

Reference:

P1.

1. Introduction to plasma physics and controlled fusion, by Francis F. Chen
2. Principles of Plasma discharges and Material Processing, by M. A. Lieberman and A. J. Lichtenberg.
3. Plasma diagnostics by electrical probes - Practicum manual and documentation by L. Sirghi, G. Popa, D. Alexandroaies, C. Costin.

Sheaths.

- In all practical plasma devices, plasma \rightarrow in a vacuum chamber w/ finite size.
- What happens to the plasma "at the wall"?
- Suppose \rightarrow NO electric field in the plasma (\because Debye shielding)

$$\phi \equiv 0$$

- electrons & ions \rightarrow hit the wall
 \rightarrow recombine & are lost.

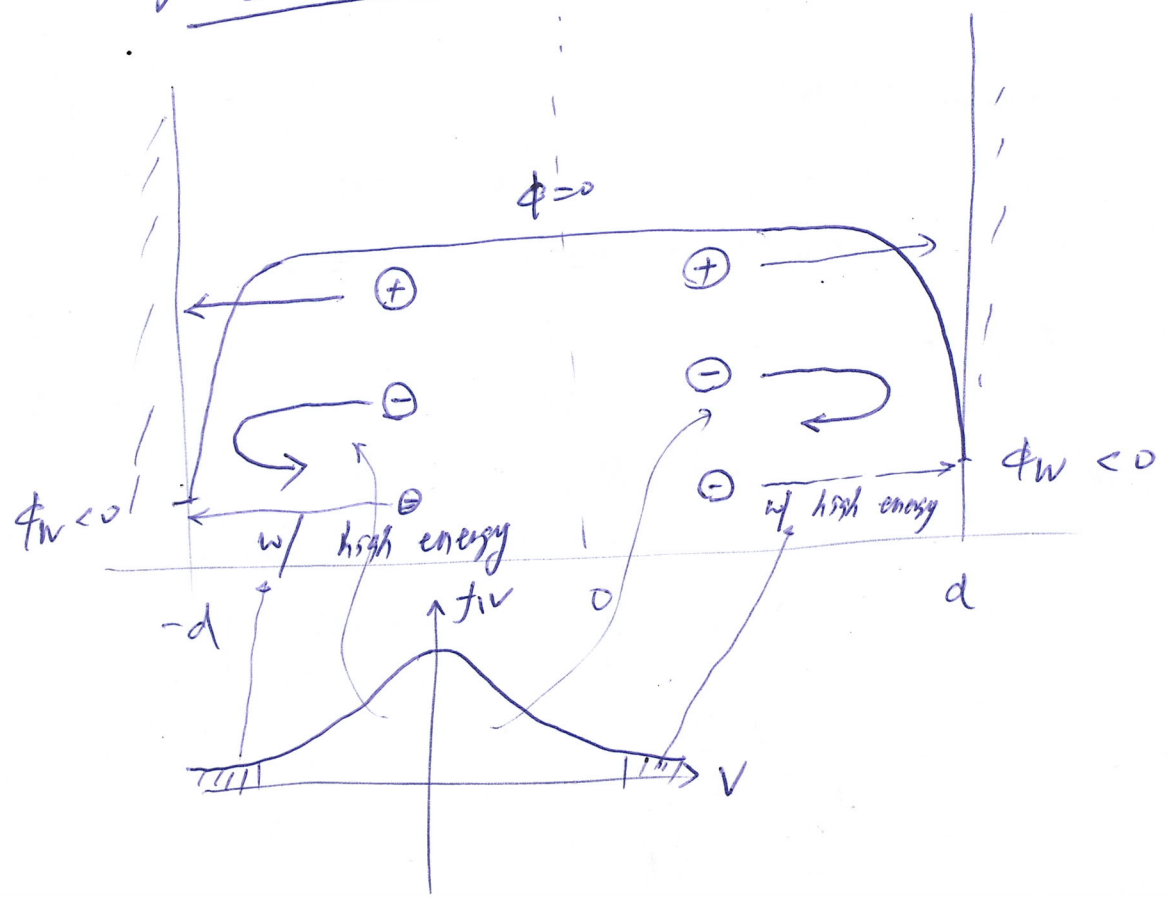
$$E = \frac{1}{2} m v^2 \Rightarrow v \propto \frac{E}{\sqrt{m}} \sim \frac{kT}{\sqrt{m}}$$

$\Rightarrow v_{th,e} \gg v_{th,i}$ ($\because m_e \ll m_i$
 $m_i = 1836 m_e$)

$\Rightarrow \bar{e}$ are lost faster and leave the plasma
w/ a net positive charge.

$$\Rightarrow \phi_w < 0$$

- However, w/ Debye shielding \Rightarrow the potential can
NOT be distributed over the entire plasma.
In fact, potential variation to a layer of the
order of several Debye lengths in thickness
 \rightarrow Sheath



* The function of a sheath is to form a potential barrier so that the more mobile species, usually electrons, are confined electrostatically. P3

> The height of the barrier adjusts itself so that the flux of electrons that have enough energy to go over the barrier to the wall is just equal to the flux of ions reaching the wall.

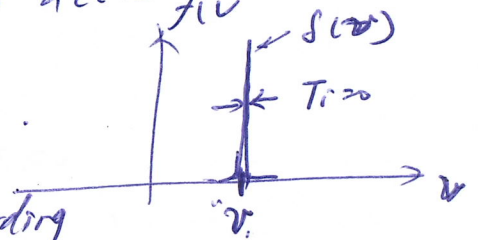
? The planar sheath equation.

* In a weakly ionized plasma, electrons are heated by the source while $T_{ion} \approx T_{gas}$. $T_e \approx \text{few eV}$

$$T_e \gg T_{ion} \approx T_{gas}$$

ions are assumed to be cold, but $T_i \approx 0$

\Rightarrow Monoenergetic ions are accelerated through the sheath potential.



Electron density decreases according to a Boltzmann factor.

$$m n \left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} \right] = z n \vec{E} - \nabla p - \underbrace{m n \nu_m \vec{u}}_{\text{collision}}$$

For steady state & no drift velocity of electrons

$$\frac{\partial \vec{u}}{\partial t} = 0, \quad (\vec{u} \cdot \nabla) \vec{u} = 0 \Rightarrow z n \vec{E} - \nabla p = 0$$

$$+ e n_e \vec{E} + \nabla p = 0$$

$$\vec{E} = -\nabla\phi, \quad p_e = n_e k T_e$$

$$\Rightarrow -e n_e \nabla\phi + k T_e \nabla n_e = 0 \quad \text{for isothermal}$$

$$\Rightarrow -\nabla\phi - \underbrace{\frac{k T_e}{e}}_{\text{const}} \underbrace{\frac{1}{n_e} \nabla n_e}_{\nabla \ln n_e} = 0$$

$$\Rightarrow \nabla (e\phi - k T_e \ln n_e) = 0$$

$$\Rightarrow e\phi - k T_e \ln n_e = \text{const} \neq n_0$$

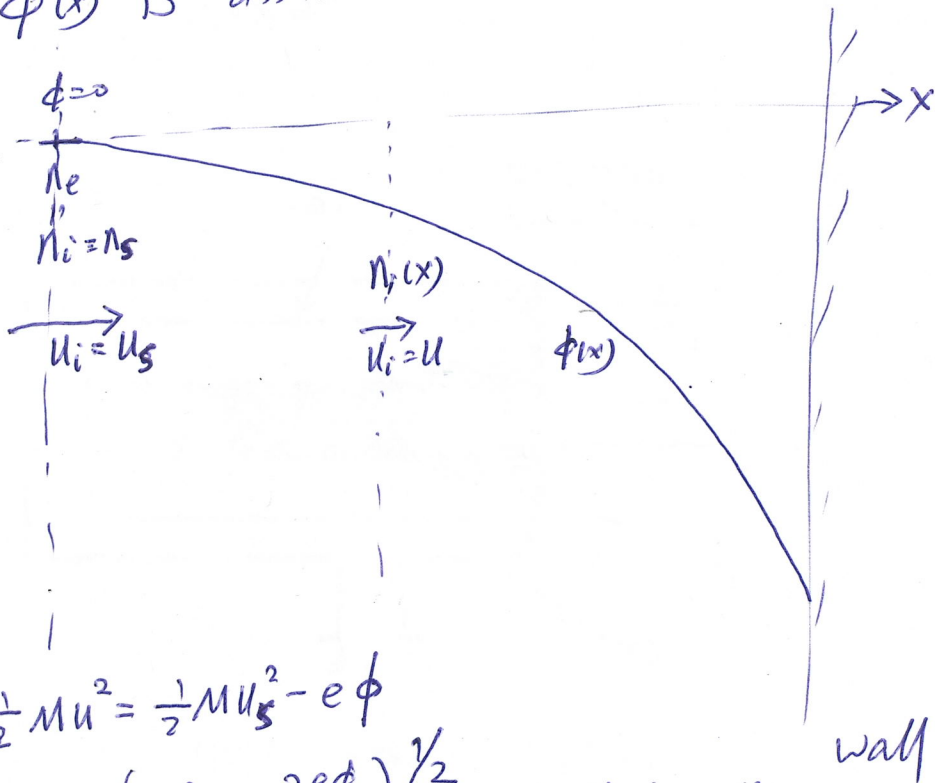
$$n_e(\vec{r}) = n_0 e^{e\phi(\vec{r})/k T_e}$$

