Introduction to Nuclear Fusion as An Energy Source

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Institute of Space and Plasma Sciences, National Cheng Kung University

Lecture 10

2024 spring semester

Wednesday 9:10-12:00

Materials:

https://capst.ncku.edu.tw/PGS/index.php/teaching/

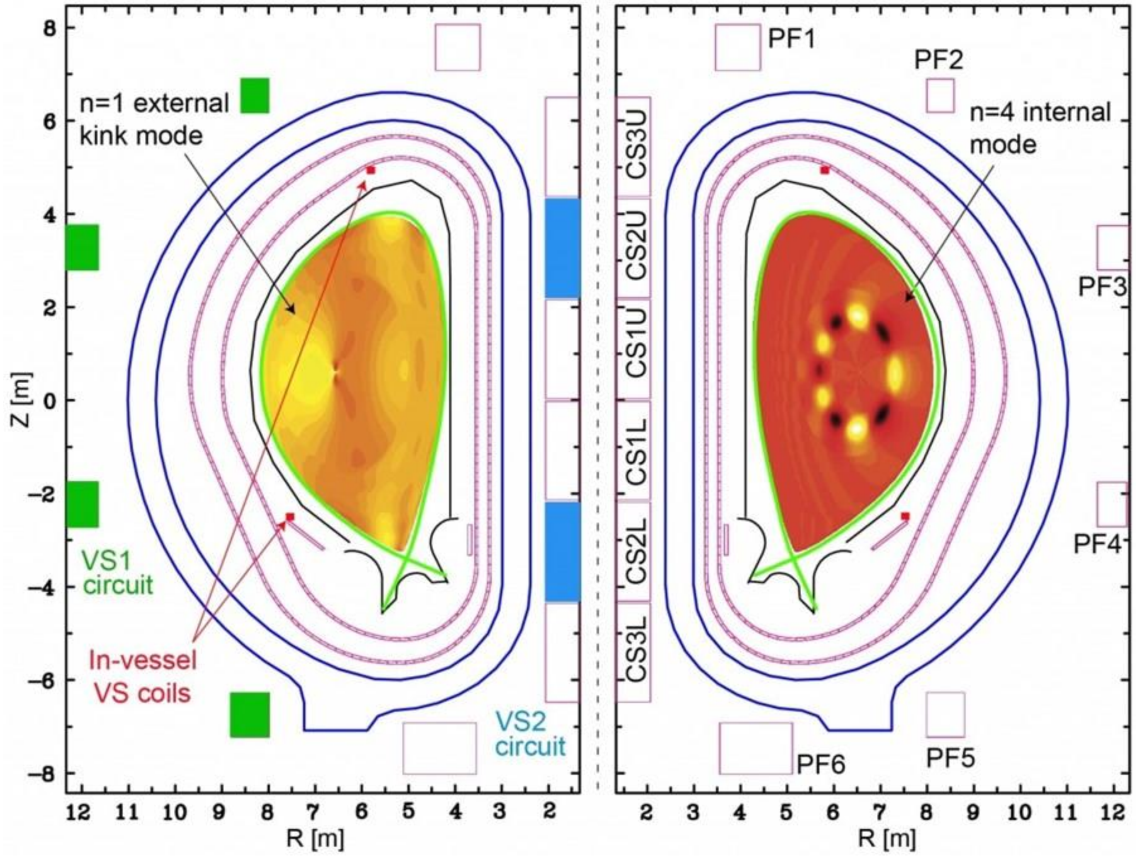
Online courses:

https://nckucc.webex.com/nckucc/j.php?MTID=ma76b50f97b1c6d72db61de9eaa9f0b27

2024/5/15 updated

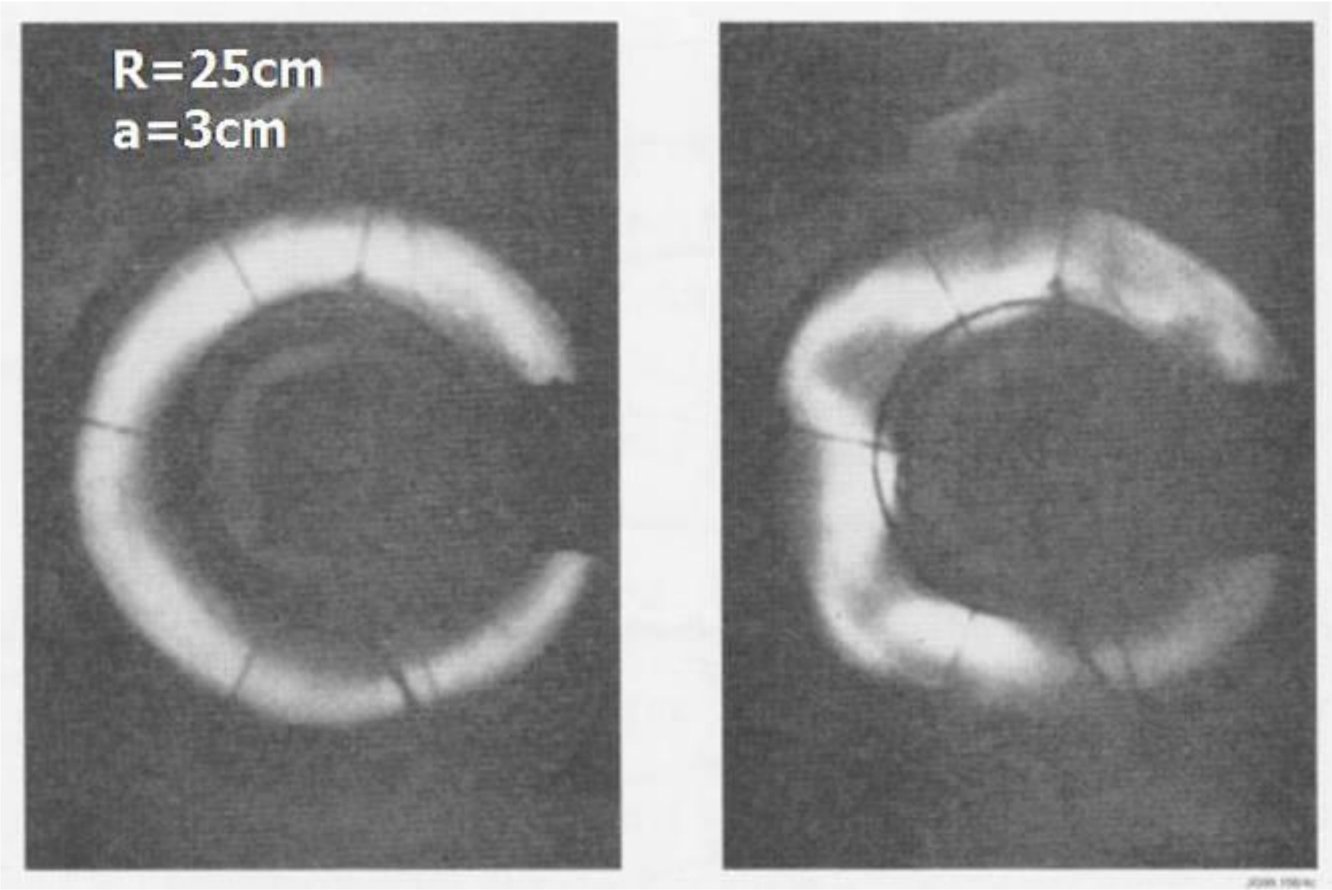
External mode vs internal mode

- **Predicted behaviors of the plasma in ITER**



https://www.iter.org/newsline/-/3401

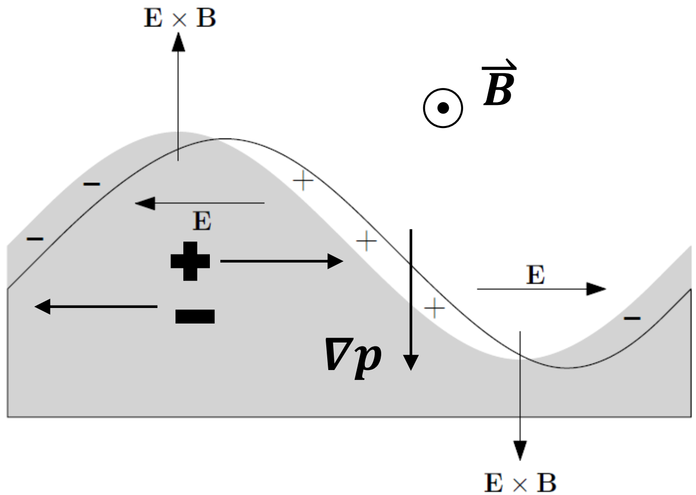
Kink instability in Tokamak

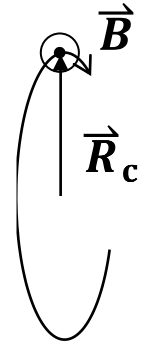


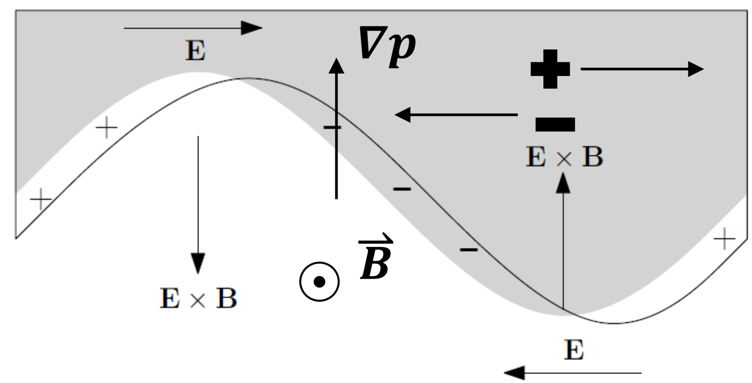
https://en.wikipedia.org/wiki/Kink_instability

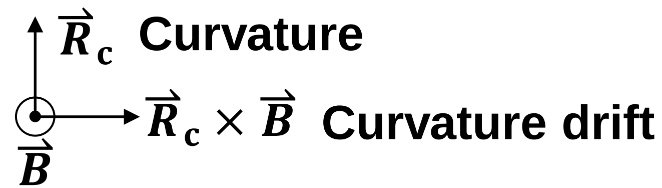
Pressure driven instability – interchange perturbations

- **Unstable: bad curvature** $\vec{R}_c \cdot \nabla p < 0$
- **stable: good curvature** $\vec{R}_c \cdot \nabla p > 0$

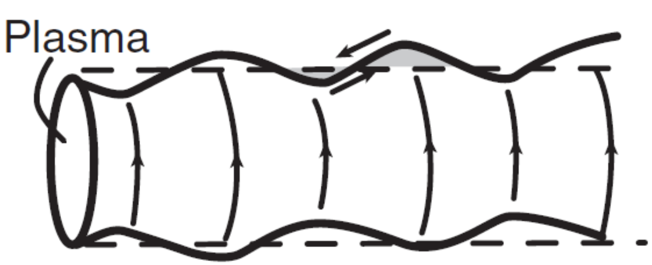








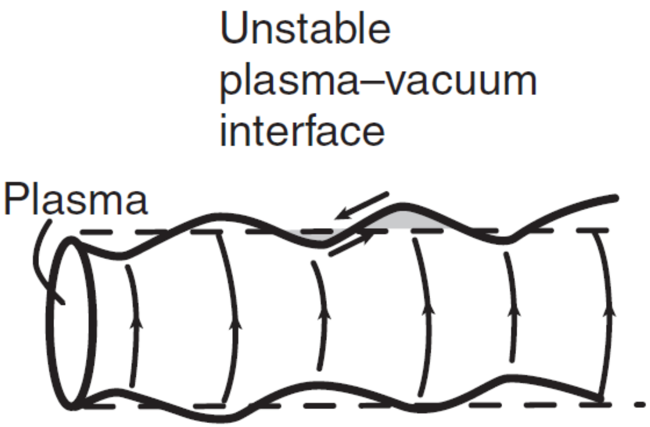
Unstable plasma–vacuum interface



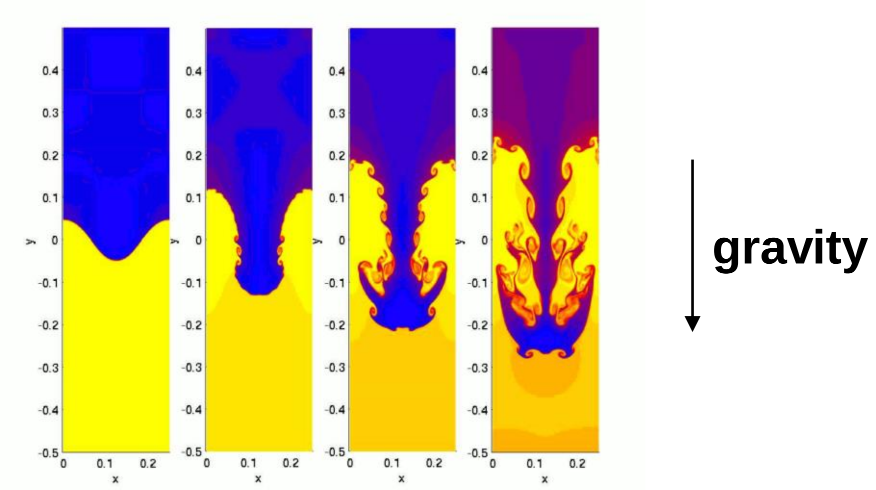
Stable plasma–vacuum interface

Ben Dudson, Lecture note of Magnetic Confinement Fusion 2014

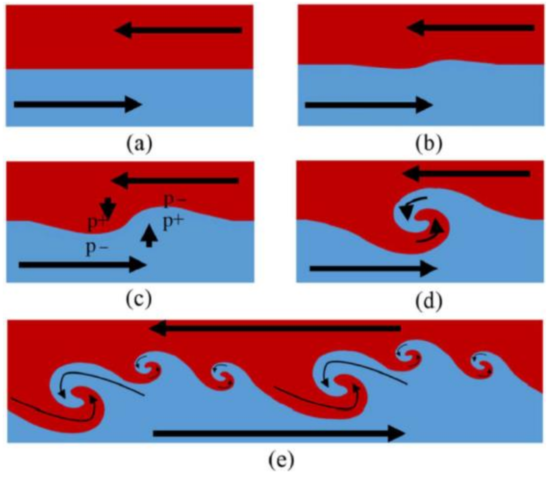
Rayleigh-Taylor instability



- **Rayleigh-Taylor instability**

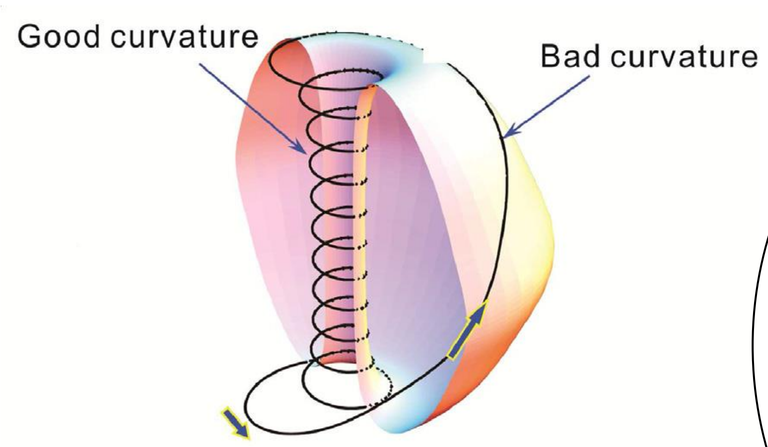


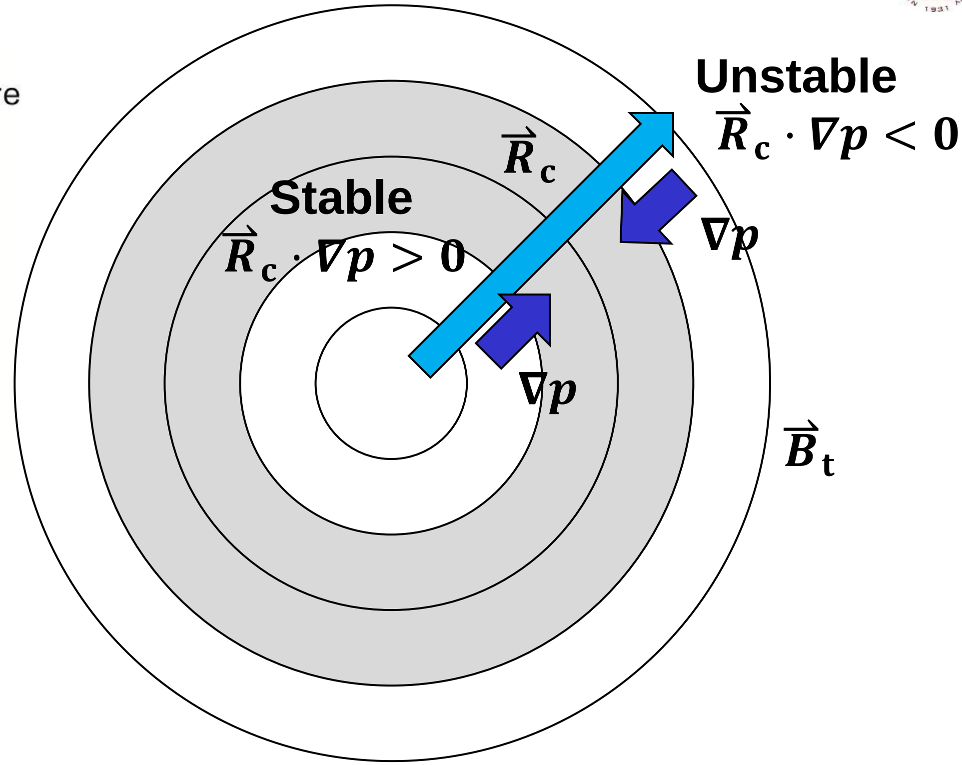
- **Kelvin-Helmholtz instability**



https://en.wikipedia.org/wiki/Rayleigh%E2%80%93Taylor_instability
https://en.wikipedia.org/wiki/Kelvin%E2%80%93Helmholtz_instability
Xie Lei et al, Energy Report **7**, 2262 (2021)

Pressure driven instability – interchange perturbations





- **Suydam criterion for cylindrical plasmas:**

$$\mu_o \frac{2r^2}{{B_\theta}^2} \frac{1}{s^2} \vec{R}_c \cdot \nabla p > -\frac{1}{4}$$

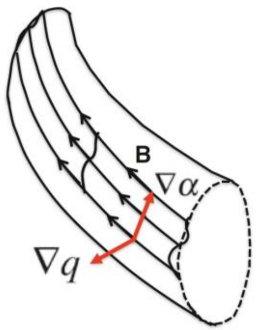
Shear: $s = \frac{r}{q} \frac{dq}{dr}$

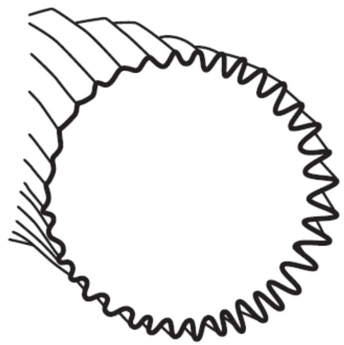
- **Mercier criterion for tokamak:**

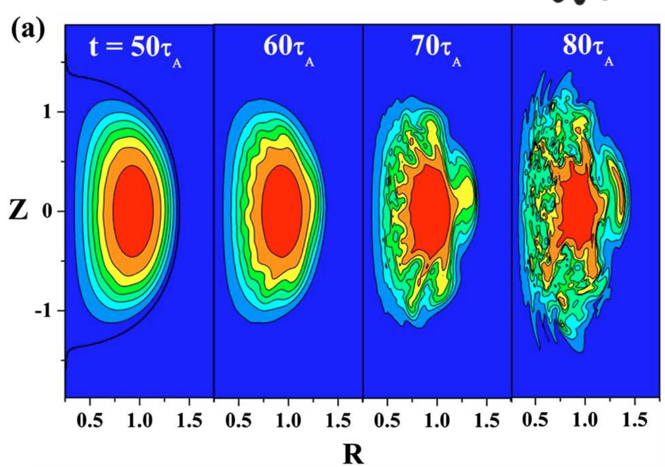
$$D = -\mu_o \frac{2r}{B^2} \frac{1}{s^2} \frac{dp}{dr} (1 - q^2) < \frac{1}{4}$$

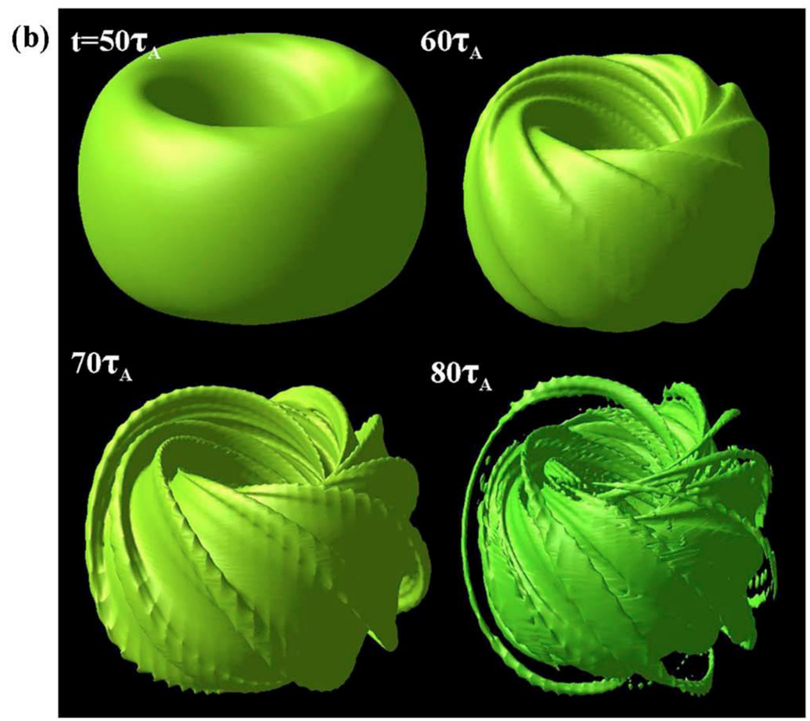
Ballooning mode – show wavelength mode

$$\delta W_F = \frac{1}{2}\int d\,\vec{r}\left[\frac{\left|\vec{Q}_\perp\right|^2}{\mu_o} + \frac{B^2}{\mu_o}\left|\nabla\cdot\vec{\xi}_\perp + 2\,\vec{\xi}_\perp\cdot\vec{\kappa}\right|^2 + \gamma p\left|\nabla\cdot\vec{\xi}\right|^2 \right.$$
$$\left. -2\left(\vec{\xi}_\perp\cdot\nabla p\right)\left(\vec{\kappa}\cdot\vec{\xi}_\perp^{\,*}\right) - J_{||}\left(\vec{\xi}_\perp^{\,*}\times\vec{b}\right)\cdot\vec{Q}_\perp\right]$$



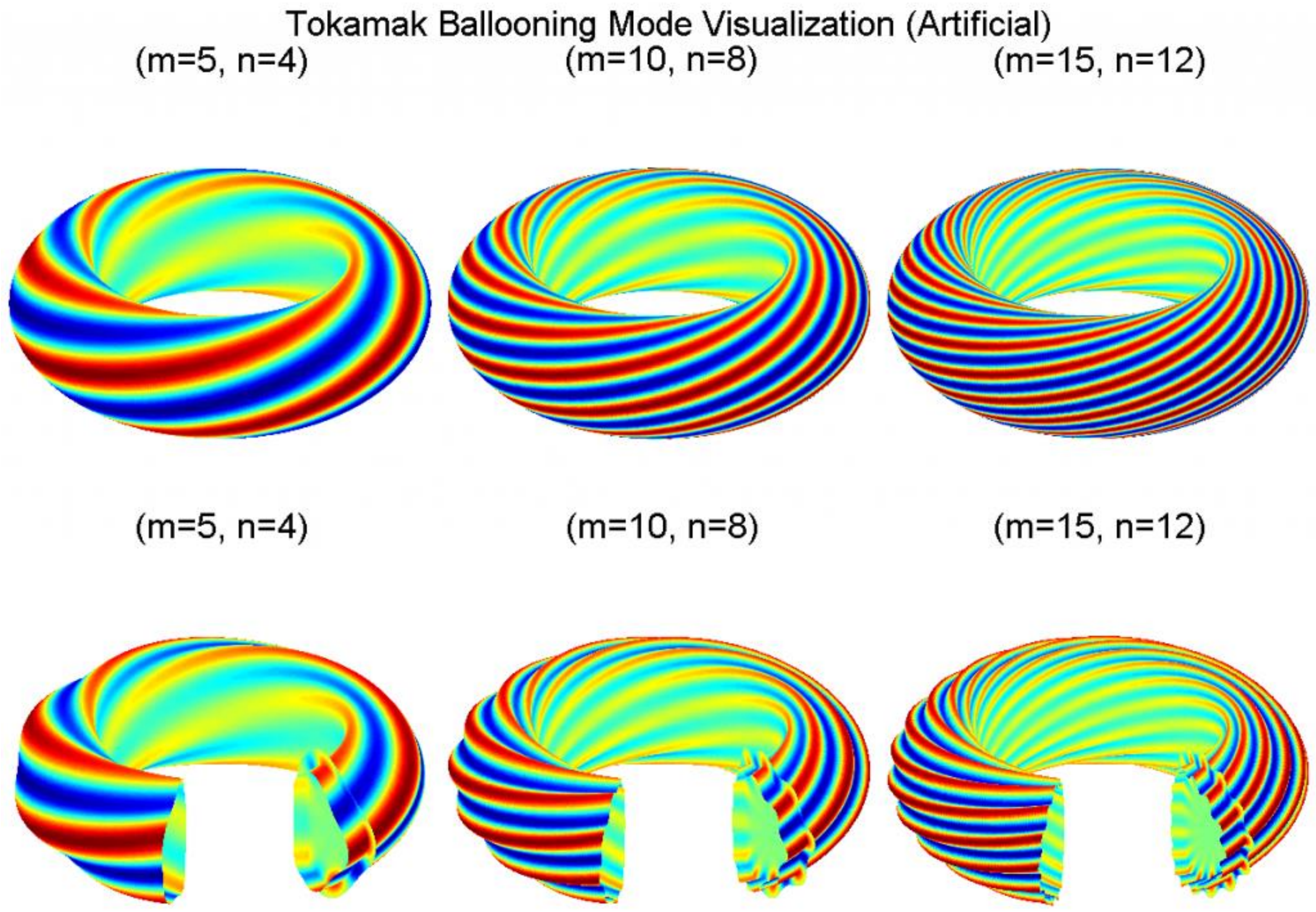






R. Khan et al, Phys. Plasmas **14**, 062302 (2007)

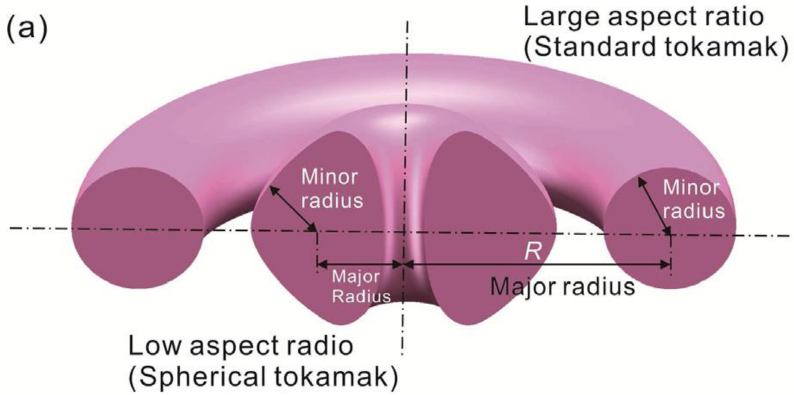
Ballooning mode – show wavelength mode

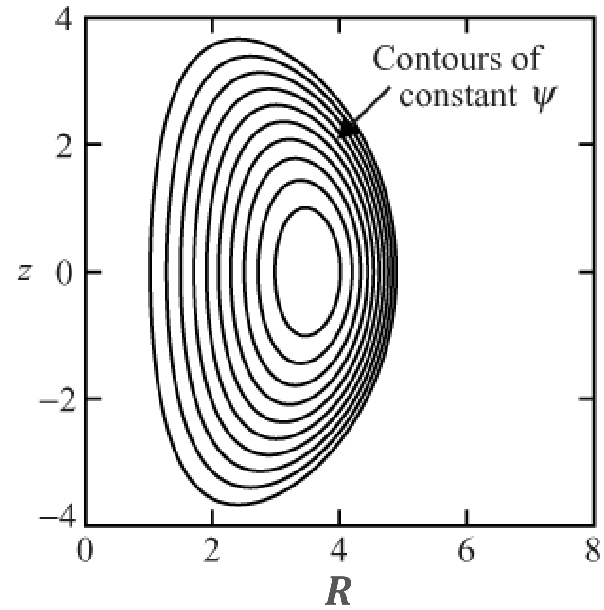


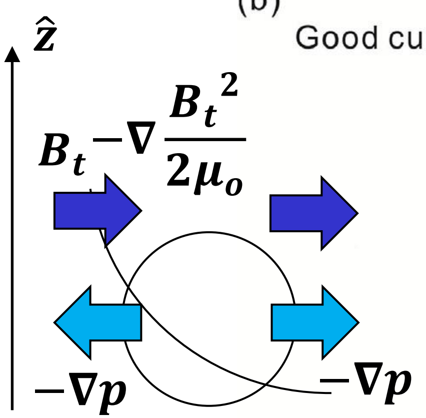
https://blog.sciencenet.cn/home.php?mod=space&uid=222053&do=album&picid=308476#pic_block

