant *η*:**

$$-\frac{\partial}{\partial t} \nabla \times \vec{B} = \eta \nabla \times \nabla \times \vec{j} = \eta\left(\nabla\left(\nabla \cdot \vec{j}\right) - \nabla^2 \vec{j}\right)$$

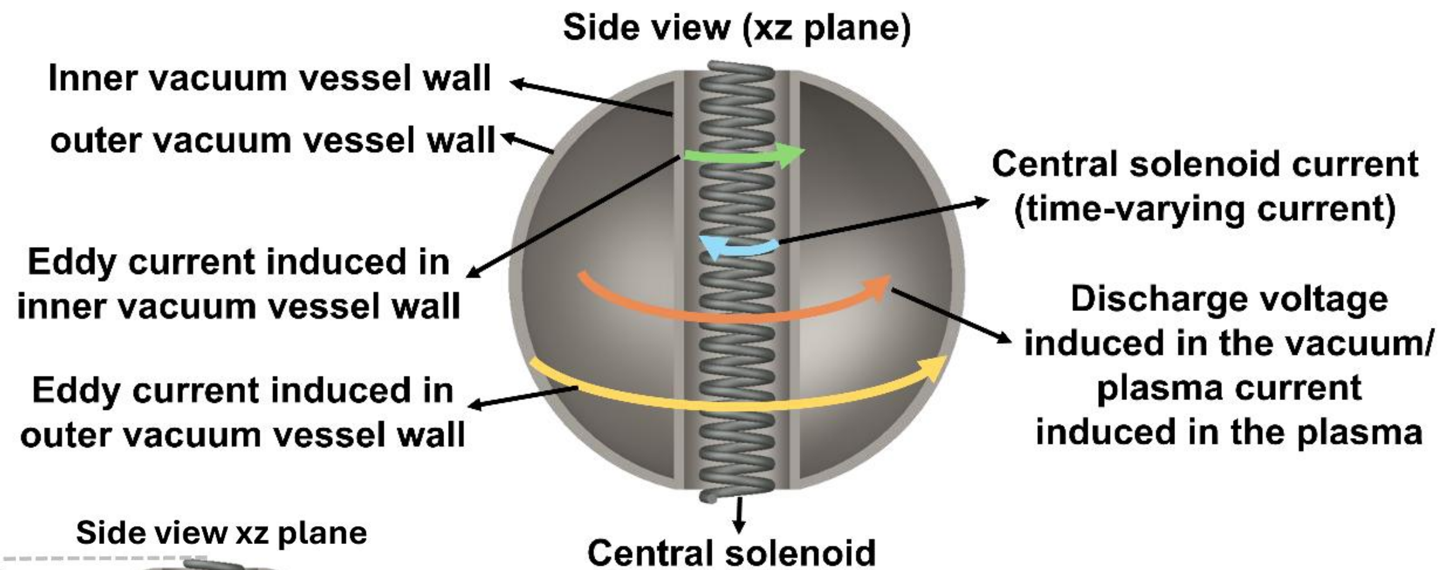
$$\frac{\partial \vec{j}}{\partial t} = \frac{\eta}{\mu_o} \nabla^2 \vec{j}$$

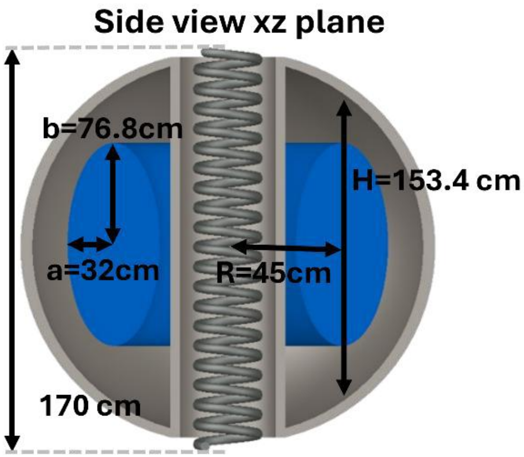
- **Assuming non-constant *η*:**

$$\frac{\partial \vec{j}}{\partial t} = \frac{1}{\mu_o} \nabla^2 (\eta \vec{j}) - \nabla\left[\nabla \cdot (\mu \vec{j})\right] \qquad \frac{\partial j_T}{\partial t} = \frac{1}{\mu_o} \nabla^2 (\eta j_T)$$

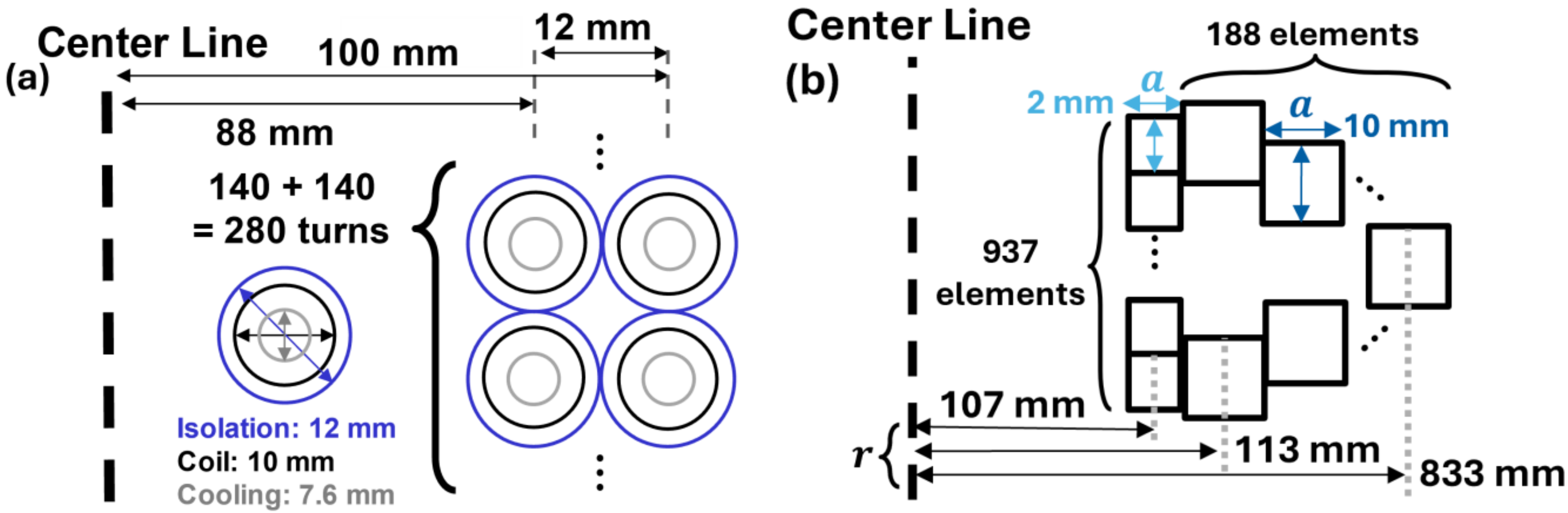
- **Since $\eta \; \alpha \; T^{-3/2}$, resistance drops with higher temperature. The typical limited temperature is ~3 keV.**

Eddy current needed to be considered





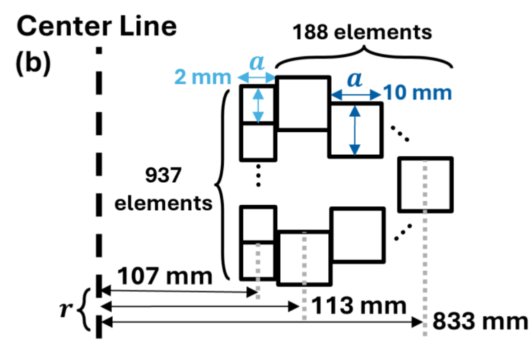
Chamber is broken into ring elements carrying eddy currents



The eddy current in each chamber element is solved numerically

$$\overleftrightarrow{M}\frac{d\,\vec{I}}{\mathrm{dt}} + \overleftrightarrow{R}_{\Omega}\,\vec{I} = \vec{V} \qquad \overleftrightarrow{M}\frac{\vec{I}' - \vec{I}}{\Delta \mathrm{t}} + \overleftrightarrow{R}_{\Omega}\,\vec{I} = \vec{V}$$

$$\vec{I}' = \left(\overleftrightarrow{1} - \Delta\mathrm{t}\,\overleftrightarrow{M}^{-1}\,\overleftrightarrow{R}_{\Omega}\right)\vec{I} + \Delta\mathrm{t}\,\overleftrightarrow{M}^{-1}\,\vec{V}$$

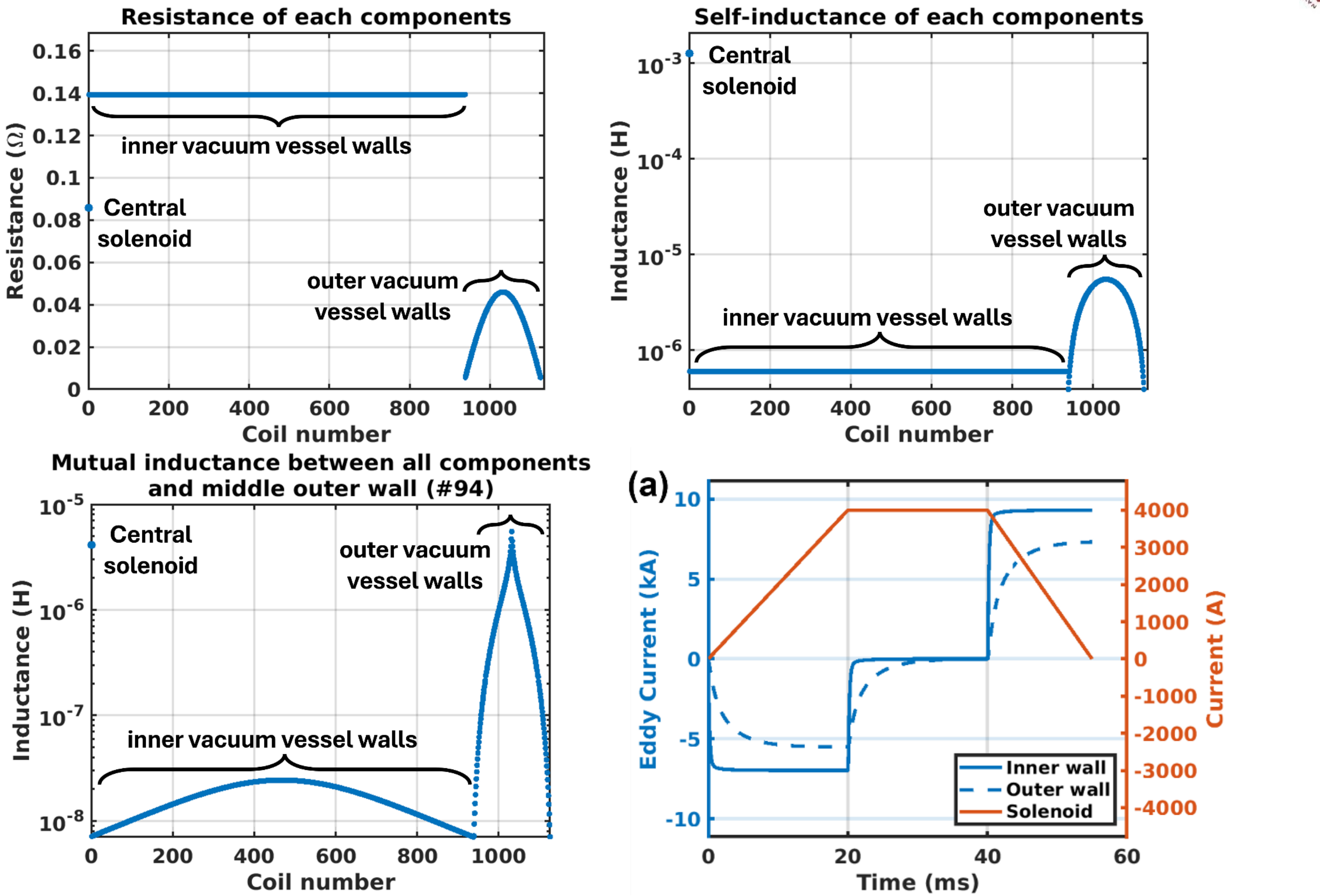


$$\vec{V} = \overleftrightarrow{M}\frac{d\vec{I}}{dt} + \overleftrightarrow{R}\vec{I}$$

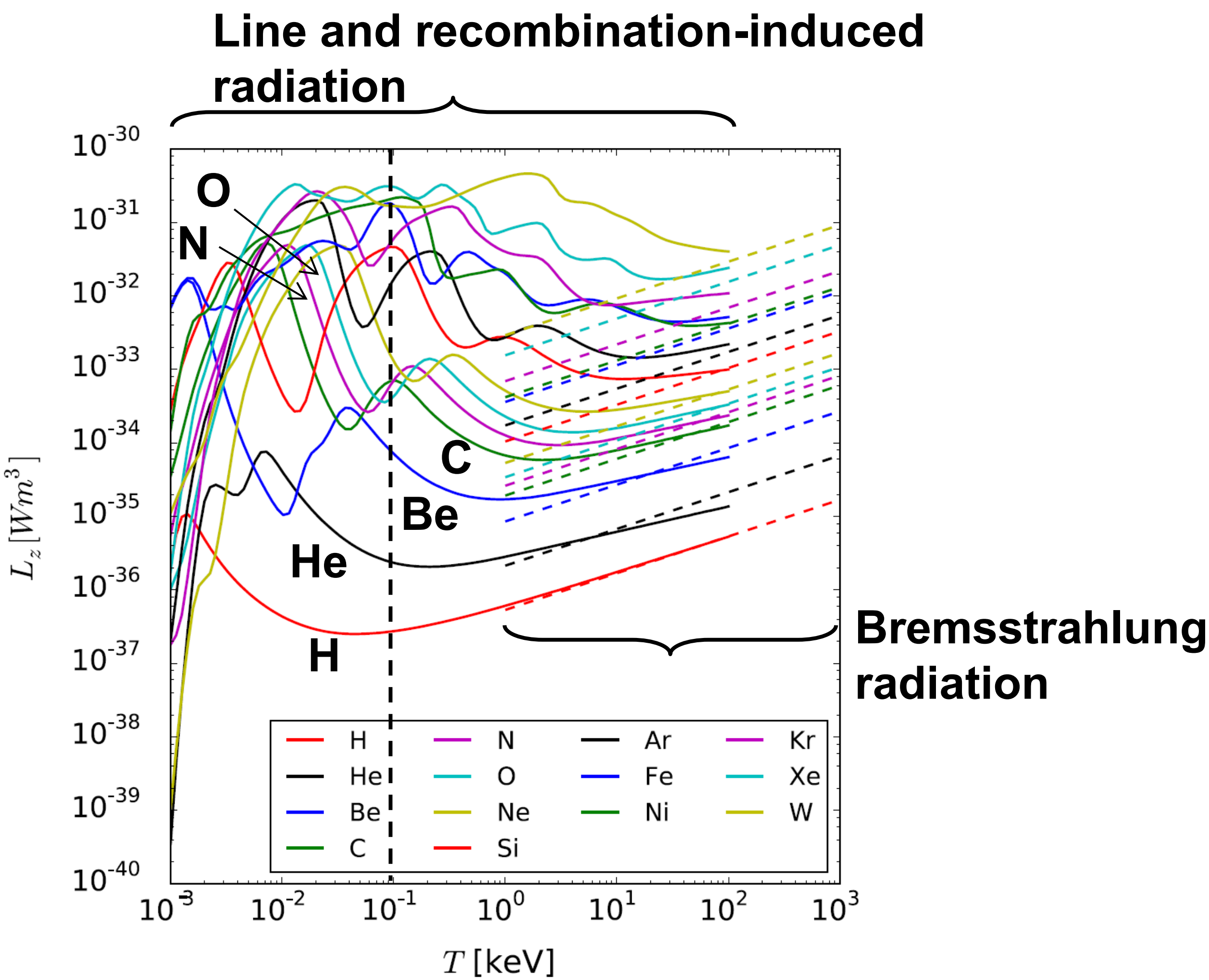
$$\Rightarrow \begin{bmatrix} V_{\mathrm{c1}} \\ V_{\mathrm{c2}} \\ \vdots \\ V_{\mathrm{v1}} \\ V_{\mathrm{v2}} \\ \vdots \\ V_{\mathrm{p}} \end{bmatrix} = \begin{bmatrix} L_{\mathrm{c1}} & M_{\mathrm{c1,c2}} & M_{\mathrm{c1,c3}} & \cdots & M_{\mathrm{c1,v1}} & M_{\mathrm{c1,v2}} & M_{\mathrm{c1,v3}} & \cdots & M_{\mathrm{c1,p}} \\ M_{\mathrm{c2,c1}} & L_{\mathrm{c2}} & M_{\mathrm{c2,c3}} & \cdots & M_{\mathrm{c2,v1}} & M_{\mathrm{c2,v2}} & M_{\mathrm{c2,v3}} & \cdots & M_{\mathrm{c2,p}} \\ M_{\mathrm{c3,c1}} & M_{\mathrm{c3,c2}} & L_{\mathrm{c3}} & \cdots & M_{\mathrm{c3,v1}} & M_{\mathrm{c3,v2}} & M_{\mathrm{c3,v3}} & \cdots & M_{\mathrm{c3,p}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ M_{\mathrm{v1,c1}} & M_{\mathrm{v1,c2}} & M_{\mathrm{v1,c3}} & \cdots & L_{\mathrm{v1}} & M_{\mathrm{v1,v2}} & M_{\mathrm{v1,v3}} & \cdots & M_{\mathrm{v1,p}} \\ M_{\mathrm{v2,c1}} & M_{\mathrm{v2,c2}} & M_{\mathrm{v2,c3}} & \cdots & M_{\mathrm{v2,v1}} & L_{\mathrm{v2}} & M_{\mathrm{v2,v3}} & \cdots & M_{\mathrm{v2,p}} \\ M_{\mathrm{v3,c1}} & M_{\mathrm{v3,c2}} & M_{\mathrm{v3,c3}} & \cdots & M_{\mathrm{v3,v1}} & M_{\mathrm{v3,v2}} & L_{\mathrm{v3}} & \cdots & M_{\mathrm{v3,p}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ M_{\mathrm{p,c1}} & M_{\mathrm{p,c2}} & M_{\mathrm{p,c3}} & \cdots & M_{\mathrm{p,v1}} & M_{\mathrm{p,v2}} & M_{\mathrm{p,v3}} & \cdots & L_{\mathrm{p}} \end{bmatrix} \begin{bmatrix} I'_{\mathrm{c1}} \\ I'_{\mathrm{c2}} \\ \vdots \\ I'_{\mathrm{v1}} \\ I'_{\mathrm{v2}} \\ \vdots \\ I'_{\mathrm{p}} \end{bmatrix}$$

$$+ \begin{bmatrix} R_{\mathrm{c1}} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & R_{\mathrm{c2}} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & R_{\mathrm{v1}} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & R_{\mathrm{v2}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & R_{\mathrm{p}} \end{bmatrix} \begin{bmatrix} I_{\mathrm{c1}} \\ I_{\mathrm{c2}} \\ \vdots \\ I_{\mathrm{v1}} \\ I_{\mathrm{v2}} \\ \vdots \\ I_{\mathrm{p}} \end{bmatrix}. \qquad (2)$$

Eddy current on the chamber wall is induced when the current of the solenoid changes with time

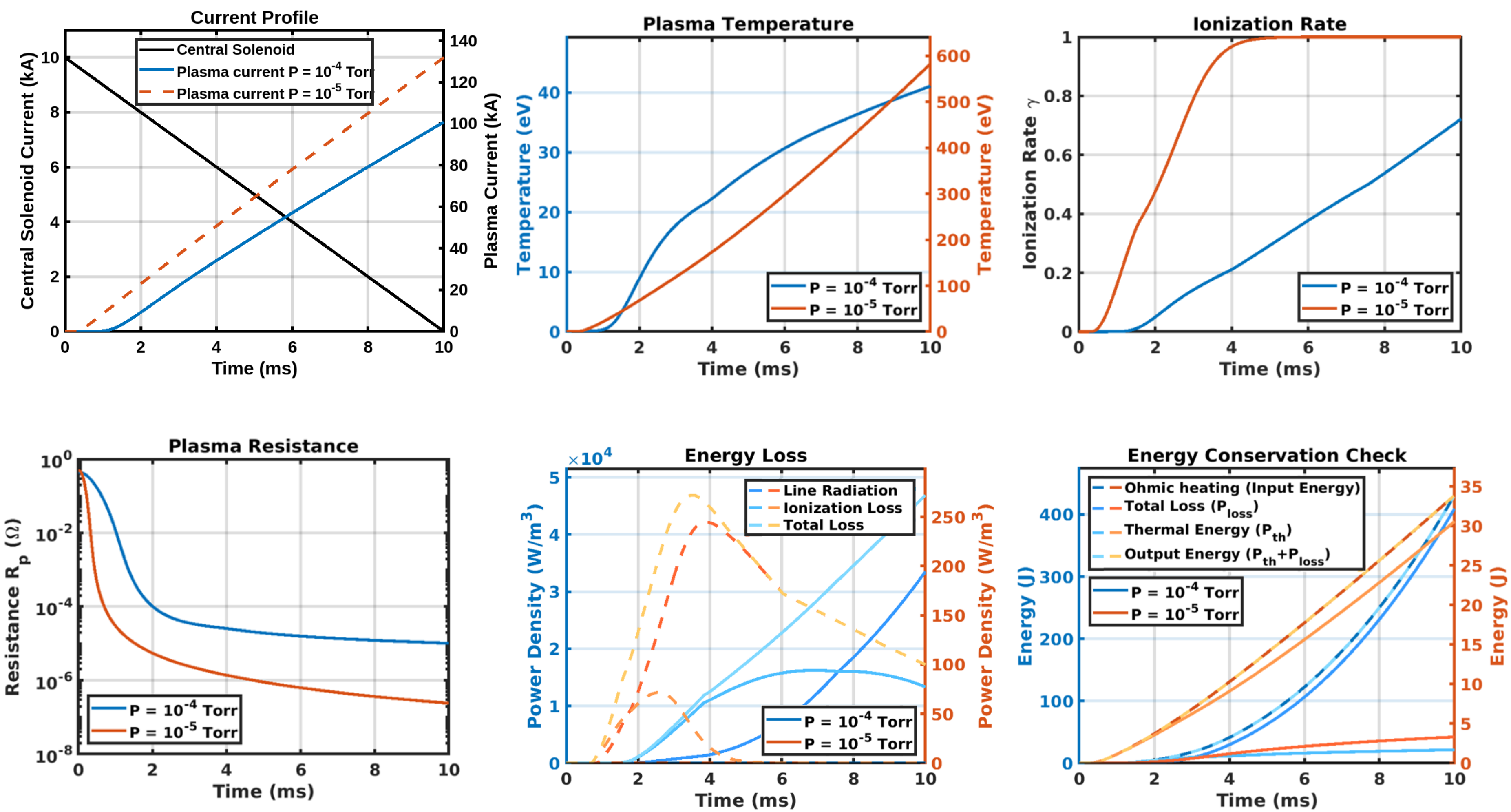


Temperature of 100 eV is the threshold of radiation barrier by impurities



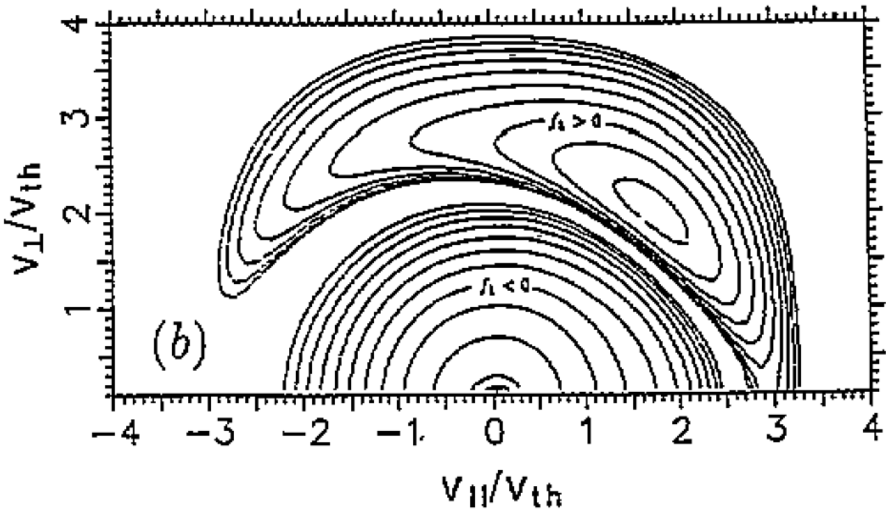
H. Lux, etc., Fusion Eng. Des. **42**, 101 (2015)

