Practice Course in Plasma

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Thursday 9:10-12:00

Material: http://capst.ncku.edu.tw/PGS/index.php/teaching/

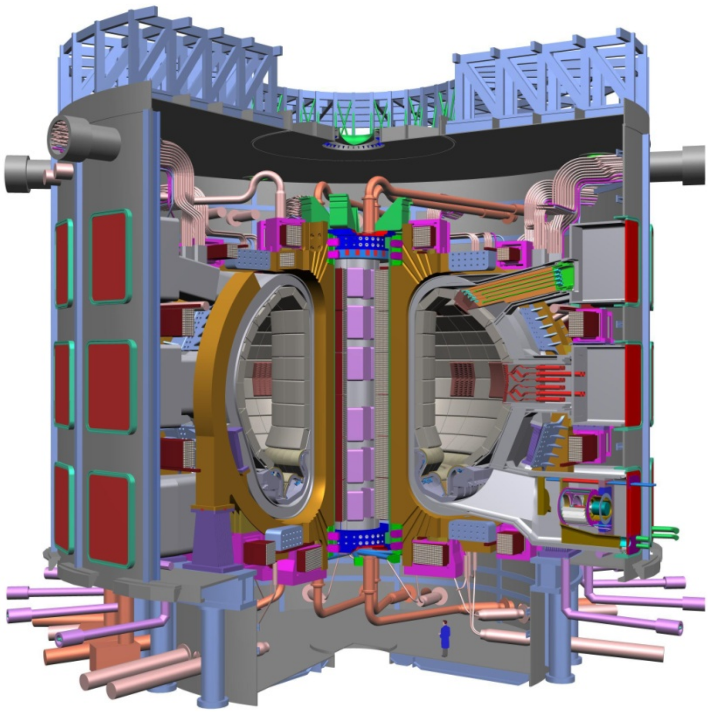
Lecture 2

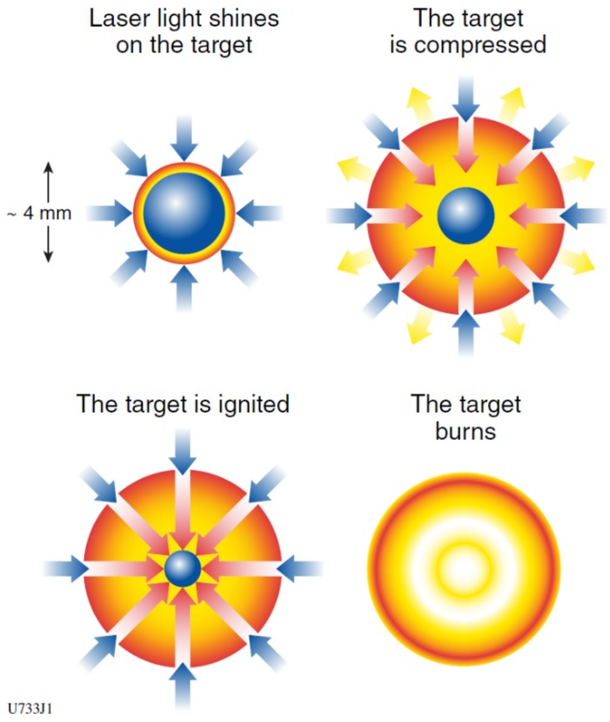
2021/3/4 updated

Reference

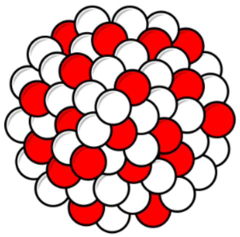
- **Riccardo Betti, University of Rochester, HEDSA HEDP summer school, San Diego, CA, August 16-21, 2015.**
- **Inertial Confinement Fusion, R. Betti, Phy558/ME533, University of Rochester.**

To Fuse, or Not to Fuse...





