氫硼核融合實作
Practical course on proton-boron nuclear fusion

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Institute of Space and Plasma Sciences, National Cheng Kung University

2025 summer break

7/14(Mon.) – 7/18(Fri.) 14:00-17:40

Lecture 2

https://capst.ncku.edu.tw/PGS/index.php/teaching/

2025/7/15 updated

Course Outline

- Formosa Integrated Research Spherical Tokamak (FIRST), the first Tokamak in Taiwan
- Physics used in Alpha-E
  - Microwave
  - ECRH
  - Spectrum of fusion products
  - Stopping power of the filter
  - Data analysis using Pulse Shaping
- Final report

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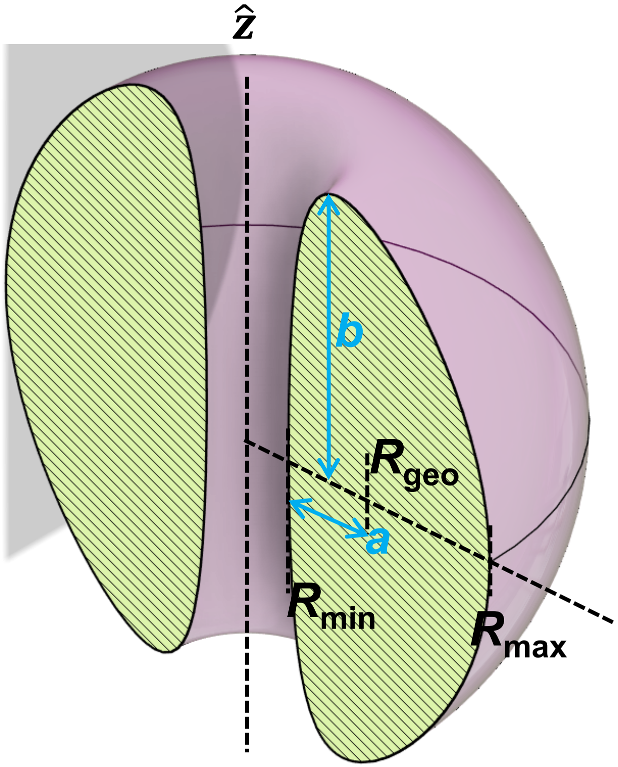
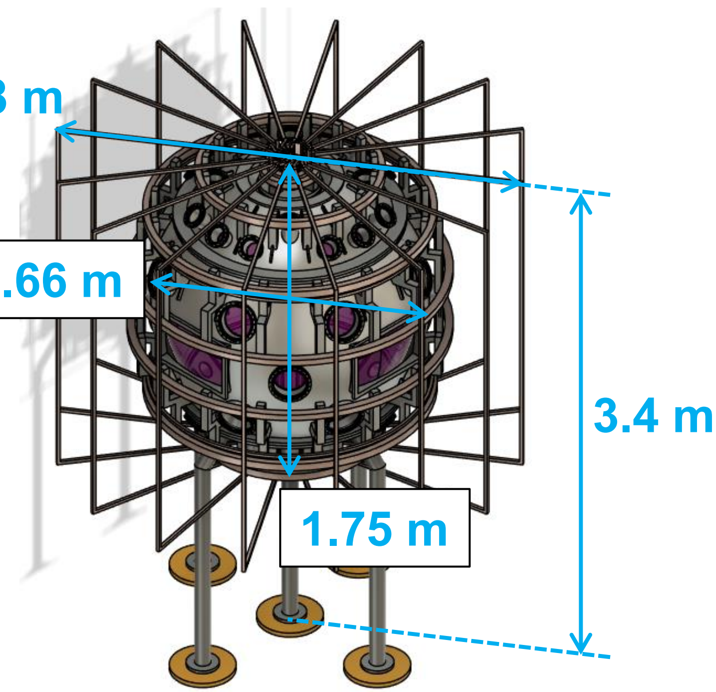
Development of Formosa Integrated Research Spherical Tokamak (FIRST), the first Tokamak in Taiwan

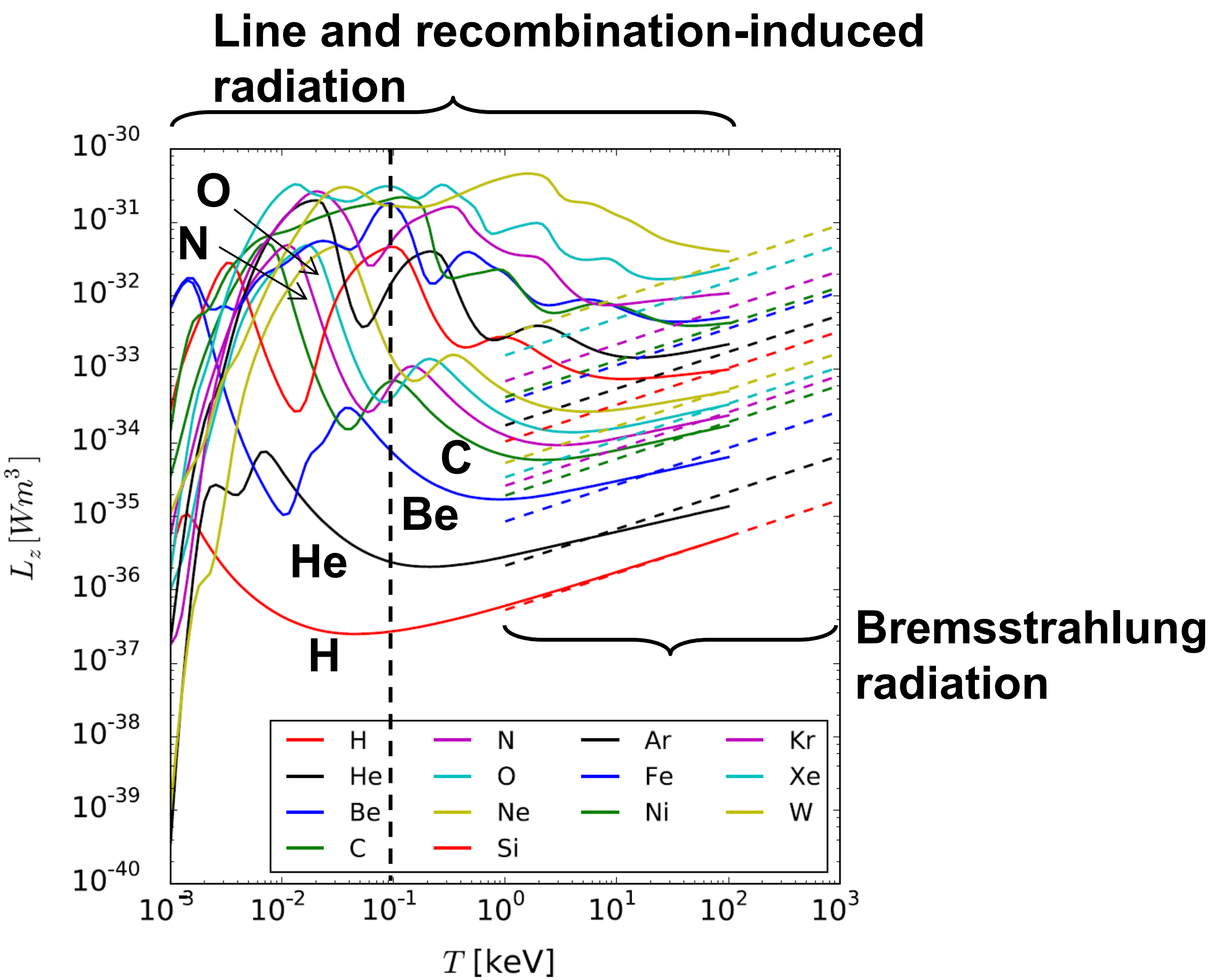


- R_{geo} = 45 cm
- a = 32 cm
- b = 76.8 cm
- T ~ 100 eV
- B_T ~ 0.1 - 0.5 T
- I_p ~ 100 kA
- Ohmic heating
- Gas: H_2
- Duration: 100 ms

- **First plasma: 2026.**



Temperature of 100 eV is the threshold of radiation barrier by impurities



H. Lux, etc., Fusion Eng. Des. **42**, 101 (2015)

Teams

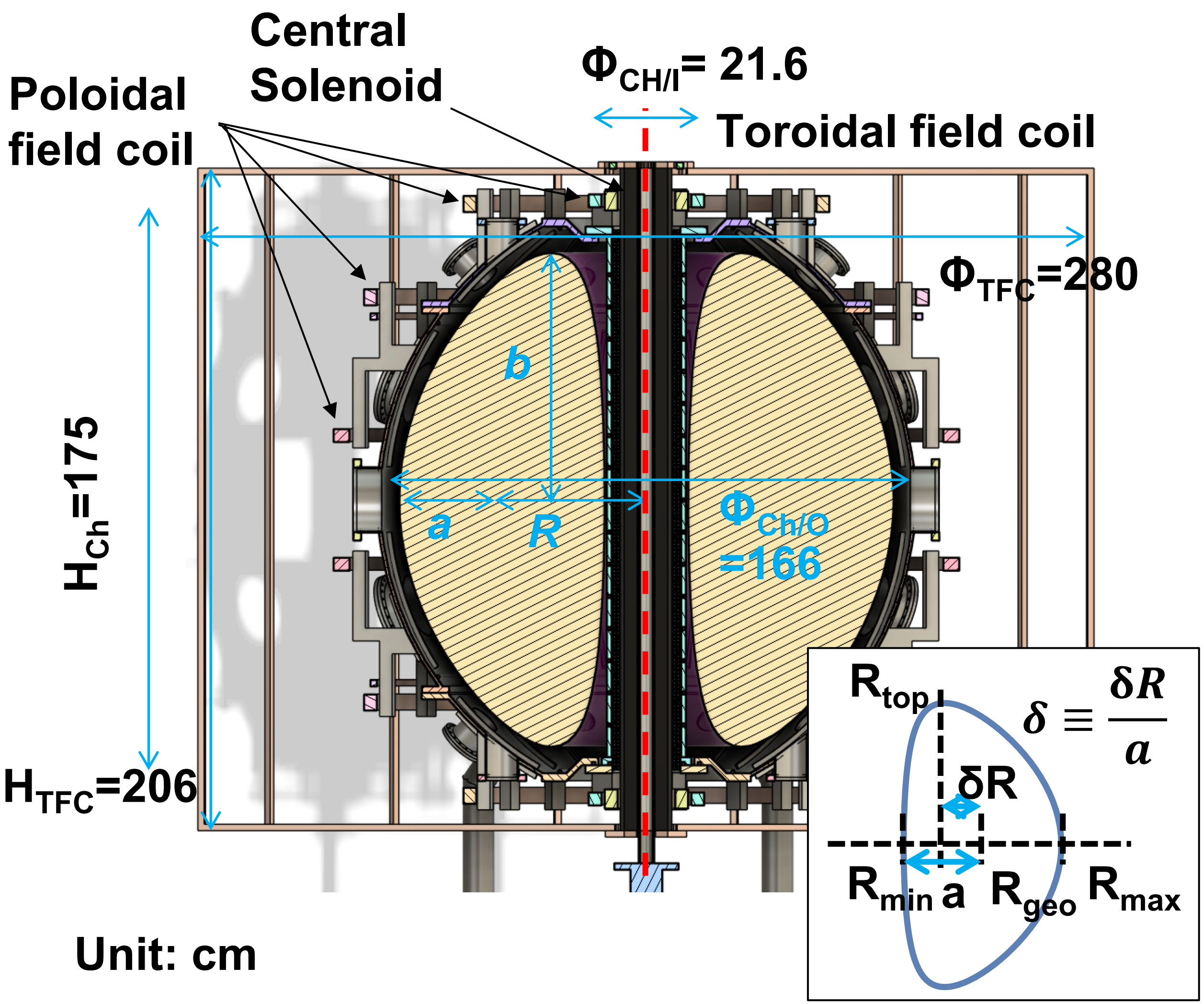
- 馬維揚團隊 @ 國家原子能科技研究院
  - Site
- 張博宇團隊 @ 成功大學 太空與電漿科學研究所
  - System design and development/diagnostics
- 向克強團隊 @ 成功大學 前瞻電漿中心
  - Theoretical design
- 河森榮一郎團隊 @ 成功大學 太空與電漿科學研究所
  - Diagnostics
- 柳克強團隊 @ 清華大學 工程與系統科學系
  - Diagnostics
- 蔡宗哲團隊 @ 國家高速網路與計算中心
  - Simulation
- 張存續團隊 @ 清華大學 物理系
  - RF startup

- **We welcome anyone interested in fusion research to join us!**

The system will be built in National Atomic Research Institute (NARI, 國家原子能科技研究院) at 龍潭

https://www.sipa.gov.tw/home.jsp?mserno=201001210037&serno=201001210041&menudata=ChineseMenu&contlink=content/introduction_4_1.jsp&serno3=201002010023
https://www.nari.org.tw/newsdetail/activity/353.html

Formosa Integrated Research Spherical Tokamak (FIRST) aiming for the first plasma in 2026



Unit: cm

- $R/a/b$=45/32/76.8 cm
- Aspect ratio = R/a = 1.5
- $k = b/a = 2.4$
- $\delta = 0.5$
- B_T = 0.1 ~ 0.5 T
- $T \geq$ 100 eV
- $I_p \geq$ 100 kA
- Gas: H_2
- Ohmic heating
- Duration: 100 ms
- FIRST is targeted for
  - Low aspect ratio
  - High beta
  - High bootstrap current

- **We welcome anyone interested in fusion research to join us!**

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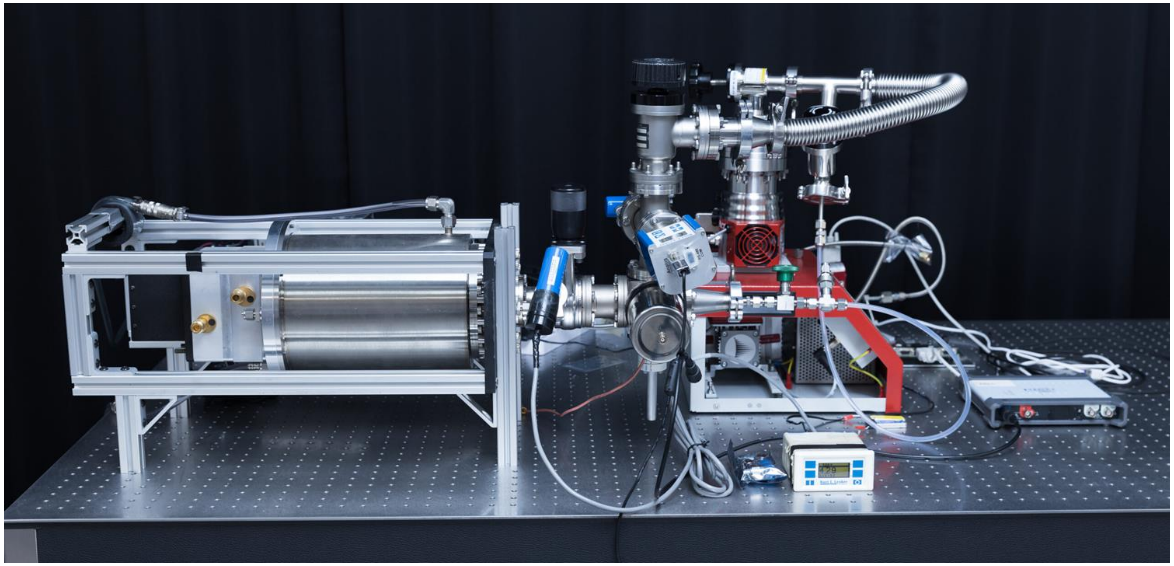
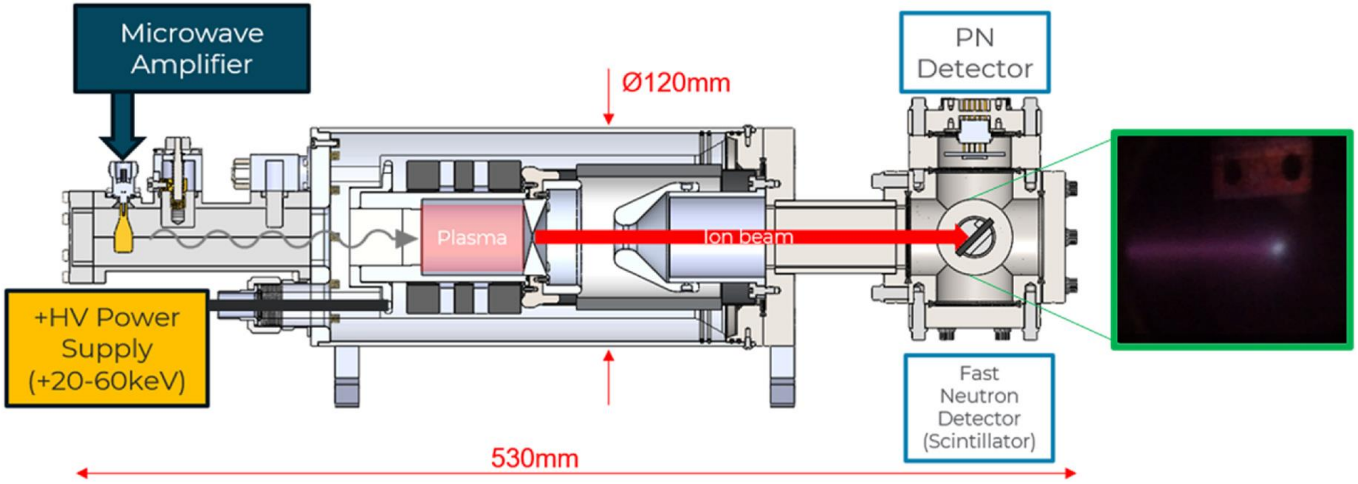
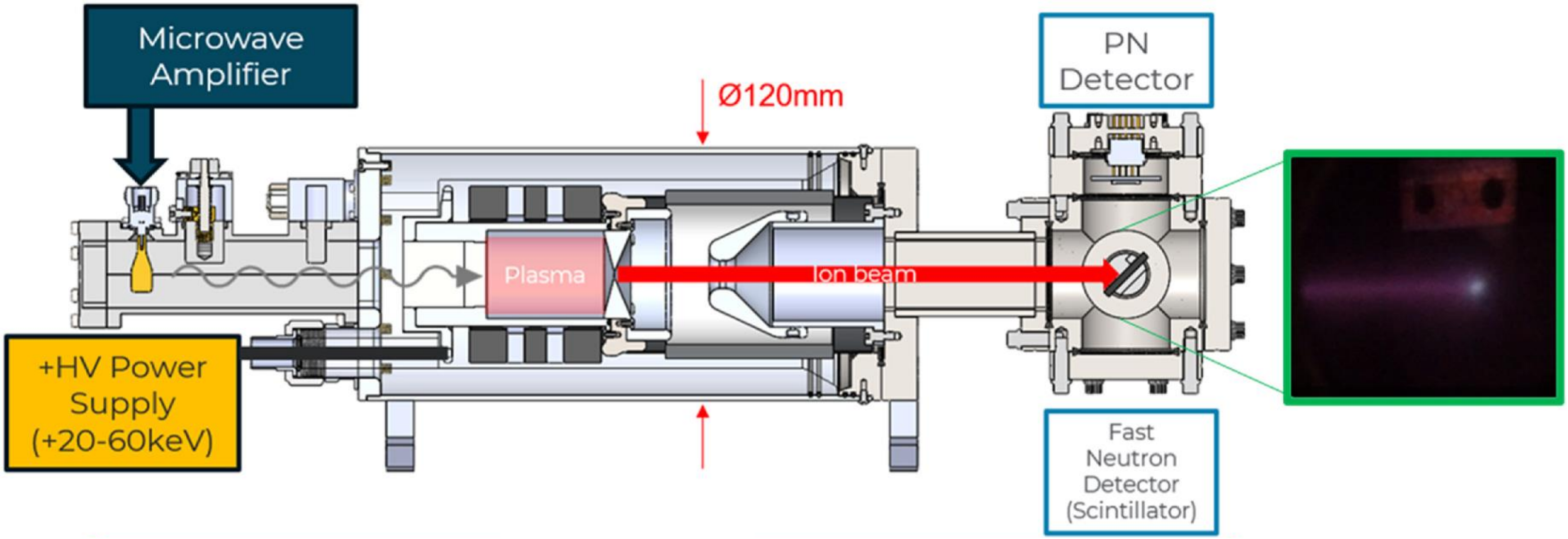
Alpha-E provided by Alpha Ring will be used to demonstrate the proton-boron nuclear fusion





https://alpharing.com/alpha-e/
Allan Xi Chen, etc., Physics Open 21, 100234(2024)

Alpha-E provided by Alpha Ring will be used to demonstrate the proton-boron nuclear fusion



Microwave source

Electron cyclotron resonance heating (ECRH)

Acceleration

Nuclear fusion / Detector

Allan Xi Chen, etc., Physics Open 21, 100234(2024)