Plasma temperature can be estimated using the simple circuit model



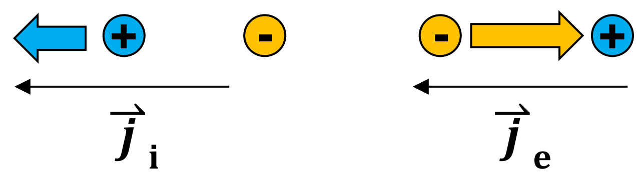
The collisional re-distribution of the ECRH-driven anisotropy in $E_\perp$ causes some parallel momentum to flow from e^- to ions

- **Coulomb collisions are more efficient at lower energies.**



Velocity: $v_2 > v_1$

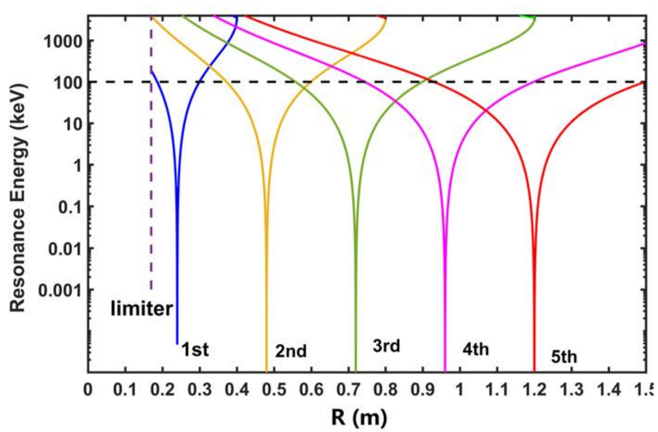
Collisions: $\nu_2 < \nu_1$



$$\vec{j}_{\mathrm{p}} = -e n_{\mathrm{e}} \vec{v}_{\mathrm{e}} + e n_{\mathrm{i}} \vec{v}_{\mathrm{i}}$$

$$\vec{P} = n_{\mathrm{e}} m_{\mathrm{e}} \vec{v}_{\mathrm{e}} + n_{\mathrm{i}} m_{\mathrm{i}} \vec{v}_{\mathrm{i}} \approx 0$$

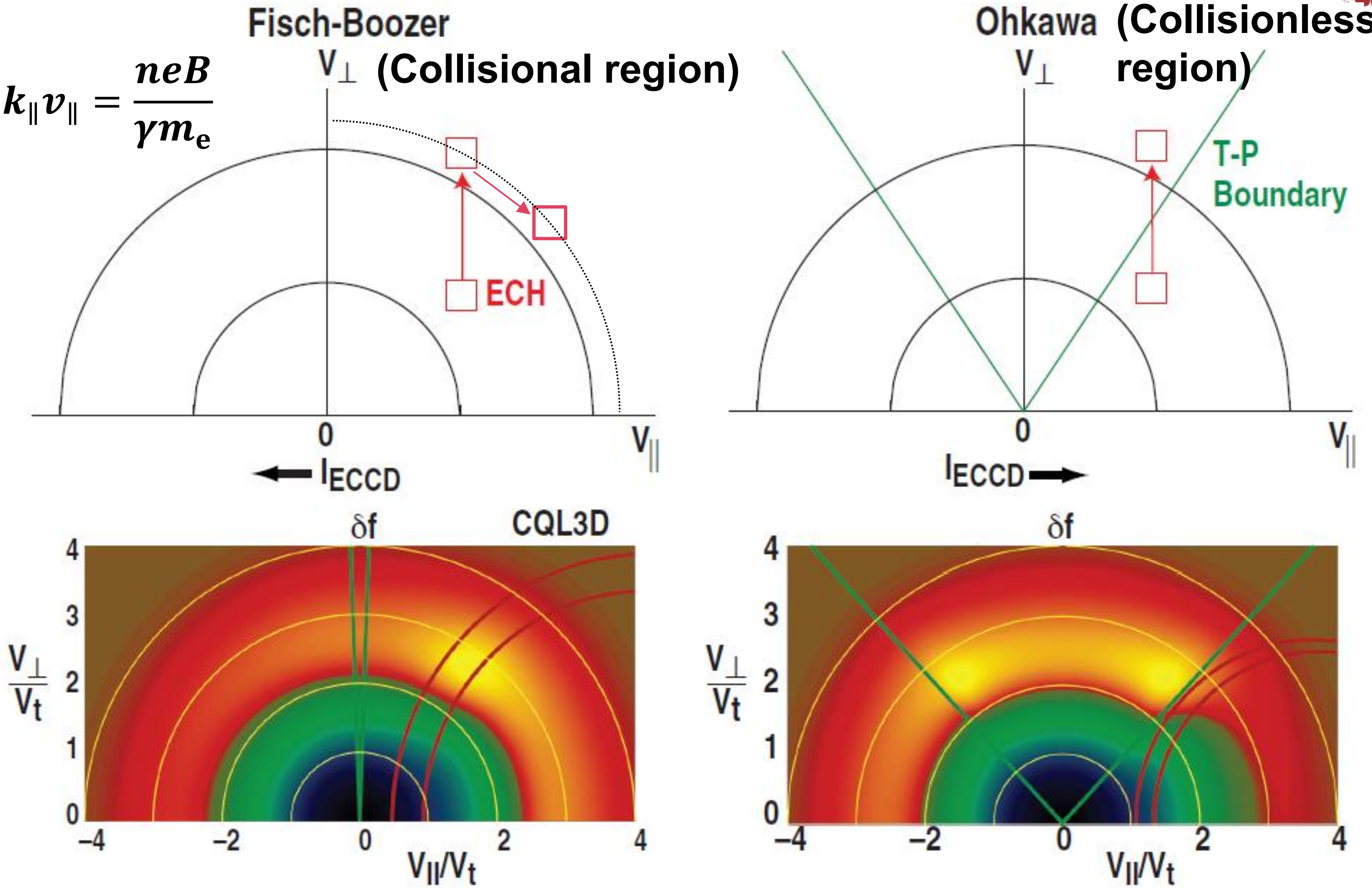
- **Electron cyclotron current drive:**



V. Erckmann and U. Gasparion, Plasma Phys. Control. Fusion **36**, 1869 (1994)
Yuejiang Shi et al, Nucl. Fusion **62**, 086047 (2022)

Passing electrons can be trapped if the $v_\perp$ is increased by heating

$$\omega - k_\parallel v_\parallel = \frac{neB}{\gamma m_e}$$



Lecture note "Physics and Applications of Electron Cyclotron Heating and Current Drive" by R. Prater from GA at 6th ITER International School 2012, Institute for Plasma Research, Gandhinagar Ahmedabad, India, December 2-6, 2012

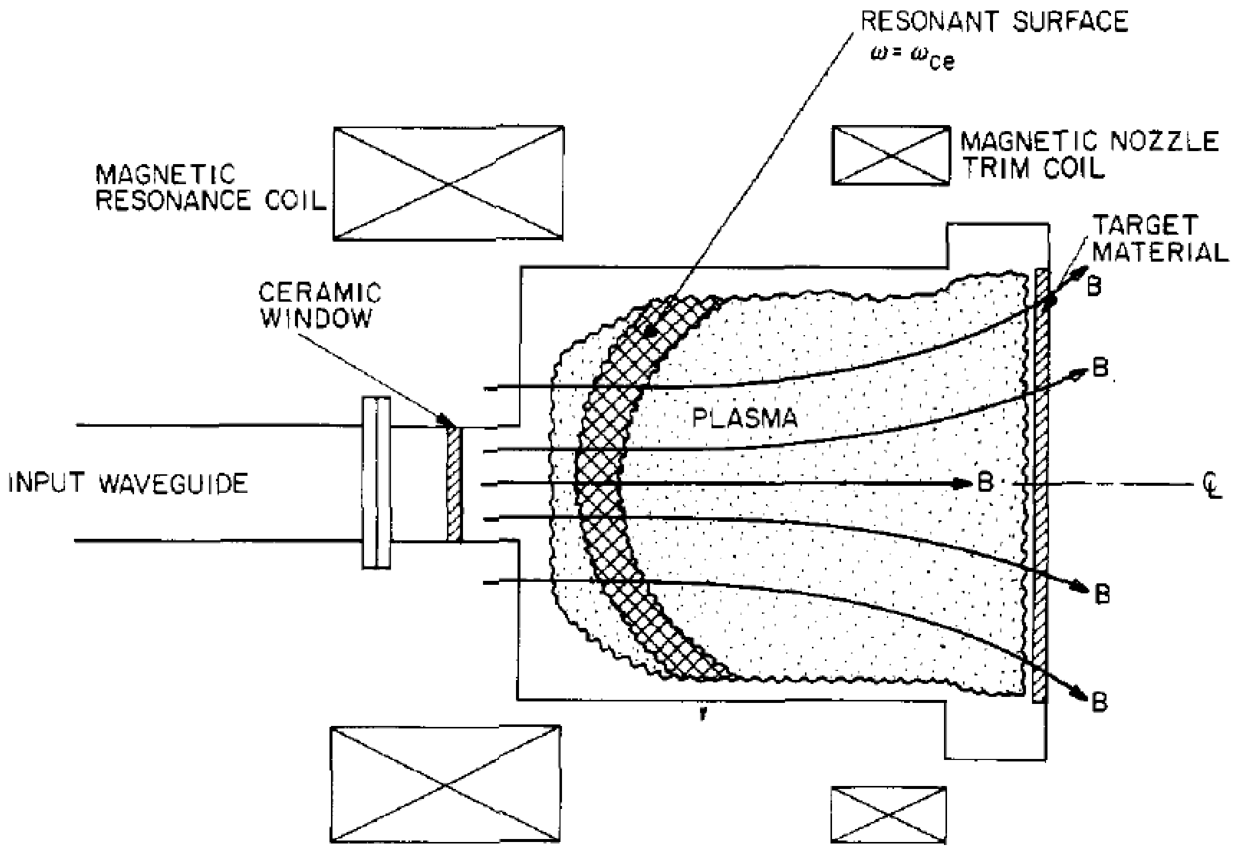
Comparison of Fisch-Boozer Mechanism and Ohkawa Mechanism

Aspect	Fisch–Boozer Mechanism[1]	Ohkawa Mechanism[2]
Physical Process	Asymmetric heating of passing electrons with subsequent collisional momentum transfer	Selective de-trapping of barely trapped electrons into passing orbits (collisionless mechanism)
Requires collisions?	Yes (collisional mechanism)	No (collisionless pitch-angle scattering)
Key Particle Population	Passing electrons	Trapped (or barely trapped) electrons
Wave absorption location	Depends on Doppler-shifted resonance; typically near magnetic axis or mid-radius	Usually near edge where barely trapped particles are abundant

1 N. J. Fisch and A. H. Boozer, Phys. Rev. Lett. 45, 720 (1980).
2 T. Ohkawa, "Steady state operation of tokamaks by rf heating," General Atomics Report No. GA-A13847 (1976).

Strong absorption occurs when the frequency matches the electron cyclotron frequency

- **Electron cyclotron resonance (ECR) plasma reactor**



Electron cyclotron frequency depends on magnetic field only

$$m_e \frac{d\vec{v}}{\mathrm{dt}} = -\frac{e}{c}\,\vec{v} \times \vec{B}$$

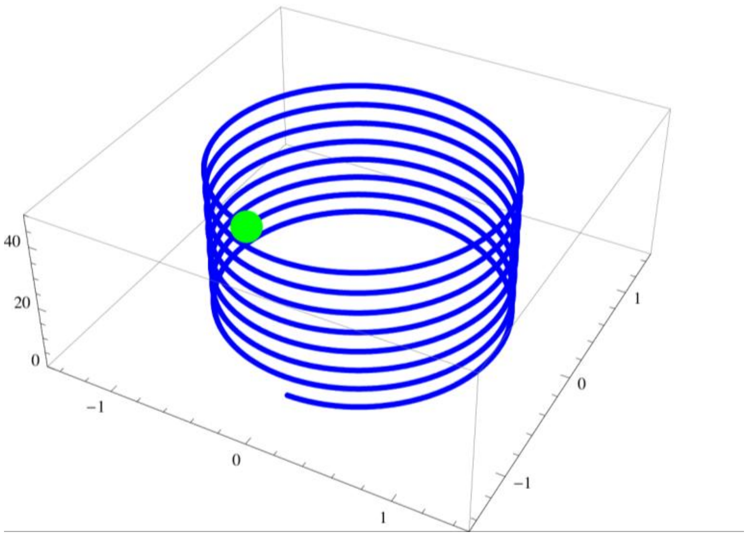
- Assuming $\vec{B} = B\hat{z}$ and the electron oscillates in x-y plane

$$m_e \dot{v}_x = -\frac{e}{c} B v_y \qquad m_e \dot{v}_z = 0$$

$$m_e \dot{v}_y = \frac{e}{c} B v_x$$

$$\ddot{v}_x = -\frac{eB}{m_e c}\dot{v}_y = -\left(\frac{eB}{m_e c}\right)^2 v_x$$

$$\ddot{v}_y = -\frac{eB}{m_e c}\dot{v}_x = -\left(\frac{eB}{m_e c}\right)^2 v_y$$



- Therefore

$$\omega_{\mathrm{ce}} = \frac{eB}{m_e c}$$

Electrons keep getting accelerated when a electric field rotates in electron's gyrofrequency

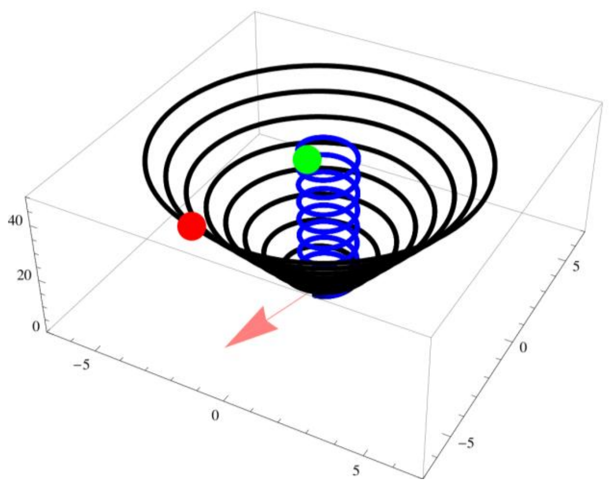
$$m_e \frac{d\vec{v}}{dt} = -\frac{e}{c}\vec{v} \times \vec{B} - e\vec{E} \qquad \vec{B} = B_0\hat{z} \qquad \vec{E} = E_0[\hat{x}\cos(\omega t) + \hat{y}\sin(\omega t)]$$

$$m_e\dot{v}_x = -\frac{e}{c}Bv_y + E_0\cos(\omega t) \qquad m_e\dot{v}_y = \frac{e}{c}Bv_x + E_0\sin(\omega t) \qquad m_e\dot{v}_z = 0$$

$$\ddot{v}_x = -\frac{eB}{m_e c}\dot{v}_y - \frac{E_0}{m_e}\omega\sin(\omega t) = -\omega_{ce}{}^2 v_x - \frac{E_0}{m_e}(\omega_{ce} + \omega)\sin(\omega t)$$

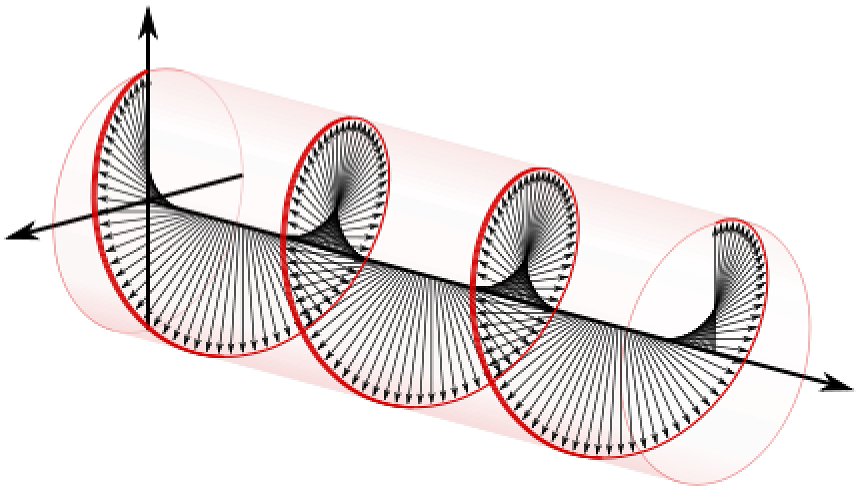
$$\ddot{v}_y = -\frac{eB}{m_e c}\dot{v}_x + \frac{E_0}{m_e}\omega\cos(\omega t) = -\omega_{ce}{}^2 v_y + \frac{E_0}{m_e}(\omega_{ce} + \omega)\cos(\omega t)$$

$$\omega_{ce} = \frac{eB}{m_e c}$$

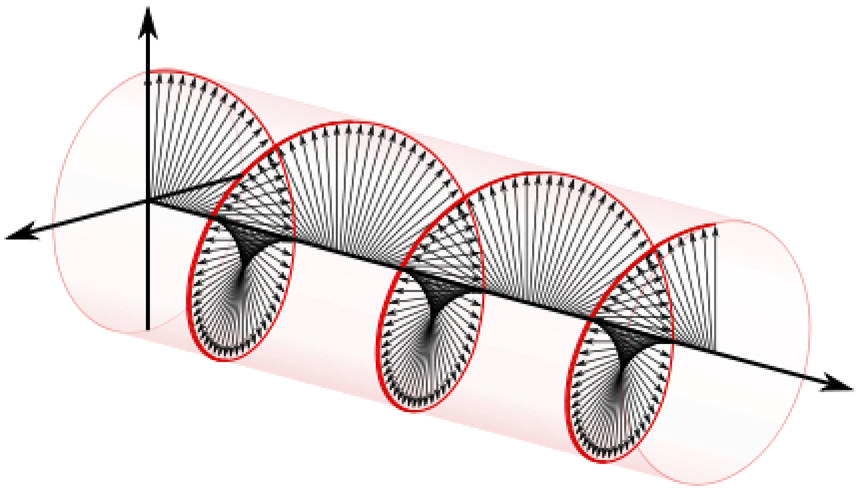


Electric field in a circular polarized electromagnetic wave keeps rotating as the wave propagates