The "iron group" of isotopes are the most tightly bound



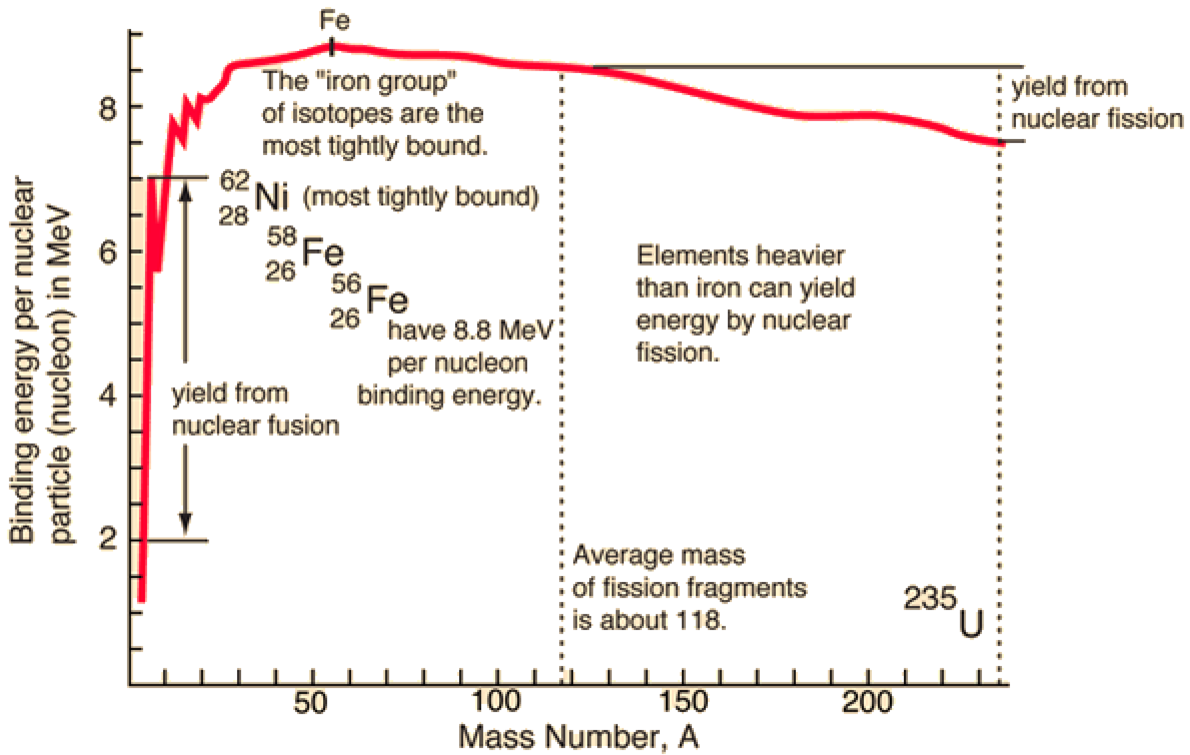
Nucleus

= i  + j 

Proton Neutron

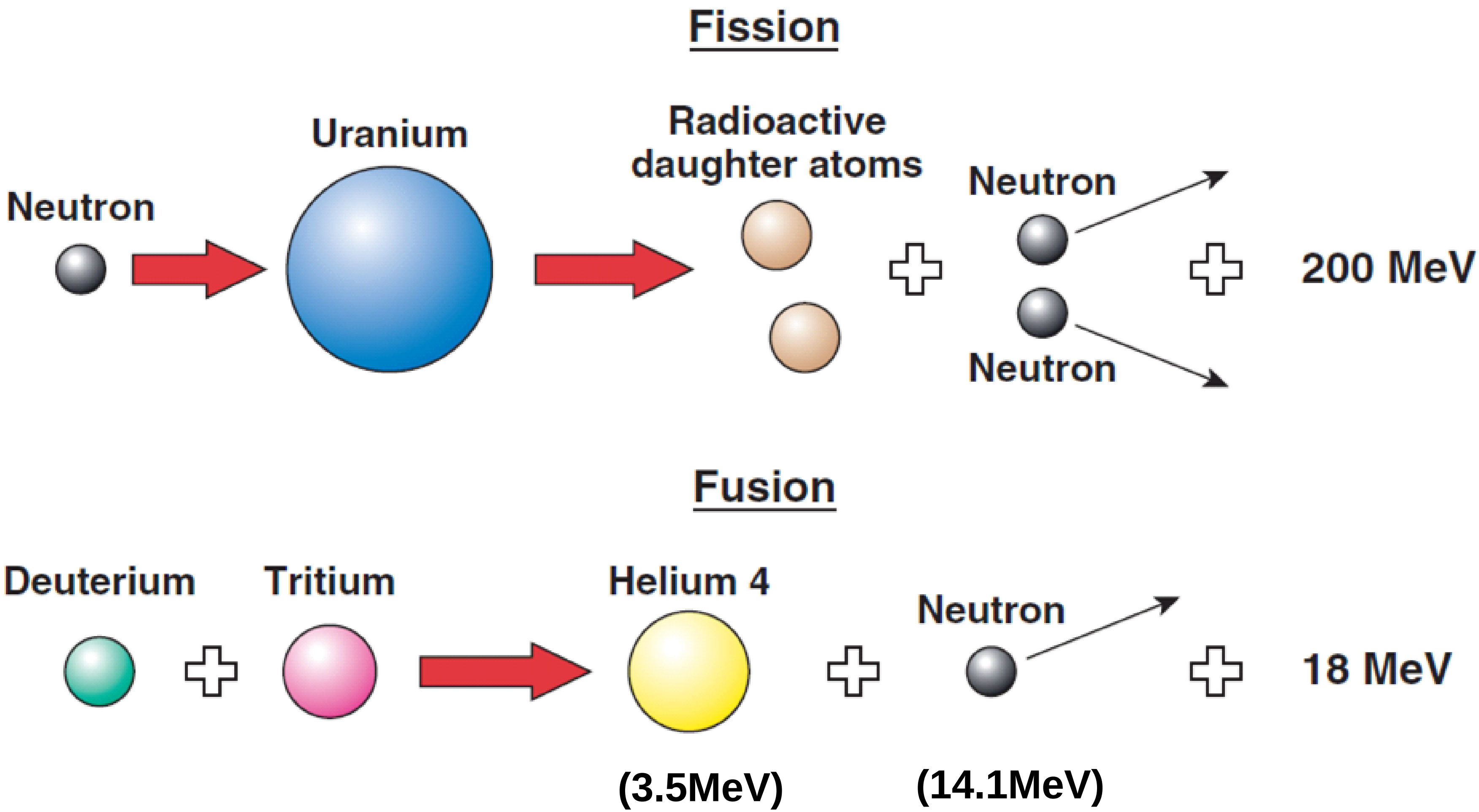
$$M_{Nucleus} < i \times M_{proton} + j \times M_{neutron}$$

$$E_{binding} = (i \times M_{proton} + j \times M_{neutron} - M_{nucleus}) C^2$$



http://hyperphysics.phy-astr.gsu.edu/hbase/nucene/nucbin.html

Nuclear fusion and fission release energy through energetic neutrons



Nuclear fusion provides more energy per atomic mass unit (amu) than nuclear fission

Fusion of $^2H + ^3H$: $$\frac{Q}{A} = \frac{17.6\ MeV}{(3+2)\ amu} = 3.5\ \frac{MeV}{amu}$$

Fission of ^{235}U: $$\frac{Q}{A} = \frac{200\ MeV}{236\ amu} = 0.85\ \frac{MeV}{amu}$$

Source	Energy density
Nuclear Fusion (50% D + 50% T)	**5.4×10^{14} J/kg**
Nuclear Fission (5% ^{235}U + 95% ^{238}U)	**1.5×10^{18} J/m^3 8×10^{13} J/kg**

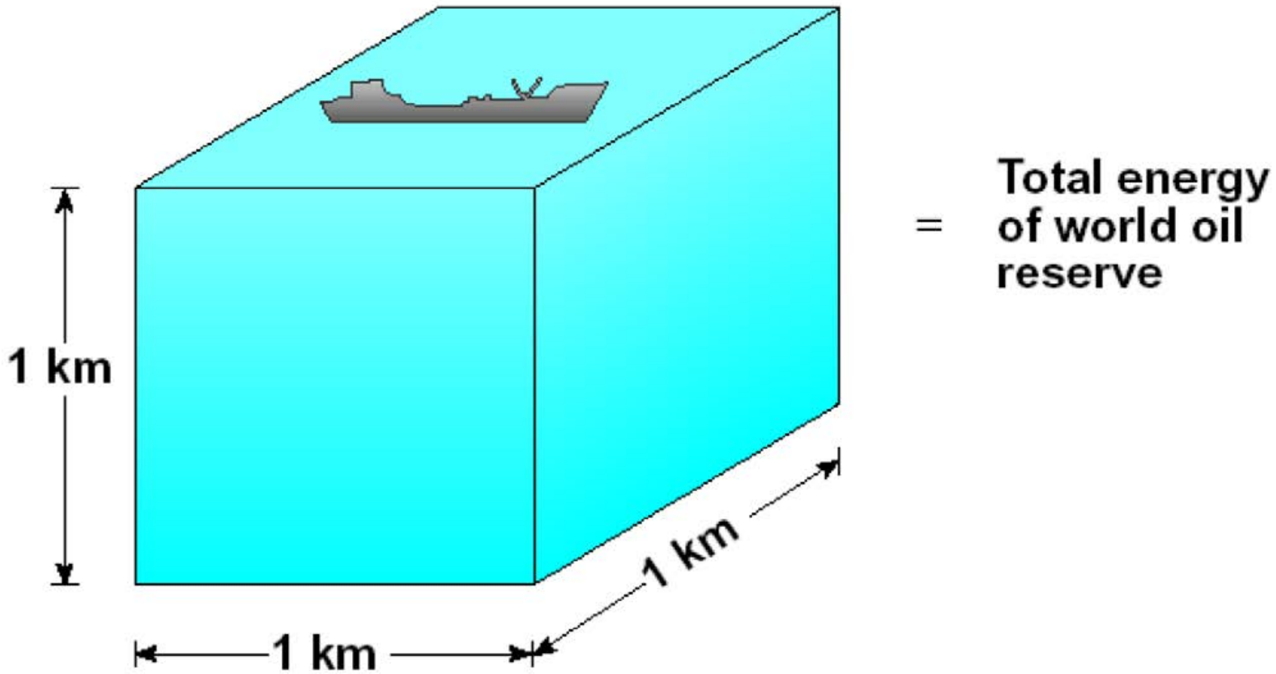
	Half-life (years)
U235	**7.04×10^8**
U238	**4.47×10^9**
…	
Tritium	**12.3**

What could you do with 1 kg DT?