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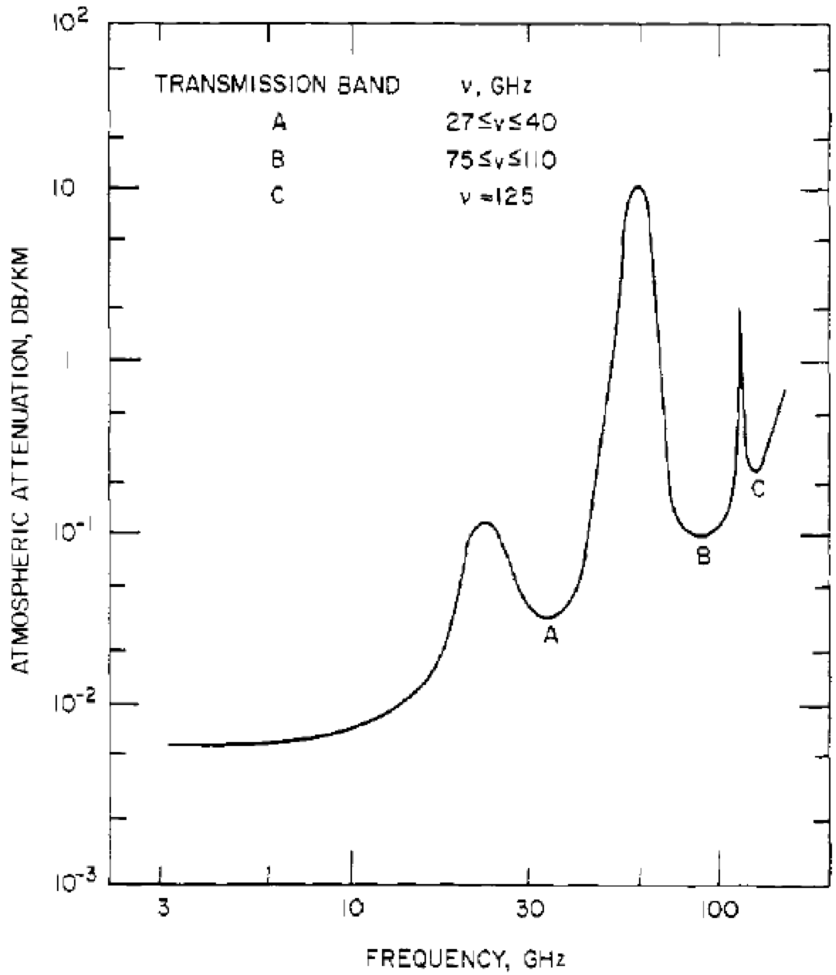
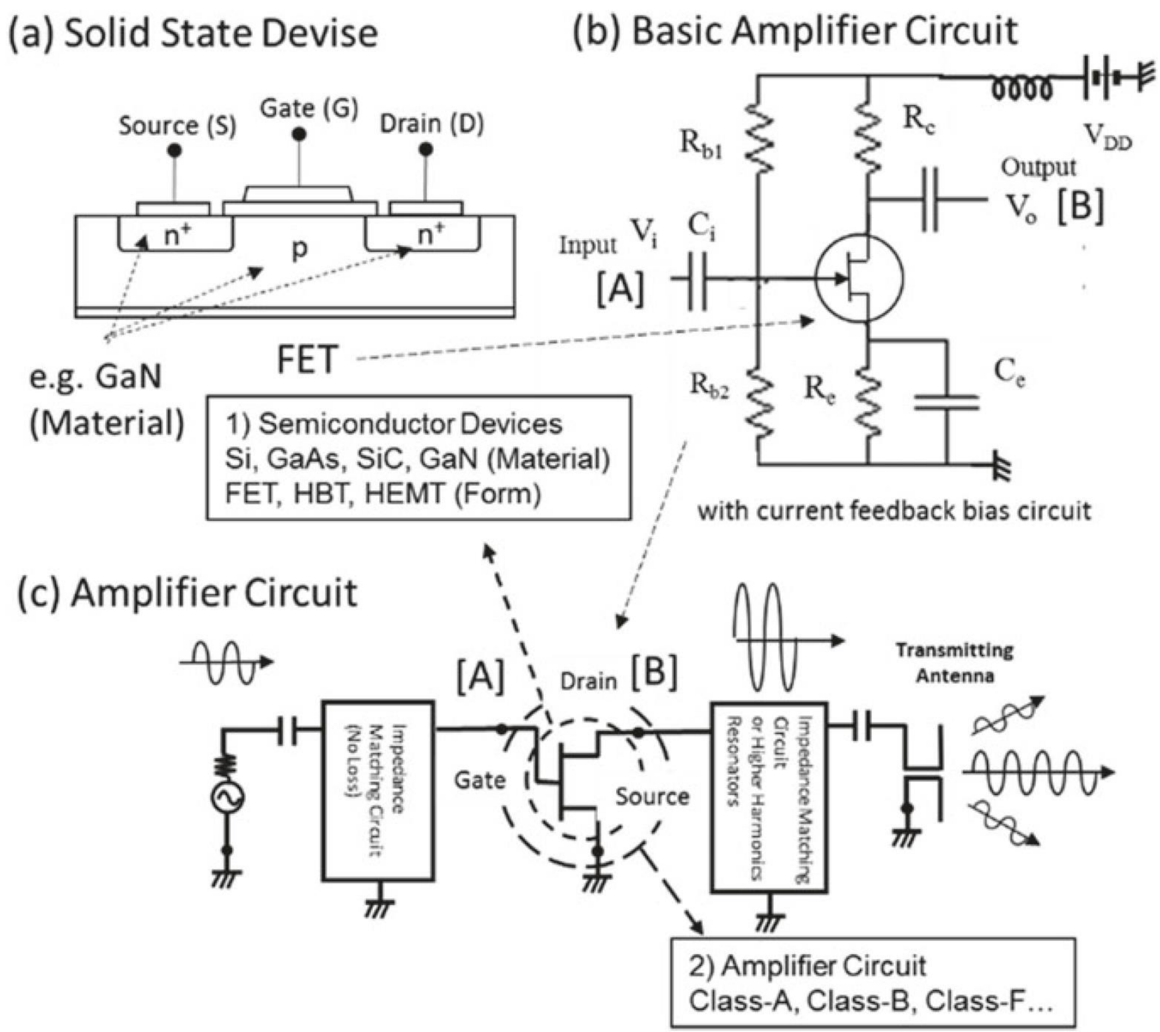
Microwave frequency is determined for those used in communications and radar purposes



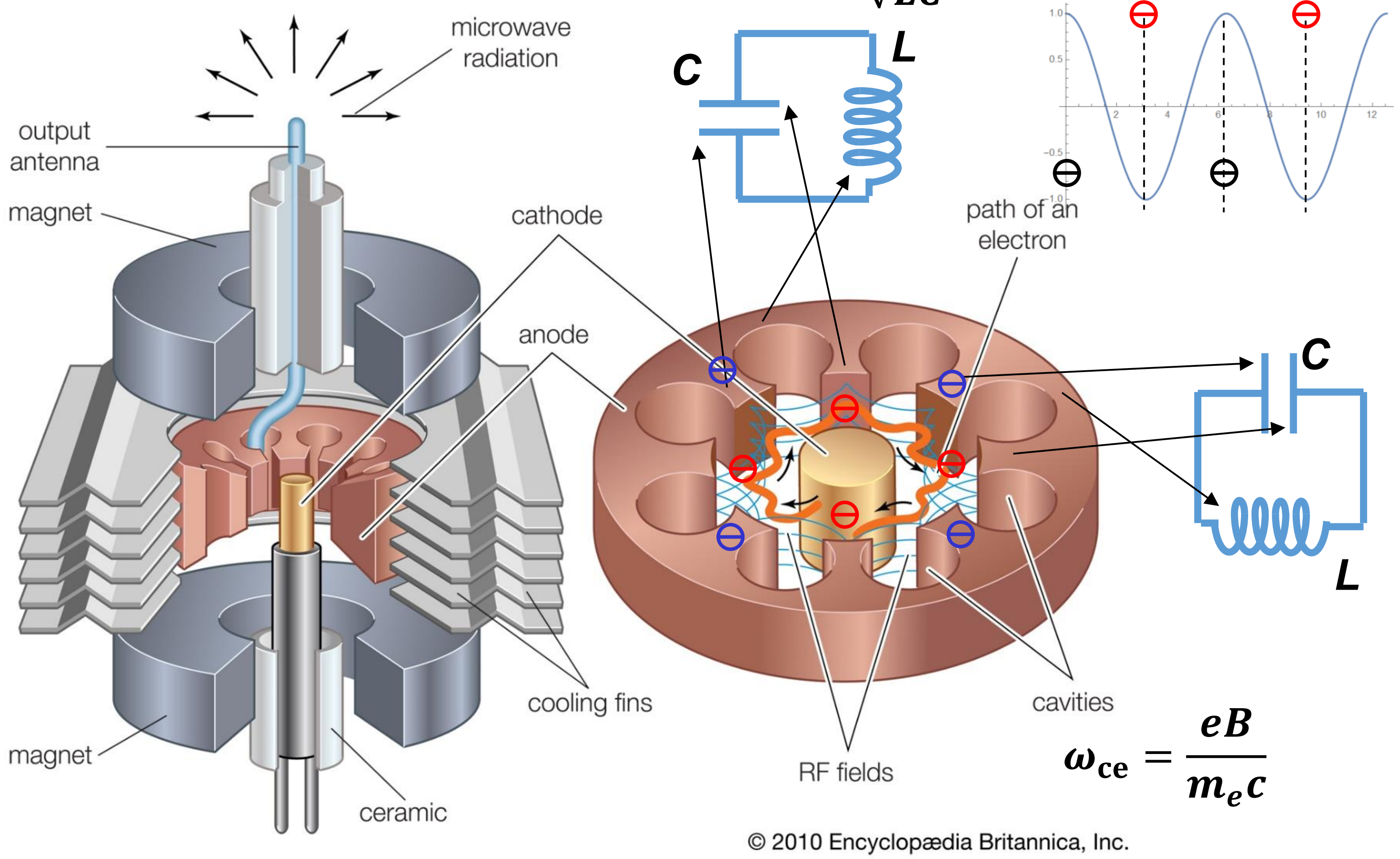
Microwave / Radio-frequency (RF) wave can be generated using solid-state devise



S. Horikoshi and N. Serpone, RF Power semiconductor generator application in heating and energy utilization, Springer

Internal of a magnetron

$$\omega_{\mathrm{LC}} = \frac{1}{\sqrt{LC}} \equiv 4\omega_{\mathrm{ce}}$$

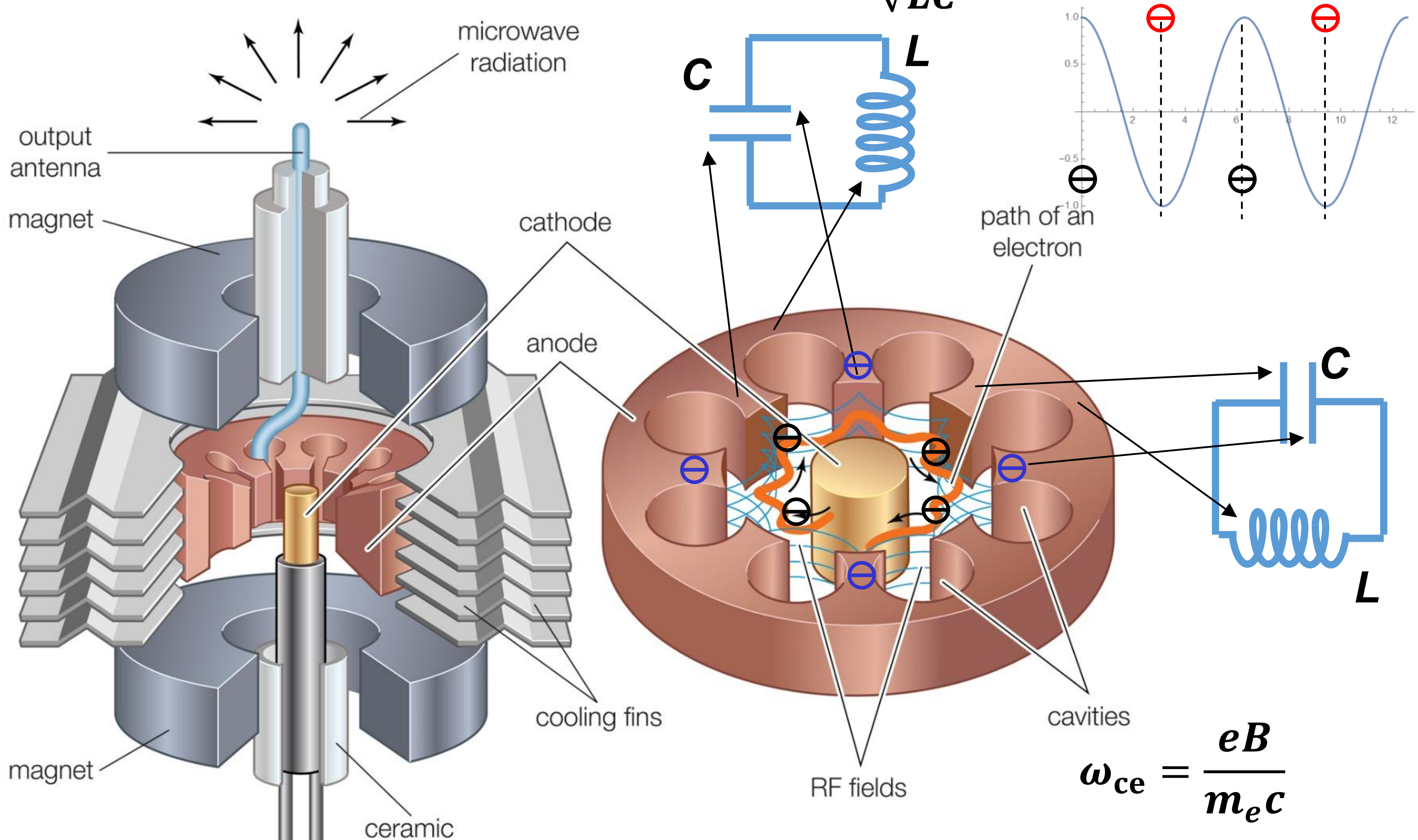


$$\omega_{\mathrm{ce}} = \frac{eB}{m_e c}$$

https://kids.britannica.com/students/article/electron-tube/106024/media?assemblyId=137

Internal of a magnetron

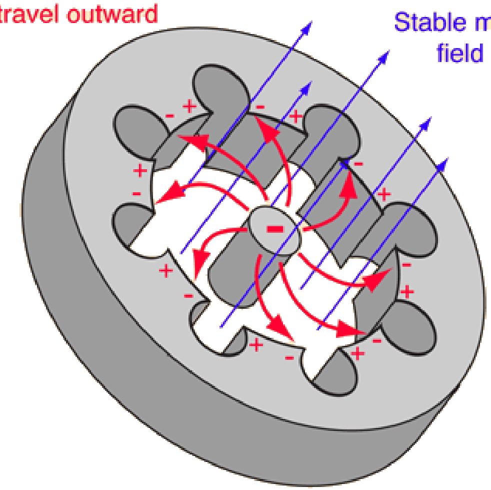
$$\omega_{LC} = \frac{1}{\sqrt{LC}} \equiv 4\omega_{ce}$$



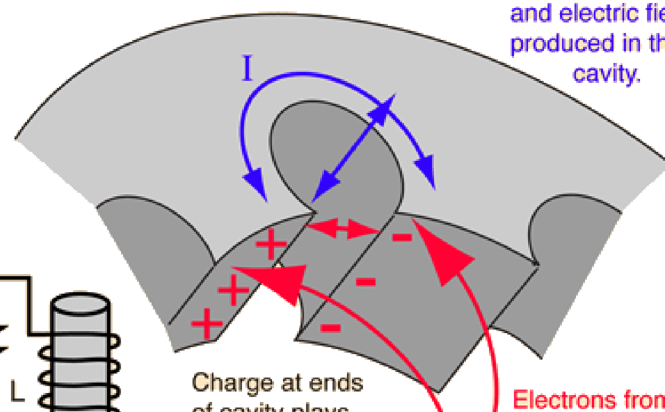
$$\omega_{ce} = \frac{eB}{m_e c}$$

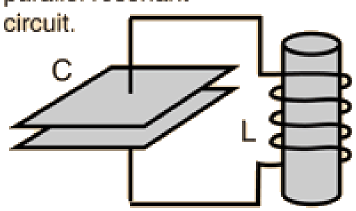
https://kids.britannica.com/students/article/electron-tube/106024/media?assemblyId=137

Magnetron is a forced oscillation driven by electrons between the gap



Electrons from a hot filament would travel radially to the outside ring if it were not for the magnetic field. The magnetic force deflects them in the sense shown and they tend to sweep around the circle. In so doing, they "pump" the natural resonant frequency of the cavities. The currents around the resonant cavities cause them to radiate electromagnetic energy at that resonant frequency.

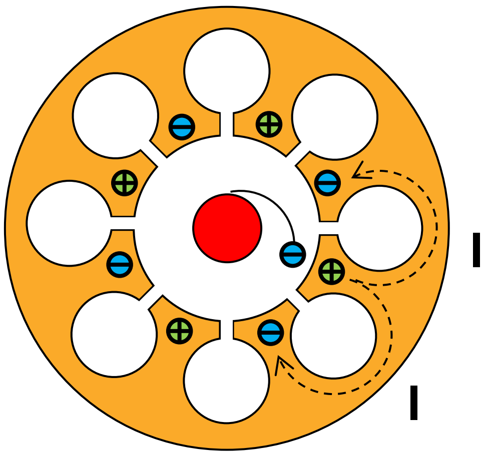


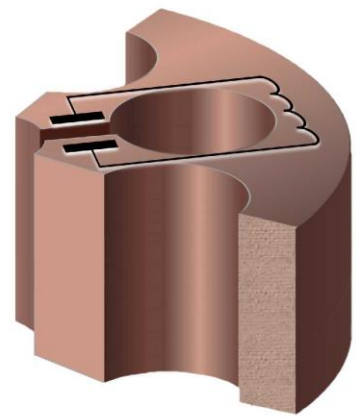


$$f_{resonance} \approx \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

http://hyperphysics.phy-astr.gsu.edu/hbase/Waves/magnetron.html

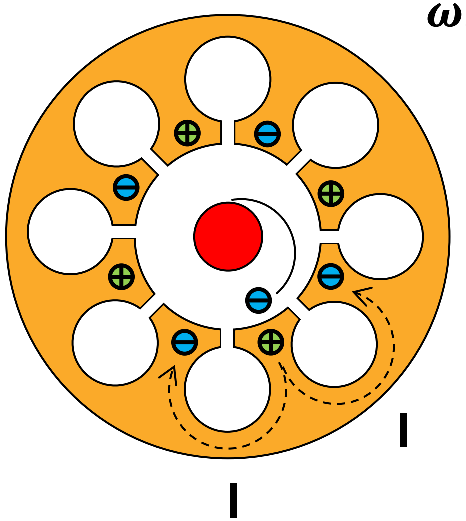
Strong oscillation occurs when the electron cyclotron frequency match the LC oscillation frequency





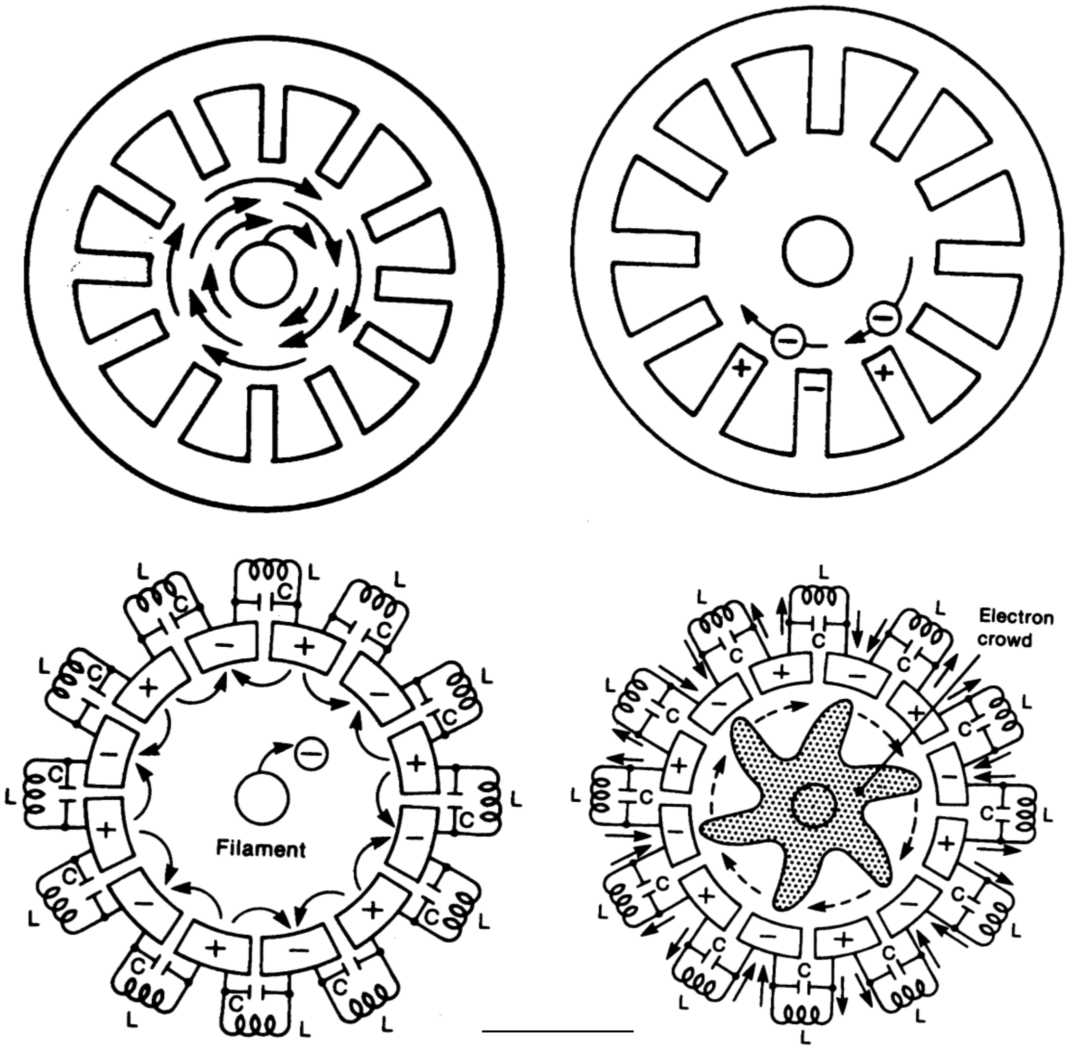
$$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega_{\mathrm{CE}} = \frac{eB}{mc}$$



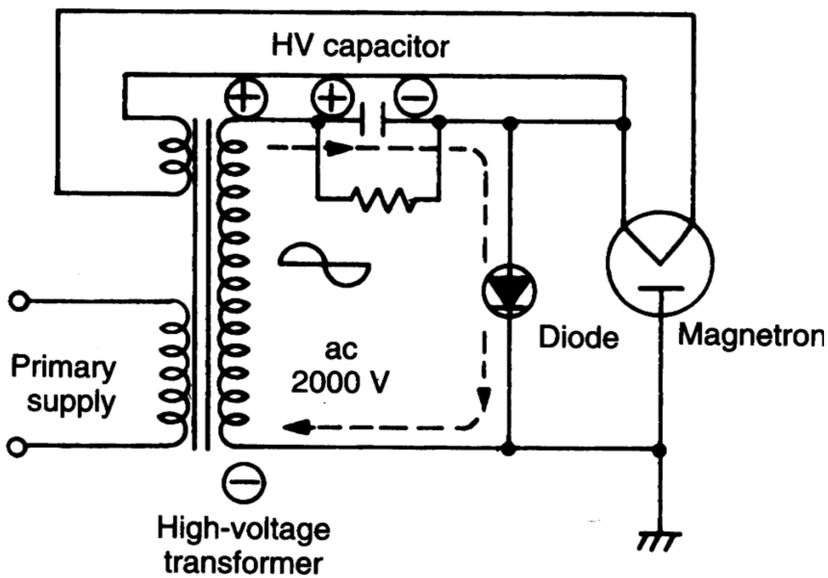
Resonance condition: $\omega_{\mathrm{CE}} = \omega$

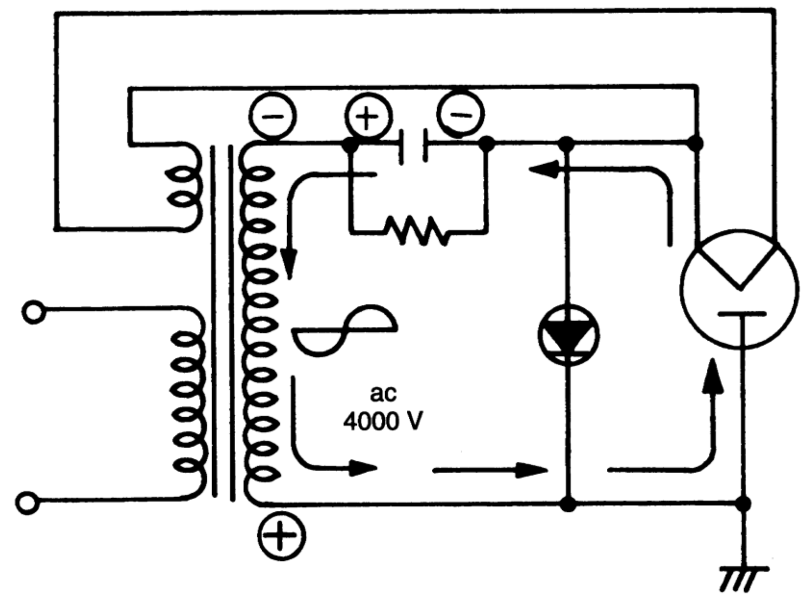
Resonance in a magnetron



http://cdn.preterhuman.net/texts/government_information/intelligence_and_espionage/homebrew.military.and.espionage.electronics/servv89pn0aj.sn.sourcedns.com/_gbpprorg/mil/herf1/index.html