- **Right-handed polarization**



- **Left-handed polarization**



https://en.wikipedia.org/wiki/Circular_polarization

Only right-handed polarization can resonance with electron's gyromotion

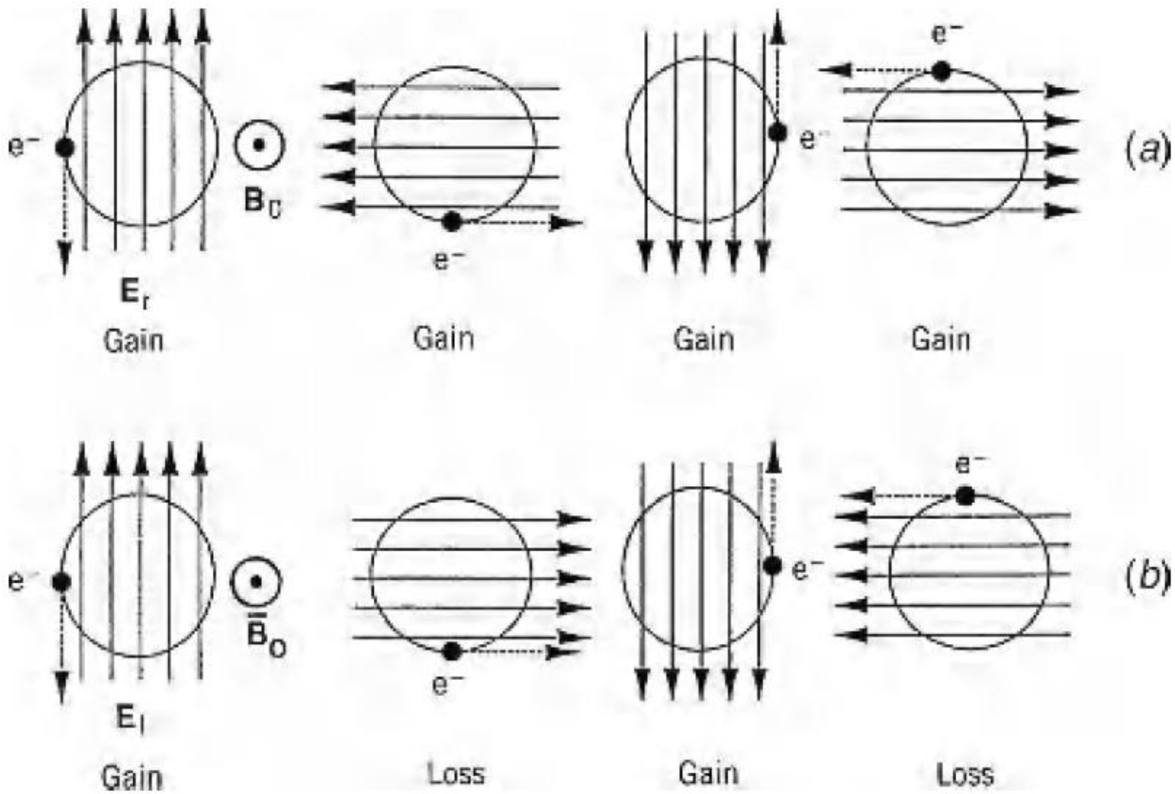
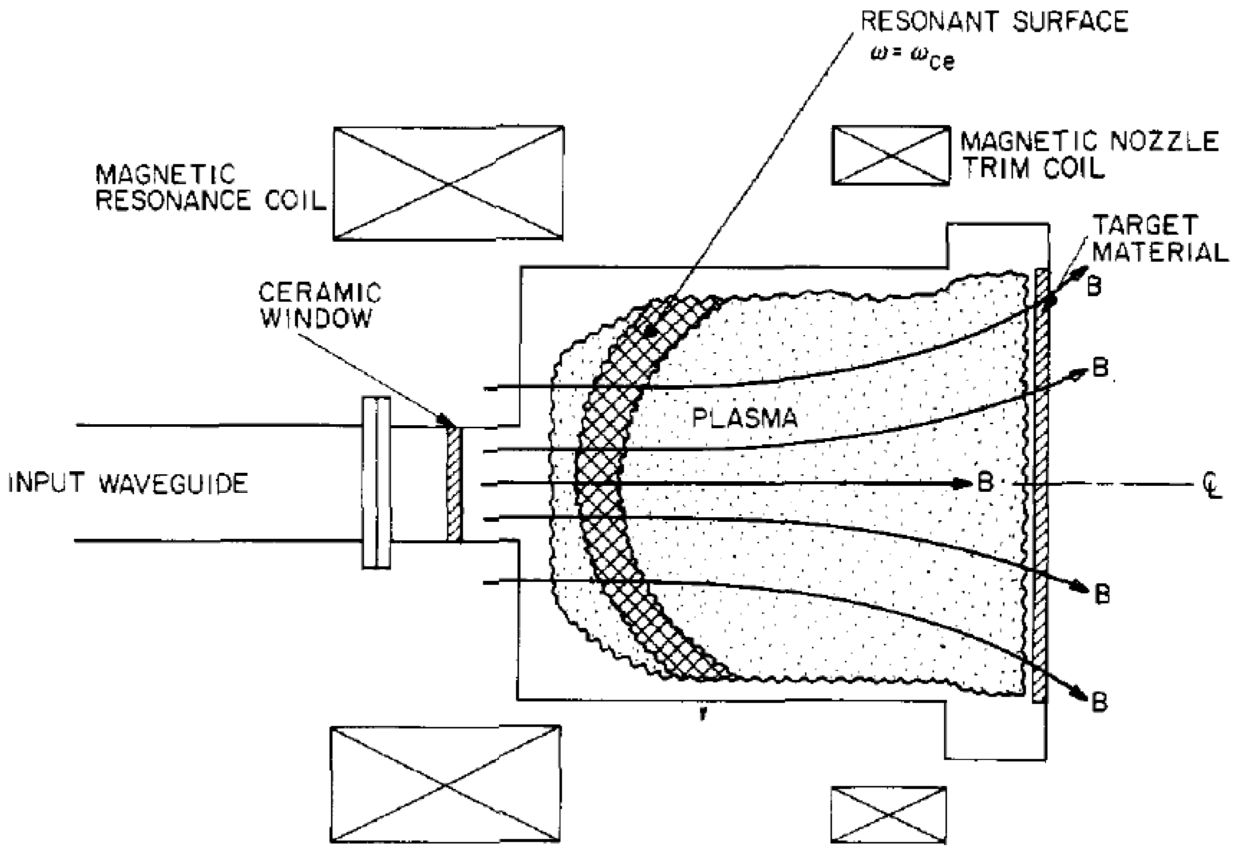


FIGURE 13.5. Basic principle of ECR heating: (*a*) continuous energy gain for right-hand polarization; (*b*) oscillating energy for left-hand polarization (after Lieberman and Gottscho, 1994).

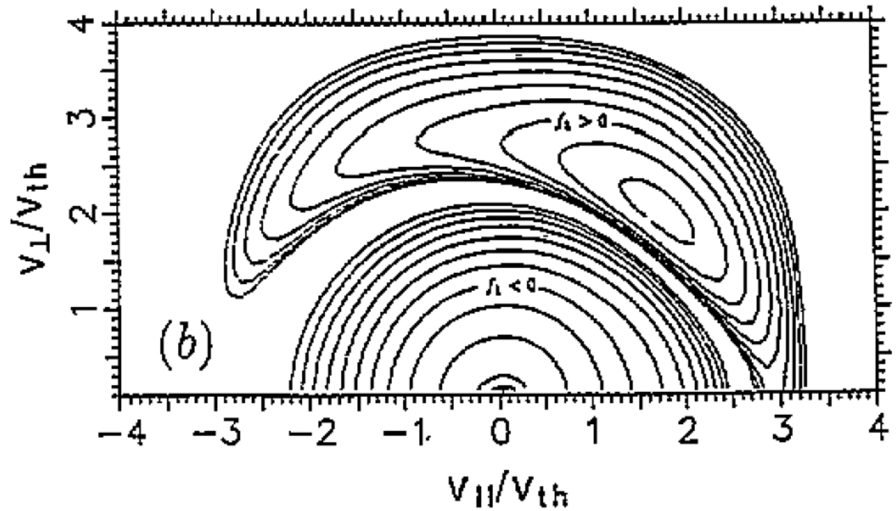
Strong absorption occurs when the frequency matches the electron cyclotron frequency

- **Electron cyclotron resonance (ECR) plasma reactor**



The collisional re-distribution of the ECRH-driven anisotropy in $E_\perp$ causes some parallel momentum to flow from e^- to ions

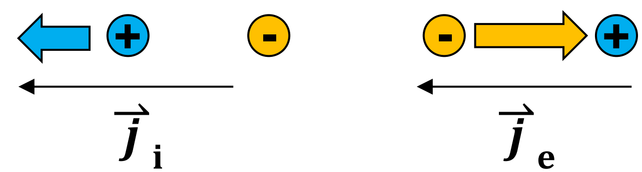
- **Coulomb collisions are more efficient at lower energies.**



Velocity: $v_2 > v_1$

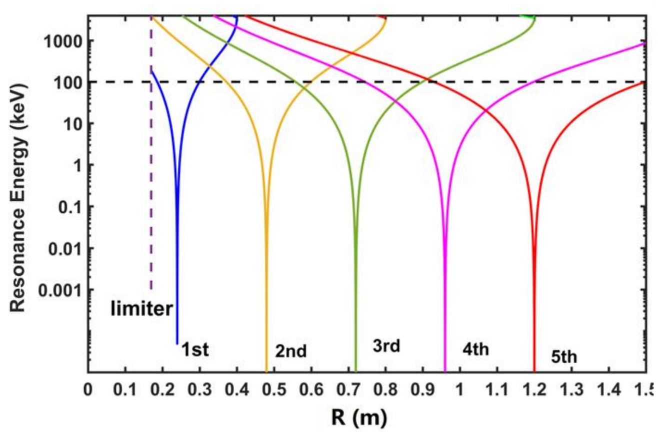
Collisions: $\nu_2 < \nu_1$

- **Electron cyclotron current drive:**



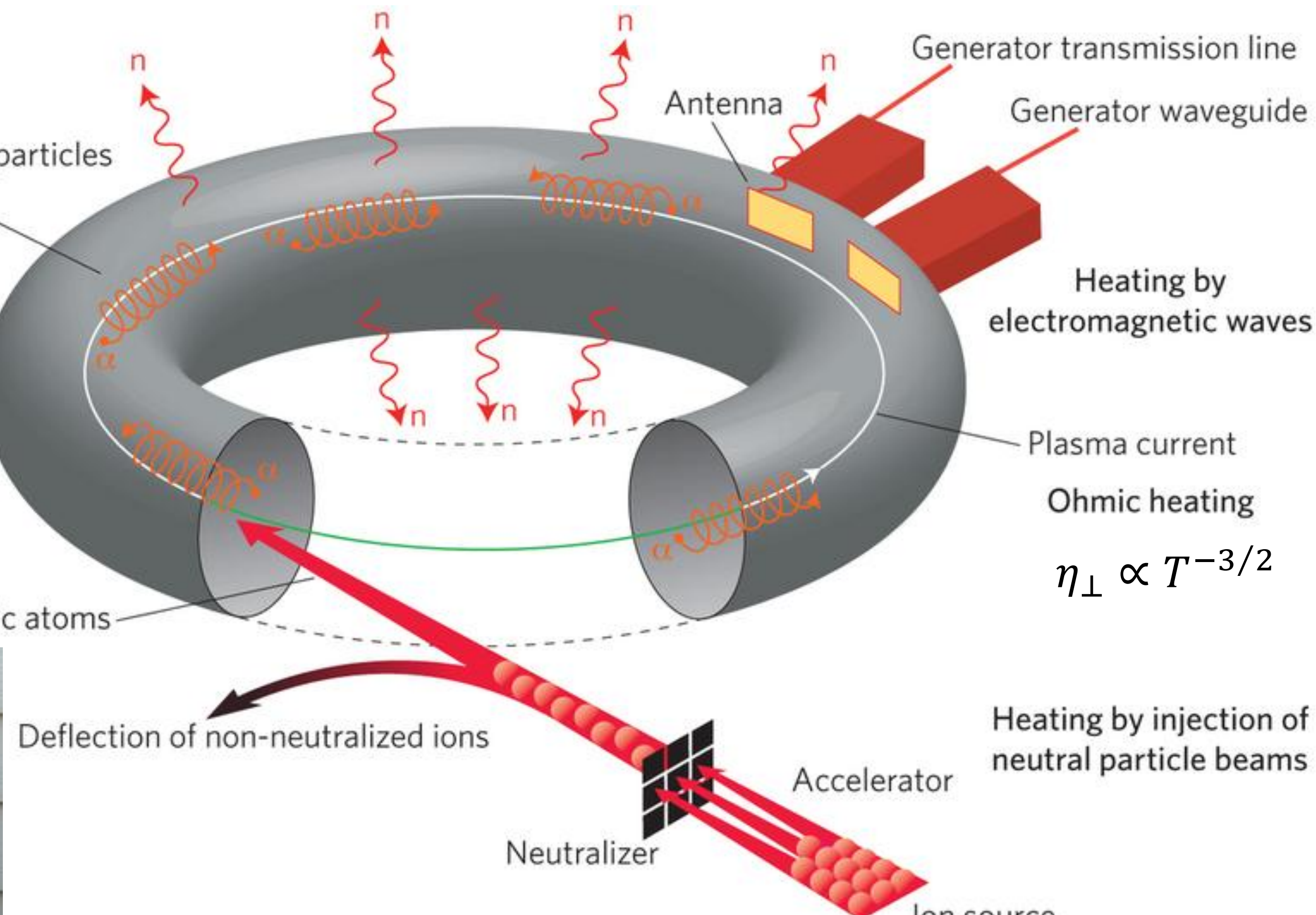
$$\vec{j}_{\mathrm{p}} = -en_{\mathrm{e}}\,\vec{v}_{\mathrm{e}} + en_{\mathrm{i}}\,\vec{v}_{\mathrm{i}}$$

$$\vec{P} = n_{\mathrm{e}}m_{\mathrm{e}}\,\vec{v}_{\mathrm{e}} + n_{\mathrm{i}}m_{\mathrm{i}}\,\vec{v}_{\mathrm{i}} \approx 0$$



V. Erckmann and U. Gasparion, Plasma Phys. Control. Fusion **36**, 1869 (1994)
Yuejiang Shi et al, Nucl. Fusion **62**, 086047 (2022)

Neutral beam injector is one of the main heat mechanisms in MCF



$$\eta_{\perp} \propto T^{-3/2}$$

D. Mazon, etc., NATURE PHYSICS, 12, 14, 2016
https://zh.wikihow.com/%E5%9C%A8%E6%89%8B%E6%8C%87%E4%B8%8A%E8%BD%AC%E7%AF%AE%E7%90%83

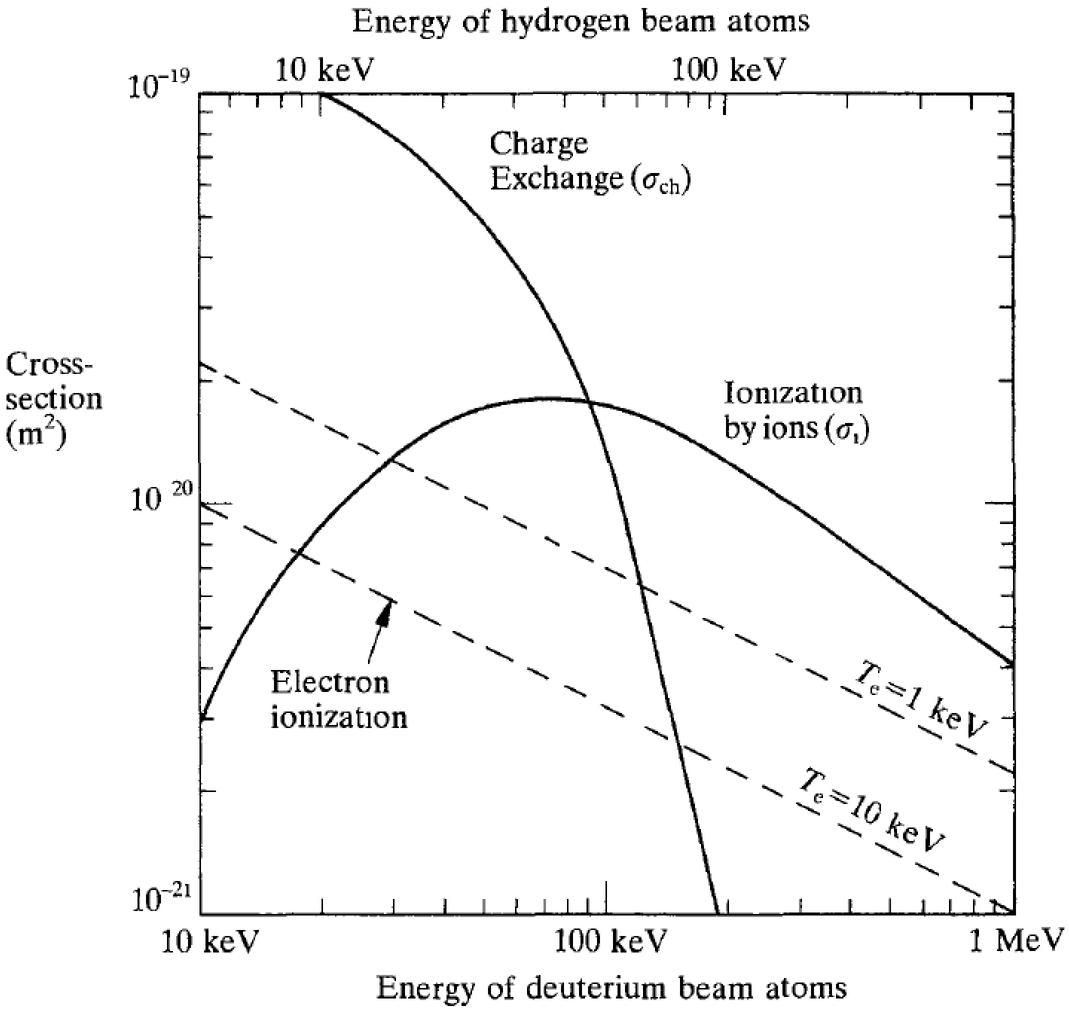
Varies way of heating a MCF device

System		Frequency/ energy	Maximum power coupled to plasma	Overall system efficiency	Development/ demonstration required	Remarks
ECRF	Demonstrated in tokamaks	28–157 GHz	2.8 MW, 0.2 s	30–40%	Power sources and windows, off-axis CD	Provides off-axis CD
	ITER needs	150–170 GHz	50 MW, S S			
ICRF	Demonstrated in tokamaks	25–120 MHz	22 MW, 3 s (L-mode); 16.5 MW, 3 s (H-mode)	50–60%	ELM tolerant system	Provides ion heating and smaller ELMs
	ITER needs	40–75 MHz	50 MW, S S			
LHRF	Demonstrated in tokamaks	1.3–8 GHz	2.5 MW, 120 s; 10 MW, 0.5 s	45–55%	Launcher, coupling to H-mode	Provides off-axis CD
	ITER needs	5 GHz	50 MW, S S			
NBI +ve ion	Demonstrated in tokamaks	80–140 keV	40 MW, 2 s; 20 MW, 8 s	35–45%	None	Not applicable
	ITER needs	None	None			
NBI −ve ion	Demonstrated in tokamaks	0.35 MeV	5.2 MW, D^-, 0.8 s (from 2 sources)			
	ITER needs	1 MeV	50 MW, S S	∼37%	System, tests on tokamak, plasma CD	provides rotation

'S S' indicates steady state

Nuclear Fusion, 39, 2495, 1999

Neutral atoms are ionized by collisions in the plasma



- **Charge exchange:**

  $H_b + H_p^+ \rightarrow H_b^+ + H_p$

- **Ionization by ions**

  $H_b + H_p^+ \rightarrow H_b^+ + H_p^+ + e^-$

- **Ionization by electrons**

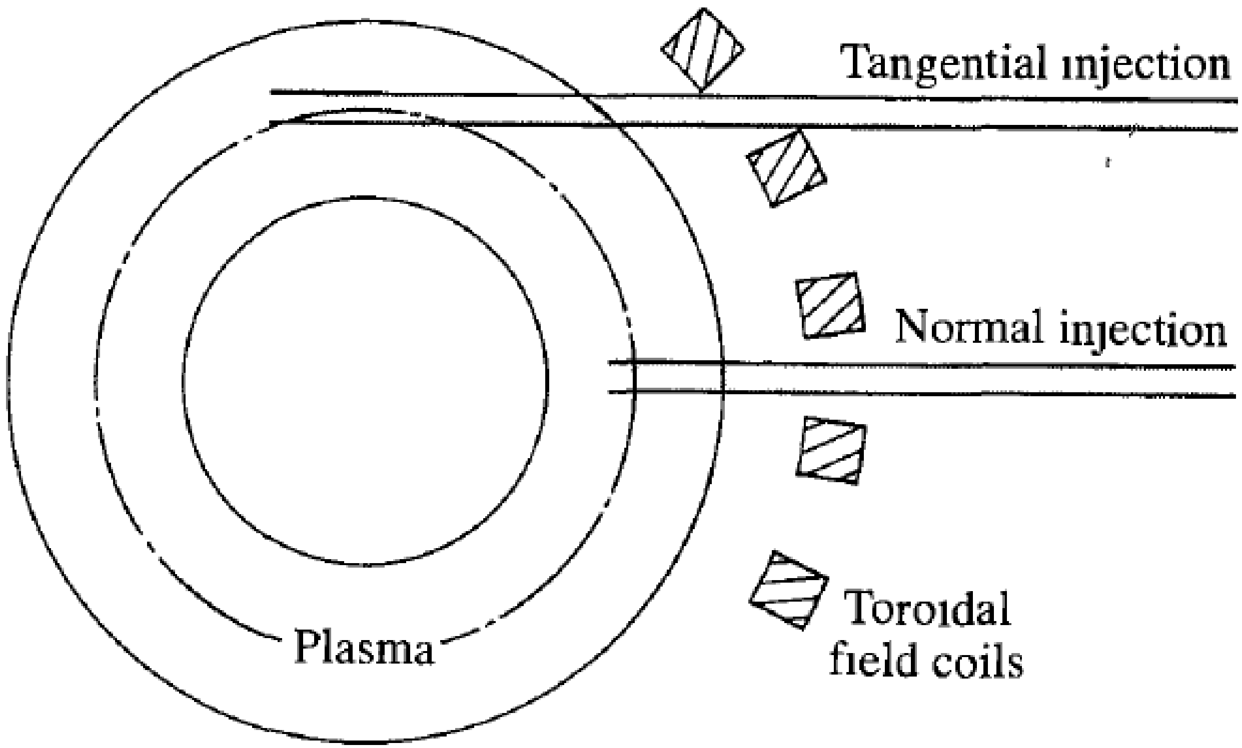
  $H_b + e^- \rightarrow H_b^+ + 2e^-$

b: beam
p: plasma

John Wesson, Tokamaks 3rd edition

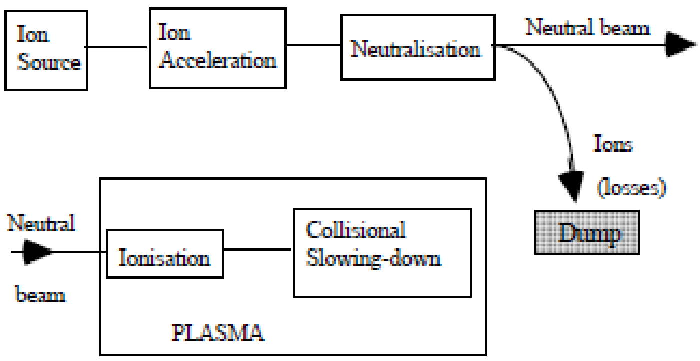
Neutral beam absorption length increases with tangential injection

