

Introduction to Nuclear Fusion as An Energy Source



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Lecture 10

2025 spring semester

Tuesday 9:00-12:00

Materials:

<https://capst.ncku.edu.tw/PGS/index.php/teaching/>

Online courses:

<https://nckucc.webex.com/nckucc/j.php?MTID=mf1a33a5dab5eb71de9da4380ae888592>

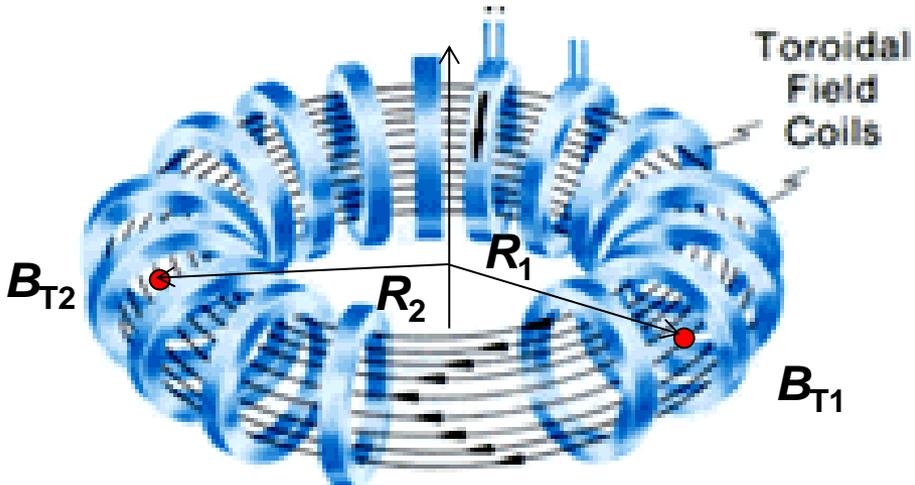
With poloidal fields, charged particles see nonuniform toroidal magnetic field



- W/o poloidal field

$$R_1 = R_2$$

$$B_{T1} = B_{T2}$$



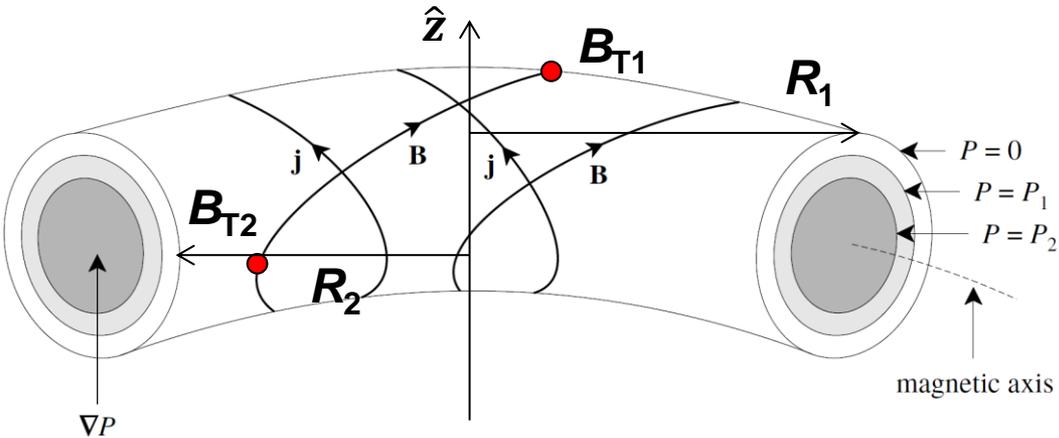
$$B_T \propto \frac{1}{R}$$

$$B_T \gg B_p$$

- W/ poloidal field

$$R_1 > R_2$$

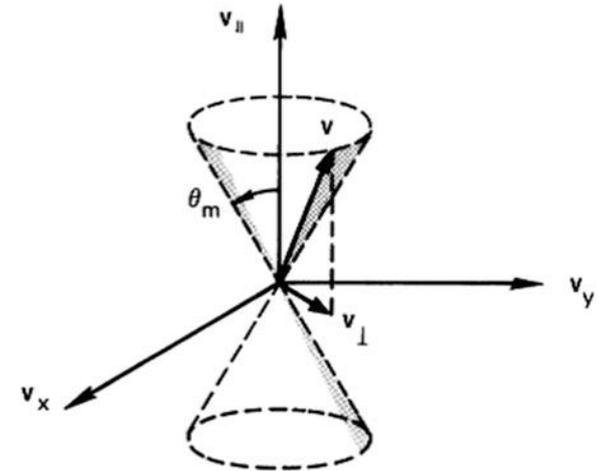
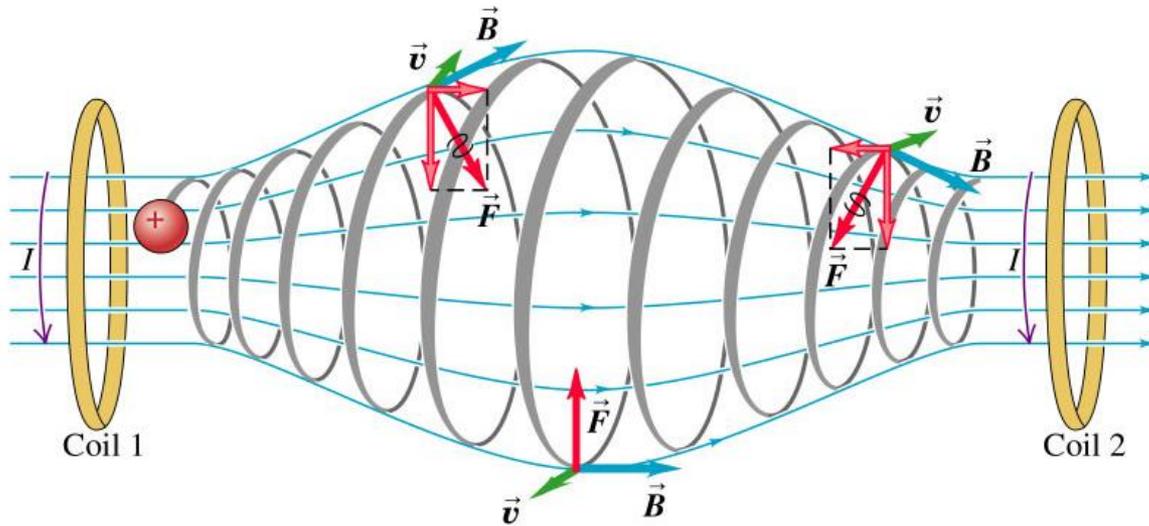
$$B_{T1} < B_{T2}$$



Charged particles can be partially confined by a magnetic mirror machine



- Charged particles with small v_{\parallel} eventually stop and are reflected while those with large v_{\parallel} escape.



$$\frac{1}{2}mv^2 = \frac{1}{2}mv_{\parallel}^2 + \frac{1}{2}mv_{\perp}^2 \quad \text{Invariant: } \mu \equiv \frac{1}{2} \frac{mv_{\perp}^2}{B}$$

$$v_{\perp}^{\prime 2} = v_{\perp 0}^2 + v_{\parallel 0}^2 \equiv v_0^2$$

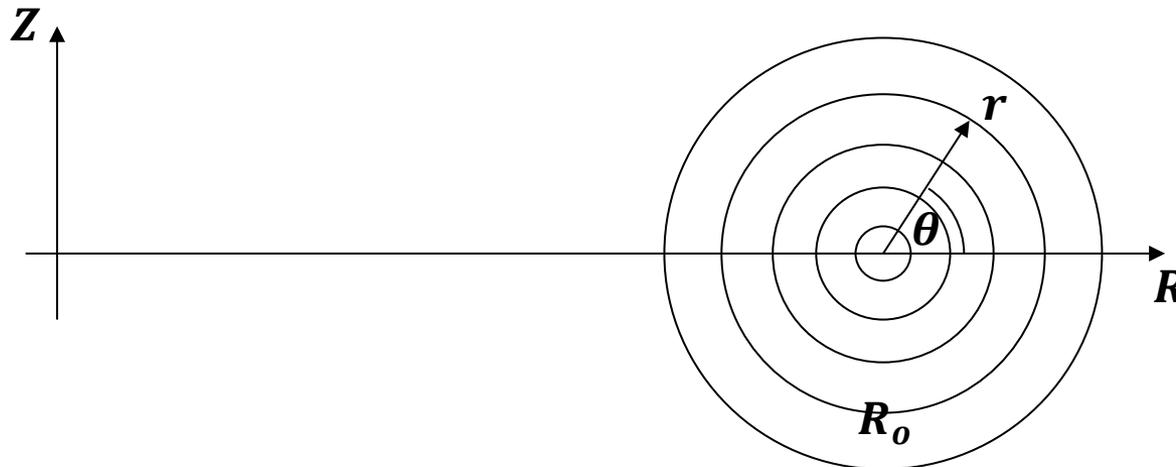
$$\frac{B_0}{B'} = \frac{v_{\perp 0}^2}{v_{\perp}^{\prime 2}} = \frac{v_{\perp 0}^2}{v_0^2} \equiv \sin^2 \theta$$

$$\frac{B_0}{B_m} \equiv \frac{1}{R_m} = \sin^2 \theta_m$$

- Large v_{\parallel} may occur from collisions between particles.

• Those confined charged particle are eventually lost due to collisions.

Parallel velocity changes when particles follow field the field line



$$R \gg r$$

$$B_T \gg B_p$$

$$R = R_0 + r \cos \theta = R_0 (1 + \epsilon \cos \theta)$$

Inverse aspect ratio:

$$\epsilon \equiv \frac{r}{R_0}$$

$$B \simeq \frac{B_0}{1 + \epsilon \cos \theta} \simeq B_0 (1 - \epsilon \cos \theta)$$

Invariant: $\mu \equiv \frac{1}{2} \frac{m v_{\perp}^2}{B} \quad \frac{v_{\perp}^2}{B_0 (1 - \epsilon \cos \theta)} = \frac{v_{\perp 0}^2}{B_0 (1 - \epsilon)} \quad v_{\perp}^2 = \frac{v_{\perp 0}^2 (1 - \epsilon \cos \theta)}{1 - \epsilon}$

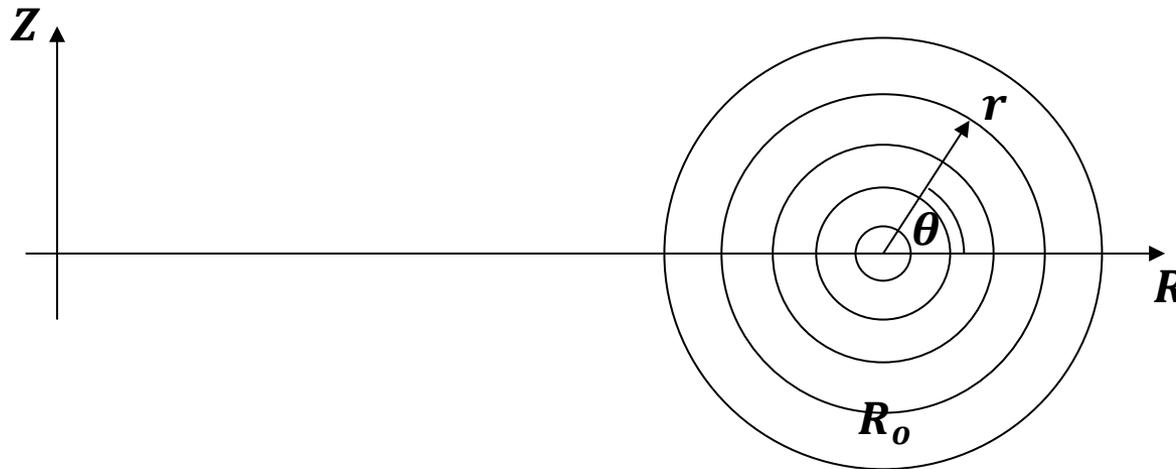
$$v^2 = v_{\perp}^2 + v_{\parallel}^2 = v_{\perp 0}^2 + v_{\parallel 0}^2$$

$$v_{\parallel}^2 = v^2 \left(1 - \frac{v_{\perp 0}^2}{v^2} \frac{(1 - \epsilon \cos \theta)}{1 - \epsilon} \right)$$

$$v_{\parallel}^2 = v^2 \left(1 - \frac{v_{\perp}^2}{v^2} \right)$$

$$\approx v^2 \left(1 - \frac{v_{\perp 0}^2}{v^2} \left(1 + 2\epsilon \sin^2 \left(\frac{\theta}{2} \right) \right) \right)$$

Particles may be trapped by nonuniform magnetic field



$$R \gg r$$
$$B_T \gg B_p$$

$$\epsilon \equiv \frac{r}{R_0}$$

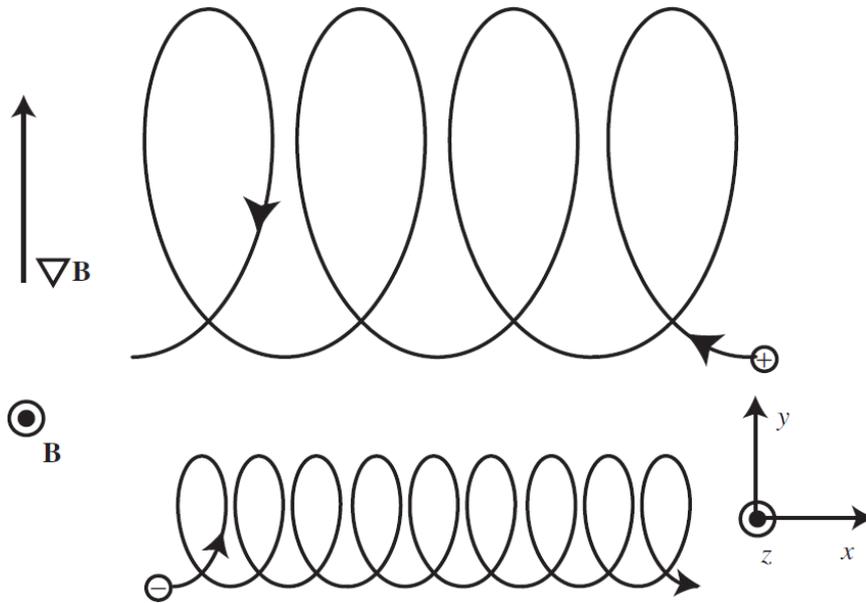
$$v_{\parallel}^2 \approx v^2 \left(1 - \frac{v_{\perp 0}^2}{v^2} \left(1 + 2\epsilon \sin^2 \left(\frac{\theta}{2} \right) \right) \right)$$

- For $v_{\parallel}^2 \geq 0$, particles are passing.
- For $v_{\parallel}^2 \leq 0$, particles are trapped.

Charge particles drift across magnetic field lines when the magnetic field is not uniform or curved

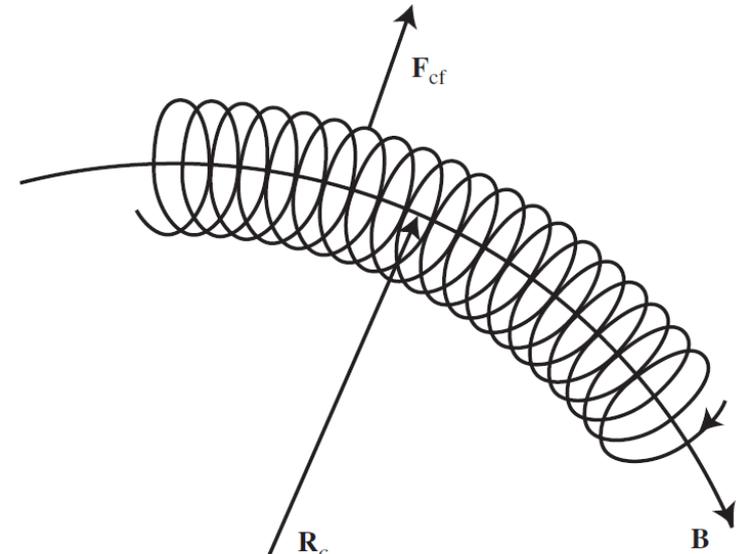


- Gradient-B drift



$$\vec{v}_{\nabla} = \frac{mv_{\perp}^2}{2q} \frac{\vec{B} \times \nabla B}{B^3}$$

- Curvature drift



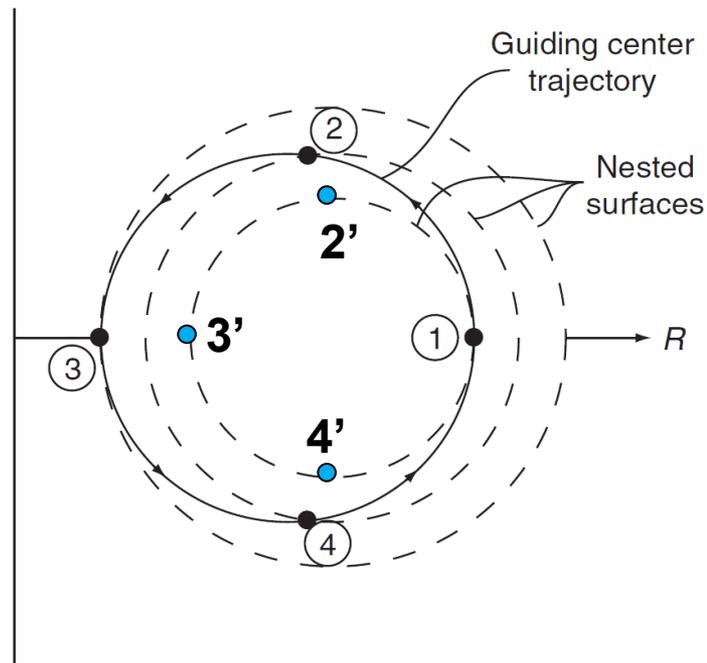
$$\vec{v}_R = \frac{mv_{\parallel}^2}{2q} \frac{\vec{R}_c \times \vec{B}}{R_c B^2}$$

$$\vec{v}_{\text{total}} = \vec{v}_R + \vec{v}_{\nabla} = \frac{\vec{B} \times \nabla B}{\omega_c B^2} \left(v_{\parallel}^2 + \frac{1}{2} v_{\perp}^2 \right) = \frac{m}{q} \frac{\vec{R}_c \times \vec{B}}{R_c^2 B^2} \left(v_{\parallel}^2 + \frac{1}{2} v_{\perp}^2 \right)$$

For passing particles, they drift back to the original position with a “semicircle” orbit



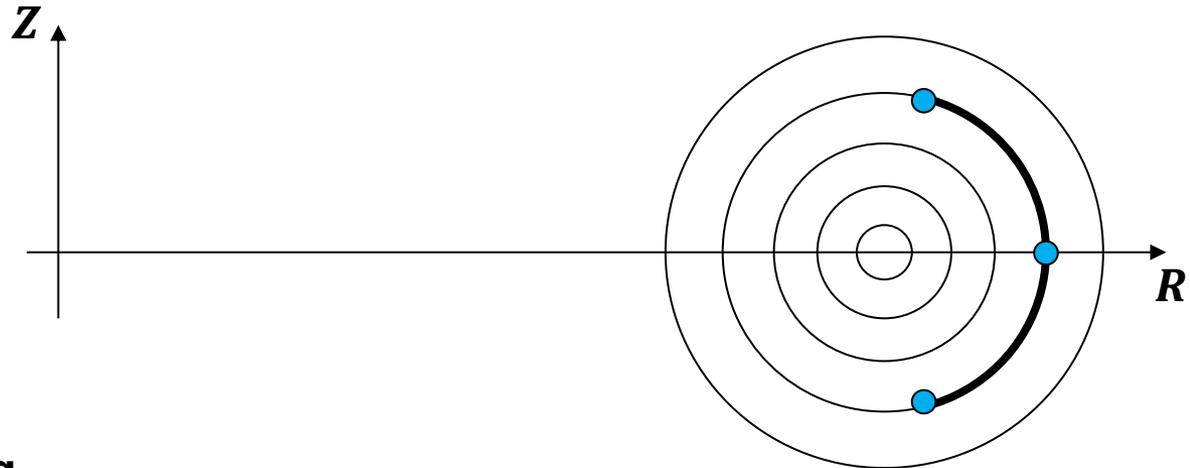
$$\vec{v}_{\text{total}} = \vec{v}_R + \vec{v}_\nabla = \frac{\vec{B} \times \nabla B}{\omega_c B^2} \left(v_{\parallel}^2 + \frac{1}{2} v_{\perp}^2 \right) = \frac{m}{q} \frac{\vec{R}_c \times \vec{B}}{R_c^2 B^2} \left(v_{\parallel}^2 + \frac{1}{2} v_{\perp}^2 \right)$$



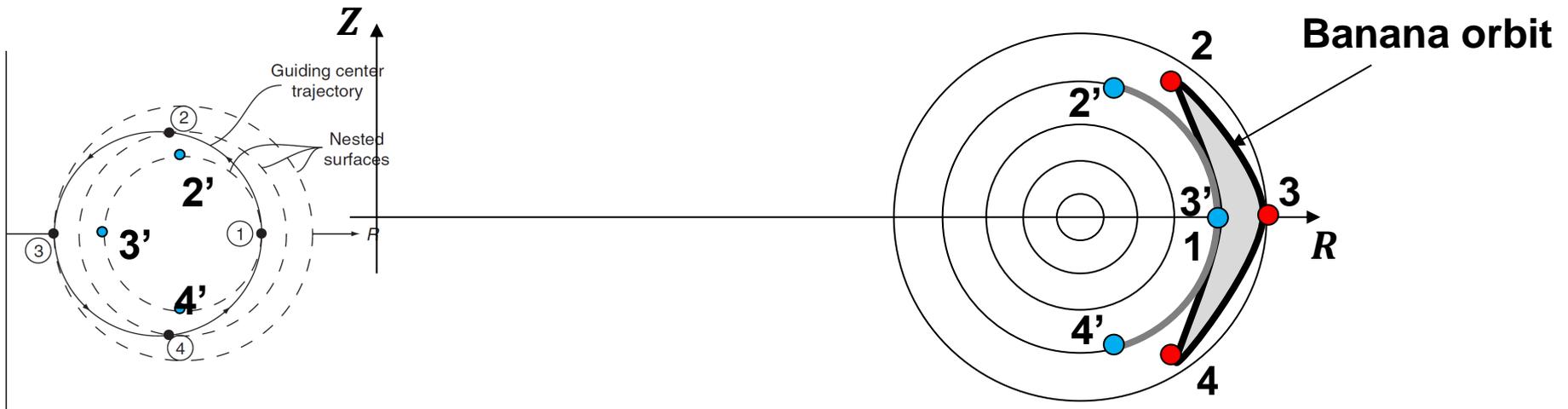
For trapped particles, they drift back to the original position with a banana orbit