Magnetron schematic diagram





http://cdn.preterhuman.net/texts/government_information/intelligence_and_espionage/homebrew.military.and.espionage.electronics/servv89pn0aj.sn.sourcedns.com/_gbpprorg/mil/herf1/index.html

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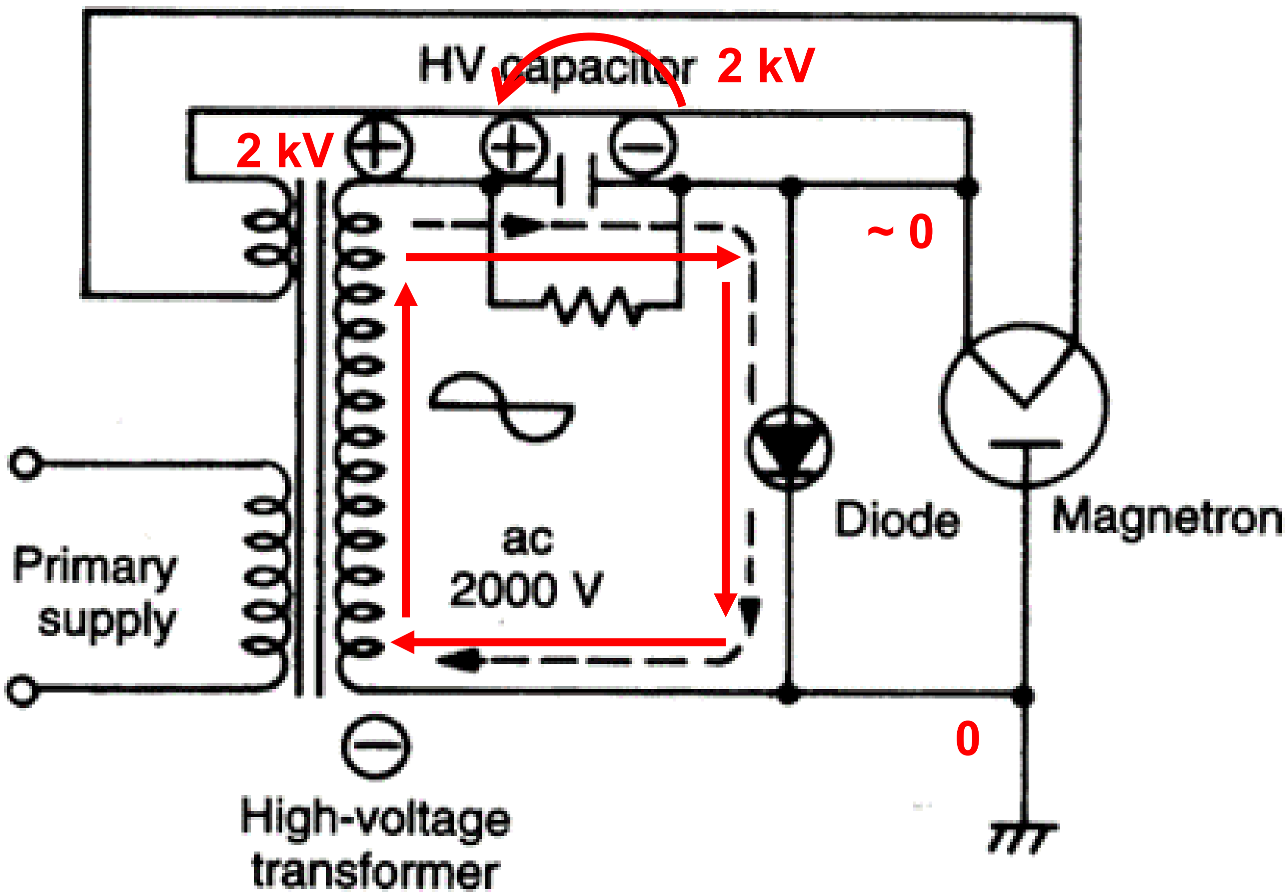

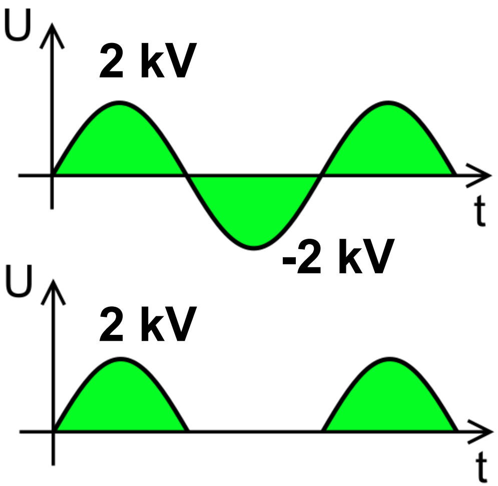

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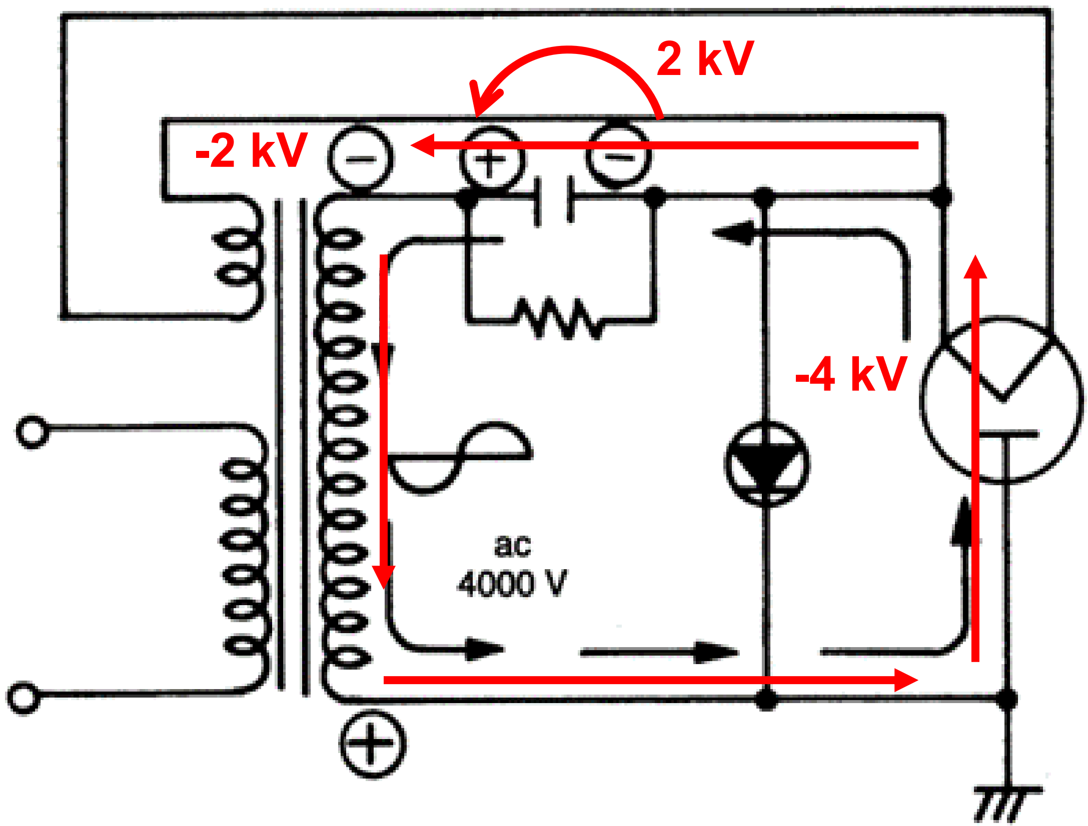
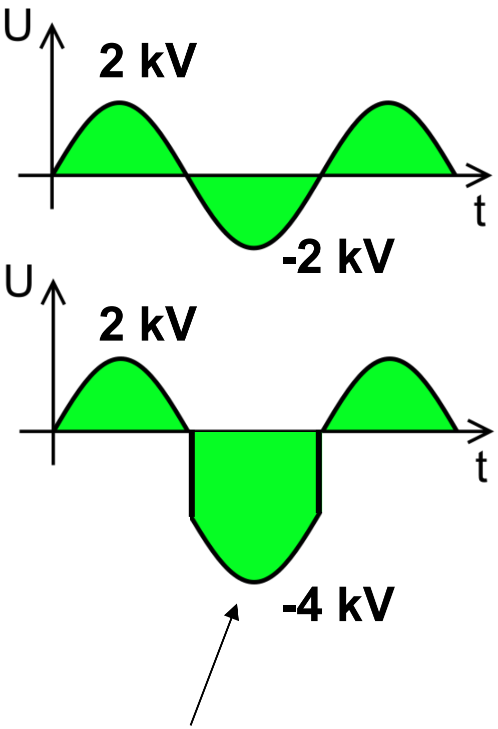
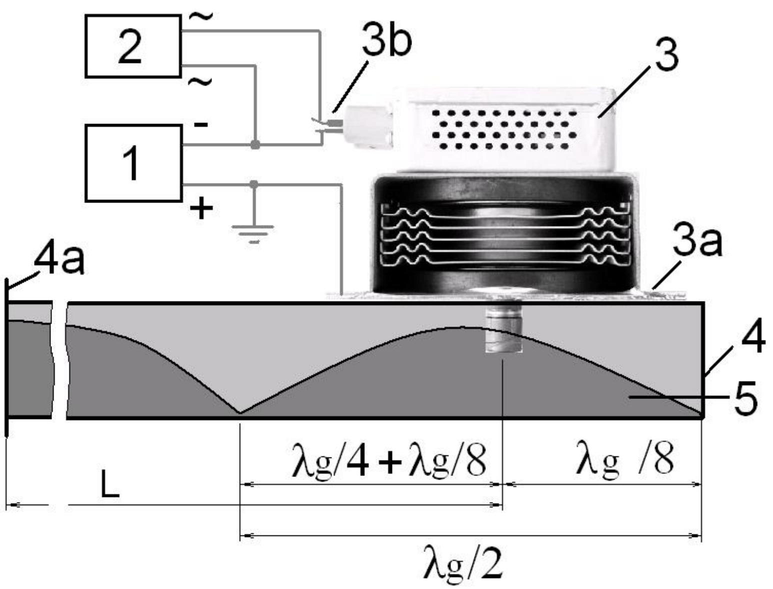


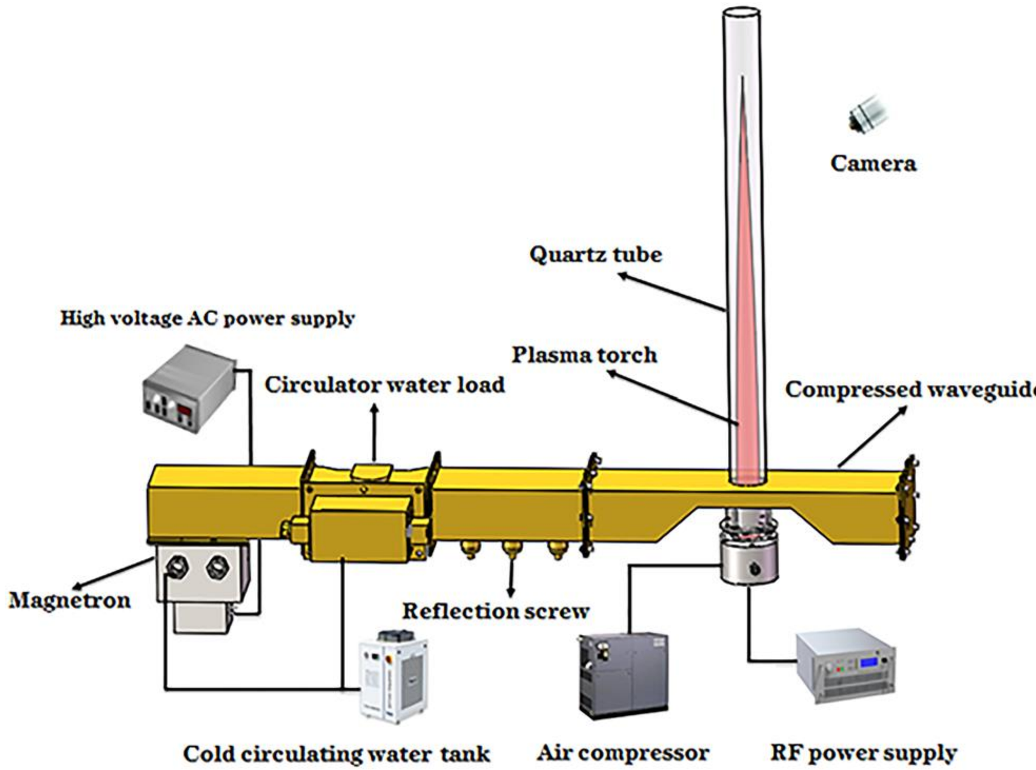


Microwave is generated.

http://cdn.preterhuman.net/texts/government_information/intelligence_and_espionage/homebrew.military.and.espionage.electronics/servv89pn0aj.sn.sourcedns.com/_gbpprorg/mil/herf1/index.html

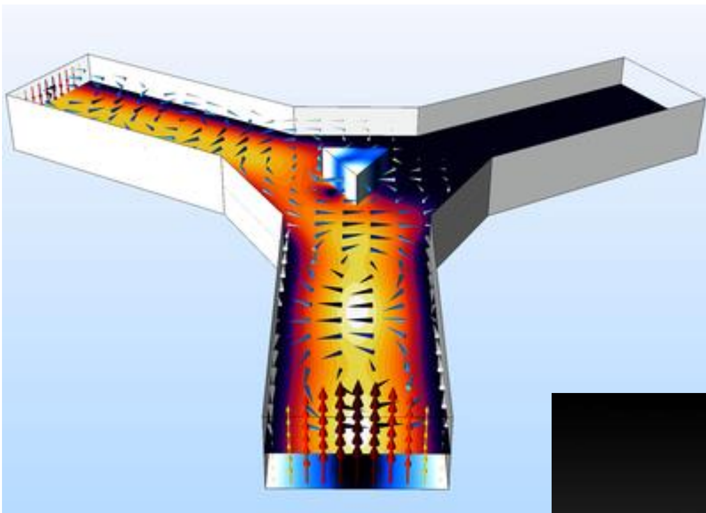
The electrode of the microwave source is located at the location with the highest electric field

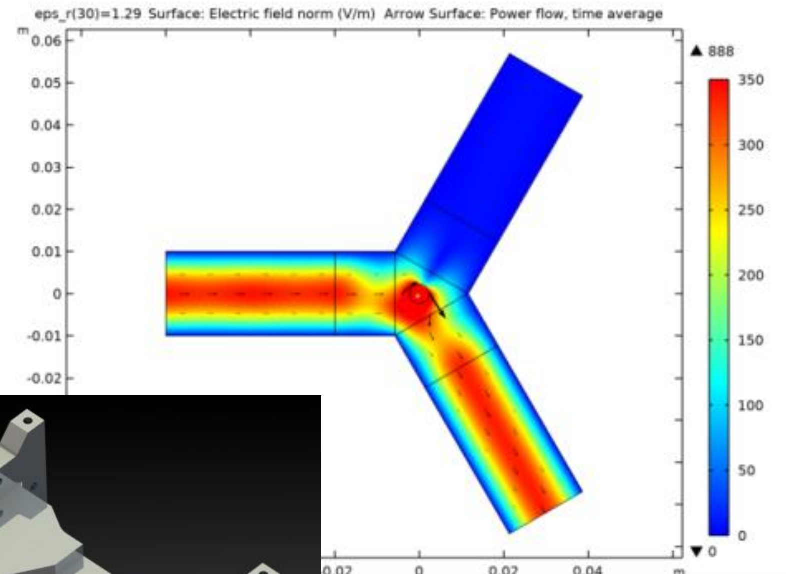


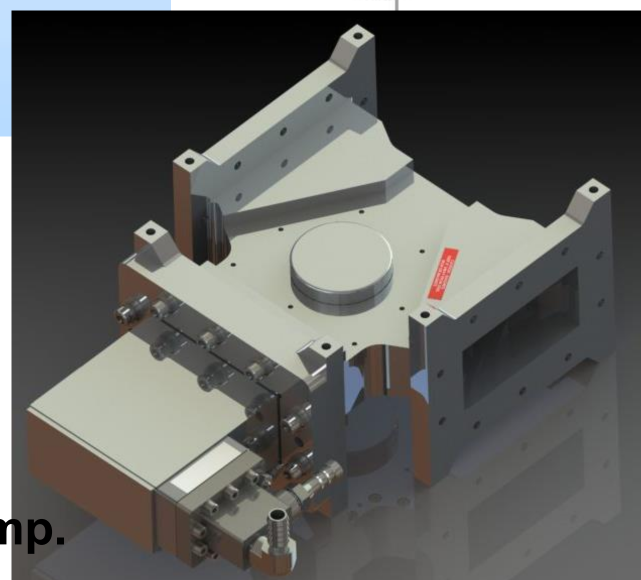


V. Surducan, etc., AIP Conference Proceedings 1425, 89 (2012)
Dan Ye, etc., AIP Advances 10, 055002 (2020

A 3-port circulator combining with a dump can be used as a isolator



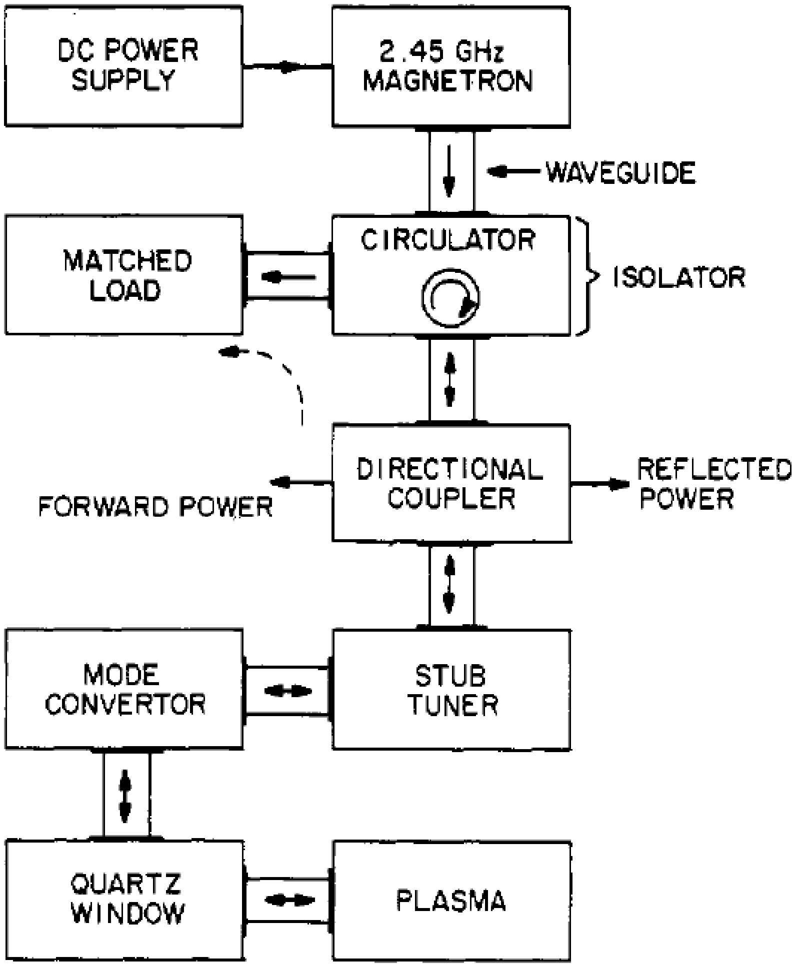




Water load as a dump.

https://cn.comsol.com/model/impedance-matching-of-a-lossy-ferrite-3-port-circulator-10302
https://doc.comsol.com/6.0/doc/com.comsol.help.models.rf.circulator/circulator.html
https://ferriteinc.com/high-power-microwave-circulators-isolators/wr340-waveguide-s-band/

Electron cyclotron resonance (ECR) microwave systems



microwave systems

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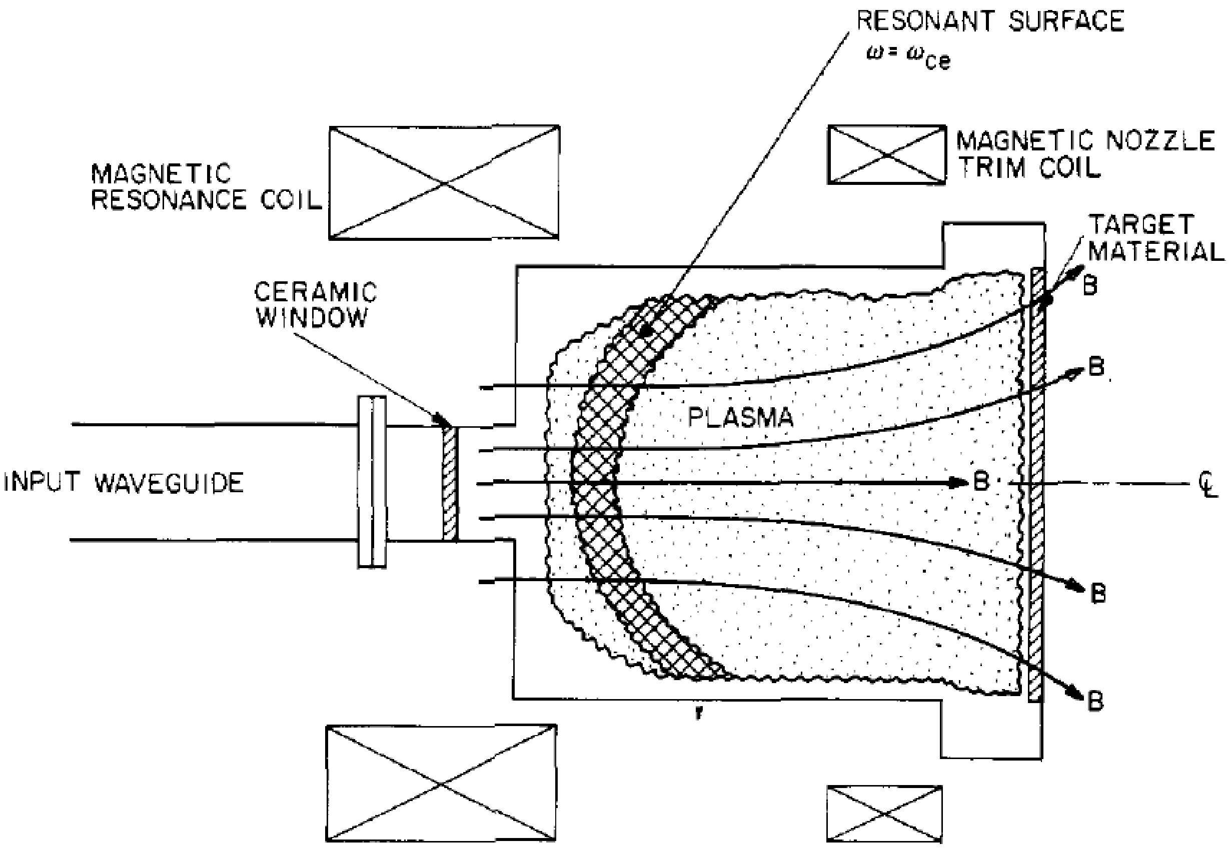
Strong absorption occurs when the frequency matches the electron cyclotron frequency