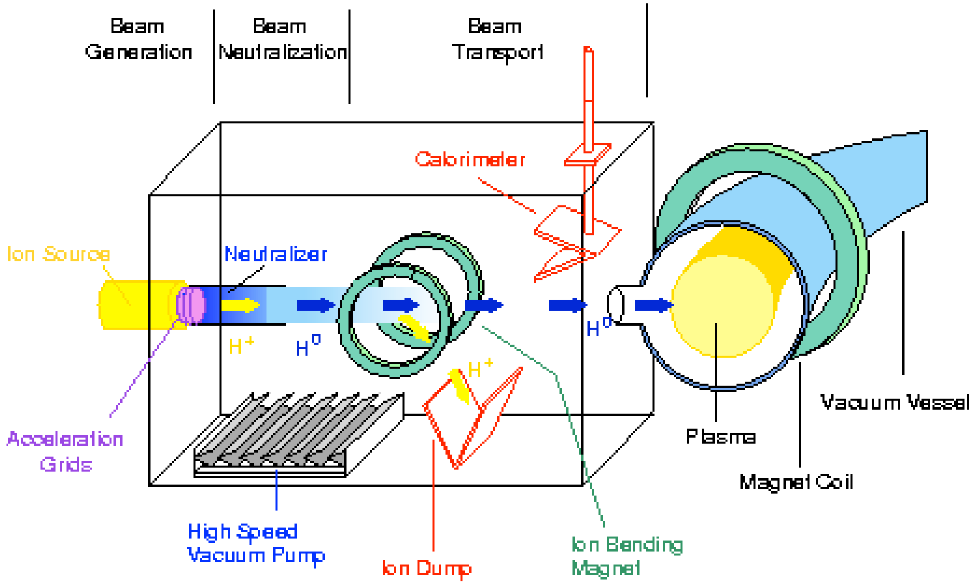
- **It is more difficult to access through the toroidal field coils with tangential injection.**



John Wesson, Tokamaks 3rd edition

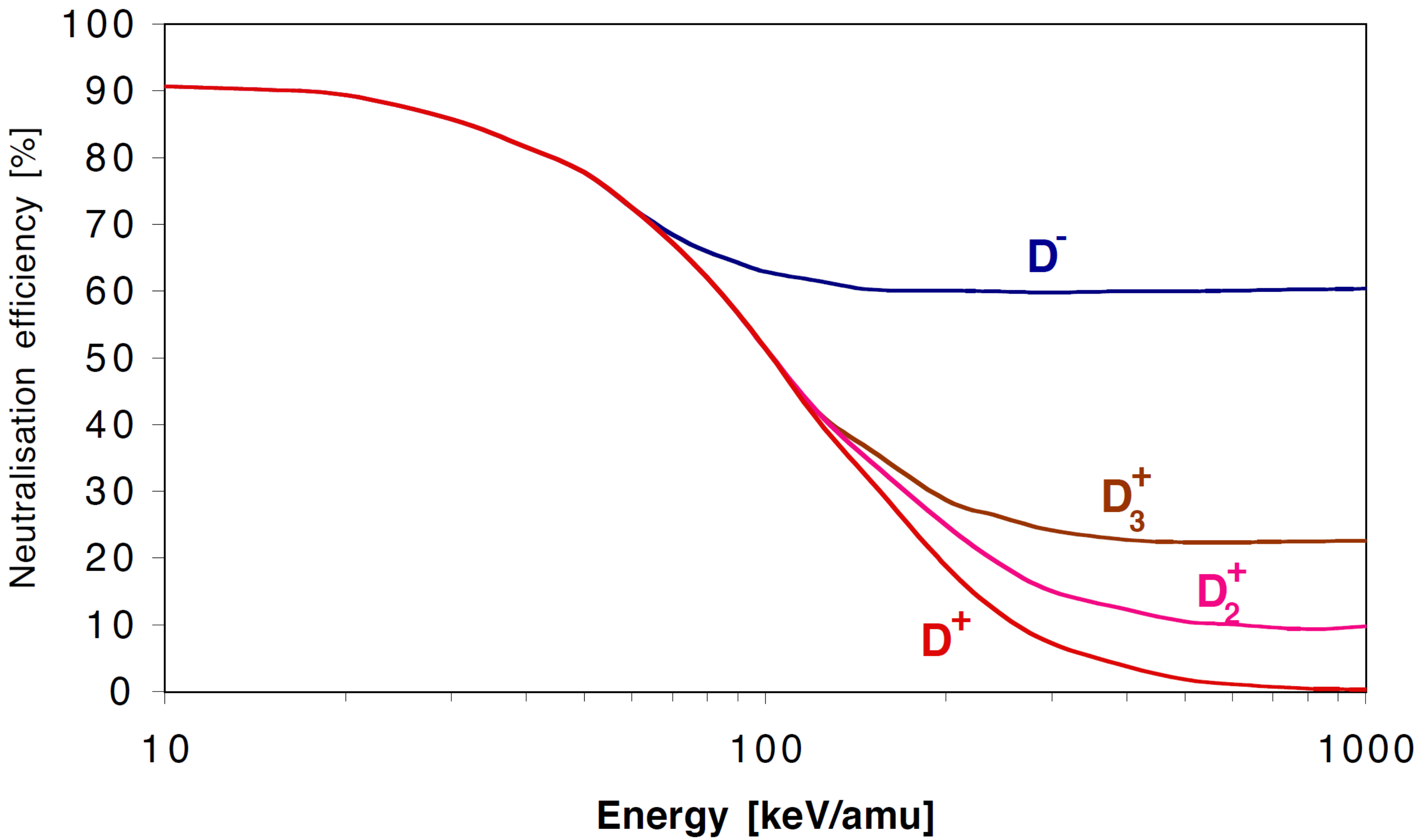
Neutral particles heat the plasma via coulomb collisions





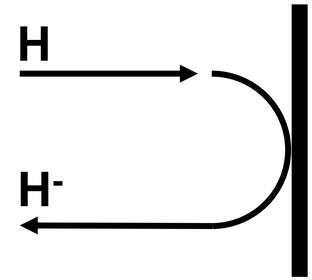
1. create energetic (fast) neutral ions
2. ionize the neutral particles
3. heat the plasma (electrons and ions) via Coulomb collisions

Negative ion source is preferred due to higher neutralization efficiency



There are two ways to make negative ions – surface and volume production

- **Surface production, depends on :**
  - **Work function Φ**
  - **Electron affinity level, 0.75 eV for H^-**
  - **Perpendicular velocity**
  - **Work function can be reduced by covering the metal surface with cesium**



$$H + e^- \longrightarrow H^-$$

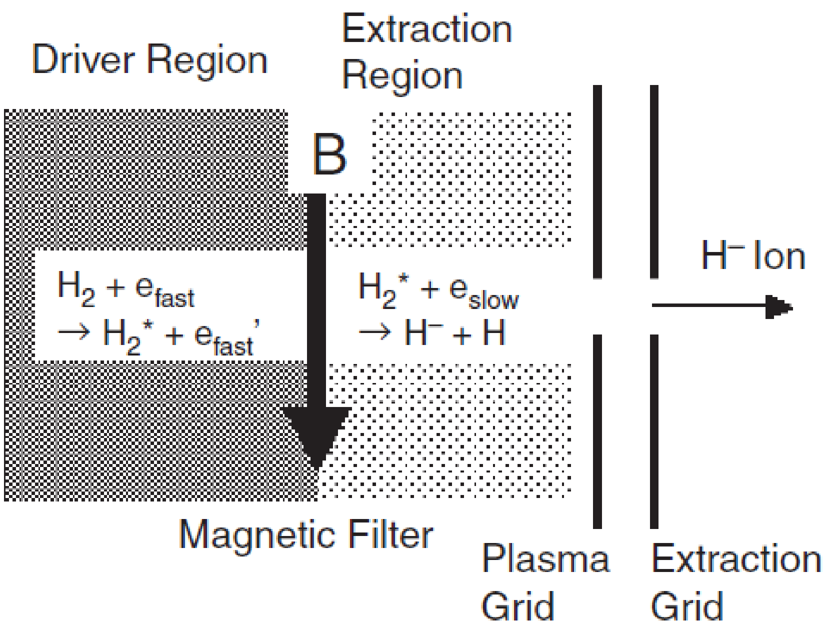
$$H^+ + 2e^- \longrightarrow H^-$$

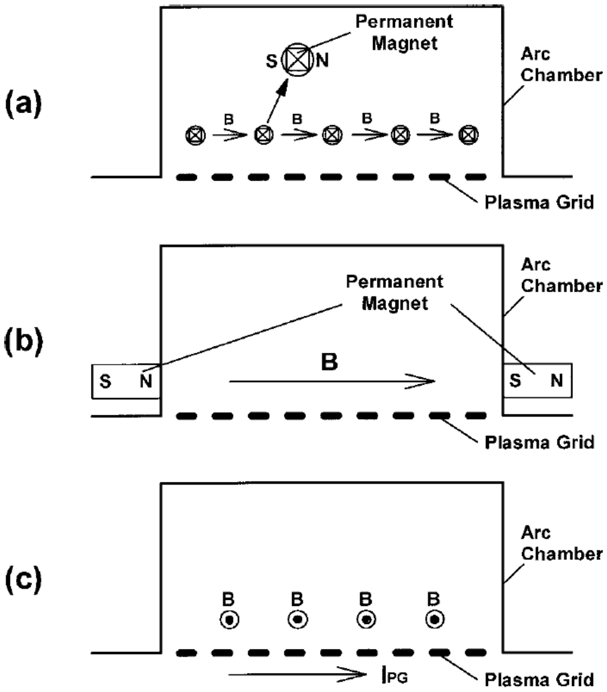
- **Volume production:**

$$H_2 + e_{fast}(>20\ eV) \rightarrow H_2^*(\text{excited state}) + e_{fast},$$

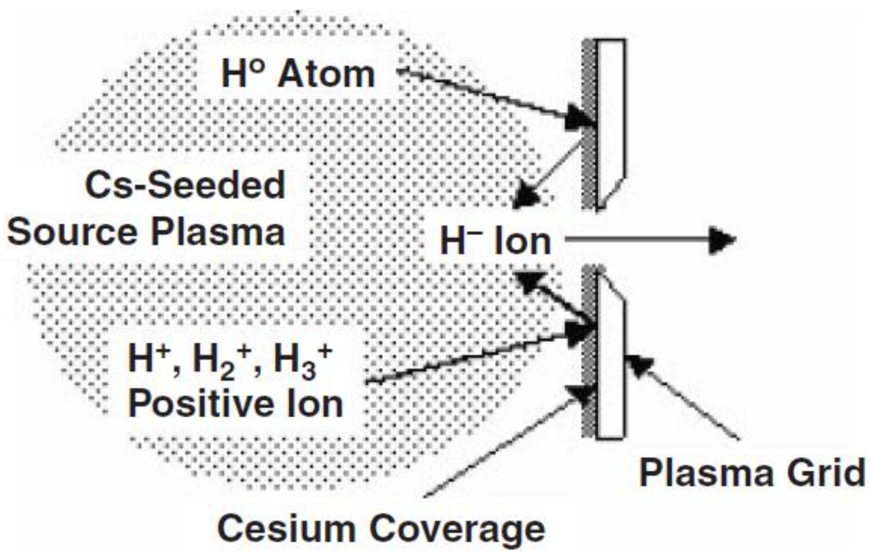
$$H_2^*(\text{excited state}) + e_{slow}(\approx 1\ eV) \rightarrow H^- + H.$$

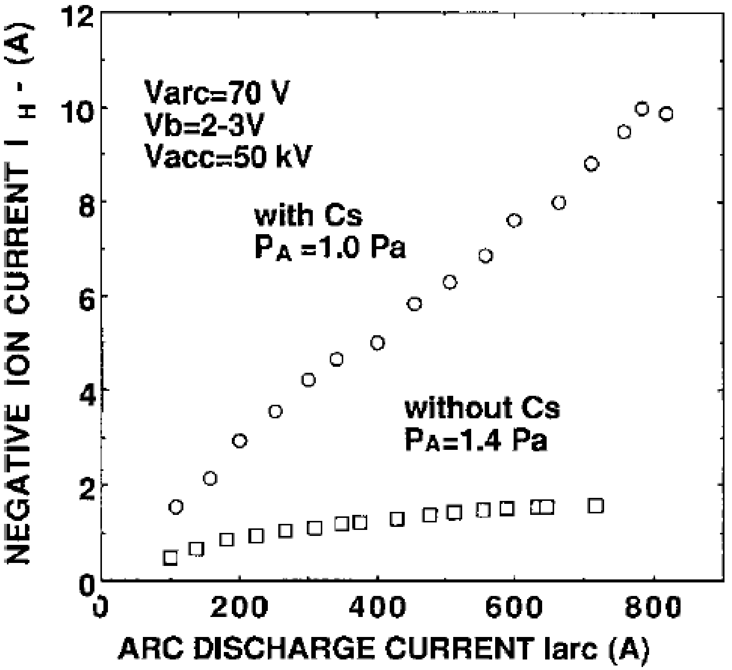
Two-chamber method of negative ions in volume production with a magnetic filter



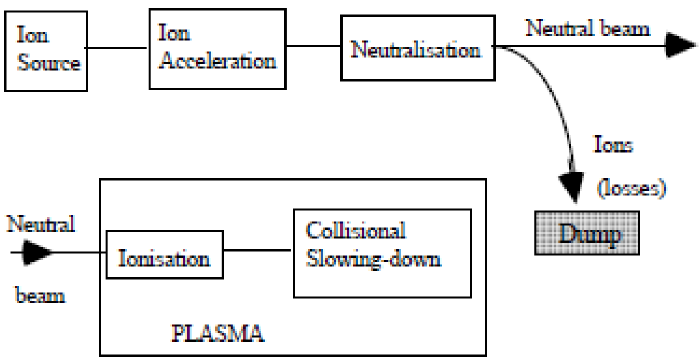


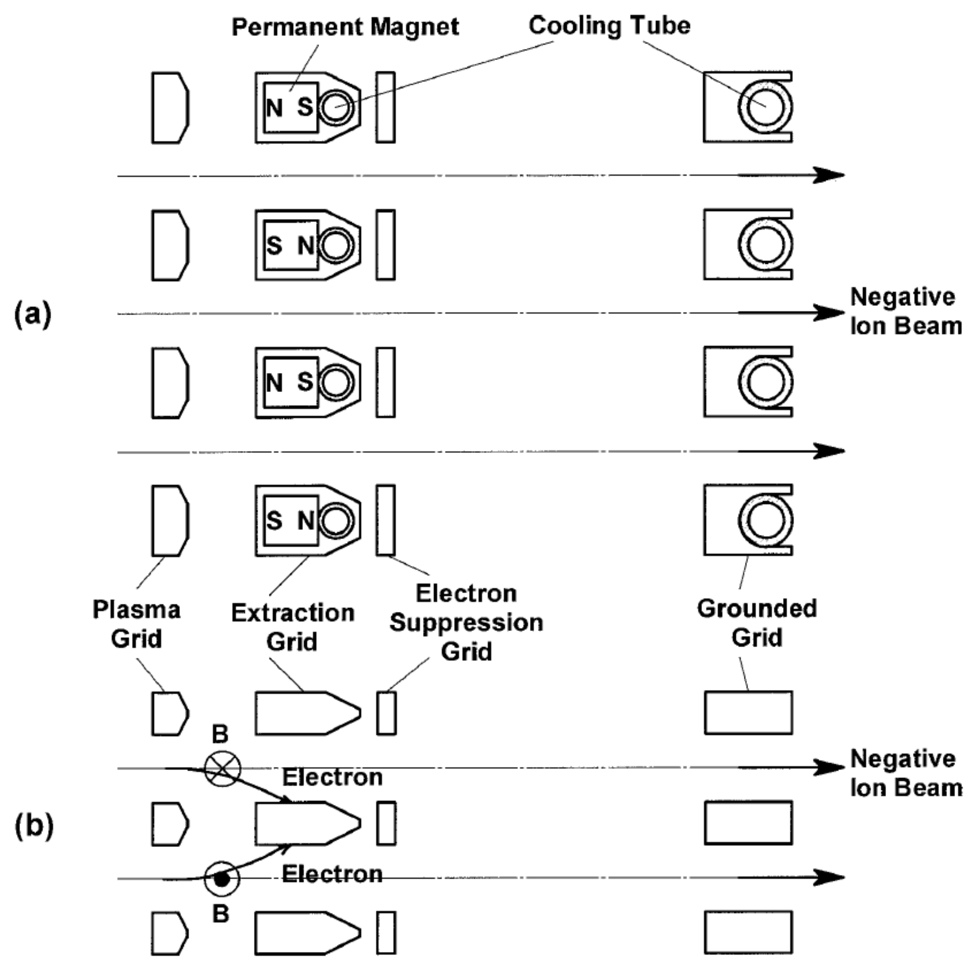
Adding cesium increases negative ion current





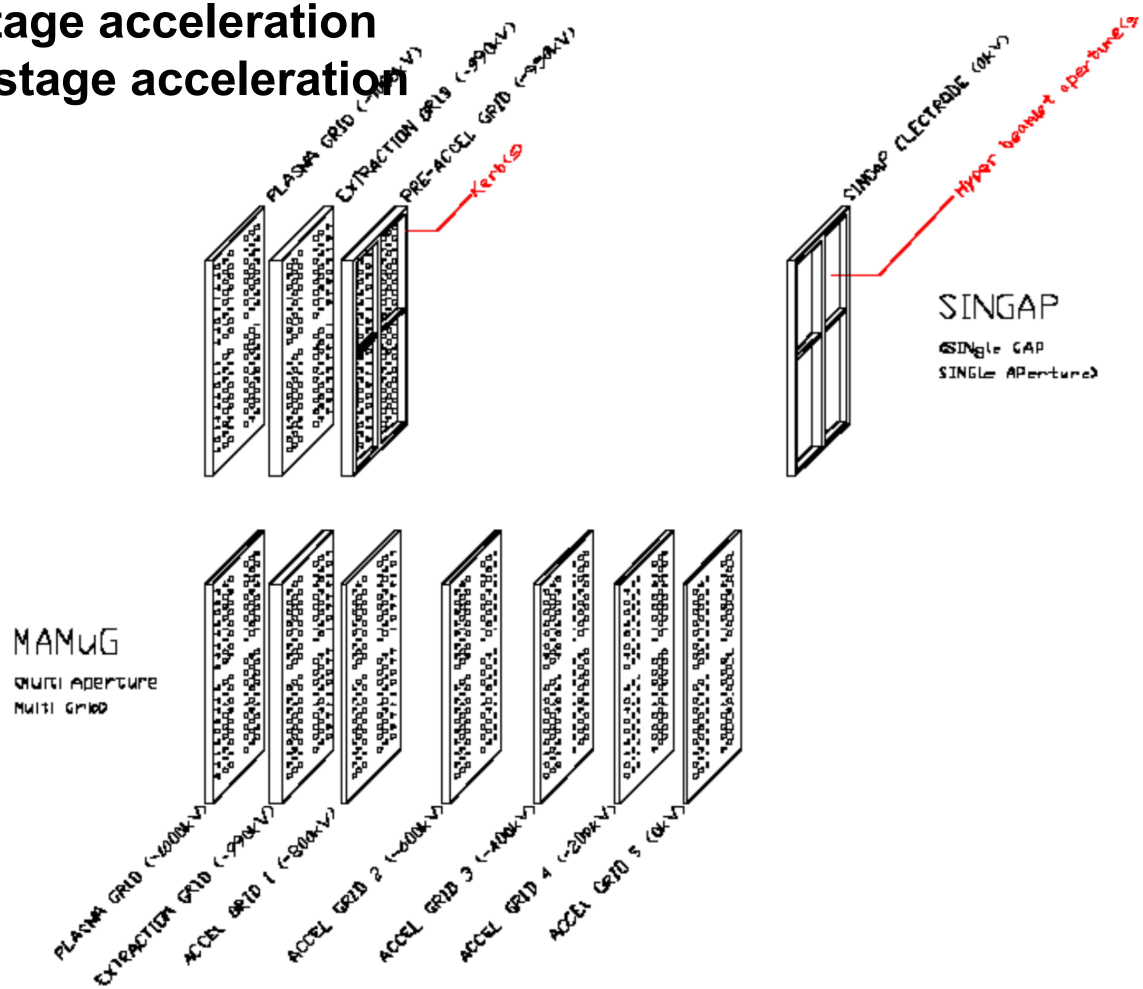
Electrons need to be filtered out since they are extracted together with negative ions





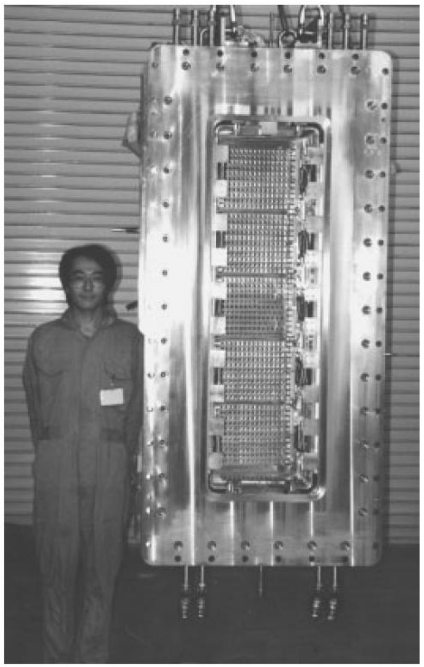
Acceleration

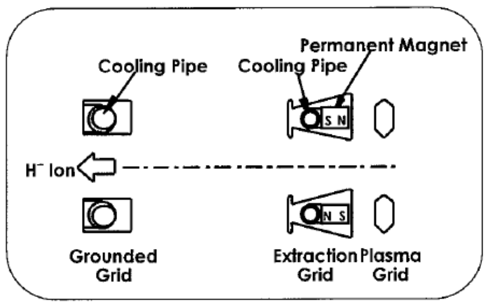
- **Multi-stage acceleration**
- **Single-stage acceleration**