- **1 kg DT -> 340 Tera joules**
  - **You can drive your car for ~40,000 km (back and forth between Keelung and Kaoshiung for 50 times).**
  - **You can keep your furnace running for 8 years.**
  - **You can blow things up! 1 TJ = 250 tons of TNT.**

*R. Betti, HEDSA HEDP Summer School, 2015

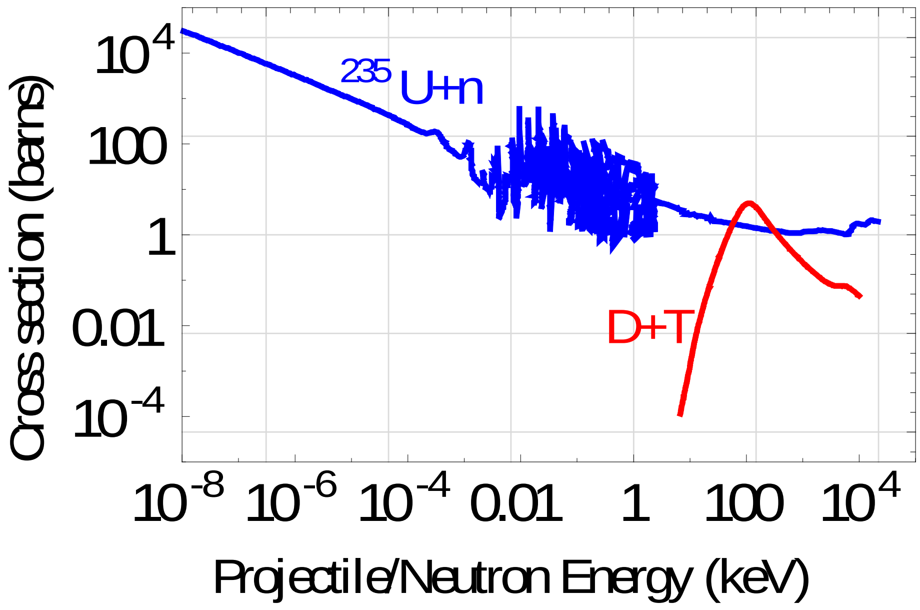
Enormous fusion fuel can be produced from sea water



*R. Betti, HEDSA HEDP Summer School, 2015

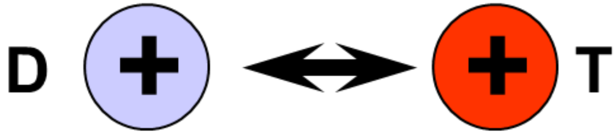
Fusion is much harder than fission

- **Fission:** $n +^{235}_{92} U \rightarrow^{236}_{92} U \rightarrow^{144}_{56} Ba +^{89}_{36} Kr + 3n + 177 \text{ MeV}$
- **Fusion:** $D + T \rightarrow He^4\,(3.5 \text{ MeV}) + n\,(14.1 \text{ MeV})$

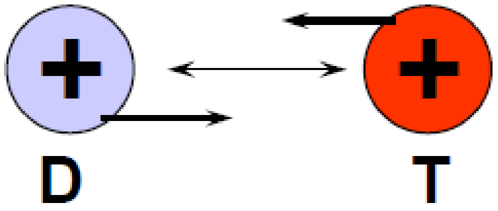


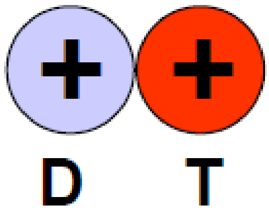
A “hot plasma” at 100M ^{0}C is needed

- **Probability for fusion reactions to occur is low at low temperatures due to the coulomb repulsion force.**



- **If the ions are sufficiently hot, i.e., large random velocity, they can collide by overcoming coulomb repulsion**



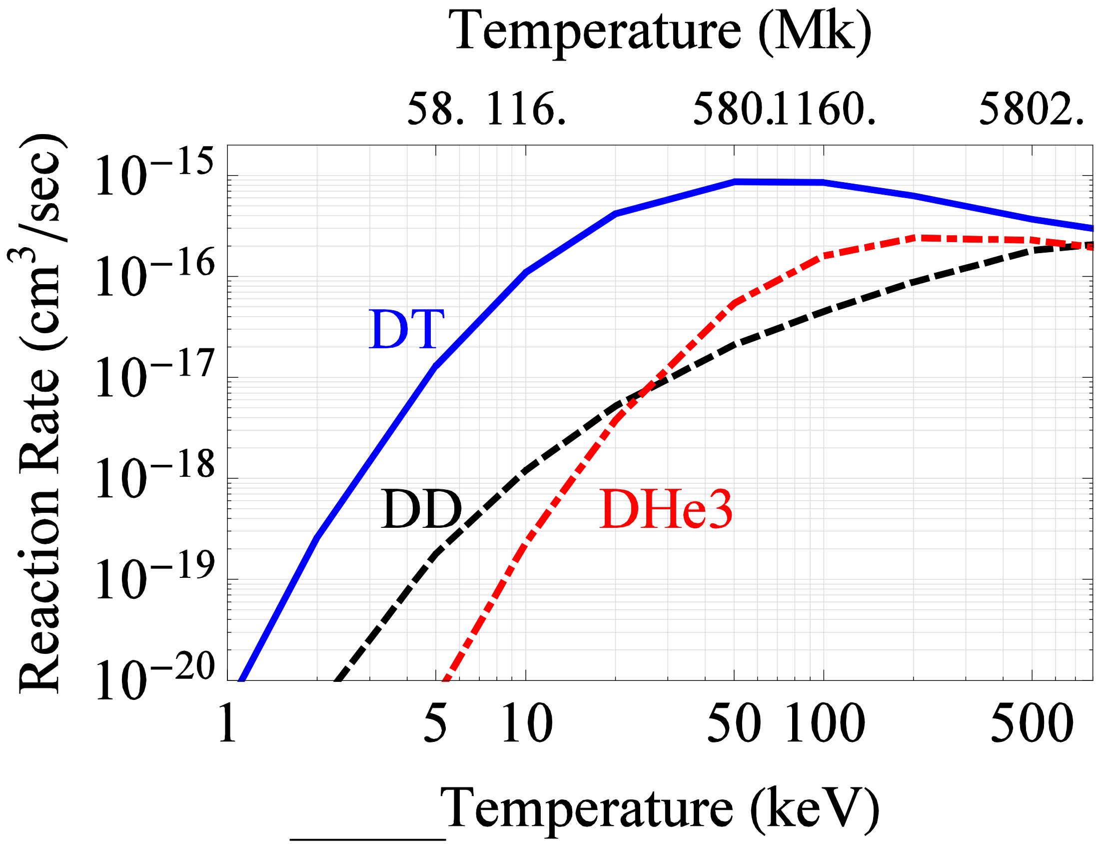


*R. Betti, HEDSA HEDP Summer School, 2015

Fusion doesn't come easy

averaged reaction rate : $\langle \sigma v \rangle = \int \int d\vec{v}_1 d\vec{v}_2 \sigma_{1,2}(v)\, v f_1(v_1)$