- **Electron cyclotron resonance (ECR) plasma reactor**



Electron cyclotron frequency depends on magnetic field only

$$m_e \frac{d\,\vec{v}}{\mathrm{dt}} = -\frac{e}{c}\,\vec{v} \times \vec{B}$$

- Assuming $\vec{B} = B\hat{z}$ and the electron oscillates in x-y plane

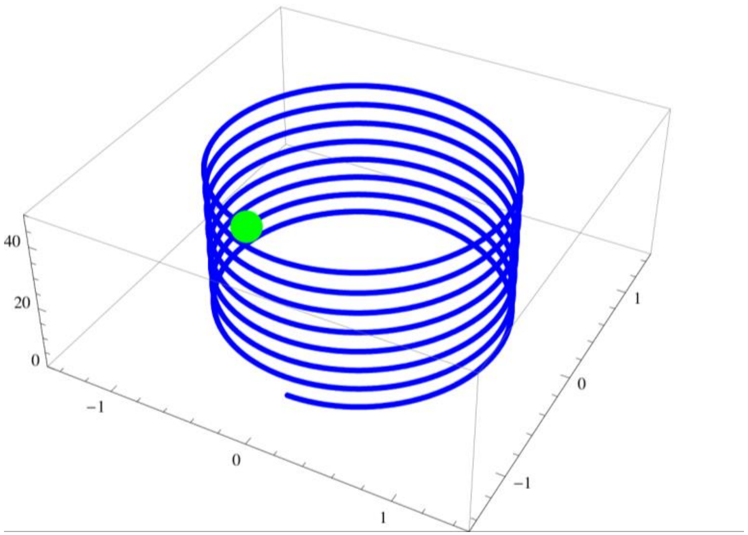
$$m_e \dot{v}_x = -\frac{e}{c} B v_y$$

$$m_e \dot{v}_y = \frac{e}{c} B v_x$$

$$m_e \dot{v}_z = 0$$

$$\ddot{v}_x = -\frac{eB}{m_e c}\dot{v}_y = -\left(\frac{eB}{m_e c}\right)^2 v_x$$

$$\ddot{v}_y = -\frac{eB}{m_e c}\dot{v}_x = -\left(\frac{eB}{m_e c}\right)^2 v_y$$



- Therefore

$$\omega_{\mathrm{ce}} = \frac{eB}{m_e c}$$

Electrons keep getting accelerated when a electric field rotates in electron's gyrofrequency

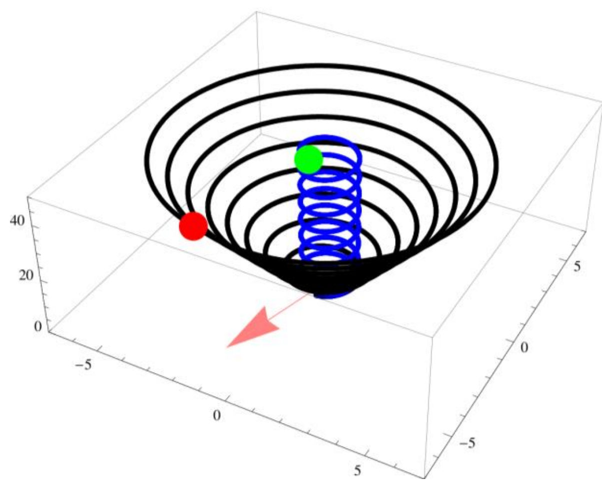
$$m_e \frac{d\vec{v}}{\mathrm{dt}} = -\frac{e}{c}\,\vec{v} \times \vec{B} - e\,\vec{E} \qquad \vec{B} = B_0\hat{z} \qquad \vec{E} = E_0[\hat{x}\cos(\omega t) + \hat{y}\sin(\omega t)]$$

$$m_e\dot{v}_x = -\frac{e}{c}Bv_y + E_0\cos(\omega t) \qquad m_e\dot{v}_y = \frac{e}{c}Bv_x + E_0\sin(\omega t) \qquad m_e\dot{v}_z = 0$$

$$\ddot{v}_x = -\frac{eB}{m_e c}\dot{v}_y - \frac{E_0}{m_e}\omega\sin(\omega t) = -\omega_{ce}{}^2 v_x - \frac{E_0}{m_e}(\omega_{ce} + \omega)\sin(\omega t)$$

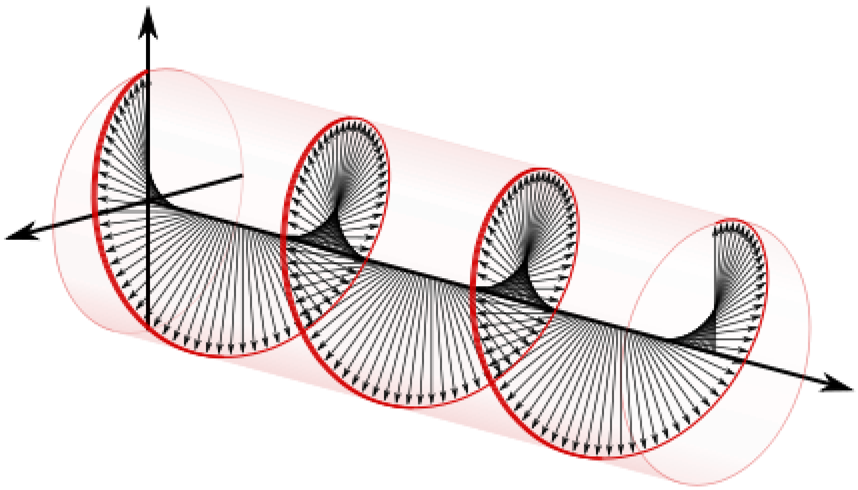
$$\ddot{v}_y = -\frac{eB}{m_e c}\dot{v}_x + \frac{E_0}{m_e}\omega\cos(\omega t) = -\omega_{ce}{}^2 v_y + \frac{E_0}{m_e}(\omega_{ce} + \omega)\cos(\omega t)$$

$$\omega_{ce} = \frac{eB}{m_e c}$$

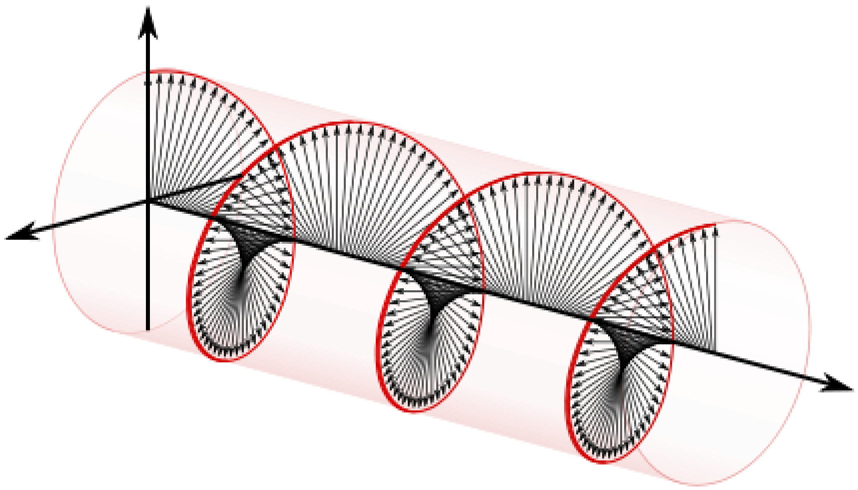


Electric field in a circular polarized electromagnetic wave keeps rotating as the wave propagates

- **Right-handed polarization**



- **Left-handed polarization**

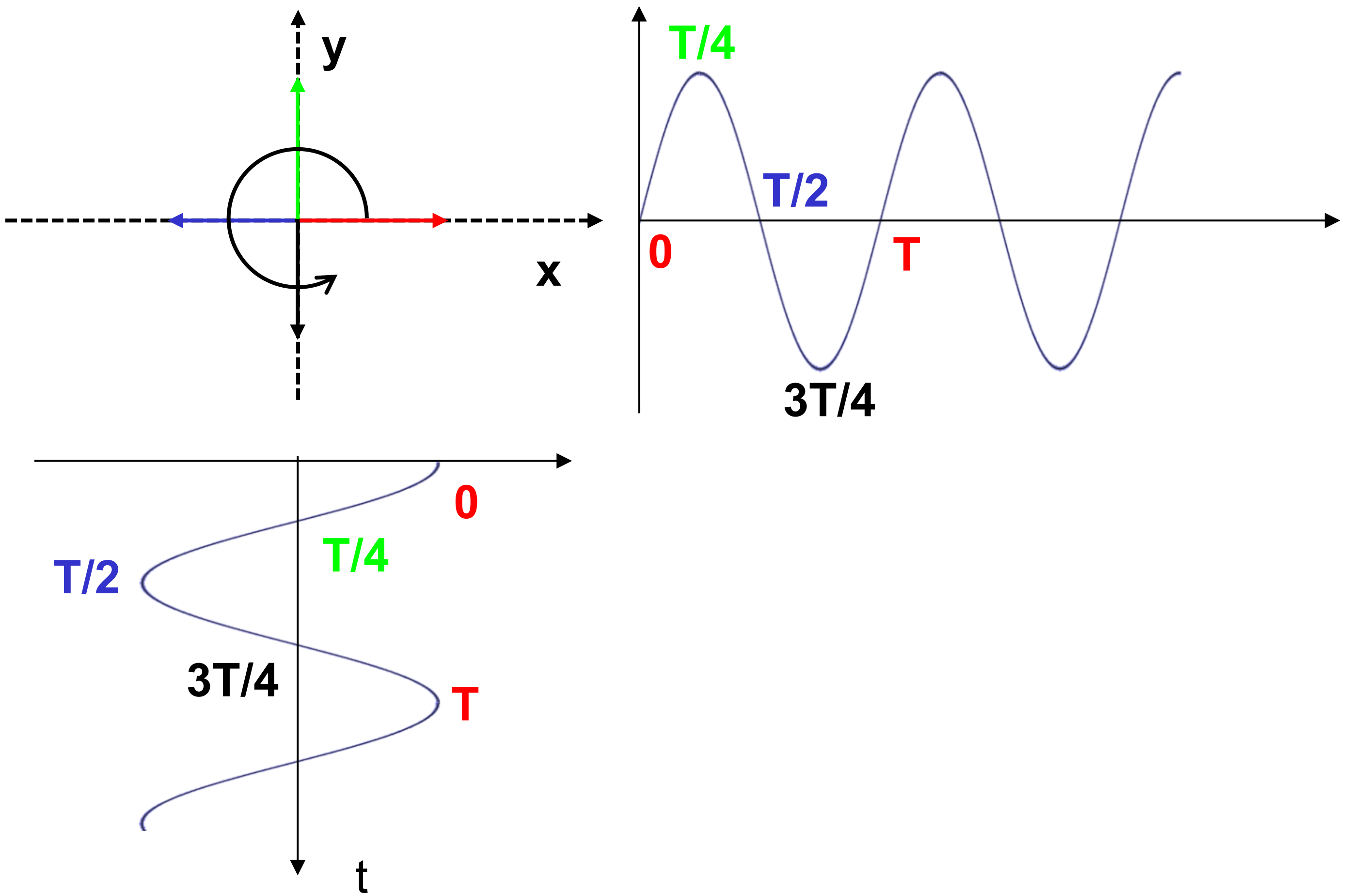


https://en.wikipedia.org/wiki/Circular_polarization

Electric field rotates in a circular polarization

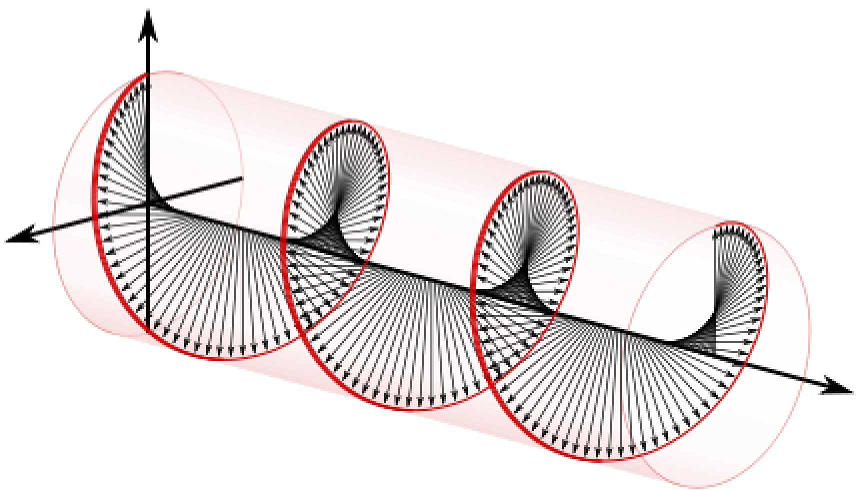
$$E_x = E_o \exp(-i\omega t)$$

$$E_y = \pm i E_x = i E_o \exp(-i\omega t) = E_o \exp\left(\pm i\frac{\pi}{2}\right)\exp(-i\omega t) = E_o \exp\left[-i\left(\omega t \pm \frac{\pi}{2}\right)\right]$$

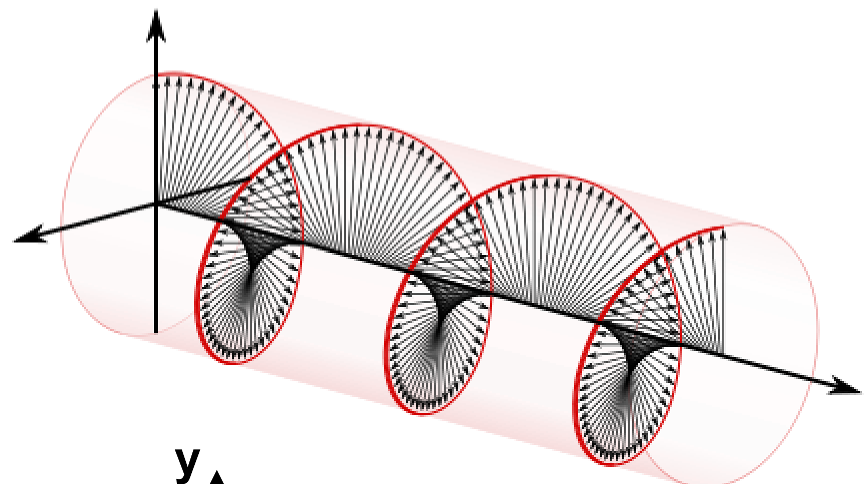


A linear polarized wave can be decomposed by a left-handed and a right-handed polarized wave

- **Right-handed polarization**

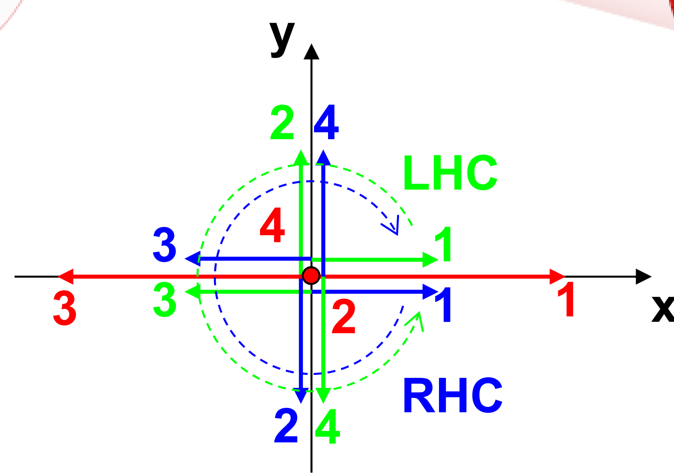


- **Left-handed polarization**



$$\vec{E} = E_o\hat{x} = \frac{E_o}{2}[(\hat{x}+i\hat{y})+(\hat{x}-i\hat{y})]$$

RHC **LHC**



https://en.wikipedia.org/wiki/Circular_polarization

Only right-handed polarization can resonance with electron’s gyromotion

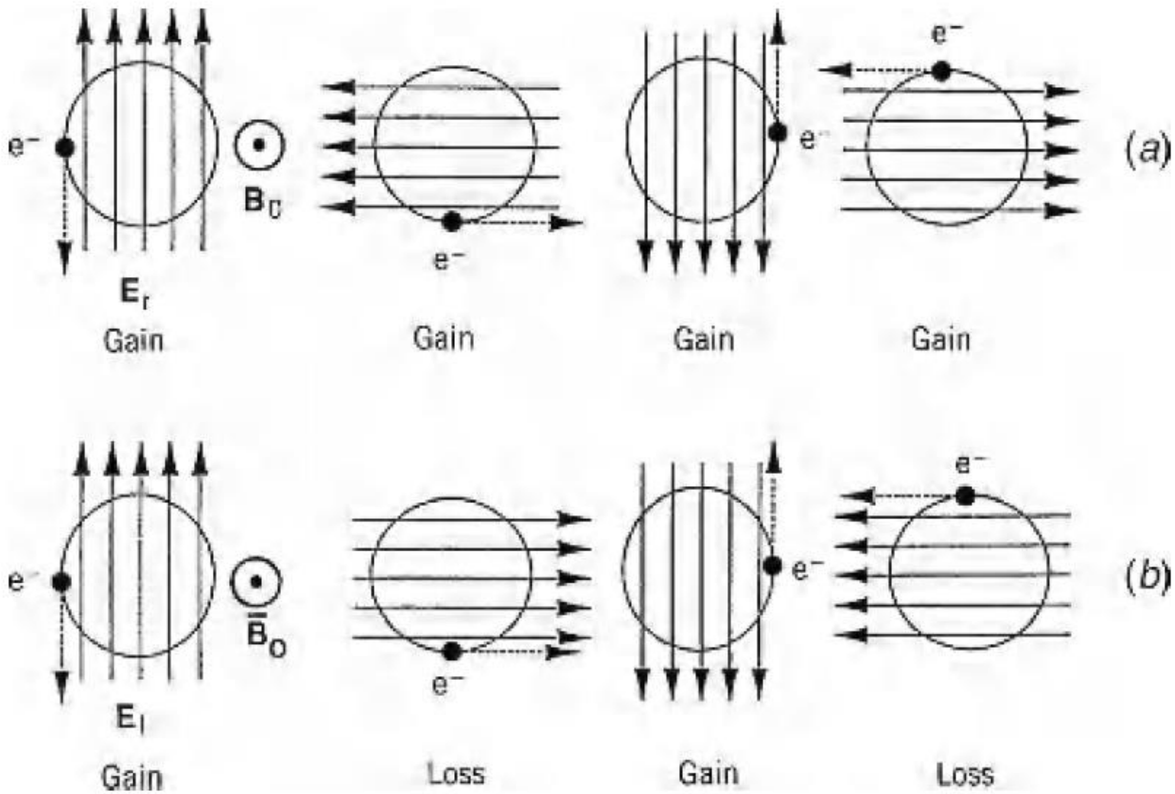
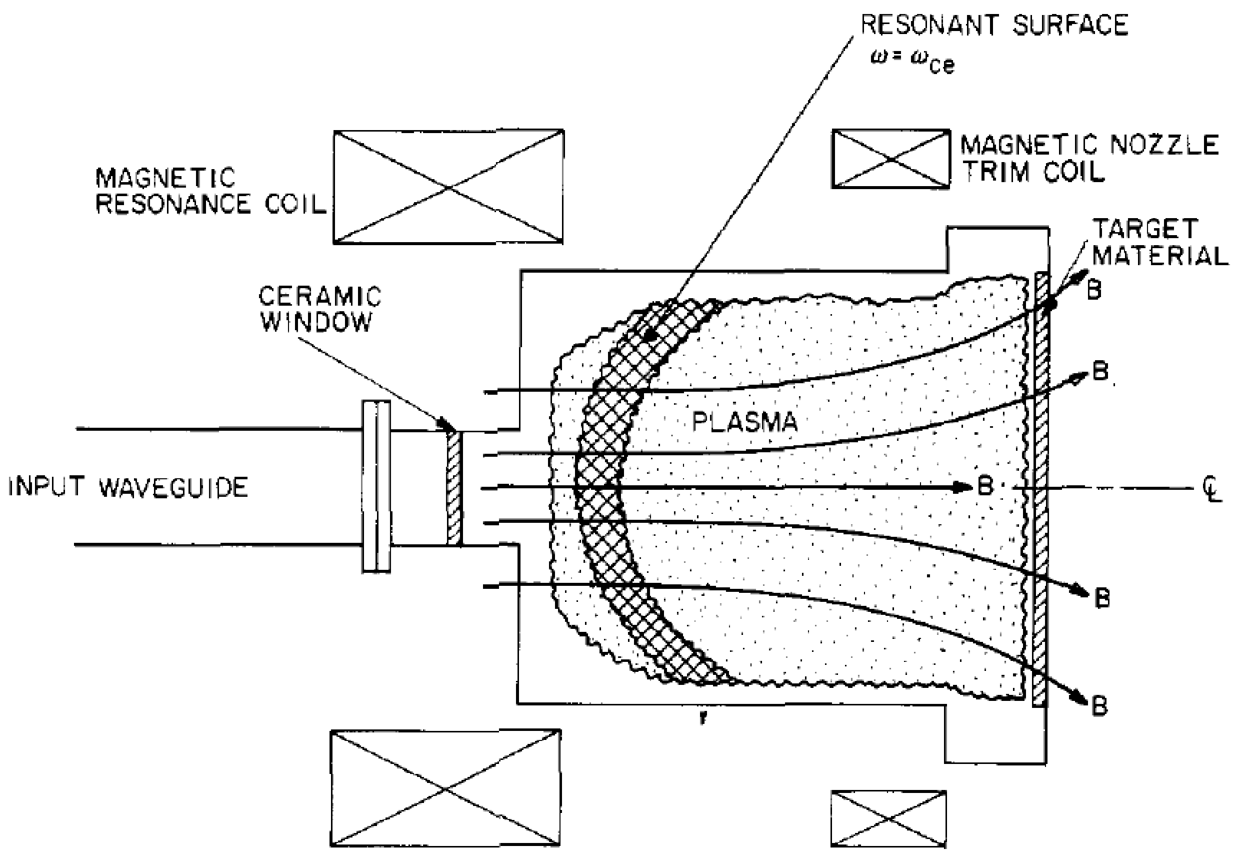


FIGURE 13.5. Basic principle of ECR heating: (*a*) continuous energy gain for right-hand polarization; (*b*) oscillating energy for left-hand polarization (after Lieberman and Gottscho, 1994).