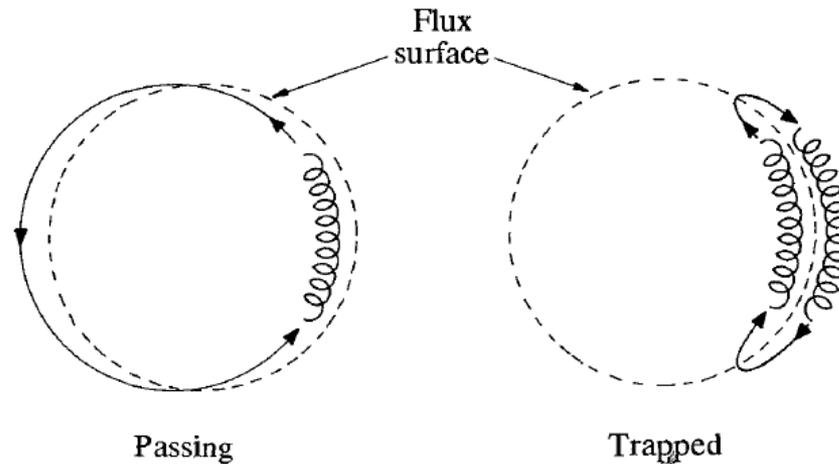
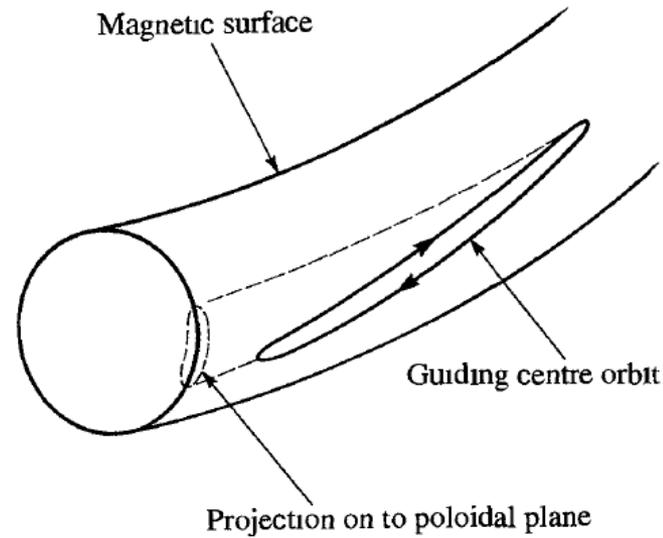
- W/o drifting



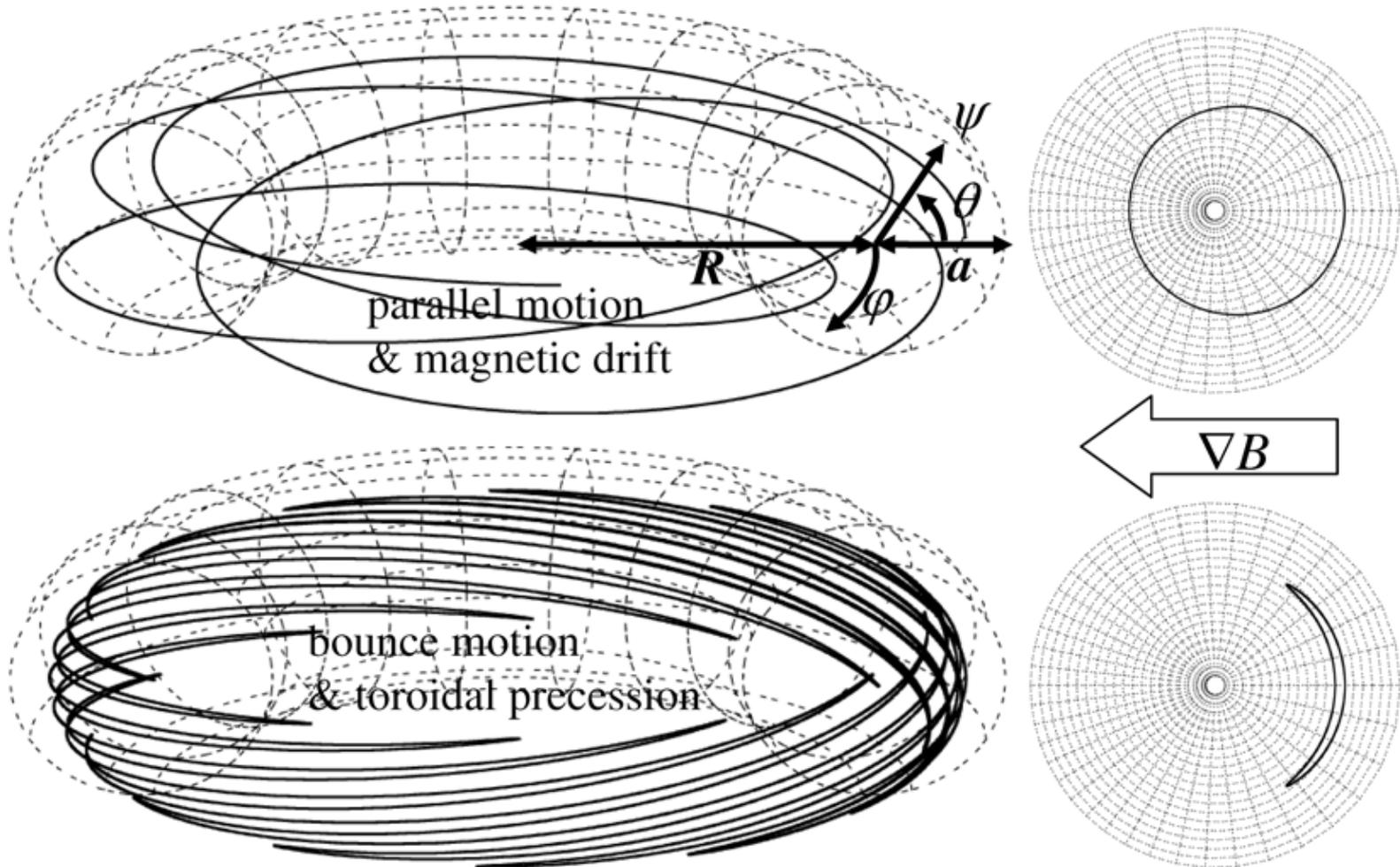
- W/ drifting



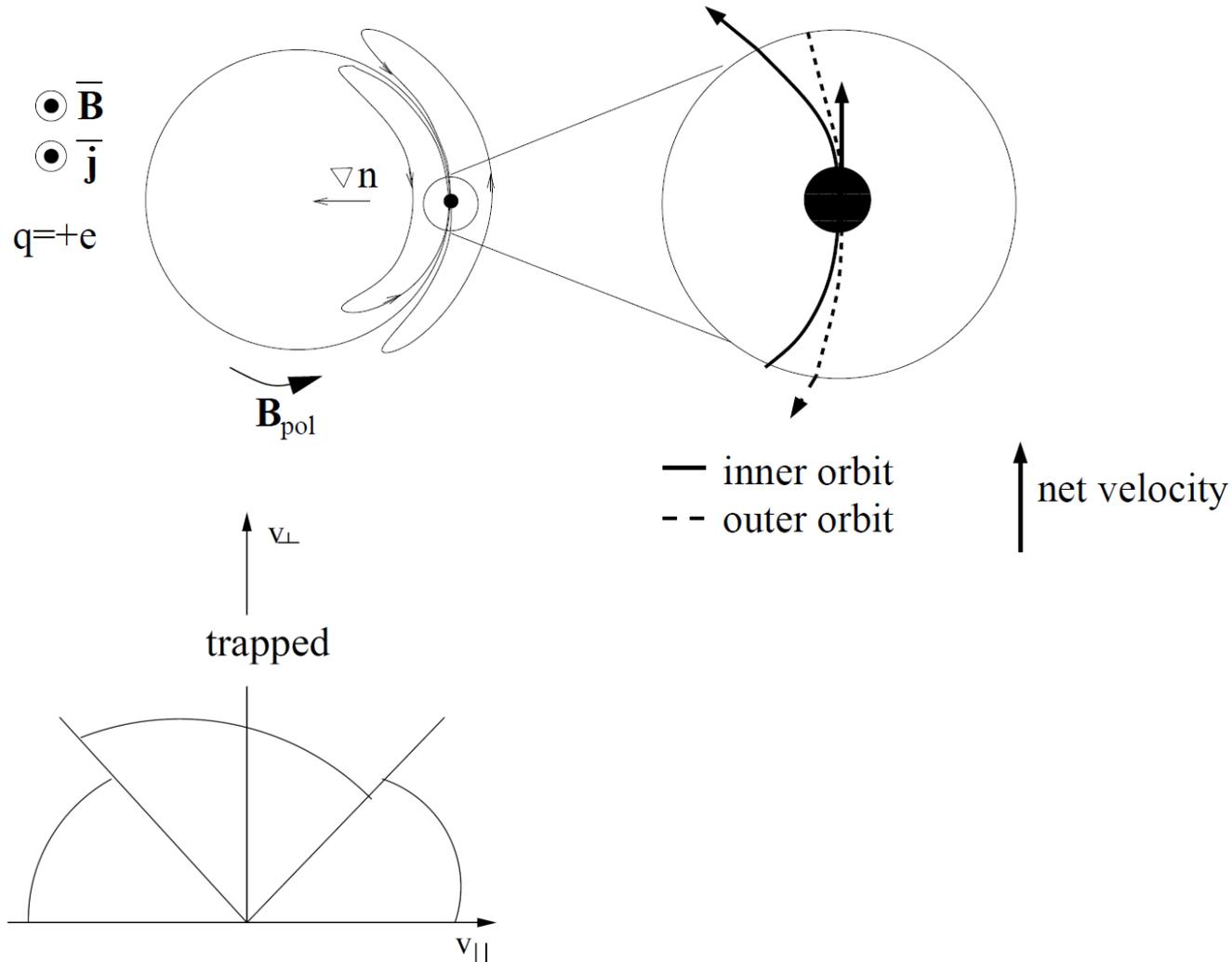
Trajectories of charged particles



The trajectories of charged particles follow the toroidal field lines



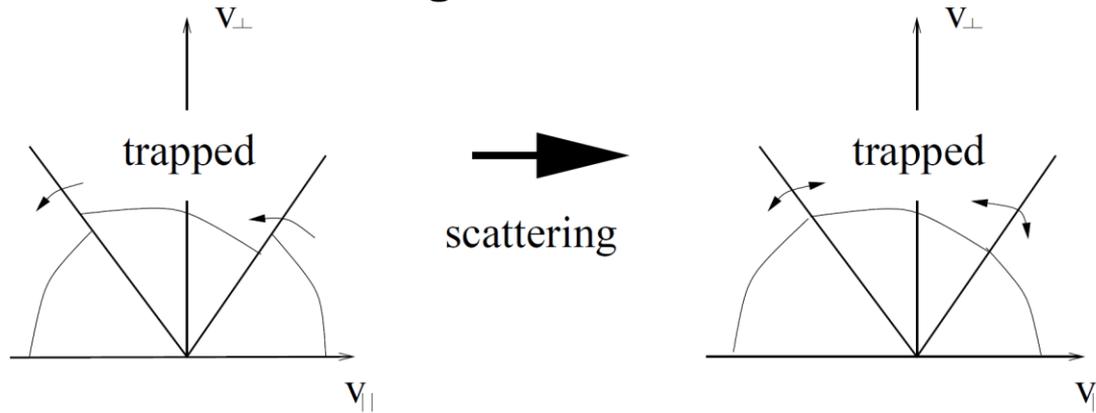
A banana current is generated when there is a pressure gradient in the plasma



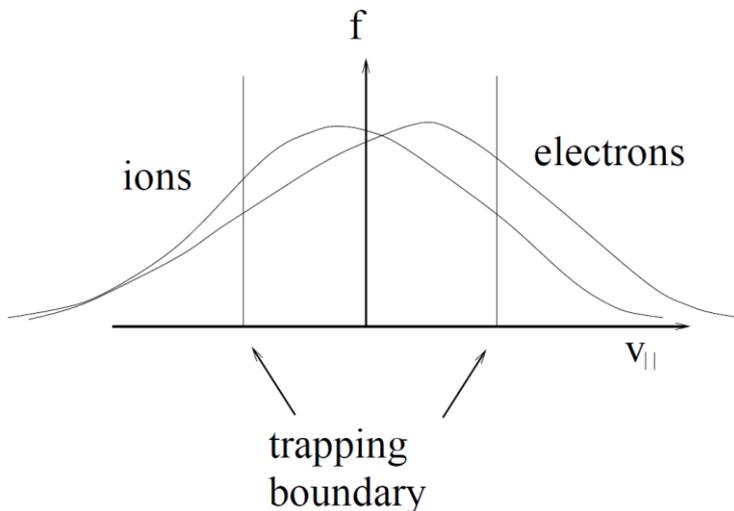
Bootstrap current is generated when passing particles are scattered by the trapped particles



- Scattering smooths the velocity distribution and shifts it in the parallel direction, i.e., a current is generated. It is called the bootstrap current.

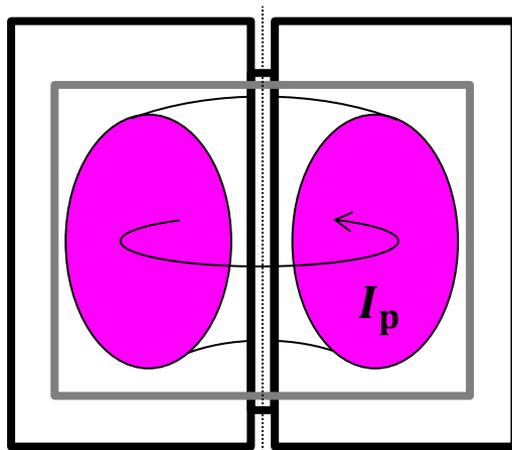


$$j = -enu_{||e} + enu_{||i} = 4\epsilon^{3/2} \frac{1}{B_p} T \frac{dn}{dr}$$

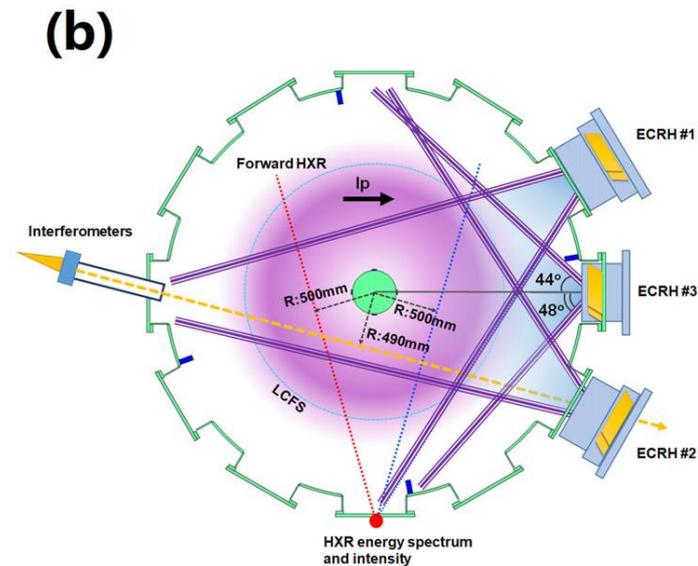
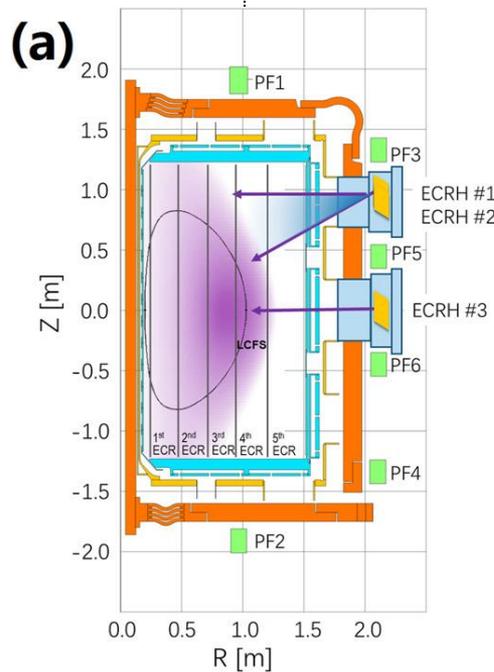


- The bootstrap current is vital for steady-state operation.

Momentum exchange may be needed to drive plasma current



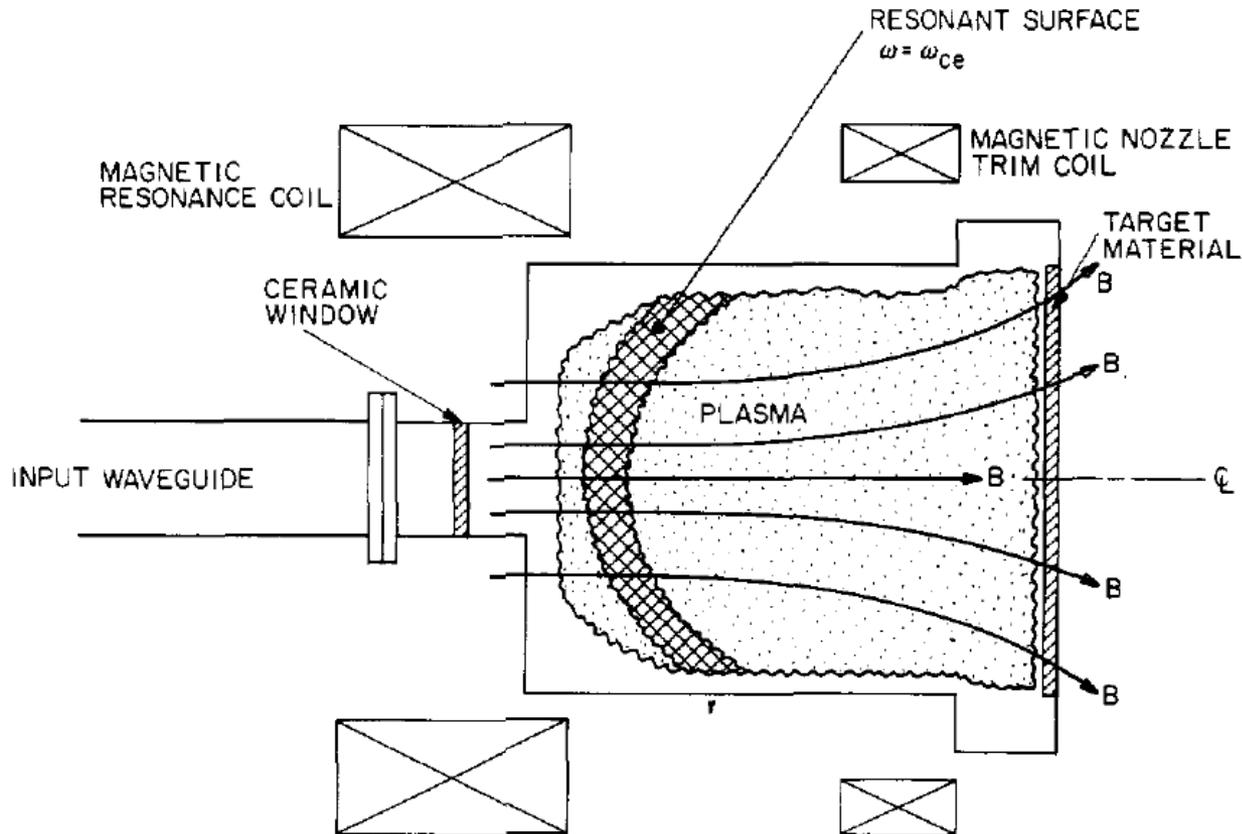
$$\vec{j}_p = \Sigma qn \vec{v} = -en_e \vec{v}_e + en_i \vec{v}_i$$



Strong absorption occurs when the frequency matches the electron cyclotron frequency



- Electron cyclotron resonance (ECR) plasma reactor



Electron cyclotron frequency depends on magnetic field only



$$m_e \frac{d\vec{v}}{dt} = -\frac{e}{c} \vec{v} \times \vec{B}$$

- Assuming $\vec{B} = B\hat{z}$ and the electron oscillates in x-y plane

$$m_e \dot{v}_x = -\frac{e}{c} B v_y \quad m_e \dot{v}_z = 0$$

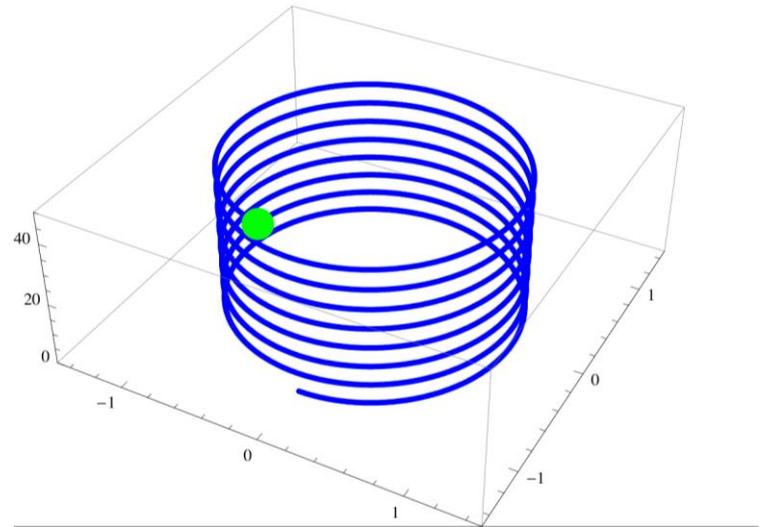
$$m_e \dot{v}_y = \frac{e}{c} B v_x$$

$$\ddot{v}_x = -\frac{eB}{m_e c} \dot{v}_y = -\left(\frac{eB}{m_e c}\right)^2 v_x$$

$$\ddot{v}_y = -\frac{eB}{m_e c} \dot{v}_x = -\left(\frac{eB}{m_e c}\right)^2 v_y$$

- Therefore

$$\omega_{ce} = \frac{eB}{m_e c}$$



Electrons keep getting accelerated when a electric field rotates in electron's gyrofrequency