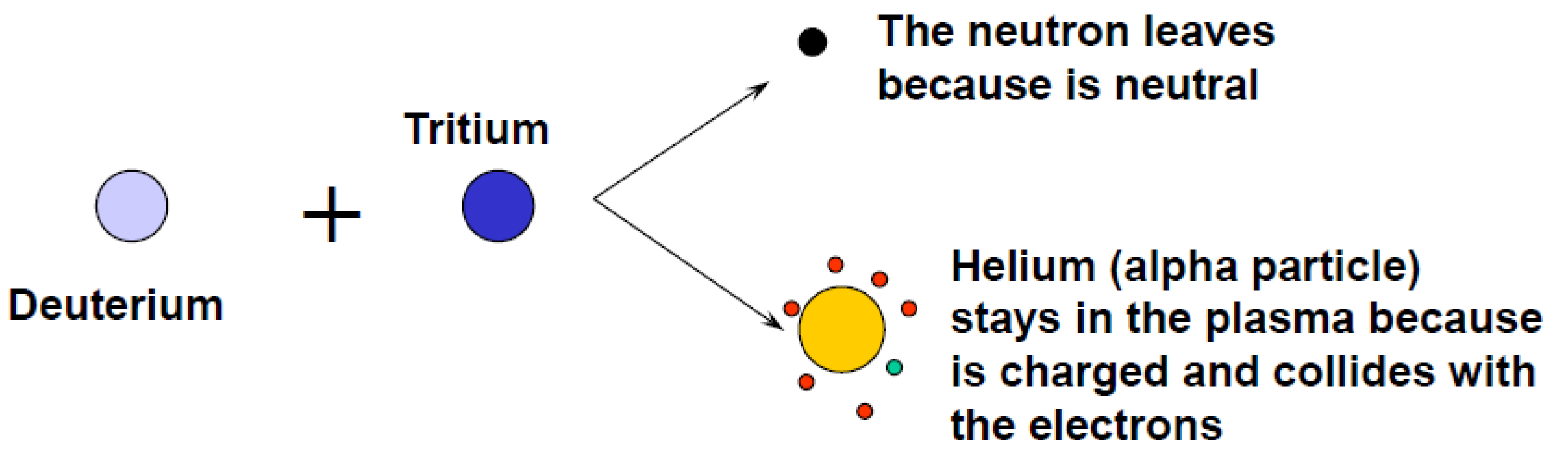
$$f_j(v_j) = \left(\frac{m_j}{2\pi k_B T} \right)^{3/2} \exp\left(-\frac{m_j v_j^2}{2 k_B T} \right)$$



*NRL Plasma Formulary, Naval Research Laboratory, Washington, DC 203785-5320

It takes a lot of energy or power to keep the plasma at 100M ^{0}C

- **Let the plasma do it itself!**



- **The α-particles heat the plasma.**

*R. Betti, HEDSA HEDP Summer School, 2015

Under what conditions the plasma keeps itself hot?

- **Steady state 0-D power balance:**

$S_\alpha + S_h = S_B + S_k$

S_α: α particle heating $\quad S_\alpha = \frac{1}{4} E_\alpha n^2 <\sigma v> = \frac{1}{16} E_\alpha p^2 \frac{<\sigma v>}{T^2}$

S_h: external heating

S_B: Bremsstrahlung radiation $\quad S_B = \frac{1}{4} C_B Z_{eff} \frac{p^2}{T^{3/2}}$

S_k: heat conduction lost $\quad S_\kappa = \frac{3}{2} \frac{p}{\tau}$

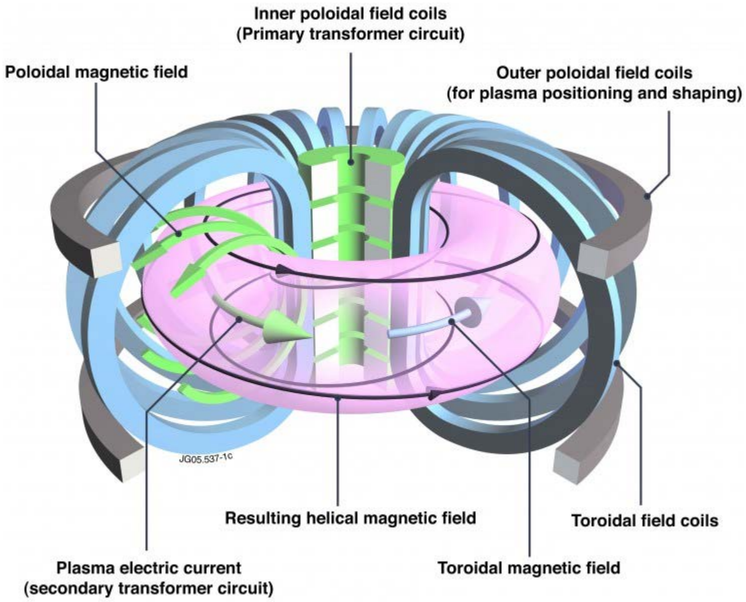
Ignition condition: Pτ > 10 atm-s = 10 Gbar - ns

- P: pressure, or called energy density
- τ is confinement time

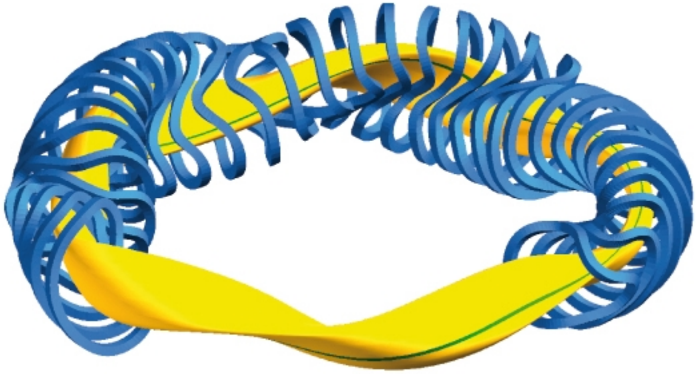
The plasma is too hot to be contained

- **Solution 1: Magnetic confinement fusion (MCF), use a magnetic field to contain it. P~atm, τ~sec, T~10 keV**

Tokamak



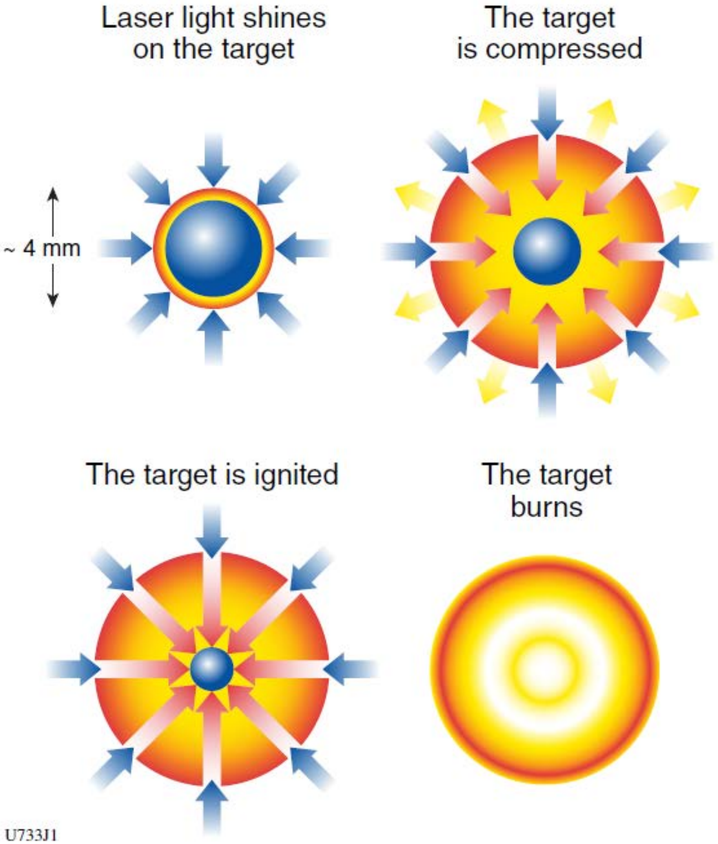
Stellarator



https://www.euro-fusion.org/2011/09/tokamak-principle-2/
https://en.wikipedia.org/wiki/Stellarator

Don’t confine it!