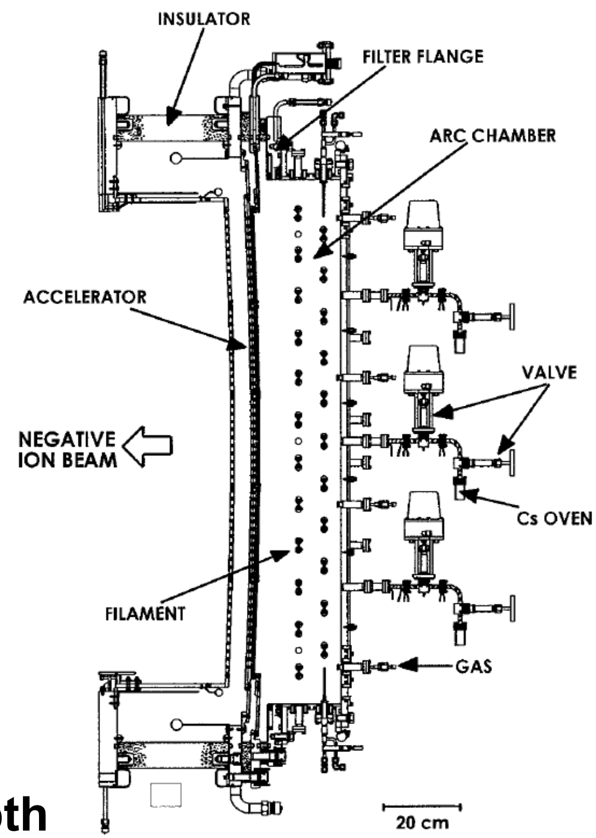
The ITER neutral beam system: status of the project and review of the main technological issues, presented by V. Antoni

NBI system of the LHD fusion machine





Negative Ion Accelerator

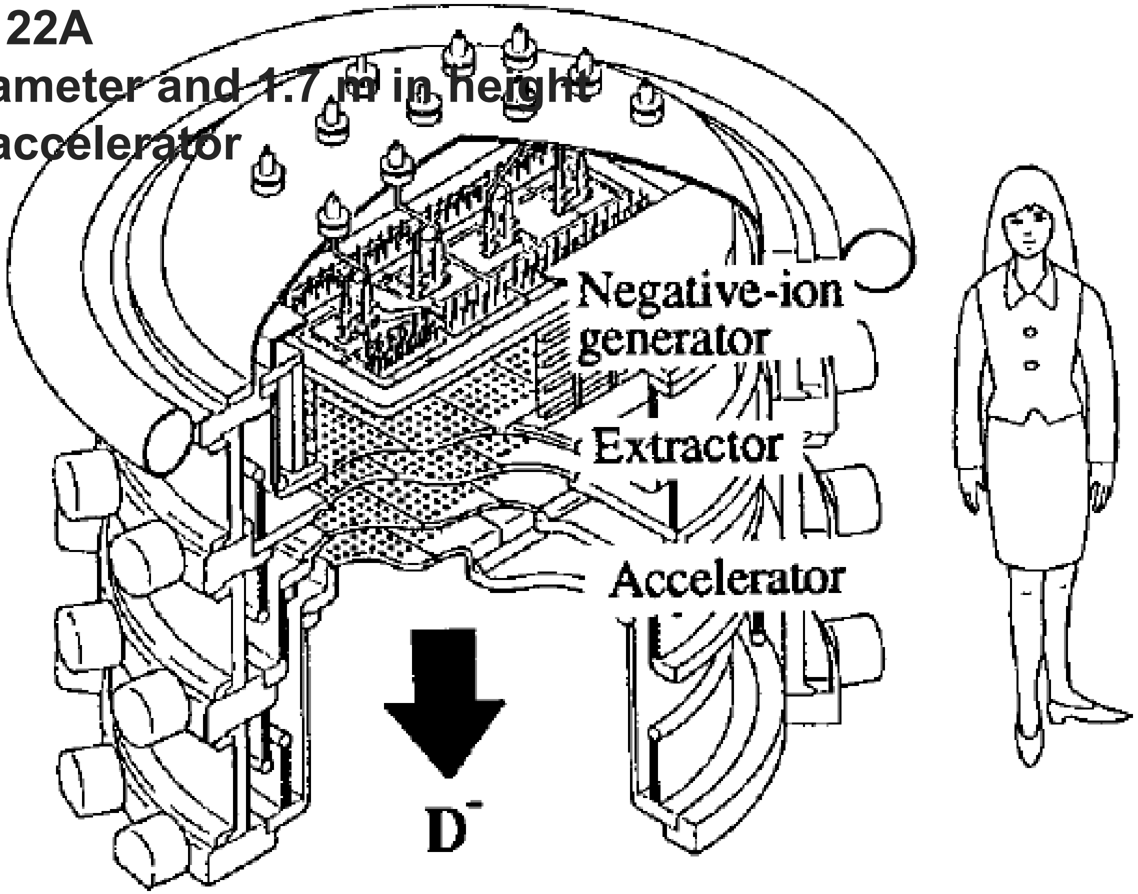


- 180 keV and 30 A
- Arc chamber: 35 cm x 145 cm, 21cm in depth
- Single stage accelerator

Y. Takeiri, etc., Nucl. Fusion, 46, S199, 2006

JT60U NBI system

- JT-60 (Japan-Torus) is a tokamak in Japan.
- 550 keV, 22A
- 2m in diameter and 1.7 m in height
- 3-stage accelerator



Neutralization

- Gas neutralization
  - Collisions between fast negative ions and atoms

$$H^- + H_2 \longrightarrow H + H_2 + e^-$$

  - Fast ions can lose another electron after neutralized

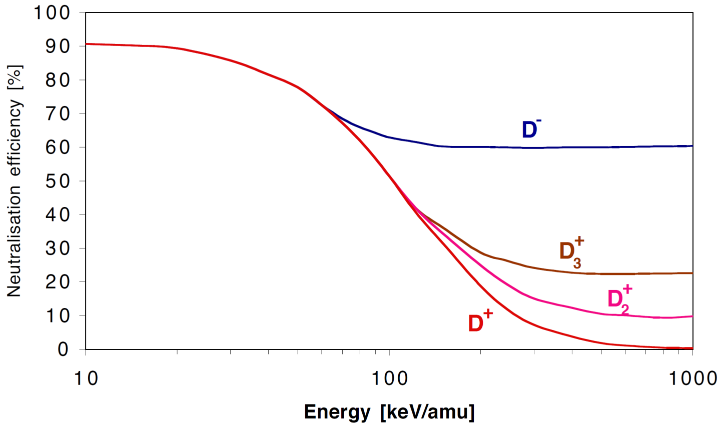
$$H + H_2 \longrightarrow H^+ + H_2 + e^-$$

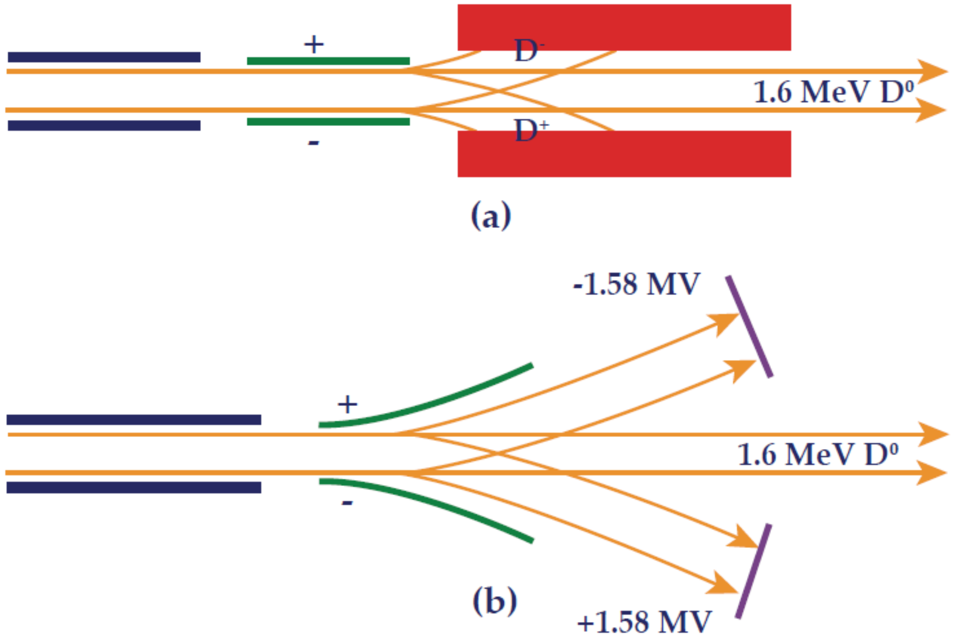
- Plasma neutralization
  - Collisions with charged particles in plasma

$$H^- + X(\,e, \mathrm{Ar}, H^+, {H_2}^+) \longrightarrow H + X + e^-$$

  - The efficiencies reach up to 85% for fully ionized hydrogen plasma

Beam dump

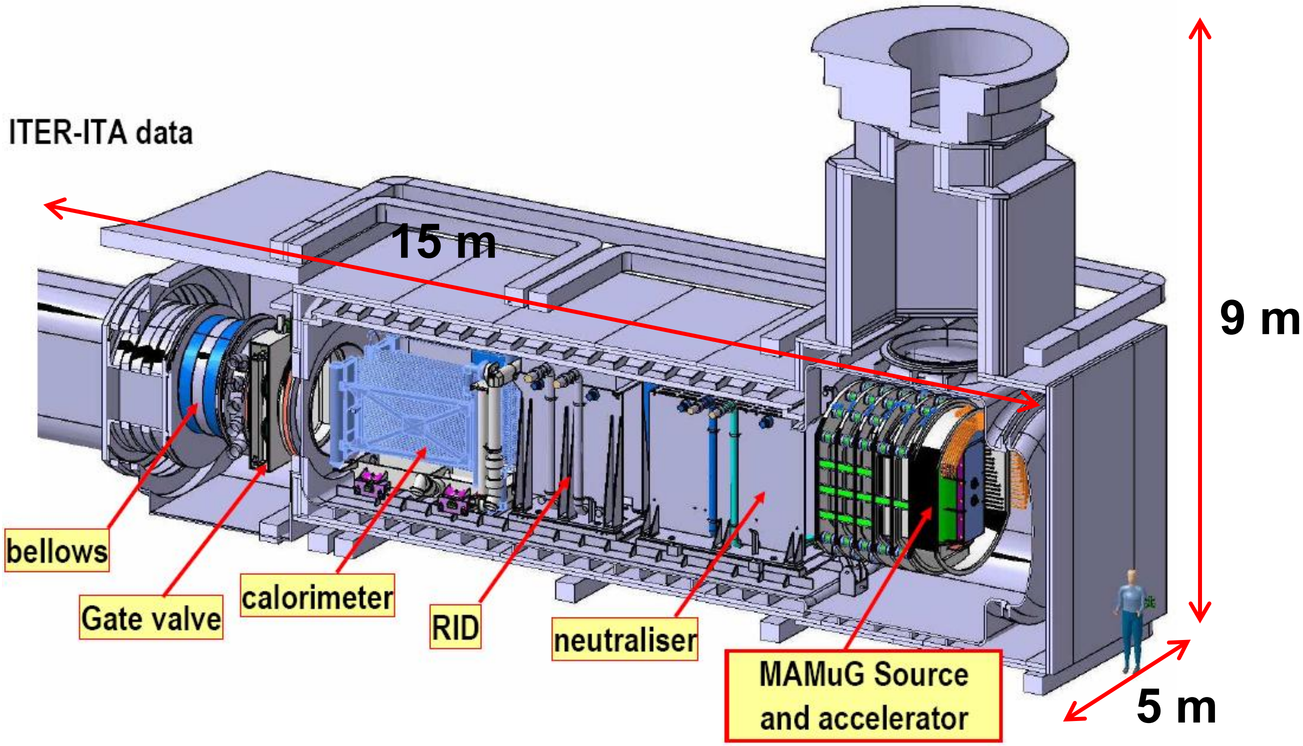




R. W. Moir, Fusion Technology, 25, 129, 1994

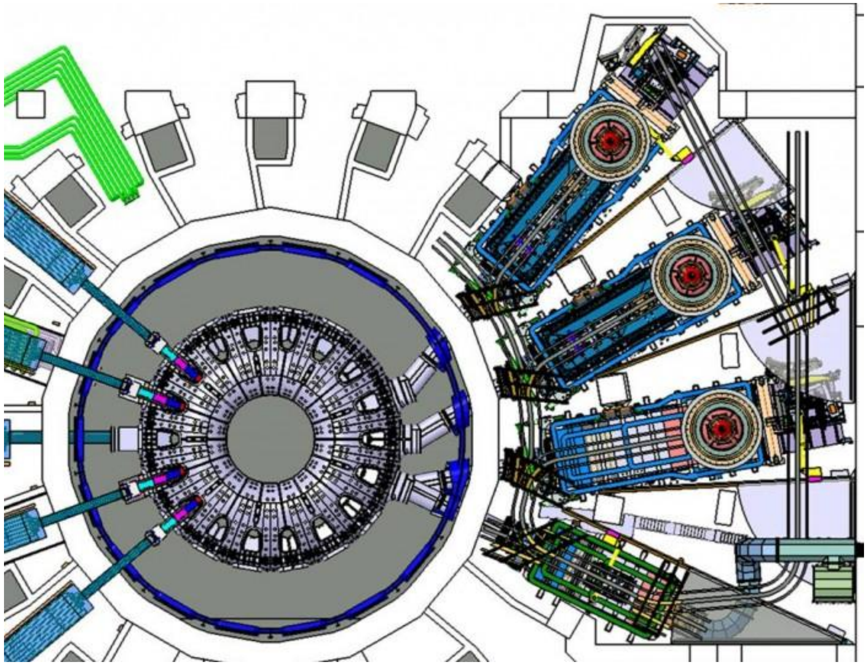
NBI for ITER

- **beam components (Ion Source, Accelerator, Neutralizer, Residual Ion Dump and Calorimeter)**
- **other components (cryo-pump, vessels, fast shutter, duct, magnetic shielding, and residual magnetic field compensating coils)**



The ITER neutral beam system: status of the project and review of the main technological issues, presented by V. Antoni

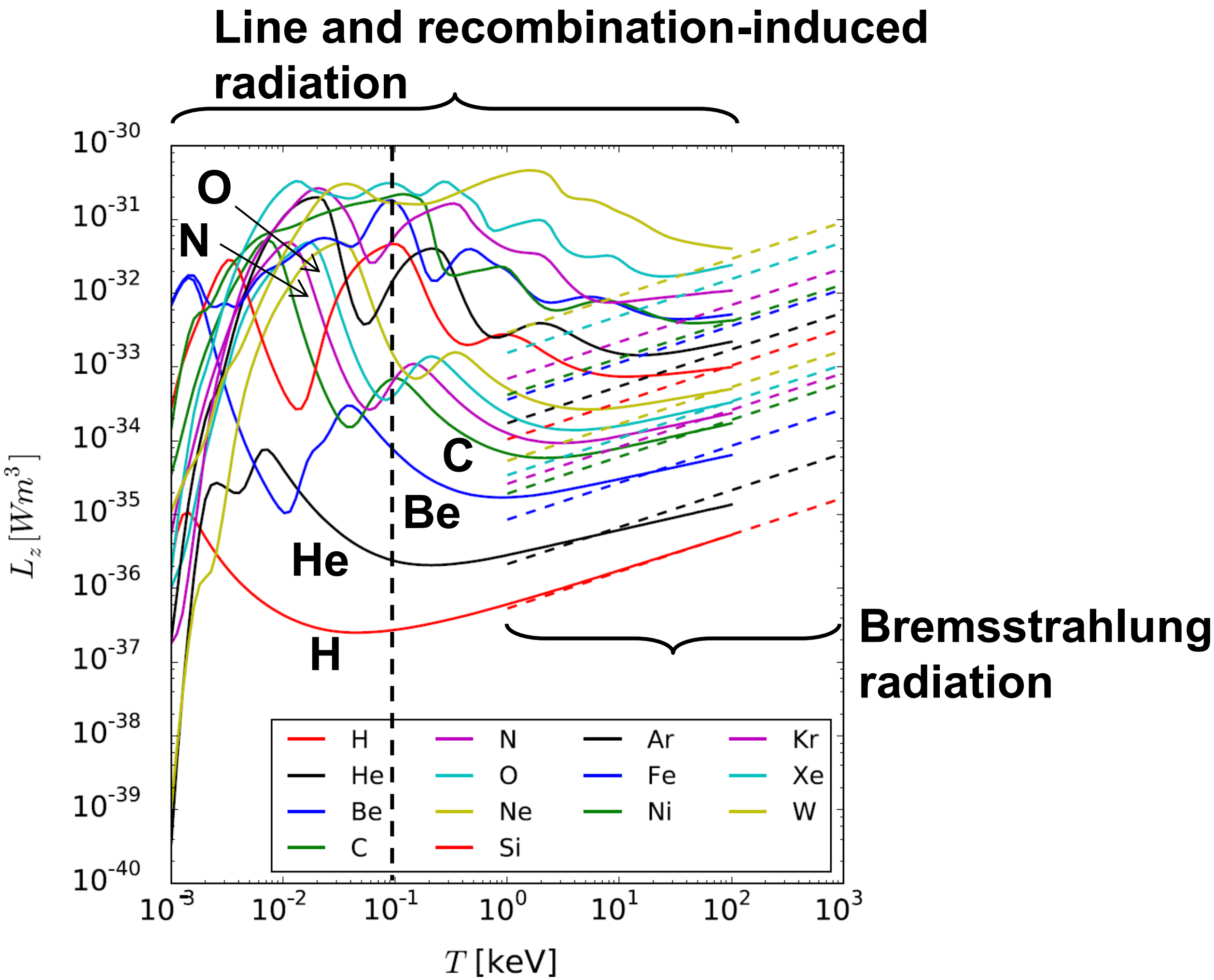
Neutral beam penetration



- **Parallel direction**
  - **Longest path through the densest part of the plasma**
  - **Harder to be built**
- **Perpendicular direction**
  - **Path is short**
  - **Larger perpendicular energies leads to larger losses**
  - **Easier to be built**

https://www.iter.org

Temperature of 100 eV is the threshold of radiation barrier by impurities



H. Lux, etc., Fusion Eng. Des. **42**, 101 (2015)

Course Outline

- Inertial confinement fusion (ICF)
  - Plasma frequency and critical density
  - Direct- and indirect- drive
  - Laser generated pressure (Inverse bremsstrahlung and Ablation pressure)
  - Burning fraction, why compressing a capsule?
  - Implosion dynamics
  - Shock (Compression with different adiabat)
  - Laser pulse shape
  - Rocket model, shell velocity
  - Laser-plasma interaction (Stimulated Raman Scattering, SRS; Stimulated Brillouin Scattering, SBS; Two-plasmon decay)
  - Instabilities (Rayleigh-taylor instability, Kelvin-Helmholtz instability, Richtmeyer-Meshkov instability)

Under what conditions the plasma keeps itself hot?