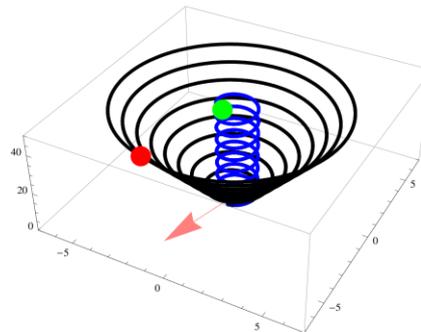
$$m_e \frac{d\vec{v}}{dt} = -\frac{e}{c} \vec{v} \times \vec{B} - e \vec{E} \quad \vec{B} = B_0 \hat{z} \quad \vec{E} = E_0 [\hat{x} \cos(\omega t) + \hat{y} \sin(\omega t)]$$

$$m_e \dot{v}_x = -\frac{e}{c} B v_y + E_0 \cos(\omega t) \quad m_e \dot{v}_y = \frac{e}{c} B v_x + E_0 \sin(\omega t) \quad m_e \dot{v}_z = 0$$

$$\ddot{v}_x = -\frac{eB}{m_e c} \dot{v}_y - \frac{E_0}{m_e} \omega \cos(\omega t) = -\omega_{ce}^2 v_x - \frac{E_0}{m_e} (\omega_{ce} + \omega) \cos(\omega t)$$

$$\ddot{v}_y = -\frac{eB}{m_e c} \dot{v}_x + \frac{E_0}{m_e} \omega \sin(\omega t) = -\omega_{ce}^2 v_y + \frac{E_0}{m_e} (\omega_{ce} + \omega) \sin(\omega t)$$

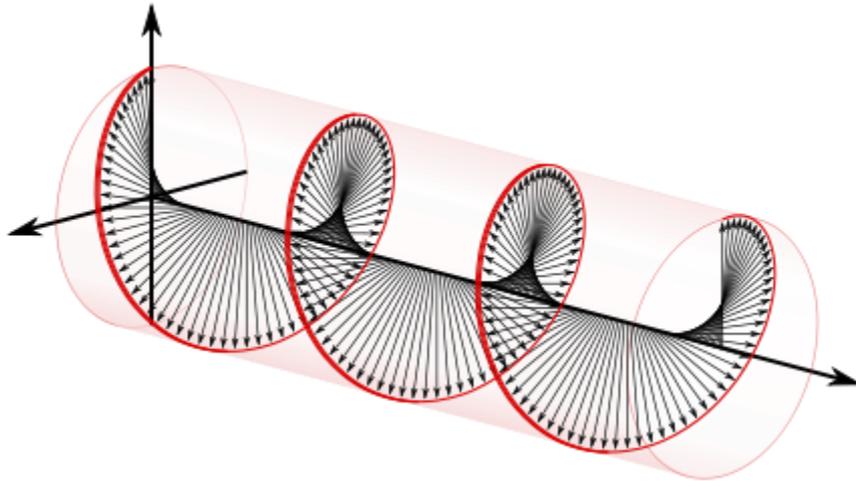
$$\omega_{ce} = \frac{eB}{m_e c}$$



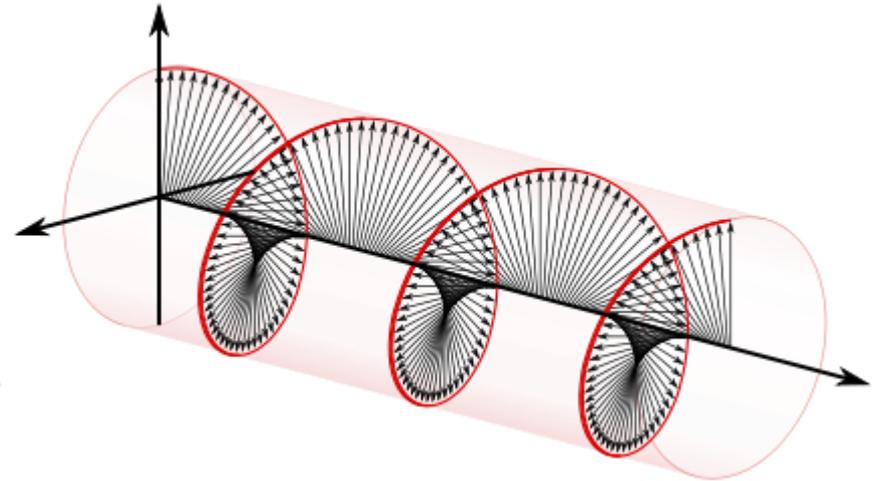
Electric field in a circular polarized electromagnetic wave keeps rotating as the wave propagates



- Right-handed polarization



- Left-handed polarization



Only right-handed polarization can resonance with electron's gyromotion

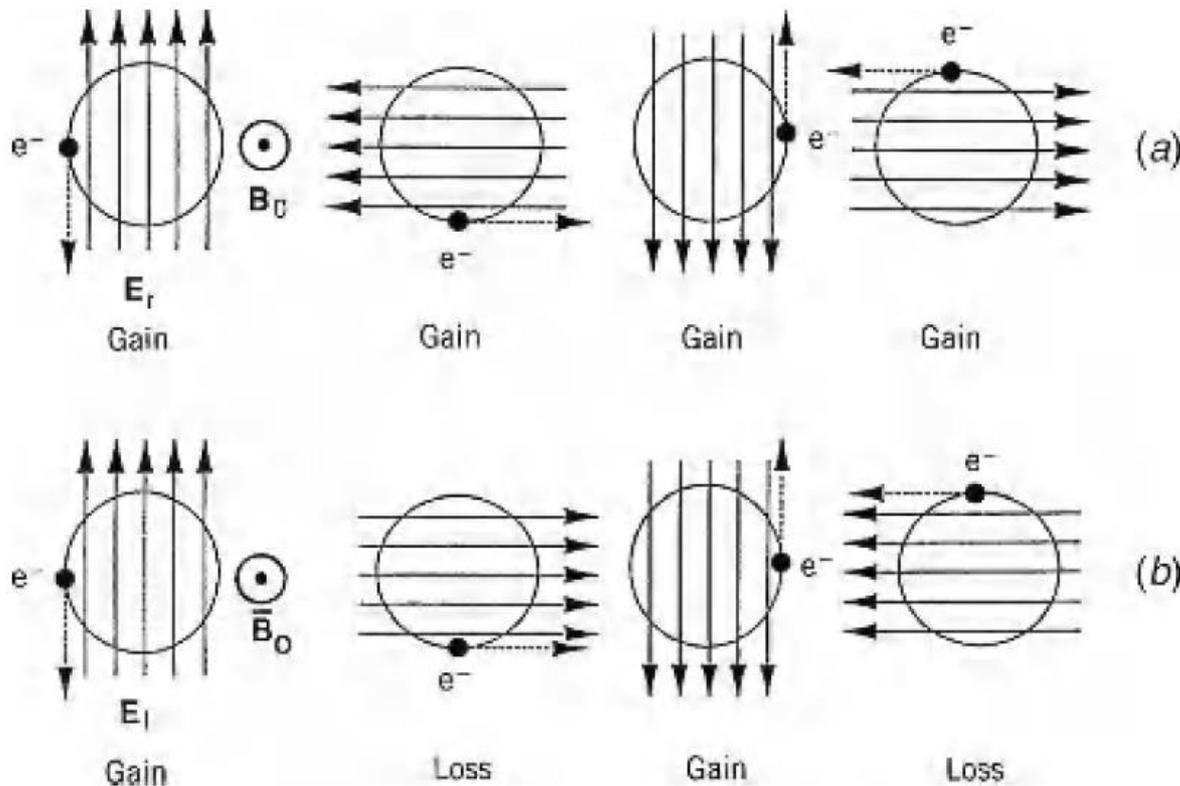
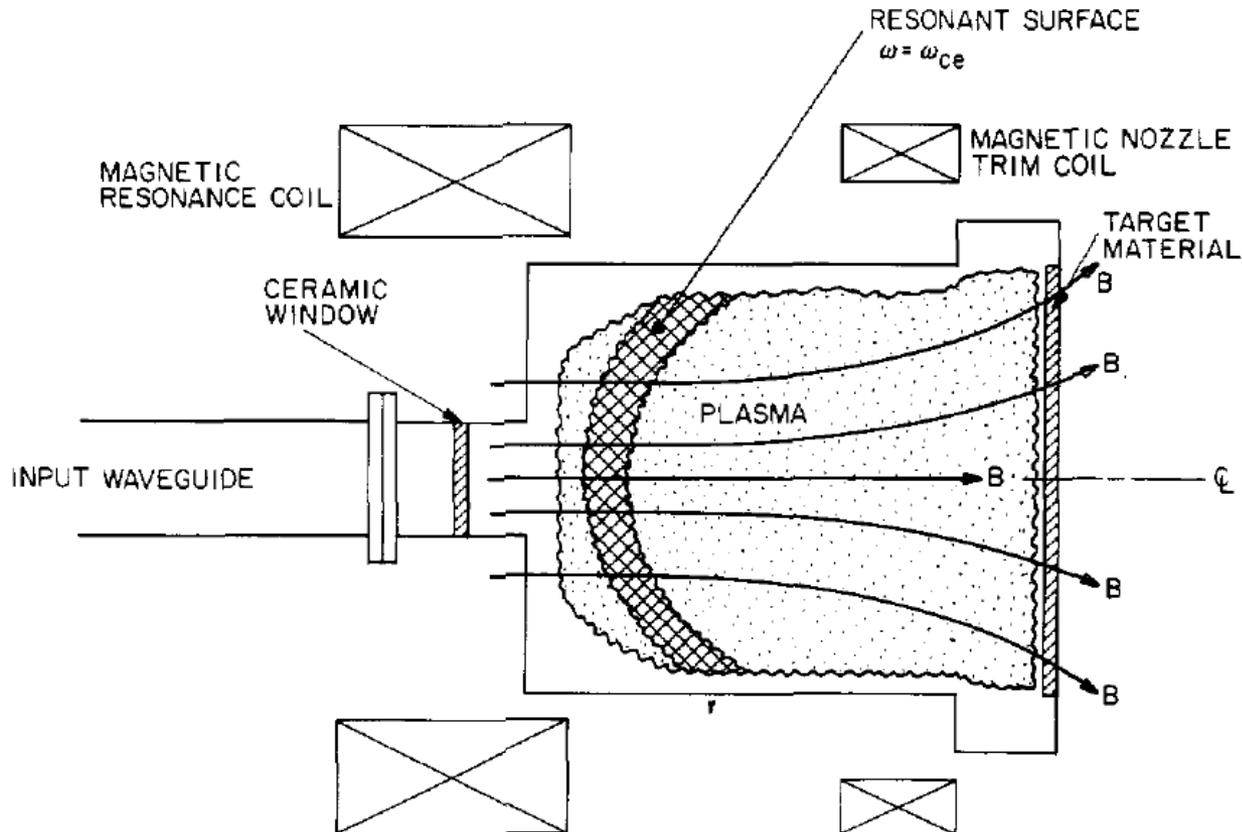


FIGURE 13.5. Basic principle of ECR heating: (a) continuous energy gain for right-hand polarization; (b) oscillating energy for left-hand polarization (after Lieberman and Gottscho, 1994).

Strong absorption occurs when the frequency matches the electron cyclotron frequency



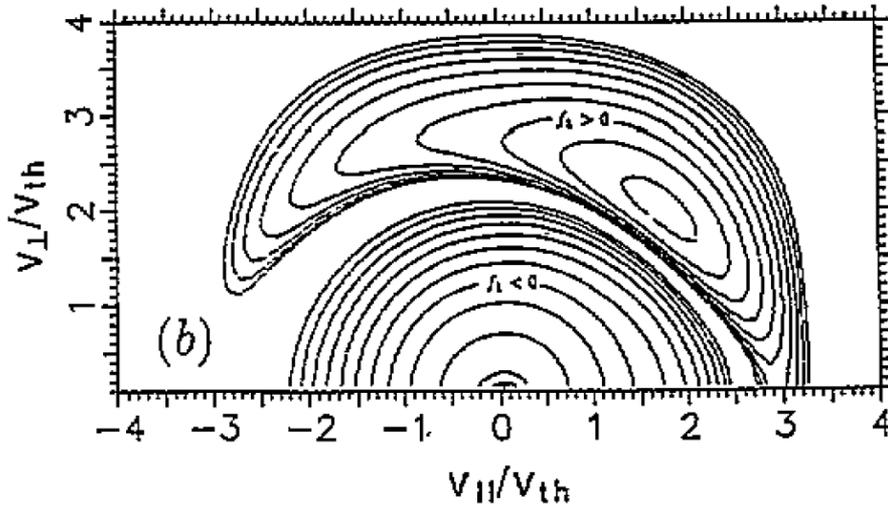
- Electron cyclotron resonance (ECR) plasma reactor



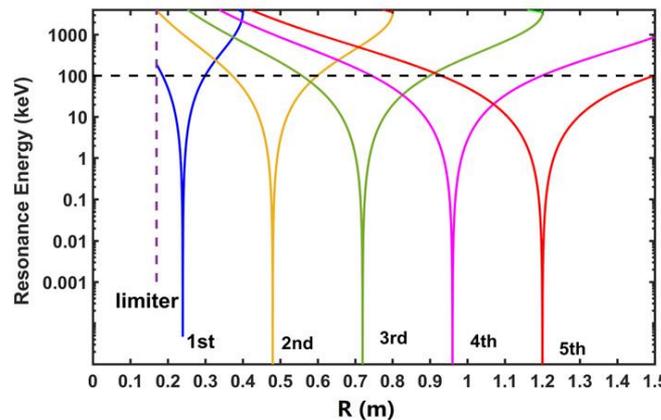
The collisional re-distribution of the ECRH-driven anisotropy in E_{\perp} causes some parallel momentum to flow from e^{-} to ions



- Coulomb collisions are more efficient at lower energies.

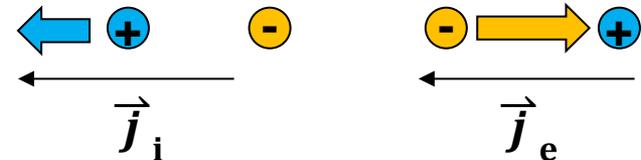


- Electron cyclotron current drive:



Velocity: $v_2 > v_1$

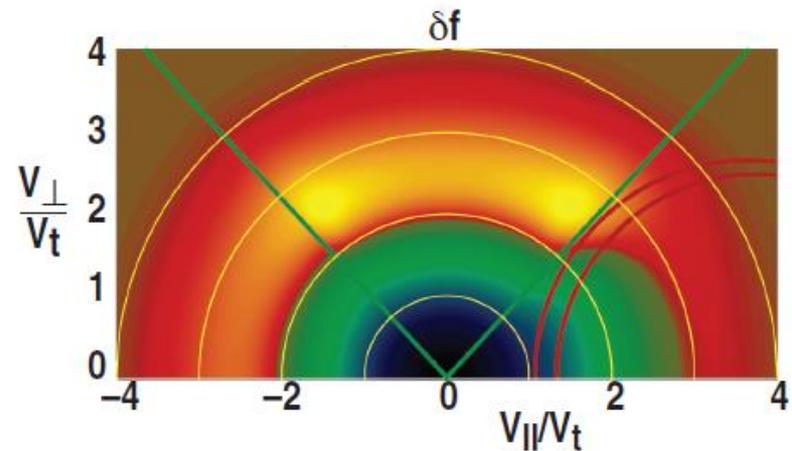
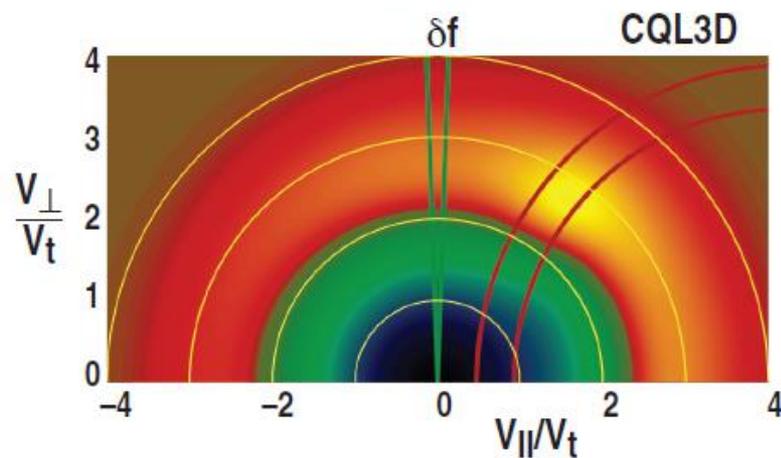
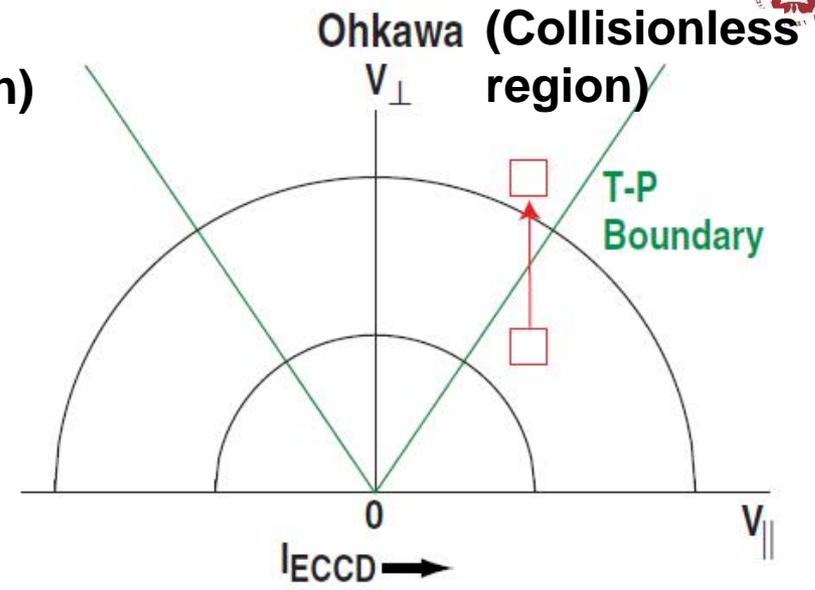
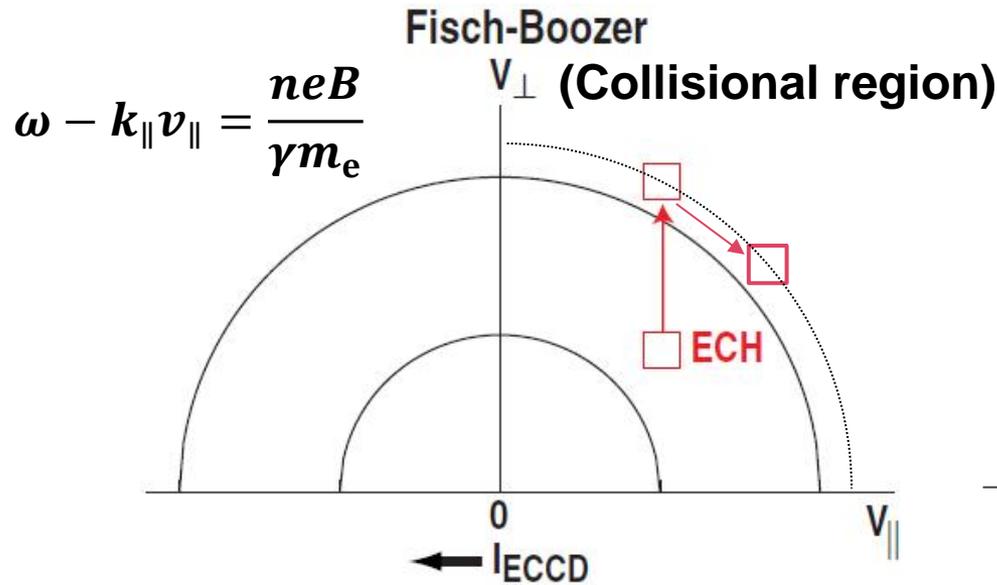
Collisions: $v_2 < v_1$



$$\vec{j}_p = -en_e \vec{v}_e + en_i \vec{v}_i$$

$$\vec{P} = n_e m_e \vec{v}_e + n_i m_i \vec{v}_i \approx 0$$

Passing electrons can be trapped if the v_{\perp} is increased by heating



Comparison of Fisch-Boozer Mechanism and Ohkawa Mechanism

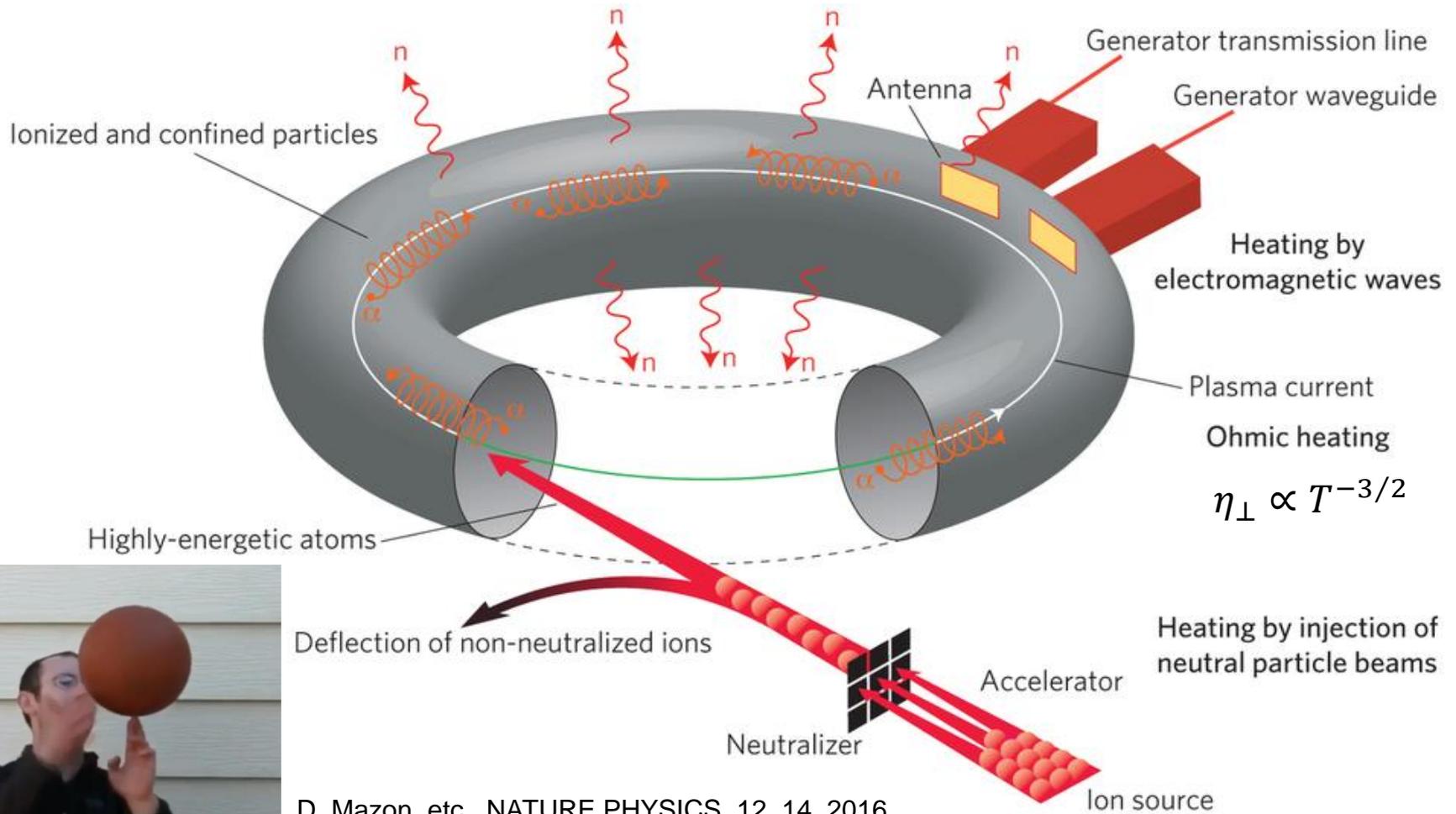


Aspect	Fisch–Boozer Mechanism ^[1]	Ohkawa Mechanism ^[2]
Physical Process	Asymmetric heating of passing electrons with subsequent collisional momentum transfer	Selective de-trapping of barely trapped electrons into passing orbits (collisionless mechanism)
Requires collisions?	Yes (collisional mechanism)	No (collisionless pitch-angle scattering)
Key Particle Population	Passing electrons	Trapped (or barely trapped) electrons
Wave absorption location	Depends on Doppler-shifted resonance; typically near magnetic axis or mid-radius	Usually near edge where barely trapped particles are abundant