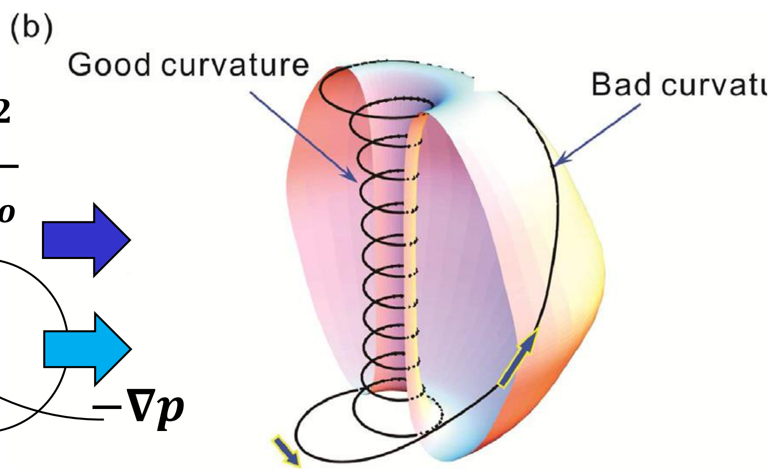
The Spherical tokamak

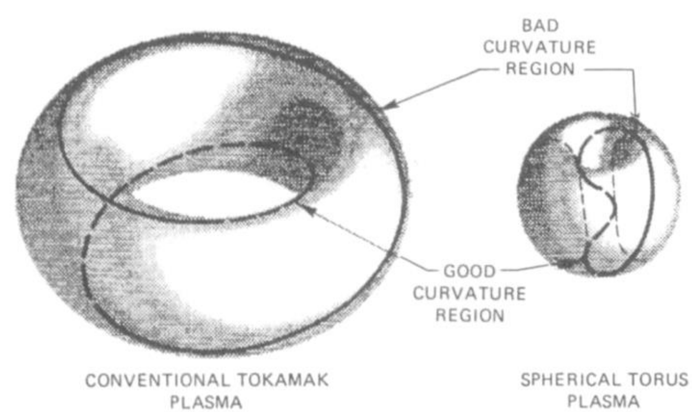
- **Aspect ratio R_o/a ~ 1.6**









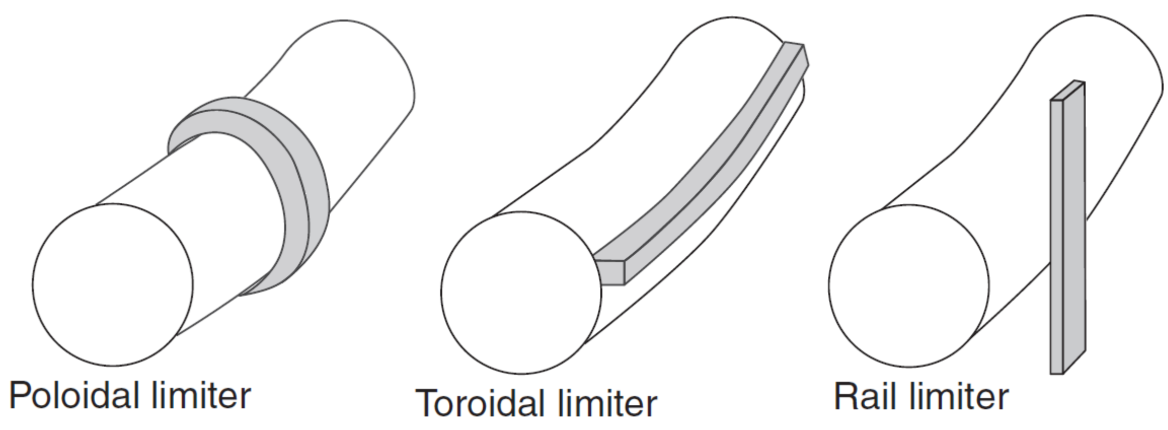


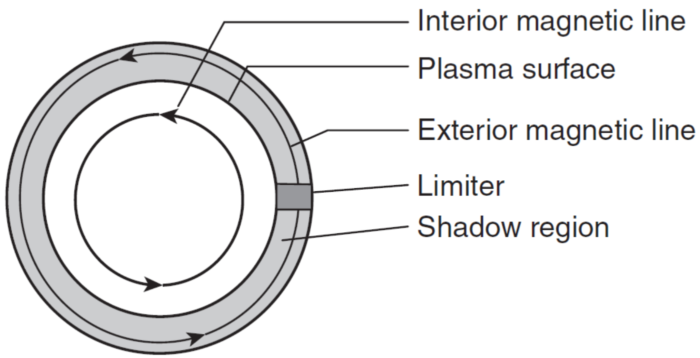
Zhe Gao, Matter and Radia. Extrem., **1**, 153 (2016)
Y-k. M.Peng and D. J. Strickler, Nucl. Fusion **26**, 769 (1986)

The Spherical tokamak

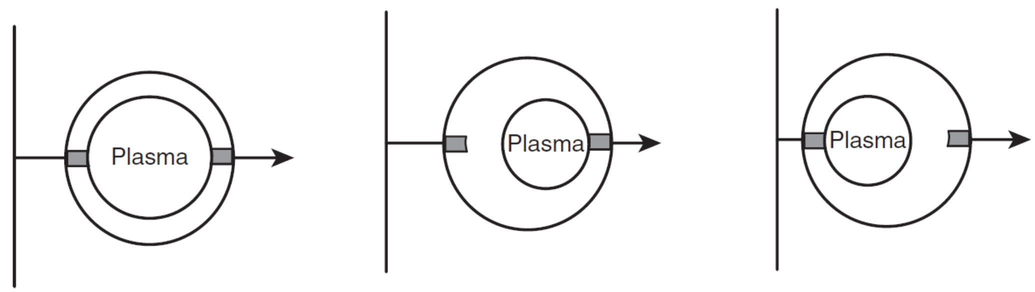
- **Aspect ratio $R_o/a \sim 1.6$**
- **Advantages:**
  - **Higher β_t limit.**
  - **A compact design almost spherical in appearance.**
- **Challenges:**
  - **Minimum space is given in the center of the torus to accommodate the toroidal field coils.**
  - **With a very compact design the technology associated with the construction and maintenance of the device may be more difficult than for a "normal" tokamak.**
  - **Large currents will have to be driven noninductively, a costly and physically difficult requirement.**

Limiter protects the vacuum chamber from plasma bombardment and defines the edge of the plasma



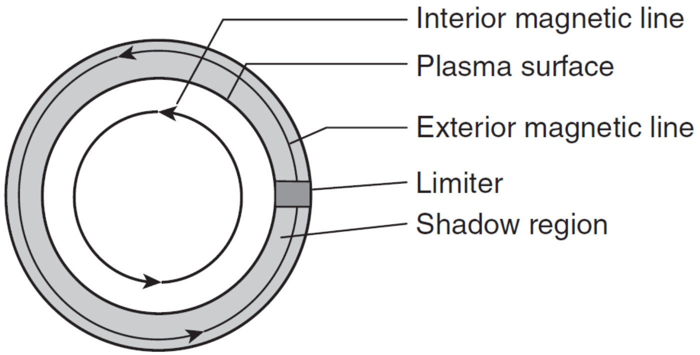


- **Vertical field is correct.**
- **Vertical field is too small.**
- **Vertical field is too large.**

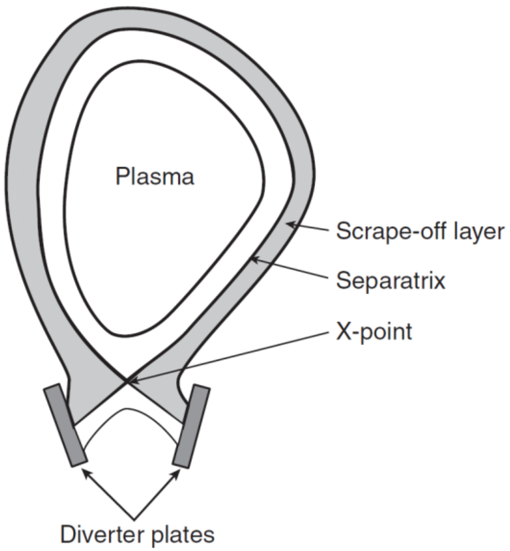


Limiter protects the vacuum chamber from plasma bombardment and defines the edge of the plasma

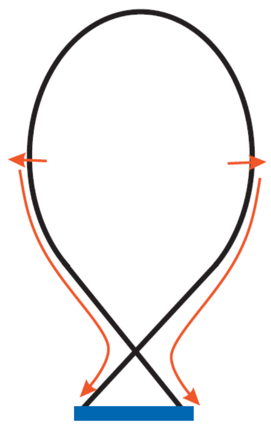
- **A mechanical limiter is a robust piece of material, often made of tungsten, molybdenum, or graphite placed inside the vacuum chamber.**
- **Some of the particles of the limiter surface may escape. Neutral particles can penetrate some distance into the plasma before being ionized.**
- **The high-z impurities can lead to significant additional energy loss in the plasma through radiation.**
- **In ignition experiments and fusion reactors, the bombardment is more intense and extends over longer periods of time. In addition, if the impurity level is too high, it may not be possible to achieve a high enough temperature to ignite.**

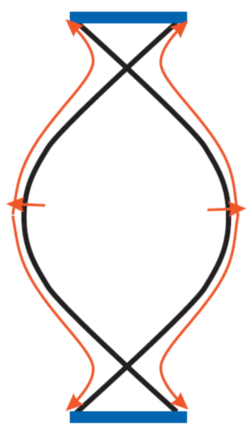


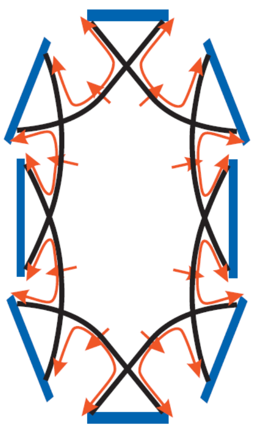
The magnetic divertor – guide a narrower layer of magnetic lines away from the edge of the plasma



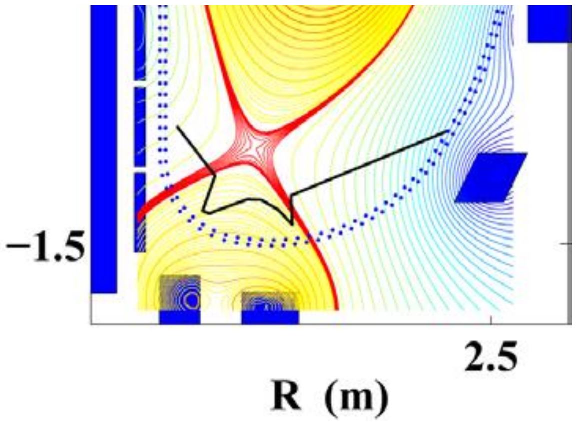
- **Single-null poloidal-field divertors for tokamak**
- **Double-null poloidal-field divertors for tokamak**
- **Island divertor for stellerators**



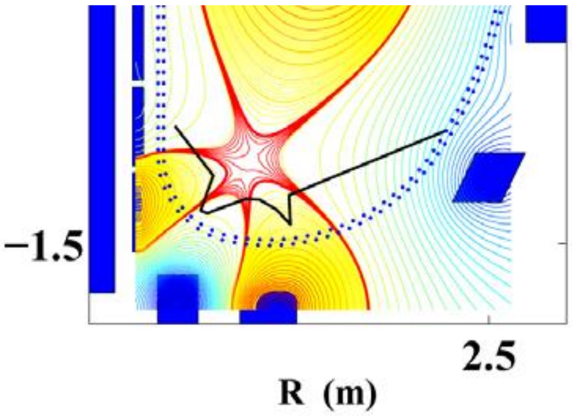




- **Standard**



- **Snowflake**



Y. Feng, Nucl. Fusion, **46**, 807 (2006)
L Xue *et al*, Plasma Phys. Control. Fusion **58**, 055005 (2016)

Pros and cons of a divertor

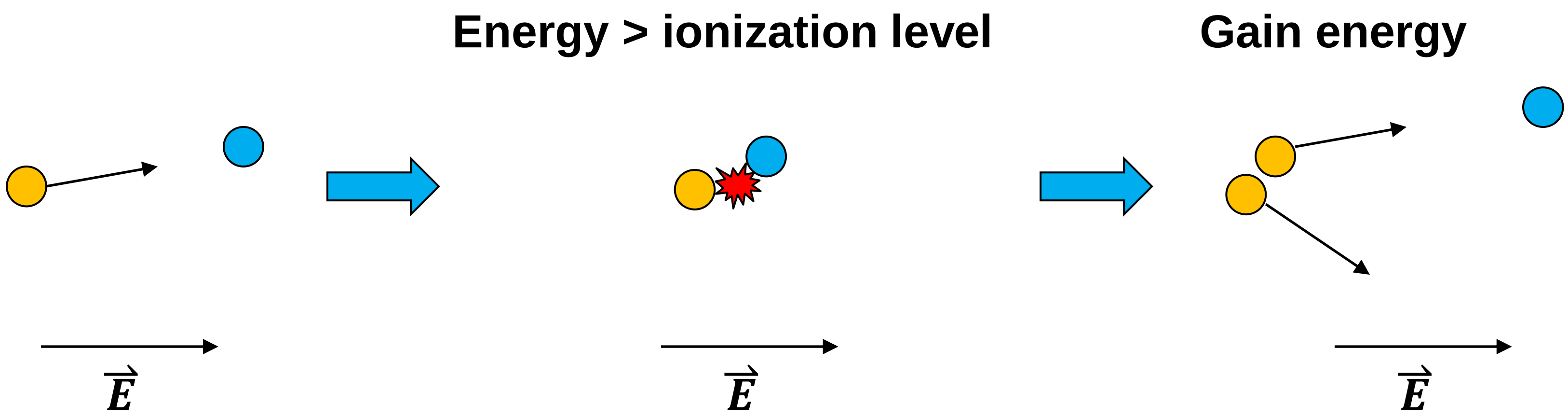
- **Advantages:**
  - **The collector plate is remote from the plasma. There is space available to spread out the magnetic lines.**
  - **A lower intensity of particles and energy bombard the collector plate leading to a longer replacement time.**
  - **It is more difficult for impurities to migrate into the plasma.**
  - **There are longer distance distances to travel and if a neutral particle becomes ionized before or during the time it crosses the divertor layer on its way toward the plasma, its parallel motion then carries it back to the collector plate.**
  - **The larger divertor chamber provides more access to pump out impurities.**
  - **The plasma edge is not in direct contact with a solid material such as a limiter.**
- **Disadvantages: larger and more complex system and more expensive.**

Course Outline

- **Magnetic confinement fusion (MCF)**
  - **Gyro motion, MHD**
  - **1D equilibrium (z pinch, theta pinch)**
  - **Drift: ExB drift, grad B drift, and curvature B drift**
  - **Tokamak, Stellarator (toroidal field, poloidal field)**
  - **Magnetic flux surface**
  - **2D axisymmetric equilibrium of a torus plasma: Grad-Shafranov equation.**
  - **Stability (Kink instability, sausage instability, Safety factor Q)**
  - **Central-solenoid (CS) start-up (discharge) and current drive**
  - **CS-free current drive: electron cyclotron current drive, bootstrap current.**
  - **Auxiliary Heating: ECRH, Ohmic heating, Neutral beam injection.**

Collisions play an important role in ionization process

- At the microscopic level, breakdown requires the presence of sufficiently energy charge particles that have acquired enough energy from the applied electric field between two energy-dissipating collisions to ionize the material and to create more charge particles.



In most cases, <u>electrons</u> dominate the breakdown process since its mobility is much larger than that of ions

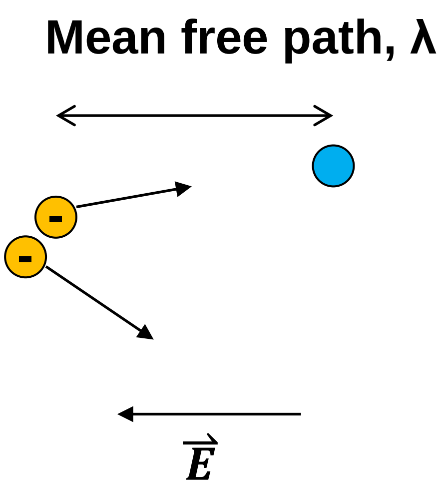
$$E_{\mathrm{k}} = \frac{1}{2} m v^2 \qquad v = \sqrt{\frac{2E_{\mathrm{k}}}{m}} \qquad E_{\mathrm{k}} \sim kT$$

Collision time: $t = \dfrac{s}{\sqrt{\dfrac{2E_{\mathrm{k}}}{m}}} \sim \dfrac{n^{-1/3}}{\sqrt{T}} \sqrt{m}$ $\qquad n = \dfrac{\#/}{V} \sim \dfrac{\#//}{S^3}$ $\qquad s \sim n^{-1/3}$

$$\frac{m_{\mathrm{i}}}{m_{\mathrm{e}}} \sim 2000 \times \text{Atomic mass} \qquad \frac{t_{\mathrm{i}}}{t_{\mathrm{e}}} \sim 45 \times \sqrt{A}$$

Mean free path is important in ionization process

- For an electron to acquire enough energy between collisions, its <u>mean free path</u> in the material must be sufficiently long.



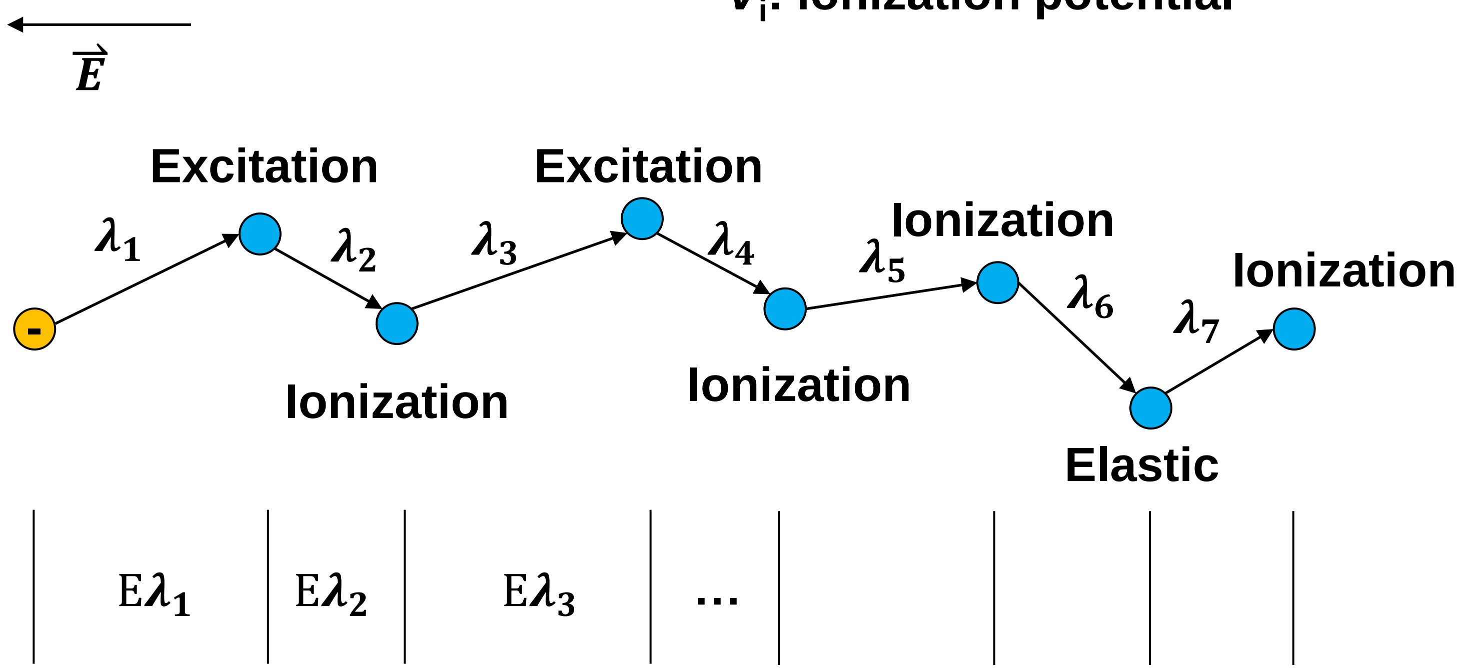
$$E_k = e \times E \times \lambda = \mathrm{eV}$$

Electron impact ionization is the most important process in a breakdown of gases

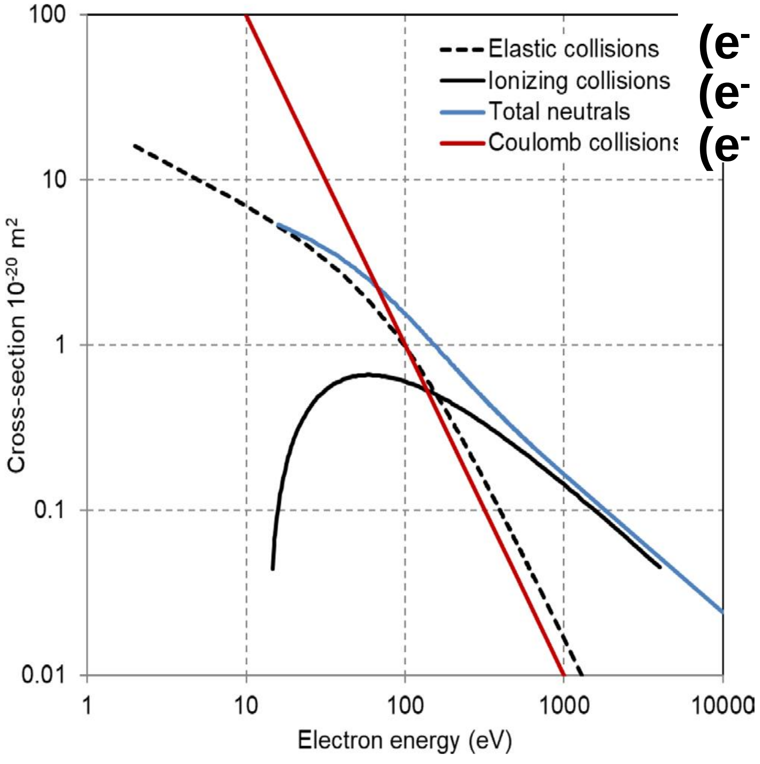
- **Electron impact ionization: A + e^- → $A^+ + e^- + e^-$**
  - **The most important process in the breakdown of gases but is not sufficient alone to result in the breakdown.**

$$eE\lambda_{e,i} \geq eV_i$$

V_i: ionization potential



Collision cross-sections of elastic, ionizing collisions between e^- and H_2 and coulomb collisions



(e^- vs H_2)
($e^- + H_2 \rightarrow 2e^- + H^+ + H$)
(e^- vs H^+)

$$\sigma_{\mathrm{elastic,m^2}} = \frac{1.75\times10^{-16}}{\left(W_{\mathrm{e,eV}}{}^{1.5}+750\right)\sqrt{W_{\mathrm{e,eV}}}}$$

$$\sigma_{\mathrm{ionizing,m^2}} \sim 3\times10^{-20}\left(ln\epsilon - 0.69 + \frac{0.66}{\epsilon}\right)\epsilon^{-1}$$

$$\epsilon = \frac{W_{\mathrm{e}}}{E_{\mathrm{ion}}} \quad E_{\mathrm{ion}} \sim 15\ \mathrm{eV\ for\ H_2}$$

$$\sigma_{\mathrm{coulomb}} = \frac{e^4}{4\pi\epsilon_{\mathrm{o}}{}^2 m_{\mathrm{e}}{}^2 v_{\mathrm{e}}{}^2}\ln\Lambda$$

$$= \frac{e^4}{16\pi\epsilon_{\mathrm{o}}{}^2 W_{\mathrm{e}}{}^2}\ln\Lambda \sim 10^{-16} W_{\mathrm{e}}{}^{-2}$$

$$\mathrm{for}\ \ln\Lambda \sim 13-15$$

$$\nu = n v_E \sigma$$

$$\lambda = \frac{1}{n\sigma}$$

P. C. de Vries and Y. Gribov, Nucl. Fusion, **59**, 096043 (2019)

Townsend avalanche process for Tokamak breakdown

- **The first Townsend coefficient α: the number of ionizing collisions made on the average by an electron as it travels 1 m along the electric field:**

$$\alpha \sim \frac{1}{\lambda_i} = \frac{\nu_{ei}}{\bar{v}_e} = \frac{n_0 \langle \sigma v_e \rangle_{ne}}{\bar{v}_e} = \frac{p}{T} \frac{\langle \sigma v \rangle_{ne}}{\bar{v}_e} \equiv Ap \qquad A \equiv \frac{1}{T} \frac{\langle \sigma v \rangle_{ne}}{\bar{v}_e}$$