- **Solution 2: Inertial confinement fusion (ICF). Don’t confine it! Or you can say it is confined by its own inertia: P~Gigabar, τ~nsec, T~10 keV**

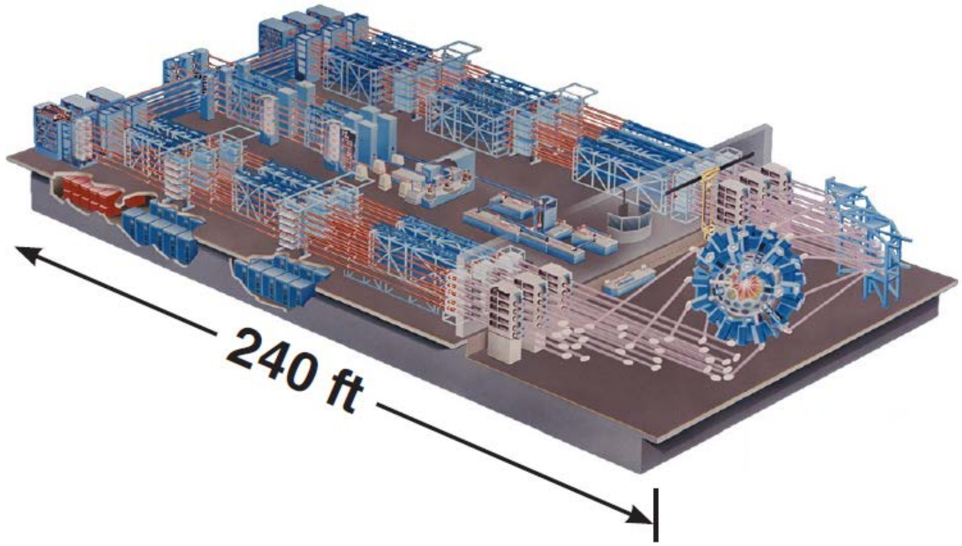


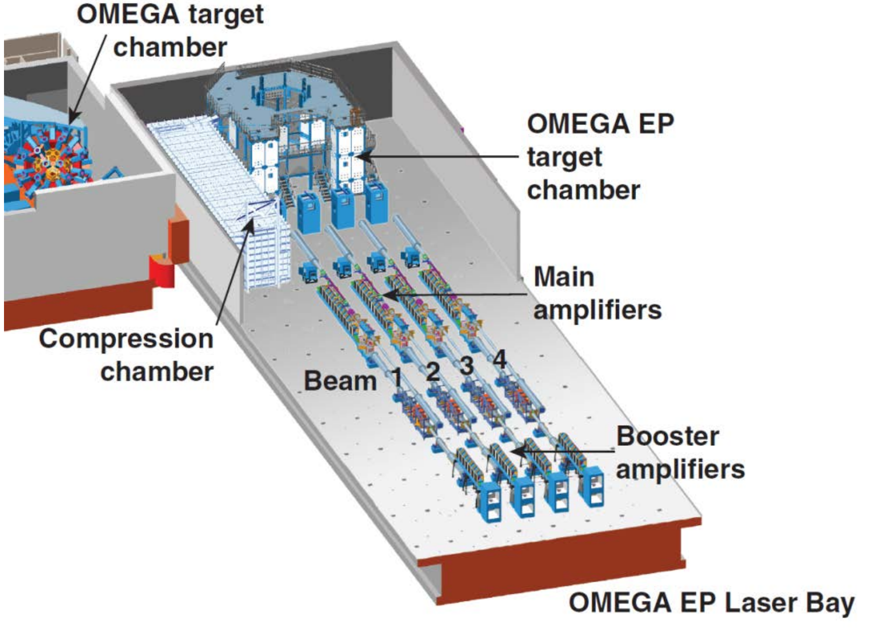
High pressures and temperatures required for inertial fusion can be achieved through laser-driven spherical implosions of a thin shell

- **Achieve extreme states of matter of interest for ICF and general high energy density plasma (HEDP)**
- **High temperature (10-100 keV)**
- **High densities (~300-1000g/cc)**
- **High pressures (Gbar-Tbar, ~10^{9-12} atm)**
- **High areal densities (ρR) (~1-3 g/cm^2)**

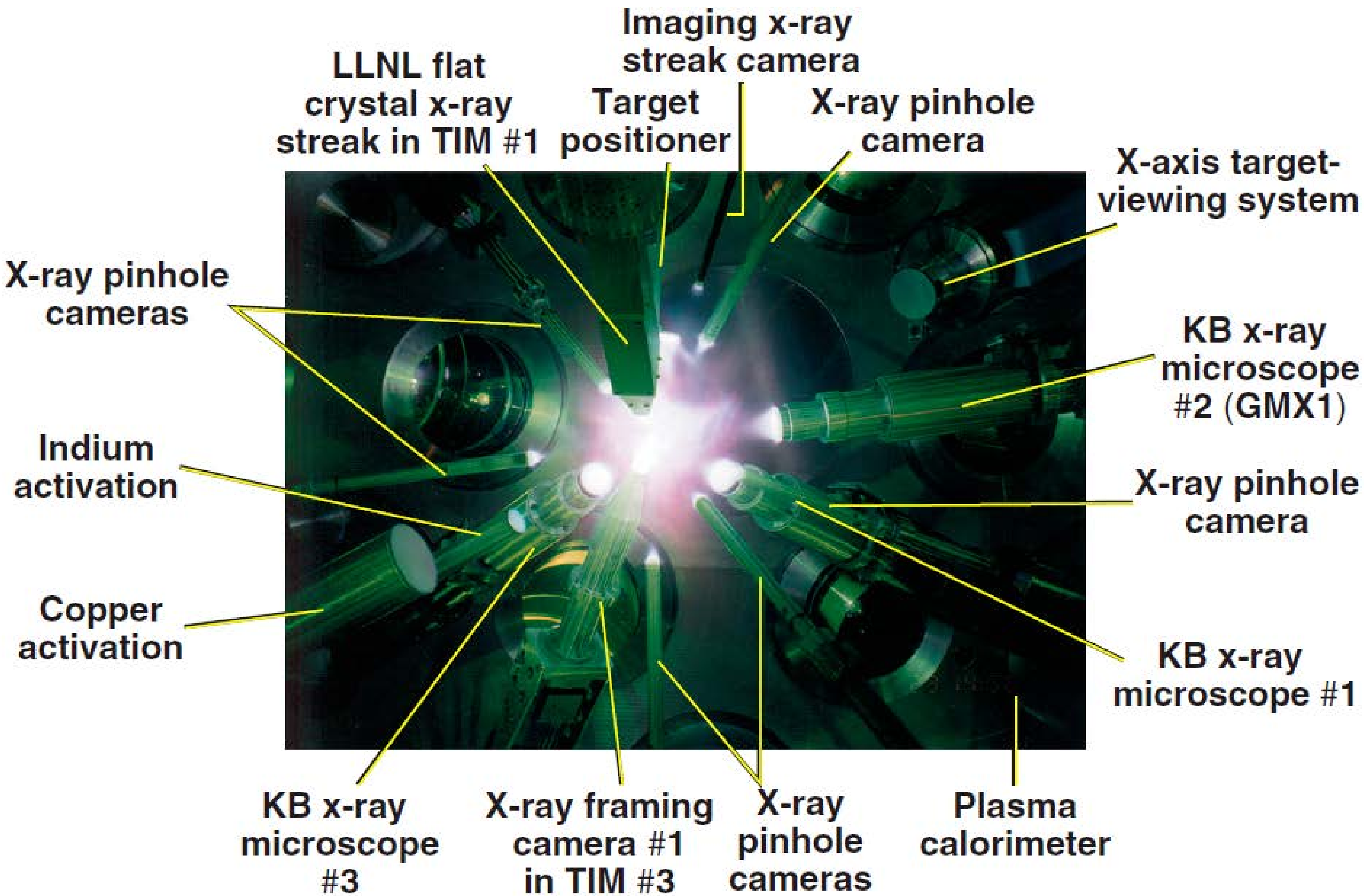
Laboratory for Laser Energetics, University of Rochester is a pioneer in laser fusion