for 1D, $\vec{r} \equiv x$

$$n_e(x) = n_0 e^{e\phi(x)/k T_e}$$

$$\underline{n_e(x) = n_0 e^{e\phi(x)/k T_e}}$$

* Consider steady state in a collisionless sheath. $\phi(x)$ is assumed to decrease monotonically w/ x .



$$\frac{1}{2} M u^2 = \frac{1}{2} M u_s^2 - e\phi$$

$$u = \left(u_s^2 - \frac{2e\phi}{M} \right)^{1/2}$$

M: ion mass

wall

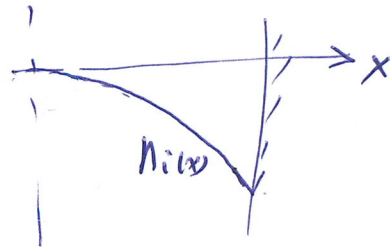
Continuity of ions.

$$n_s u_s = n_i u$$

$$n_i(x) = n_s u_s u^{-1} = n_s u_s \left(u_s^2 - \frac{2e\phi}{m} \right)^{-1/2}$$

$$= n_s \left(1 - \frac{2e\phi}{m u_s^2} \right)^{-1/2}$$

$$|\phi| \uparrow \Rightarrow \left(1 - \frac{2e\phi}{m u_s^2} \right)^{-1/2} \uparrow \Rightarrow n_i \downarrow$$



For electrons in steady state:

$$n_e(x) = n_s \exp\left(\frac{e\phi}{kT_e}\right)$$

Poisson's Eq: $\nabla \cdot \vec{E} = \frac{\sigma}{\epsilon_0}$ $\vec{E} = -\nabla\phi$

$$\Rightarrow \epsilon_0 \nabla^2 \phi = -\sigma$$

$$\Rightarrow \epsilon_0 \frac{d^2 \phi}{dx^2} = -\left(e n_i - e n_e \right)$$

$$= e (n_e - n_i)$$

$$\epsilon_0 \frac{d^2 \phi}{dx^2} = e n_s \left[\exp\left(\frac{e\phi}{kT_e}\right) - \left(1 - \frac{2e\phi}{m u_s^2} \right)^{-1/2} \right]$$

Let $e \epsilon_s \equiv \frac{1}{2} m u_s^2$

$$\frac{d^2 \phi}{dx^2} = \frac{e n_s}{\epsilon_0} \left[\exp\left(\frac{e\phi}{kT_e}\right) - \left(1 - \frac{\phi}{\epsilon_s} \right)^{-1/2} \right]$$

3 The Bohn sheath criterion.

x $\frac{d\phi}{dx}$ on both side.

$$\frac{d\phi}{dx} \cdot \frac{d}{dx} \left(\frac{d\phi}{dx} \right) = \frac{ens}{\epsilon_0} \left[\exp\left(\frac{e\phi}{kT_e}\right) - \left(1 - \frac{\phi}{\xi_s}\right)^{-1/2} \right] \frac{d\phi}{dx}$$

$$\frac{1}{2} \frac{d}{dx} \left[\left(\frac{d\phi}{dx} \right)^2 \right]$$

$$\Rightarrow \frac{1}{2} \int d \left[\left(\frac{d\phi}{dx} \right)^2 \right] = \frac{ens}{\epsilon_0} \int_0^\phi d\phi \left[\exp\left(\frac{e\phi}{kT_e}\right) - \left(1 - \frac{\phi}{\xi_s}\right)^{-1/2} \right]$$

LHS: $= \frac{1}{2} \left[\left(\frac{d\phi}{dx} \right)^2 \right]_{\phi=0}^{\phi=\phi} = \frac{1}{2} \left(\frac{d\phi}{dx} \right)^2$ assuming $\phi=0, \frac{d\phi}{dx} \rightarrow 0$ @ $x=0$
field free plasma.

$$RHS = \frac{ens}{\epsilon_0} \left[\frac{kT_e}{e} \exp\left(\frac{e\phi}{kT_e}\right) + 2\xi_s \left(1 - \frac{\phi}{\xi_s}\right)^{1/2} \right]_0^\phi$$

$$= \frac{ens}{\epsilon_0} \left[\frac{kT_e}{e} \exp\left(\frac{e\phi}{kT_e}\right) - \frac{kT_e}{e} + 2\xi_s \left(1 - \frac{\phi}{\xi_s}\right)^{1/2} - 2\xi_s \right]$$

$$\frac{1}{2} \left(\frac{d\phi}{dx} \right)^2 = \frac{ens}{\epsilon_0} \left[\frac{kT_e}{e} \exp\left(\frac{e\phi}{kT_e}\right) - \frac{kT_e}{e} + 2\xi_s \left(1 - \frac{\phi}{\xi_s}\right)^{1/2} - 2\xi_s \right]$$

It can be integrated numerically, to obtain $\phi(x)$

Expand RHS to 2nd order of Taylor series.

$$f(x) = e^x \quad f(0) = 1$$

$$f'(x) = e^x \quad f'(0) = 1$$

$$f''(x) = e^x \quad f''(0) = 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\approx 1 + x + \frac{1}{2}x^2$$

$$f(x) = (1-x)^{1/2} \quad f(0) = 1$$

$$f'(x) = -\frac{1}{2}(1-x)^{-1/2} \quad f'(0) = -1/2$$

$$f''(x) = \frac{1}{4}(1-x)^{-3/2} \quad f''(0) = 1/4$$

$$(1-x)^{1/2} = 1 - \frac{1}{2}x - \frac{1}{4} \frac{x^2}{2!} + \dots$$

$$\approx 1 - \frac{x}{2} - \frac{x^2}{8}$$

$$\approx \frac{en_s}{\epsilon_0} \left[\frac{kT_e}{e} \left(1 + \frac{e\phi}{kT_e} + \frac{1}{2} \frac{e^2\phi^2}{(kT_e)^2} \right) - \frac{kT_e}{e} + 2\epsilon_s \left(1 - \frac{\phi}{2\epsilon_s} - \frac{\phi^2}{8\epsilon_s^2} \right) - 2\epsilon_s \right] \quad P7$$

$$= \frac{en_s}{\epsilon_0} \left[\frac{kT_e}{e} + \cancel{\phi} + \frac{1}{2} \frac{e\phi^2}{kT_e} - \frac{kT_e}{e} + 2\epsilon_s - \cancel{\phi} - \frac{\phi^2}{4\epsilon_s} - 2\epsilon_s \right]$$

$$= \frac{en_s}{\epsilon_0} \left[\frac{1}{2} \frac{e\phi^2}{kT_e} - \frac{\phi^2}{4\epsilon_s} \right] = \frac{1}{2} \left(\frac{d\phi}{dx} \right)^2 \geq 0$$

$$\Rightarrow \frac{1}{2} \frac{e\phi^2}{kT_e} - \frac{\phi^2}{4\epsilon_s} \geq 0$$