- **Number of primary electrons with energy higher than the ionization potential:**

$$dn_e = -n_e \frac{dx_i}{\lambda_i} \quad \Rightarrow \quad \frac{n_e(x_i)}{n_{e0}} = \exp\left(-\frac{x_i}{\lambda_i}\right)$$

$$\alpha \equiv \frac{\#/\text{ ionization collisions}}{\text{per electron}} \times (\#/\text{electron with E} > \text{ionization potential})$$

$$= \frac{1}{\lambda_i} \frac{n_e(x_i)}{n_{e0}} = \frac{1}{\lambda_i} \exp\left(-\frac{x_i}{\lambda_i}\right)$$

$$\alpha = Ap \exp(-Apx_i)$$

***A* = 3.83 m^{-1}Pa^{-1} = 1060 m^{-1}Torr^{-1}**

***B* = 96.6 Vm^{-1}Pa^{-1} = 35000 m^{-1}Torr^{-1}**

$$\alpha = Ap\exp\left(-\frac{AV^*}{E/p}\right) \equiv Ap\exp\left(-\frac{B}{E/p}\right) \qquad x_i \approx \frac{V^*}{E} \text{ where } V^* > V_i$$

- **The parameters *A* and *B* must be experimentally determined.**

Paschen's curve for minimum breakdown voltage

$$\alpha \sim \frac{1}{\lambda_{\mathrm{i}}} = Ap \quad \alpha = Ap\exp\left(-\frac{B}{E/p}\right)$$

A = 3.83 $m^{-1}Pa^{-1}$ = 1060 $m^{-1}Torr^{-1}$

B = 96.6 $Vm^{-1}Pa^{-1}$ = 35000 $m^{-1}Torr^{-1}$

- For p=1 mPa, $\lambda_{\mathrm{i}} \sim 262\ \mathrm{m}$, for ITER, $2\pi r_{\mathrm{o}} \sim 38\ \mathrm{m}$ $\quad \frac{\lambda_{\mathrm{i}}}{2\pi r_{\mathrm{o}}} \sim 7$

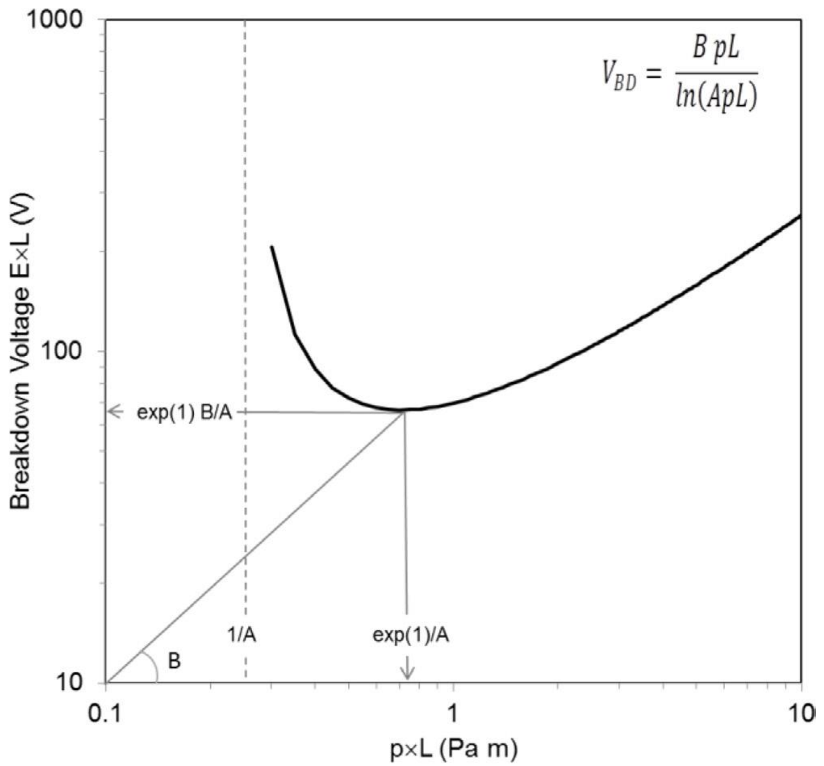
- For breakdown to happen:

$$\alpha L > 1 \quad \alpha L = ApL\exp\left(-\frac{BpL}{V_{\mathrm{BD}}}\right) > 1$$

$$\exp\left(-\frac{BpL}{V_{BD}}\right) > \frac{1}{ApL}$$

$$-\frac{BpL}{V_{\mathrm{BD}}} > -\ln(ApL)$$

$$V_{\mathrm{BD}} > \frac{BpL}{\ln(ApL)} \quad E_{\mathrm{BD}} > \frac{Bp}{\ln(ApL)}$$



P. C. de Vries and Y. Gribov, Nucl. Fusion, **59**, 096043 (2019)

Perpendicular stray-field (B_z) needs to be as small as possible

- For *p*=1 mPa, $\lambda_i \sim 262$ m , for ITER, $2\pi r_o \sim 38$ m $\frac{\lambda_i}{2\pi r_o} \sim 7$

$$\frac{B_z}{B_T} \sim 10^{-3} \qquad \lambda_i \times \frac{B_z}{B_T} = 0.26\ \mathrm{m}$$

- For ITER,

$$E \sim E_{\mathrm{loop}} = 0.3\ \mathrm{V/m}$$

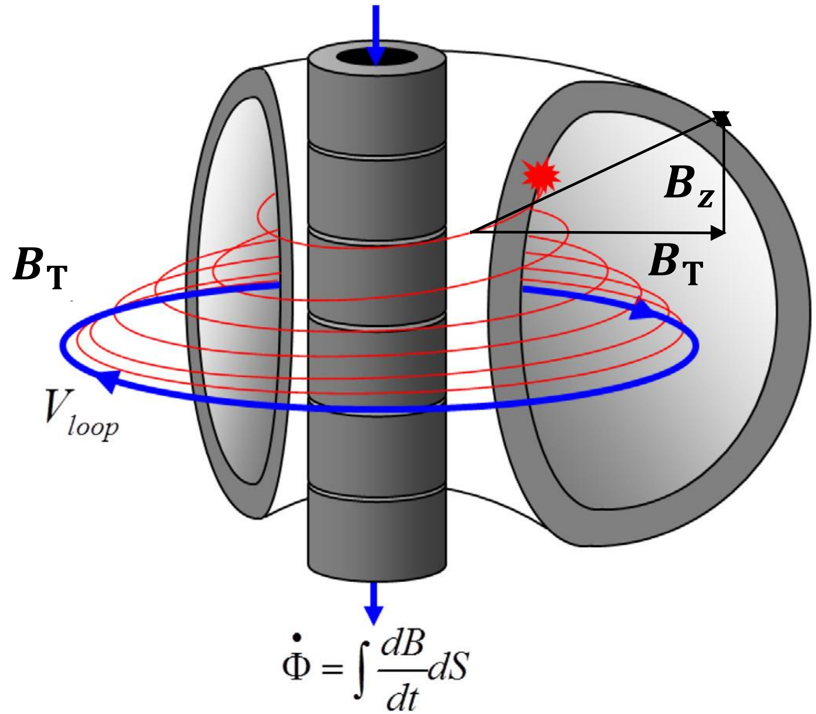
$$p = 1\ \mathrm{mPa} \qquad L_{\mathrm{BD}} = 357\ \mathrm{m}$$

- Required loop field:

$$E_{\mathrm{BD}} > \frac{Bp}{\ln(ApL)}$$

$$E_{\mathrm{BD}} > \frac{1.25 \times 10^4 P_{\mathrm{Torr}}}{\ln(510PL_{\mathrm{c}})}$$

$$L_{\mathrm{c}} = 0.25 a_{\mathrm{eff}} \left(\frac{B_z}{B_T}\right)$$



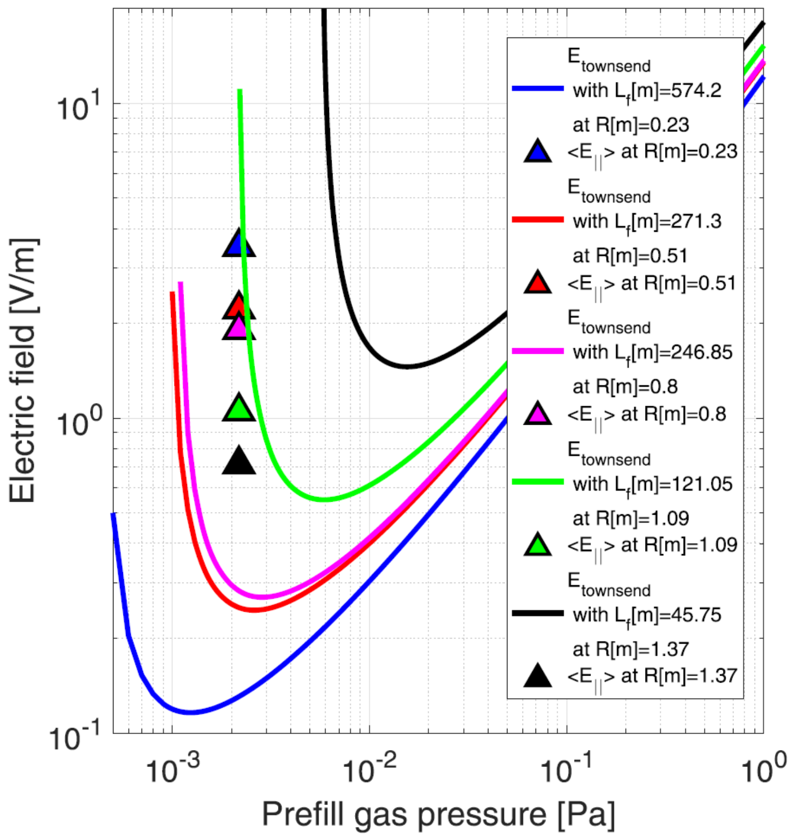
- W/ preionization: $E_T \frac{B_T}{B_z} \geq 100\ \mathrm{V/m}$
- Purely Ohmic discharges: $E_T \frac{B_T}{B_z} \geq 100\ \mathrm{V/m}$

P. C. de Vries and Y. Gribov, Nucl. Fusion, **59**, 096043 (2019)
S. J. Doyle et al, Fusion Eng. Des. **171**, 112706 (2021)

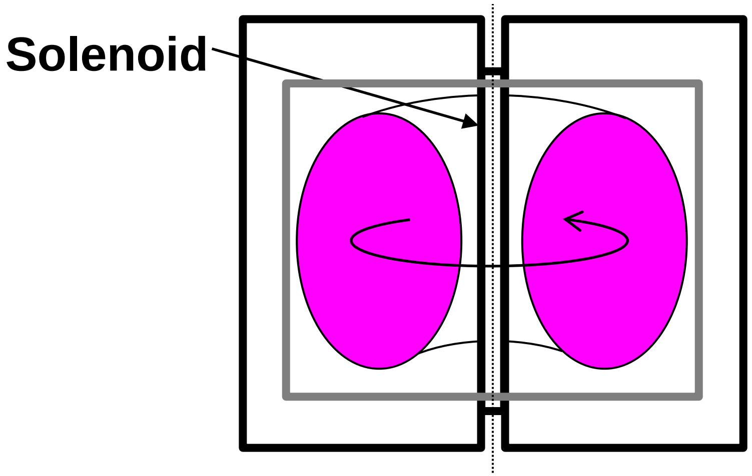
Examples or required loop electric fields

$$E_{BD} > \frac{1.25\times10^4 P_{Torr}}{\ln(510PL_c)}$$

$$L_c = 0.25a_{eff}\left(\frac{B_T}{B_z}\right)$$



- W/ preionization: $E_T\frac{B_T}{B_z} \geq 100\,\mathrm{V/m}$
- Purely Ohmic discharges: $E_T\frac{B_T}{B_z} \geq 100\,\mathrm{V/m}$



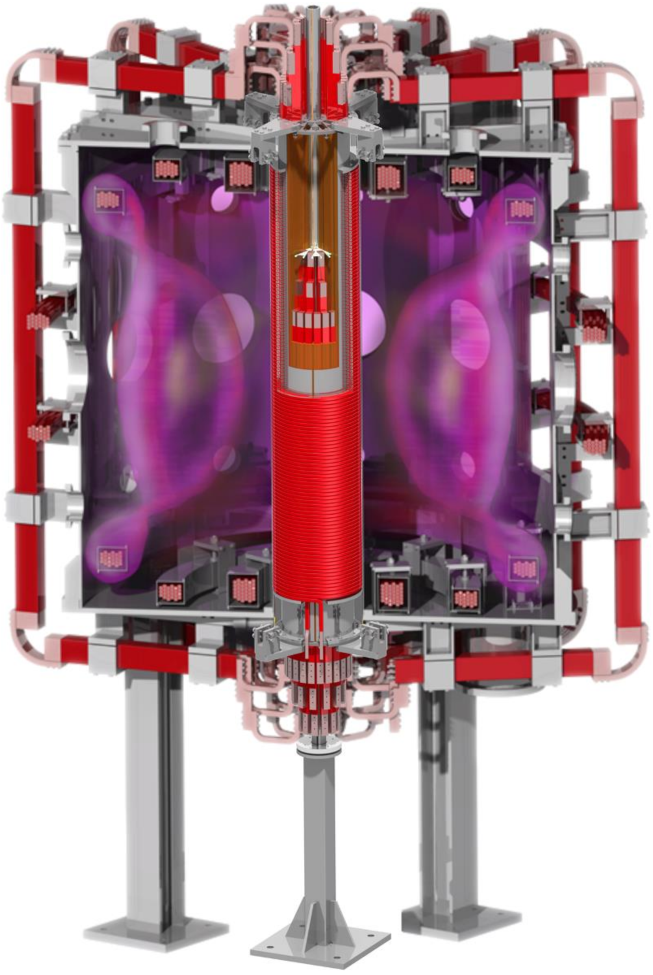
$$V_\phi = -\frac{\partial\phi}{\partial t} \equiv M\frac{\partial I_{CS}}{\partial t}$$

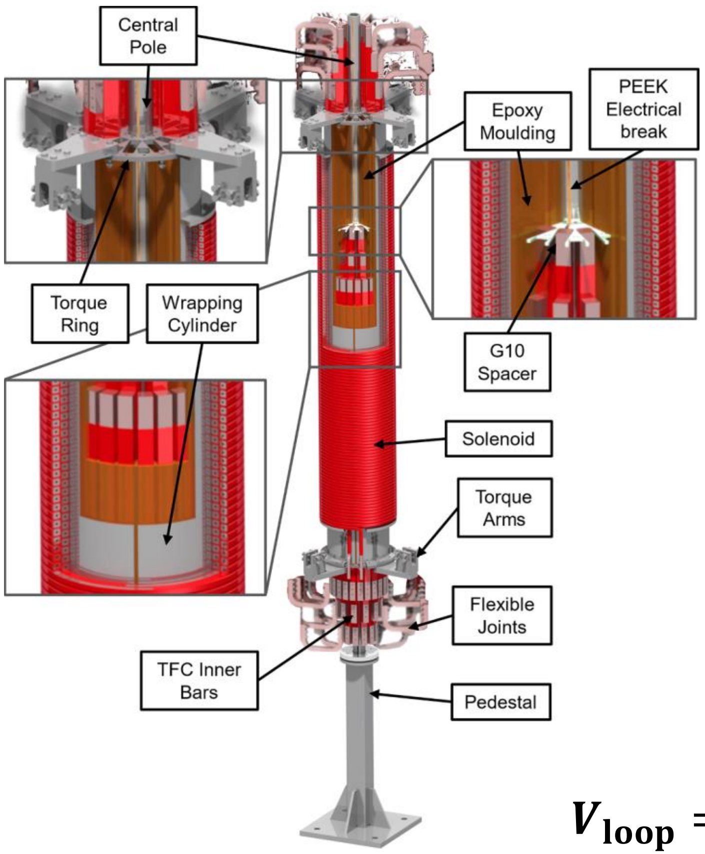
$$E_\phi = -\frac{1}{2\pi r}\frac{\partial\phi}{\partial t}$$

H.-T. Kim, etc., Nucl. Fusion **62**, 126012 (2022)
S. J. Doyle et al, Fusion Eng. Des. **171**, 112706 (2021)

Central solenoid can be used to provide the required loop voltage for breakdown

- **SMART:**





$$V_{\mathrm{loop}} = \frac{A_{\mathrm{sol}} \mu N_{\mathrm{sol}}}{L_{\mathrm{sol}}} \frac{dI_{\mathrm{sol}}}{dt}$$

J. Segado-Fernandez et al, Fusion Eng. Des. **193**, 113832 (2023)

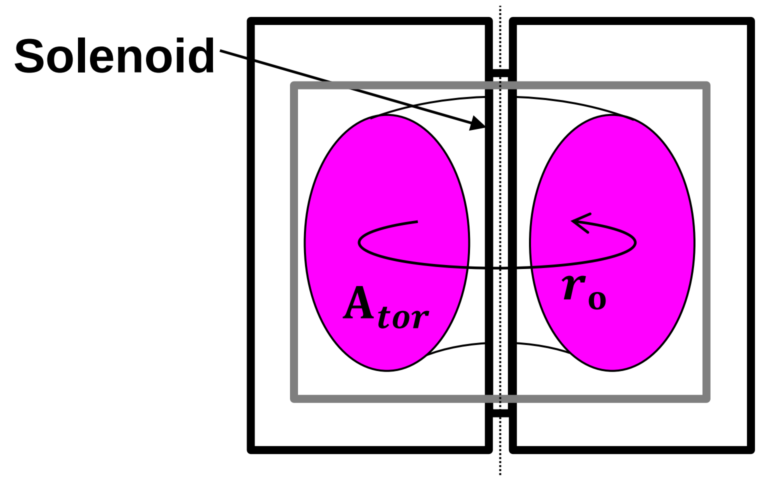
Solenoid can be used to drive the plasma current

$$L_{\mathrm{tor}}\frac{dI}{dt}+IR=V_{\mathrm{loop}}=M\frac{dI_{\mathrm{sol}}}{dt}$$

$$L_{\mathrm{tor}}=\mu_{\mathrm{o}}r_{\mathrm{o}}\left(\ln\left(\frac{8r_{\mathrm{o}}}{a}\right)-1.5\right)$$

$$R_{\mathrm{spitzer}}=\eta_{\mathrm{spiter}}\frac{2\pi r_{\mathrm{o}}}{A_{\mathrm{tor}}}$$

$$\eta_{\mathrm{spiter}}=5.2\times10^{-3}Z\ln\Lambda\,T_{e,(\mathrm{eV})}^{-3/2}$$



$$\Rightarrow\quad IR=V_{\mathrm{loop}}=M\frac{dI_{\mathrm{sol}}}{dt}$$

For constant plasma current,
i.e., *dI/dt*=0

Current is initially driven at the surface and then diffuses into the plasma

- **Simplified Ohm's law:** $\vec{E} + \vec{v} \times \vec{B} = \eta \vec{j}$

- **Assuming a stationary plasma:** $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\eta \vec{j}\right)$

- **Assuming a constant *η*:**

$$-\frac{\partial}{\partial t} \nabla \times \vec{B} = \eta \nabla \times \nabla \times \vec{j} = \eta\left(\nabla\left(\nabla \cdot \vec{j}\right) - \nabla^2 \vec{j}\right)$$

$$\frac{\partial \vec{j}}{\partial t} = \frac{\eta}{\mu_o} \nabla^2 \vec{j}$$

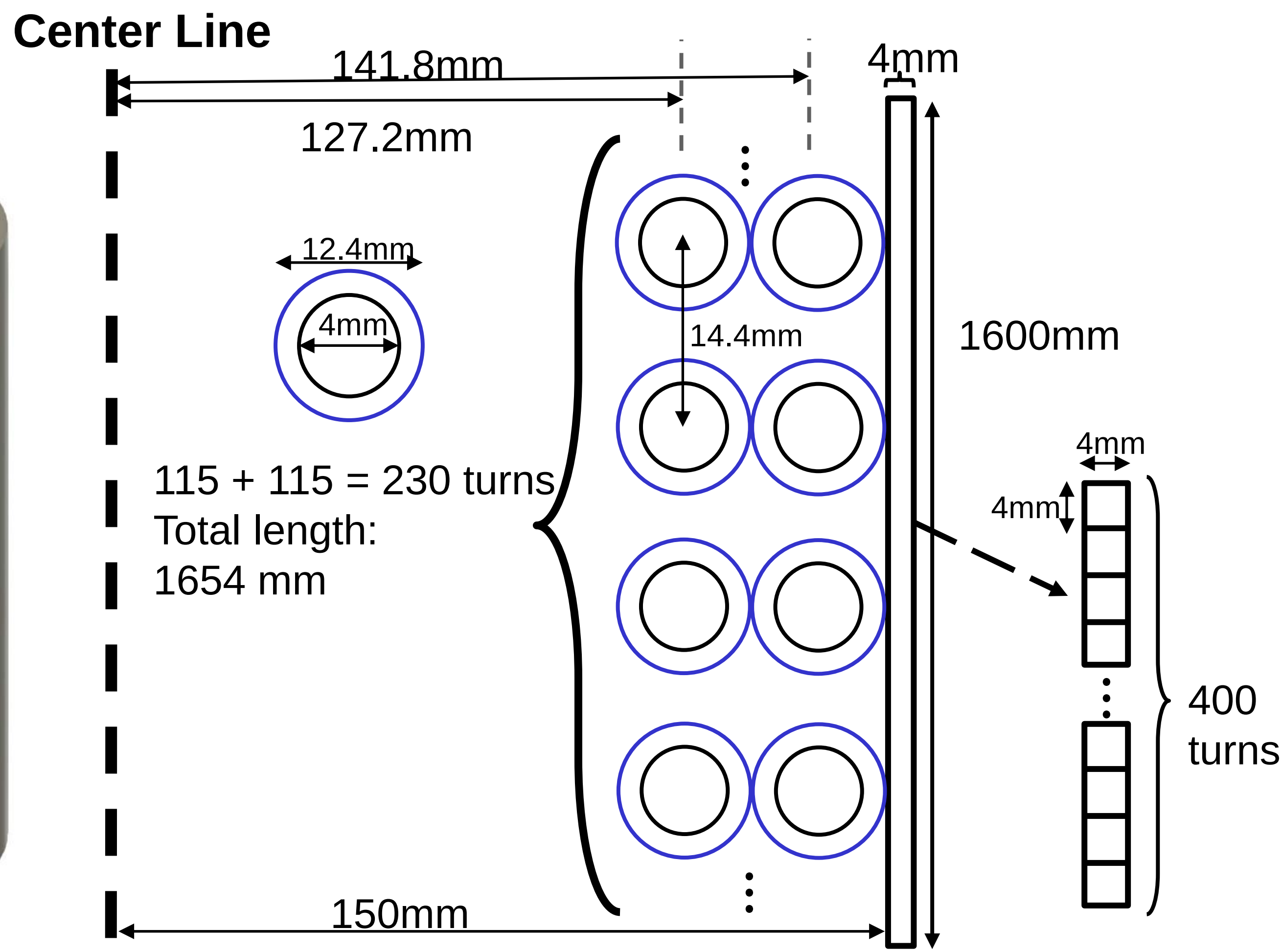
- **Assuming non-constant *η*:**

$$\frac{\partial \vec{j}}{\partial t} = \frac{1}{\mu_o} \nabla^2\left(\eta \vec{j}\right) - \nabla\left(\nabla \cdot \left(\mu \vec{j}\right)\right) \qquad \frac{\partial j_T}{\partial t} = \frac{1}{\mu_o} \nabla^2\left(\eta j_T\right)$$

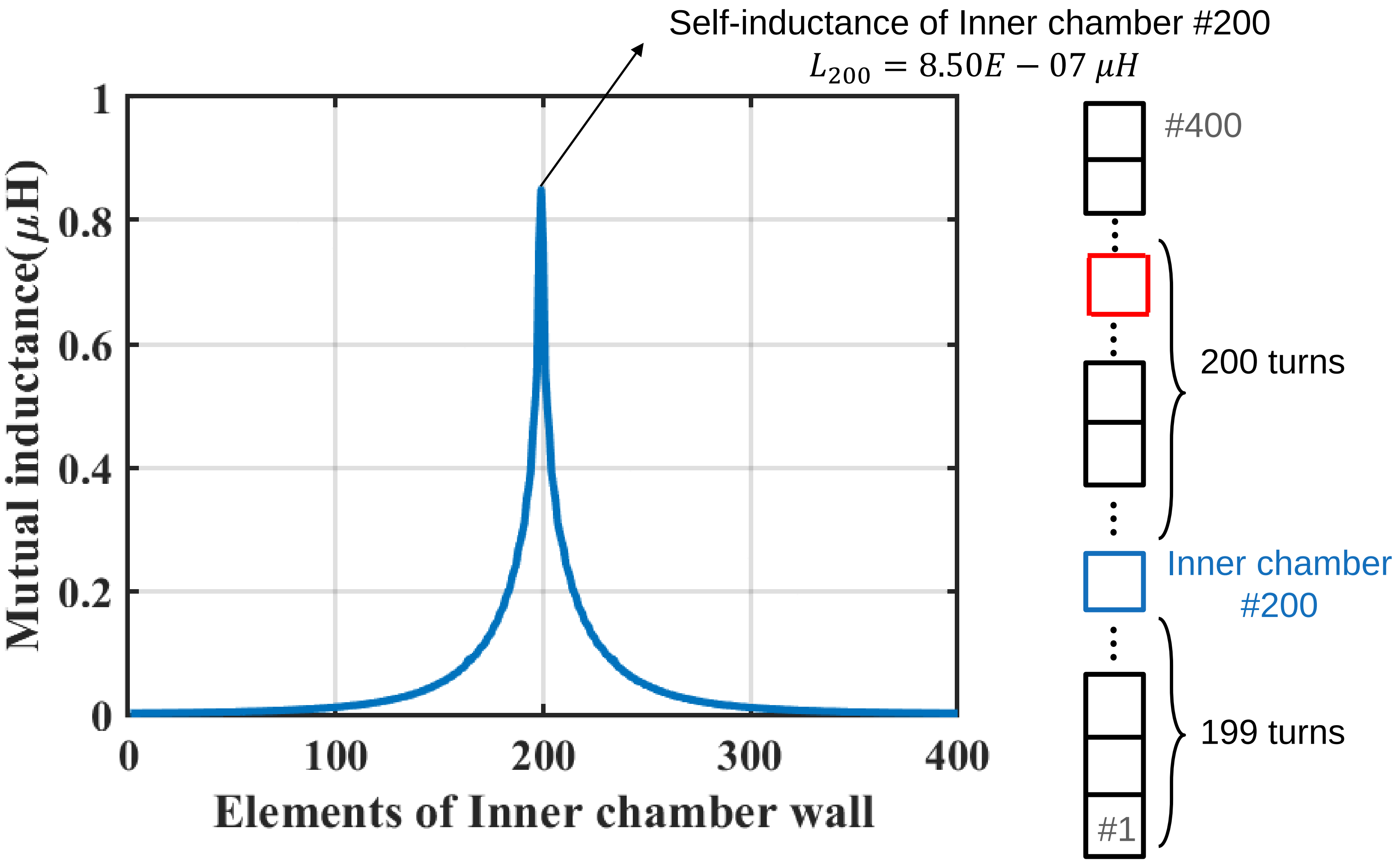
- **Since $\eta \propto T^{-3/2}$, resistance drops with higher temperature. The typical limited temperature is ~3 keV.**