1 N. J. Fisch and A. H. Boozer, Phys. Rev. Lett. 45, 720 (1980).

2 T. Ohkawa, “Steady state operation of tokamaks by rf heating,” General Atomics Report No. GA-A13847 (1976).

Neutral beam injector is one of the main heat mechanisms in MCF



D. Mazon, etc., NATURE PHYSICS, 12, 14, 2016

<https://zh.wikihow.com/%E5%9C%A8%E6%89%8B%E6%8C%87%E4%B8%8A%E8%BD%AC%E7%AF%AE%E7%90%83>

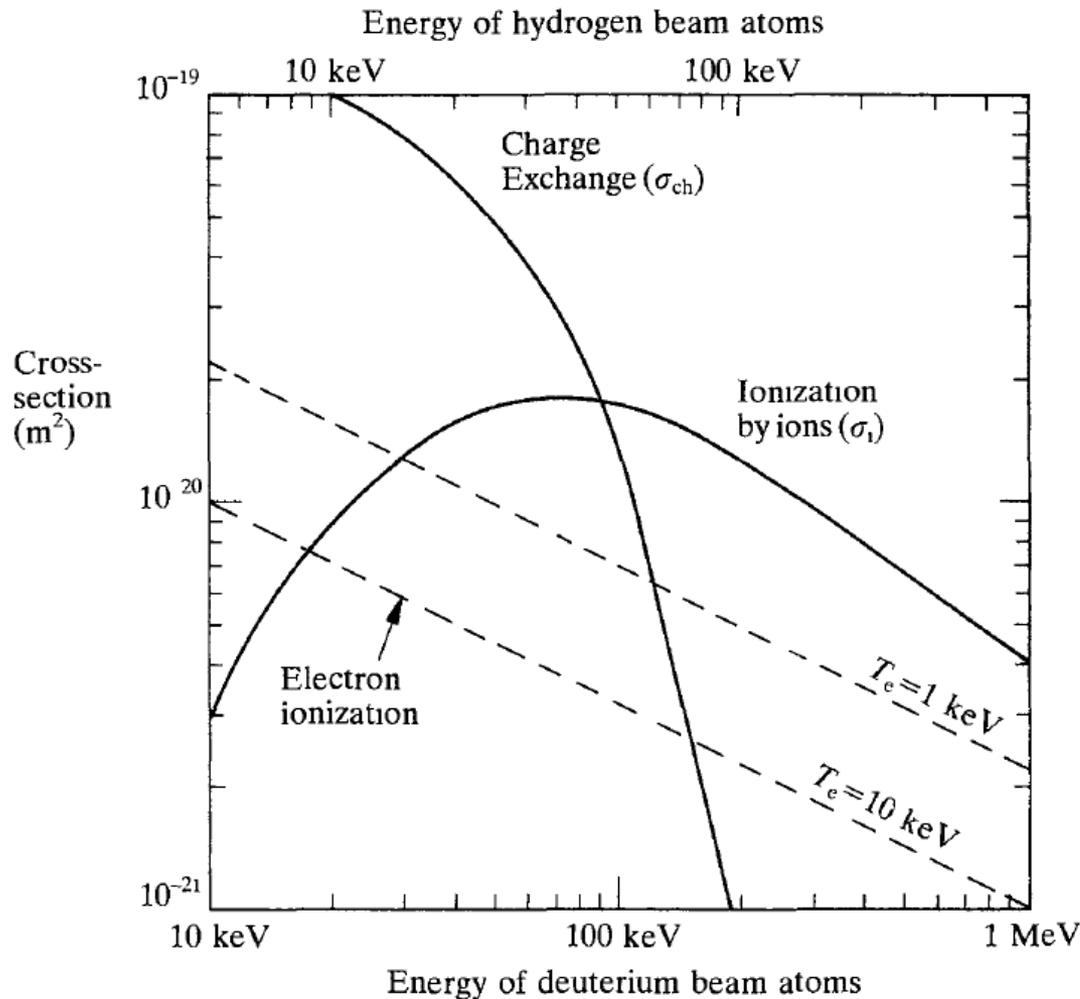
Varies way of heating a MCF device



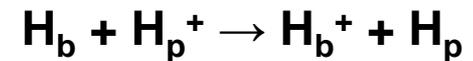
	System	Frequency/ energy	Maximum power coupled to plasma	Overall system efficiency	Development/ demonstration required	Remarks
ECRF	Demonstrated in tokamaks	28–157 GHz	2.8 MW, 0.2 s	30–40%	Power sources and windows, off-axis CD	Provides off-axis CD
	ITER needs	150–170 GHz	50 MW, S S			
ICRF	Demonstrated in tokamaks	25–120 MHz	22 MW, 3 s (L-mode); 16.5 MW, 3 s (H-mode)	50–60%	ELM tolerant system	Provides ion heating and smaller ELMs
	ITER needs	40–75 MHz	50 MW, S S			
LHRF	Demonstrated in tokamaks	1.3–8 GHz	2.5 MW, 120 s; 10 MW, 0.5 s	45–55%	Launcher, coupling to H-mode	Provides off-axis CD
	ITER needs	5 GHz	50 MW, S S			
NBI	+ve ion Demonstrated in tokamaks	80–140 keV	40 MW, 2 s; 20 MW, 8 s	35–45%	None	Not applicable
	ITER needs	None	None			
NBI	–ve ion Demonstrated in tokamaks	0.35 MeV	5.2 MW, D [–] , 0.8 s (from 2 sources)	~37%	System, tests on tokamak, plasma CD	provides rotation
	ITER needs	1 MeV	50 MW, S S			

‘SS’ indicates steady state

Neutral atoms are ionized by collisions in the plasma



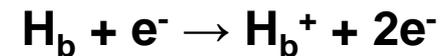
- Charge exchange:



- Ionization by ions



- Ionization by electrons



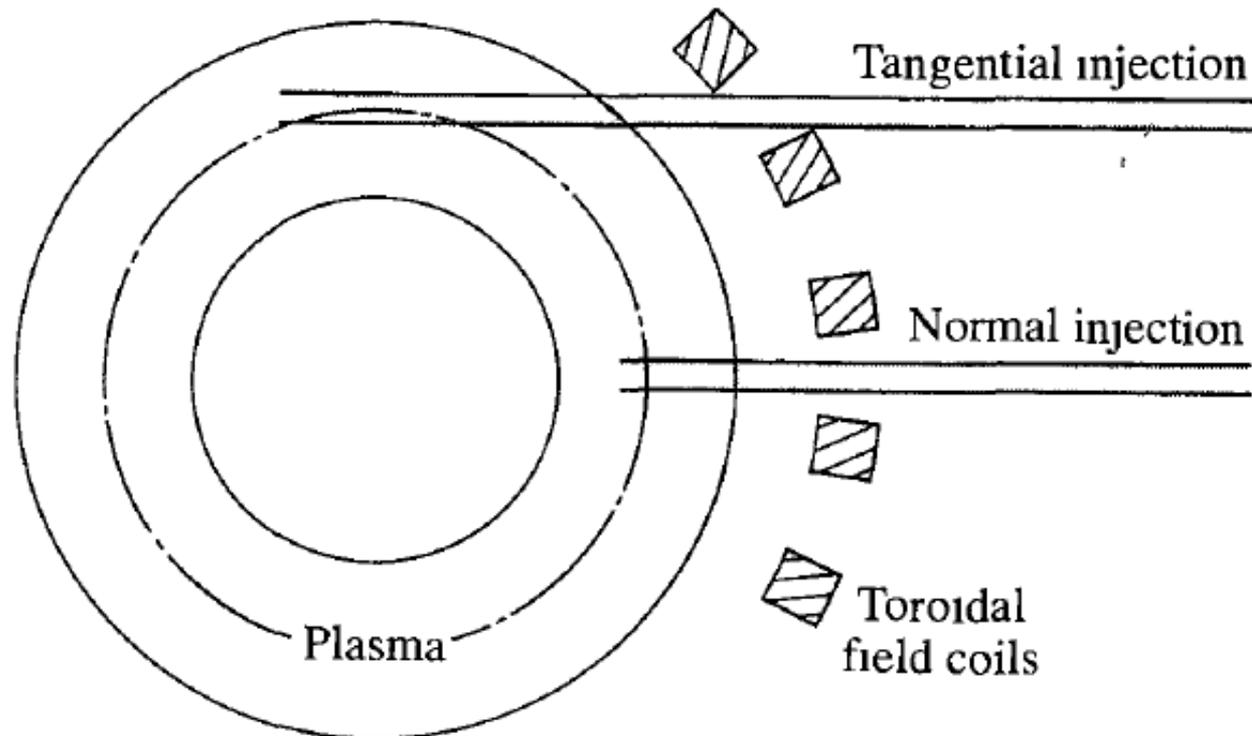
b: beam

p: plasma

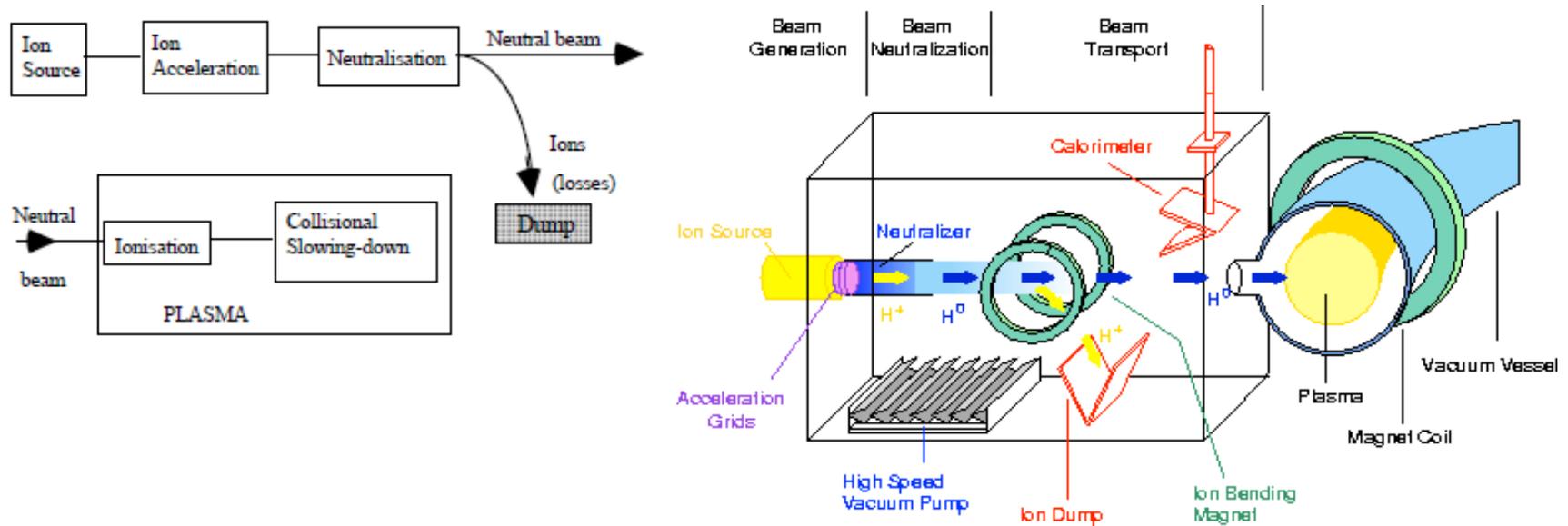
Neutral beam absorption length increases with tangential injection



- It is more difficult to access through the toroidal field coils with tangential injection.

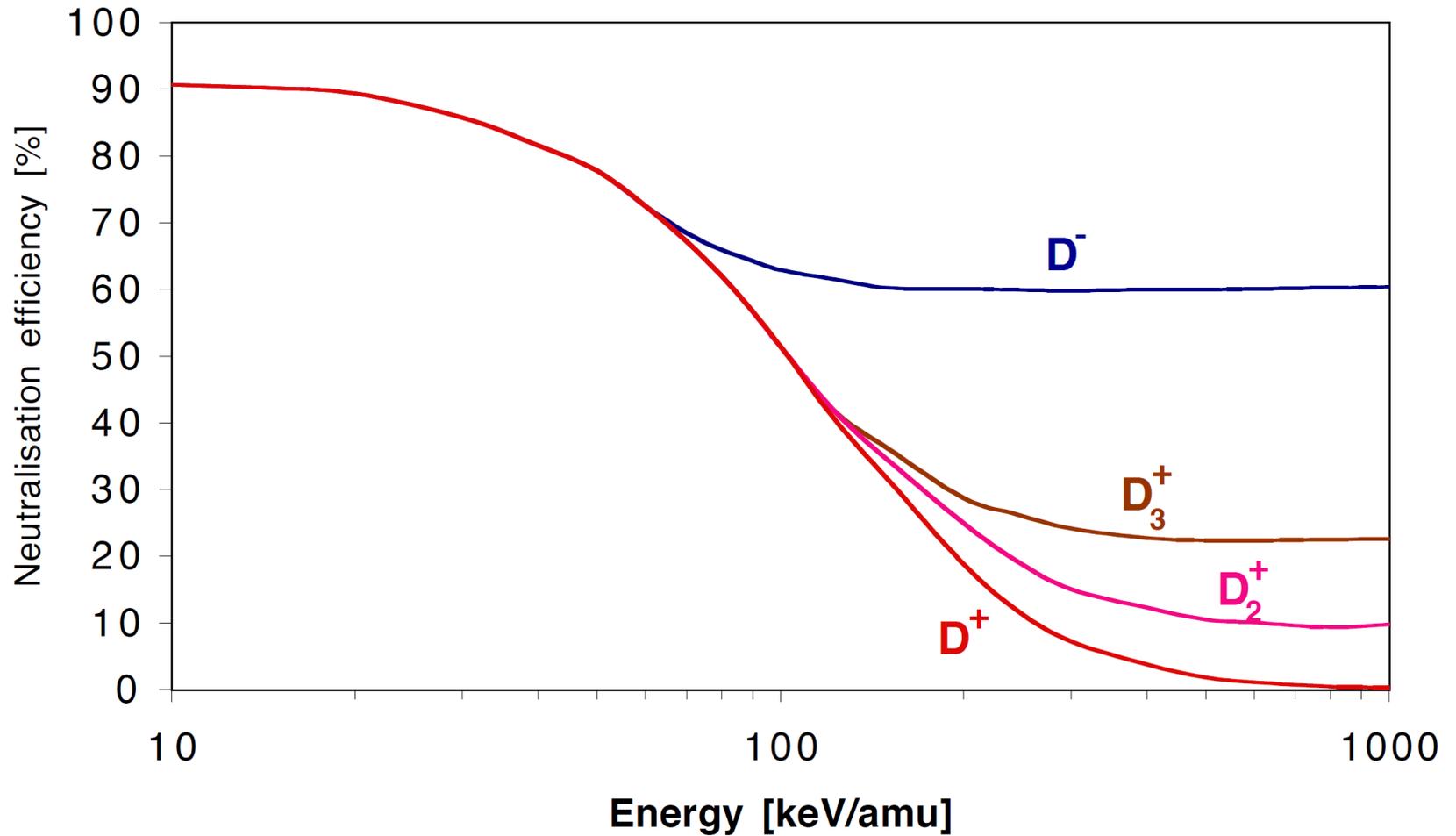


Neutral particles heat the plasma via coulomb collisions



1. create energetic (fast) neutral ions
2. ionize the neutral particles
3. heat the plasma (electrons and ions) via Coulomb collisions

Negative ion source is preferred due to higher neutralization efficiency

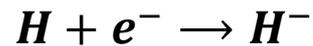
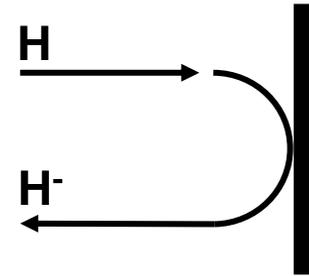


There are two ways to make negative ions – surface and volume production

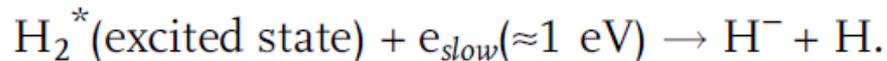
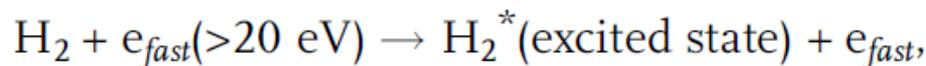


- **Surface production, depends on :**

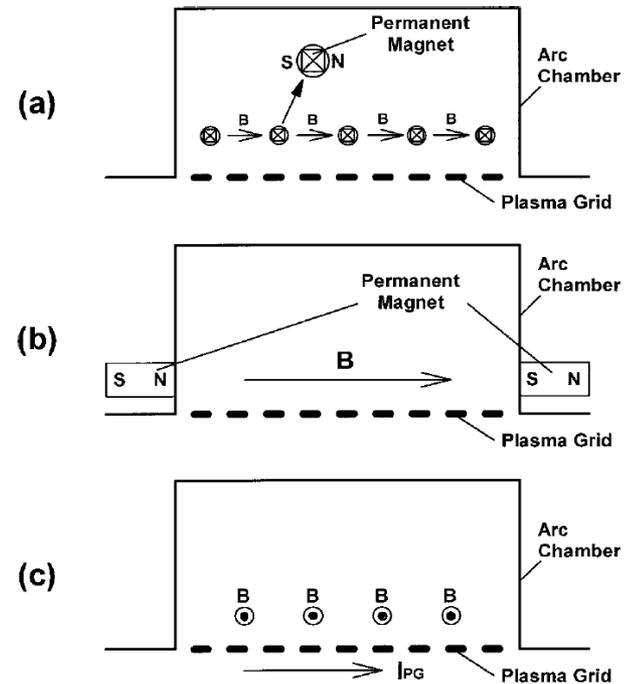
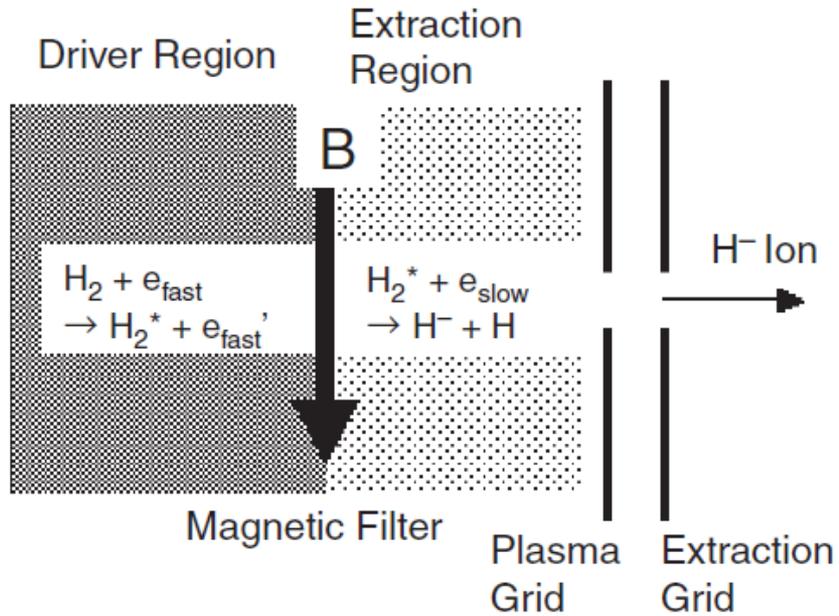
- **Work function Φ**
- **Electron affinity level, 0.75 eV for H^-**
- **Perpendicular velocity**
- **Work function can be reduced by covering the metal surface with cesium**