$$\Rightarrow e\epsilon_s \geq \frac{kT_e}{2}$$

Note that $e\epsilon_s \equiv \frac{1}{2} M U_s^2$

$$\frac{1}{2} M U_s^2 \geq \frac{kT_e}{2}$$

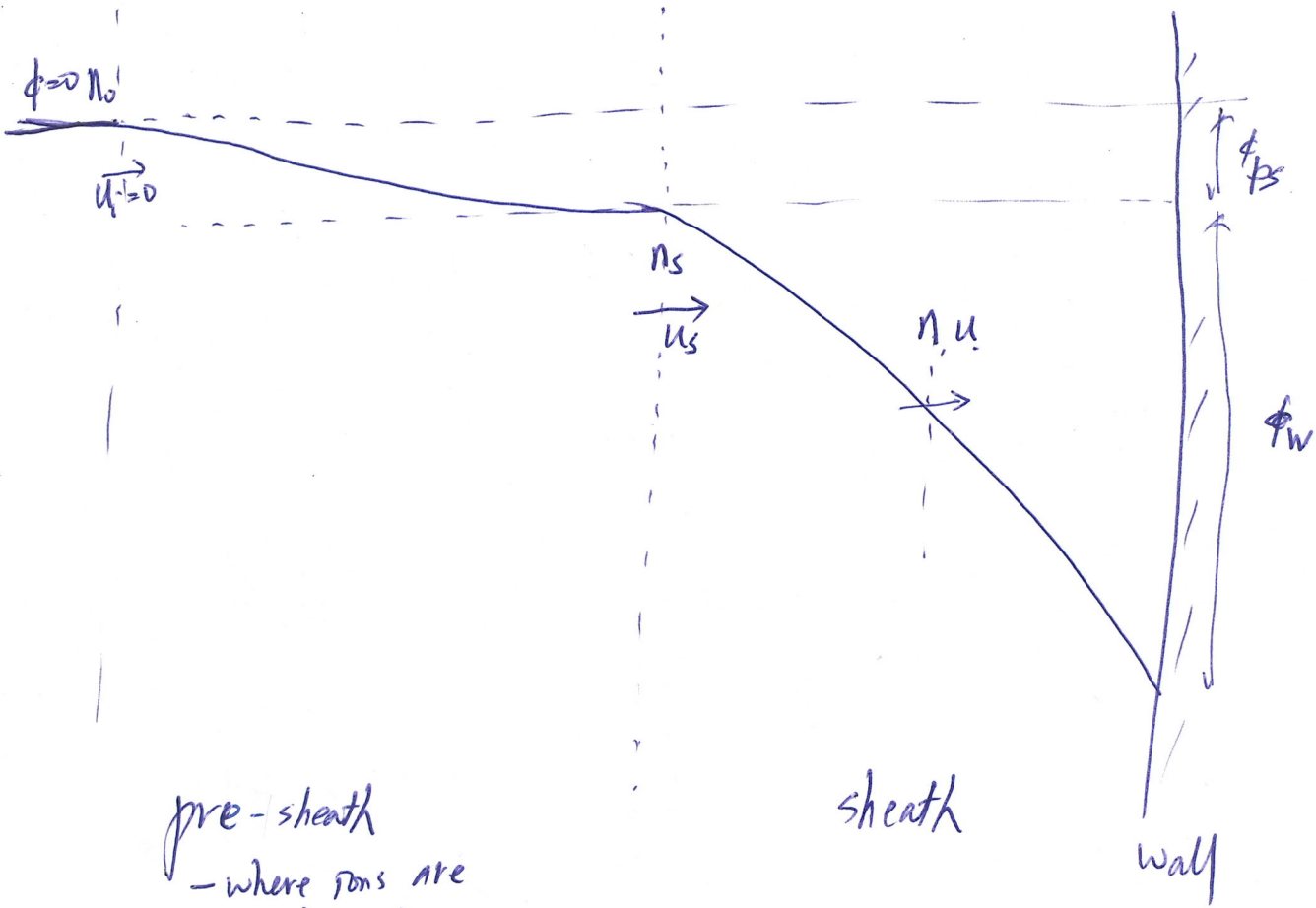
$$U_s \geq \left(\frac{kT_e}{M} \right)^{1/2} \equiv U_B$$

Bohn sheath criterion

* Ions must enter the sheath region w/ a velocity greater than the acoustic velocity U_B

\Rightarrow There must be a finite electric field in the plasma.

$\Rightarrow \frac{d\phi}{dx} \neq 0 @ x=0$ is only an approximation



pre-sheath
- where ions are accelerated.

sheath

wall

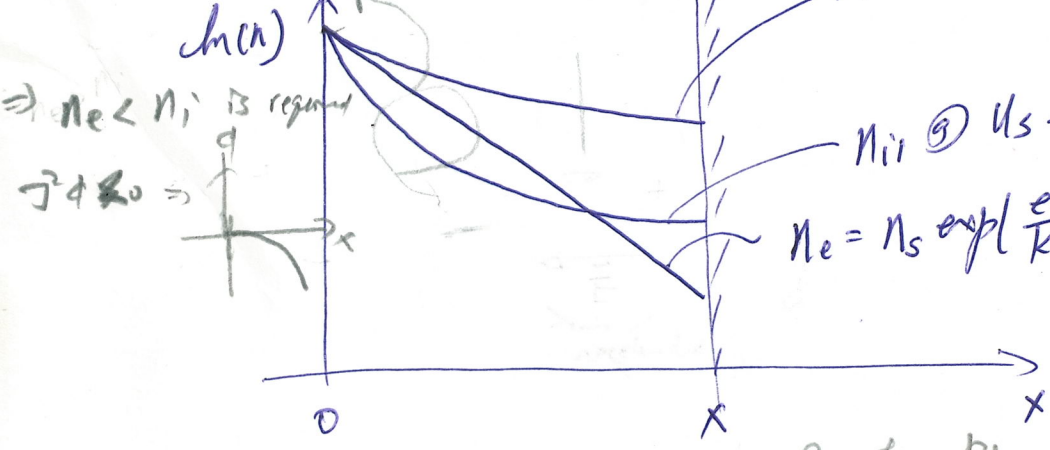
Note that $n_s u_s$ is fixed by the ion production rate.

let $u_s = u_B$. $\frac{1}{2} m u_B^2 = -e \phi_{ps}$

$\Rightarrow n_s = n_{es} = n_0 e^{\frac{e \phi_{ps}}{k T_e} - \frac{1}{2} \frac{m u_s^2}{k T_e}} = n_0 e^{-1/2} = 0.61 n_0$

$\nabla^2 \phi = e(n_e - n_i)$

for $n_e > n_i$ $\nabla^2 \phi > 0 \Rightarrow$ $n_s = 0.61 n_0$



$n_{i2} \text{ @ } u_s > \left(\frac{k T_e}{m}\right)^{1/2}$

$n_{i1} \text{ @ } u_s < \left(\frac{k T_e}{m}\right)^{1/2}$

$n_e = n_s \exp\left(\frac{e \phi}{k T_e}\right)$

$n_i = n_s \left(1 - \frac{2e \phi}{m u_s^2}\right)^{1/2}$

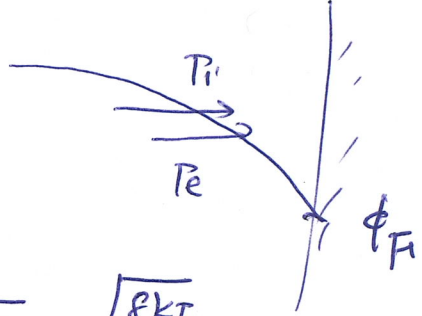
For larger u_s^2 , $\frac{2e \phi}{m u_s^2}$ is small
 $\Rightarrow n_i$ changes slower w/ respect to ϕ .

go to p12

step 3) sheath potential @ a floating wall. p8a

for practice
course on
jump to p12

$T_i = T_e \rightarrow$ NO current



$T_i = n_s \cdot u_s = n_s u_B$

$T_e = \frac{1}{4} n_{ew} \cdot \bar{v}_e$, where $\bar{v}_e = \sqrt{\frac{8kT}{\pi m}}$

For 3D: $f_m = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{v^2}{v_{th}^2}\right)$

$v^2 = v_x^2 + v_y^2 + v_z^2$, $v_{th}^2 = \frac{2kT}{m}$
 $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dv_x dv_y dv_z = 4\pi v^2 dv$