Eddy current needed to be considered

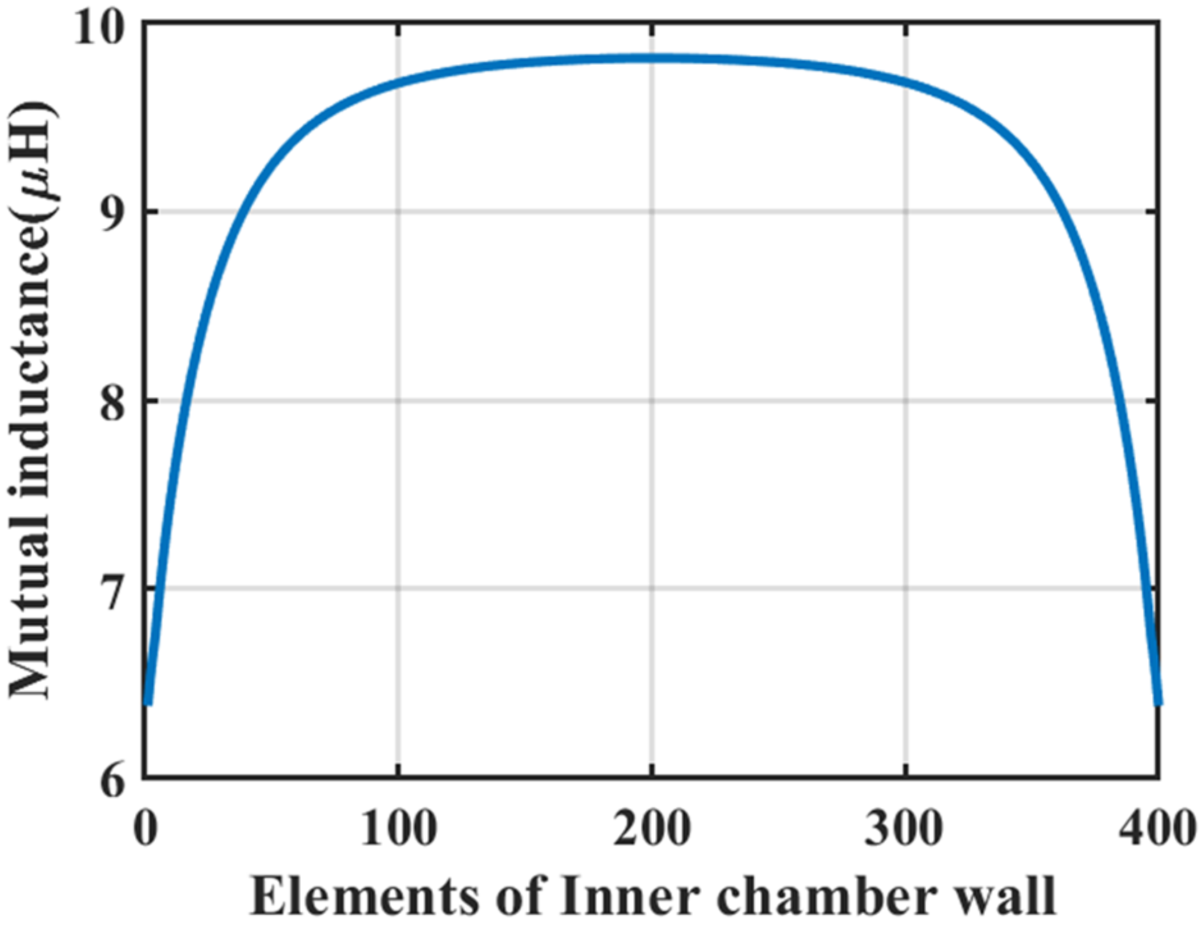
$$\overleftrightarrow{\boldsymbol{M}}\frac{d\vec{\boldsymbol{I}}}{\mathbf{dt}}+\overleftrightarrow{\boldsymbol{R}}_{\boldsymbol{\Omega}}\vec{\boldsymbol{I}}=\vec{\boldsymbol{V}} \qquad \overleftrightarrow{\boldsymbol{M}}\frac{\vec{\boldsymbol{I}'}-\vec{\boldsymbol{I}}}{\boldsymbol{\Delta}\mathbf{t}}+\overleftrightarrow{\boldsymbol{R}}_{\boldsymbol{\Omega}}\vec{\boldsymbol{I}}=\vec{\boldsymbol{V}} \qquad \vec{\boldsymbol{I}'}=\left(\overleftrightarrow{\mathbf{1}}-\boldsymbol{\Delta}\mathbf{t}\,\overleftrightarrow{\boldsymbol{M}}^{-1}\overleftrightarrow{\boldsymbol{R}}_{\boldsymbol{\Omega}}\right)\vec{\boldsymbol{I}}+\boldsymbol{\Delta}\mathbf{t}\,\overleftrightarrow{\boldsymbol{M}}^{-1}\vec{\boldsymbol{V}}$$

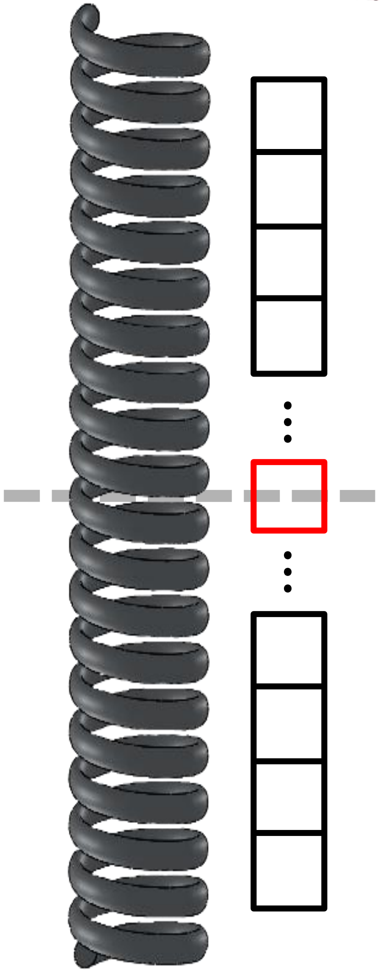


The mutual inductance between inner chamber element #200 and other elements

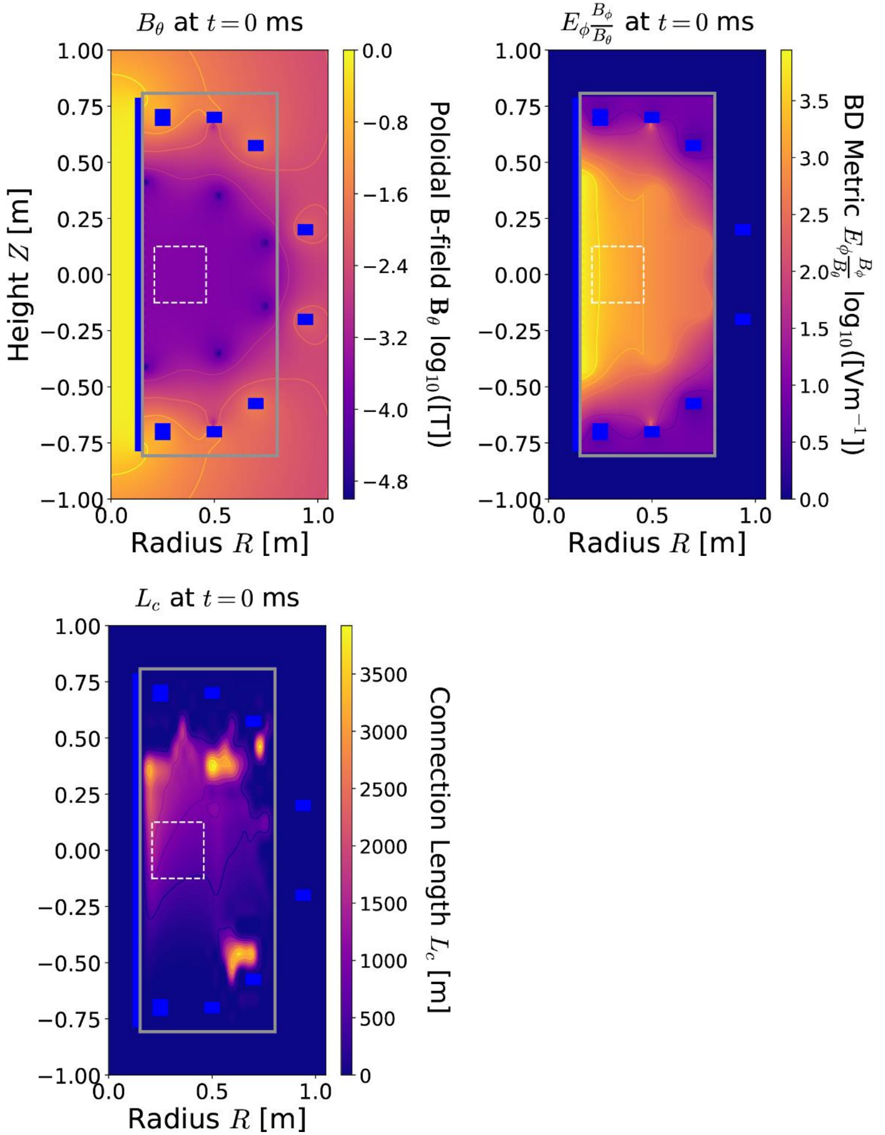


The mutual inductance between the central solenoid and other chamber wall elements

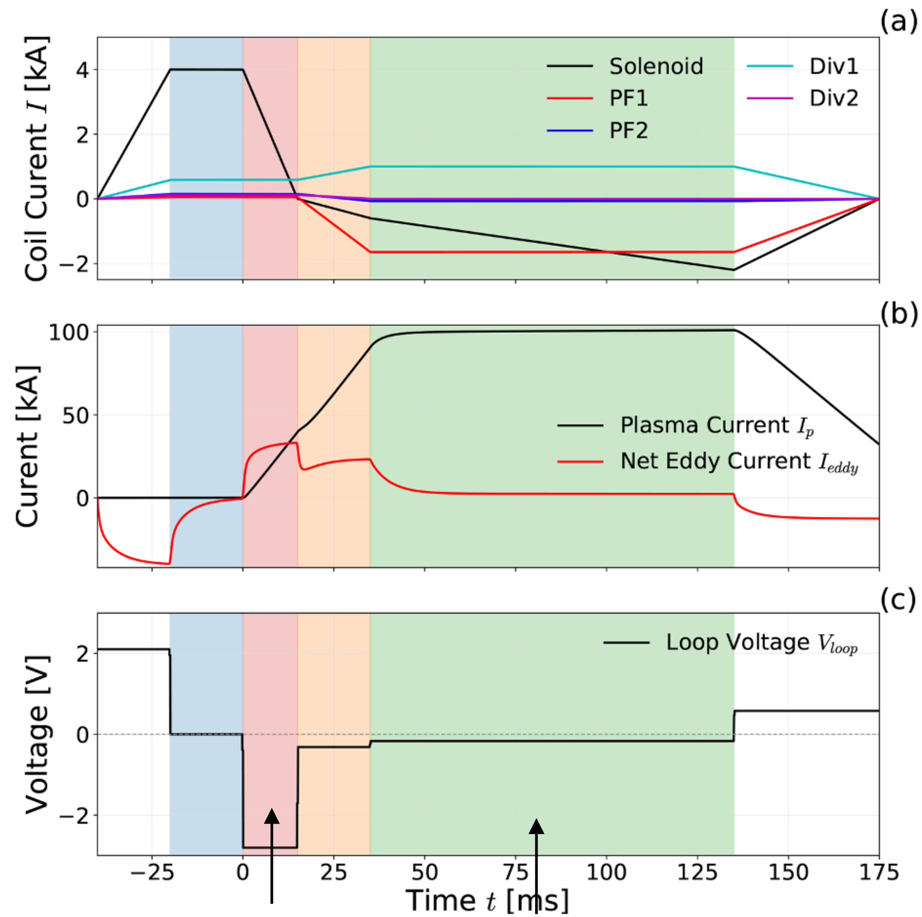




Poloidal coils are used to reduce the stray field during breakdown



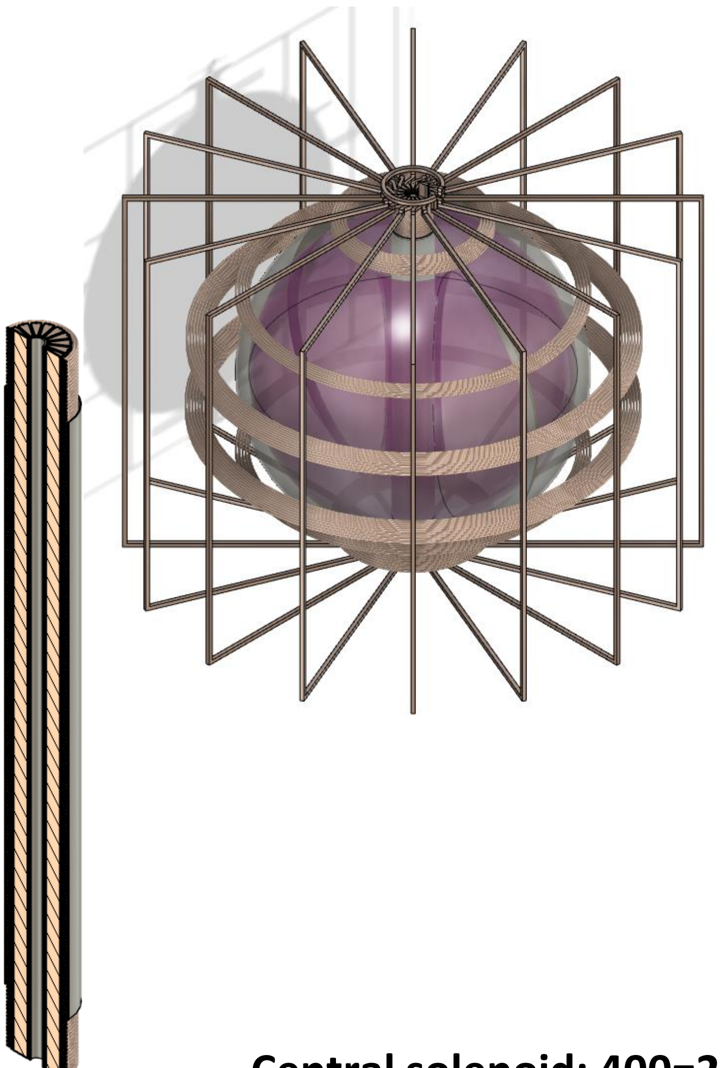
- **Currents of SMART**

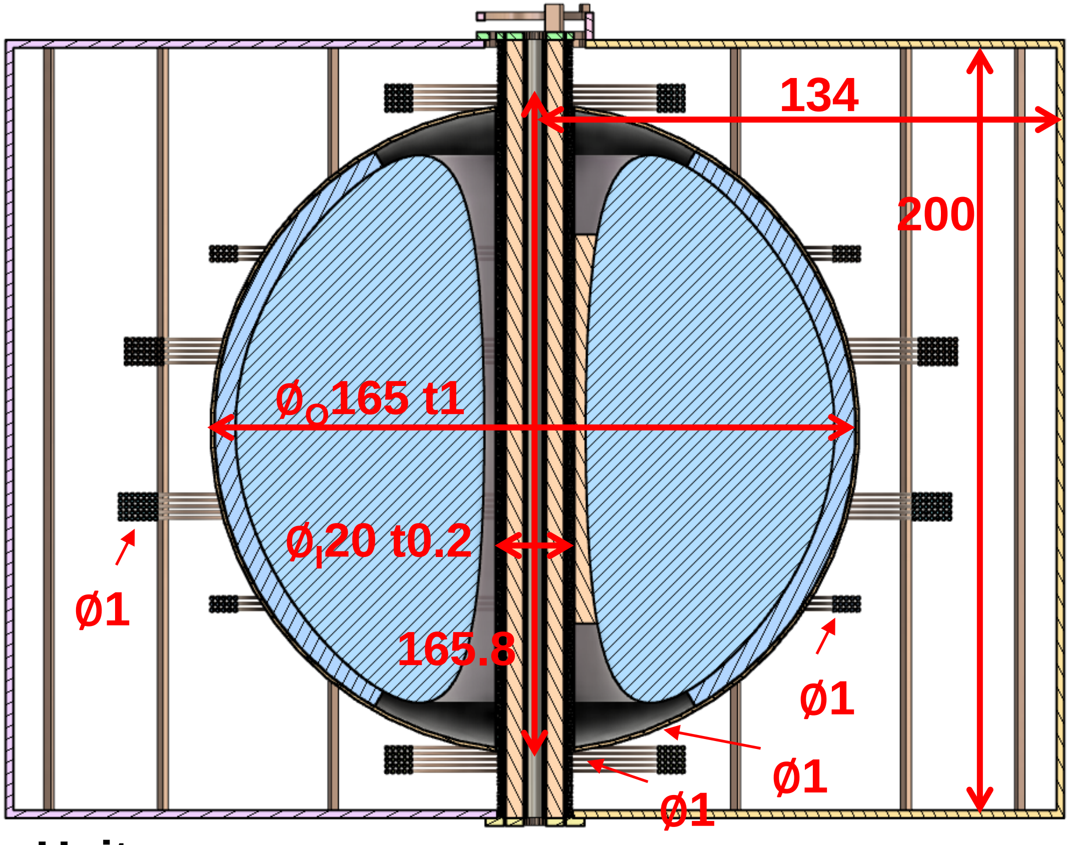


Breakdown **Plasma current is driven**

S. J. Doyle, etc., Fusion Eng. Des. **171**, 112706 (2021)

Potential design of Formosa Integrated Research Spherical Tokamak (FIRST)

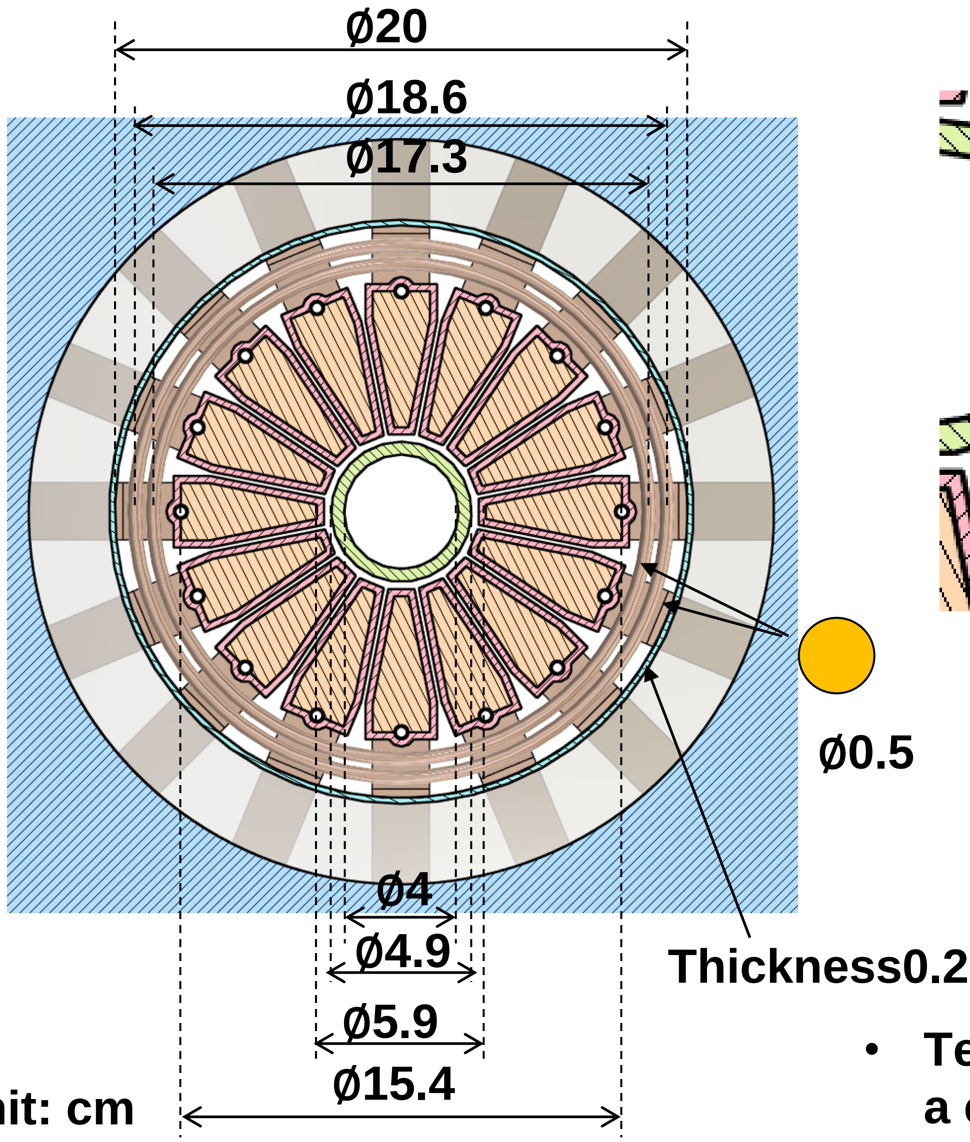


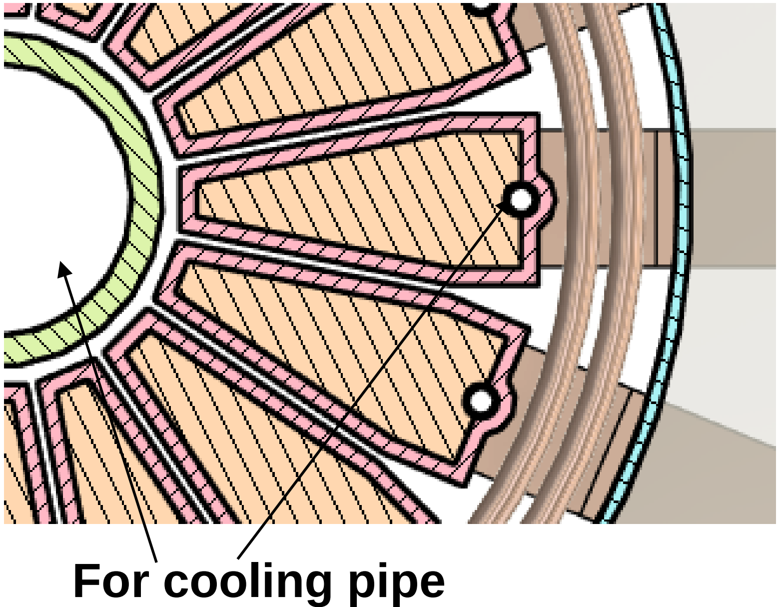


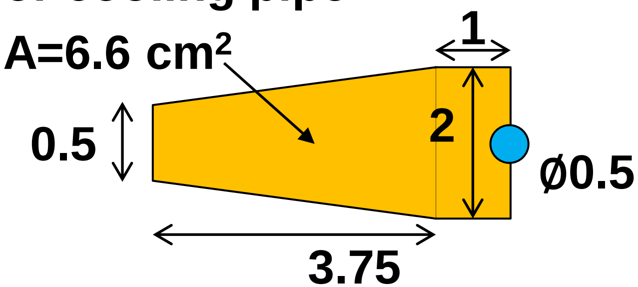
Unit: cm

Central solenoid: 400=2x200 turns

There are limited space in the central stack for solenoid



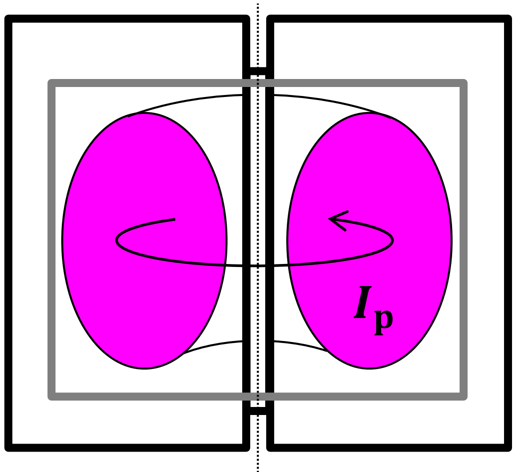




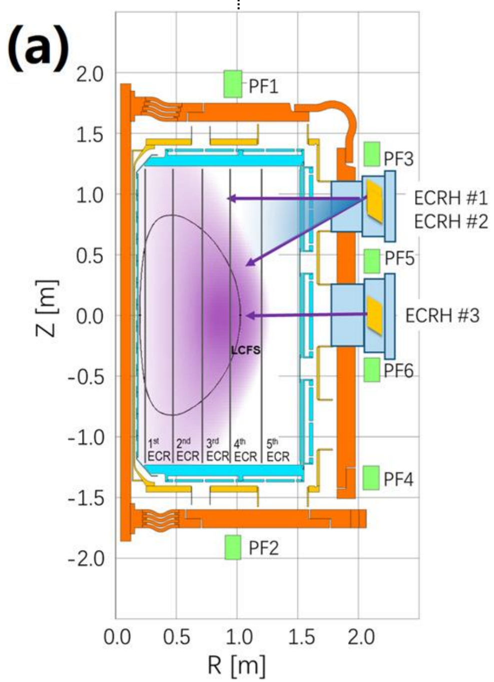
Unit: cm

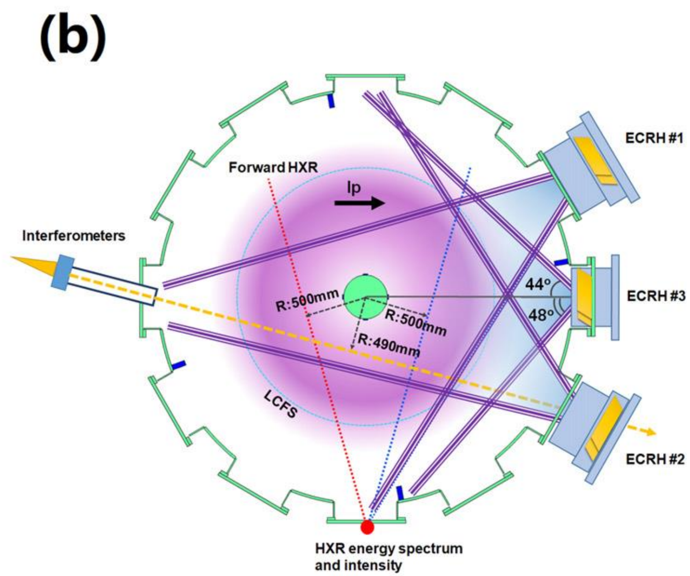
- Temperature increase ~ 80 ℃ for a current of 70 kA lasting 500 ms.