- **Steady state 0-D power balance:**

$S_\alpha + S_h = S_B + S_k$

S_α: α particle heating

S_h: external heating

S_B: Bremsstrahlung radiation

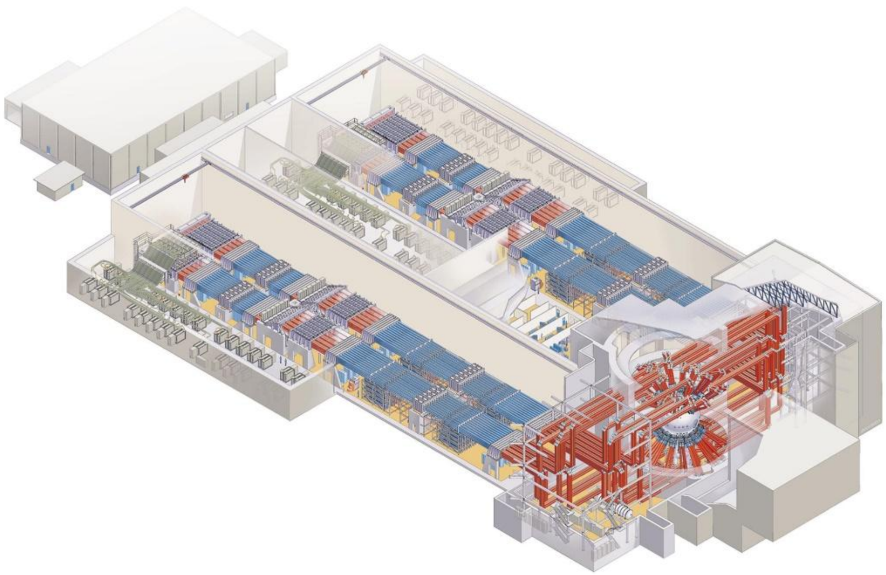
S_k: heat conduction lost

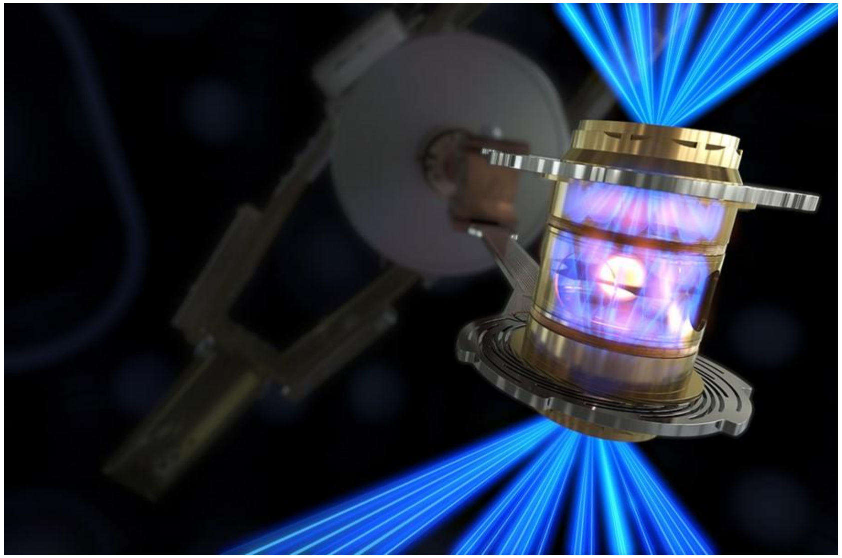
Ignition condition: $P\tau$ > 10 atm-s = 10 Gbar - ns

- **P: pressure, or called energy density**
- **τ is confinement time**

Significant breakthrough was achieved in ICF recently

- **Inertial confinement fusion (ICF)**





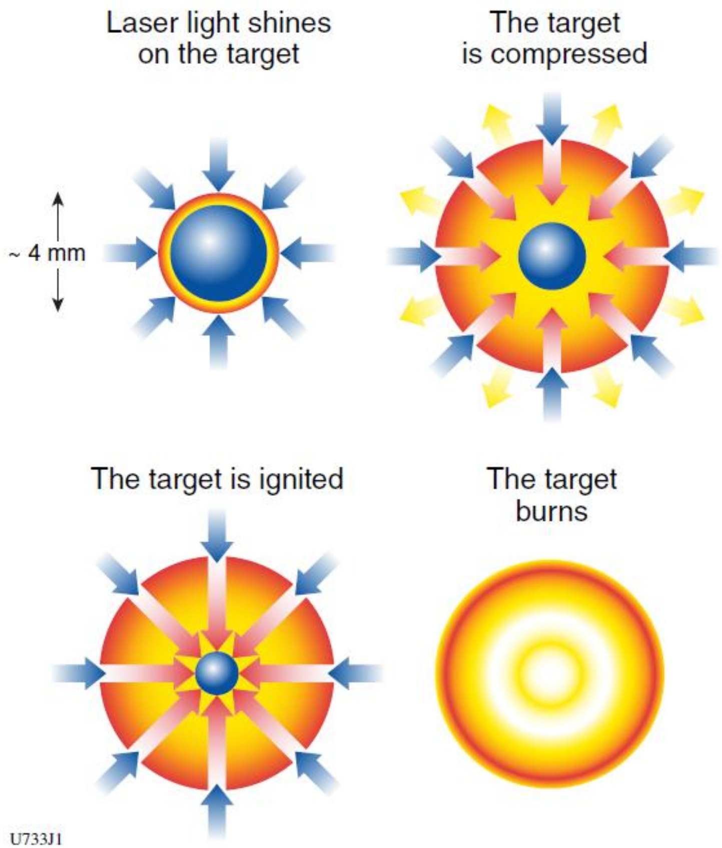
- **National Ignition Facility (NIF) demonstrated a gain grater than 1 for the first time on 2022/12/5. The yield of 3.15 MJ from the 2.05-MJ input laser energy, i.e., Q=1.5.**

---

https://zh.wikipedia.org/wiki/國家點火設施
https://www.science.org/content/article/historic-explosion-long-sought-fusion-breakthrough

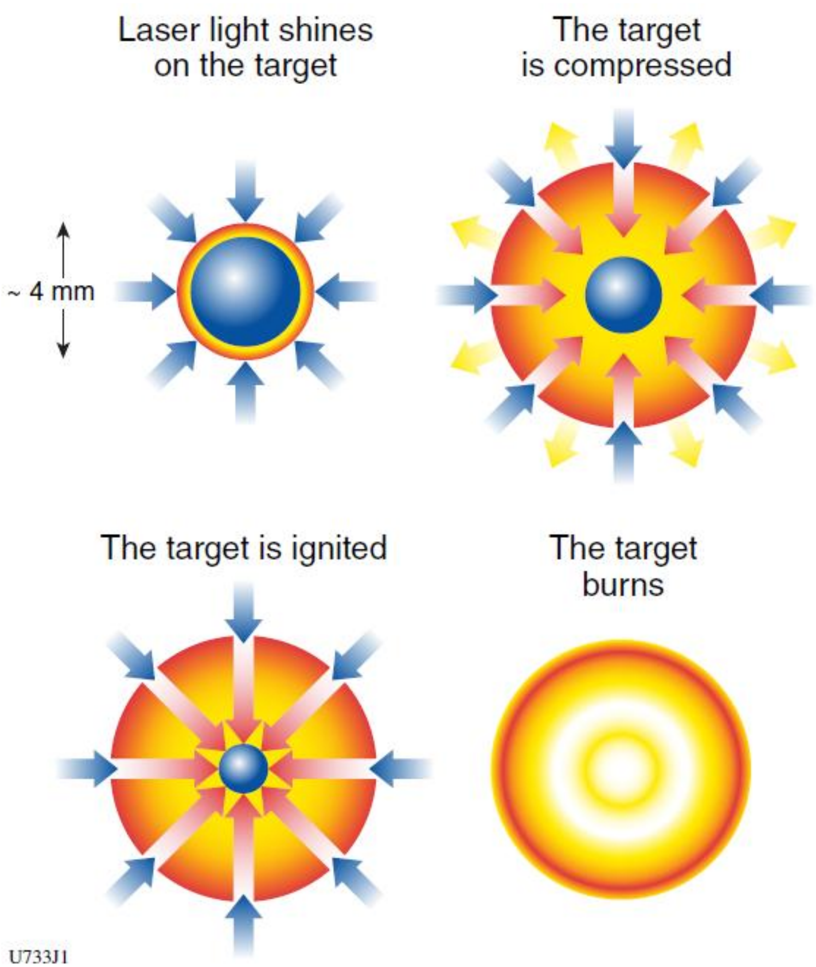
Don’t confine it!

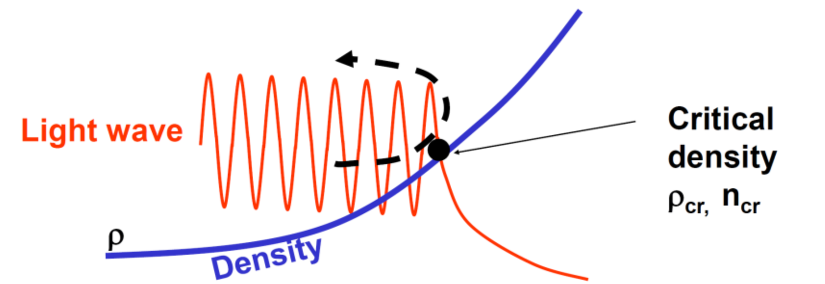
- **Solution 2: Inertial confinement fusion (ICF). Or you can say it is confined by its own inertia: P~Gigabar, τ~nsec, T~10 keV (10^8 ^{0}C)**

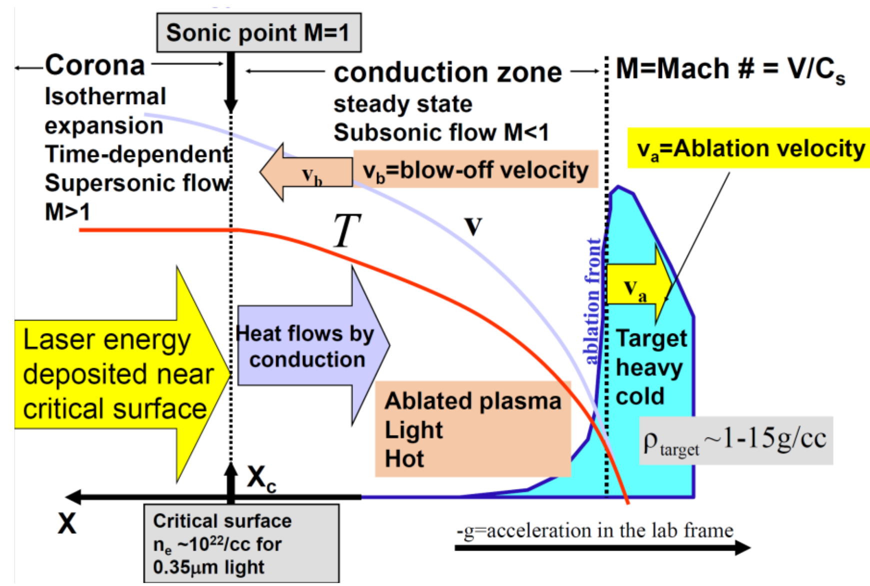


Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester

Compression happens when outer layer of the target is heated by laser and ablated outward

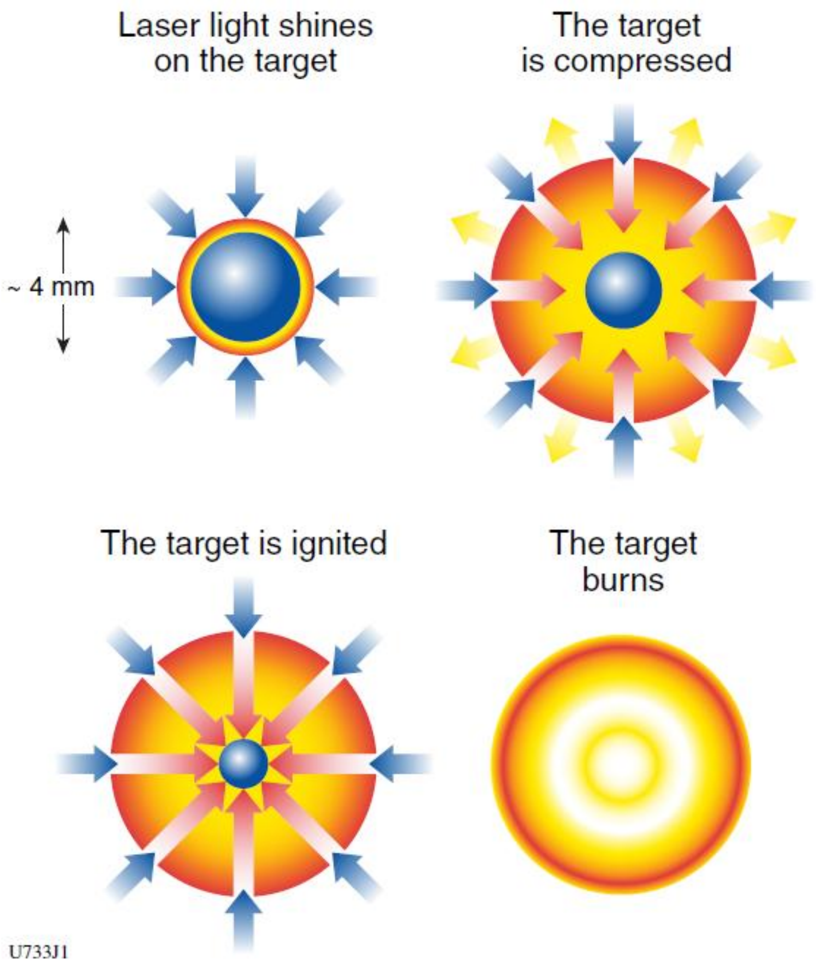




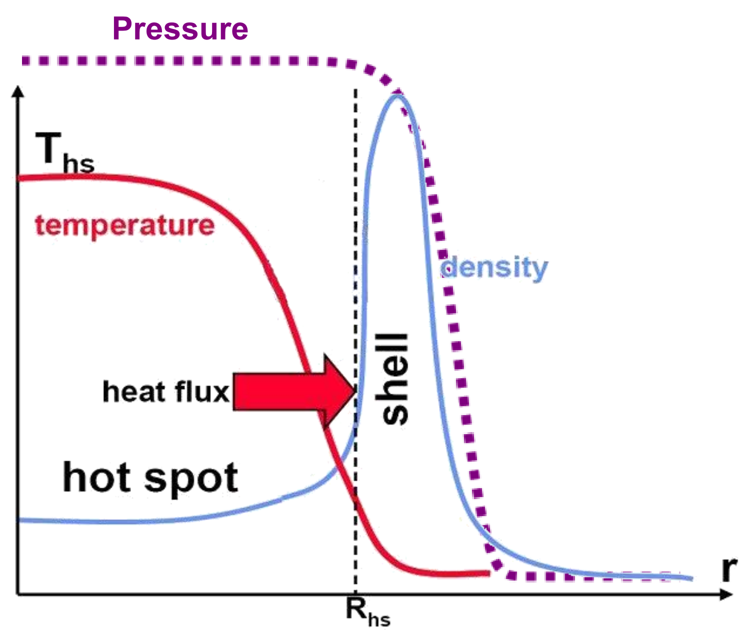


Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester
R. Betti, HEDSA HEDP Summer School, 2015

Plasma is confined by its own inertia in inertial confinement fusion (ICF)



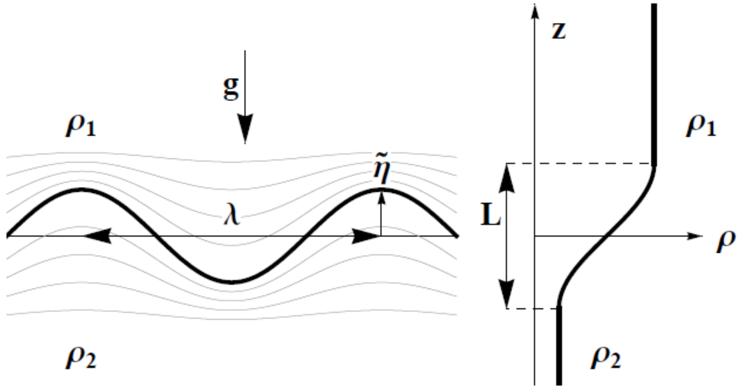
Spatial profile at stagnation



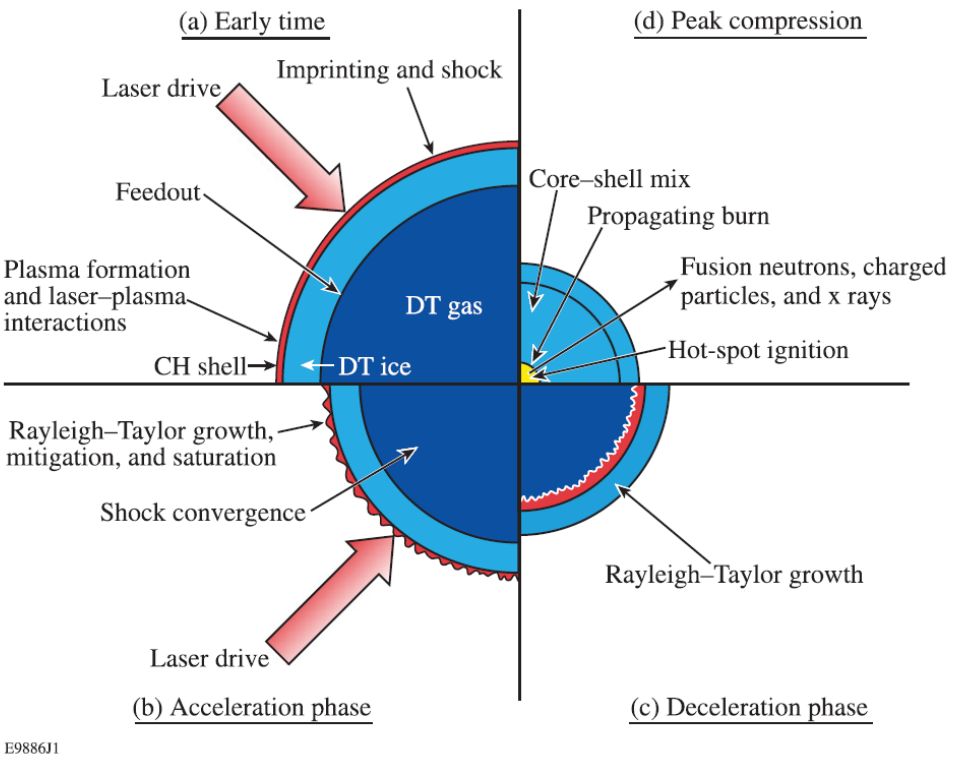
Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester

A ball can not be compressed uniformly by being squeezed between several fingers

- **Rayleigh-Taylor instability**

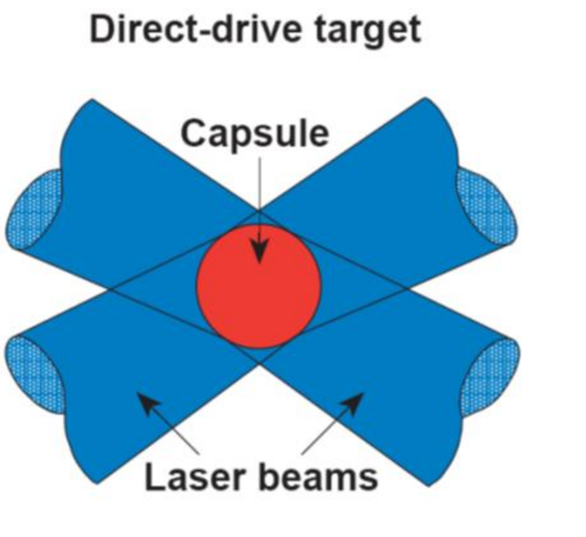


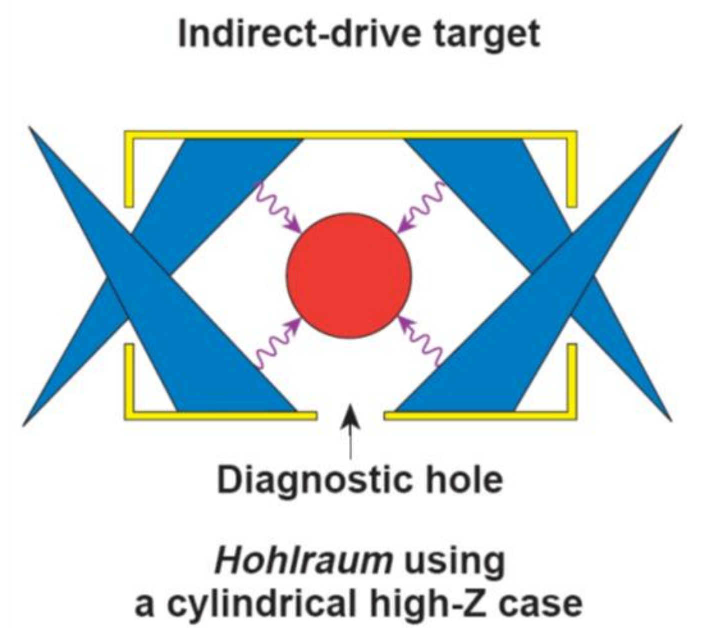
- **Stages of a target implosion**



P.-Y. Chang, PhD Thesis, U of Rochester (2013)
R. S. Craxton, etc., *Phys. Plasmas* **22**, 110501 (2015)

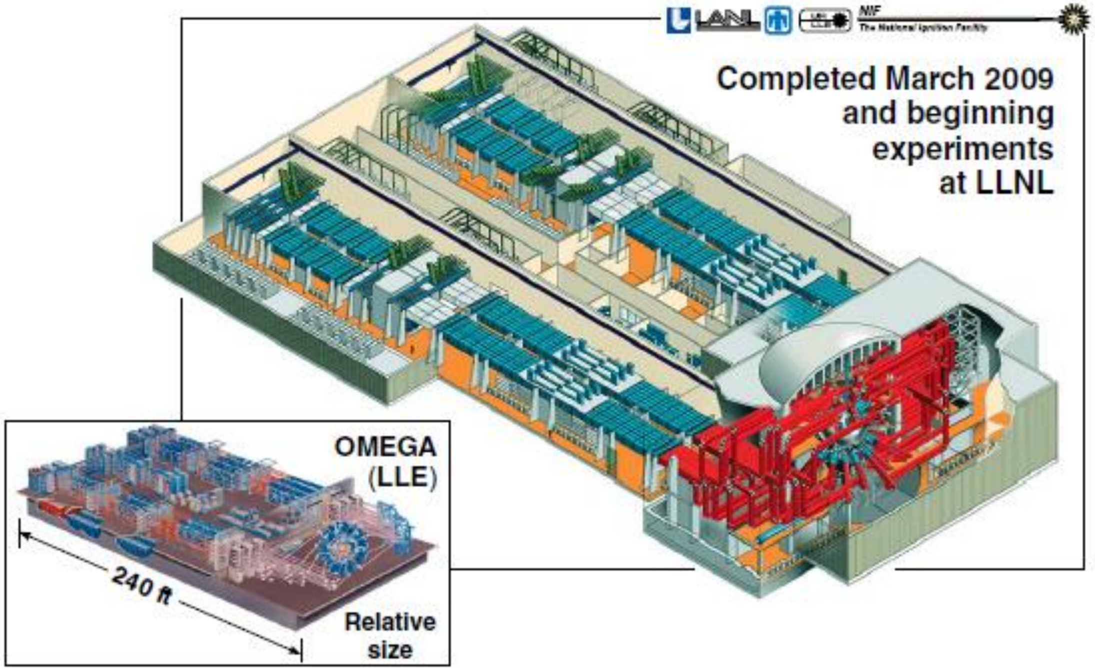
A spherical capsule can be imploded through directly or indirectly laser illumination





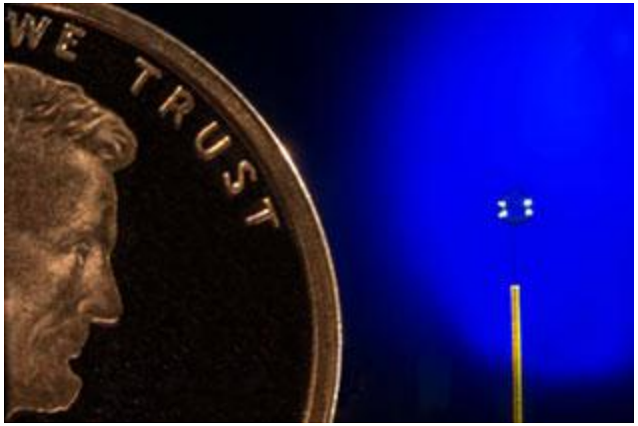
*R. Betti, HEDSA HEDP Summer School, 2015