Strong absorption occurs when the frequency matches the electron cyclotron frequency

- **Electron cyclotron resonance (ECR) plasma reactor**



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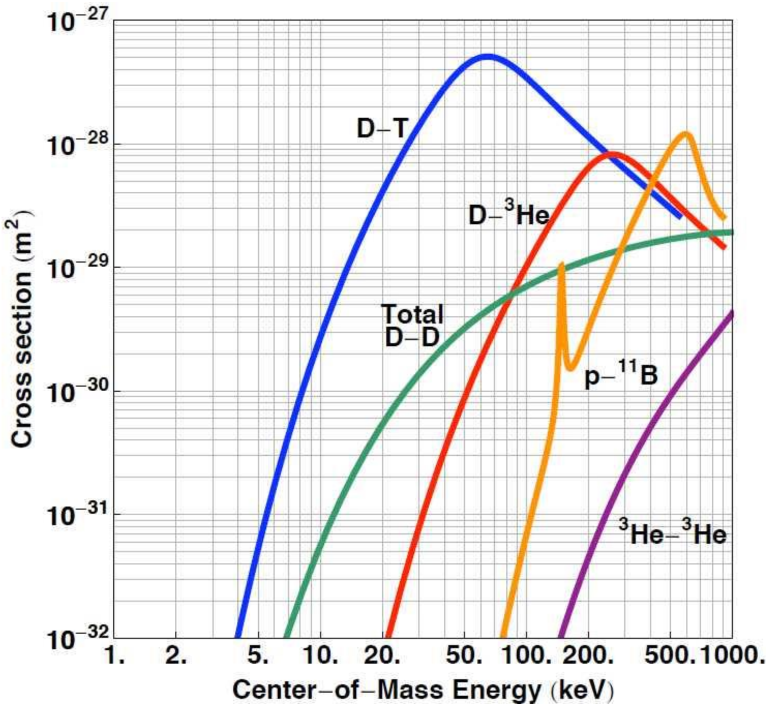
Two nuclear fusion reactions will be observed in experiments



- **Local experiment:**

p + B^{11} → 3He^4 (8.7 MeV)

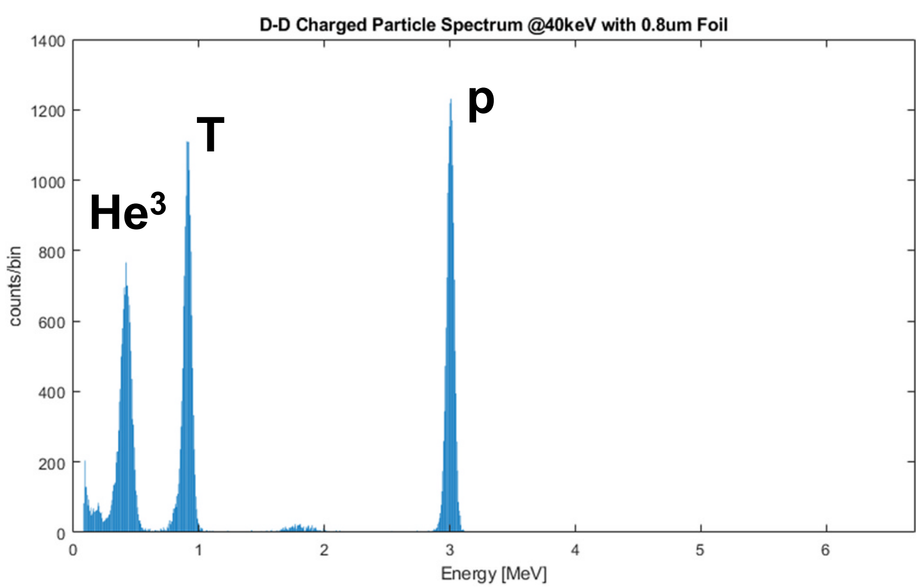
- **Remote experiment:**

D + D → T (1.01 MeV) + p (3.02 MeV) (50 %)

→ He^3 (0.82 MeV) + n (2.45 MeV) (50 %)

Monoenergetic particles will be generated in D-D fusion

D + D → T (1.01 MeV) + p (3.02 MeV)
(50 %)
→ He^3 (0.82 MeV) + n (2.45 MeV)
(50 %)



Alphas with wide spectrum will be generated

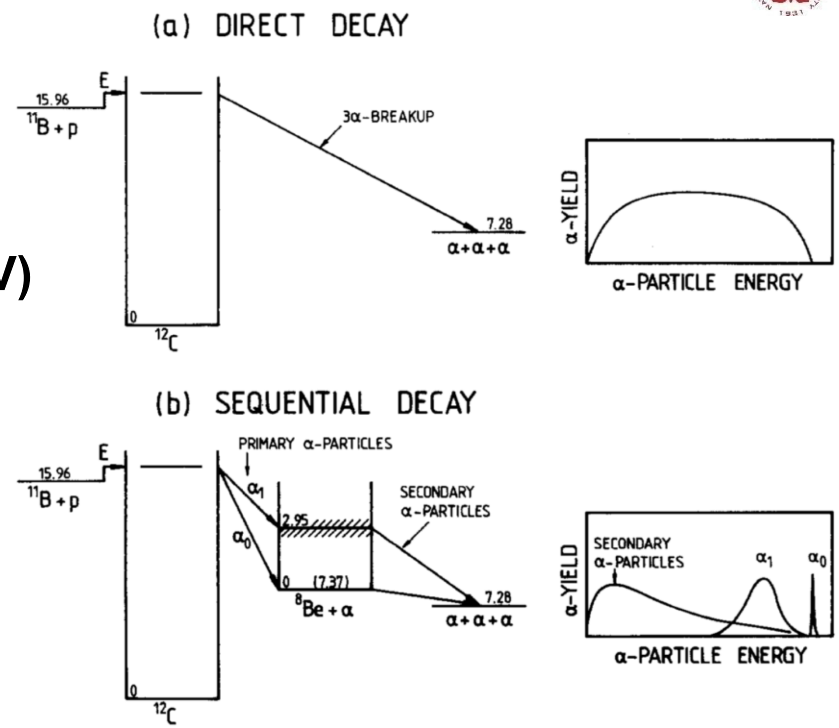
$p + {}^{11}B \rightarrow {}^{12}C^* \rightarrow \alpha_0 + {}^{8}Be$ (8.586 MeV)

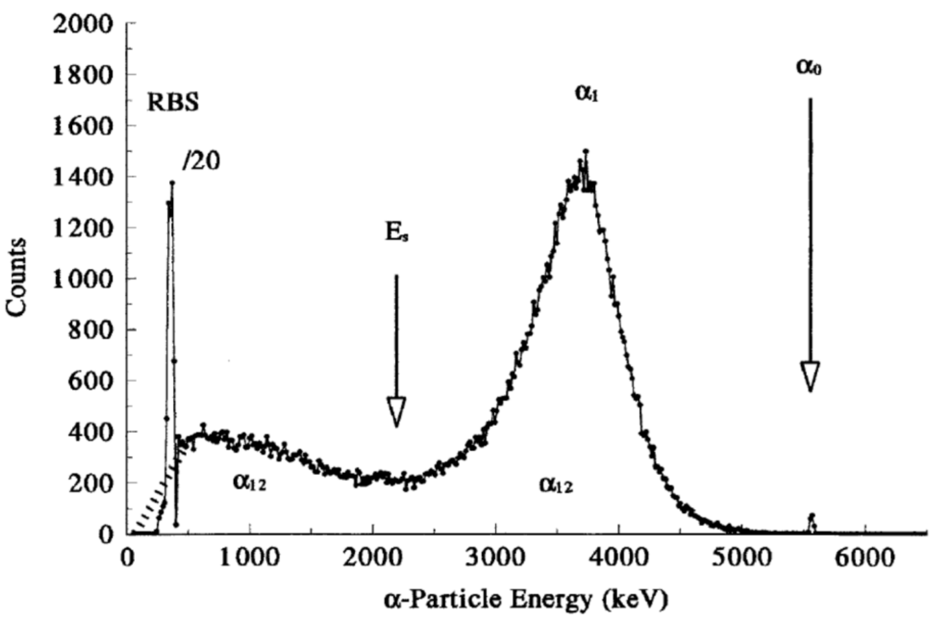
$\rightarrow \alpha_0 + \alpha_{01} + \alpha_{02}$

$\rightarrow {}^{12}C^* \rightarrow \alpha_1 + {}^{8}Be^*$ (5.65 MeV)

$\rightarrow \alpha_1 + \alpha_{12} + \alpha_{12}$ (3.028 MeV)

$p + {}^{11}B \rightarrow {}^{12}C^* \rightarrow \alpha_2 + \alpha_3 + \alpha_4$





RBS: Rutherford backscattering spectrometry

H. W. Becker, etc, Z. Phys. A – Atomic Nuclei 327, 341 (1987)
Jiarui Liu, etc., Nucl. Instr. And Meth. B 190, 107 (2002)
F. A. Geser and M. Valente, Radiat. Phys. Chem 167, 108224 (2020)

Course Outline

- Formosa Integrated Research Spherical Tokamak (FIRST), the first Tokamak in Taiwan
- **Physics used in Alpha-E**
  - Microwave
  - ECRH
  - Spectrum of fusion products
  - **Stopping power of the filter**
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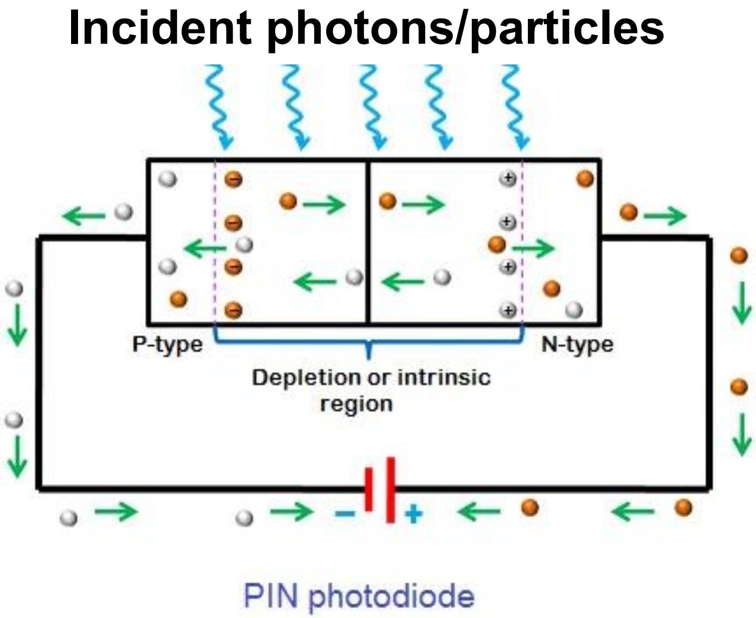
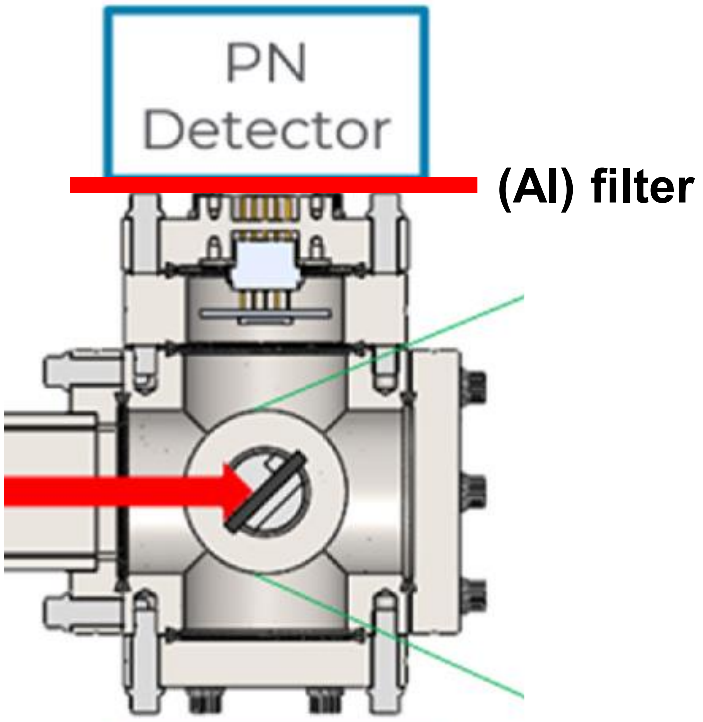
A filter is used to block the photon so that the detector only sees alphas



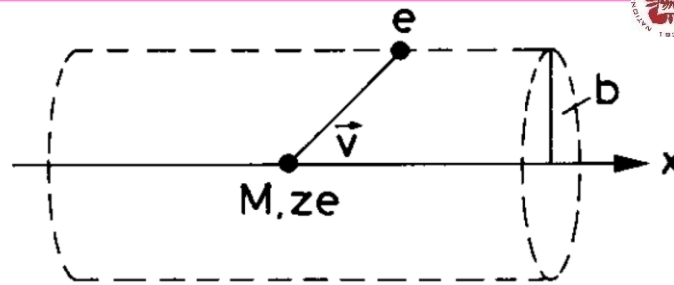


https://www.quarktwin.com/blogs/diode/the-working-principle-and-applications-of-photodiodes/125
Allan Xi Chen, etc., Physics Open 21, 100234(2024)

Energetic charged particles losses most of its energy right before it stops

- **Momentum transfer:**

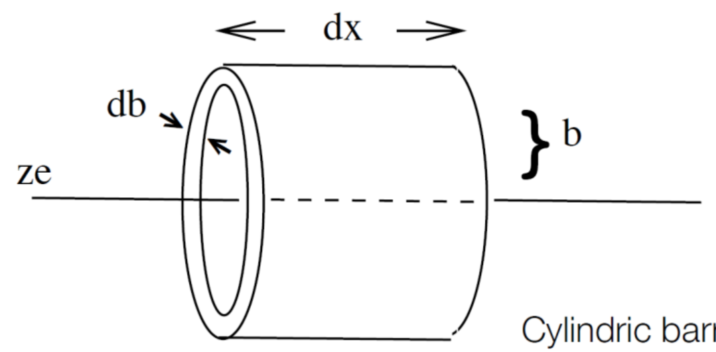
$$\Delta p_\perp = \int F_\perp dt = \int F_\perp \frac{dt}{dx} dx = \int F_\perp \frac{dx}{v}$$



$$= \int_{-\infty}^{\infty} \frac{ze^2}{(x^2+b^2)} \frac{b}{\sqrt{x^2+b^2}} \frac{1}{v} dx = \frac{ze^2 b}{v} \left[\frac{x}{b^2\sqrt{x^2+b^2}} \right]_{-\infty}^{\infty} = \frac{2ze^2}{bv}$$

$\Delta p_\parallel$: **averages to zero due to symmetry**

$$\Delta E(b) = \frac{\Delta p^2}{2m_e} \qquad N_e = n(2\pi b) db dx$$



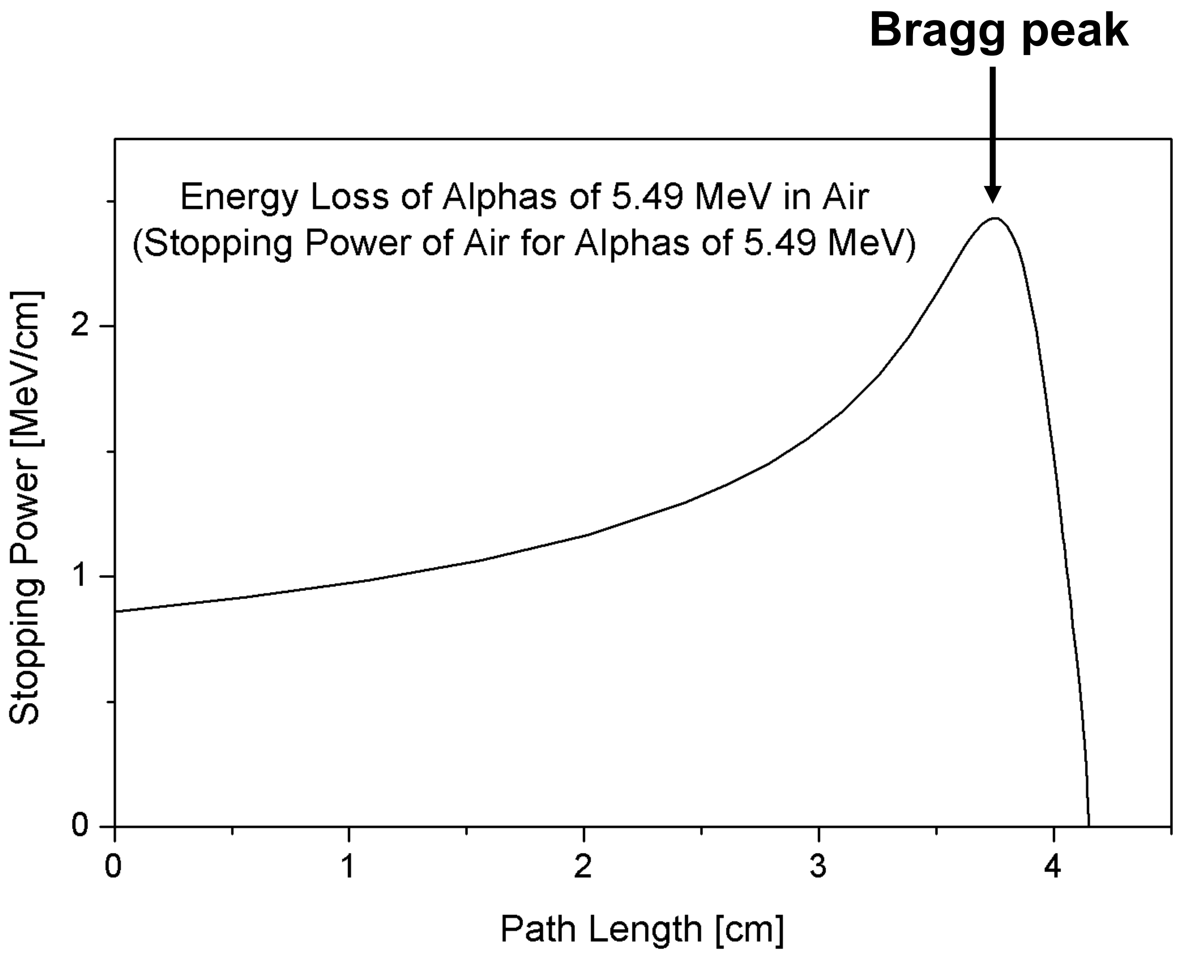
Cylindric barrel with N_e electrons

$$-dE(b) = \frac{\Delta p^2}{2m_e} 2\pi n b db dx$$

$$-\frac{dE}{dx} = \frac{4\pi n z^2 e^4}{m_e v^2} \int_{b_{min}}^{b_{max}} \frac{db}{b} = \frac{4\pi n z^2 e^4}{m_e v^2} \ln\left(\frac{b_{max}}{b_{min}}\right) \propto \frac{1}{E}$$

http://www.kip.uni-heidelberg.de/~coulon/Lectures/DetectorsSoSe10/

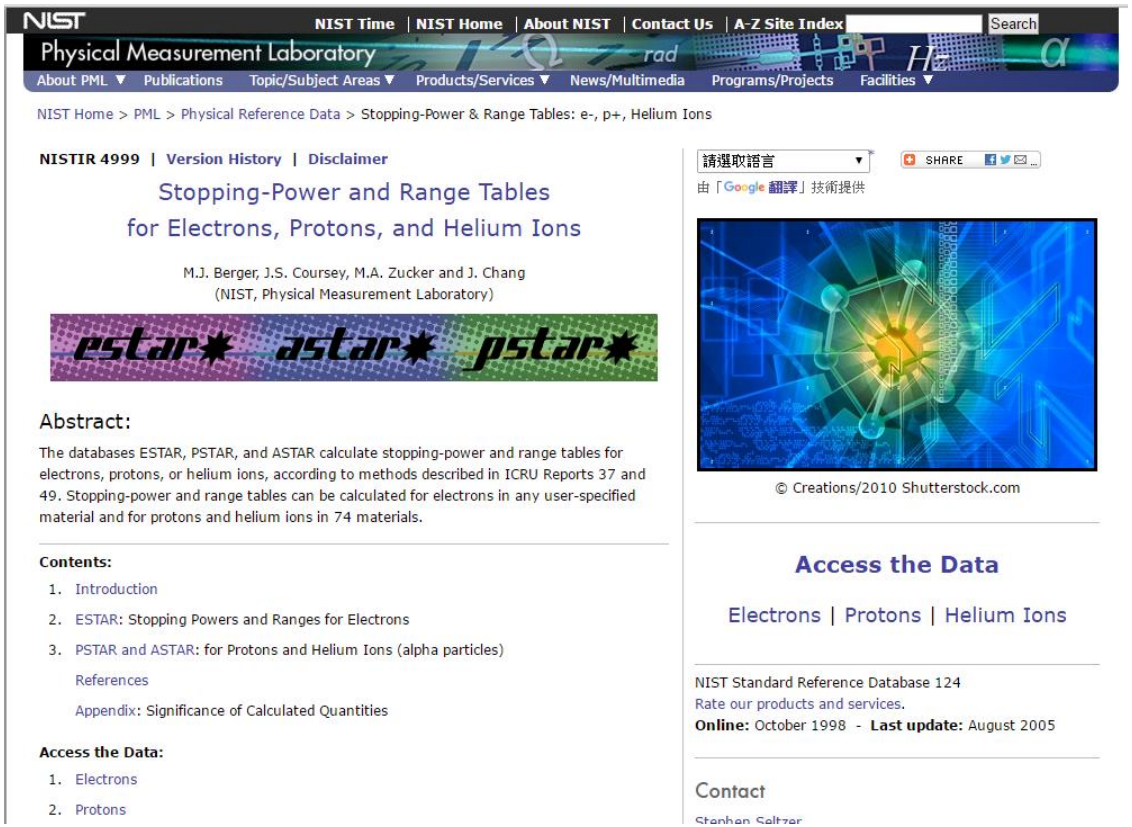
A particle loses most of its energy right before it stops



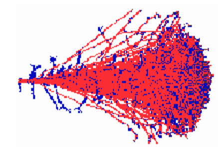
http://www.shi.co.jp/quantum/eng/product/proton/proton.html

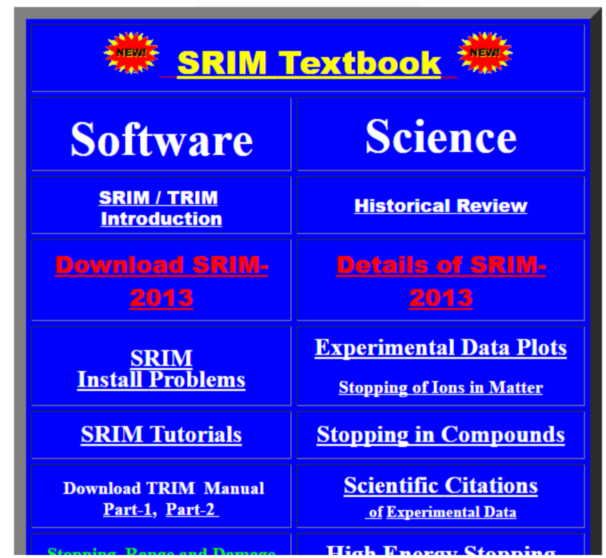
There are two suggested website for getting the information of proton stopping power in different materials

http://www.nist.gov/pml/data/star/



http://www.srim.org/





The thickness of a filter can be decided from the range data from NIST website

COPPER

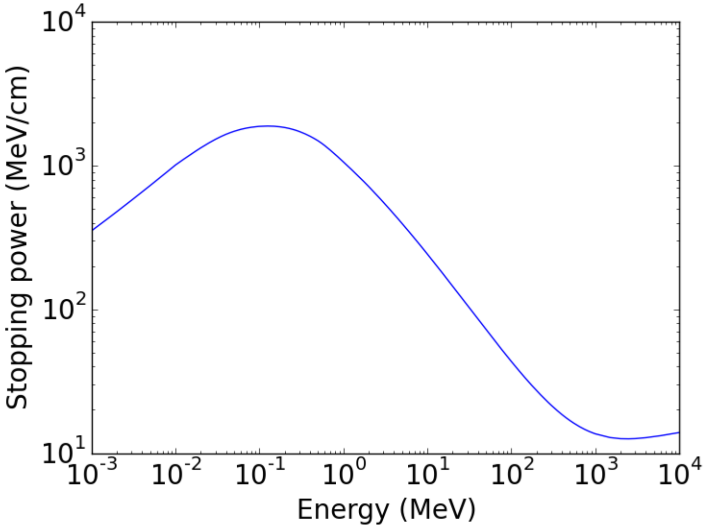
To download data in spreadsheet (array) form, choose a delimiter and use the checkboxes in the table heading. After downloading, save the output by using your browser's Save As feature.