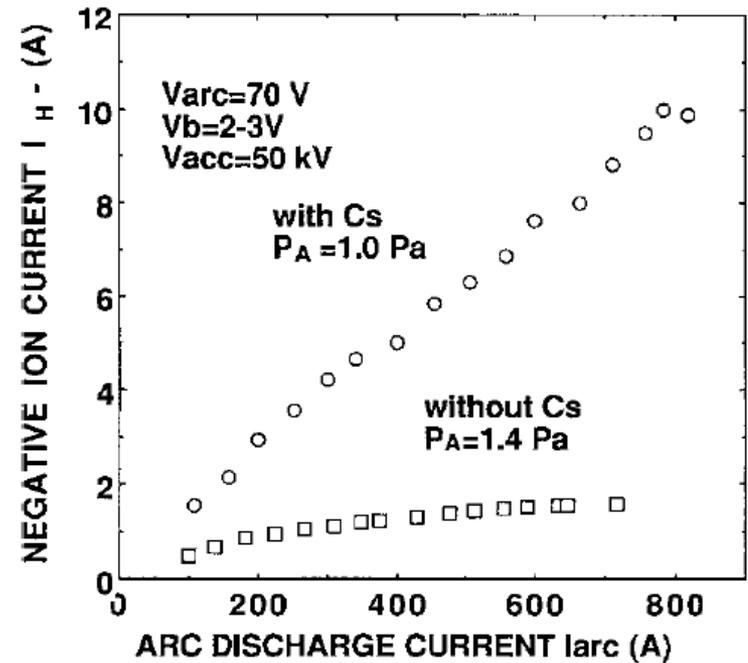
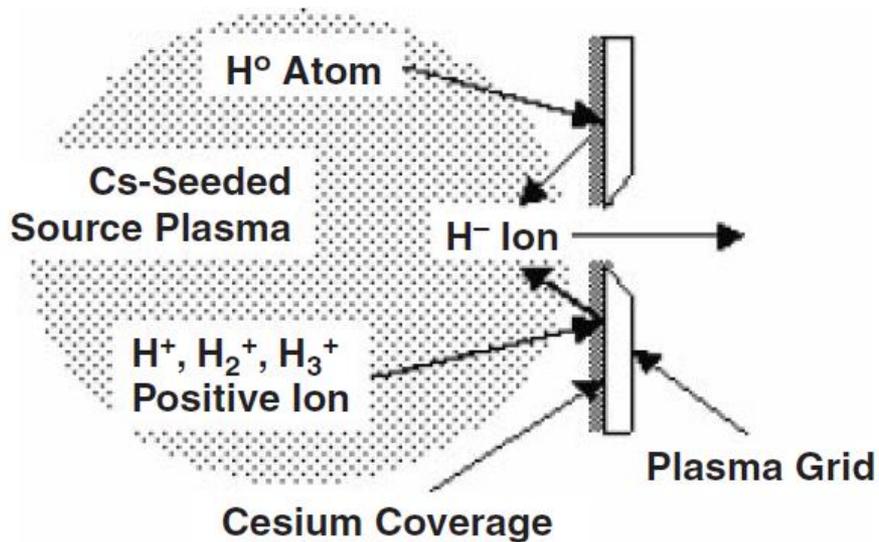
- **Volume production:**



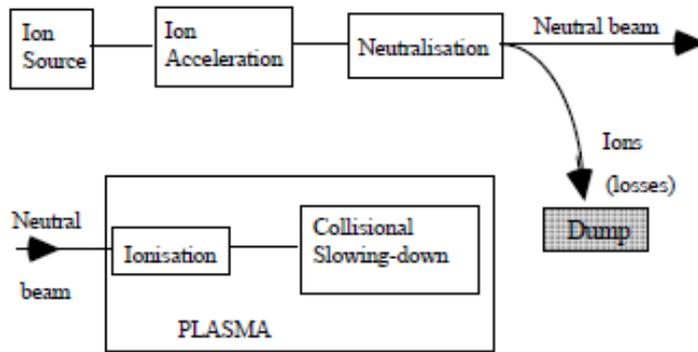
Two-chamber method of negative ions in volume production with a magnetic filter



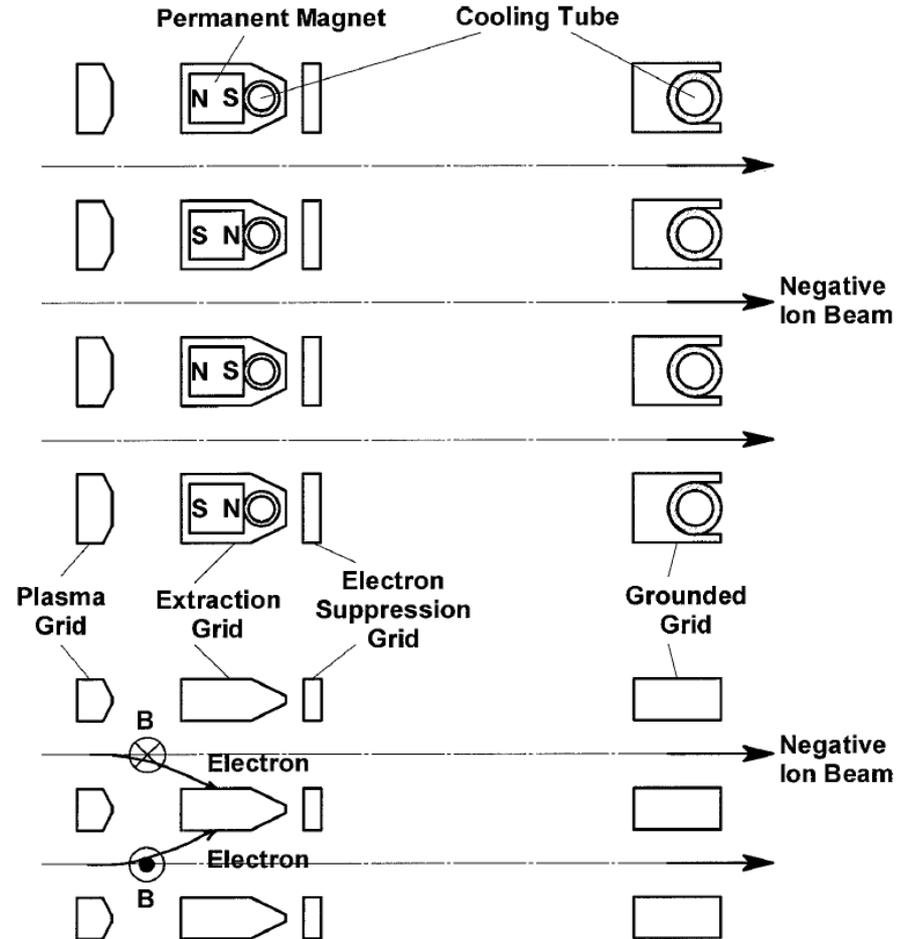
Adding cesium increases negative ion current



Electrons need to be filtered out since they are extracted together with negative ions



(a)

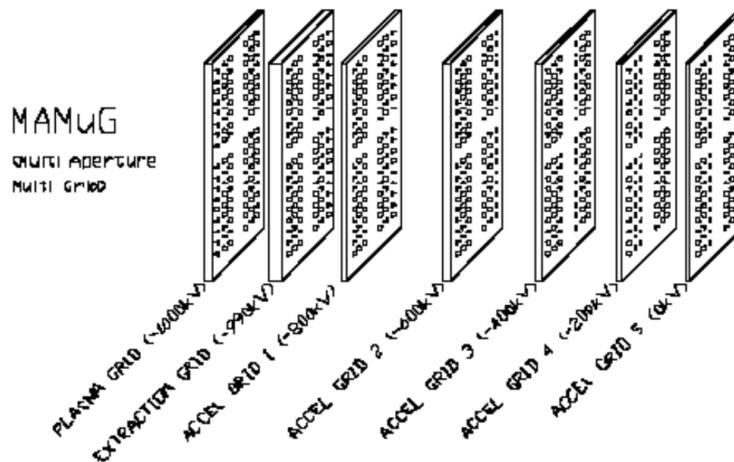
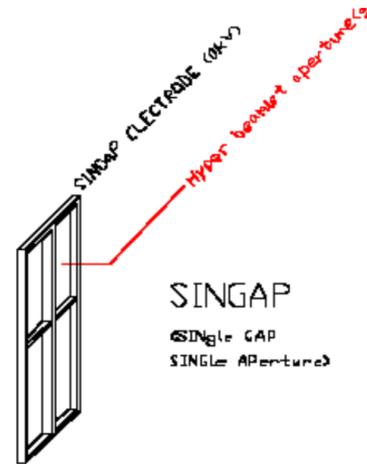
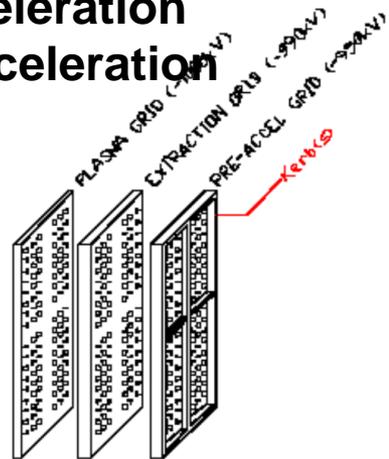


(b)

Acceleration

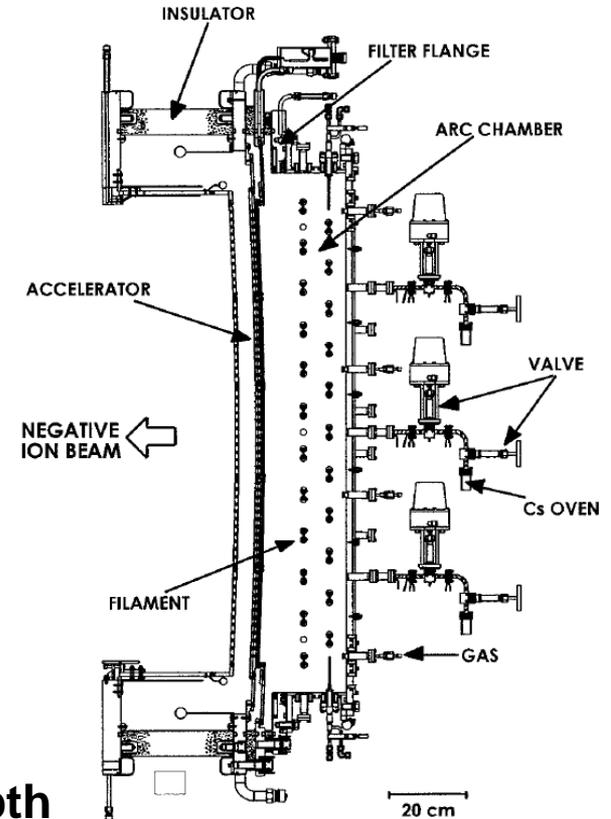
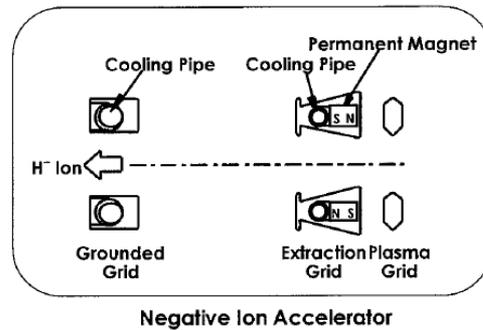
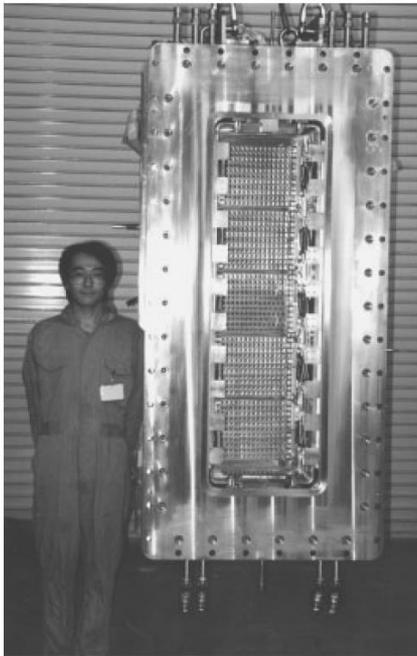


- Multi-stage acceleration
- Single-stage acceleration



The ITER neutral beam system: status of the project and review of the main technological issues, presented by V. Antoni

NBI system of the LHD fusion machine

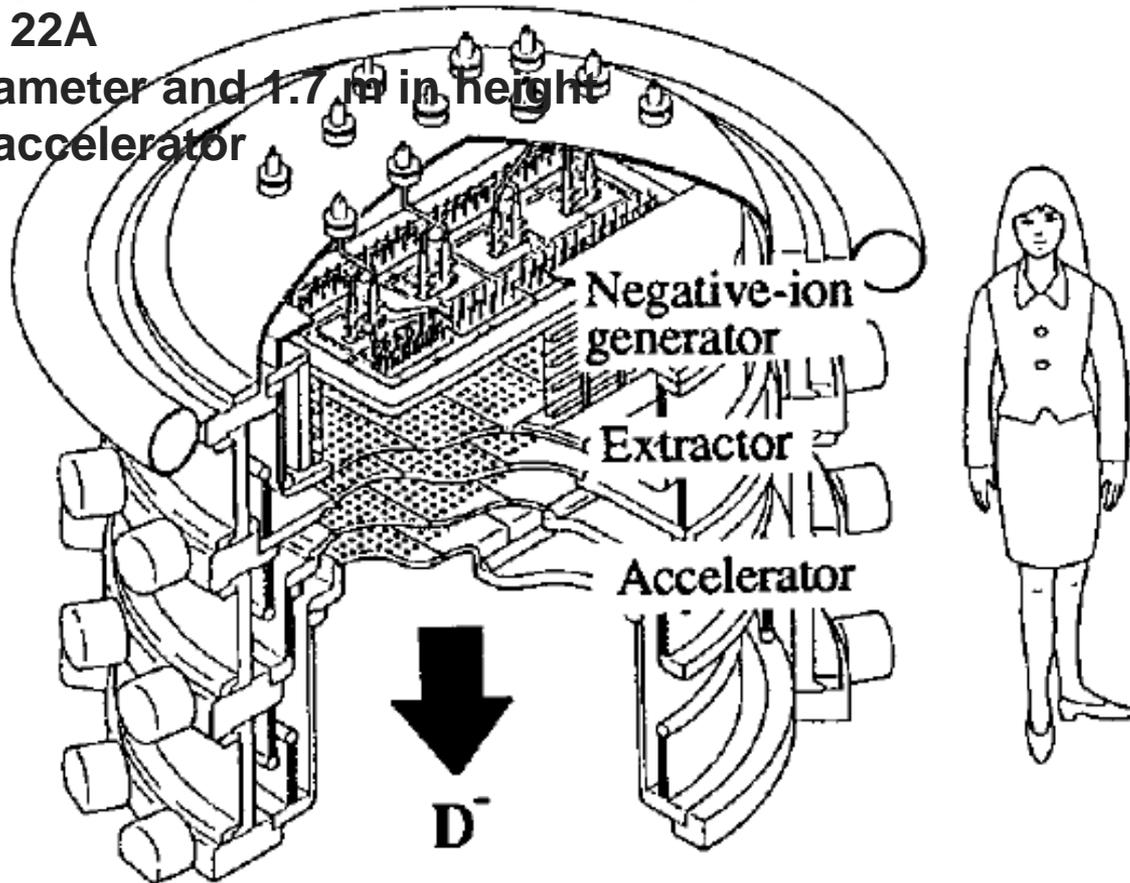


- 180 keV and 30 A
- Arc chamber: 35 cm x 145 cm, 21cm in depth
- Single stage accelerator

JT60U NBI system



- JT-60 (Japan-Torus) is a tokamak in Japan.
- 550 keV, 22A
- 2m in diameter and 1.7 m in height
- 3-stage accelerator

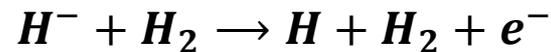


Neutralization



- **Gas neutralization**

- **Collisions between fast negative ions and atoms**



- **Fast ions can lose another electron after neutralized**



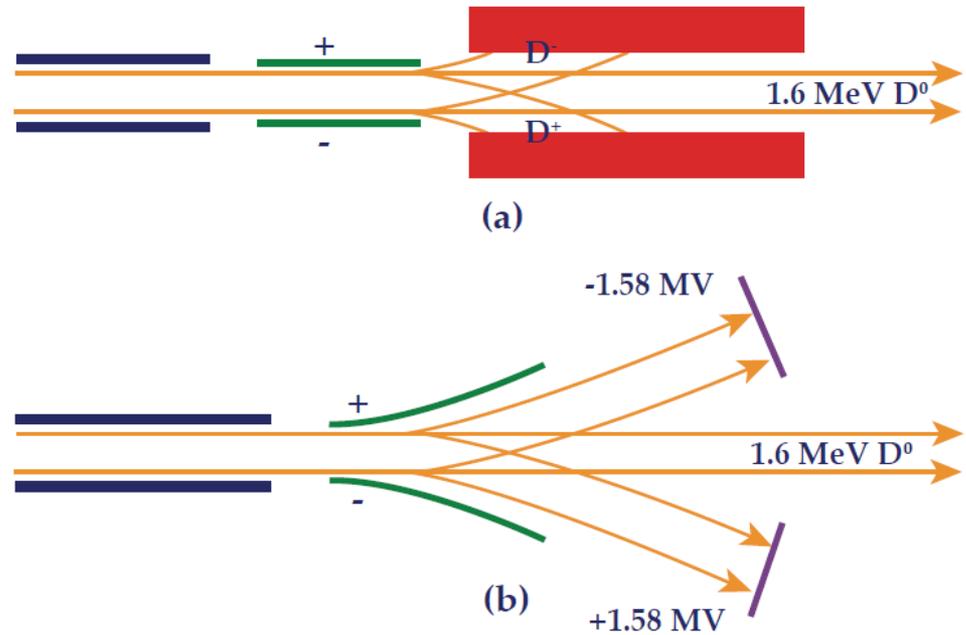
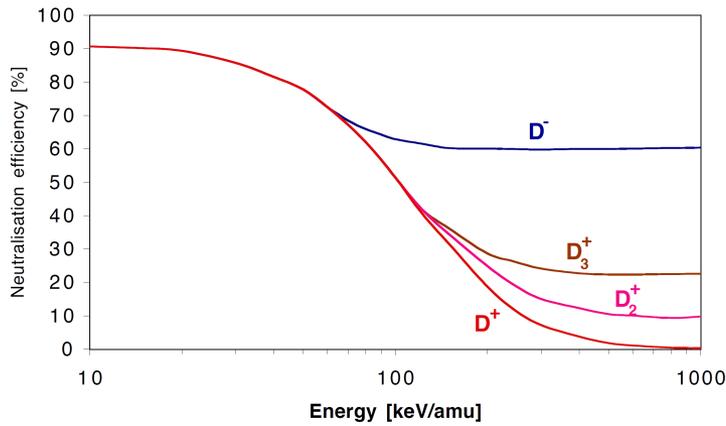
- **Plasma neutralization**

- **Collisions with charged particles in plasma**



- **The efficiencies reach up to 85% for fully ionized hydrogen plasma**

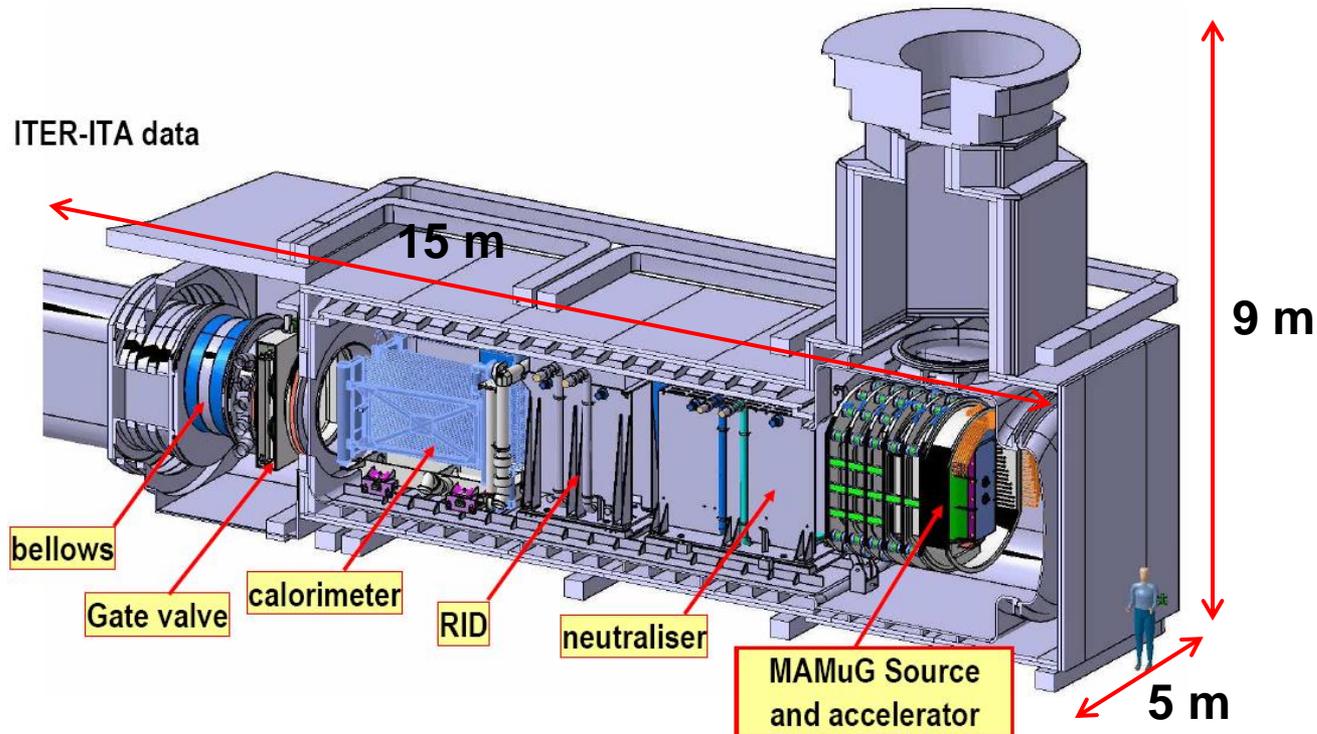
Beam dump



NBI for ITER

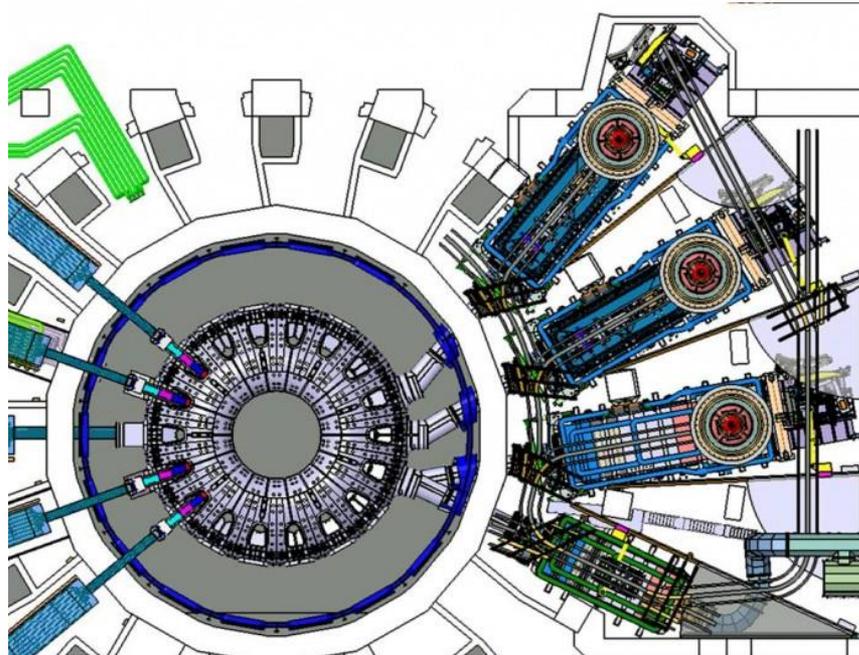


- beam components (Ion Source, Accelerator, Neutralizer, Residual Ion Dump and Calorimeter)
- other components (cryo-pump, vessels, fast shutter, duct, magnetic shielding, and residual magnetic field compensating coils)