$\bar{v}^2 = \left(\frac{m}{2\pi kT}\right)^{3/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} v^2 \exp\left(-\frac{v^2}{v_{th}^2}\right) dv^3 = \frac{3kT}{m}$

$\bar{v} = \int v f(v) dv^3$

$= \left(\frac{m}{2\pi kT}\right)^{3/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} v \cdot \exp\left(-\frac{v^2}{v_{th}^2}\right) 4\pi v^2 dv$

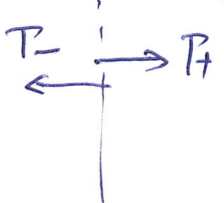
$\int_0^{\infty} v^4 \exp\left(-\frac{v^2}{v_{th}^2}\right) dv = \frac{3\sqrt{\pi}}{8} (v_{th}^2)^{5/2}$
 $= \frac{4\pi}{\pi^2} \frac{3\sqrt{\pi}}{8} \left(\frac{2kT}{m}\right)^{5/2} = \frac{3}{4\pi^2} \frac{2\pi kT}{m} = \frac{3kT}{2\pi m}$

$= 2\sqrt{\frac{2kT}{\pi m}} \cdot 4\pi \frac{(v_{th}^2)^4}{2} = \frac{2}{\pi} \left(\frac{2\pi kT}{m}\right)^{1/2} = \left(\frac{8kT}{\pi m}\right)^{1/2}$

$|\bar{v}_x| = \int |v_x| f(v) dv^3 = \left(\frac{m}{2\pi kT}\right)^{3/2} \int_{-\infty}^{\infty} dv_y \exp\left(-\frac{v_y^2}{v_{th}^2}\right) \int_{-\infty}^{\infty} dv_z \exp\left(-\frac{v_z^2}{v_{th}^2}\right) \int_0^{\infty} 2v_x \exp\left(-\frac{v_x^2}{v_{th}^2}\right) dv_x$

$= \left(\frac{m}{2\pi kT}\right)^{1/2} \int_0^{\infty} \exp\left(-\frac{v_x^2}{v_{th}^2}\right) dv_x^2 = \left(\frac{m}{2\pi kT}\right)^{1/2} \left(-v_{th}^2\right) \exp\left(-\frac{v_x^2}{v_{th}^2}\right) \Big|_0^{\infty}$

$= \left(\frac{m}{2\pi kT}\right)^{1/2} \left(\frac{2kT}{m}\right)^{1/2} = \left(\frac{2kT}{\pi m}\right)^{1/2} = \frac{1}{2} \bar{v}$



$T_+ = \left(\frac{1}{2}n\right) \bar{v}_x$, $T_- = -\left(\frac{1}{2}n\right) \bar{v}_x$
 $\frac{1}{2}\bar{v}$

$$n_{ew} = n_s e^{e\phi_F/kT_e}$$

$$\Rightarrow P_i = P_e$$

$$n_s \cdot u_s = n_s \left(\frac{kT_e}{m}\right)^{1/2} = \frac{1}{4} n_s e^{e\phi_F/kT_e} \cdot \sqrt{\frac{8kT_e}{\pi m}}$$

$$e^{e\phi_F/kT_e} = 4 \left(\frac{\pi m}{8m}\right)^{1/2} = \left(\frac{2\pi m}{m}\right)^{1/2}$$

$$\frac{e\phi_F}{kT_e} = \ln\left(\frac{2\pi m}{m}\right)^{1/2} = -\ln\left(\frac{m}{2\pi m}\right)^{1/2}$$

$$\phi_F = -\frac{kT_e}{e} \ln\left(\frac{m}{2\pi m}\right)^{1/2}$$

$$= -\frac{kT_e}{2e} \ln\left(\frac{M}{2\pi m}\right)$$

M: ion mass
m: e⁻ mass

For Argon, M = 40 amu. *1

$$\ln\left(\frac{M}{2\pi m}\right) \approx 4.7$$

Hydrogen, M = 1.

$$\ln\left(\frac{M}{2\pi m}\right) \approx 2.8$$

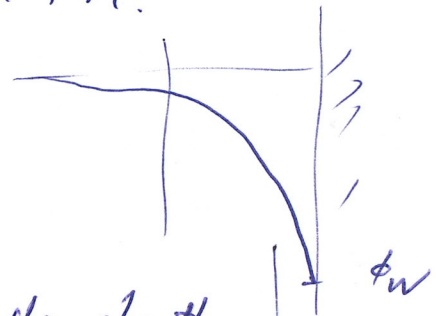
kT_e ~ 10 eV,
 $\Rightarrow \phi_F \sim \underline{5-10 \text{ V}}$

3 The high-voltage sheath.

$|e\phi_w| \gg kT_e$ in general

$\Rightarrow n_e = n_s e^{e\phi_w/kT_e} \sim 0$

\Rightarrow only ions are present in the sheath.



A dark layer where no e^- are present to excite atoms to emission.

3 Matrix Sheath.

Assuming a uniform ion density (simplest solution) sheath.

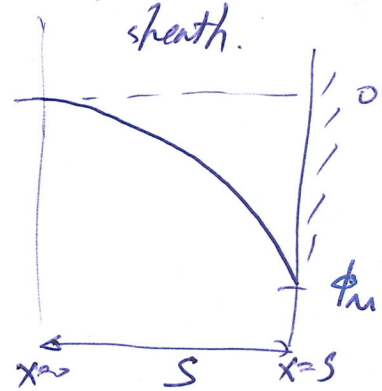
Let $n_i = n_s = \text{const.}$

$\epsilon_0 \nabla \cdot \vec{E} = \rho \approx e n_i$ ($n_e \approx 0$)

$\frac{dE}{dx} = \frac{e n_i}{\epsilon_0} \approx \frac{e n_s}{\epsilon_0}$

$E = \frac{e n_s}{\epsilon_0} x$ $\nabla \phi = -\vec{E} \Rightarrow \frac{d\phi}{dx} = -E$

$\frac{d\phi}{dx} = -\frac{e n_s}{\epsilon_0} x \Rightarrow \phi \Big|_0^{\phi_m} = -\frac{1}{2} \frac{e n_s}{\epsilon_0} x^2 \Big|_0^S$



$+|\phi_m| \equiv \phi_m = +\frac{1}{2} \frac{e n_s}{\epsilon_0} S^2$

or $S = \left(\frac{2 \epsilon_0 |\phi_m|}{e n_s} \right)^{1/2}$

Debye length: $\lambda_D = \left(\frac{\epsilon_0 kT_e}{e n_s} \right)^{1/2}$

$S = \lambda_D \left(\frac{2 |\phi_m|}{kT_e} \right)^{1/2} \approx \text{tens of Debye length}$



Child Law Sheath

- * Keep the high voltage assumption, i.e., $n_e \approx 0$
- * Ion density should decrease as the ions accelerated across the sheath.

Assuming initial ion energy $\frac{1}{2} m U_s^2 = e \phi_s \ll |\phi|$
 that the $\frac{1}{2} m U_s^2$ in the following eq.:

$$\frac{1}{2} m U^2 = \frac{1}{2} m U_s^2 - e \phi \approx -e \phi$$

$$n_s U_s = n_i U$$

$$U = \left(-\frac{2e\phi}{m} \right)^{1/2}$$

($J_e \rightarrow 0$
 $n_e \rightarrow 0$)

Current density:

$$J_0 = e \cdot n_i U = e n_i \left(-\frac{2e\phi}{m} \right)^{1/2}$$

$$\Rightarrow n_i = \frac{J_0}{e} \left(-\frac{2e\phi}{m} \right)^{-1/2}$$

Poisson's Eq:

$$\frac{d^2 \phi}{dx^2} = -\frac{e n_i}{\epsilon_0} = -\frac{J_0}{\epsilon_0} \left(-\frac{2e\phi}{m} \right)^{-1/2}$$