Momentum exchange may be needed to drive plasma current



$$\vec{j}_p = \Sigma q n \vec{v} = -e n_e \vec{v}_e + e n_i \vec{v}_i$$

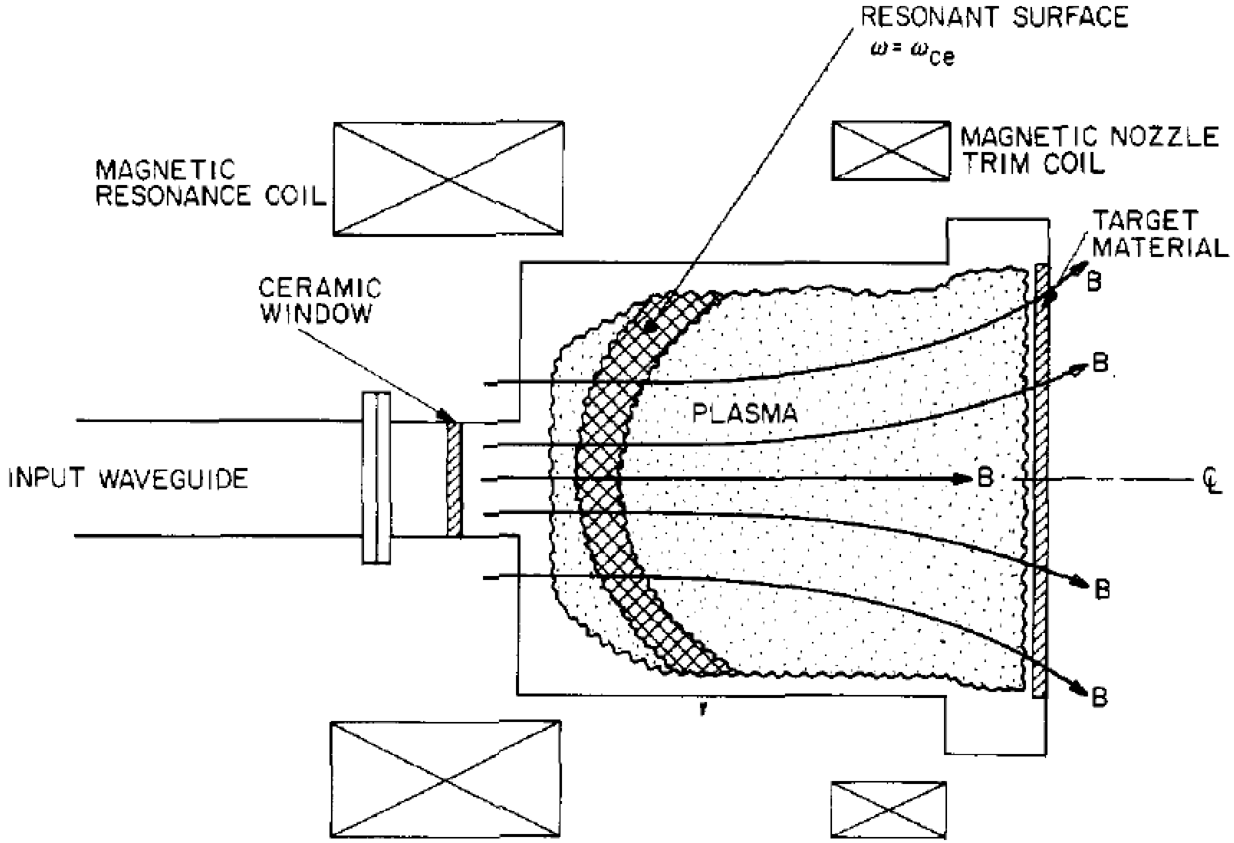




Yuejiang Shi et al, Nucl. Fusion **62**, 086047 (2022)

Strong absorption occurs when the frequency matches the electron cyclotron frequency

- **Electron cyclotron resonance (ECR) plasma reactor**



Electron cyclotron frequency depends on magnetic field only

$$m_e \frac{d\vec{v}}{dt} = -\frac{e}{c}\vec{v} \times \vec{B}$$

- **Assuming** $\vec{B} = B\hat{z}$ **and the electron oscillates in x-y plane**

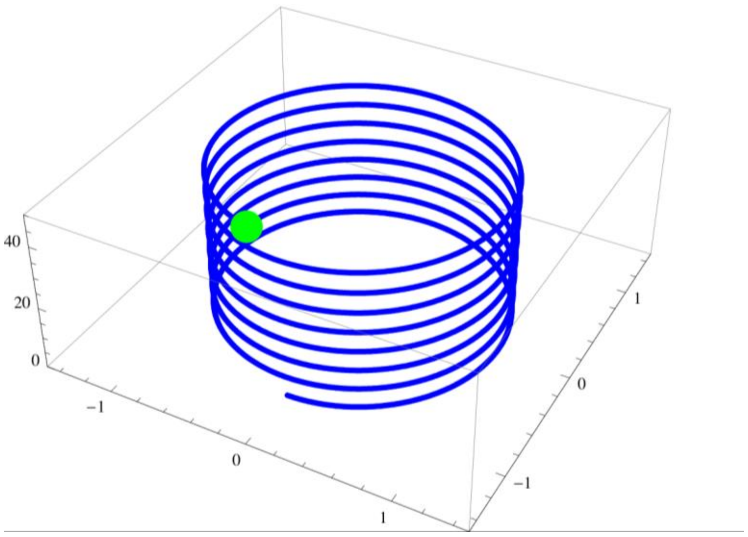
$$m_e\dot{v}_x = -\frac{e}{c}Bv_y$$

$$m_e\dot{v}_y = \frac{e}{c}Bv_x$$

$$m_e\dot{v}_z = 0$$

$$\ddot{v}_x = -\frac{eB}{m_e c}\dot{v}_y = -\left(\frac{eB}{m_e c}\right)^2 v_x$$

$$\ddot{v}_y = -\frac{eB}{m_e c}\dot{v}_x = -\left(\frac{eB}{m_e c}\right)^2 v_y$$



- **Therefore**

$$\omega_{ce} = \frac{eB}{m_e c}$$

Electrons keep getting accelerated when a electric field rotates in electron's gyrofrequency

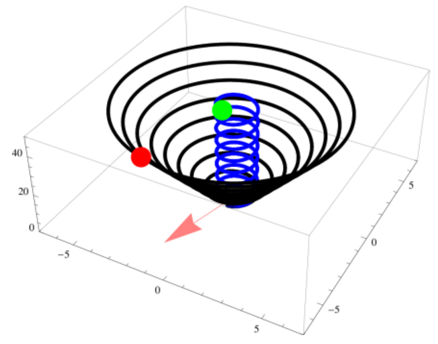
$$m_e \frac{d\vec{v}}{dt} = -\frac{e}{c}\,\vec{v} \times \vec{B} - e\,\vec{E} \qquad \vec{B} = B_0\hat{z} \qquad \vec{E} = E_0[\hat{x}\cos(\omega t) + \hat{y}\sin(\omega t)]$$

$$m_e\dot{v}_x = -\frac{e}{c}Bv_y + E_0\cos(\omega t) \qquad m_e\dot{v}_y = \frac{e}{c}Bv_x + E_0\cos(\omega t) \qquad m_e\dot{v}_z = 0$$

$$\ddot{v}_x = -\frac{eB}{m_e c}\dot{v}_y - \frac{E_0}{m_e}\omega\cos(\omega t) = -\omega_{ce}{}^2 v_x - \frac{E_0}{m_e}(\omega_{ce} + \omega)\cos(\omega t)$$

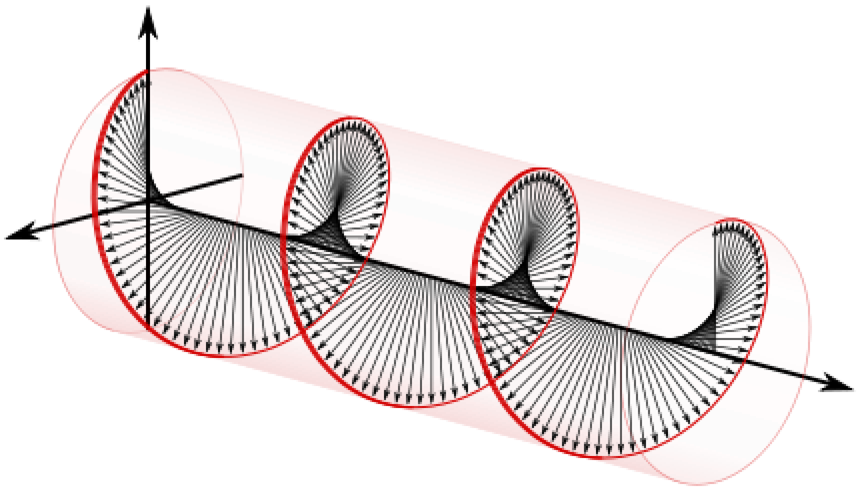
$$\ddot{v}_y = -\frac{eB}{m_e c}\dot{v}_x + \frac{E_0}{m_e}\omega\sin(\omega t) = -\omega_{ce}{}^2 v_y + \frac{E_0}{m_e}(\omega_{ce} + \omega)\sin(\omega t)$$

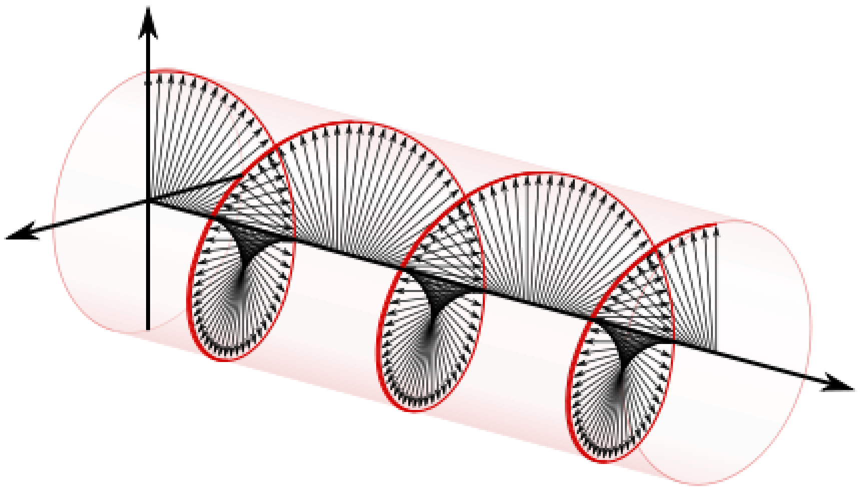
$$\omega_{ce} = \frac{eB}{m_e c}$$



Electric field in a circular polarized electromagnetic wave keeps rotating as the wave propagates

- **Right-handed polarization**
- **Left-handed polarization**





https://en.wikipedia.org/wiki/Circular_polarization

Only right-handed polarization can resonance with electron's gyromotion

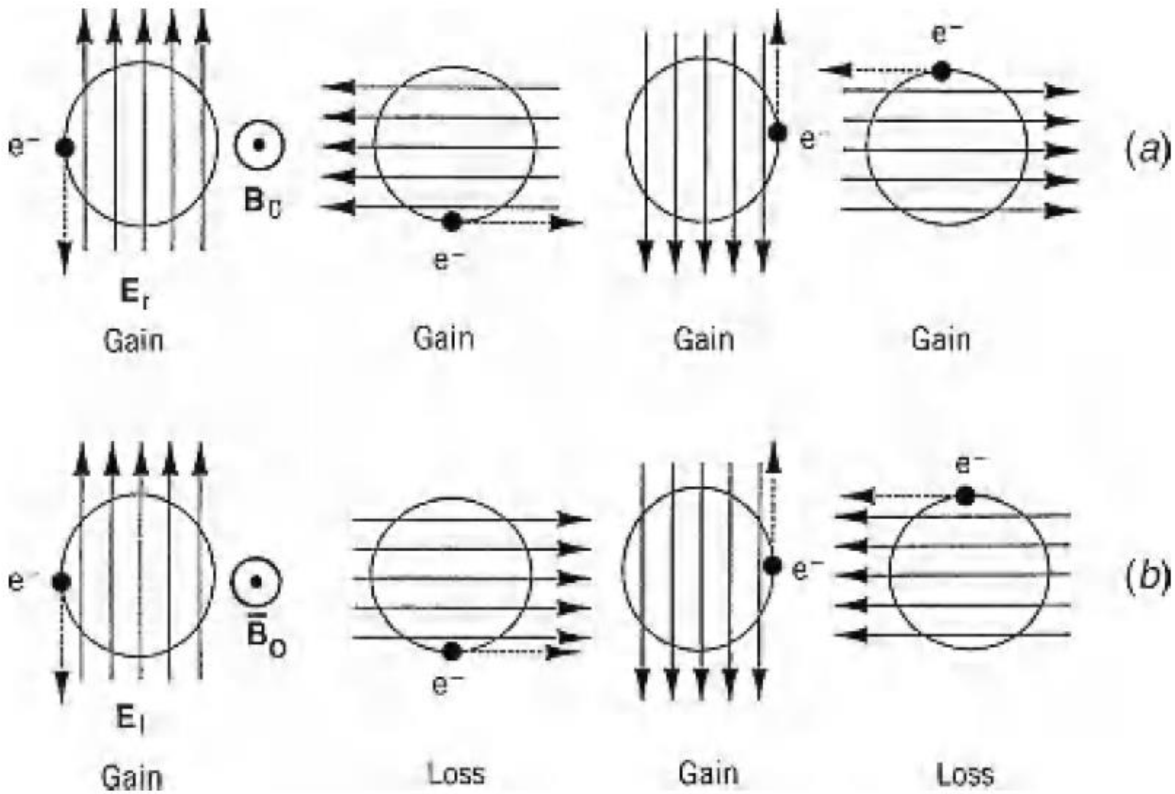
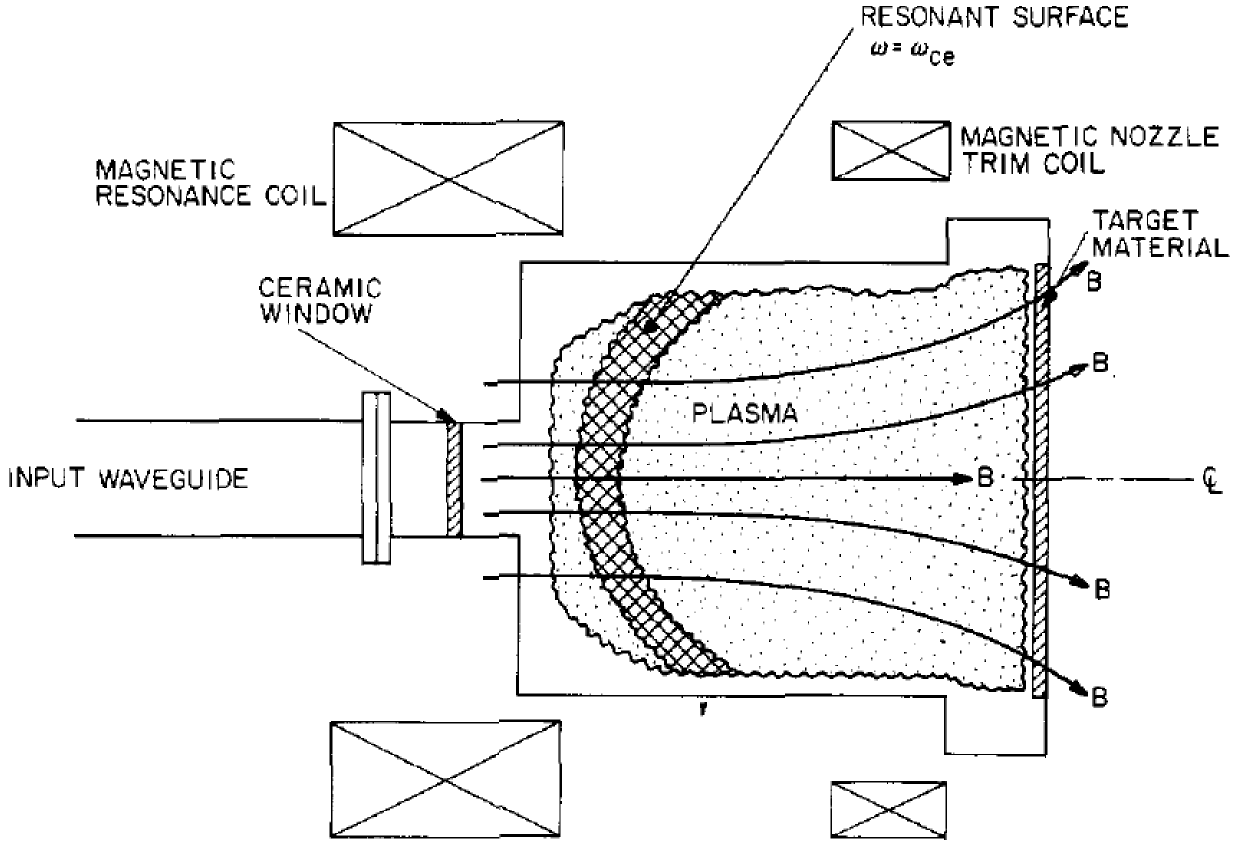


FIGURE 13.5. Basic principle of ECR heating: (*a*) continuous energy gain for right-hand polarization; (*b*) oscillating energy for left-hand polarization (after Lieberman and Gottscho, 1994).

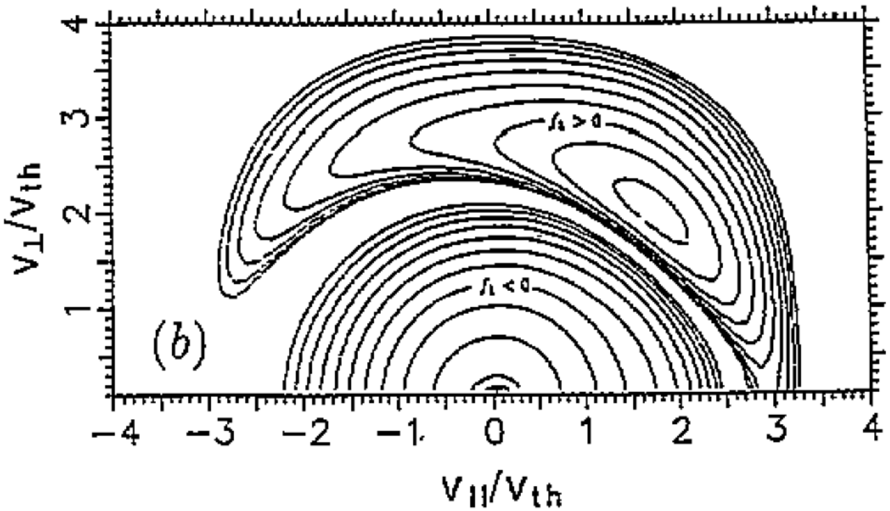
Strong absorption occurs when the frequency matches the electron cyclotron frequency

- **Electron cyclotron resonance (ECR) plasma reactor**



The collisional re-distribution of the ECRH-driven anisotropy in $E_\perp$ causes some parallel momentum to flow from e^- to ions

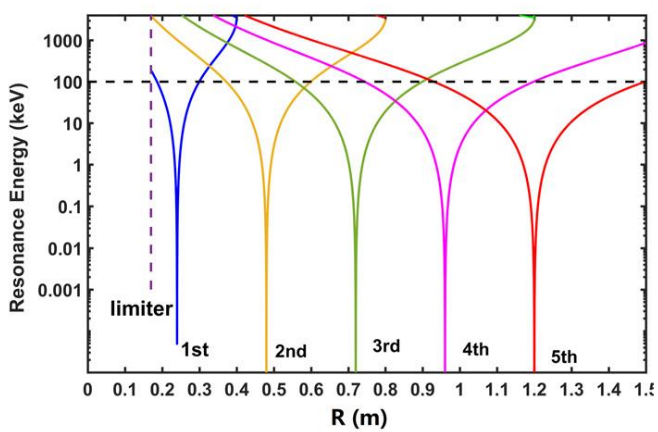
- **Coulomb collisions are more efficient at lower energies.**

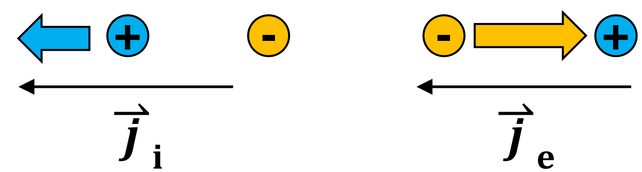


Velocity: $v_2 > v_1$

Collisions: $\nu_2 < \nu_1$

- **Electron cyclotron current drive:**





$$\vec{j}_p = -en_e\vec{v}_e + en_i\vec{v}_i$$

$$\vec{P} = n_e m_e\vec{v}_e + n_i m_i\vec{v}_i \approx 0$$

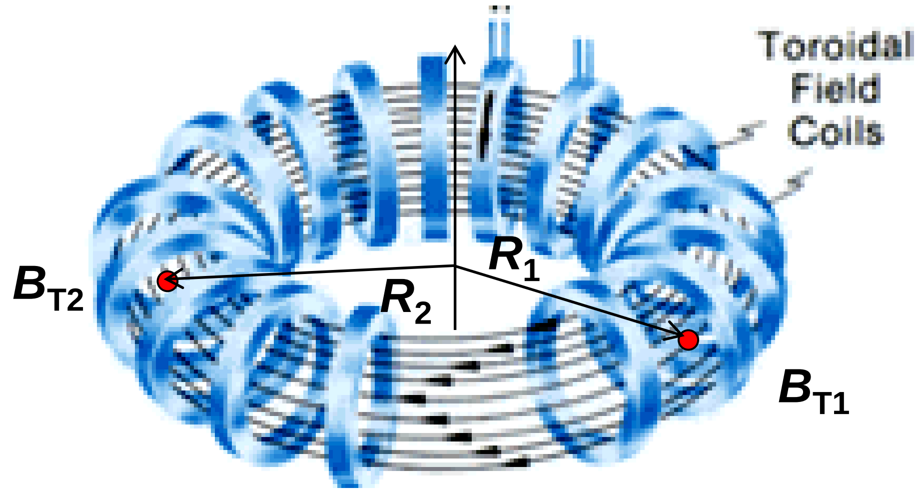
V. Erckmann and U. Gasparion, Plasma Phys. Control. Fusion **36**, 1869 (1994)
Yuejiang Shi et al, Nucl. Fusion **62**, 086047 (2022)

With poloidal fields, charged particles see nonuniform toroidal magnetic field

- **W/o poloidal field**

$$R_1 = R_2$$

$$B_{T1} = B_{T2}$$



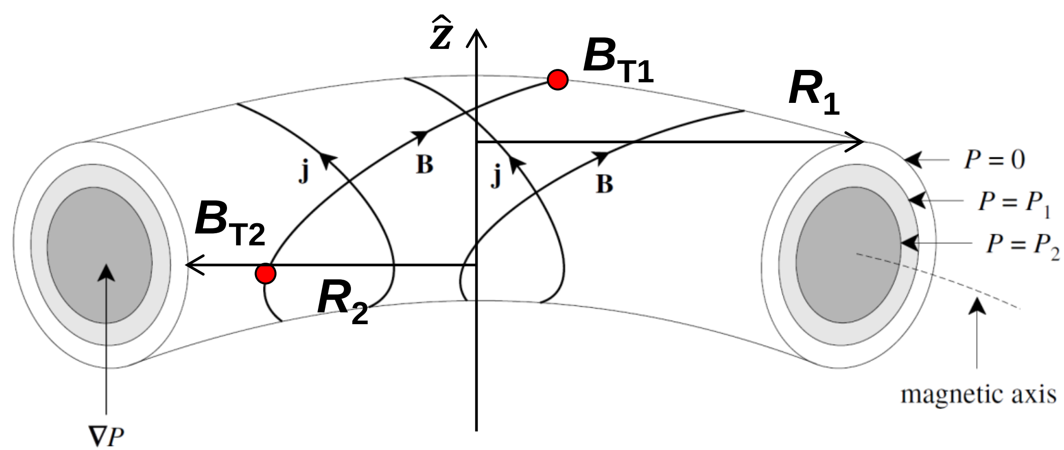
$$B_T \propto \frac{1}{R}$$

$$B_T \gg B_p$$

- **W/ poloidal field**

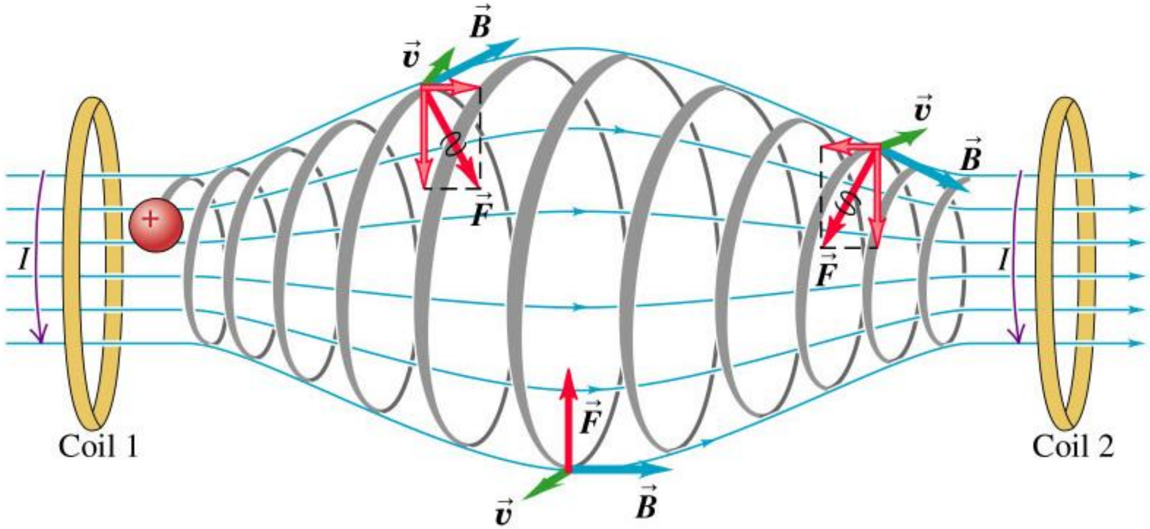
$$R_1 > R_2$$

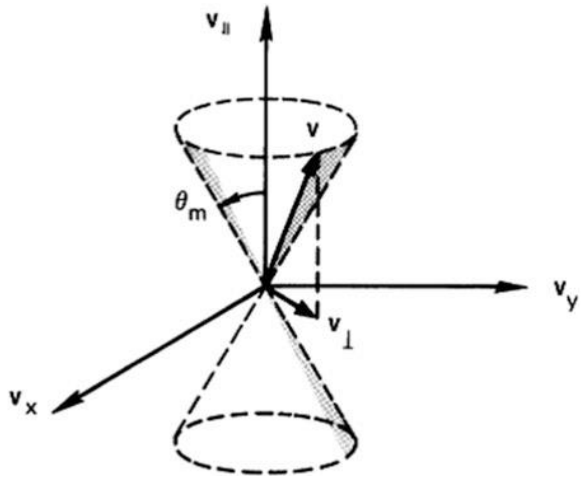
$$B_{T1} < B_{T2}$$



Charged particles can be partially confined by a magnetic mirror machine

- **Charged particles with small $v_{||}$ eventually stop and are reflected while those with large $v_{||}$ escape.**





$$v'_{\perp}{}^2 = v_{\perp o}{}^2 + v_{||o}{}^2 \equiv v_o{}^2$$

$$\frac{B_o}{B'} = \frac{v_{\perp o}{}^2}{v'_{\perp}{}^2} = \frac{v_{\perp o}{}^2}{v_o{}^2} \equiv sin^2\theta$$