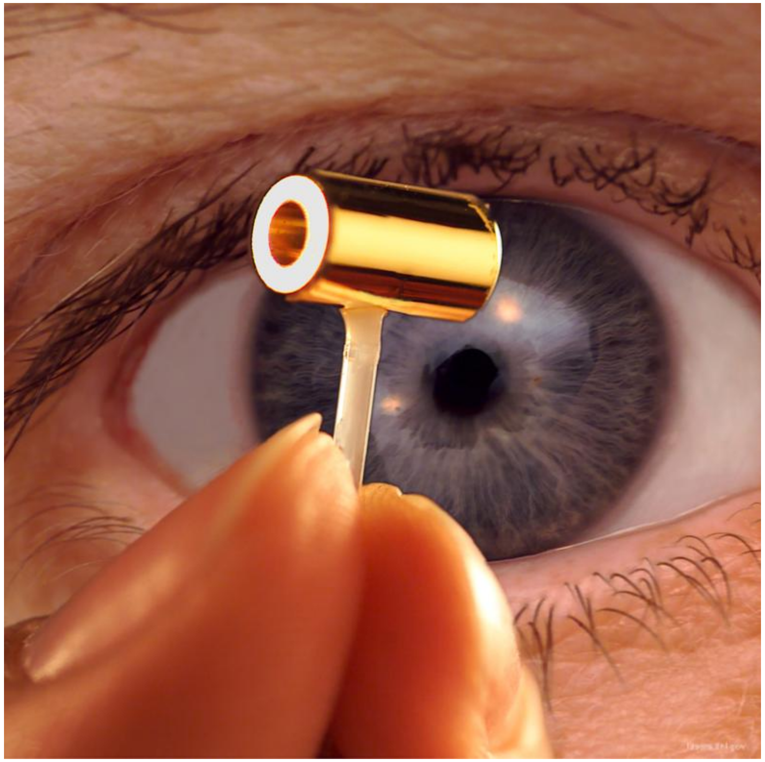
The 1.8-MJ National Ignition Facility (NIF) will demonstrate ICF ignition and modest energy gain



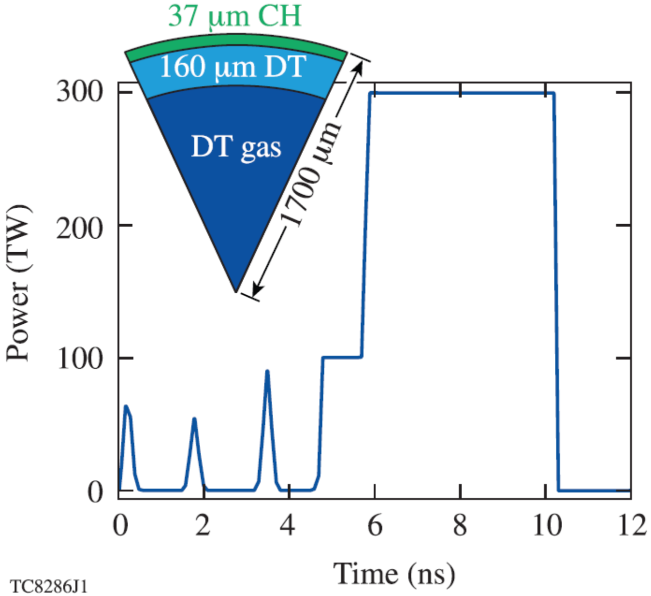
OMEGA experiments are integral to an ignition demonstration on the NIF.

Targets used in ICF





- **Triple-point temperature : 19.79 K**

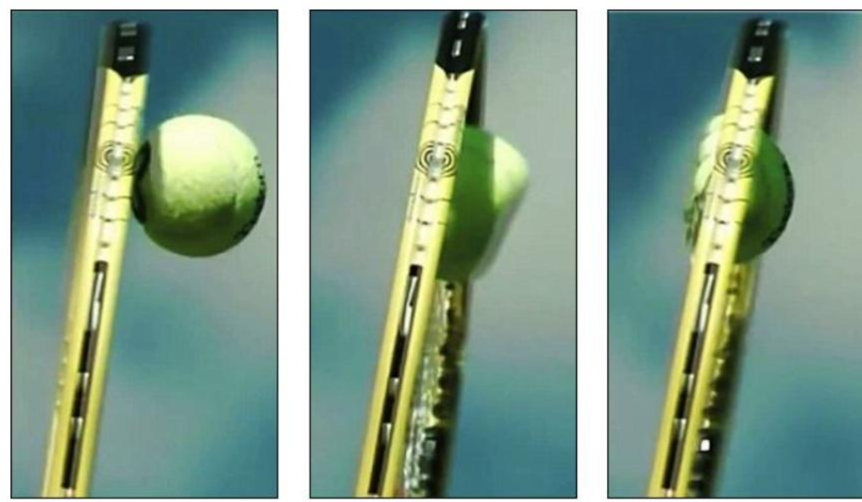


http://www.lle.rochester.ed
https://en.wikipedia.org/wiki/Inertial_confinement_fusion
R. S. Craxton, etc., *Phys. Plasmas* **22**, 110501 (2015)

Softer material can be compressed to higher density

- **Compression of a baseball**

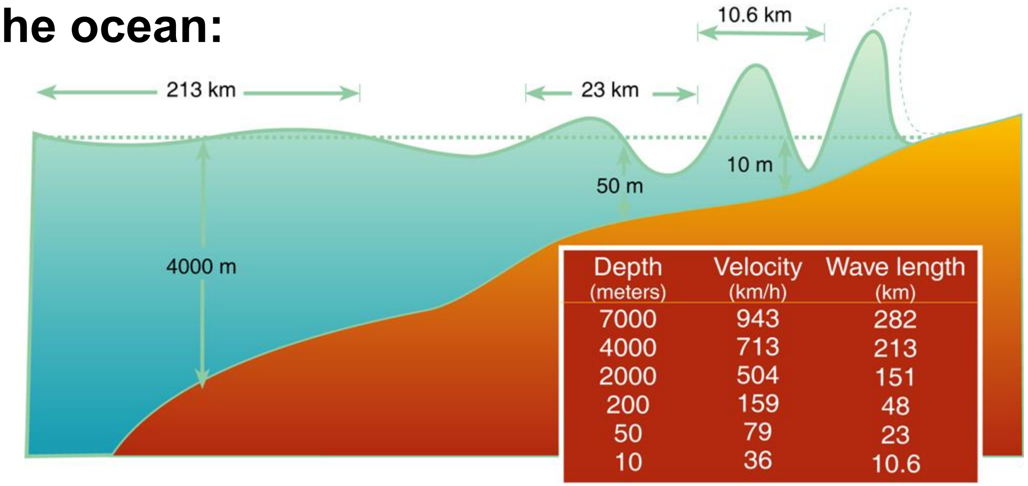
- **Compression of a tennis ball**



https://www.youtube.com/watch?v=uxlIdMoAwbY
https://newsghana.com.gh/wimbledon-slow-motion-video-of-how-a-tennis-ball-turns-to-goo-after-serve/

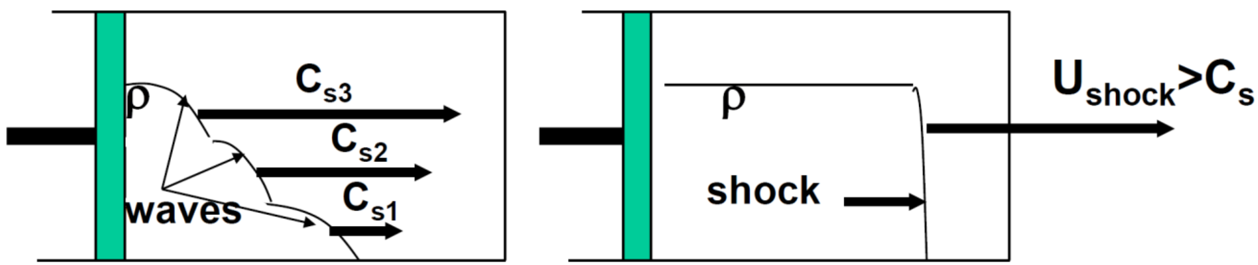
A shock is formed due to the increasing sound speed of a compressed gas/plasma

- **Wave in the ocean:**



Depth (meters)	Velocity (km/h)	Wave length (km)
7000	943	282
4000	713	213
2000	504	151
200	159	48
50	79	23
10	36	10.6

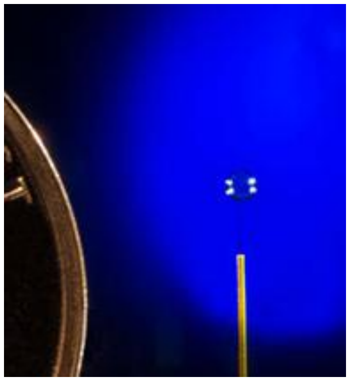
- **Acoustic/compression wave driven by a piston:**



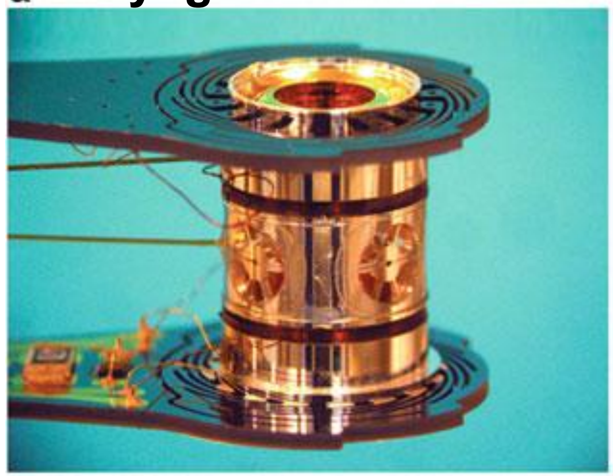
$$C_s \sim \sqrt{\frac{p}{\rho}} \sim \sqrt{\frac{\alpha \rho^{5/3}}{\rho}} \sim \sqrt{\alpha}\rho^{1/3}$$

http://neamtic.ioc-unesco.org/tsunami-info/the-cause-of-tsunamis
*R. Betti, HEDSA HEDP Summer School, 2015

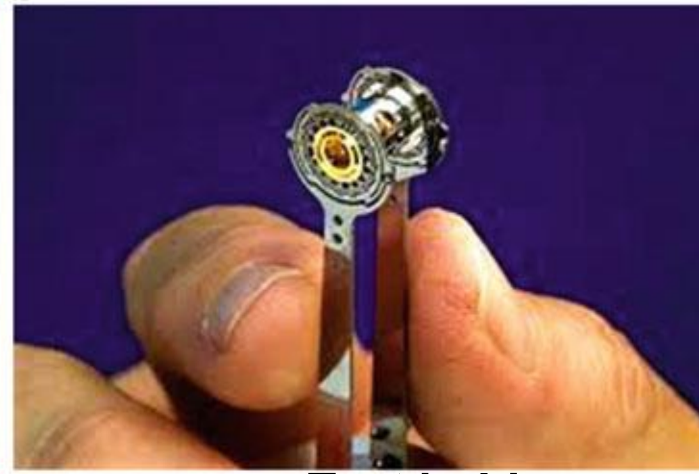
Targets used in ICF



a **Cryogenic hohlraum**



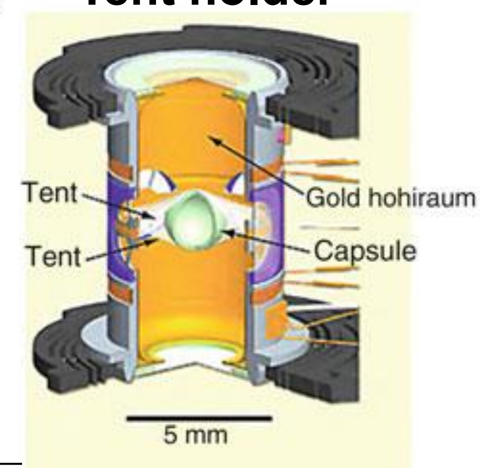
b



Cryogenic shroud

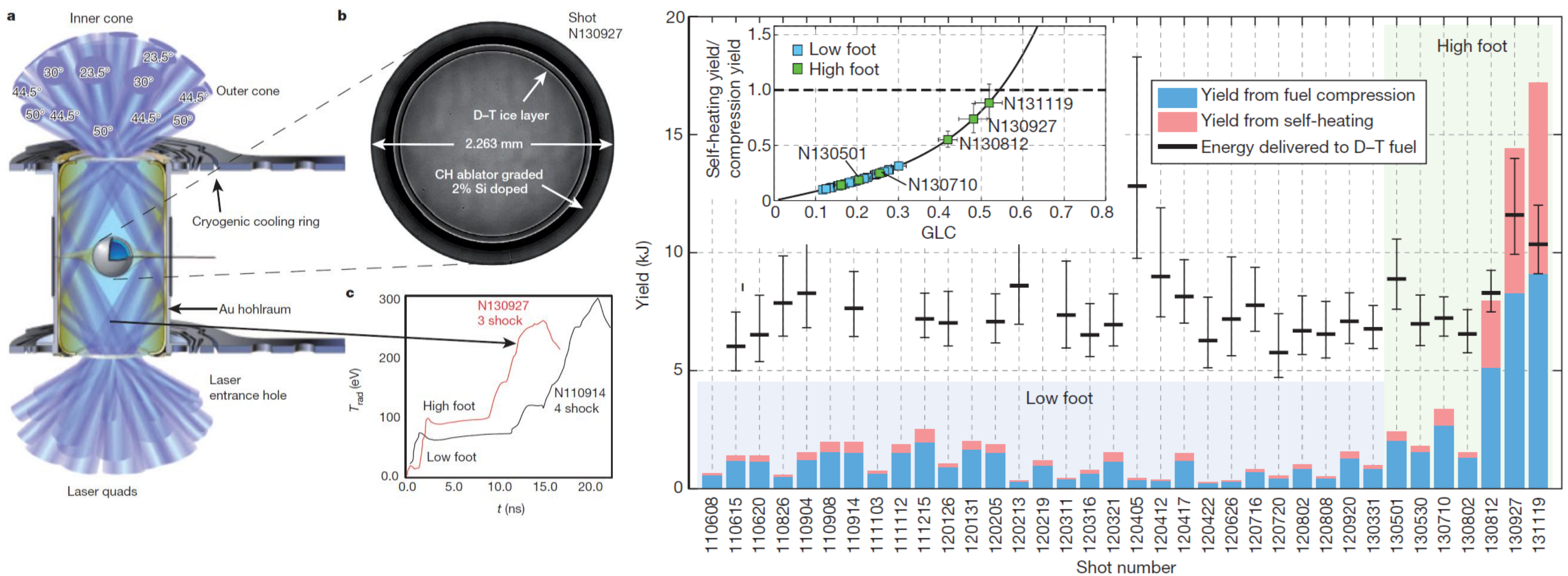
c **Rugby hohlraum**

d **Tent holder**



https://www.lle.rochester.edu/index.php/2014/11/10/next-generation-cryo-target/
Introduction to Plasma Physics and Controlled Fusion 3rd Edition, by Francis F. Chen
https://www.llnl.gov/news/nif-shot-lights-way-new-fusion-ignition-phase

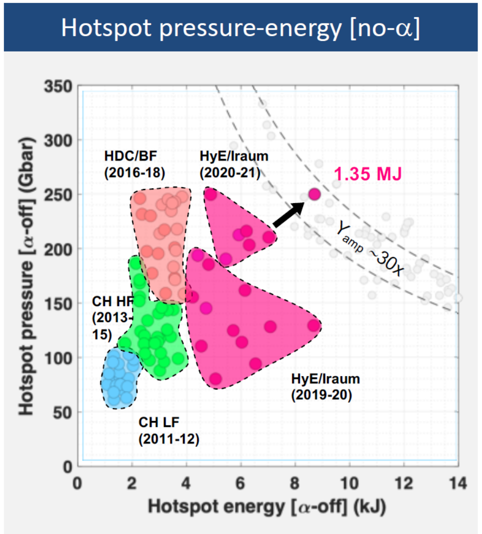
Nature letter "Fuel gain exceeding unity in an inertially confined fusion implosion"

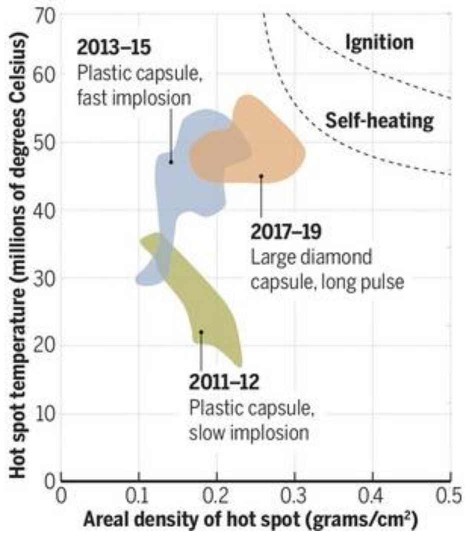


- **Fuel gain exceeding unity was demonstrated for the first time.**

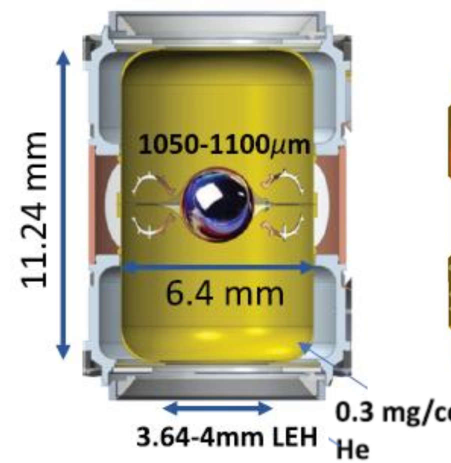
Nature 506, p343, 2014

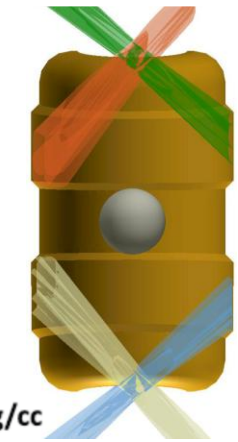
The hot spot has entered the burning plasma regime

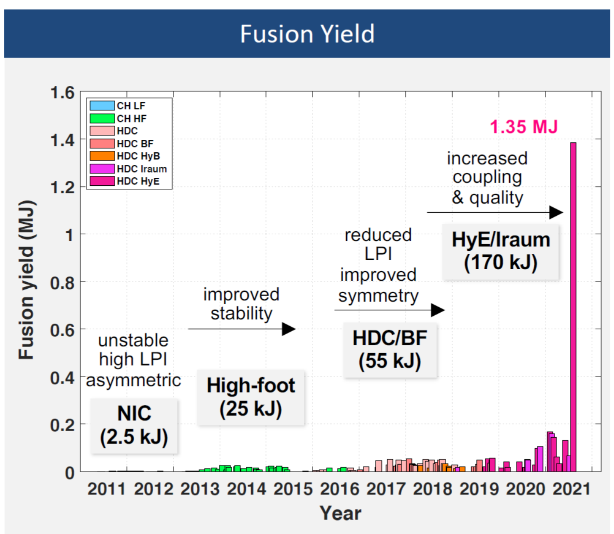




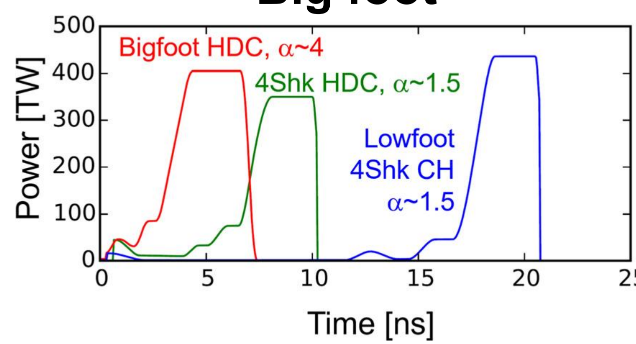
- **Hybrid-E**
- **I-raum**

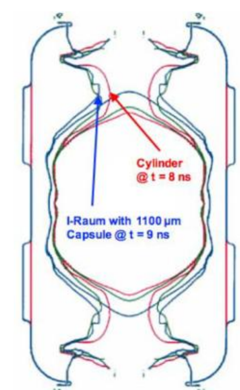






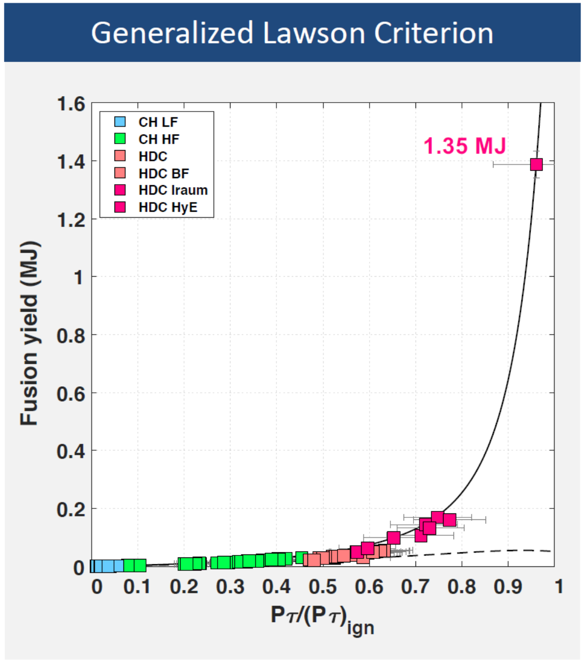
- **Big foot**

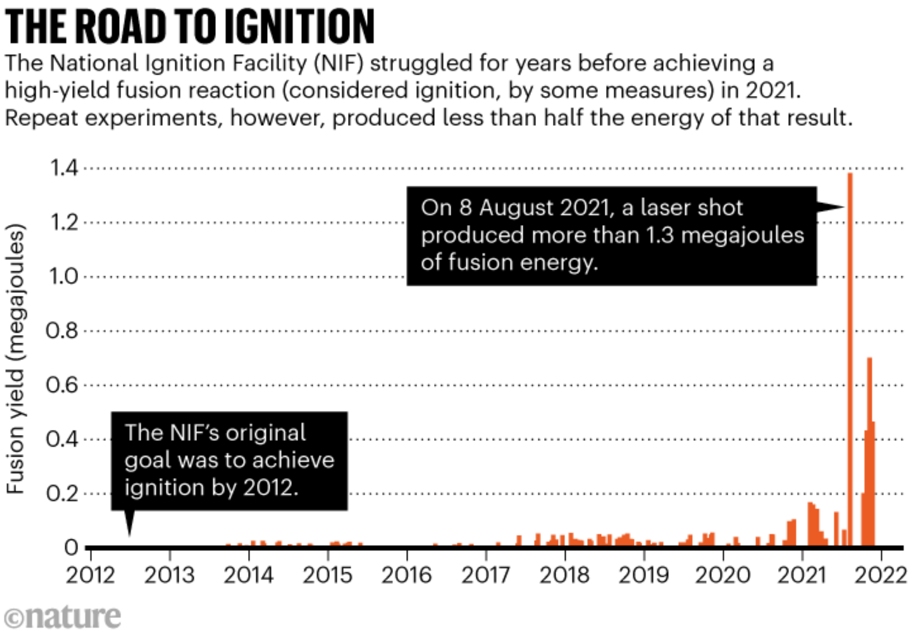




T. Ma, ARPA-E workshop, April 26, 2022
Science 370, p1019, 2020
D. T. Casey, etc., Phys. Plasmas, 25, 056308 (2018)
A. L. Kritcher, etc., Phys. Plasmas, 28, 072706 (2021)
H. F. Robey, etc., Phys. Plasmas, 25, 012711 (2018)