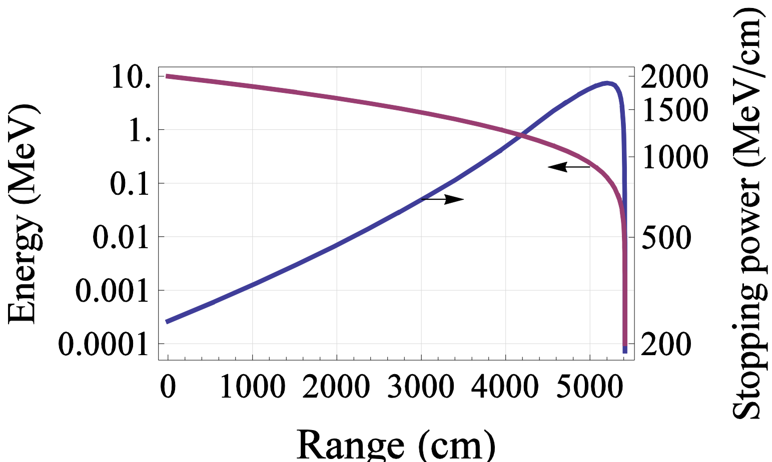
Delimiter:
- space
- | (vertical bar)
- tab (some browsers may use spaces instead)
- newline

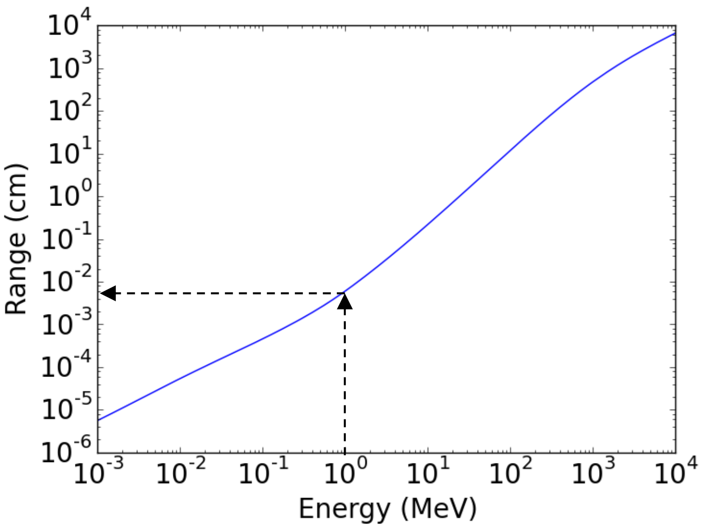
Download data | Reset

$\frac{dE}{\rho dx}$ $\qquad \rho x$

(required) Kinetic Energy (MeV)	Stopping Power (MeV cm^2/g)			Range		
	Electronic	Nuclear	Total	CSDA (g/cm^2)	Projected (g/cm^2)	Detour Factor Projected / CSDA
1.000E-03	3.490E+01	4.408E+00	3.931E+01	4.116E-05	5.620E-06	0.1365
1.500E-03	4.274E+01	4.231E+00	4.697E+01	5.267E-05	8.301E-06	0.1576
2.000E-03	4.935E+01	4.049E+00	5.340E+01	6.263E-05	1.101E-05	0.1759
2.500E-03	5.518E+01	3.876E+00	5.906E+01	7.152E-05	1.374E-05	0.1921
3.000E-03	6.045E+01	3.718E+00	6.416E+01	7.964E-05	1.647E-05	0.2068
4.000E-03	6.980E+01	3.440E+00	7.324E+01	9.419E-05	2.194E-05	0.2329
5.000E-03	7.804E+01	3.207E+00	8.124E+01	1.071E-04	2.739E-05	0.2556
6.000E-03	8.548E+01	3.010E+00	8.849E+01	1.189E-04	3.280E-05	0.2758
7.000E-03	9.233E+01	2.840E+00	9.517E+01	1.298E-04	3.817E-05	0.2940
8.000E-03	9.871E+01	2.692E+00	1.014E+02	1.400E-04	4.347E-05	0.3106
9.000E-03	1.047E+02	2.561E+00	1.073E+02	1.496E-04	4.872E-05	0.3258
1.000E-02	1.104E+02	2.445E+00	1.128E+02	1.587E-04	5.391E-05	0.3398

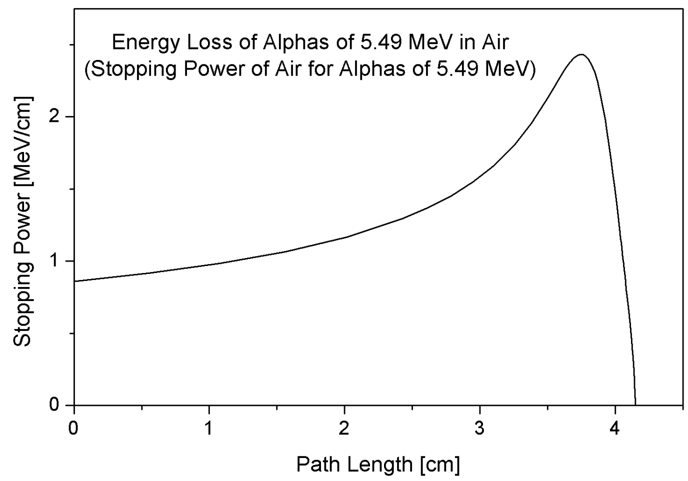


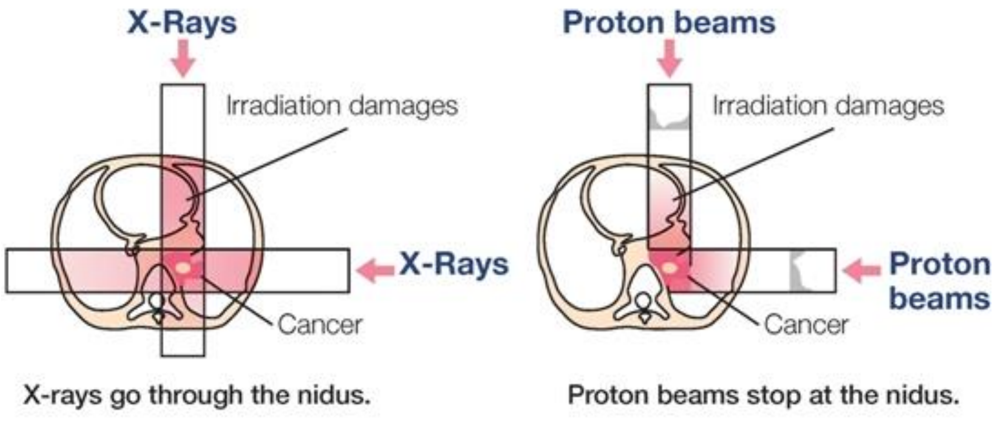




$$\frac{dE}{dx} = f(E) \Rightarrow x = \int_{E_i}^{E_f} \frac{dE}{f(E)}$$

Proton therapy takes the advantage of using Bragg peak

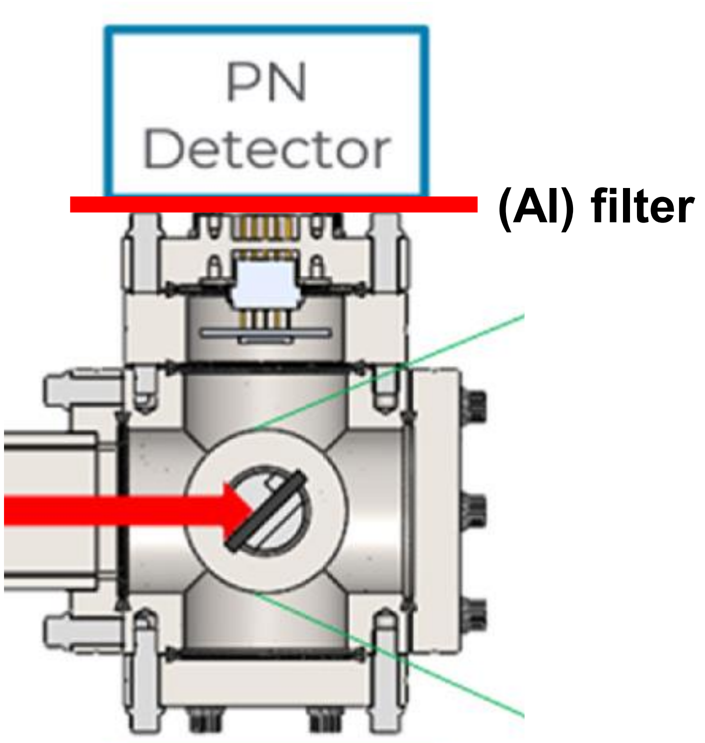




http://www.shi.co.jp/quantum/eng/product/proton/proton.html

Alpha particles loss energies when passing through the (Al) filter

- **A filter is used to block the photon so that the detector only sees alphas.**
- **Alphas in p-^{11}B, which are mostly in the range of 3–5 MeV, would be attenuated by ~130–180 keV in 0.8 μm of Al.**



Allan Xi Chen, etc., Physics Open 21, 100234(2024)

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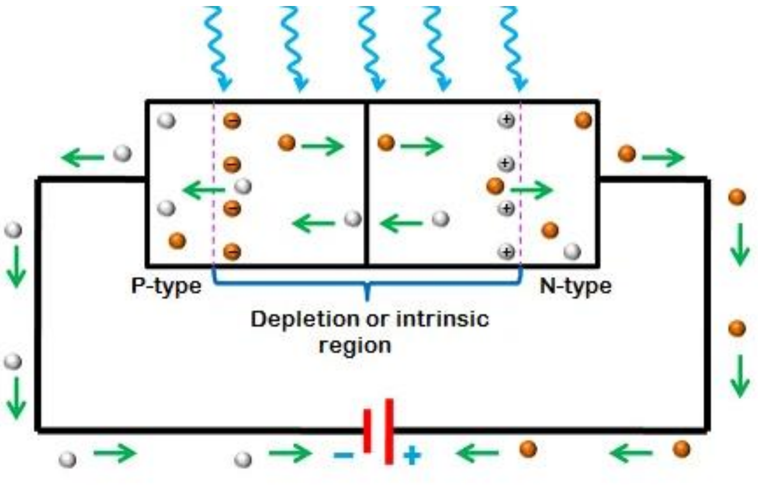
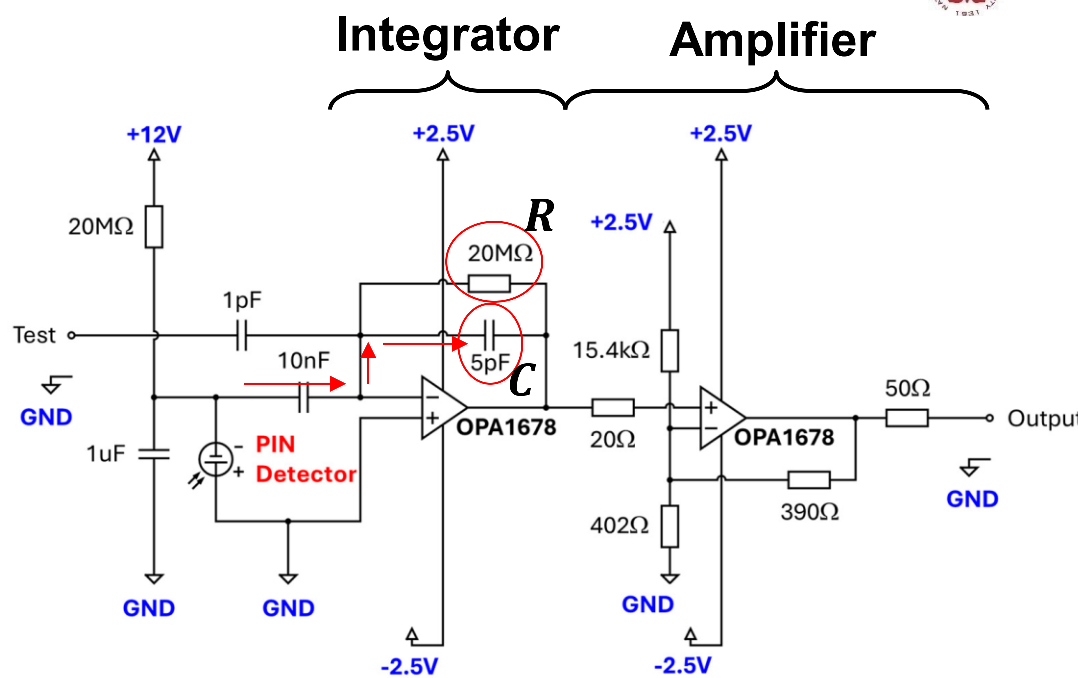
A pulse signal is converted to a voltage signal using an integrator

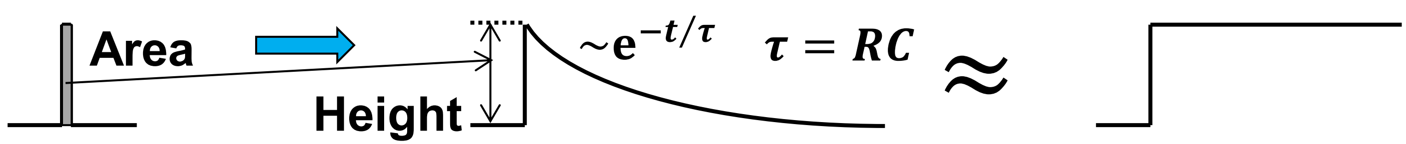
Incident photons/particles



PIN photodiode



- **The 5-pF capacitor is charged by the current from the PIN detector.**
- **The 5-pF capacitor is discharged by the 20-MΩ resistor.**



https://www.quarktwin.com/blogs/diode/the-working-principle-and-applications-of-photodiodes/125
Allan Xi Chen, etc., J. Nucl. Eng. 6, 15 (2025)

Pulse Shaping is commonly used in nuclear and particle physics electronics

- **Improve Signal-to-noise ratio S/N - Restrict bandwidth to match measurement time.**

  => Increase pulse width.

- **Improve pulse-pair resolution**

  => Decrease pulse width.

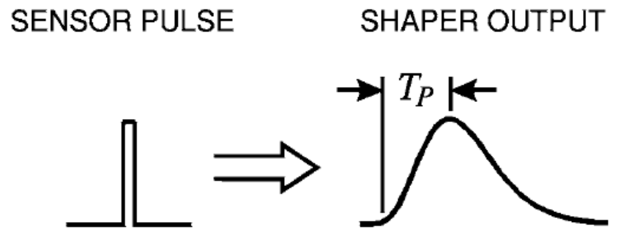


FIGURE 13. A pulse shaper transforms a short sensor pulse into a longer pulse with a rounded cusp and peaking time T_P.

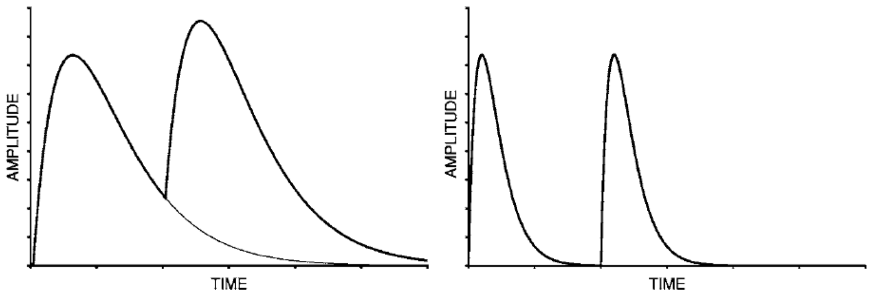
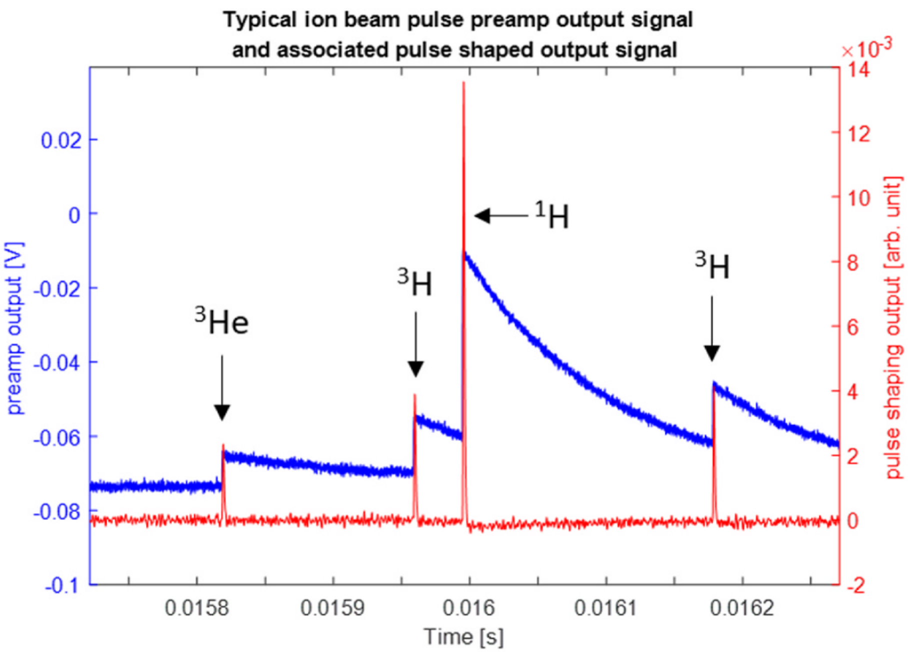
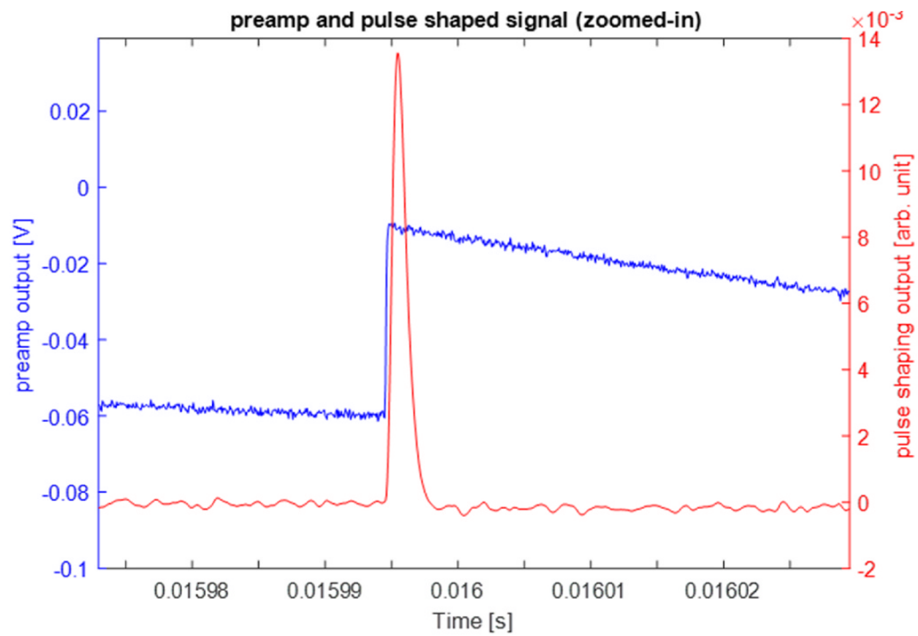


FIGURE 14. Amplitude pileup when two successive pulses overlap (left). Reducing the shaping time allows the first pulse to return to the baseline before the second arrives.