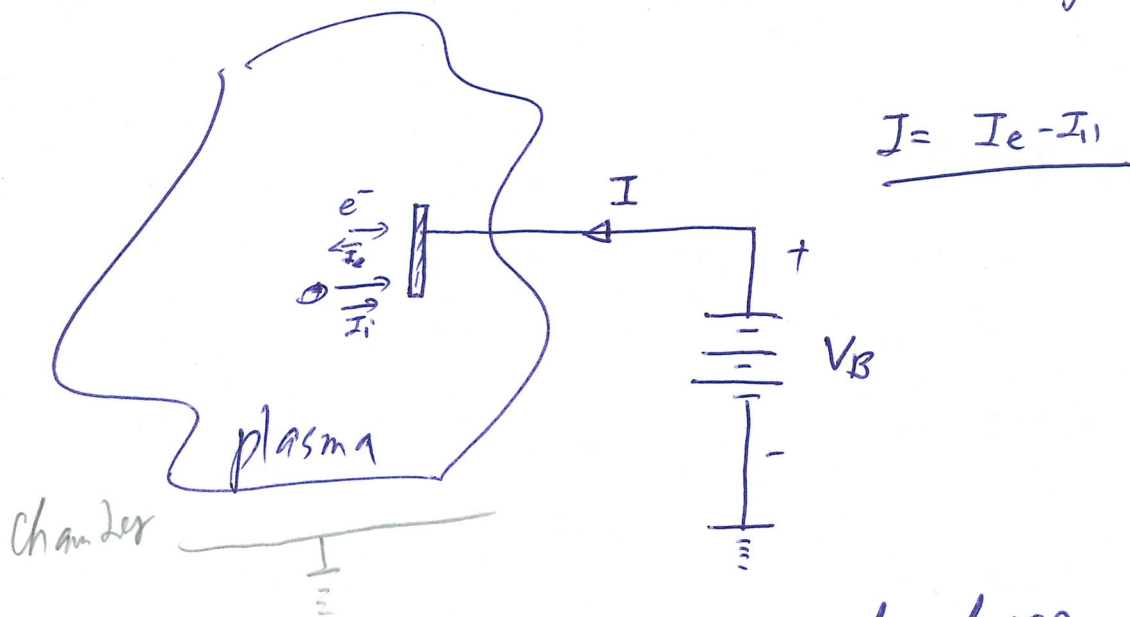
$$x \frac{d\phi}{dx} \quad \frac{d\phi}{dx} \frac{d}{dx} \left(\frac{d\phi}{dx} \right) = -\frac{J_0}{\epsilon_0} \left(-\frac{2e\phi}{m} \right)^{-1/2} \frac{d\phi}{dx}$$

$$\frac{1}{2} \frac{d}{dx} \left[\left(\frac{d\phi}{dx} \right)^2 \right]$$

$$\Rightarrow \frac{1}{2} \int_0^\phi d \left(\frac{d\phi}{dx} \right)^2 = -\frac{J_0}{\epsilon_0} \int_0^\phi \left(-\frac{2e\phi}{m} \right)^{-1/2} d\phi$$

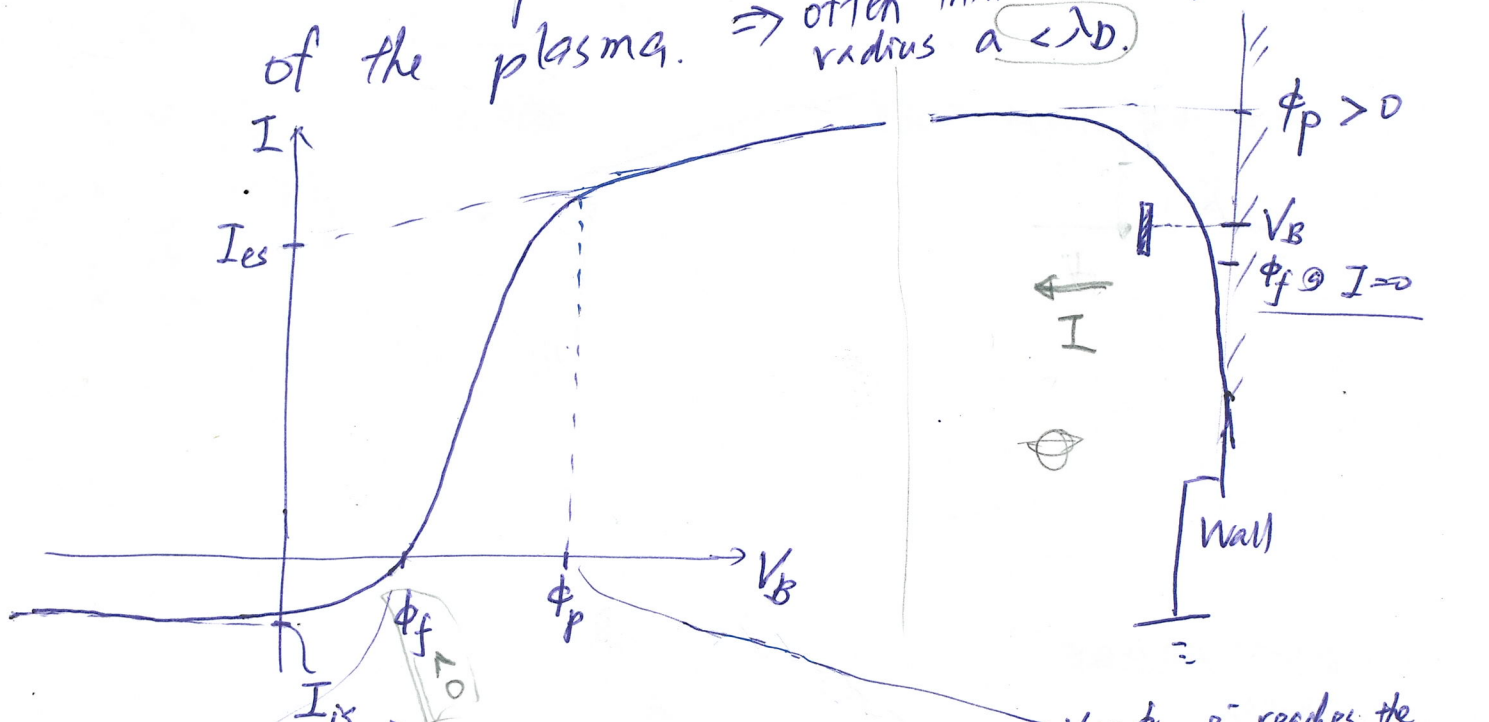
Let $\frac{d\phi}{dx} = -E \approx 0$, $\phi \approx 0$ @ $x=0$

Electrostatic probe (Langmuir probe) p12



* A metal probe, inserted in a discharge and biased positively or negatively to draw electron or ion current.

→ Probe are usually quite small and under suitable conditions, produce only minor local perturbations of the plasma. \Rightarrow often thin wires w/ the wire radius $a < \lambda_D$.



Floating potential where e^- & ion flux are balanced & no current flow through the probe.

Negative relative to plasma so that all e^- are repelled. only ion current, i.e., I_{is}

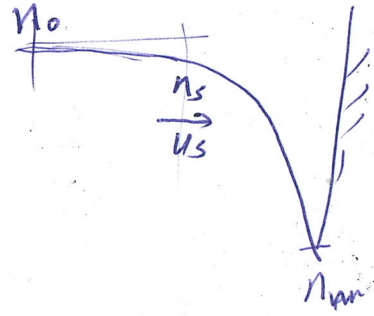
$V_B = \phi_p$, e^- reaches the probe freely, current is mainly from mobile electrons i.e. I_{es} .