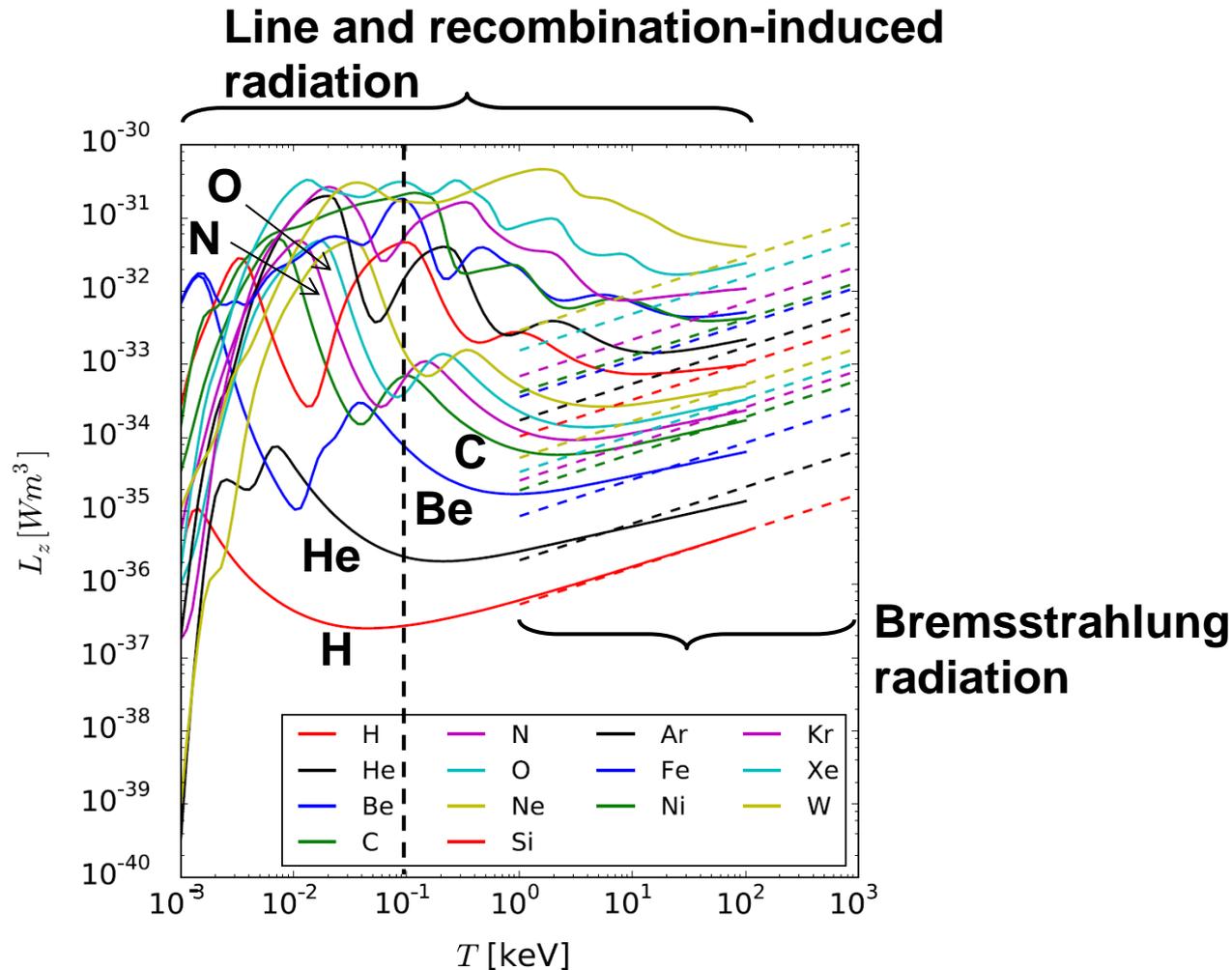
The ITER neutral beam system: status of the project and review of the main technological issues, presented by V. Antoni

Neutral beam penetration



- **Parallel direction**
 - Longest path through the densest part of the plasma
 - Harder to be built
- **Perpendicular direction**
 - Path is short
 - Larger perpendicular energies leads to larger losses
 - Easier to be built

Temperature of 100 eV is the threshold of radiation barrier by impurities



Course Outline



- **Inertial confinement fusion (ICF)**
 - **Plasma frequency and critical density**
 - **Direct- and indirect- drive**
 - **Laser generated pressure (Inverse bremsstrahlung and Ablation pressure)**
 - **Burning fraction, why compressing a capsule?**
 - **Implosion dynamics**
 - **Shock (Compression with different adiabat)**
 - **Laser pulse shape**
 - **Rocket model, shell velocity**
 - **Laser-plasma interaction (Stimulated Raman Scattering, SRS; Stimulated Brillouin Scattering, SBS; Two-plasmon decay)**
 - **Instabilities (Rayleigh-taylor instability, Kelvin-Helmholtz instability, Richtmeyer-Meshkov instability)**

Under what conditions the plasma keeps itself hot?



- **Steady state 0-D power balance:**

$$S_{\alpha} + S_h = S_B + S_k$$

S_{α} : α particle heating

S_h : external heating

S_B : Bremsstrahlung radiation

S_k : heat conduction lost

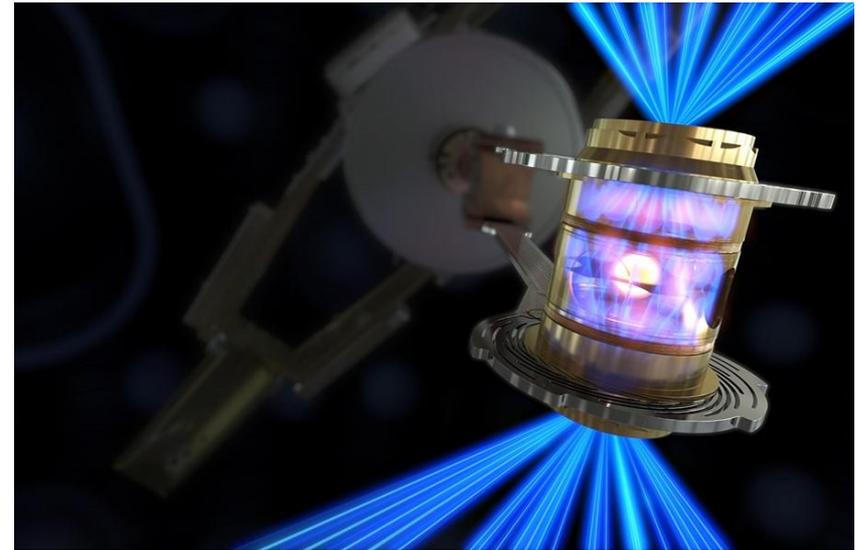
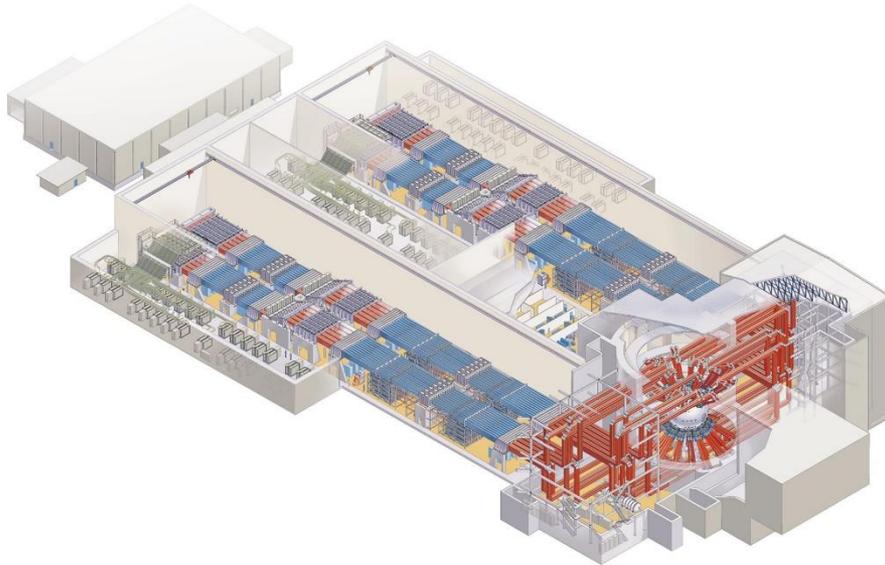
Ignition condition: $P\tau > 10 \text{ atm-s} = 10 \text{ Gbar} \cdot \text{ns}$

- **P: pressure, or called energy density**
- **τ is confinement time**

Significant breakthrough was achieved in ICF recently



- Inertial confinement fusion (ICF)



- **National Ignition Facility (NIF) demonstrated a gain greater than 1 for the first time on 2022/12/5. The yield of 3.15 MJ from the 2.05-MJ input laser energy, i.e., $Q=1.5$.**

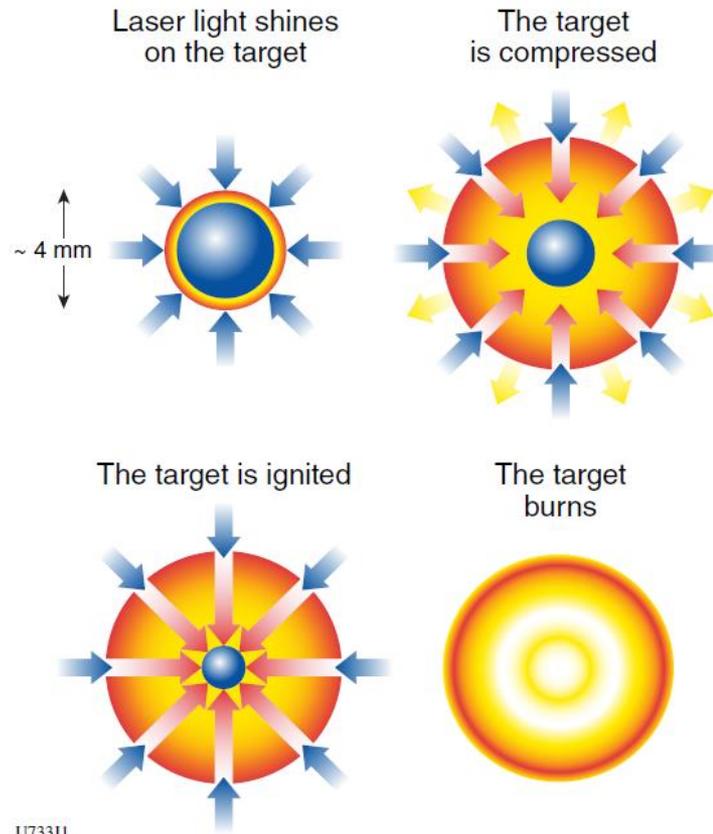
<https://zh.wikipedia.org/wiki/國家點火設施>

<https://www.science.org/content/article/historic-explosion-long-sought-fusion-breakthrough>

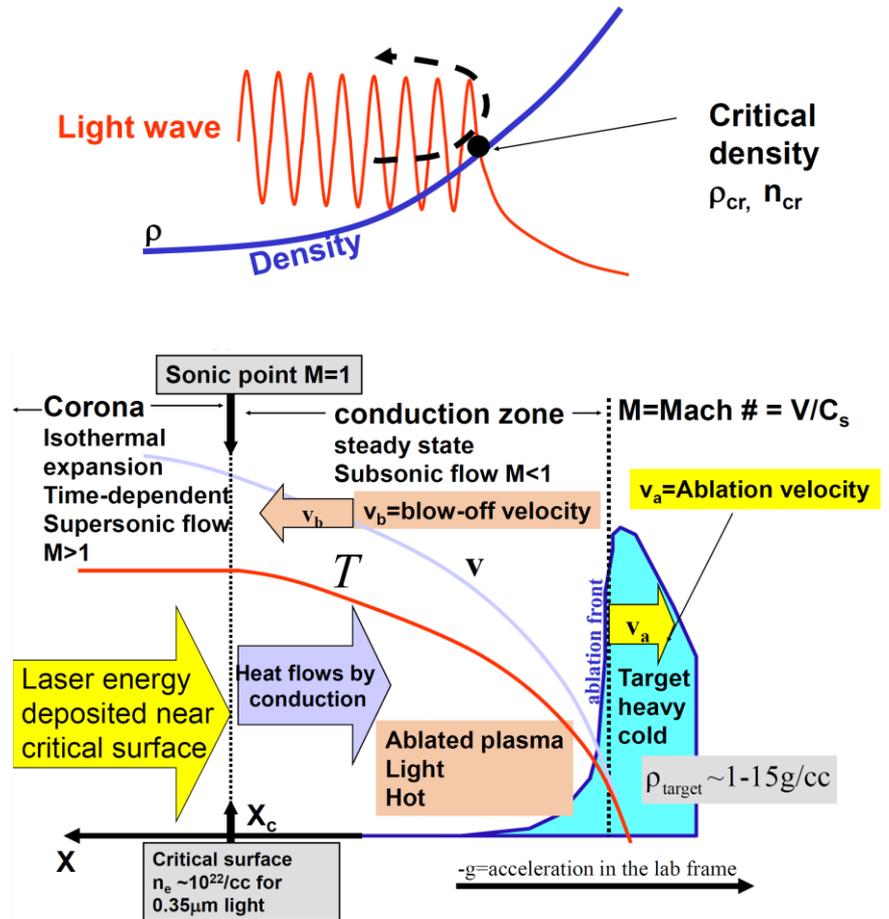
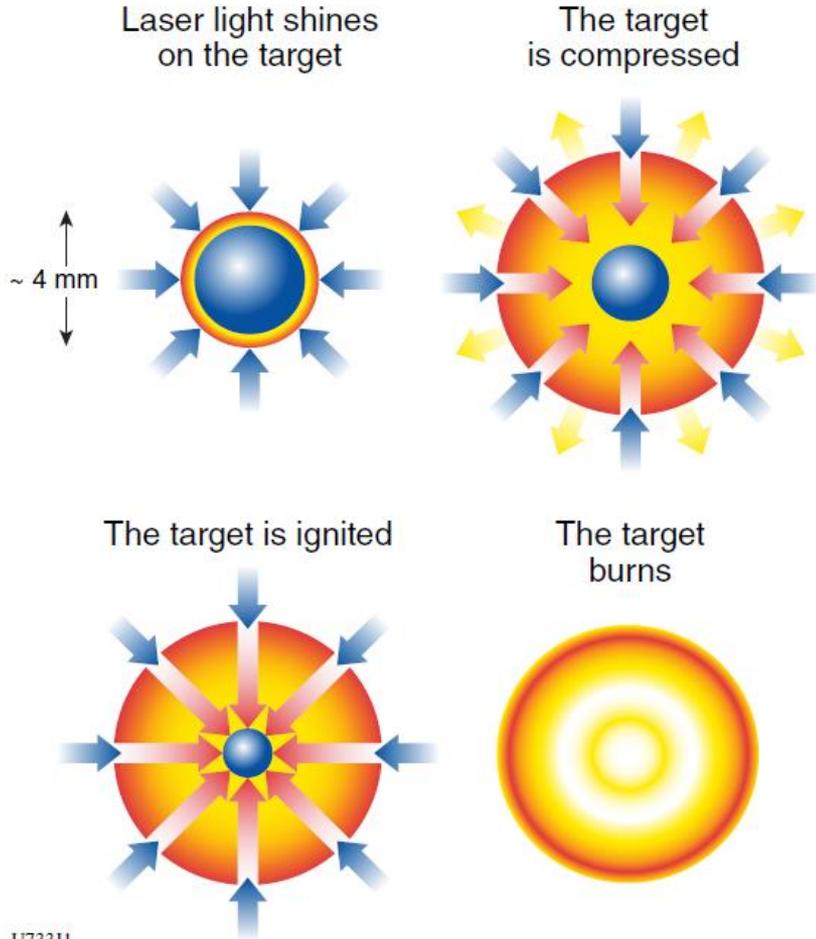
Don't confine it!



- **Solution 2: Inertial confinement fusion (ICF). Or you can say it is confined by its own inertia: $P \sim \text{Gigabar}$, $\tau \sim \text{nsec}$, $T \sim 10 \text{ keV}$ ($10^8 \text{ }^\circ\text{C}$)**

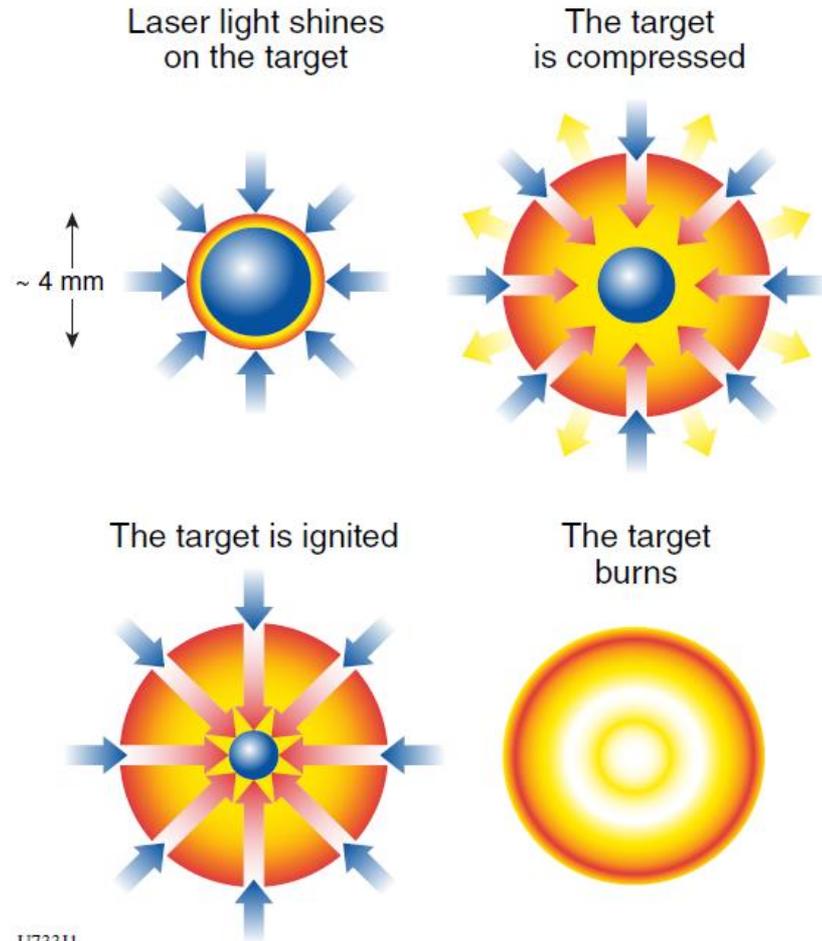


Compression happens when outer layer of the target is heated by laser and ablated outward

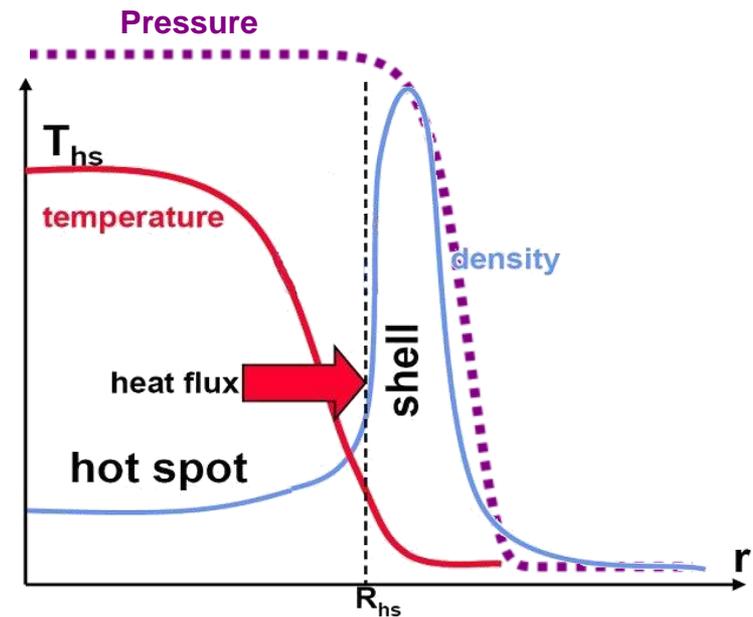


U733J1

Plasma is confined by its own inertia in inertial confinement fusion (ICF)



Spatial profile at stagnation

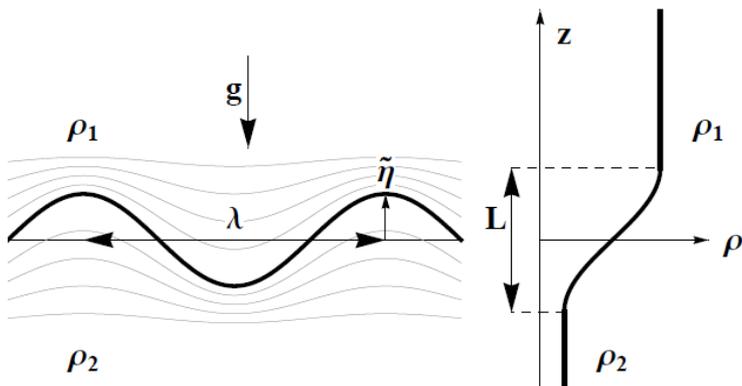


U733J1

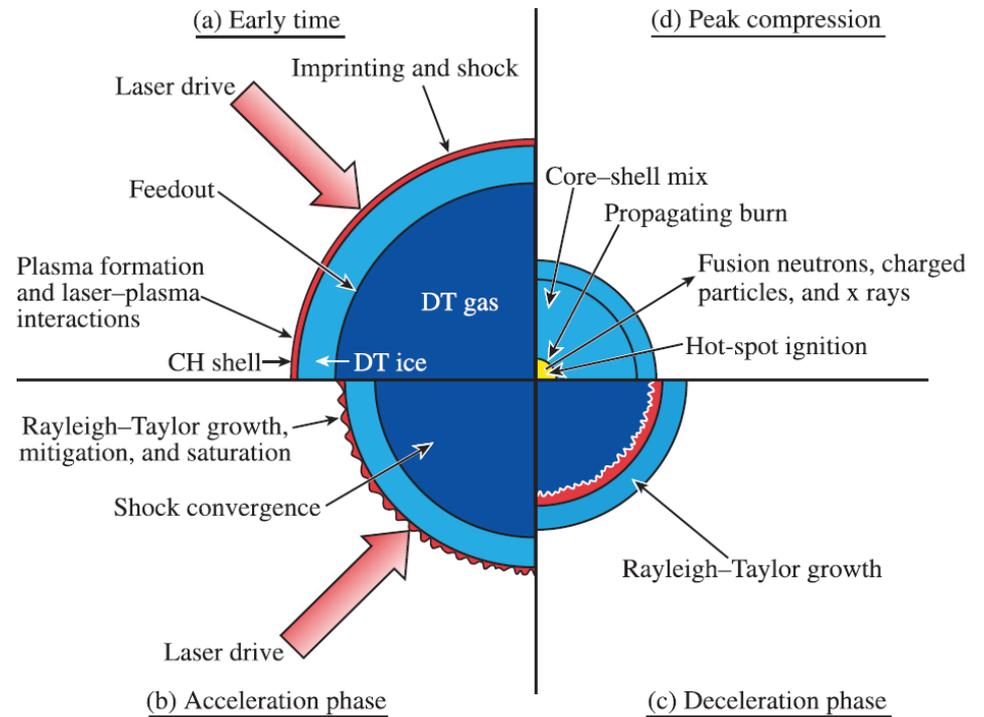
A ball can not be compressed uniformly by being squeezed between several fingers