H. Spieler, AIP Conf. Proc. 674, 76 (2003)

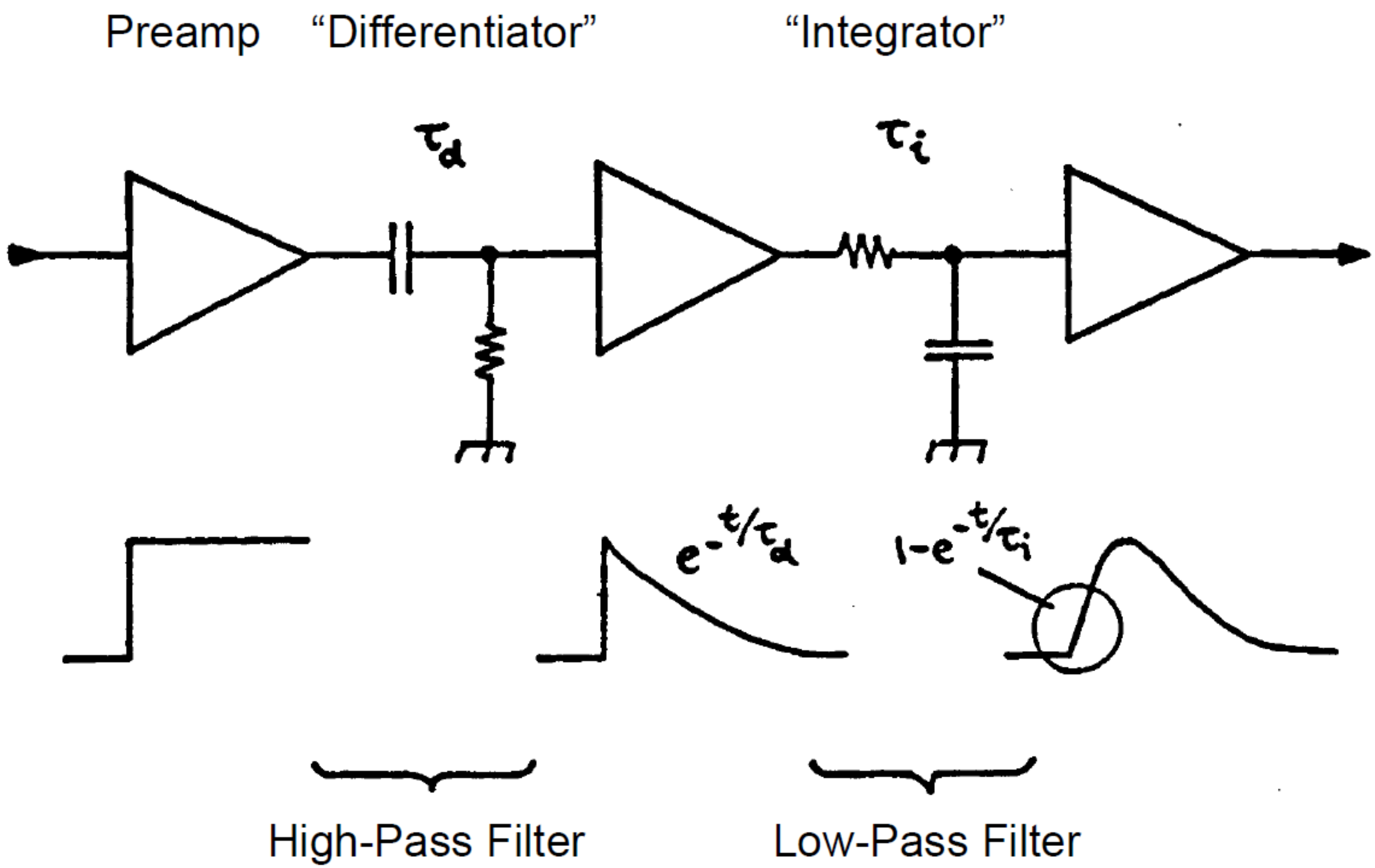
Expected data profile





Allan Xi Chen, etc., Physics Open 21, 100234(2024)

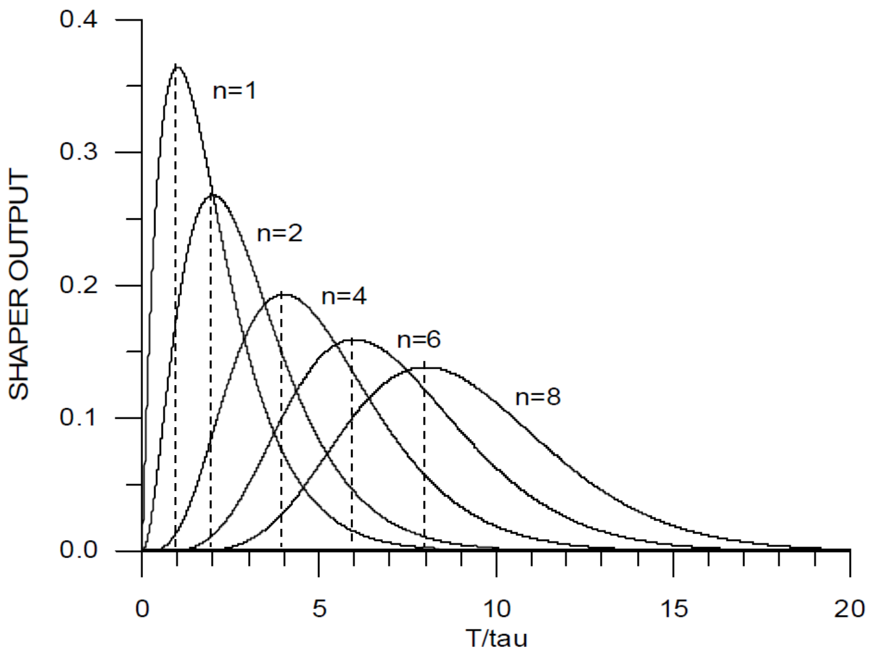
CR-RC^n pulse shaping combines a high-pass filter and n low-pass filter



Lecture Notes - VII. Heidelberger Graduate Lectures in Physics by H. Spieler (2001)
https://www-physics.lbl.gov/%7Espieler/Heidelberg_Notes_2001/index.html

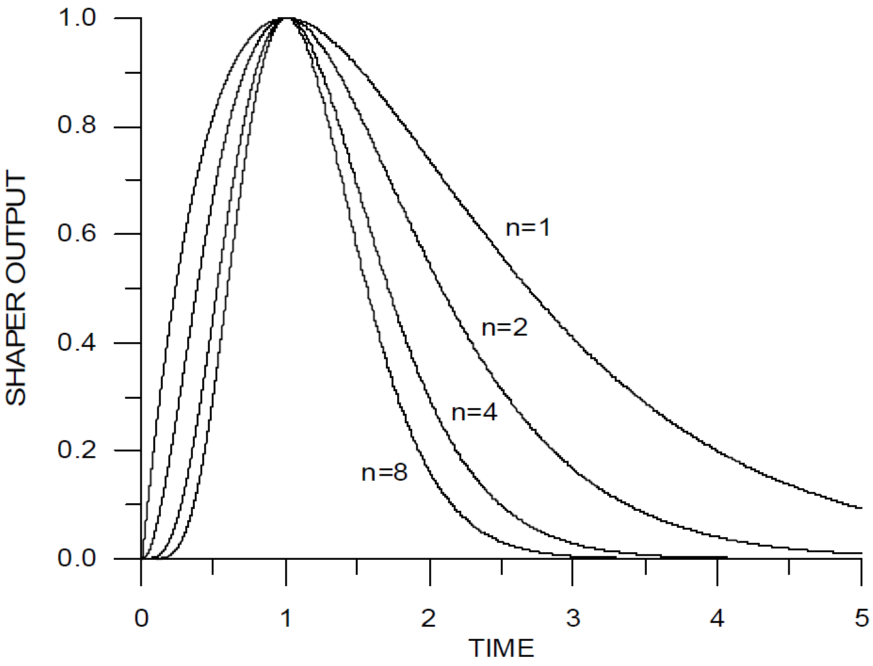
Pulses become gaussian-like after multiple integrators

- **Integrating with the same time constant** τ



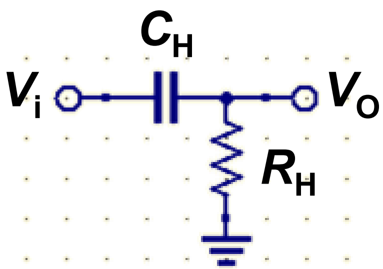
$$T_p = n\tau$$

- **Integrating with the reduced time constant** $\tau_n = \frac{\tau_{n-1}}{n}$



CR is the high pass filter while RC is the low pass filter

- **CR circuit:**



$$\tau_{\mathrm{H}} = R_{\mathrm{H}} C_{\mathrm{H}}$$

$$V_{\mathrm{o}} = V_{\mathrm{i}} \frac{R_{\mathrm{H}}}{\frac{1}{j\omega C_{\mathrm{H}}} + R_{\mathrm{H}}} = V_{\mathrm{i}} \frac{1}{\frac{1}{j\omega R_{\mathrm{H}} C_{\mathrm{H}}} + 1}$$

$$= V_i \frac{1}{\frac{1}{j\omega \tau_{\mathrm{H}}} + 1}$$

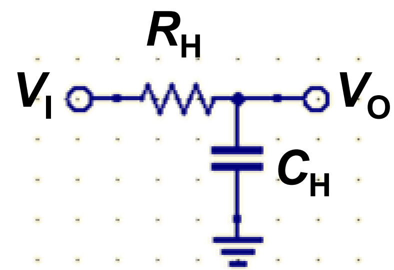
For $\omega\tau_{\mathrm{H}} \gg 1$, i.e.,

$$\frac{2\pi f}{f_{\mathrm{H}}} \gg 1 \text{ or } f \gg f_{\mathrm{H}} \quad \text{or} \quad \tau_{\mathrm{H}} \gg t$$

$V_{\mathrm{o}} \approx V_{\mathrm{i}}$

- **High-pass filter!**

- **RC circuit:**



$$\tau_{\mathrm{L}} = R_{\mathrm{L}} C_{\mathrm{L}}$$

$$V_{\mathrm{o}} = V_{\mathrm{i}} \frac{\frac{1}{j\omega C_{\mathrm{L}}}}{\frac{1}{j\omega C_{\mathrm{L}}} + R_{\mathrm{L}}} = V_{\mathrm{i}} \frac{\frac{1}{j\omega R_{\mathrm{L}} C_{\mathrm{L}}}}{\frac{1}{j\omega R_{\mathrm{L}} C_{\mathrm{L}}} + 1}$$

$$= V_{\mathrm{i}} \frac{\frac{1}{j\omega \tau_{\mathrm{L}}}}{\frac{1}{j\omega \tau_{\mathrm{L}}} + 1}$$

For $\omega\tau_{\mathrm{L}} \ll 1$, i.e.,

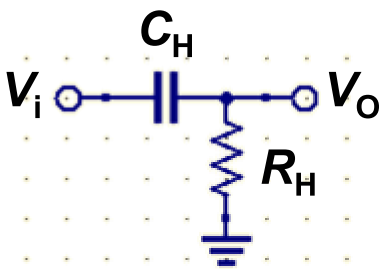
$$\frac{2\pi f}{f_{\mathrm{L}}} \ll 1 \text{ or } f \ll f_{\mathrm{L}} \quad \text{or} \quad t \ll \tau_{\mathrm{L}}$$

$V_{\mathrm{o}} \approx V_{\mathrm{i}}$

- **Low-pass filter!**

CR is the high pass filter while RC is the low pass filter

- **CR circuit:**



$$\tau_H = R_H C_H$$

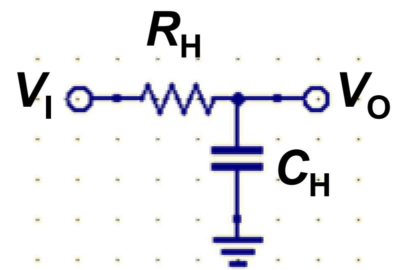
$$V_i - V_c - V_o = 0 \qquad V_o = iR_H$$

$$V_c = \frac{1}{C_H}\int i\,dt = \frac{1}{C_H}\int \frac{V_o}{R_H}dt$$

$$\dot{V}_i = \frac{V_o}{\tau_H} + \dot{V}_o$$

$$V_o(t) = e^{-t/\tau_H}\int_0^t \dot{V}_i(t')e^{t'/\tau_H}dt'$$

- **RC circuit:**



$$\tau_L = R_L C_L$$

$$V_i - V_R - V_o = 0 \qquad i = C_L\dot{V}_o$$

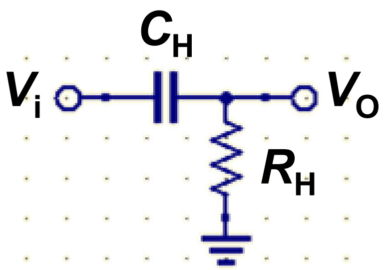
$$V_R = iR_L = R_L C_L \dot{V}_o$$

$$V_i = \tau_L \dot{V}_o + V_o$$

$$V_o(t) = e^{-t/\tau_L}\frac{1}{\tau_L}\int_0^t V_i(t')e^{t'/\tau_L}dt'$$

A step function becomes an exponential decay after the high-pass filter

- **CR circuit:**



$$\tau_H = R_H C_H$$

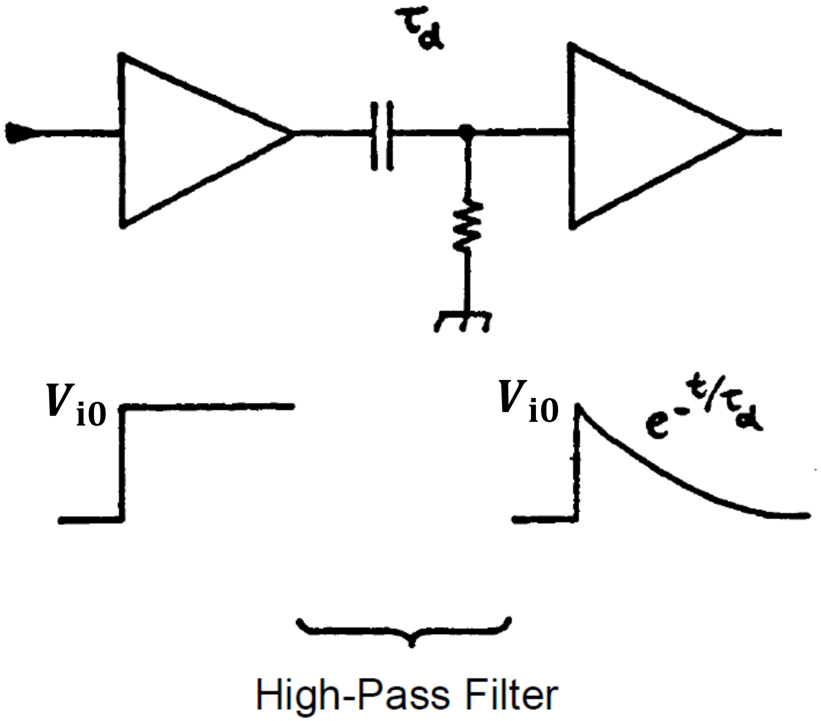
$$V_o(t) = e^{-t/\tau_H} \int_0^t \dot{V}_i(t') e^{t'/\tau_H} dt'$$

$$V_i(t) = \begin{cases} 0 & t < 0 \\ V_{i0} & t \geq 0 \end{cases}$$

$$\dot{V}_i(t) = V_{i0}\delta(t)$$

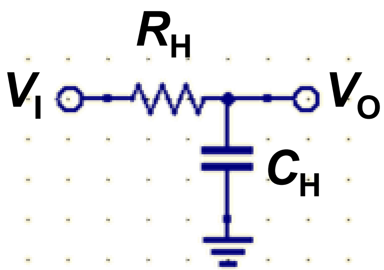
$$V_o(t) = e^{-t/\tau_H} \int_0^t V_{i0}\delta(t') e^{t'/\tau_H} dt'$$

$$= V_{i0} e^{-t/\tau_H} \, e^{t'/\tau_H}\Big|_{t'=0} = V_{i0} e^{-t/\tau_H}$$



CR is the high pass filter while RC is the low pass filter

- **RC circuit:**



$$\tau_L = R_L C_L$$

$$V_o(t) = e^{-t/\tau_L}\frac{1}{\tau_L}\int_0^t V_i(t')e^{t'/\tau_L}dt'$$

$$V_i(t) = V_{i0}e^{-t/\tau_H}$$

For $t \ll \tau_L$ $\qquad e^{-t/\tau_L} \sim e^{t/\tau_L} \sim 1$

$$V_o(t) \sim \frac{1}{\tau_L}\int_0^t V_i(t')dt' = \frac{1}{\tau_L}\int_0^t V_{i0}e^{-t'/\tau_H}dt'$$

$$= \frac{\tau_H}{\tau_L}V_{i0}\left(1 - e^{-t/\tau_H}\right)$$

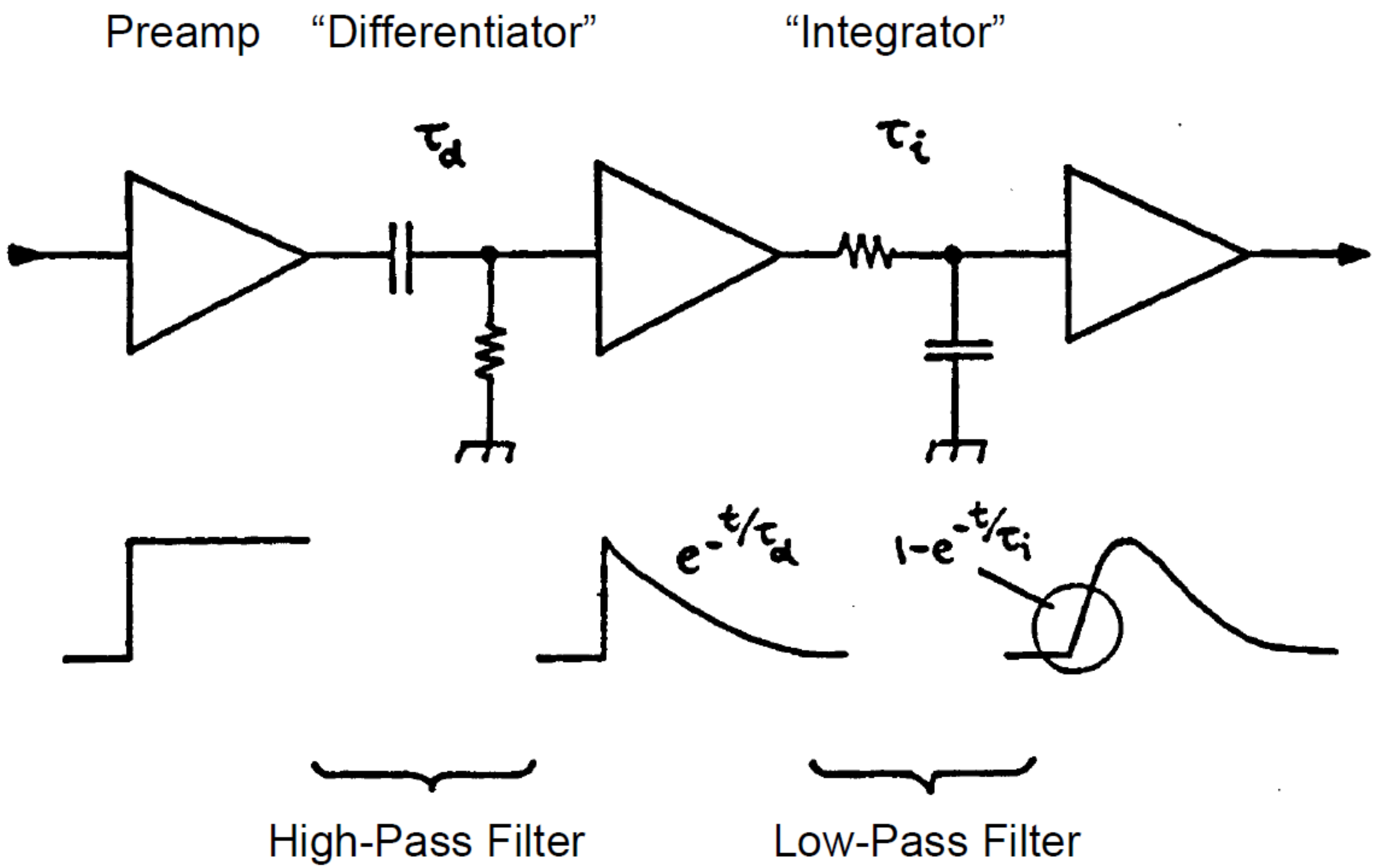
$$V_o(t) = e^{-t/\tau_L}\frac{1}{\tau_L}\int_0^t V_{i0}e^{-t'/\tau_H}e^{t'/\tau_L}dt'$$

$$= e^{-t/\tau_L}\frac{1}{\tau_L}\int_0^t V_{i0}e^{t'/\tau_{eff}}dt'$$

$$= e^{-t/\tau_L}\frac{\tau_{eff}}{\tau_L}V_{i0}\left(e^{t/\tau_H} - 1\right)$$

$$\frac{1}{\tau_{eff}} = \frac{1}{\tau_L} - \frac{1}{\tau_H} < \frac{1}{\tau_L}$$

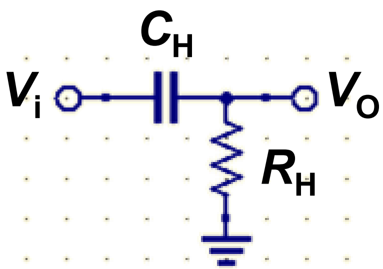
CR-RC^n pulse shaping combines a high-pass filter and n low-pass filter



Lecture Notes - VII. Heidelberger Graduate Lectures in Physics by H. Spieler (2001)
https://www-physics.lbl.gov/%7Espieler/Heidelberg_Notes_2001/index.html

Finite difference method is used to numerically applying the CR-RC pulse shaping

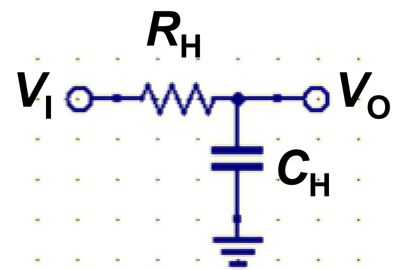
- **CR circuit:**



$$\tau_H = R_H C_H$$

$$\dot{V}_i = \frac{V_o}{\tau_H} + \dot{V}_o$$

- **RC circuit:**



$$\tau_L = R_L C_L$$

$$V_i = \tau_L \dot{V}_o + V_o$$

$$\frac{V_i(t_j) - V_i(t_{j-1})}{\Delta t} \approx \frac{V_o(t_{j-1})}{\tau_H} + \frac{V_o(t_j) - V_o(t_{j-1})}{\Delta t}$$

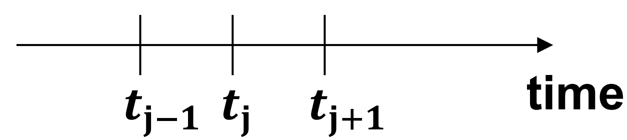
$$\approx \frac{V_o(t_j)}{\tau_H} + \frac{V_o(t_j) - V_o(t_{j-1})}{\Delta t}$$

$$V_o(t_j) = \alpha_{CR} V_o(t_{j-1}) + \alpha_{CR}\left[V_i(t_j) - V_i(t_{j-1})\right]$$

$$\alpha_{CR} \equiv \frac{\frac{\tau_H}{\Delta t}}{\frac{\tau_H}{\Delta t} + 1} = \frac{\frac{R_H C_H}{\Delta t}}{\frac{R_H C_H}{\Delta t} + 1}$$

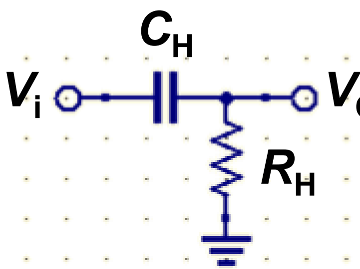
$$\frac{dV}{dt} = \lim_{\Delta t \to 0} \frac{V(t_j) - V(t_{j-1})}{\Delta t}$$

$$\approx \frac{V(t_j) - V(t_{j-1})}{\Delta t}$$



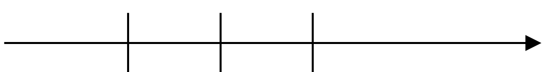
Finite difference method is used to numerically applying the CR-RC pulse shaping

- **CR circuit:**



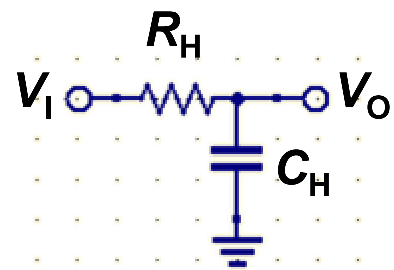
$$\tau_\mathrm{H} = R_\mathrm{H} C_\mathrm{H}$$

$$\dot{V}_\mathrm{i} = \frac{V_\mathrm{o}}{\tau_\mathrm{H}} + \dot{V}_\mathrm{o}$$