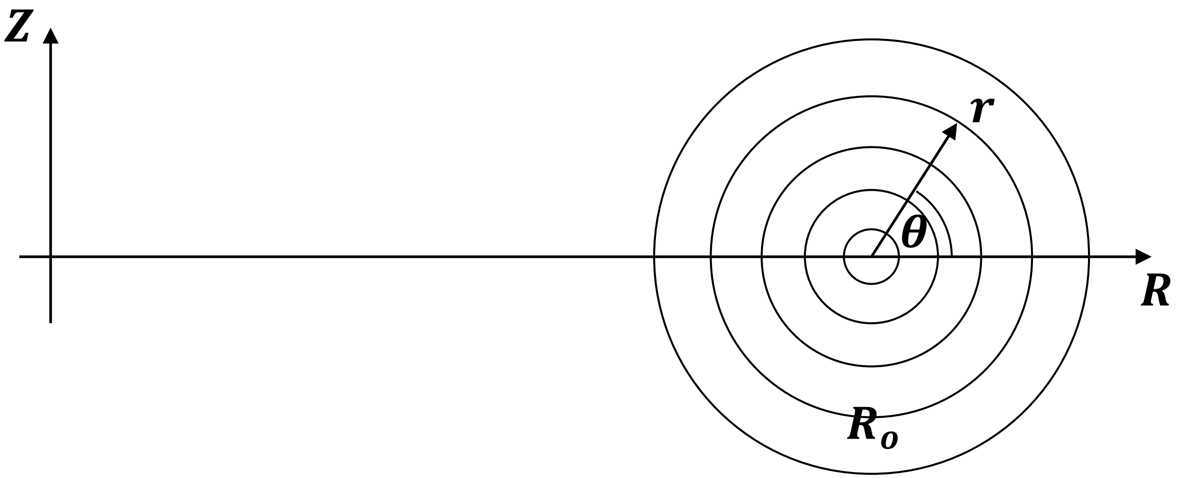
$$\frac{1}{2}mv^2 = \frac{1}{2}mv_{||}{}^2 + \frac{1}{2}mv_{\perp}{}^2 \quad \text{Invarient: } \mu \equiv \frac{1}{2}\frac{mv_{\perp}{}^2}{B}$$

$$\frac{B_o}{B_m} \equiv \frac{1}{R_m} = sin^2\theta_m$$

- **Large $v_{||}$ may occur from collisions between particles.**
- **Those confined charged particle are eventually lost due to collisions.**

https://i.stack.imgur.com/GlzGZ.jpg

Parallel velocity changes when particles follow field the field line



$R \gg r$

$B_{\mathrm{T}} \gg B_{\mathrm{p}}$

$R = R_{\mathrm{o}} + r\cos\theta = R_{\mathrm{o}}(1 + \epsilon\cos\theta)$ **Inverse aspect ratio:** $\epsilon \equiv \dfrac{r}{R_{\mathrm{o}}}$

$$B \simeq \frac{B_{\mathrm{o}}}{1 + \epsilon\cos\theta} \simeq B_{\mathrm{o}}(1 - \epsilon\cos\theta)$$

Invarient: $\mu \equiv \dfrac{1}{2}\dfrac{m{v_\perp}^2}{B}$ $\quad \dfrac{{v_\perp}^2}{B_o(1-\epsilon cos\theta)} = \dfrac{{v_{\perp 0}}^2}{B_o(1-\epsilon)}$ $\quad {v_\perp}^2 = \dfrac{{v_{\perp 0}}^2(1-\epsilon cos\theta)}{1-\epsilon}$

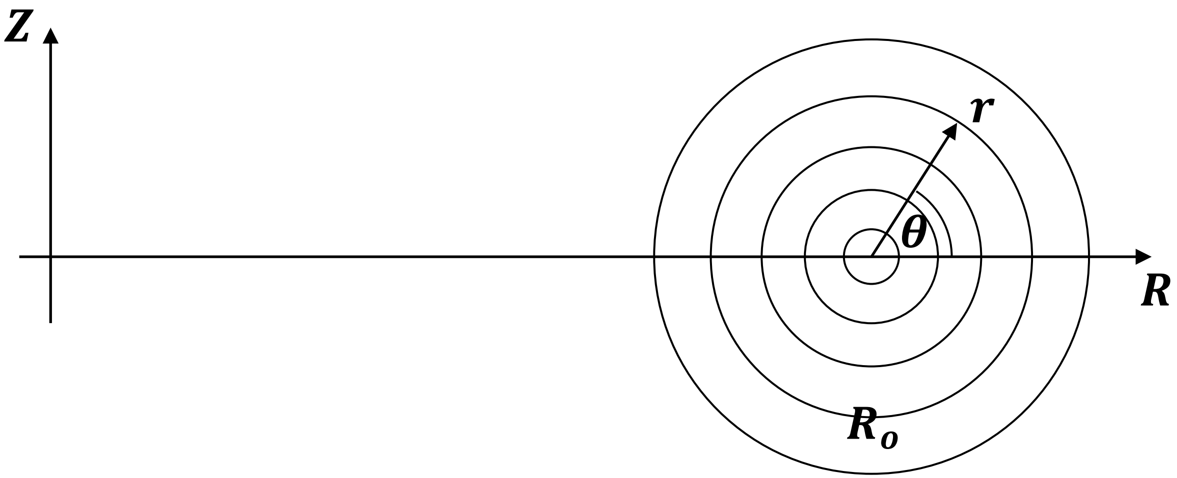
$$v^2 = {v_\perp}^2 + {v_{||}}^2 = {v_{\perp o}}^2 + {v_{||o}}^2$$

$$ {v_{||}}^2 = v^2\left(1 - \frac{{v_\perp}^2}{v^2}\right)$$

$$ {v_{||}}^2 = v^2\left(1 - \frac{{v_{\perp 0}}^2}{v^2}\frac{(1-\epsilon\cos\theta)}{1-\epsilon}\right)$$

$$\approx v^2\left(1 - \frac{{v_{\perp 0}}^2}{v^2}\left(1 + 2\epsilon\sin^2\left(\frac{\theta}{2}\right)\right)\right)$$

Particles may be trapped by nonuniform magnetic field



$$R \gg r$$

$$B_T \gg B_p$$

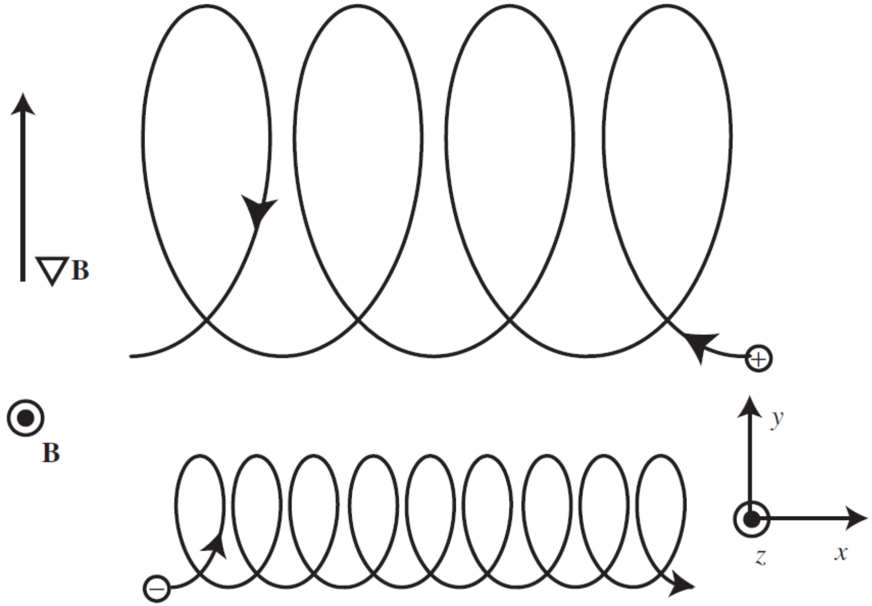
$$\epsilon \equiv \frac{r}{R_o}$$

$$v_{||}{}^2 \approx v^2\left(1-\frac{v_{\perp 0}{}^2}{v^2}\left(1+2\epsilon\sin^2\left(\frac{\theta}{2}\right)\right)\right)$$

- For $v_{||}{}^2 \geq 0$, particles are passing.
- For $v_{||}{}^2 \leq 0$, particles are trapped.

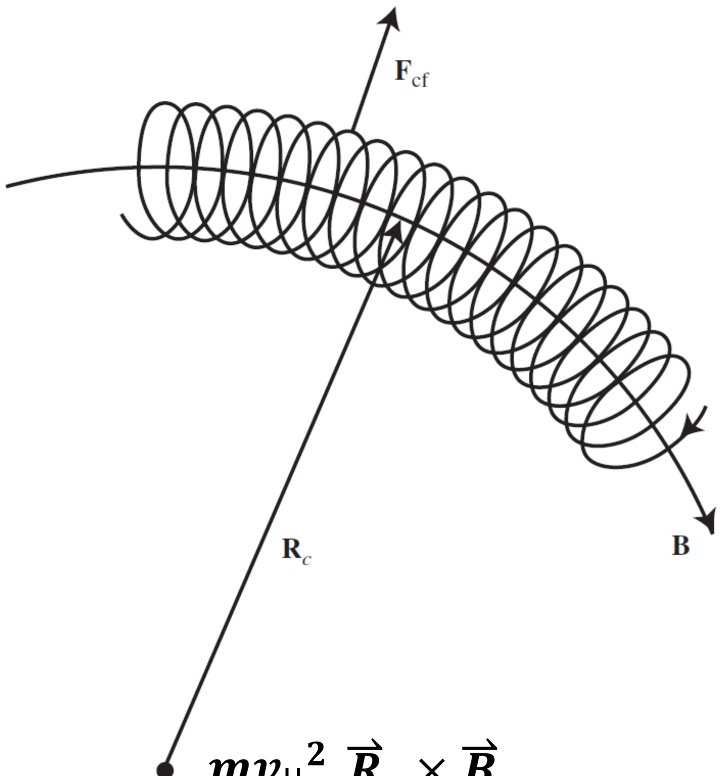
Charge particles drift across magnetic field lines when the magnetic field is not uniform or curved

- **Gradient-B drift**



$$\vec{v}_{\nabla} = \frac{m{v_{\perp}}^2}{2q}\frac{\vec{B}\times\nabla B}{B^3}$$

- **Curvature drift**

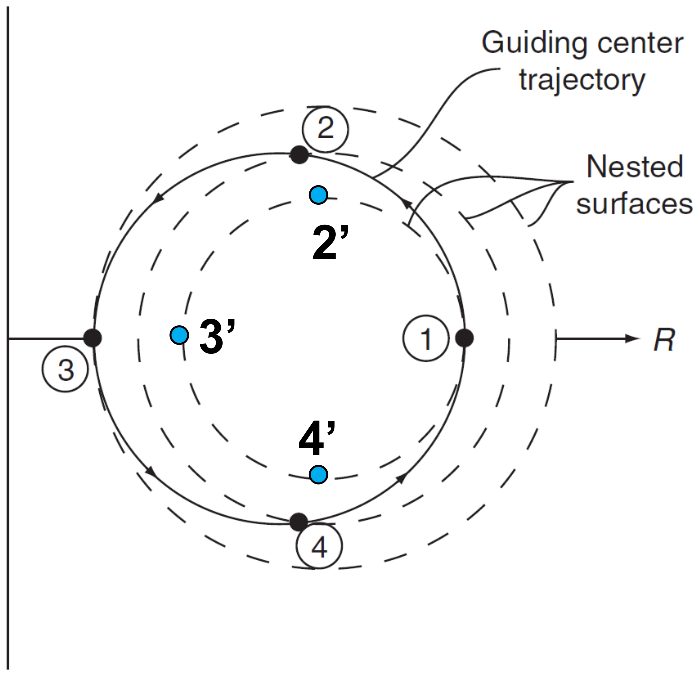


$$\vec{v}_{R} = \frac{m{v_{||}}^2}{2q}\frac{\vec{R}_c\times\vec{B}}{R_c B^2}$$

$$\vec{v}_{\text{total}} = \vec{v}_{\text{R}} + \vec{v}_{\nabla} = \frac{\vec{B}\times\nabla B}{\omega_c B^2}\left({v_{||}}^2 + \frac{1}{2}{v_{\perp}}^2\right) = \frac{m}{q}\frac{\vec{R}_c\times\vec{B}}{{R_c}^2 B^2}\left({v_{||}}^2 + \frac{1}{2}{v_{\perp}}^2\right)$$

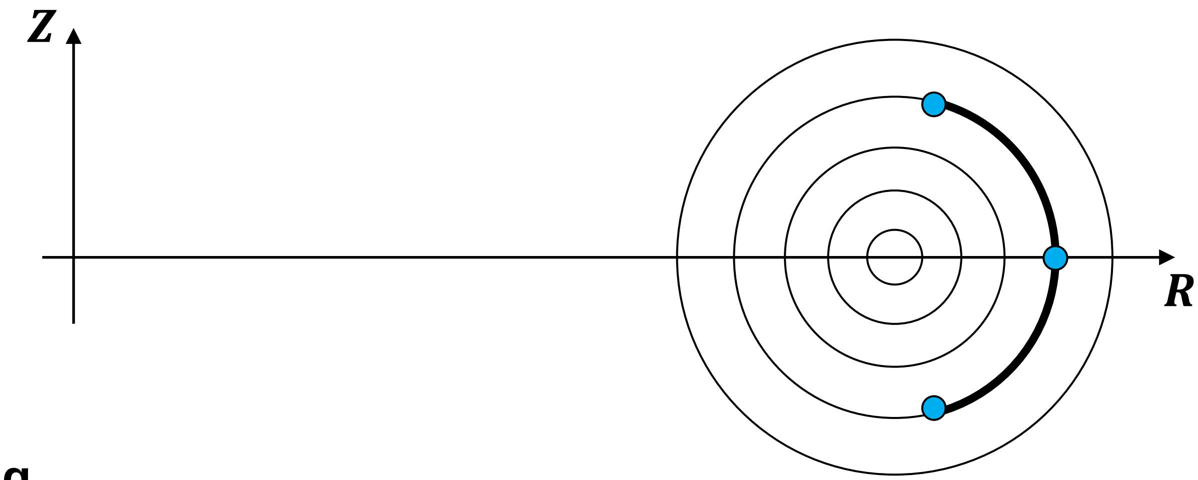
For passing particles, they drift back to the original position with a "semicircle" orbit

$$\vec{v}_{\text{total}} = \vec{v}_R + \vec{v}_\nabla = \frac{\vec{B} \times \nabla B}{\omega_c B^2}\left(v_{||}{}^2 + \frac{1}{2} v_\perp{}^2\right) = \frac{m}{q} \frac{\vec{R}_c \times \vec{B}}{R_c{}^2 B^2}\left(v_{||}{}^2 + \frac{1}{2} v_\perp{}^2\right)$$

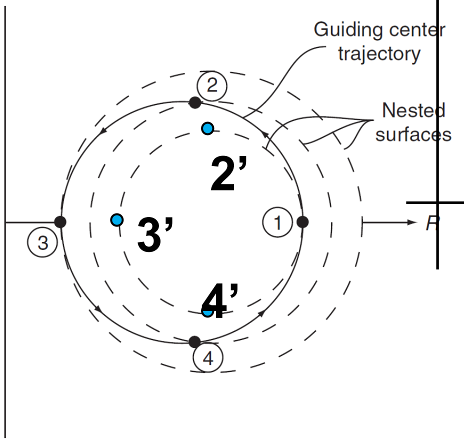


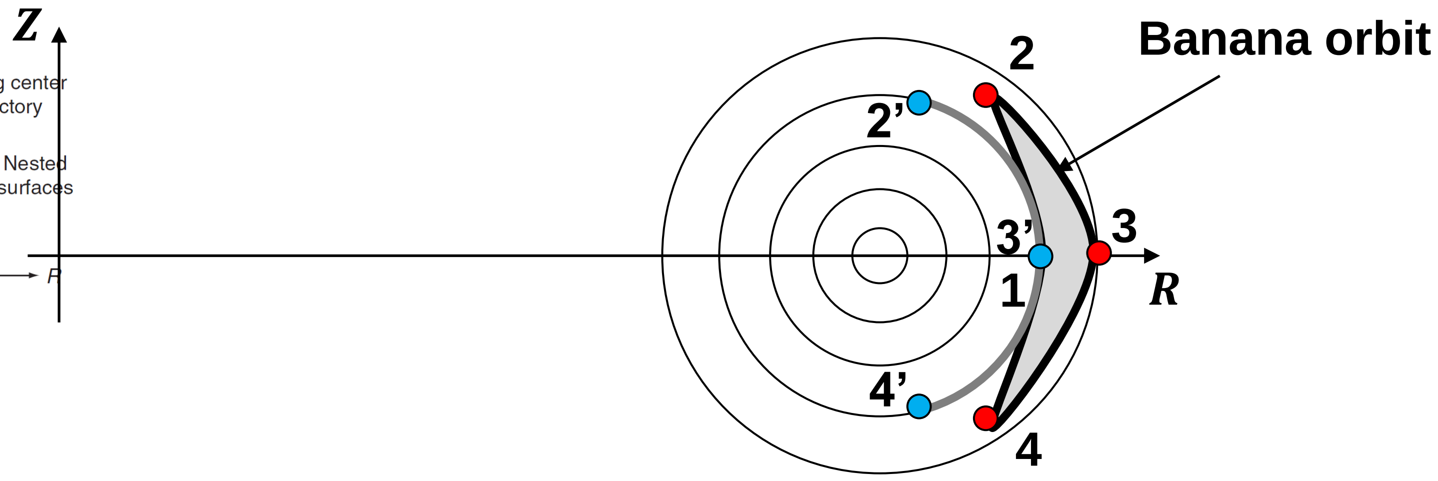
For trapped particles, they drift back to the original position with a banana orbit

- W/o drifting

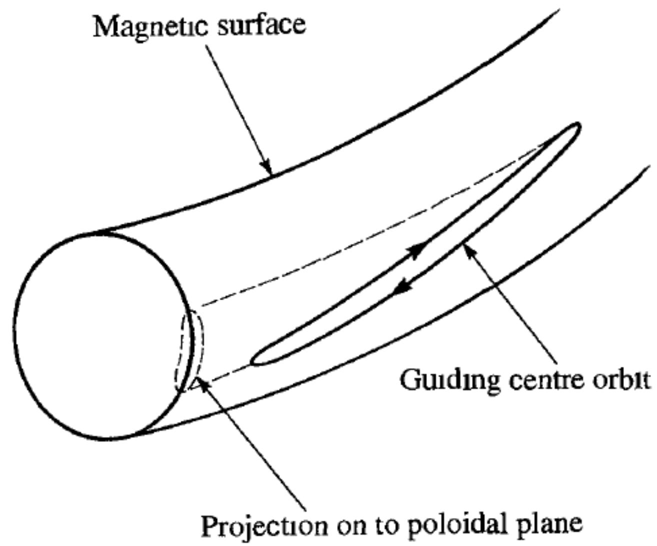


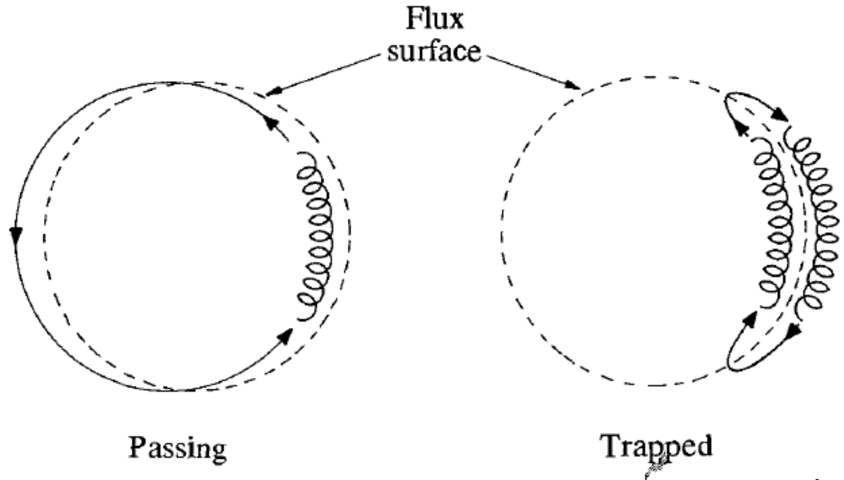
- W/ drifting





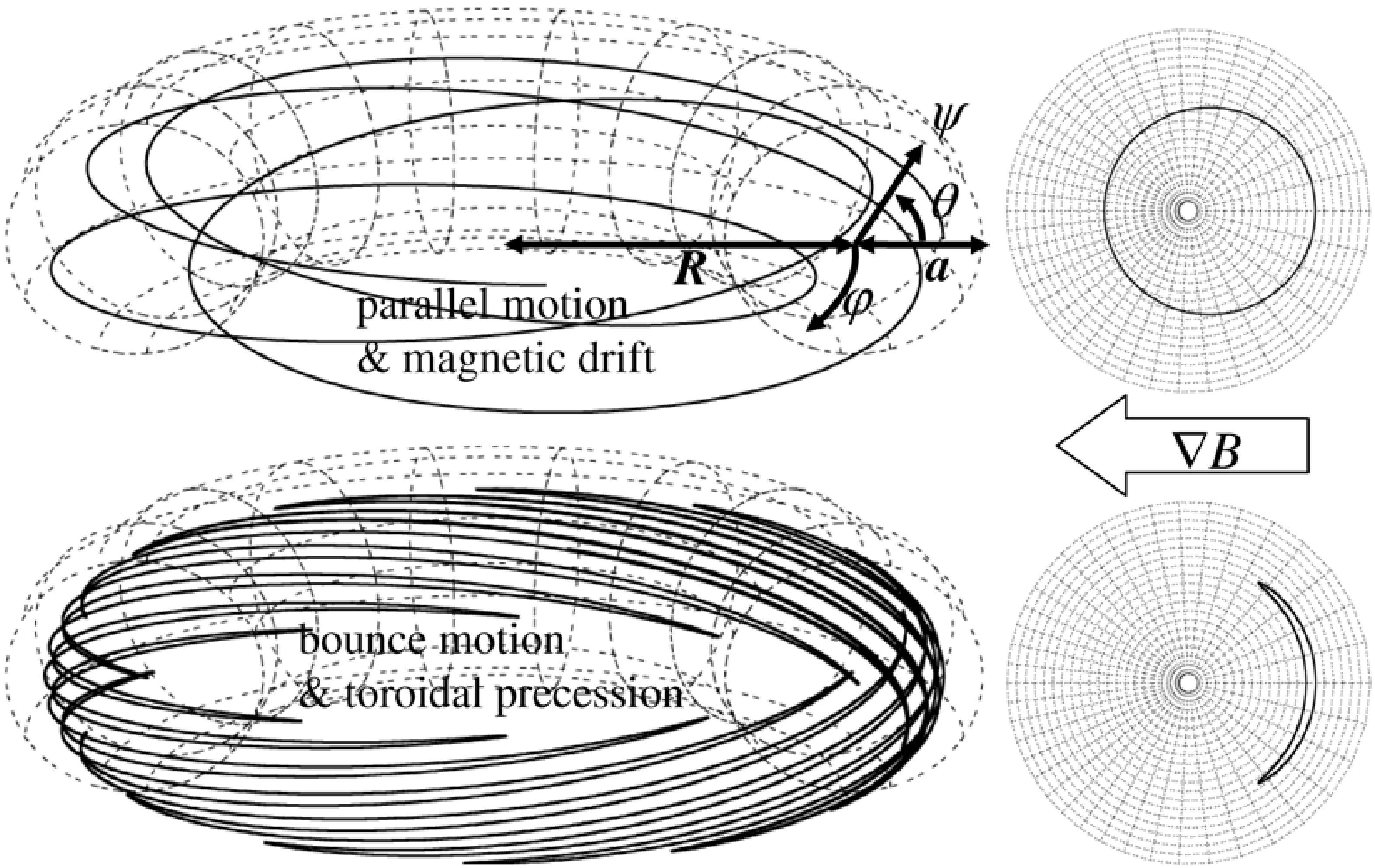
Trajectories of charged particles





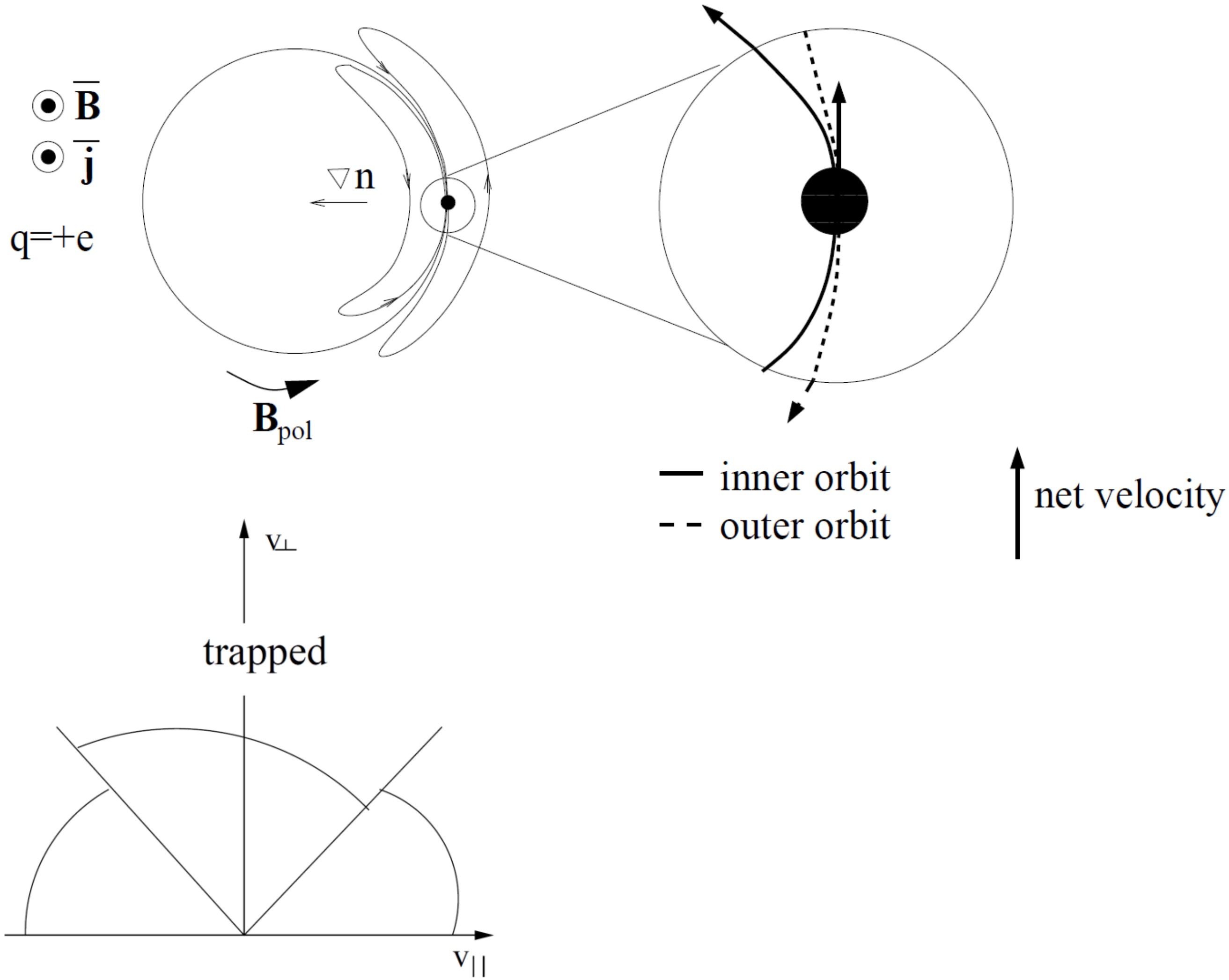
John Wesson, Tokamaks 3rd edition

The trajectories of charged particles follow the toroidal field lines



X. Garbet et al, Nucl. Fusion **50**, 043002 (2010)

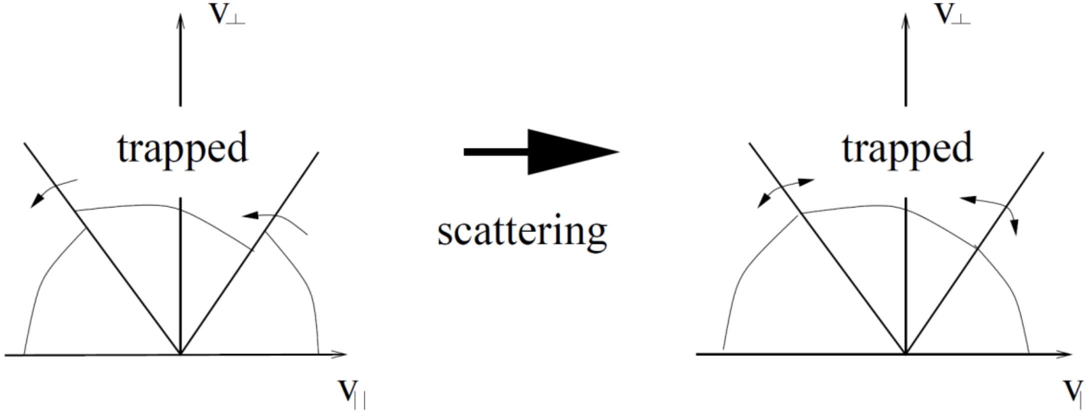
A banana current is generated when there is a pressure gradient in the plasma