• Rayleigh-Taylor instability



• Stages of a target implosion

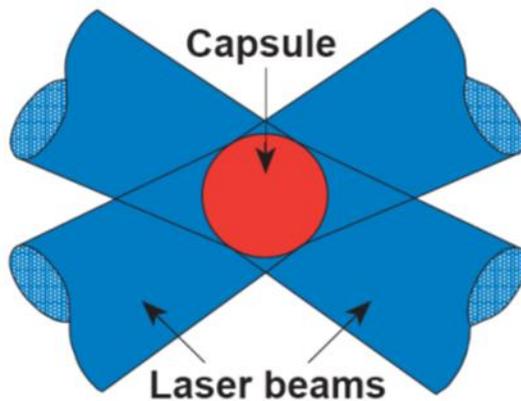


E9886J1

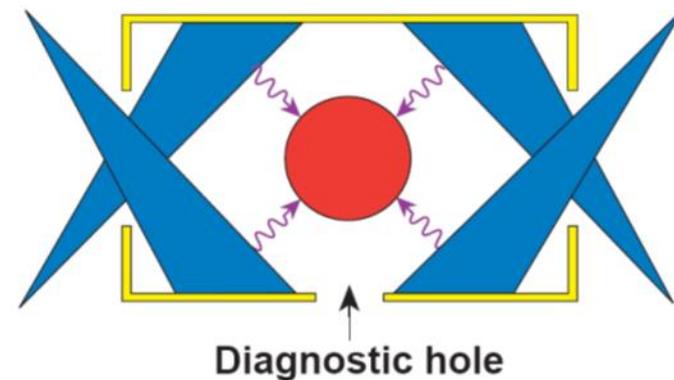
A spherical capsule can be imploded through directly or indirectly laser illumination



Direct-drive target

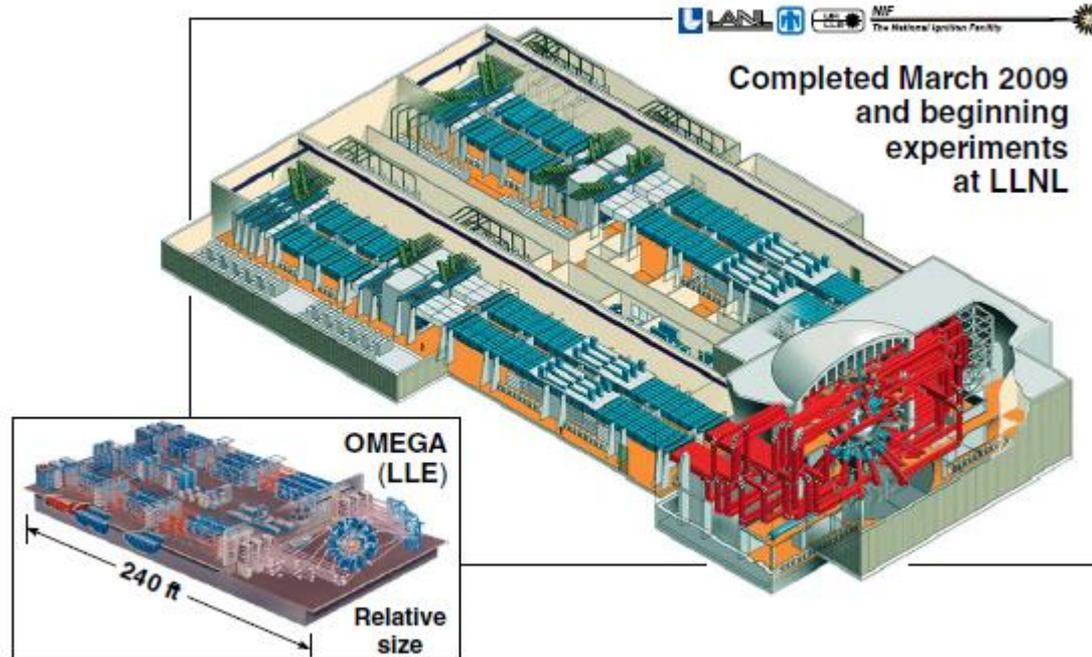


Indirect-drive target



Hohlraum using
a cylindrical high-Z case

The 1.8-MJ National Ignition Facility (NIF) will demonstrate ICF ignition and modest energy gain

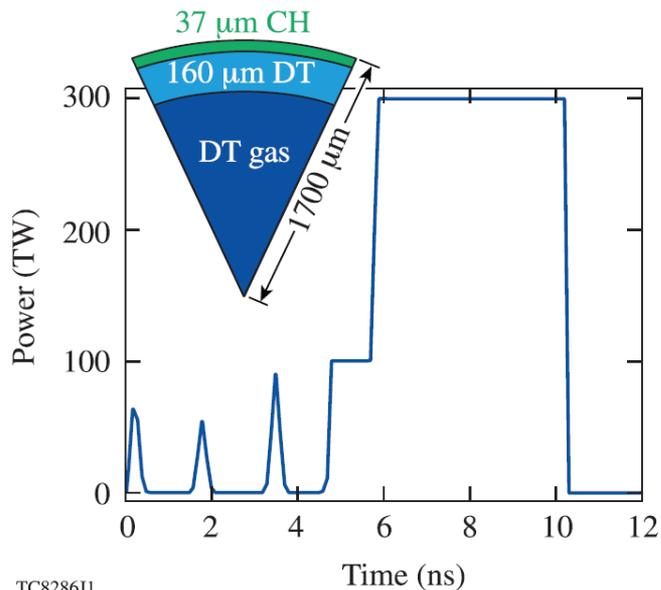


OMEGA experiments are integral to an ignition demonstration on the NIF.

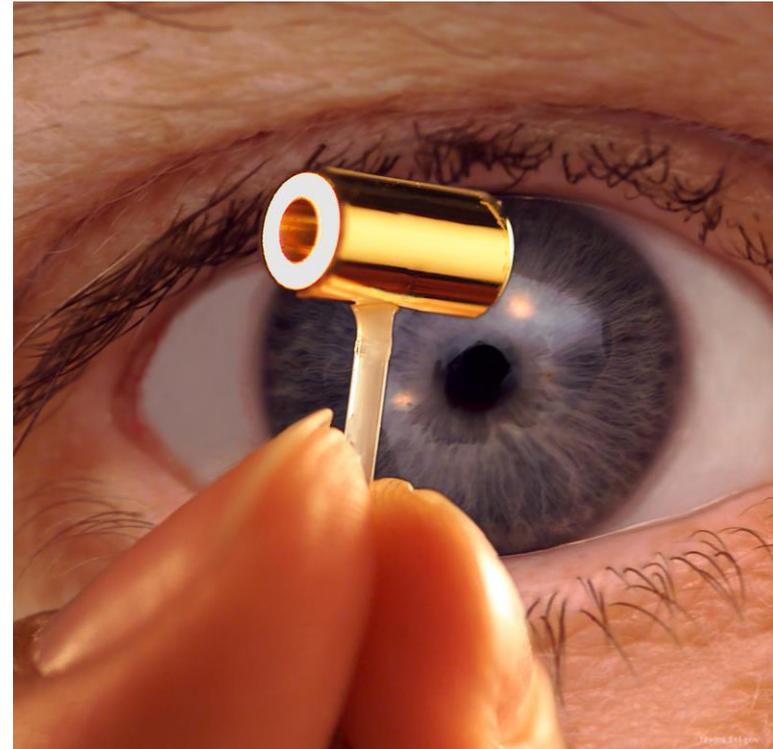
Targets used in ICF



- **Triple-point temperature : 19.79 K**



TC8286J1



<http://www.lle.rochester.edu>
https://en.wikipedia.org/wiki/Inertial_confinement_fusion
R. S. Craxton, et al., *Phys. Plasmas* **22**, 110501 (2015)

Softer material can be compressed to higher density



- **Compression of a baseball**



- **Compression of a tennis ball**



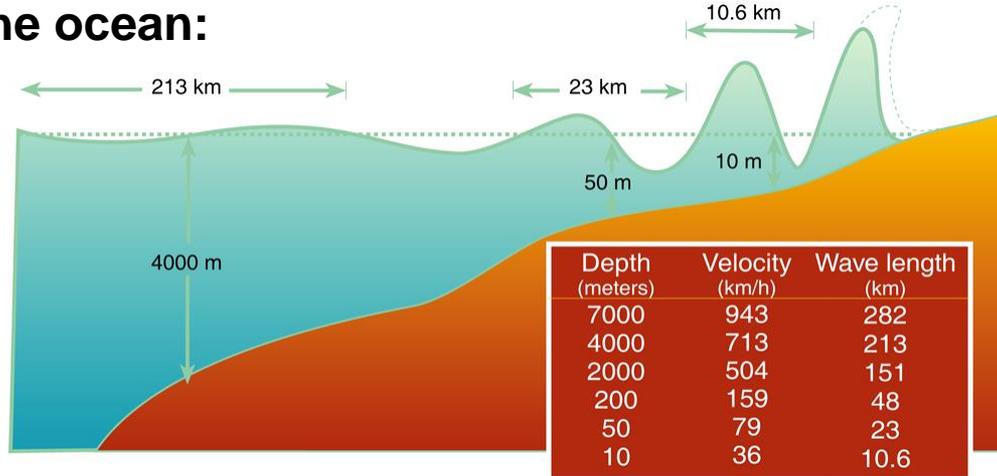
<https://www.youtube.com/watch?v=uxlldMoAwbY>

<https://newsghana.com.gh/wimbledon-slow-motion-video-of-how-a-tennis-ball-turns-to-goo-after-serve/>

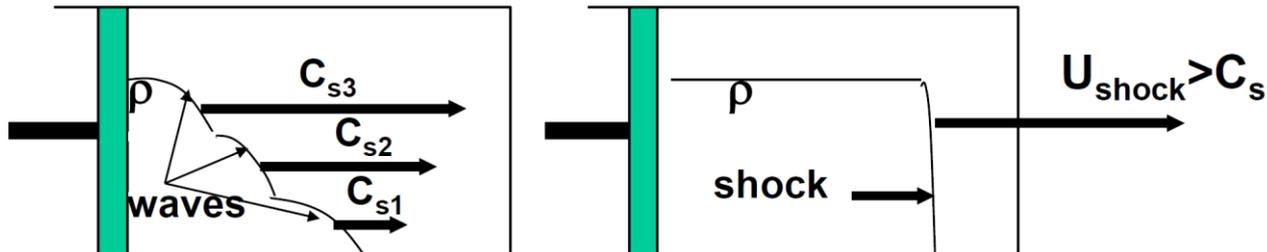
A shock is formed due to the increasing sound speed of a compressed gas/plasma



- **Wave in the ocean:**

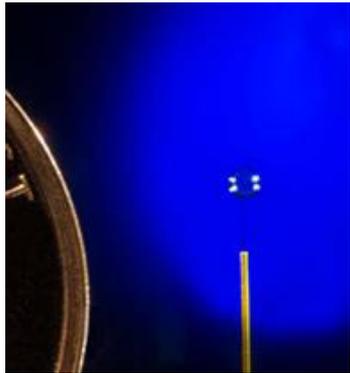


- **Acoustic/compression wave driven by a piston:**



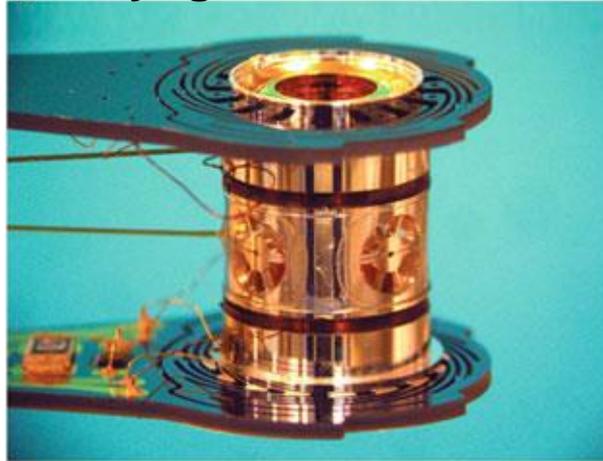
$$C_s \sim \sqrt{\frac{p}{\rho}} \sim \sqrt{\frac{\alpha \rho^{5/3}}{\rho}} \sim \sqrt{\alpha} \rho^{1/3}$$

Targets used in ICF

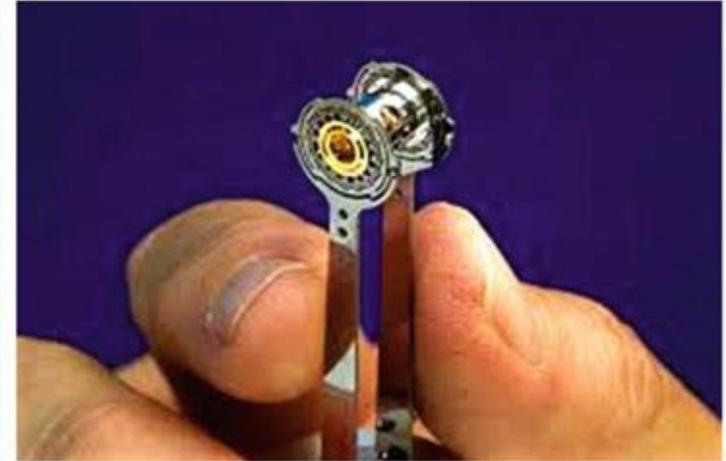


Cryogenic shroud

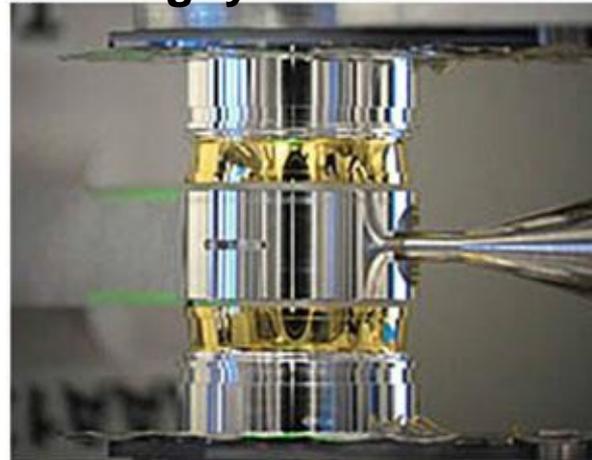
a Cryogenic hohlraum



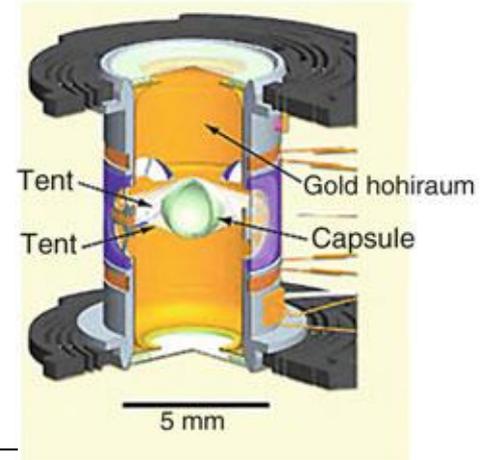
b



c Rugby hohlraum

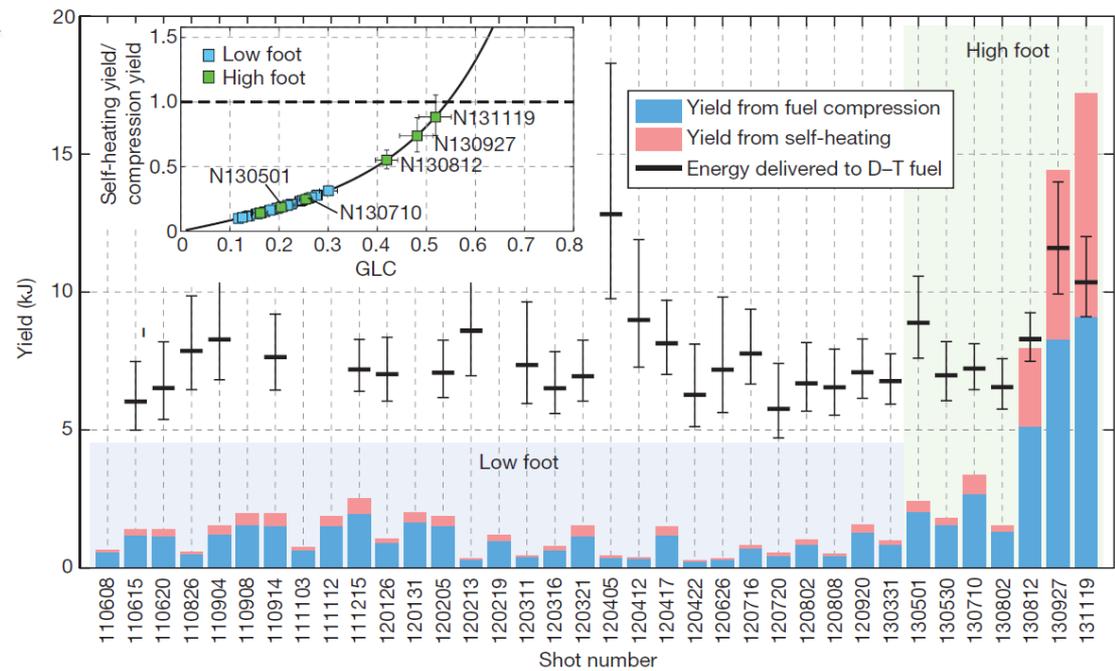
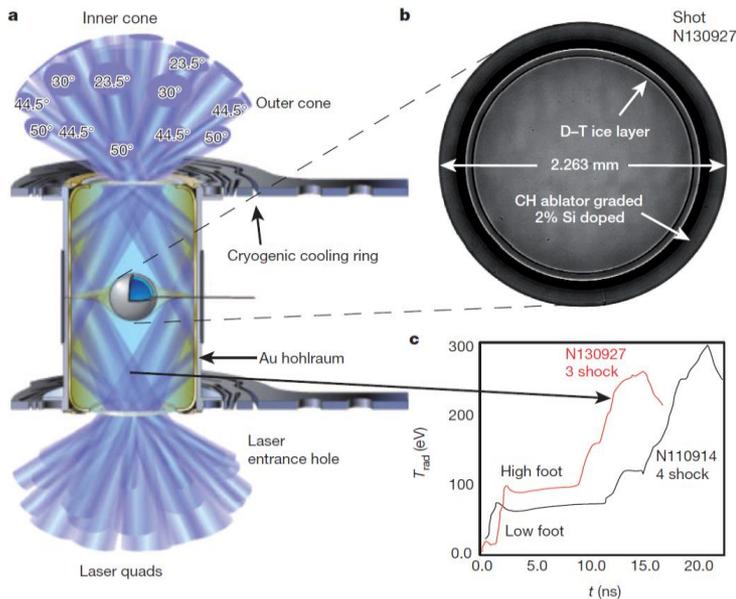


d Tent holder



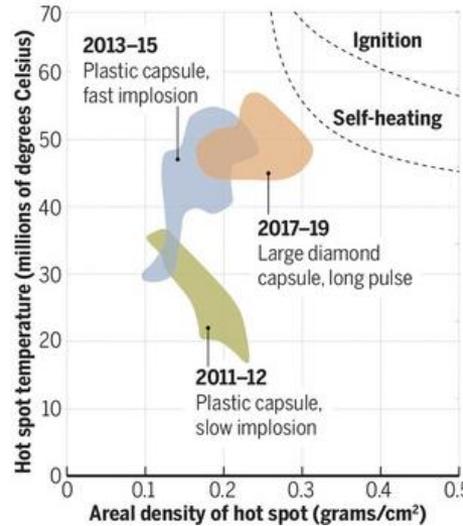
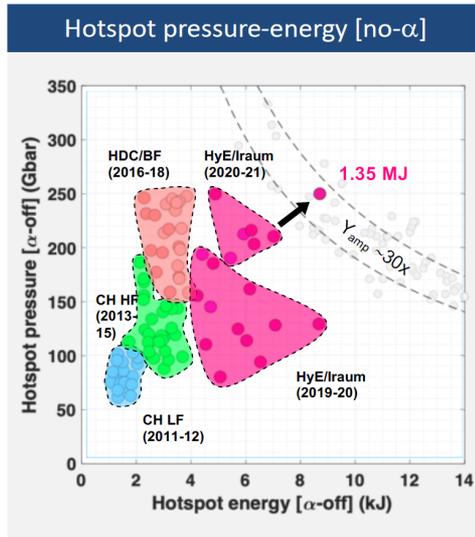
<https://www.lle.rochester.edu/index.php/2014/11/10/next-generation-cryo-target/>
Introduction to Plasma Physics and Controlled Fusion 3rd Edition, by Francis F. Chen
<https://www.llnl.gov/news/nif-shot-lights-way-new-fusion-ignition-phase>

Nature letter “Fuel gain exceeding unity in an inertially confined fusion implosion”

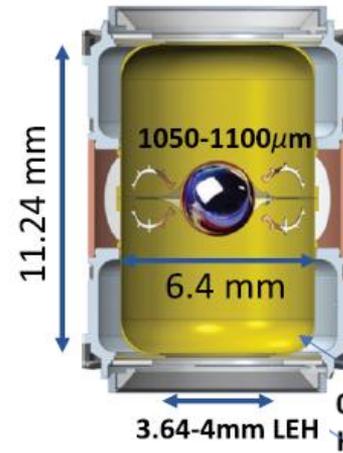


• Fuel gain exceeding unity was demonstrated for the first time.