$t_{\mathrm{j}-1}$ t_j $t_{\mathrm{j}+1}$

- **RC circuit:**



$$\tau_\mathrm{L} = R_\mathrm{L} C_\mathrm{L}$$

$$V_\mathrm{i} = \tau_\mathrm{L} \dot{V}_\mathrm{o} + V_\mathrm{o}$$

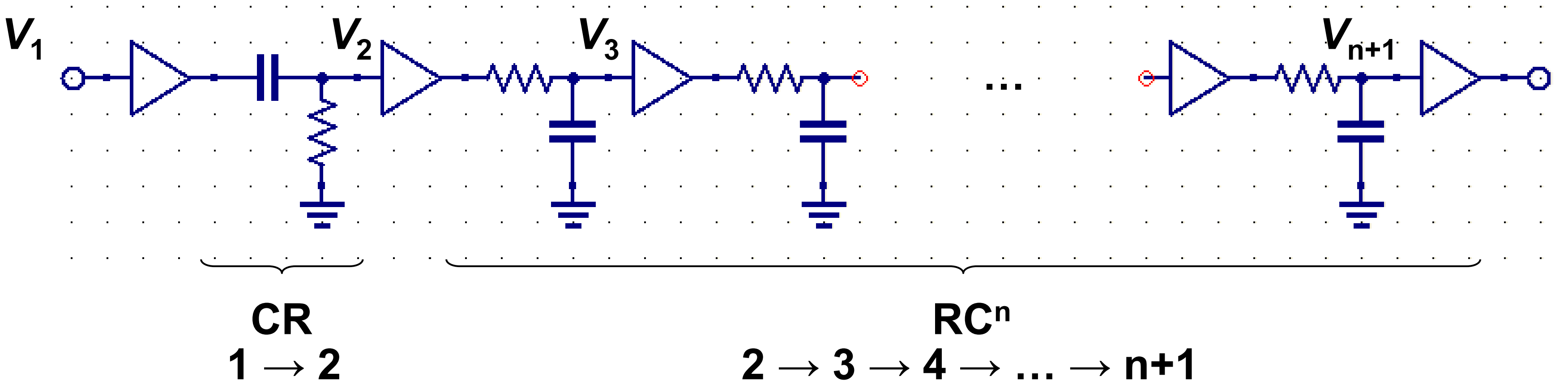
$$V_\mathrm{i}(t_\mathrm{j}) \approx \tau_\mathrm{L} \frac{V_\mathrm{o}(t_\mathrm{j}) - V_\mathrm{o}(t_{\mathrm{j}-1})}{\Delta t} + V_\mathrm{o}(t_{\mathrm{j}-1})$$

$$\approx \tau_\mathrm{L} \frac{V_\mathrm{o}(t_\mathrm{j}) - V_\mathrm{o}(t_{\mathrm{j}-1})}{\Delta t} + V_\mathrm{o}(t_\mathrm{j})$$

$$V_\mathrm{o}(t_\mathrm{j}) = (1 - \alpha_\mathrm{RC}) V_\mathrm{o}(t_{\mathrm{j}-1}) + \alpha_\mathrm{RC} V_\mathrm{i}(t_\mathrm{j})$$

$$\alpha_\mathrm{RC} \equiv \frac{1}{\frac{\tau_\mathrm{L}}{\Delta t} + 1} = \frac{1}{\frac{R_\mathrm{L} C_\mathrm{L}}{\Delta t} + 1}$$

Iteration can be used to have more orders of CR-RC^n pulse shaping



$$V_2(t_j) = \alpha_{CR} V_2(t_{j-1}) + \alpha_{CR}[V_1(t_j) - V_1(t_{j-1})]$$

$$\alpha_{CR} \equiv \frac{\frac{\tau_H}{\Delta t}}{\frac{\tau_H}{\Delta t}+1} = \frac{\frac{R_H C_H}{\Delta t}}{\frac{R_H C_H}{\Delta t}+1}$$

$$V_3(t_j) = (1-\alpha_{RC,3})V_3(t_{j-1}) + \alpha_{RC,3}V_2(t_j)$$

$$\vdots$$

$$\alpha_{RC,m} \equiv \frac{1}{\frac{\tau_{L,m}}{\Delta t}+1}$$

$$V_m(t_j) = (1-\alpha_{RC,m})V_m(t_{j-1}) + \alpha_{RC,m}V_{m-1}(t_j)$$

$$\tau_L = R_L C_L$$

$$\vdots$$

$$\tau_{L,m} = \frac{\tau_{L,m-1}}{m}$$

$$V_{n+1}(t_j) = (1-\alpha_{RC,n+1})V_{n+1}(t_{j-1}) + \alpha_{RC,n+1}V_n(t_j)$$

Let's try using excel first

$$\alpha_{\mathrm{CR}} \equiv 0.85$$

$$\alpha_{\mathrm{RC},2} \equiv 0.15 \qquad \Delta t = 64\ \mathrm{ns}$$

$$\alpha_{\mathrm{CR}} \equiv \frac{\frac{\tau_{\mathrm{H}}}{\Delta t}}{\frac{\tau_{\mathrm{H}}}{\Delta t}+1} = 0.85$$

$$\alpha_{\mathrm{RC,m}} \equiv \frac{1}{\frac{\tau_{\mathrm{L,m}}}{\Delta t}+1} = 0.15$$

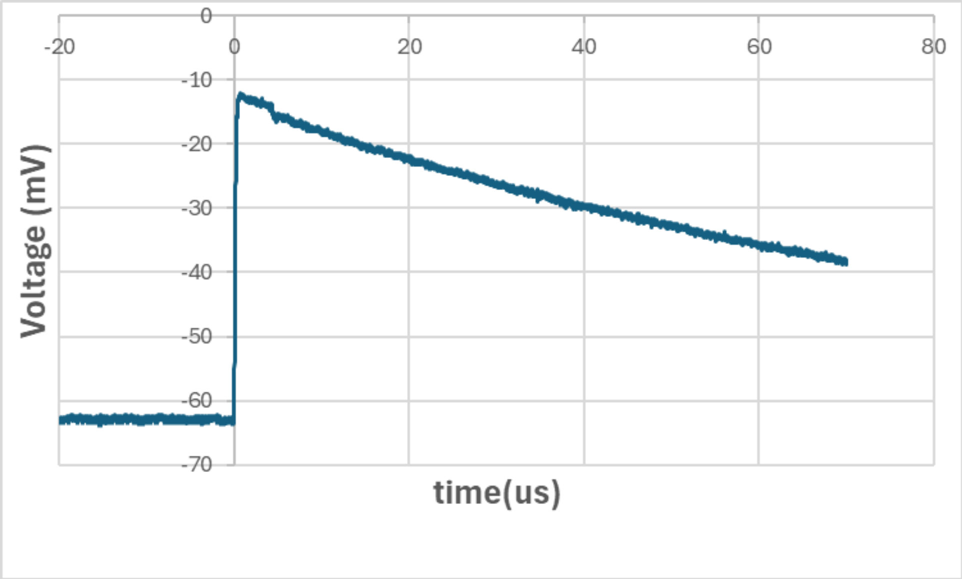
$$\tau_{\mathrm{n}} = \frac{\tau_{\mathrm{n-1}}}{n}$$

$$\tau_{\mathrm{H}} = 362.7\ \mathrm{ns}$$

$$\tau_{\mathrm{L,m}} = 362.7/m\ \mathrm{ns}$$

$$\frac{\tau_{\mathrm{L,m}}}{\Delta t} = \frac{0.85}{0.15}$$

$$\alpha_{\mathrm{RC,m}} \equiv \frac{1}{\frac{0.85}{0.15}\frac{1}{m}+1}$$



$$V_2(t_{\mathrm{j}}) = \alpha_{\mathrm{CR}}V_2(t_{\mathrm{j-1}}) + \alpha_{\mathrm{CR}}[V_1(t_{\mathrm{j}}) - V_1(t_{\mathrm{j-1}})]$$

$$V_3(t_{\mathrm{j}}) = (1-\alpha_{\mathrm{RC},3})V_3(t_{\mathrm{j-1}}) + \alpha_{\mathrm{RC},3}V_2(t_{\mathrm{j}})$$

$$V_{\mathrm{m}}(t_{\mathrm{j}}) = (1-\alpha_{\mathrm{RC,m}})V_{\mathrm{m}}(t_{\mathrm{j-1}}) + \alpha_{\mathrm{RC,m}}V_{\mathrm{m-1}}(t_{\mathrm{j}})$$

$$V_{\mathrm{n+1}}(t_{\mathrm{j}}) = (1-\alpha_{\mathrm{RC,n+1}})V_{\mathrm{n+1}}(t_{\mathrm{j-1}}) + \alpha_{\mathrm{RC,n+1}}V_{\mathrm{n}}(t_{\mathrm{j}})$$

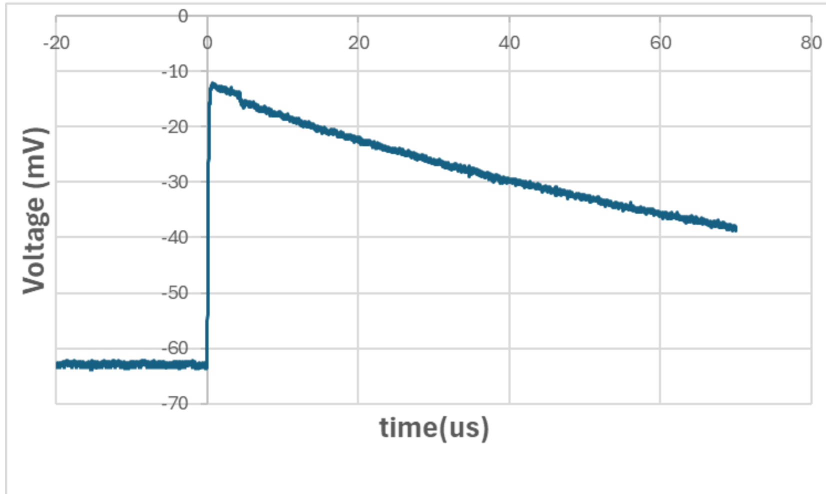
Let's try using excel first

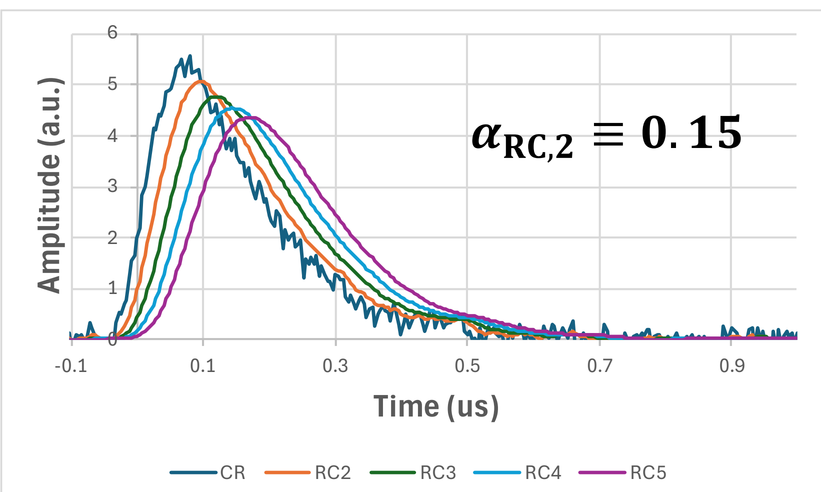
$$\Delta t = 64\ \text{ns}$$

$$\alpha_{\text{CR}} \equiv 0.85$$

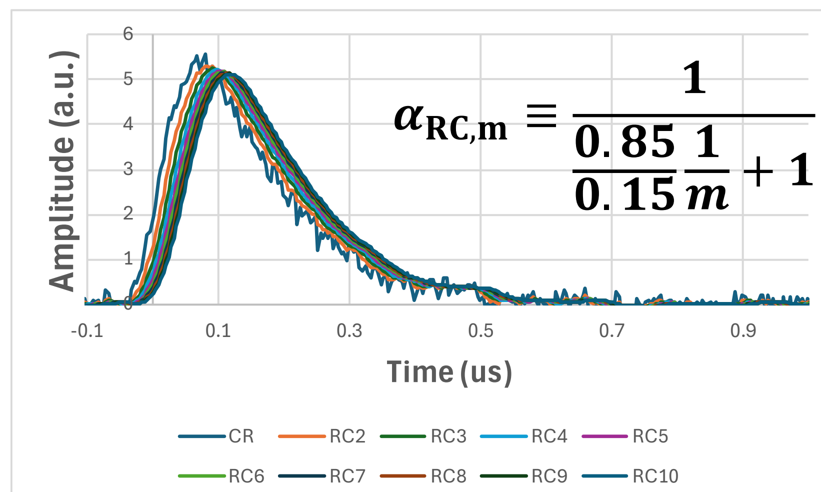
$$\tau_{\text{H}} = 362.7\ \text{ns}$$

$$\tau_{\text{L,m}} = 362.7/m\ \text{ns}$$





$$\alpha_{\text{RC,2}} \equiv 0.15$$



$$\alpha_{\text{RC,m}} \equiv \frac{1}{\frac{0.85}{0.15}\frac{1}{m} + 1}$$

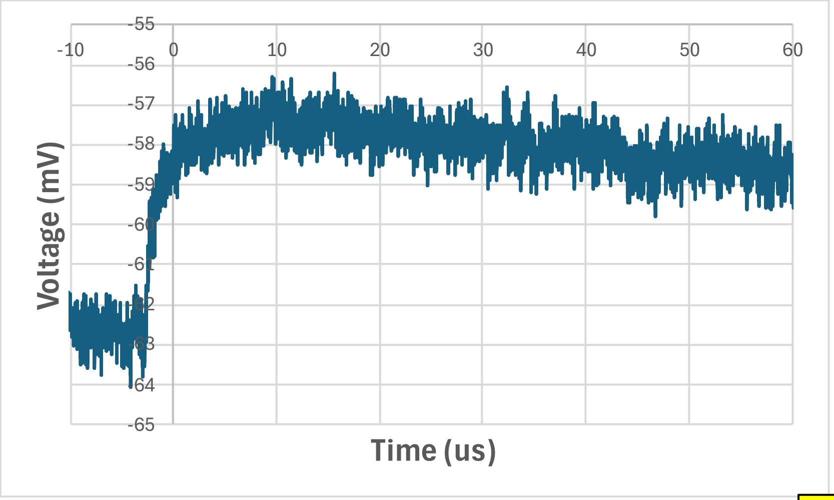
When the pulse is not sharp enough, no significant gaussian feature is observed

$\Delta t = 64\ \mathrm{ns}$

$\alpha_{\mathrm{CR}} \equiv 0.85$

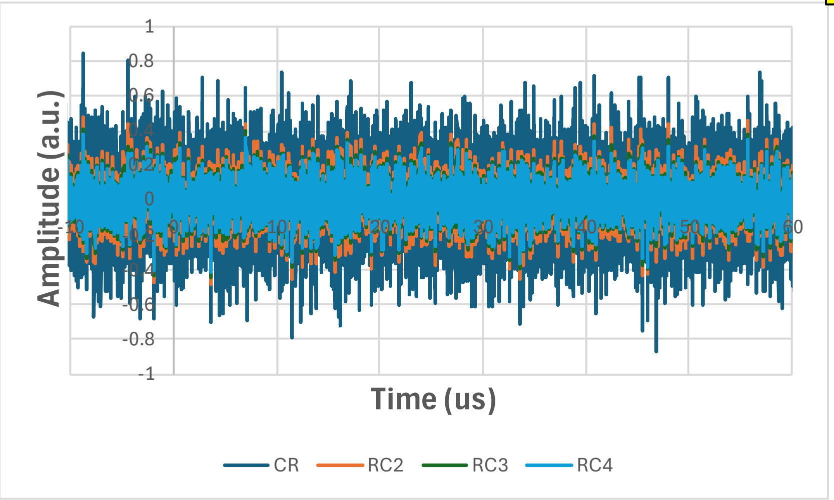
$\tau_{\mathrm{H}} = 362.7\ \mathrm{ns}$

$\tau_{\mathrm{L,m}} = 362.7/m\ \mathrm{ns}$



- What's the source?

$$\alpha_{\mathrm{RC,m}} \equiv \frac{1}{\frac{0.85}{0.15}\frac{1}{m} + 1}$$



Let's try using python

- You may install: IDLE / Visual Studio / Anaconda

```
import csv
acr = 0.85
arc = 0.15
brc = 1-arc
tauOvert = 0.85/0.15

filename = '20250610 (5)_0015'
fn2='{}.csv'.format(filename,j)
fn3='{}f2.csv'.format(filename,j)
with      open(fn2    ) as i:
    with  open(fn3,'x',encoding='utf-8', newline='') as o:
        r = csv.reader(i)
        w = csv.writer(o)
        w.writerow(next(r))     # First three rows are header names
        w.writerow(next(r))
        w.writerow(next(r))
```

	A	B	C	D	E
1	time	voltage			
2	(us)	(mV)			
3					
4	-30.0079	-62.7857			
5	-30.0039	-62.9475			
6	-29.9999	-63.1062			
7	-29.9959	-62.7857			
8	-29.9919	-63.0543			
9	-29.9879	-63.1611			
10	-29.9839	-63.1062			
11	-29.9799	-62.5172			
12	-29.9759	-62.8407			

	A	B	C	D	E	F	G	H	I	J	K
1	time	voltage	CR	RC2	RC3	RC4	RC5				
2	(us)	(mV)									
3											
4	-30.0079	-62.7857	0	0	0	0	0				
5	-30.0039	-62.9475	-0.13749	-0.03587	-0.01242	-0.00514	-0.00241				
6	-29.9999	-63.1062	-0.25176	-0.09219	-0.04003	-0.01958	-0.01046				
7	-29.9959	-62.7857	0.058388	-0.05291	-0.04449	-0.02988	-0.01956				
8	-29.9919	-63.0543	0.17965	0.08571	0.05876	0.04183	0.03				

Let’s try using python

```python
first = next(r)
lastR = float(first[1])
last0 = 0 # lastR        # Later discovered that these starting
last1 = 0 # last0 * brc # values are all not very important
last2 = 0 # last1 * brc # so we can simply choose to start
last3 = 0 # last2 * brc # with all of them set to zero
last4 = 0 # last3 * brc # and things will still work fine
w    .writerow(  [first[0],lastR,last0,last1,last2,last3,last4])
for row in r:
    last0 = acr * ( last0 + float(row[1]) - lastR )
    last1 = (1-1/(tauOvert/2+1)) *   last1 +      1/(tauOvert/2+1) * last0
    last2 = (1-1/(tauOvert/3+1)) *   last2 +      1/(tauOvert/3+1) * last1
    last3 = (1-1/(tauOvert/4+1)) *   last3 +      1/(tauOvert/4+1) * last2
    last4 = (1-1/(tauOvert/5+1)) *   last4 +      1/(tauOvert/5+1) * last3
    lastR = float(   row[1])
    w.writerow(  [  row[0],lastR,last0,last1,last2,last3,last4])
```

$$V_2(t_j) = \alpha_{CR} V_2(t_{j-1}) + \alpha_{CR}\left[V_1(t_j) - V_1(t_{j-1})\right]$$

$$V_m(t_j) = (1 - \alpha_{RC,m}) V_m(t_{j-1}) + \alpha_{RC,m} V_{m-1}(t_j)$$

$$\frac{\tau_{L,m}}{\Delta t} = \frac{0.85}{0.15}$$

$$\tau_n = \frac{\tau_{n-1}}{n}$$

$$\alpha_{RC,m} \equiv \frac{1}{\frac{0.85}{0.15}\frac{1}{m} + 1}$$

Course Outline

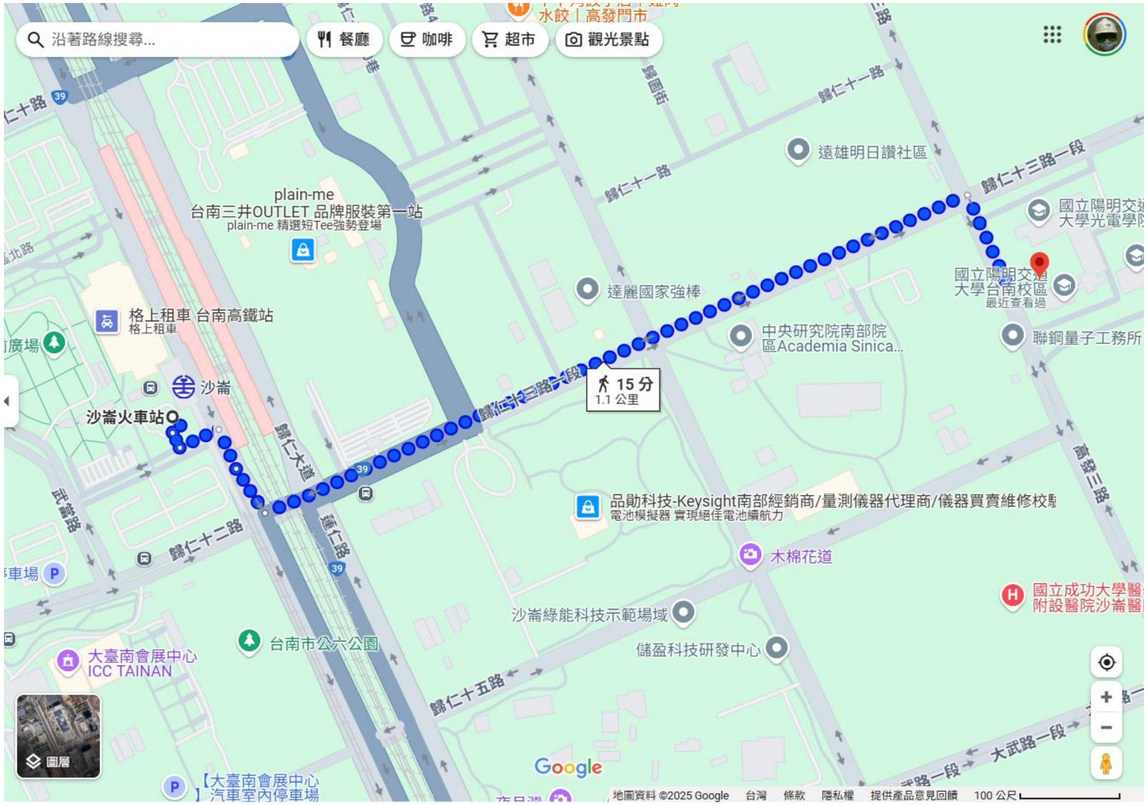
- Formosa Integrated Research Spherical Tokamak (FIRST), the first Tokamak in Taiwan
- Physics used in Alpha-E
  - Microwave
  - ECRH
  - Spectrum of fusion products
  - Stopping power of the filter
  - Data analysis using Pulse Shaping
- **Final report**

Final presentation

1. 實驗設備之原理。
2. 被分配的實驗項目的原理。
3. 數據分析原理說明。
4. 數據分析結果。
5. 討論。

若週四實驗課缺課，則報告為**0**分。

We will visit Alpha Ring’s laboratory located in Shalun



- **Alpha Ring @ Shalun**
- **Meet at Tainan Station at 12:40.**
- **Train 373, departing at 12:55.**
- **Walk to Alpha Ring arriving at 13:40.**