⑨ Saturated ion current:

PM =

$$I = -I_{iS} = -eZn_s u_s \cdot A$$

$$\left(\frac{kT_e}{m} \right)^{1/2}$$



$$n_0 \approx \frac{n_s}{0.61} \Rightarrow n_s = 0.61 n_0$$

$$I_i = 0.61 e Z n_0 \left(\frac{kT_e}{m} \right)^{1/2} A$$

$$n_0 = \frac{1}{0.61} \frac{I_{iS}}{e Z A} \left(\frac{m}{kT_e} \right)^{1/2}$$

$n_0 = n_{ew}$

⑩ Saturated e^- current:

$$I = I_{es} - I_{im} \approx I_{es} = e \frac{1}{4} n_{ew} \bar{v}_e \cdot A$$

$$n_{ew} = n_0 e^{e\phi_p/kT_e}$$

$$I_{es} = \frac{1}{4} e n_0 A$$

⑪ Saturated e^- current

$$I = I_{es} - I_{im} \approx I_{es} = e \cdot \frac{1}{4} \underbrace{n_0}_{Te} \bar{v}_e \cdot A$$

all electrons arrive the probe.

$$n_{ew} = n_0$$

$$I_{es} = \frac{1}{4} n_0 e A \sqrt{\frac{8kT_e}{2\pi m}}$$

$$n_0 = \frac{I_{es}}{e \cdot A} \sqrt{\frac{2\pi m}{kT_e}}$$

5) Transition;

$$I = I_e - I_i$$

$$I + I_i = I_e = \frac{1}{4} e n_p \bar{v}_e \cdot A \quad \text{prob}$$

$$n_p = n_0 \exp\left(e \frac{V_B - \phi_p}{kT_e}\right) \quad \underline{V_B - \phi_p < 0}$$

$$\bar{v}_e = \sqrt{\frac{8kT_e}{\pi m}}$$

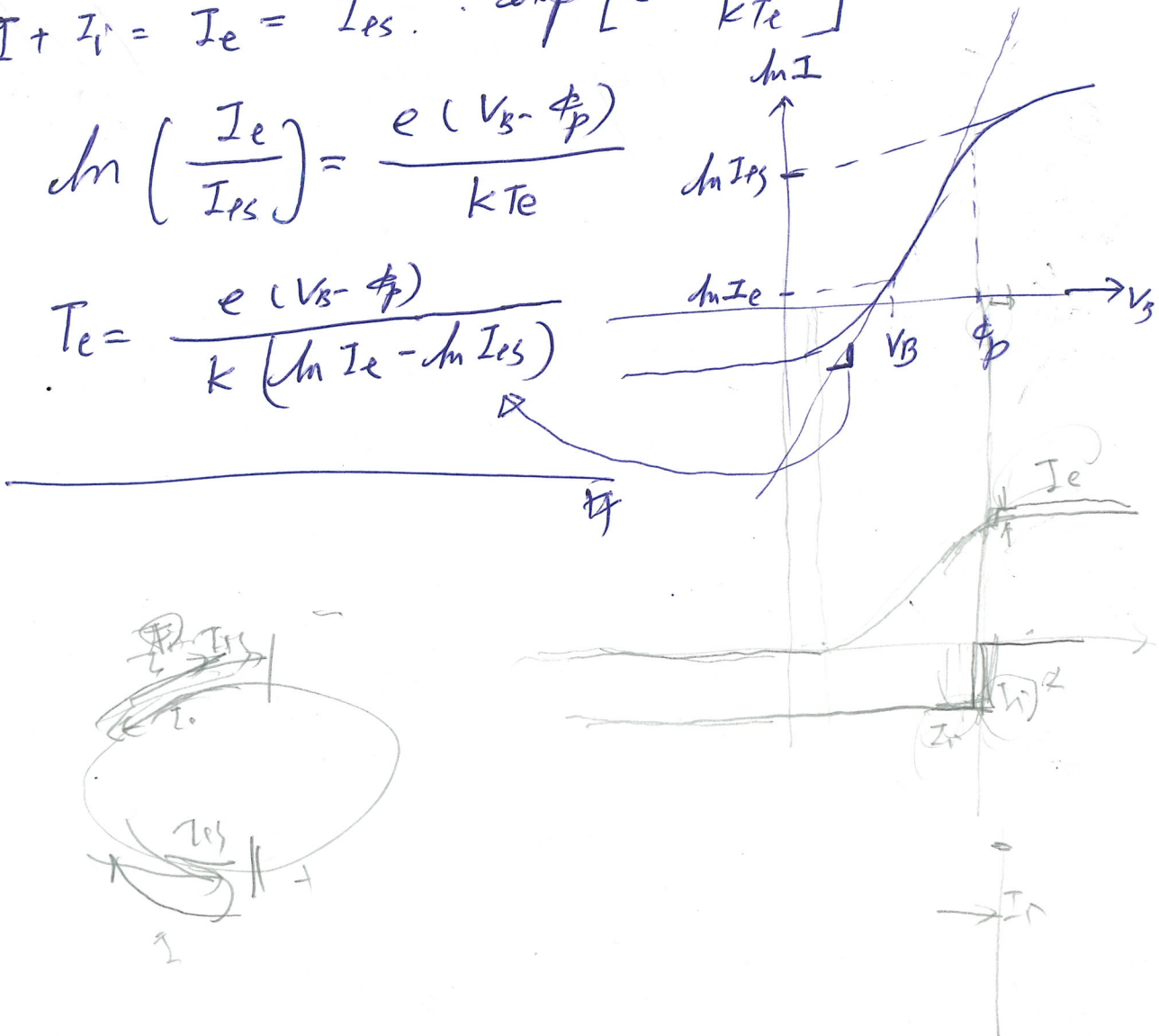
$$I + I_i = I_e = \frac{1}{4} e n_0 \exp\left(e \frac{V_B - \phi_p}{kT_e}\right) \cdot \frac{\sqrt{\frac{8kT_e}{\pi m}}}{\bar{v}_e} \cdot A$$

$$I_{es} = \frac{1}{4} e n_0 \bar{v}_e \cdot A$$

$$I + I_i = I_e = I_{es} \cdot \exp\left[e \frac{V_B - \phi_p}{kT_e}\right]$$

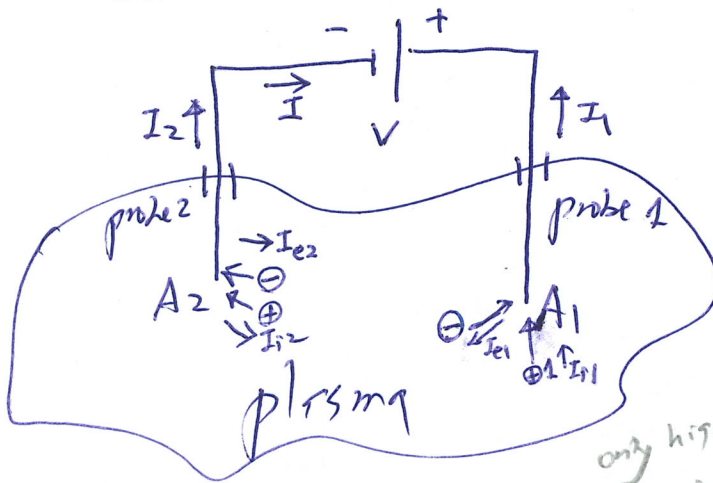
$$\ln\left(\frac{I_e}{I_{es}}\right) = \frac{e(V_B - \phi_p)}{kT_e}$$

$$T_e = \frac{e(V_B - \phi_p)}{k(\ln I_e - \ln I_{es})}$$



3 Double probes

* When there is no well-defined ground.



$\phi_2 < \phi_1$
 for large V ,
 $I_2 \rightarrow I_{sat}$
 $I_1 \rightarrow I_{sat} + I_{e2} \approx I_{e1}$
 I_e is NOT saturated

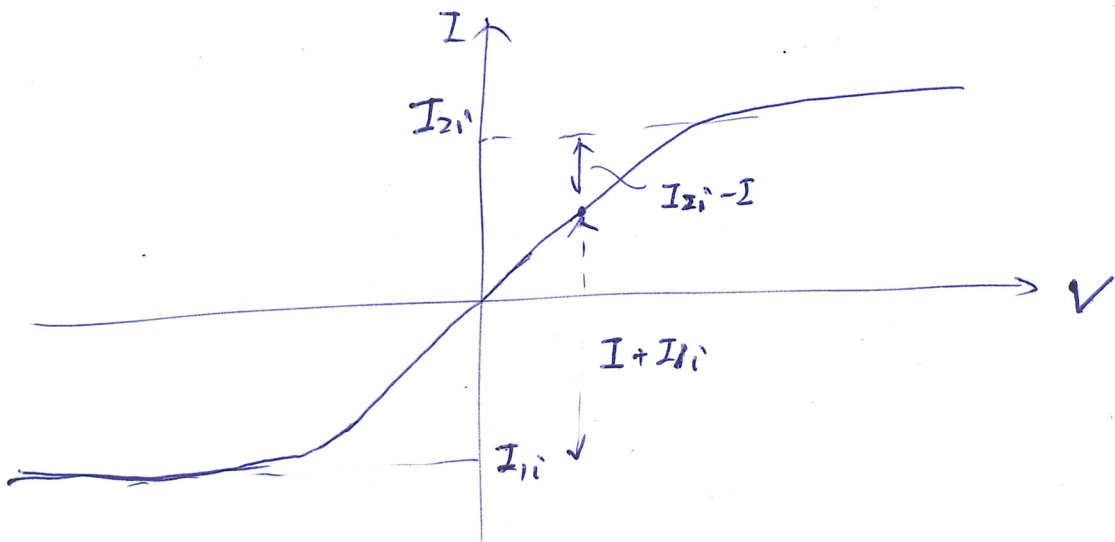
only high-energy tail is collected

* Since the two probes draw no net current, they will both be negative w/ respect to the plasma.