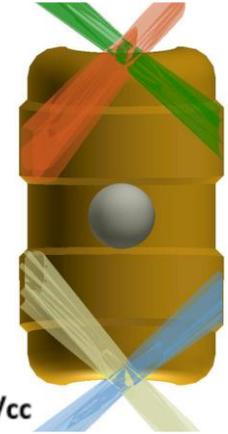
The hot spot has entered the burning plasma regime



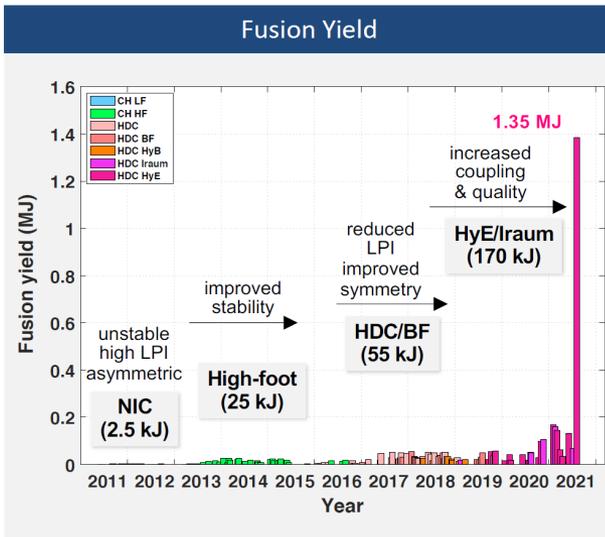
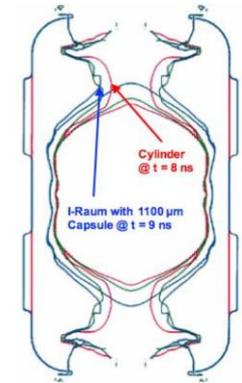
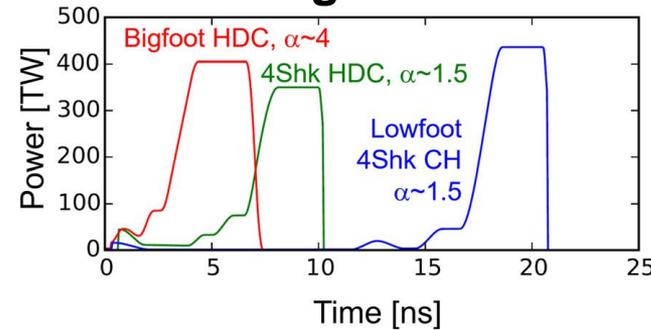
• Hybrid-E



• I-raum



• Big foot



T. Ma, ARPA-E workshop, April 26, 2022

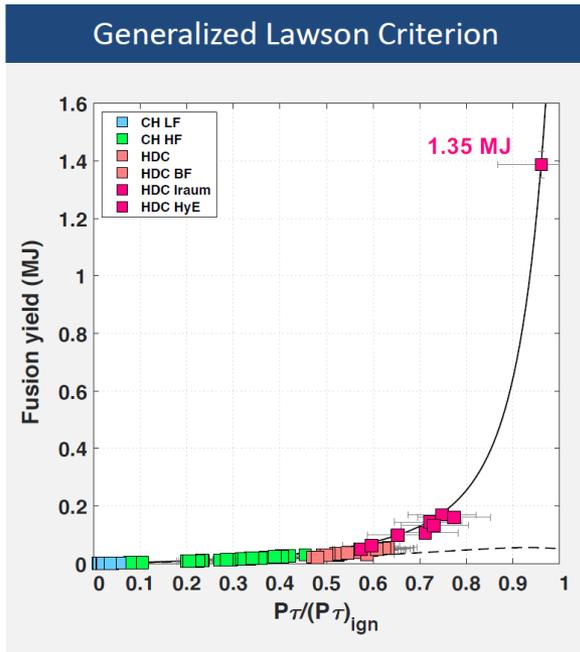
Science 370, p1019, 2020

D. T. Casey, etc., Phys. Plasmas, 25, 056308 (2018)

A. L. Kritcher, etc., Phys. Plasmas, 28, 072706 (2021)

H. F. Robey, etc., Phys. Plasmas, 25, 012711 (2018)

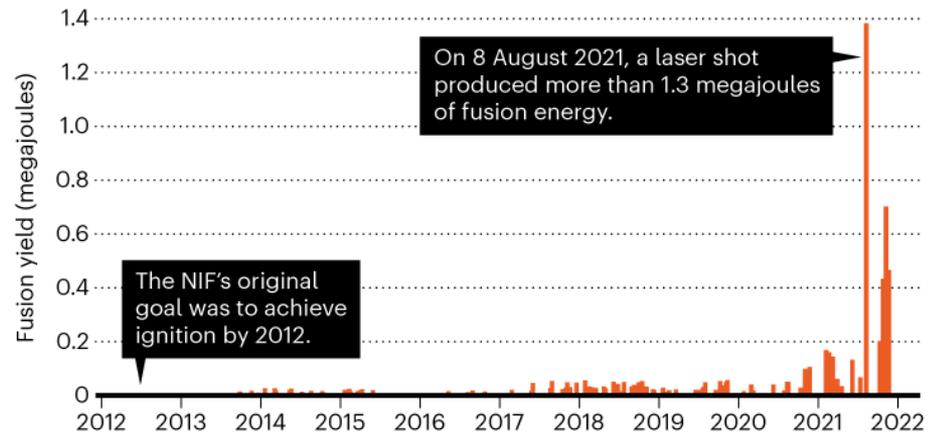
National Ignition Facility (NIF) achieved a yield of more than 1.3 MJ from ~1.9 MJ of laser energy in 2021 (Q~0.7)



- National Ignition Facility (NIF) achieved a yield of more than 1.3 MJ (Q~0.7). This advancement puts researchers at the threshold of fusion ignition.

THE ROAD TO IGNITION

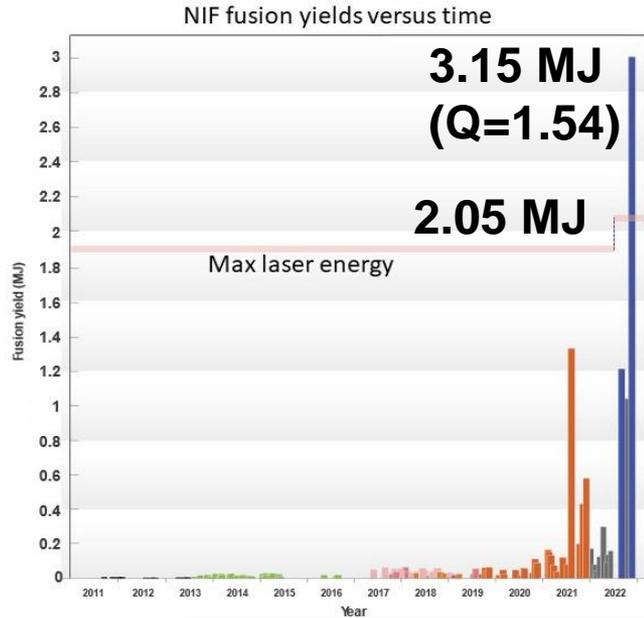
The National Ignition Facility (NIF) struggled for years before achieving a high-yield fusion reaction (considered ignition, by some measures) in 2021. Repeat experiments, however, produced less than half the energy of that result.



©nature

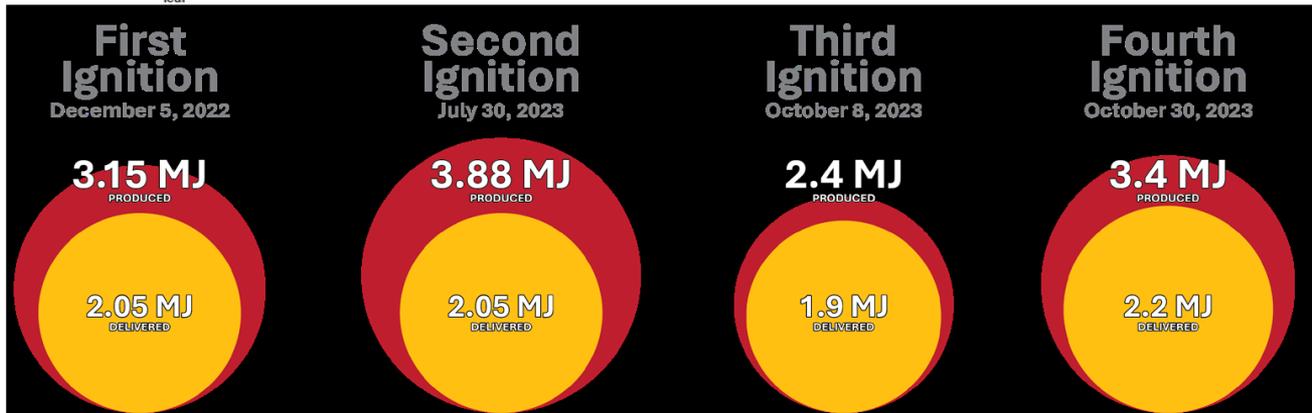
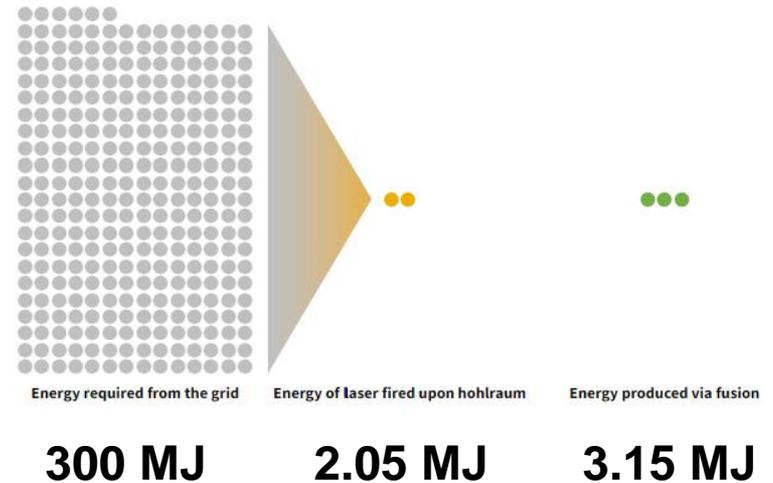
- Laser-fusion facility heads back to the drawing board.

“Ignition” (target yield larger than one) was achieved in NIF on 2022/12/5



NIF's ignition achievement in perspective

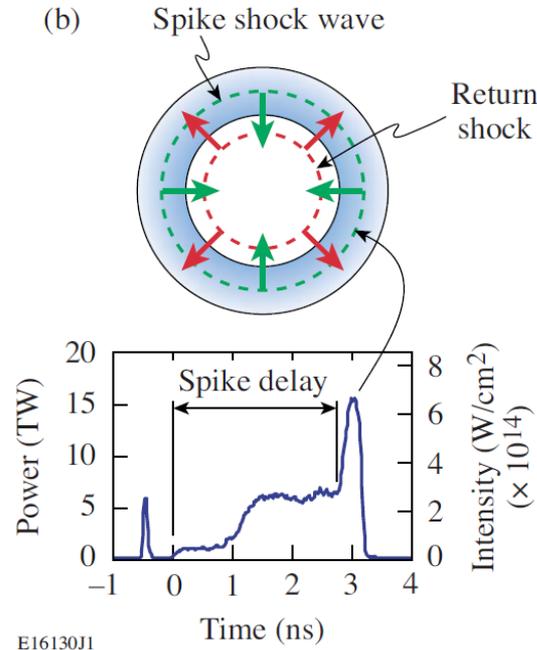
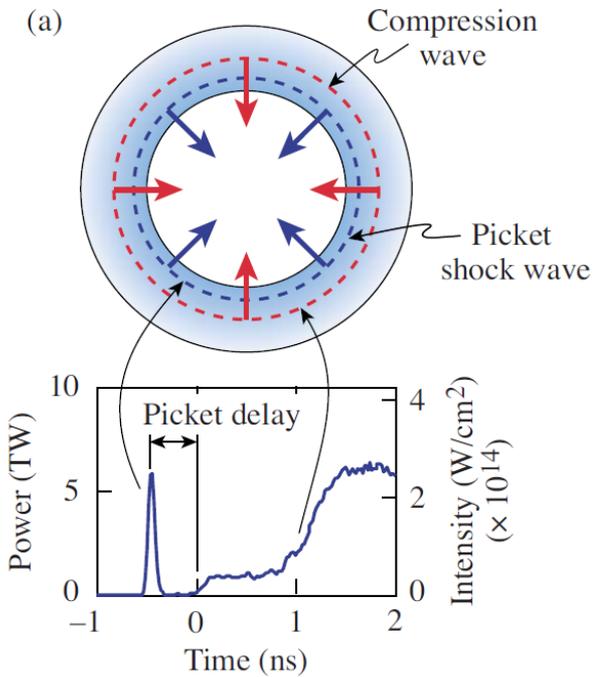
Energy in megajoules ● = 1



External “spark” can be used for ignition

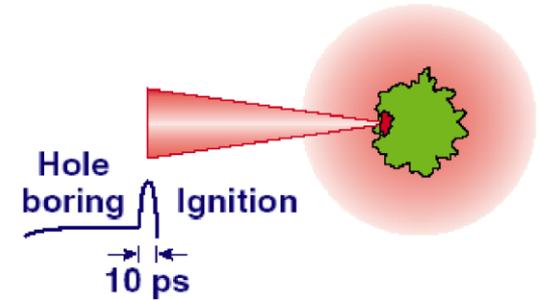


- Shock ignition

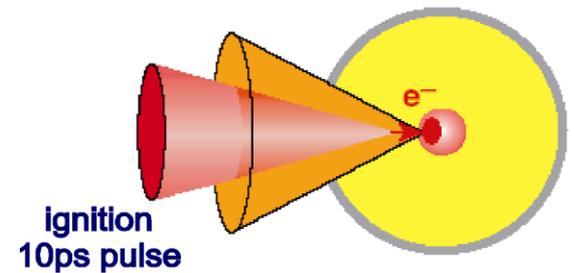


- Fast ignition

- a) channeling FI concept



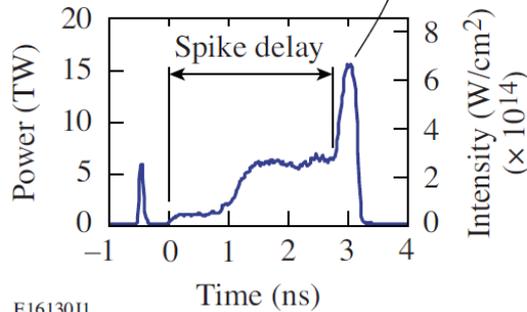
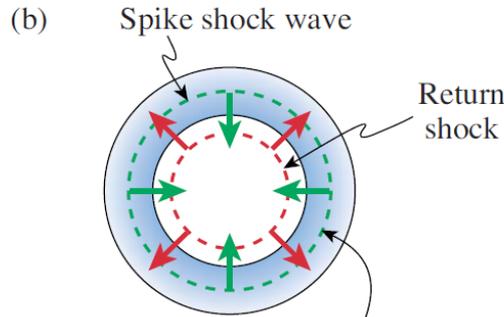
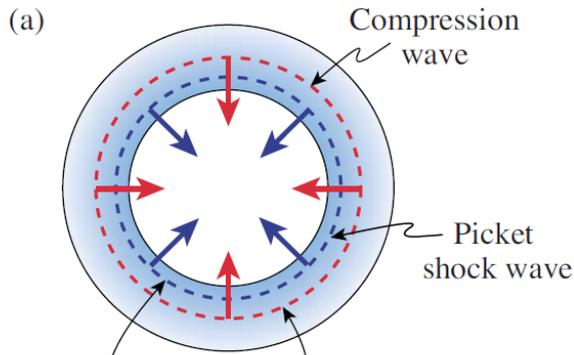
- b) cone-in-shell FI concept



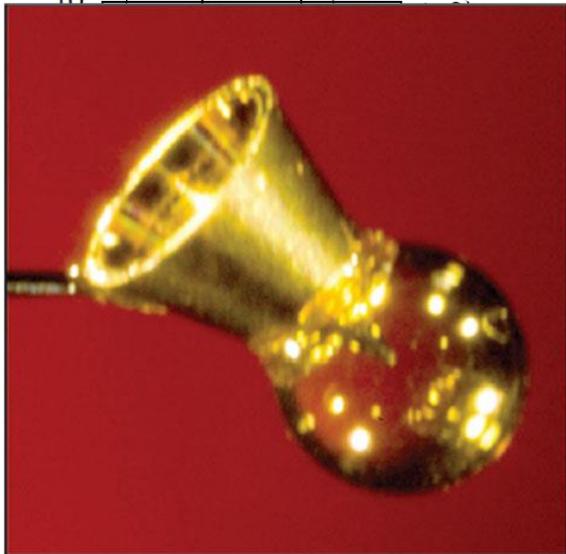
External “spark” can be used for ignition



- Shock ignition

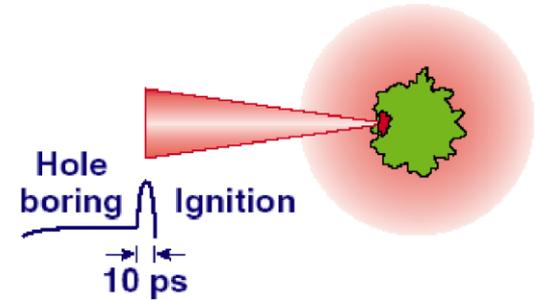


E16130J1

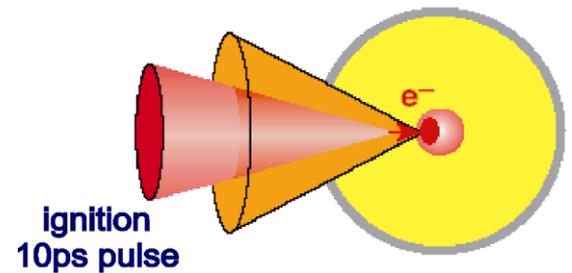


- Fast ignition

- a) channeling FI concept



- b) cone-in-shell FI concept

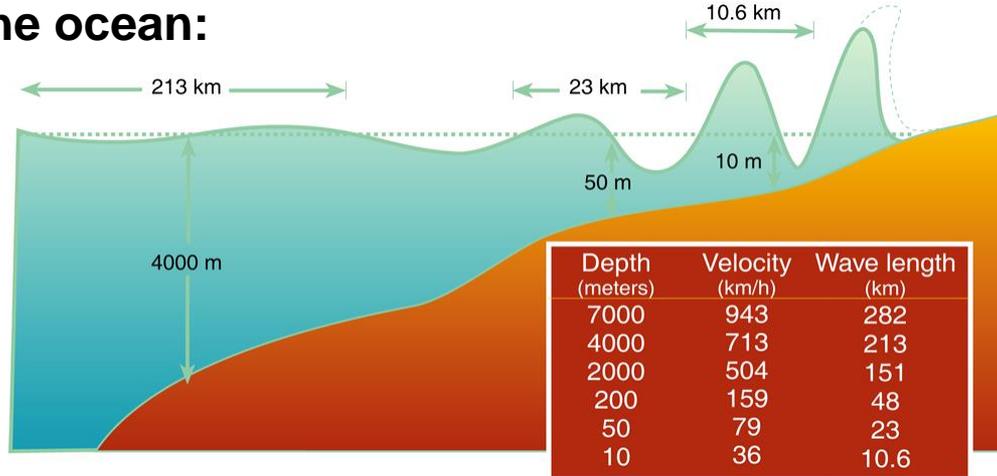


J. Badziak, Bull.Polish Acad. Sci. Tech. Sci.Phys. Plasmas 15, 056306 (2008)
 T. Ditmire, etc., J. Fusion Energy 42, 27 (2023)

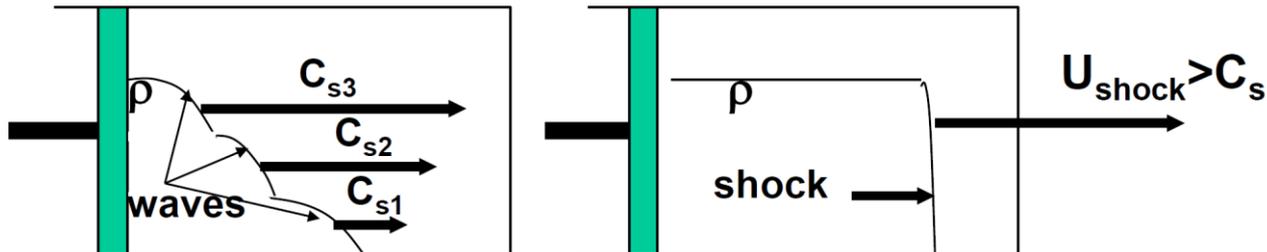
A shock is formed due to the increasing sound speed of a compressed gas/plasma



- **Wave in the ocean:**



- **Acoustic/compression wave driven by a piston:**

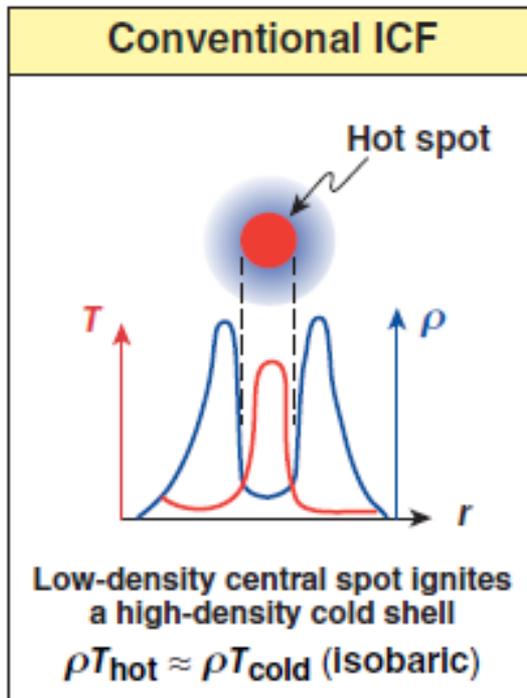


$$C_s \sim \sqrt{\frac{p}{\rho}} \sim \sqrt{\frac{\alpha \rho^{5/3}}{\rho}} \sim \sqrt{\alpha} \rho^{1/3}$$

Ignition can happen by itself or being triggered externally



Self-ignition



External “spark” for fast ignition