- OMEGA Laser System
  - 60 beams
  - >30 kJ UV on target
  - 1%~2% irradiation nonuniformity
  - Flexible pulse shaping
- OMEGA EP Laser System
  - 4 beams; 6.5 kJ UV (10ns)
  - Two beams can be high-energy petawatt
    - 2.6 kJ IR in 10 ps
    - Can propagate to the OMEGA or OMEGA EP target chamber





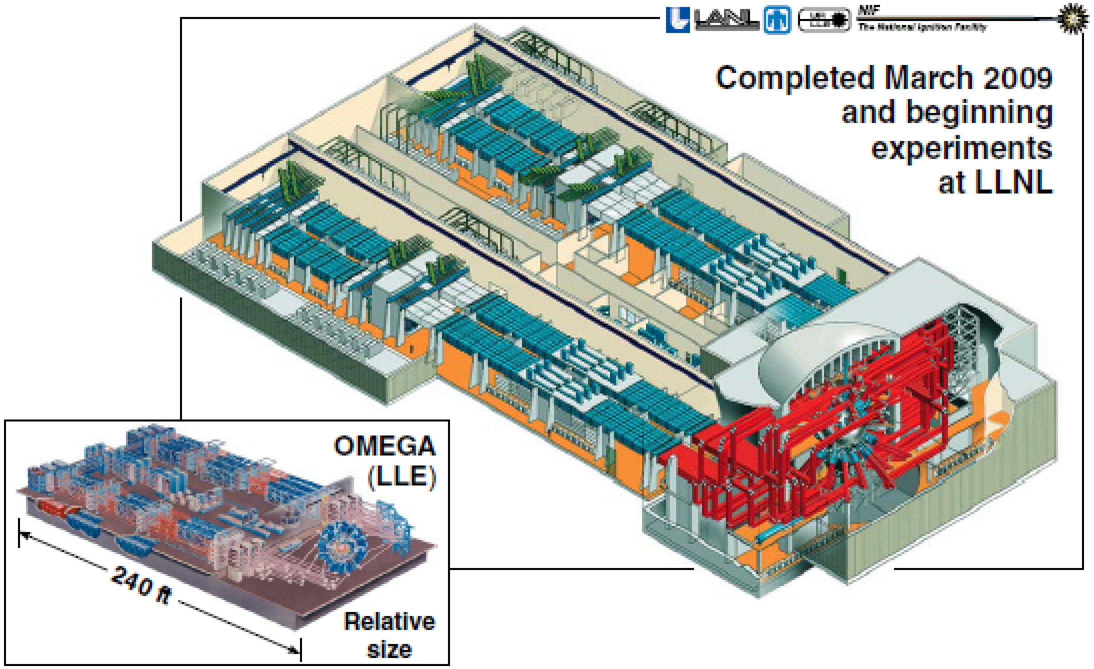
The OMEGA Facility is carrying out ICF experiments using a full suite of target diagnostics



E8012b

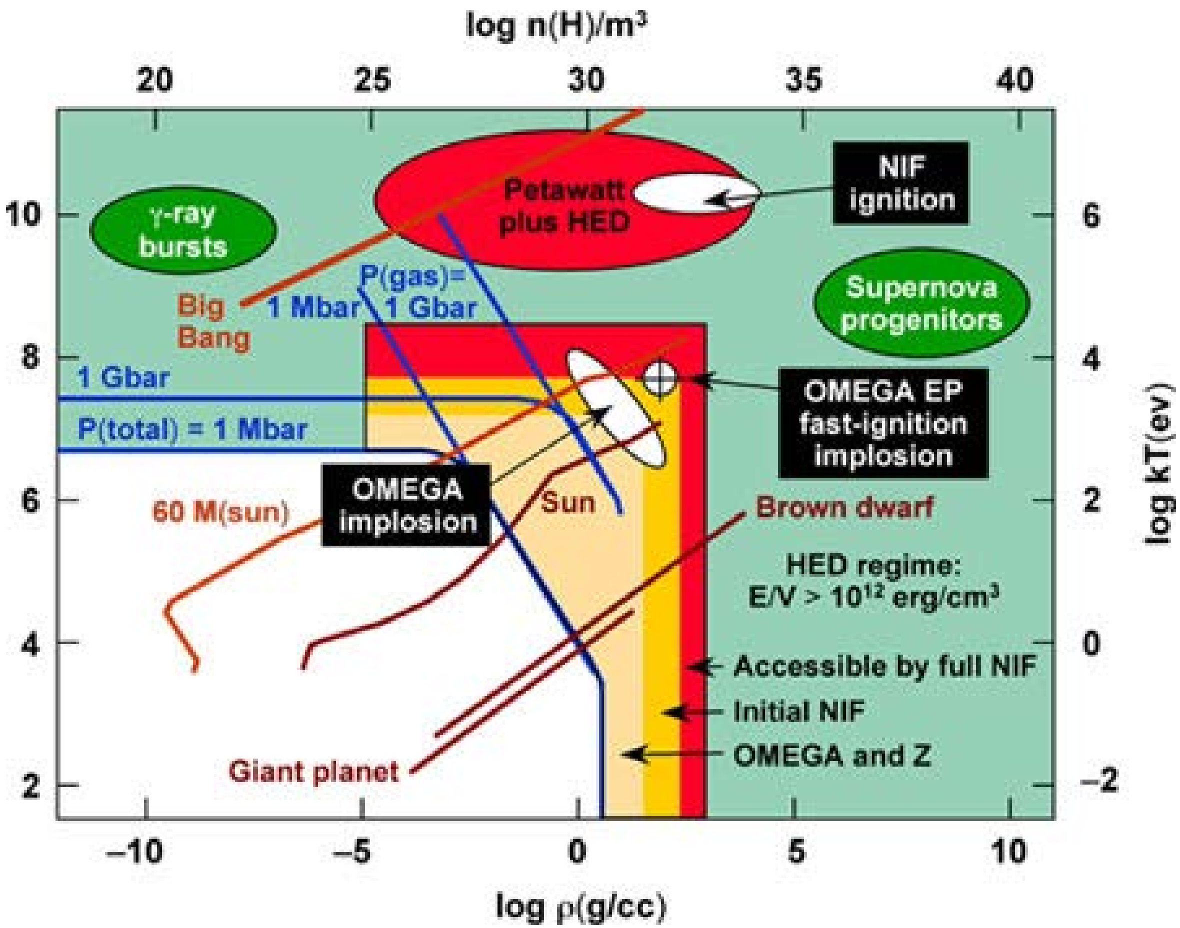
Photo taken from port H11B

The 1.8-MJ National Ignition Facility (NIF) will demonstrate ICF ignition and modest energy gain



OMEGA experiments are integral to an ignition demonstration on the NIF.