* $I \neq 0$ if $V \neq 0$.

* Advantage: the net current never exceeds the ion saturation current, minimizing the disturbance to the discharge.

Disadvantage: only the high-energy tail of the e^- distribution is collected by either probe. It may not be representative of the distribution of bulk e^- in the discharge.



$$I_1 = I_{i1} - I_{e1}, \quad I_2 = I_{i2} - I_{e2}$$

No current (system is floating)

$$I_1 + I_2 = 0 \Rightarrow I_{i1} - I_{e1} + I_{i2} - I_{e2} = 0$$

$$I_{i1} + I_{i2} - I_{e1} - I_{e2} = 0$$

$$I = -I_1 = I_{e1} - I_{i1} = I_2 = I_{i2} - I_{e2}$$

$$V_1 - \phi_p < 0$$

$$V_2 - \phi_p < 0$$

$$\begin{cases} I_{e1} = A_1 J_{es} \cdot e^{e(V_1 - \phi_p)/kT_e} = I + I_{i1} \\ I_{e2} = A_2 J_{es} \cdot e^{e(V_2 - \phi_p)/kT_e} = I_{i2} - I \end{cases} \quad) \div$$

$$\Rightarrow \frac{I + I_{i1}}{I_{i2} - I} = \frac{A_1}{A_2} e^{e(V_1 - V_2)/kT_e} \quad V = V_1 - V_2$$

$$= \frac{A_1}{A_2} e^{eV/kT_e}$$

For $A_1 = A_2$, $I_{i1} = I_{i2} = I_i$

$$\frac{I + I_i}{I_i - I} = e^{eV/kT_e} \Rightarrow I + I_i = I_i e^{eV/kT_e} - I e^{-eV/kT_e}$$

$$I = I_i \frac{e^{eV/kT_e} - 1}{1 + e^{eV/kT_e}} = I_i \frac{e^{eV/kT_e} - e^{-eV/kT_e}}{e^{eV/kT_e} + e^{-eV/kT_e}}$$

by fitting the curve, both T_e & $I_i \rightarrow$ no are obtained.

$$I = I_i \tanh\left(\frac{eV}{2kT_e}\right)$$

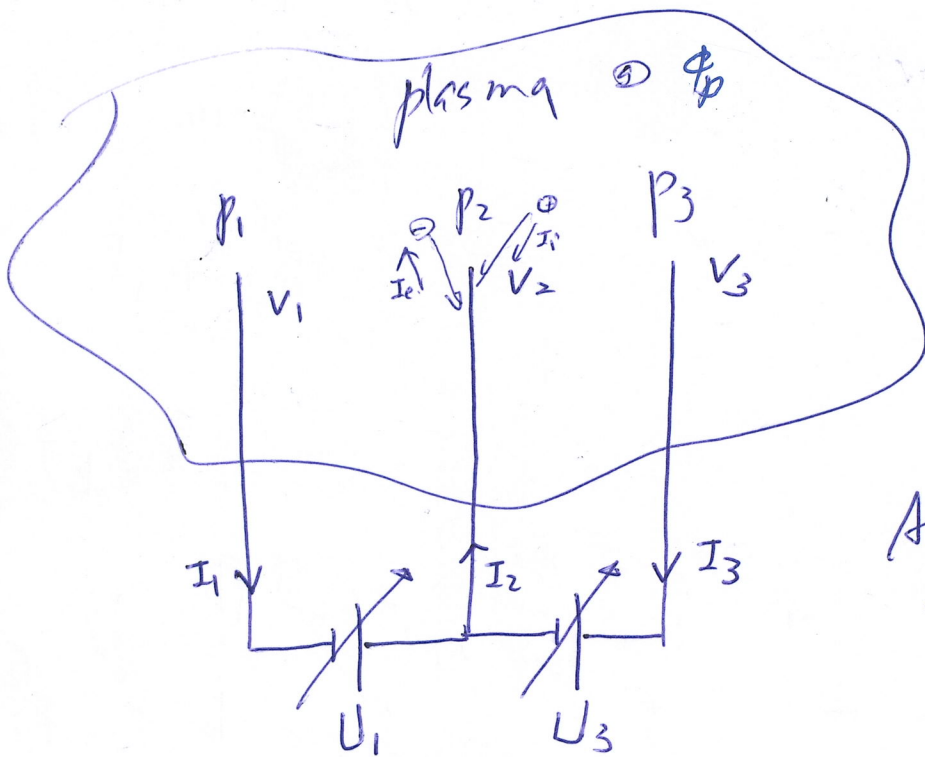
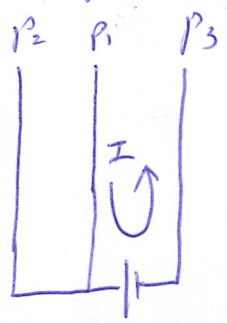
$$\textcircled{3} \quad V \gg 0 \quad \left. \frac{dI}{dV} \right|_{V \gg 0} = I_i \frac{e}{2kT_e} \frac{d}{dX} \left[\tanh(X) \right] \Big|_{X = \frac{eV}{2kT_e}} = I_i \frac{e}{kT_e} \operatorname{sech}\left(\frac{eV}{2kT_e}\right) \Big|_{V \gg 0}$$

$$= I_i \frac{e}{kT_e} \frac{2}{e^{\frac{eV}{kT_e}} + e^{-\frac{eV}{kT_e}}} \Big|_{V \gg 0}$$

$$= \frac{e}{2kT_e} I_i$$

3 Triple probe. (Current mode)

Voltage mode:



$$T_e = \frac{1}{4} n_{e0} \bar{v}_e$$

$$\bar{v}_e = \sqrt{\frac{8kT_e}{\pi m}}$$

$$I_{e0} = \frac{1}{4} n_{e0} e \bar{v}_e A$$

$$= \frac{1}{4} n_{e0} e \sqrt{\frac{8kT_e}{\pi m}} A$$

$$I_{i0} = \frac{1}{4} n_{i0} \bar{v}_i$$

$$\bar{v}_i = \sqrt{\frac{8kT_i}{\pi m}}$$

$$I_{i0} = \frac{1}{4} n_{i0} e Z \sqrt{\frac{8kT_i}{\pi m}} A$$

$$I_1 = I_{i0} - I_{e0} \exp\left(e \frac{V_1 - \phi_p}{kT_e}\right)$$

$$I_2 = -I_{i0} + I_{e0} \exp\left(e \frac{V_2 - \phi_p}{kT_e}\right)$$

$$I_3 = I_{i0} - I_{e0} \exp\left(e \frac{V_3 - \phi_p}{kT_e}\right)$$

$$I_{i0} = \frac{1}{4} n_{i0} e Z \sqrt{\frac{8kT_i}{\pi m}} \cdot A$$

$$I_{e0} = \frac{1}{4} n_{e0} e \sqrt{\frac{8kT_e}{\pi m}} \cdot A$$

$$U_1 = V_2 - V_1, \quad U_3 = V_3 - V_2$$

$$I_1 + I_2 = I_{e0} \left[\exp\left(e \frac{V_2 - \phi_p}{kT_e}\right) - \exp\left(e \frac{V_1 - \phi_p}{kT_e}\right) \right]$$

$$I_3 + I_2 = I_{e0} \left[\exp\left(e \frac{V_2 - \phi_p}{kT_e}\right) - \exp\left(e \frac{V_3 - \phi_p}{kT_e}\right) \right]$$