National Ignition Facility (NIF) achieved a yield of more than 1.3 MJ from ~1.9 MJ of laser energy in 2021 (Q~0.7)

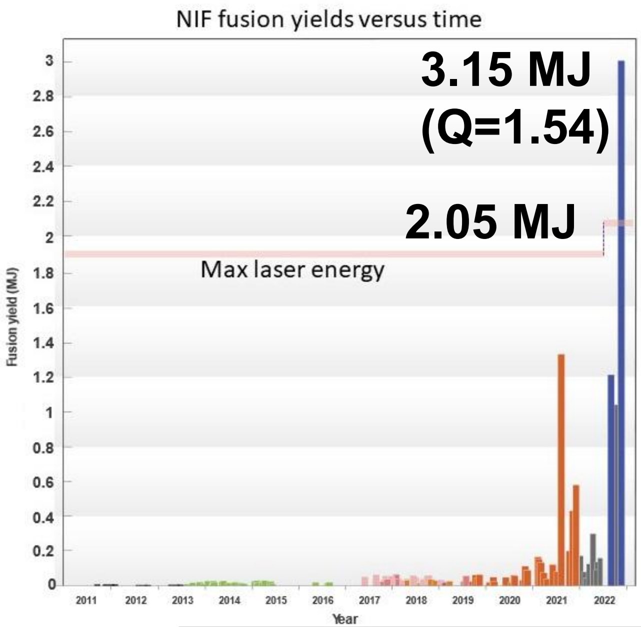


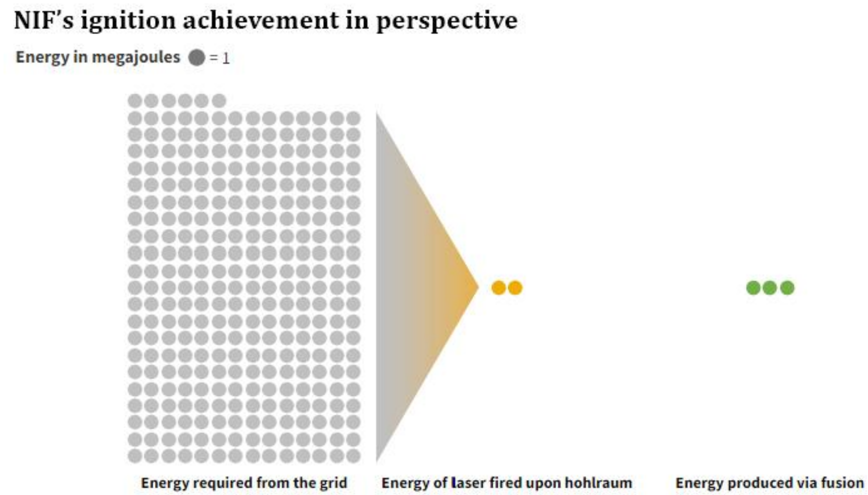


- **National Ignition Facility (NIF) achieved a yield of more than 1.3 MJ (Q~0.7). This advancement puts researchers at the threshold of fusion ignition.**
- **Laser-fusion facility heads back to the drawing board.**

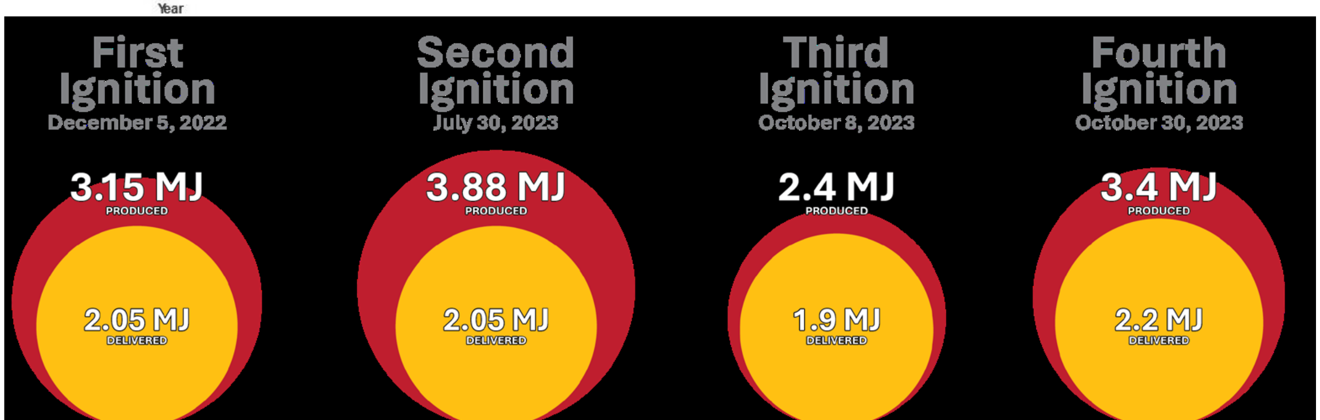
T. Ma, ARPA-E workshop, April 26, 2022
J. Tollefson, Nature (News) 608, 20 (2022)

“Ignition” (target yield larger than one) was achieved in NIF on 2022/12/5





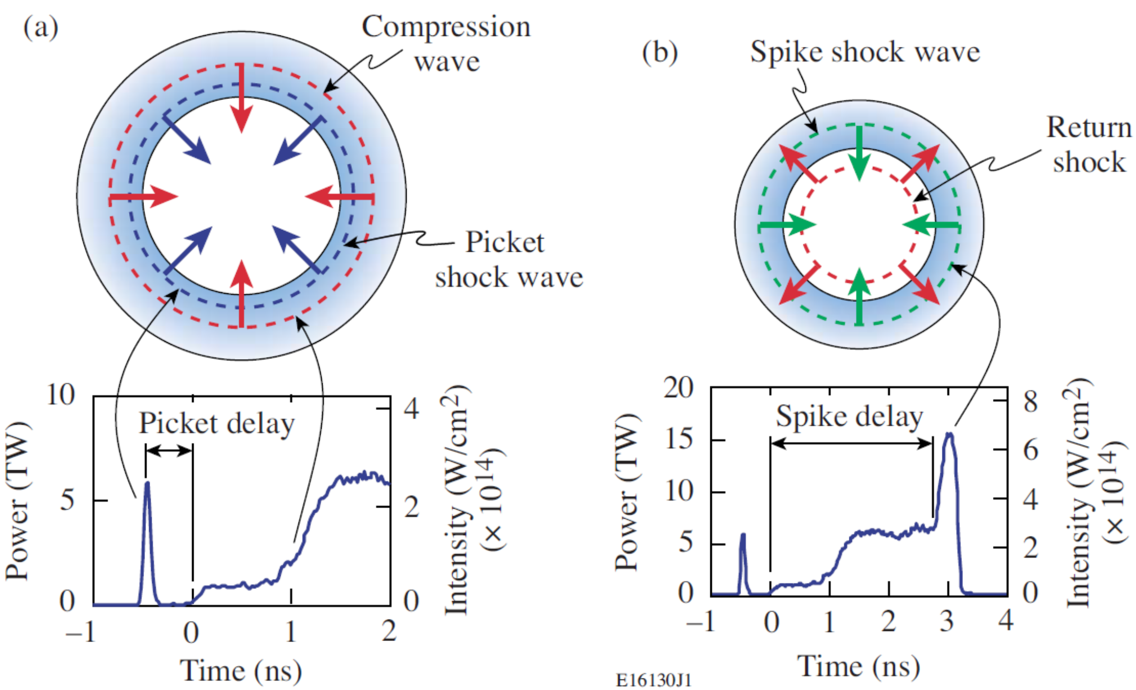
300 MJ 2.05 MJ 3.15 MJ



https://physicstoday.scitation.org/do/10.1063/PT.6.2.20221213a/full/
The age of ignition: anniversary edition, LLNL-BR-857901

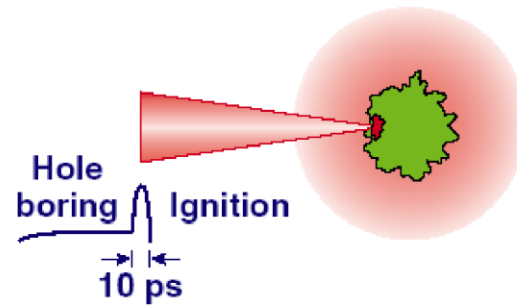
External “spark” can be used for ignition

- **Shock ignition**

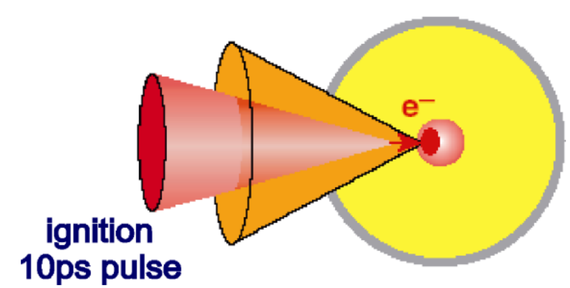


- **Fast ignition**

a) channeling FI concept



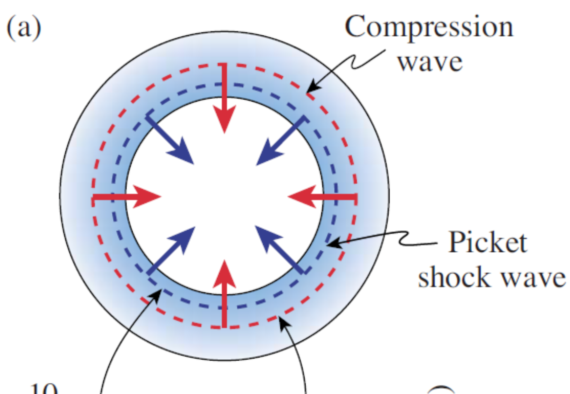
b) cone-in-shell FI concept

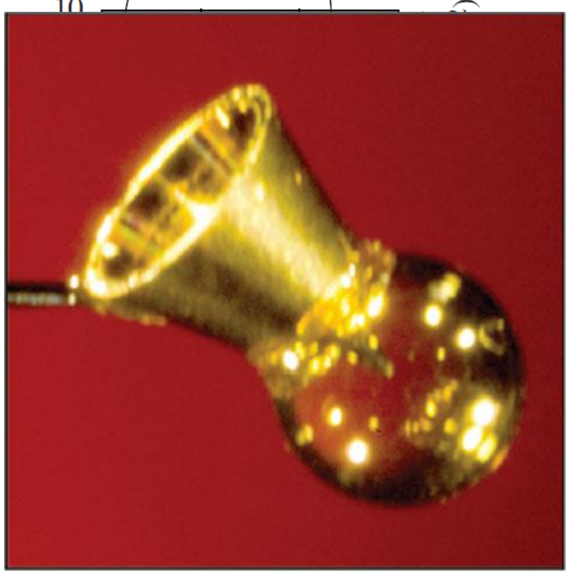


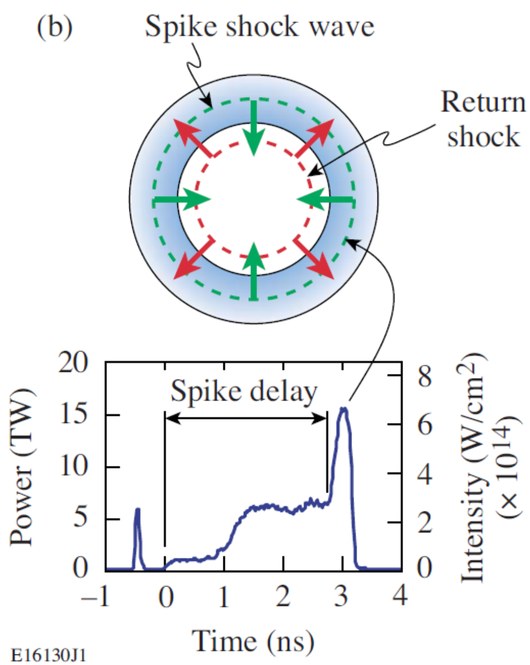
J. Badziak, Bull.Polish Acad. Sci. Tech. Sci.Phys. Plasmas 15, 056306 (2008)

External “spark” can be used for ignition

- **Shock ignition**

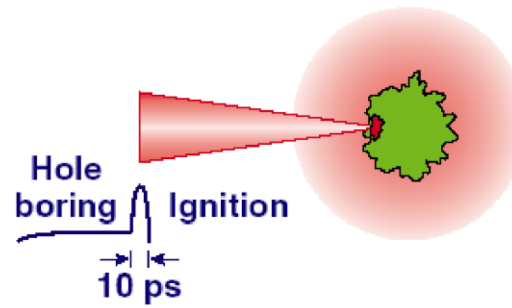




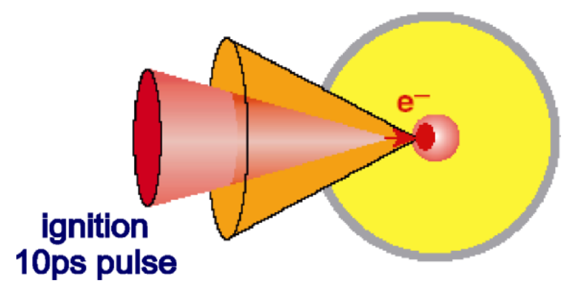


- **Fast ignition**

a) channeling FI concept



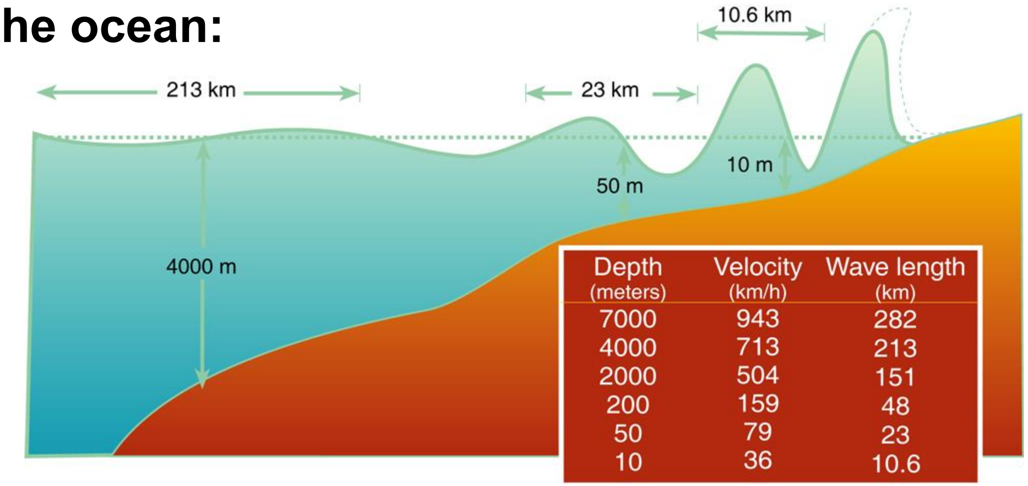
b) cone-in-shell FI concept



J. Badziak, Bull.Polish Acad. Sci. Tech. Sci.Phys. Plasmas 15, 056306 (2008)
T. Ditmire, etc., J. Fusion Energy **42**, 27 (2023)

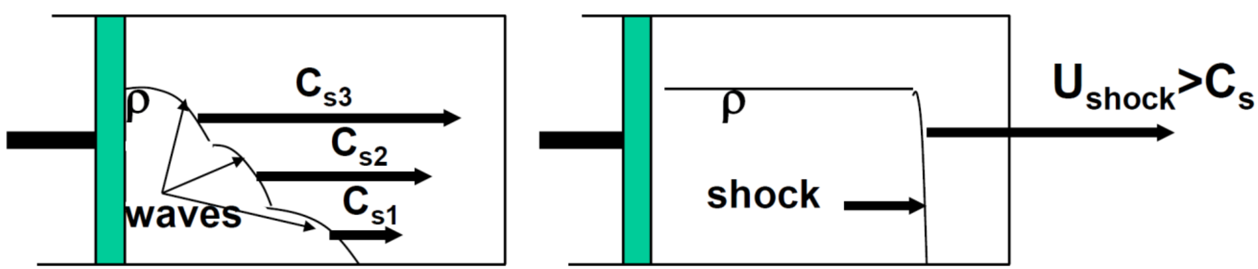
A shock is formed due to the increasing sound speed of a compressed gas/plasma

- **Wave in the ocean:**



Depth (meters)	Velocity (km/h)	Wave length (km)
7000	943	282
4000	713	213
2000	504	151
200	159	48
50	79	23
10	36	10.6

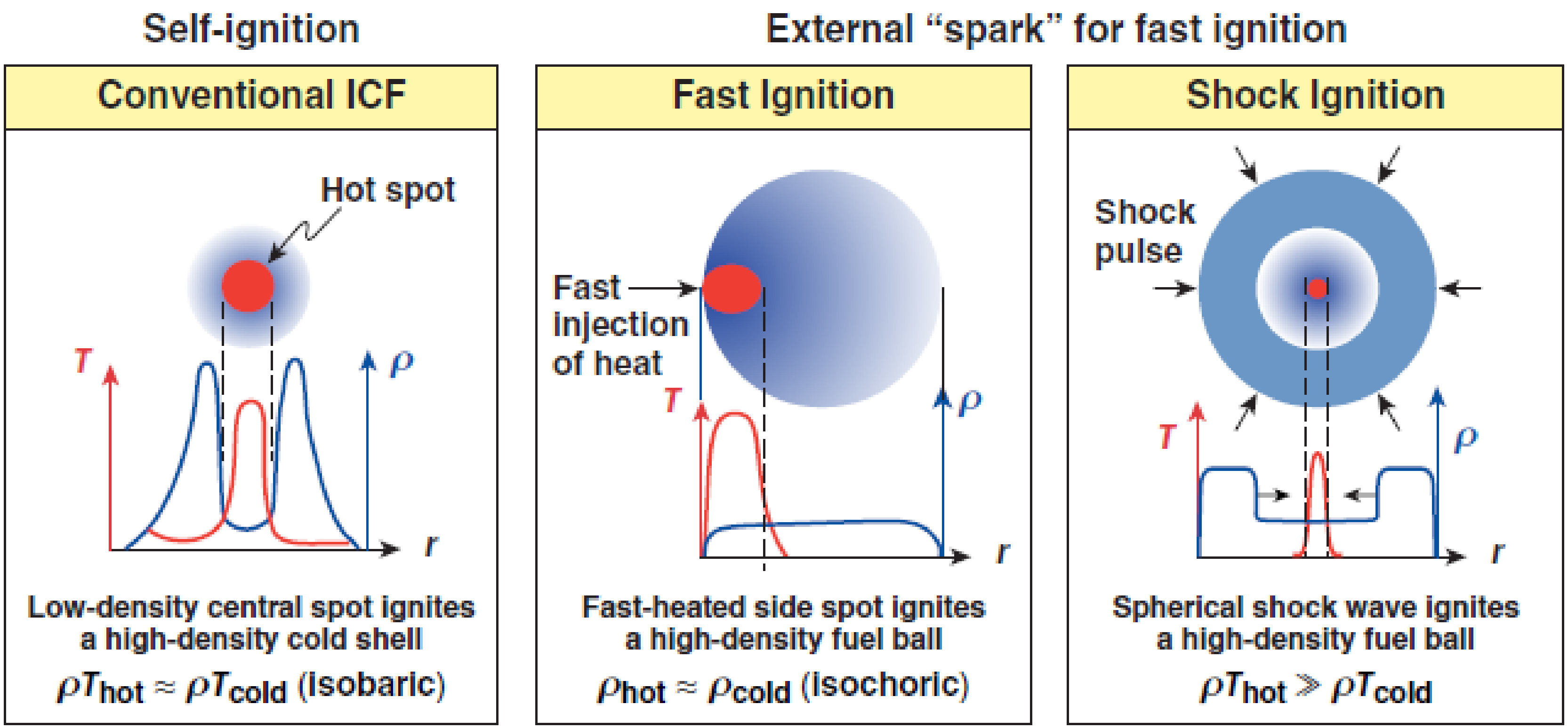
- **Acoustic/compression wave driven by a piston:**



$$C_s \sim \sqrt{\frac{p}{\rho}} \sim \sqrt{\frac{\alpha\rho^{5/3}}{\rho}} \sim \sqrt{\alpha}\rho^{1/3}$$

http://neamtic.ioc-unesco.org/tsunami-info/the-cause-of-tsunamis
*R. Betti, HEDSA HEDP Summer School, 2015

Ignition can happen by itself or being triggered externally



P. B. Radha, Fusion Energy Conference,2018