High energy density plasma is the regime that p > 1 Mbar



http://fsc.lle.rochester.edu/hedp.php

Conservation equations of gas-dynamics and ideal gas EOS are used for DT plasma

mass conservation : $\partial_t \rho + \partial_x (\rho \vec{v}) = 0$

momentum conservation : $\partial_t (\rho \vec{v}) + \partial_x \left(p + \rho v^2 \right) = \vec{F}$

energy conservation : $\partial_t \varepsilon + \partial_x \left[\vec{v} (\varepsilon + p) - \kappa \partial_x T \right] = \text{sources} + \text{sinks}$

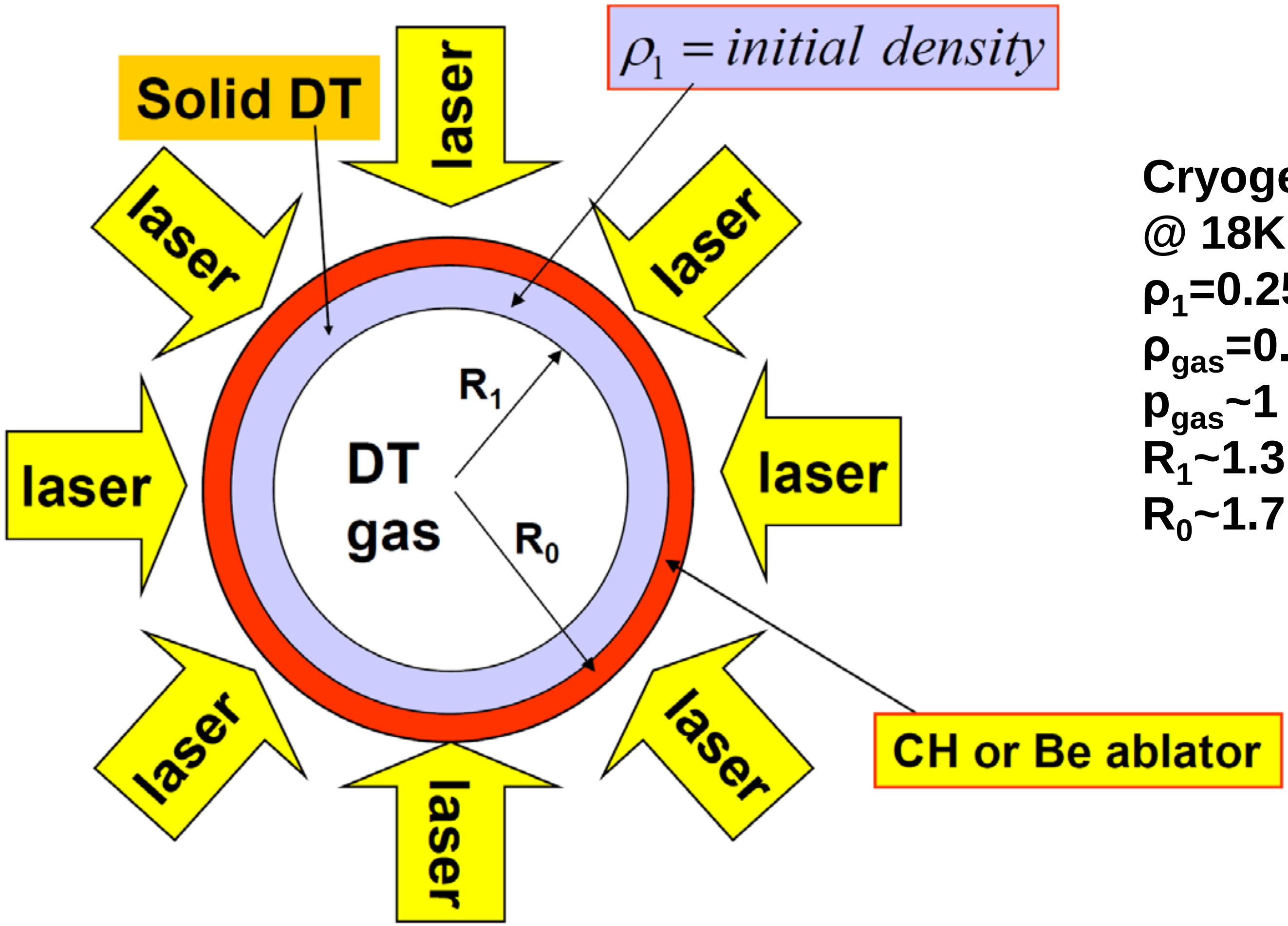
ideal gas EOS : $p = \text{pressure} = (n_\mathrm{e} T_\mathrm{e} + n_\mathrm{i} T_\mathrm{i}) = 2nT = \frac{2}{m_\mathrm{i}} \rho_\mathrm{i} T$

Total energy per unit volume: $$\varepsilon = \frac{3}{2} p + \rho \frac{v^2}{2}$$

Mass density: $\rho = n_\mathrm{i} m_\mathrm{i}$

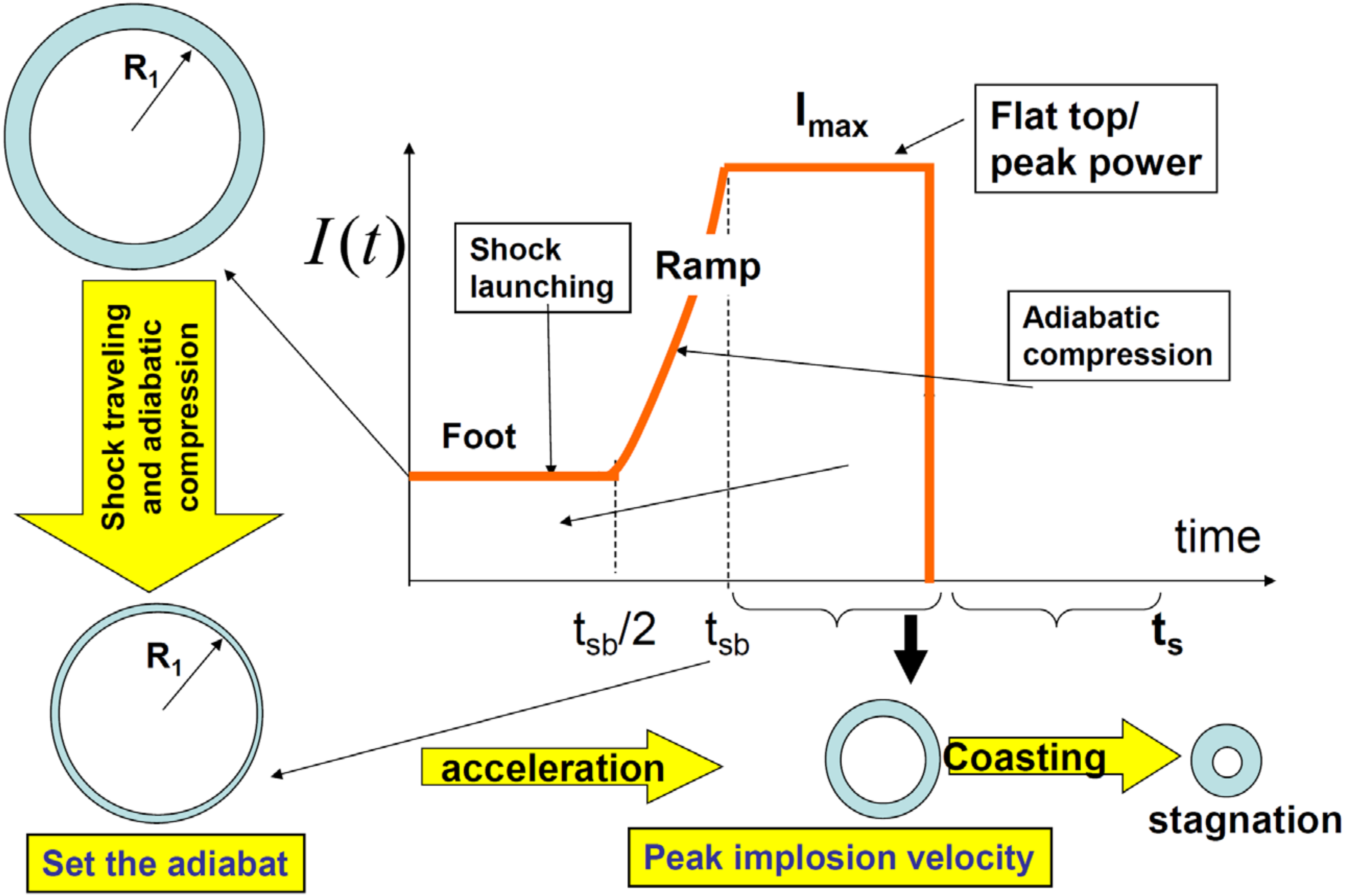
Plasma thermal conductivity: κ

Laser-driven imploding capsules are mm-size shells with hundreds of μm thick layers of cryogenic solid DT