Measure I_1, I_2, I_3 with given U_1, U_2

$\Rightarrow T_e$ is solved \Leftarrow

$$= \frac{1 - \exp\left(-e \frac{V_2 - V_1}{kT_e}\right)}{1 - \exp\left(e \frac{V_3 - V_2}{kT_e}\right)} = \frac{1 - \exp\left(-\frac{eU_1}{kT_e}\right)}{1 - \exp\left(\frac{eU_3}{kT_e}\right)}$$

$$I_1 \exp\left(e \frac{V_3}{kT_e}\right) = I_{I_0} \exp\left(e \frac{V_3}{kT_e}\right) - I_{e0} \exp\left(e \frac{V_1 + V_3 - \phi}{kT_e}\right) \quad \text{P18}$$

$$\rightarrow I_3 \exp\left(e \frac{V_1}{kT_e}\right) = I_{I_0} \exp\left(e \frac{V_1}{kT_e}\right) - I_{e0} \exp\left(e \frac{V_1 + V_3 - \phi}{kT_e}\right)$$

$$I_1 \exp\left(e \frac{V_3}{kT_e}\right) - I_3 \exp\left(e \frac{V_1}{kT_e}\right) = I_{I_0} \left[\exp\left(e \frac{V_3}{kT_e}\right) - \exp\left(e \frac{V_1}{kT_e}\right) \right]$$

$$I_1 \exp\left[e \frac{V_3 - V_1}{kT_e}\right] - I_3 = I_{I_0} \left[\exp\left(\frac{e(V_3 - V_1)}{kT_e}\right) - 1 \right]$$

$$V_3 = V_2 + U_3 \quad \Rightarrow \quad V_3 - V_1 = U_1 + U_3 \equiv \Delta U$$

$$V_1 = V_2 - U_1$$

$$I_{I_0} = \frac{I_1 \exp\left(e \frac{\Delta U}{kT_e}\right) - I_3}{\exp\left[e \frac{\Delta U}{kT_e}\right] - 1} = \frac{I_1 - I_3 \exp\left(-\frac{e\Delta U}{kT_e}\right)}{1 - \exp\left(-\frac{e\Delta U}{kT_e}\right)}$$

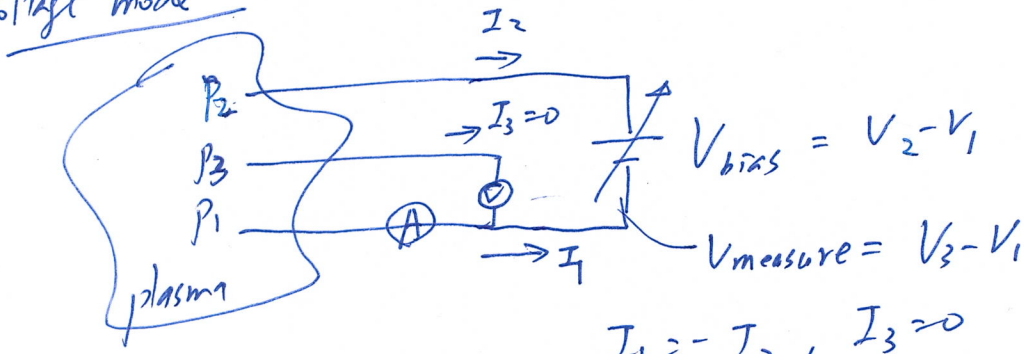
By measuring I_1, I_2, I_3 , with given U_1 & U_3 , T_e , & $I_{I_0} \rightarrow I_0$ can be obtained directly.

$$\frac{I_1 + I_2}{I_3 + I_2} \equiv \Pi = \frac{1 - \exp\left(-\frac{eU_1}{kT_e}\right)}{1 - \exp\left(-\frac{eU_3}{kT_e}\right)} \Rightarrow \text{solve for } T_e$$

~~$$\Pi \left[\Pi \exp\left(\frac{eU_3}{kT_e}\right) - 1 - \exp\left(-\frac{eU_1}{kT_e}\right) \right]$$~~

Solve for I_{I_0}
 \downarrow
 No

Voltage mode



$$I_1 = -I_2, I_3 = 0$$

$$V_i - \phi_p \ll 0$$

$$\tilde{I}_{e0} = I_{e0} \exp\left(-\frac{e\phi_p}{kT_e}\right)$$

Chamber wall

$$\begin{cases} I_1 = I_{s0} - I_{e0} \exp\left(e \frac{V_1 - \phi_p}{kT_e}\right) = I_{s0} - \tilde{I}_{e0} \exp\left(e \frac{V_1}{kT_e}\right) \quad \text{--- 1} \\ I_2 = I_{s0} - I_{e0} \exp\left(e \frac{V_2 - \phi_p}{kT_e}\right) = I_{s0} - \tilde{I}_{e0} \exp\left(e \frac{V_2}{kT_e}\right) \quad \text{--- 2} \\ I_3 = I_{s0} - I_{e0} \exp\left(e \frac{V_3 - \phi_p}{kT_e}\right) = I_{s0} - \tilde{I}_{e0} \exp\left(e \frac{V_3}{kT_e}\right) \quad \text{--- 3} \end{cases}$$

From (3) and $I_3 = 0$

$$I_{s0} - \tilde{I}_{e0} \exp\left(e \frac{V_3}{kT_e}\right) = 0$$

$$\Rightarrow \tilde{I}_{e0} = I_{s0} \exp\left(-e \frac{V_3}{kT_e}\right) \rightarrow \text{into D8 (2)}$$

$$I_1 = I_{s0} - I_{s0} \exp\left(-e \frac{V_3 - V_1}{kT_e}\right) = I_{s0} \left[1 - \exp\left(-e \frac{V_3 - V_1}{kT_e}\right) \right] \quad \text{--- 4}$$

$$I_2 = I_{s0} - I_{s0} \exp\left(-e \frac{V_3 - V_2}{kT_e}\right) = I_{s0} \left[1 - \exp\left(-e \frac{V_3 - V_2}{kT_e}\right) \right]$$

$$\frac{I_1}{I_2} = -1 = \frac{1 - \exp\left(-e \frac{V_3 - V_1}{kT_e}\right)}{1 - \exp\left(-e \frac{V_3 - V_2}{kT_e}\right)} = \frac{1 - \exp\left(-e \frac{V_3 - V_1}{kT_e}\right)}{1 - \exp\left(-e \frac{V_3 - V_1}{kT_e}\right) \exp\left(e \frac{V_2 - V_1}{kT_e}\right)}$$

$$V_3 - V_2 = V_3 - V_1 - (V_2 - V_1) \Rightarrow \frac{\exp\left(e \frac{V_3 - V_1}{kT_e}\right) - 1}{\exp\left(e \frac{V_3 - V_1}{kT_e}\right) - \exp\left(e \frac{V_2 - V_1}{kT_e}\right)}$$

$$\Rightarrow -\exp\left(e \frac{V_3 - V_1}{kT_e}\right) + \exp\left(e \frac{V_2 - V_1}{kT_e}\right) = \exp\left(e \frac{V_3 - V_1}{kT_e}\right) - 1$$

$$2 \cdot \exp\left(e \frac{V_3 - V_1}{kT_e}\right) = \exp\left(e \frac{V_2 - V_1}{kT_e}\right) + 1$$

$$2 \exp\left(e \frac{V_{\text{measure}}}{kT_e}\right) = \exp\left(e \frac{V_{\text{bias}}}{kT_e}\right) + 1$$

For a given V_{bias} & measured V_{measure} .

$\Rightarrow T_e$ can be obtained by solving the above eq.

$$\begin{aligned} \text{From (4): } I_1 &= I_{i0} \left[1 - \exp\left(-e \frac{V_3 - V_1}{kT_e}\right) \right] \\ &= I_{i0} \left[1 - \exp\left(-e \frac{V_{\text{measure}}}{kT_e}\right) \right] \end{aligned}$$

$$I_{i0} = \frac{I_1}{1 - \exp\left(-e \frac{V_{\text{measure}}}{kT_e}\right)}$$

$$= \frac{1}{4} n_0 e \sqrt{\frac{8kT_i}{\pi m}} \cdot A$$

$$\equiv \frac{1}{4} n_0 e \sqrt{\frac{8kT_e}{\pi m}} \cdot A \quad (\text{where } T_e = T_i)$$

$$\Rightarrow n_0 = \frac{4 \cdot I_0}{e \cdot A} \sqrt{\frac{\pi m}{8kT_e}} \frac{1}{1 - \exp\left(-e \frac{V_{\text{measure}}}{kT_e}\right)}$$