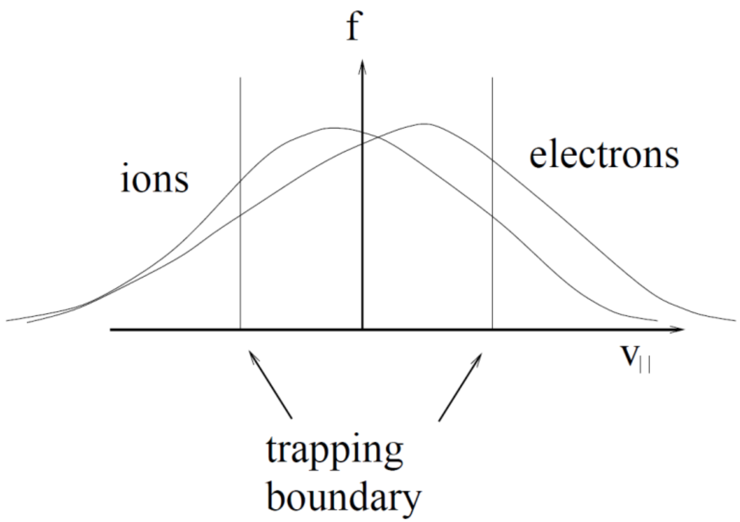


Samuli Saarelma, Helsinki University of Technology, PhD Thesis 2015

Bootstrap current is generated when passing particles are scattered by the trapped particles

- **Scattering smooths the velocity distribution and shifts it in the parallel direction, i.e., a current is generated. It is called the bootstrap current.**



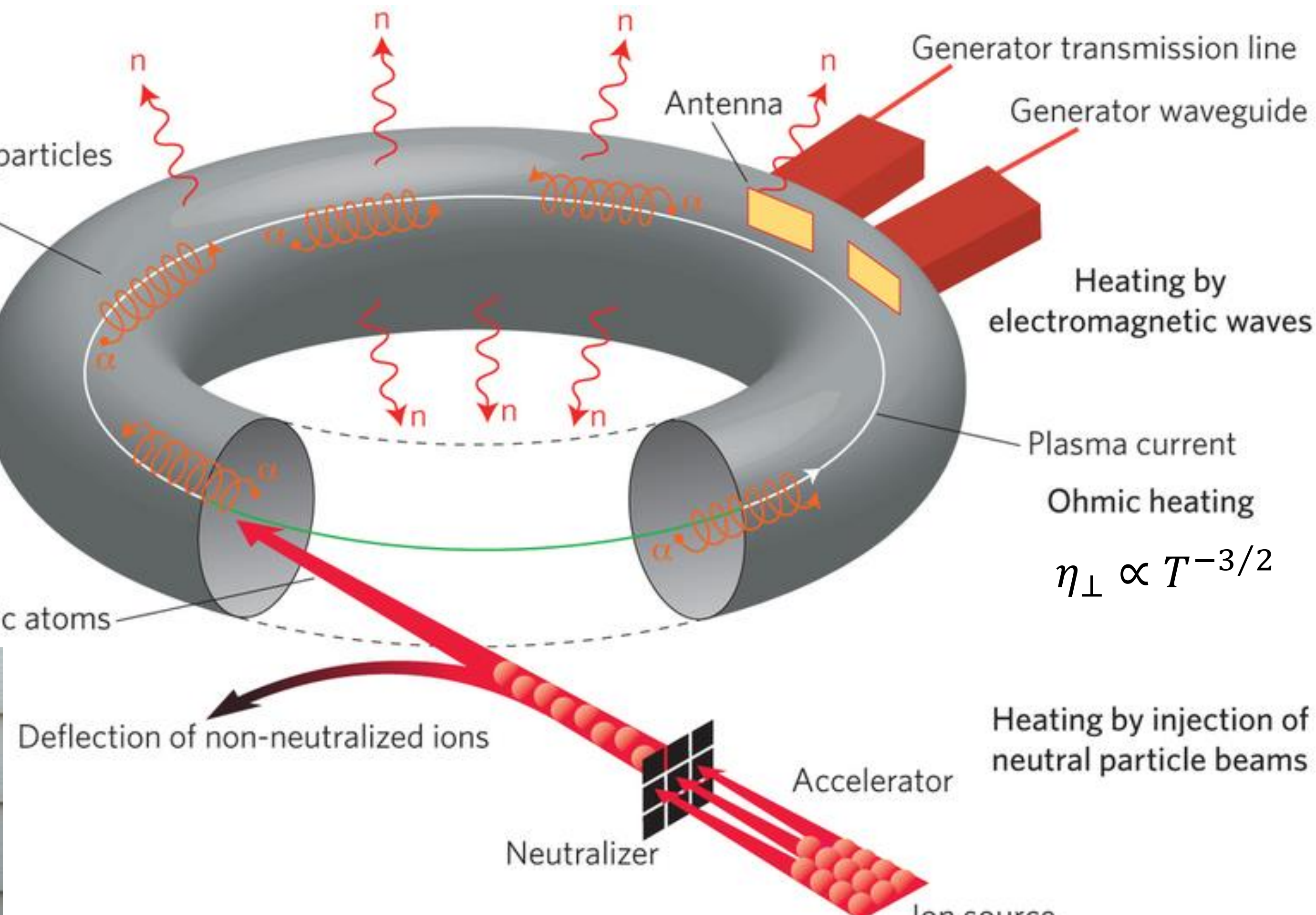


$$j = -enu_{||\mathrm{e}} + enu_{||\mathrm{i}} = 4\epsilon^{3/2}\frac{1}{B_{\mathrm{p}}}T\frac{dn}{dr}$$

- **The bootstrap current is vital for steady-state operation.**

Samuli Saarelma, Helsinki University of Technology, PhD Thesis 2015

Neutral beam injector is one of the main heat mechanisms in MCF



$$\eta_{\perp} \propto T^{-3/2}$$

D. Mazon, etc., NATURE PHYSICS, 12, 14, 2016
https://zh.wikihow.com/%E5%9C%A8%E6%89%8B%E6%8C%87%E4%B8%8A%E8%BD%AC%E7%AF%AE%E7%90%83

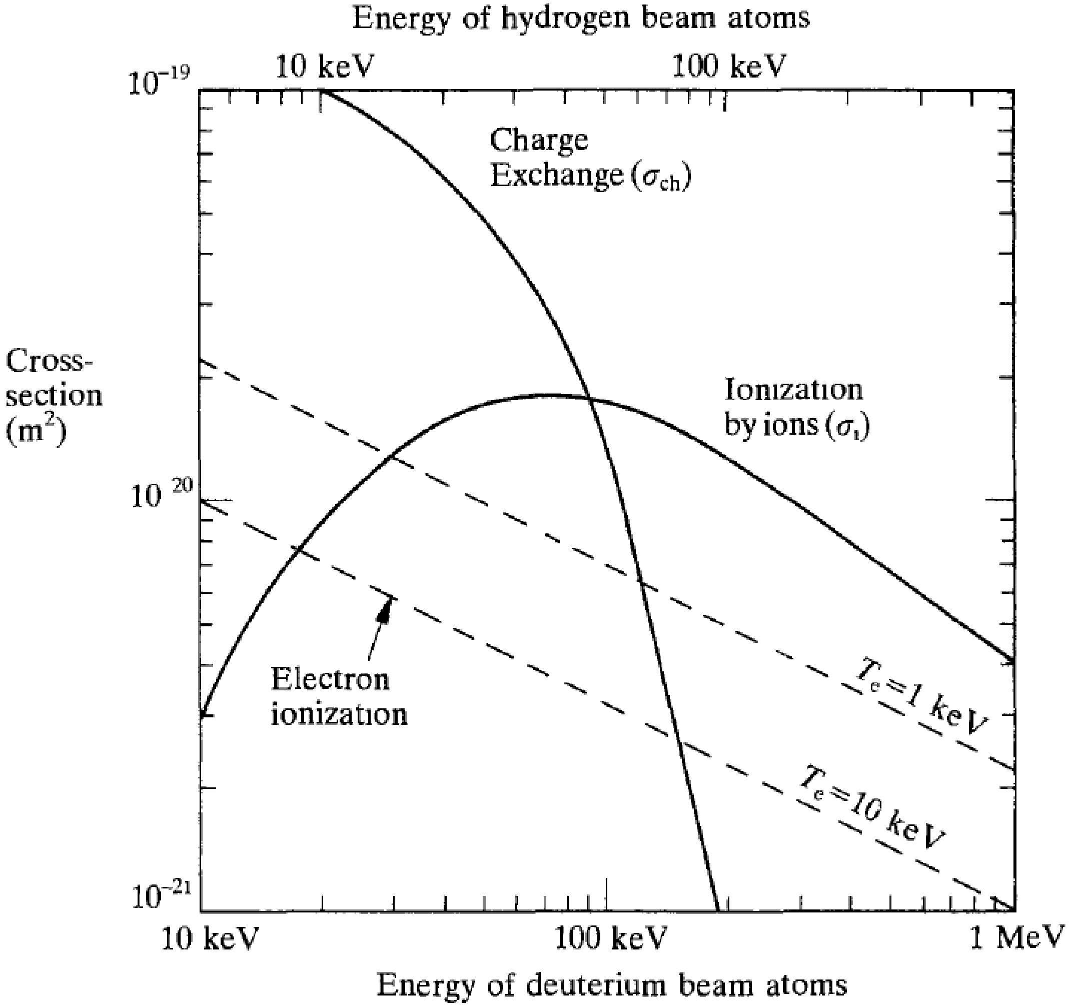
Varies way of heating a MCF device

System		Frequency/ energy	Maximum power coupled to plasma	Overall system efficiency	Development/ demonstration required	Remarks
ECRF	Demonstrated in tokamaks	28–157 GHz	2.8 MW, 0.2 s	30–40%	Power sources and windows, off-axis CD	Provides off-axis CD
	ITER needs	150–170 GHz	50 MW, S S			
ICRF	Demonstrated in tokamaks	25–120 MHz	22 MW, 3 s (L-mode); 16.5 MW, 3 s (H-mode)	50–60%	ELM tolerant system	Provides ion heating and smaller ELMs
	ITER needs	40–75 MHz	50 MW, S S			
LHRF	Demonstrated in tokamaks	1.3–8 GHz	2.5 MW, 120 s; 10 MW, 0.5 s	45–55%	Launcher, coupling to H-mode	Provides off-axis CD
	ITER needs	5 GHz	50 MW, S S			
NBI +ve ion	Demonstrated in tokamaks	80–140 keV	40 MW, 2 s; 20 MW, 8 s	35–45%	None	Not applicable
	ITER needs	None	None			
NBI −ve ion	Demonstrated in tokamaks	0.35 MeV	5.2 MW, D^-, 0.8 s (from 2 sources)			
	ITER needs	1 MeV	50 MW, S S	~37%	System, tests on tokamak, plasma CD	provides rotation

'S S' indicates steady state

Nuclear Fusion, 39, 2495, 1999

Neutral atoms are ionized by collisions in the plasma



- **Charge exchange:**

  $$H_b + H_p^+ \rightarrow H_b^+ + H_p$$

- **Ionization by ions**

  $$H_b + H_p^+ \rightarrow H_b^+ + H_p^+ + e^-$$

- **Ionization by electrons**

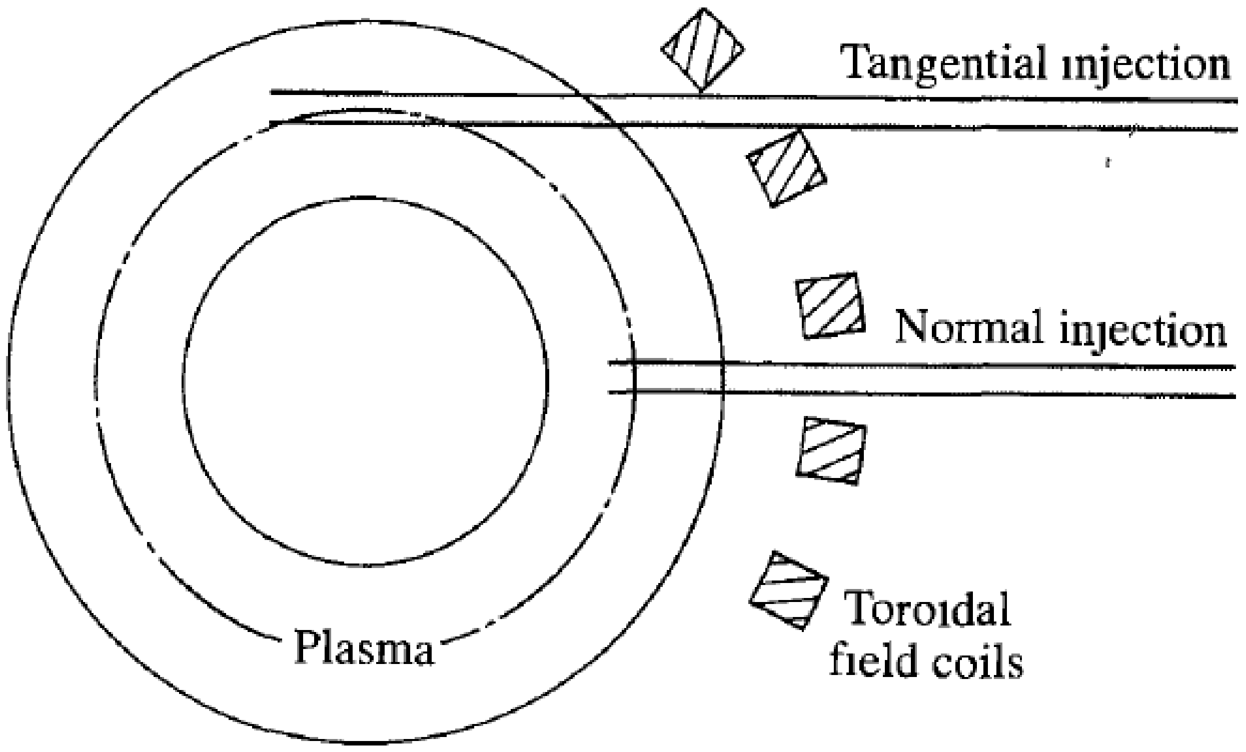
  $$H_b + e^- \rightarrow H_b^+ + 2e^-$$

b: beam
p: plasma

John Wesson, Tokamaks 3rd edition

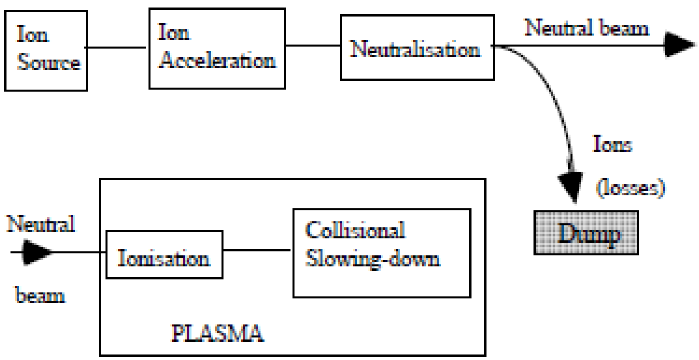
Neutral beam absorption length increases with tangential injection

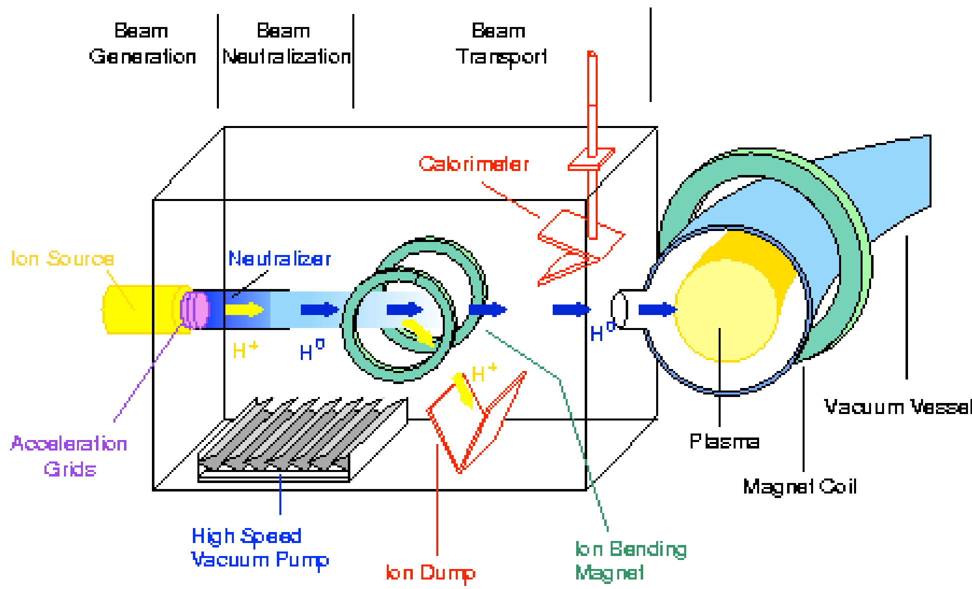
- **It is more difficult to access through the toroidal field coils with tangential injection.**



John Wesson, Tokamaks 3rd edition

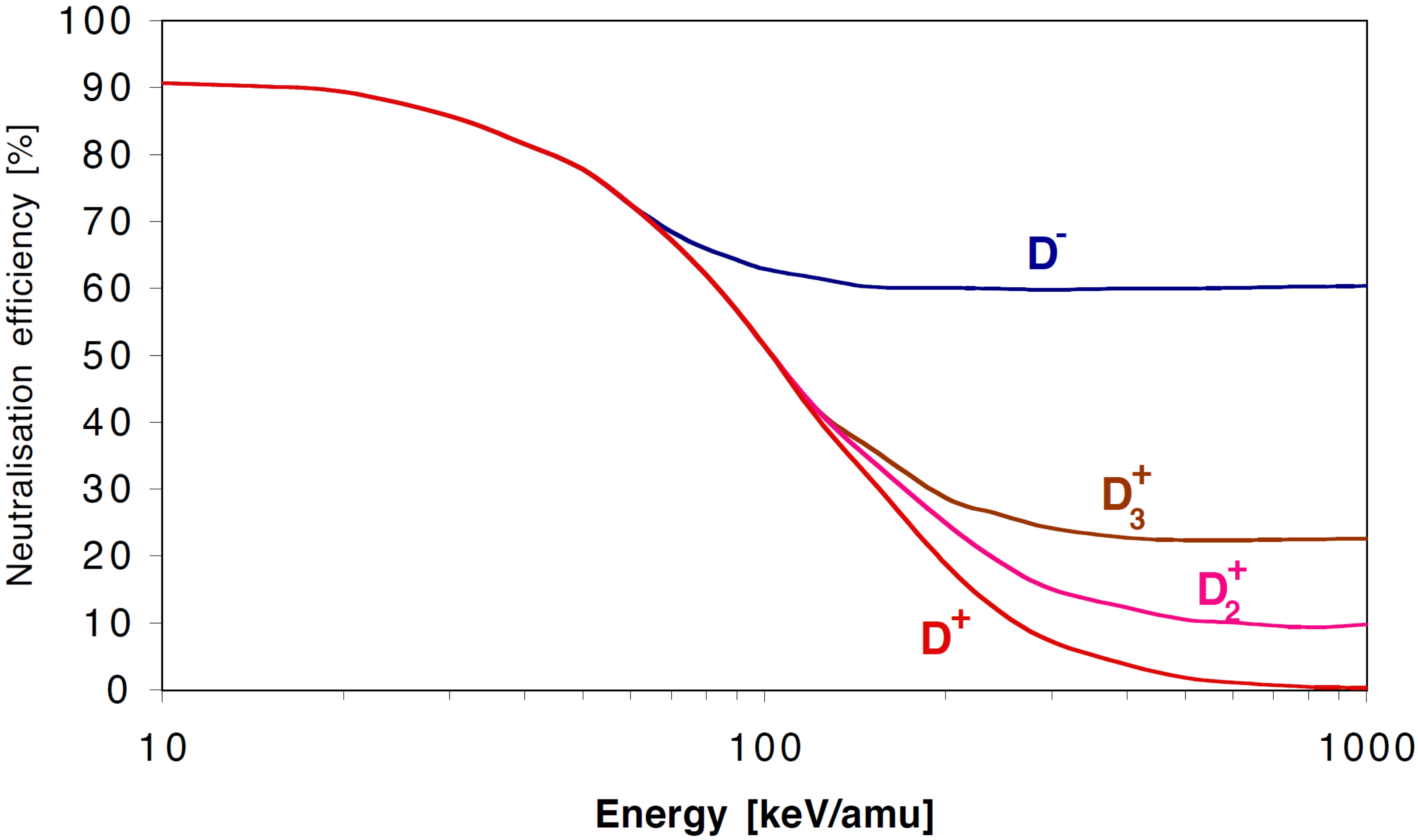
Neutral particles heat the plasma via coulomb collisions





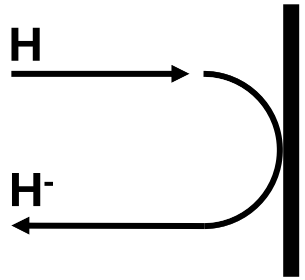
1. create energetic (fast) neutral ions
2. ionize the neutral particles
3. heat the plasma (electrons and ions) via Coulomb collisions

Negative ion source is preferred due to higher neutralization efficiency



There are two ways to make negative ions – surface and volume production

- **Surface production, depends on :**
  - **Work function Φ**
  - **Electron affinity level, 0.75 eV for H^-**
  - **Perpendicular velocity**
  - **Work function can be reduced by covering the metal surface with cesium**



$$H + e^- \longrightarrow H^-$$

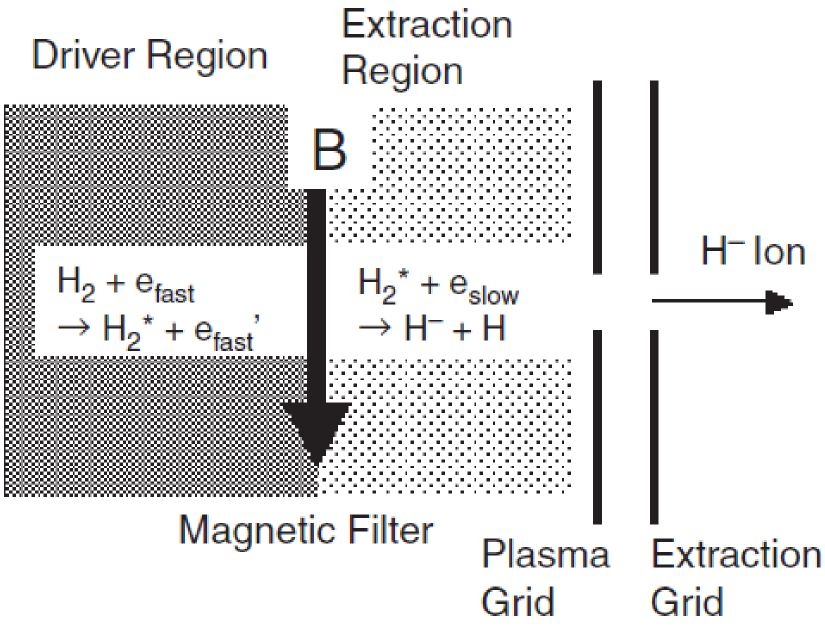
$$H^+ + 2e^- \longrightarrow H^-$$

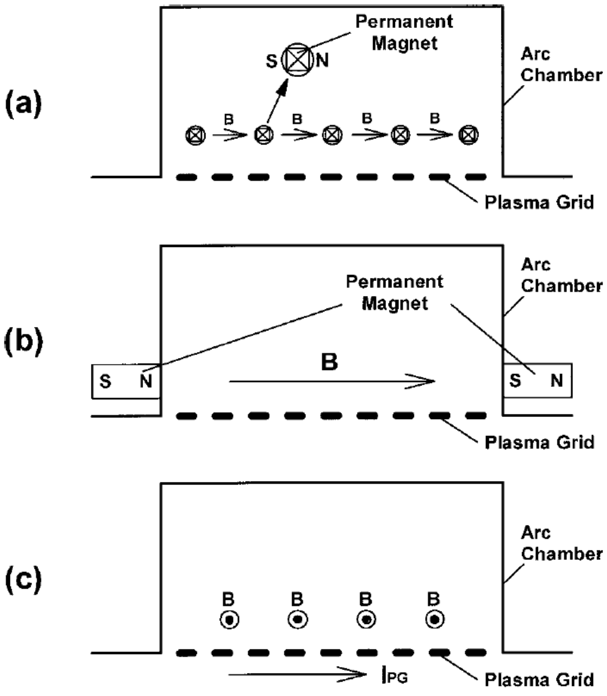
- **Volume production:**

$$H_2 + e_{fast}(>20\ \text{eV}) \rightarrow H_2^*(\text{excited state}) + e_{fast},$$

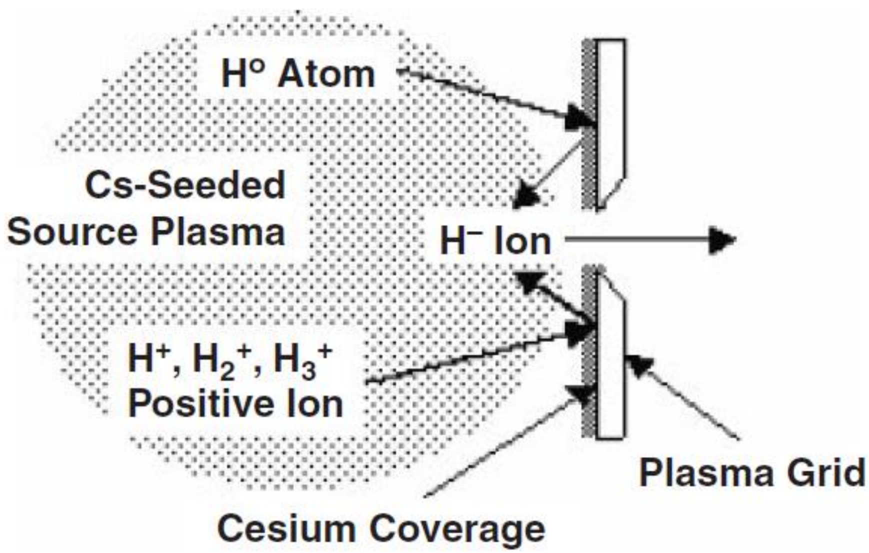
$$H_2^*(\text{excited state}) + e_{slow}(\approx 1\ \text{eV}) \rightarrow H^- + H.$$

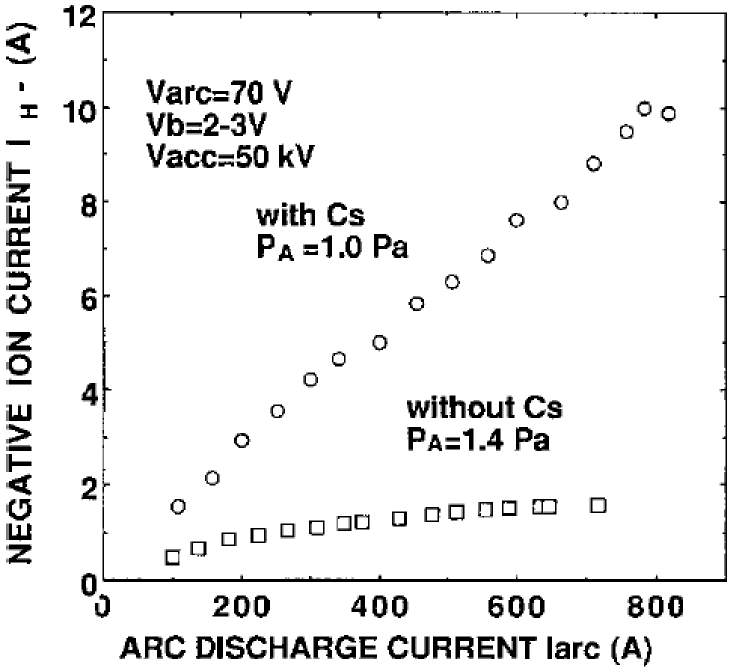
Two-chamber method of negative ions in volume production with a magnetic filter



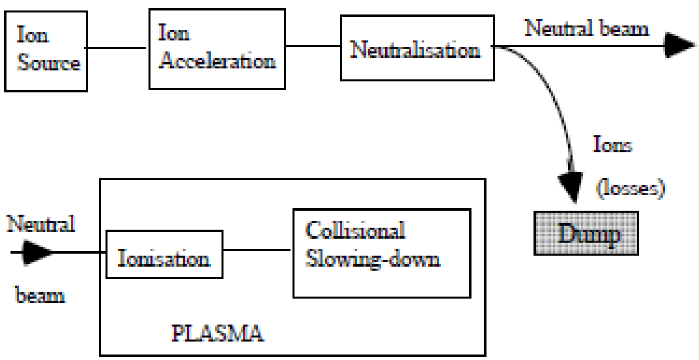


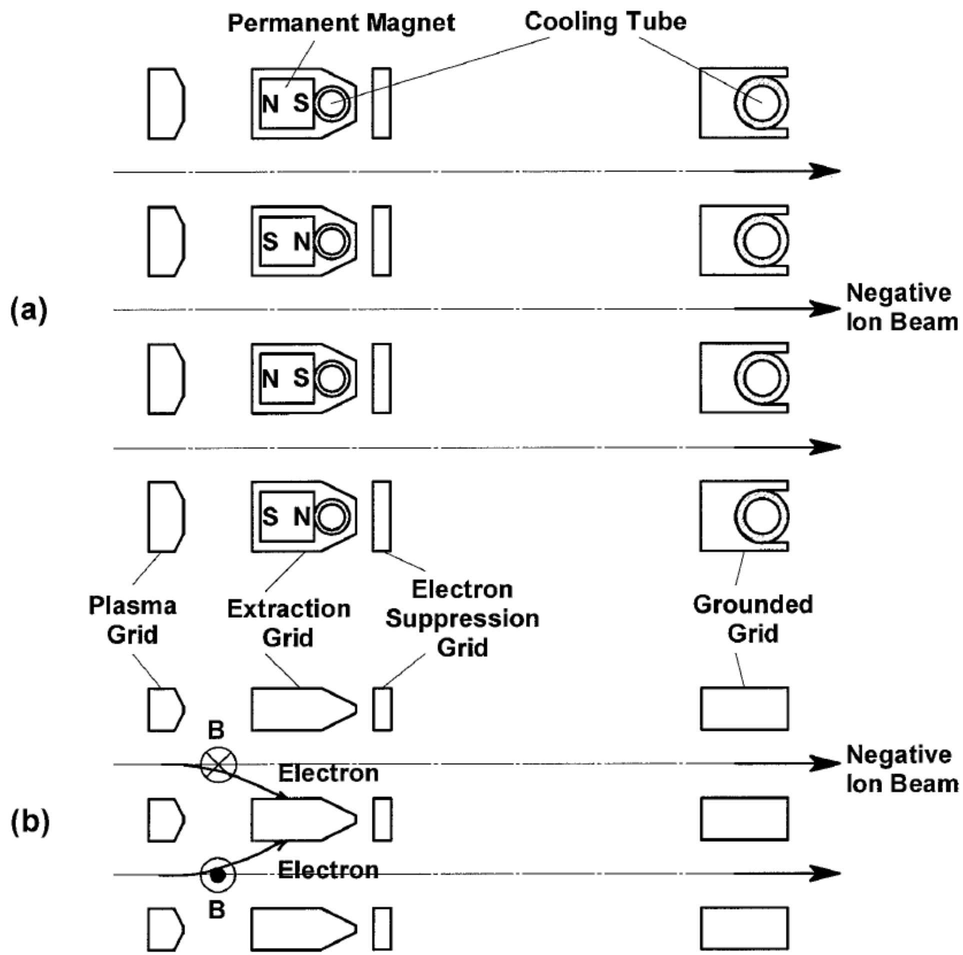
Adding cesium increases negative ion current





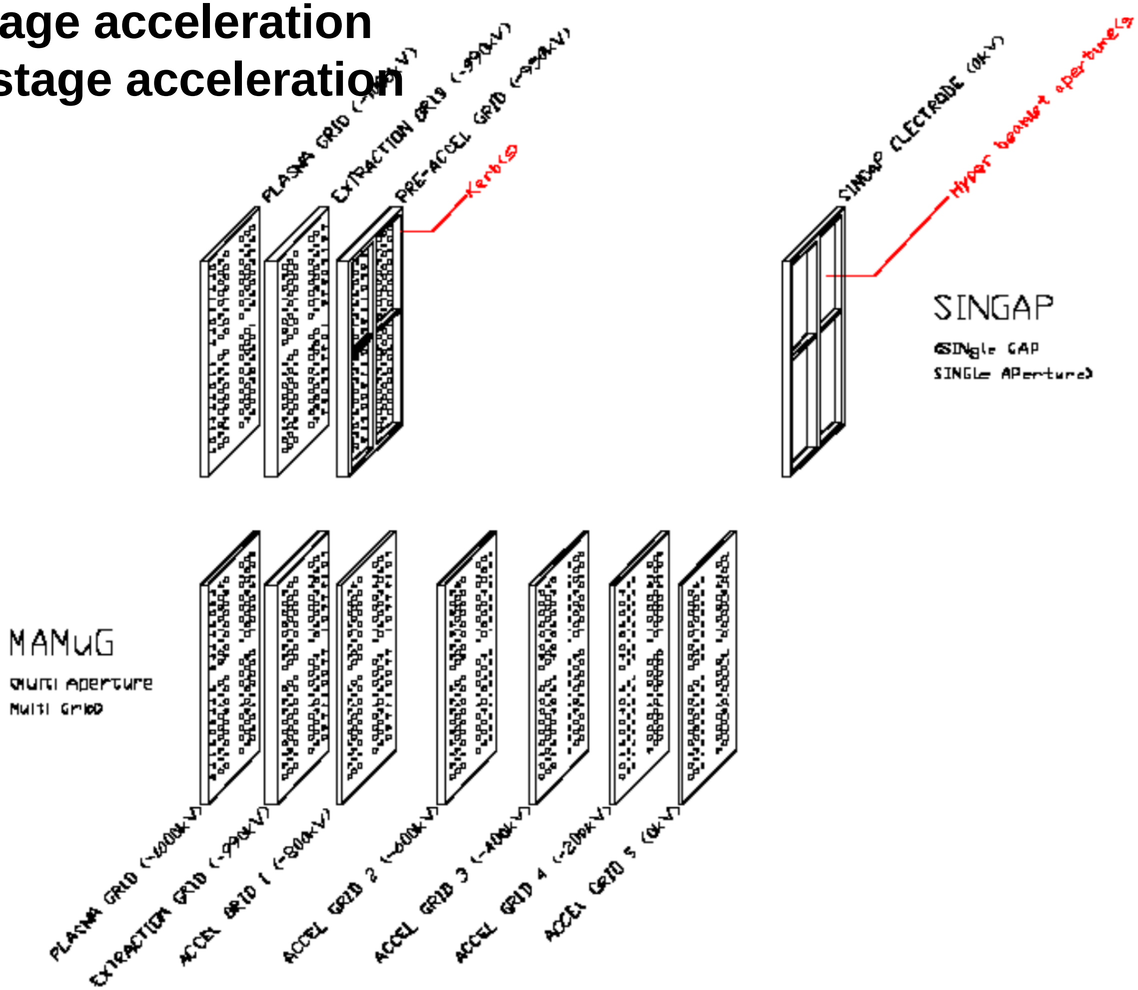
Electrons need to be filtered out since they are extracted together with negative ions





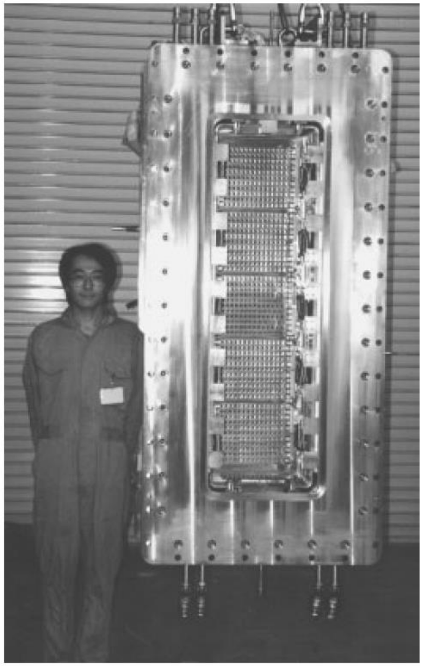
Acceleration

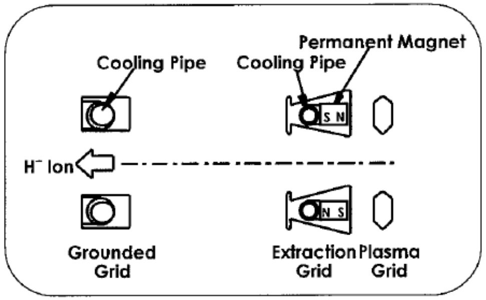
- **Multi-stage acceleration**
- **Single-stage acceleration**



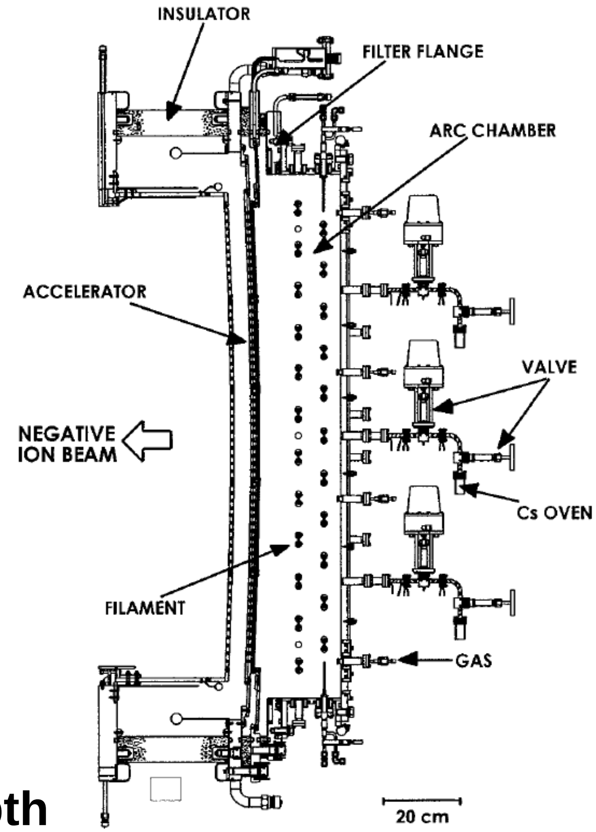
The ITER neutral beam system: status of the project and review of the main technological issues, presented by V. Antoni

NBI system of the LHD fusion machine





Negative Ion Accelerator

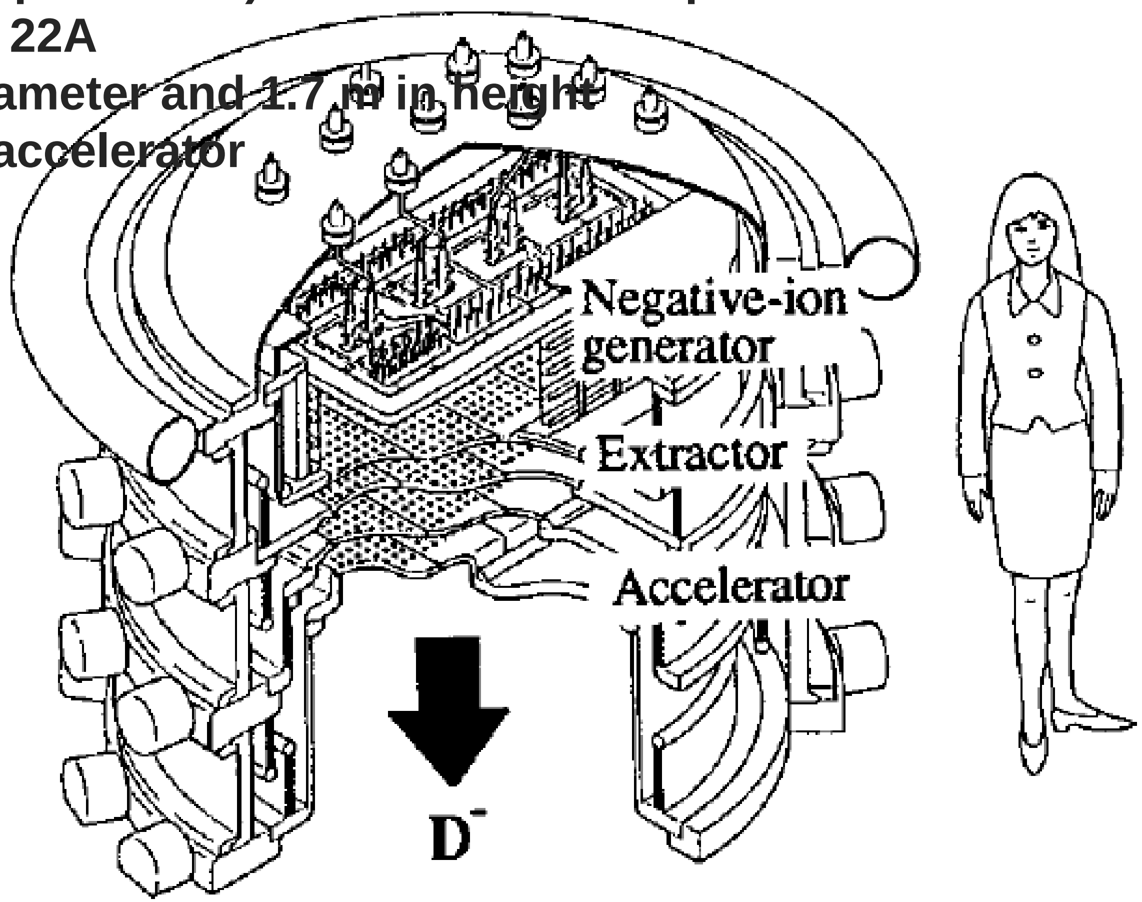


- **180 keV and 30 A**
- **Arc chamber: 35 cm x 145 cm, 21cm in depth**
- **Single stage accelerator**

Y. Takeiri, etc., Nucl. Fusion, 46, S199, 2006

JT60U NBI system

- JT-60 (Japan-Torus) is a tokamak in Japan.
- 550 keV, 22A
- 2m in diameter and 1.7 m in height
- 3-stage accelerator



Neutralization

- **Gas neutralization**
  - **Collisions between fast negative ions and atoms**

$$H^- + H_2 \longrightarrow H + H_2 + e^-$$

  - **Fast ions can lose another electron after neutralized**

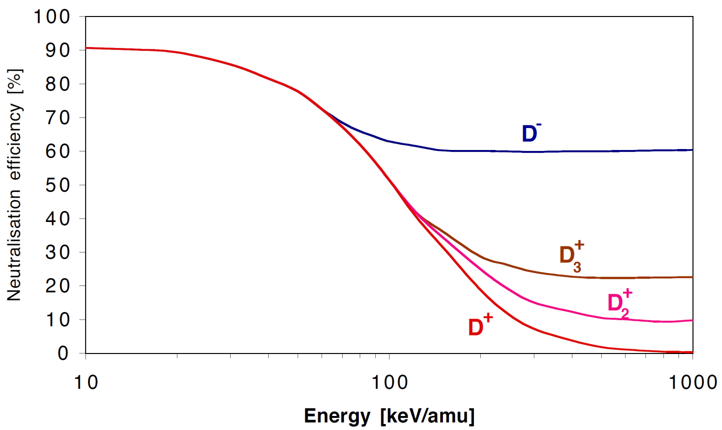
$$H + H_2 \longrightarrow H^+ + H_2 + e^-$$

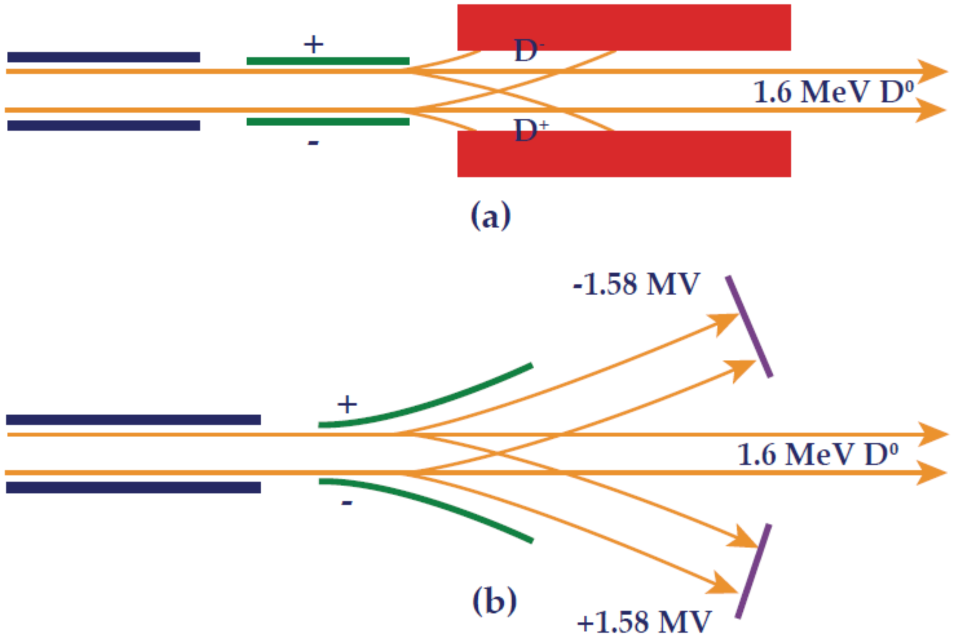
- **Plasma neutralization**
  - **Collisions with charged particles in plasma**

$$H^- + X(e, \mathrm{Ar}, H^+, {H_2}^+) \longrightarrow H + X + e^-$$

  - **The efficiencies reach up to 85% for fully ionized hydrogen plasma**

Beam dump

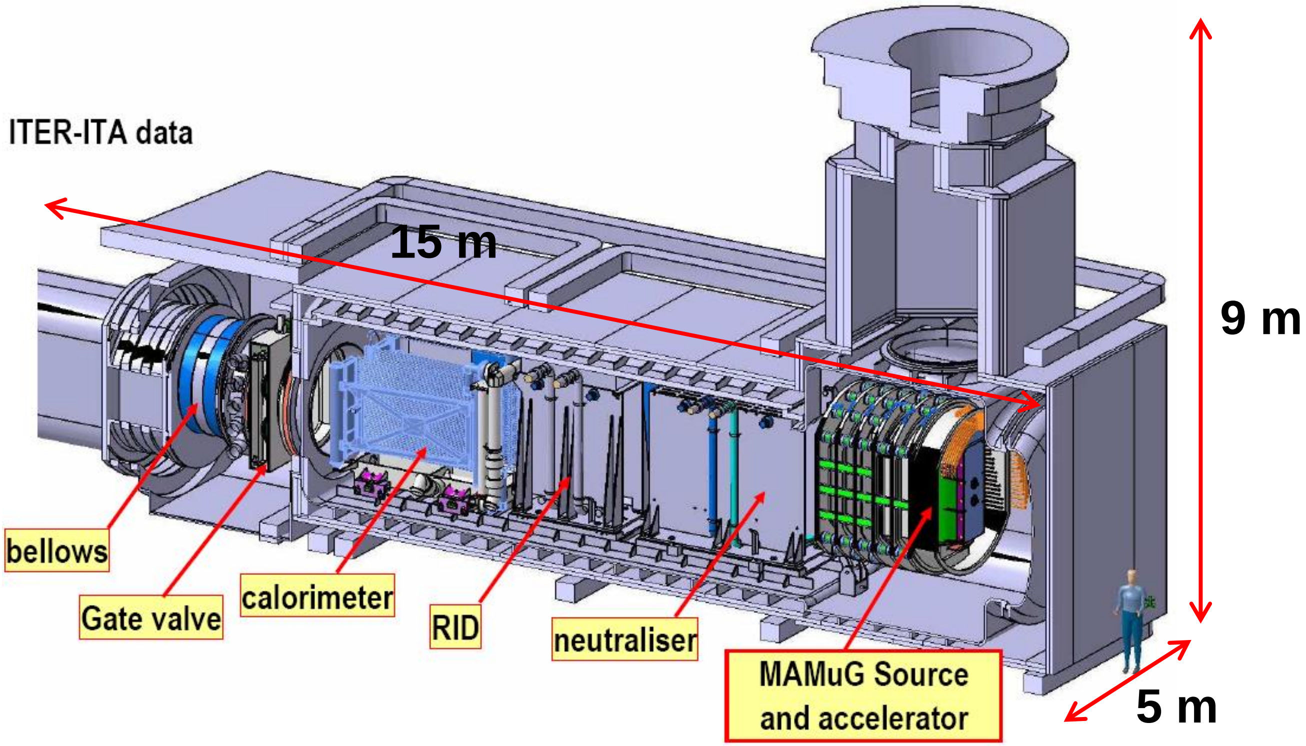




R. W. Moir, Fusion Technology, 25, 129, 1994

NBI for ITER

- **beam components (Ion Source, Accelerator, Neutralizer, Residual Ion Dump and Calorimeter)**
- **other components (cryo-pump, vessels, fast shutter, duct, magnetic shielding, and residual magnetic field compensating coils)**



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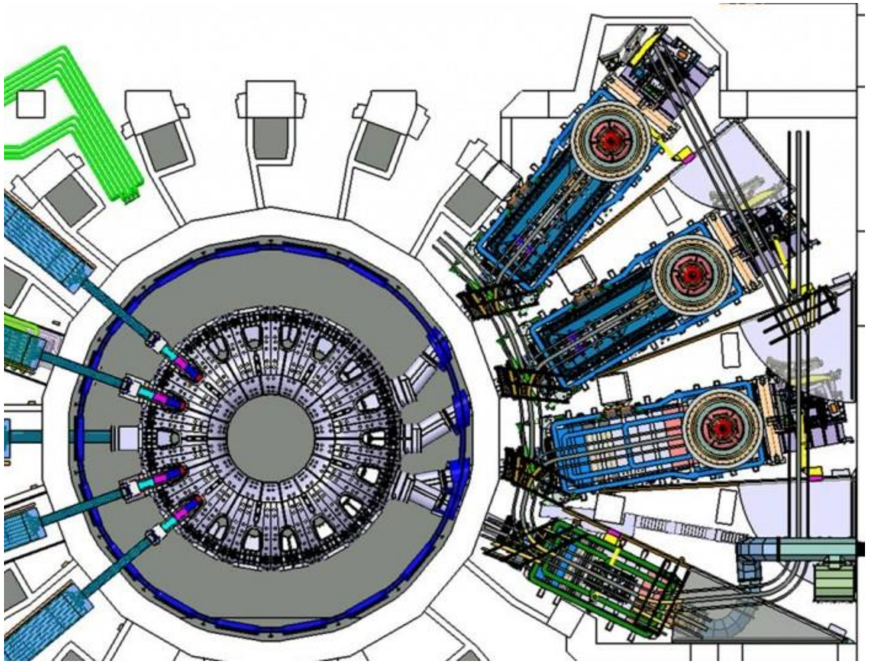
Neutral beam penetration



- **Parallel direction**
  - **Longest path through the densest part of the plasma**
  - **Harder to be built**
- **Perpendicular direction**
  - **Path is short**
  - **Larger perpendicular energies leads to larger losses**
  - **Easier to be built**

https://www.iter.org