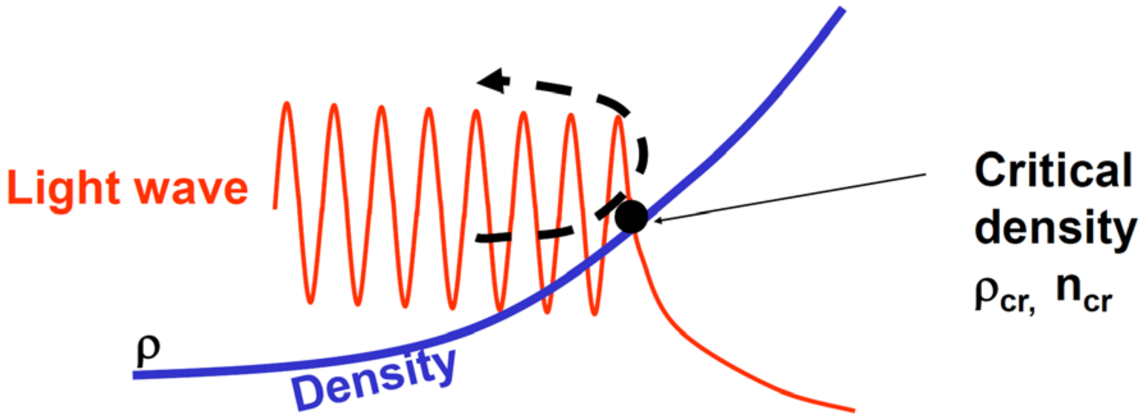


Cryogenic solid DT ice @ 18K
ρ_1=0.25 g/cc
ρ_{gas}=0.3-0.6 mg/cc
p_{gas}~1 atm
R_1~1.3 mm
R_0~1.7 mm

There are three stages in the laser pulse: foot, ramp, and flat top



The laser light cannot propagate past a critical density

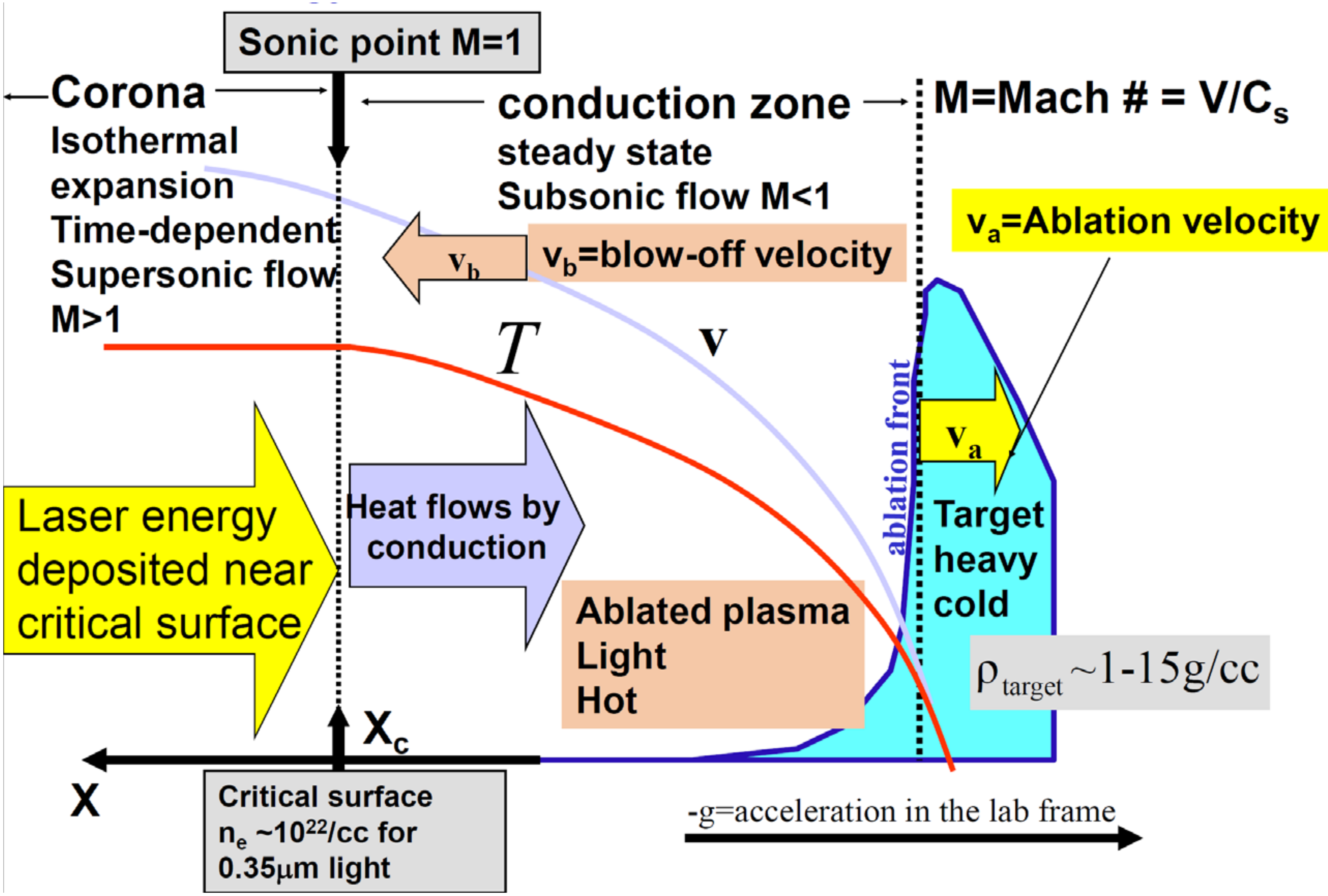


- **Critical density is given by plasma frequency=laser frequency**

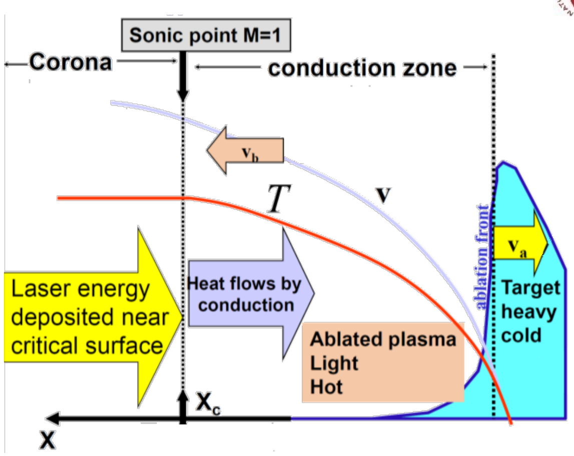
$$\omega_L = \frac{2\pi c}{\lambda_L} \qquad \omega_{pe} = \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

$$\omega_L^2 = \omega_{pe}^2 \qquad n_e^{cr} = \frac{1.1 \times 10^{21}}{\lambda_{L,\mu m}{}^2} \mathrm{cm}^{-3}$$

The laser generates a pressure by depositing energy at the critical surface



Pressure generated by a laser is obtained using energy conservation equation



- **Energy conservation equation:**

$$\partial_t \epsilon + \partial_x[\vec{v}\,(\epsilon + p) - \kappa \partial_x T] = \underbrace{\mathrm{I}\delta(x - x_{cr})}_{\text{w/cm}^2\text{s -1/cm}}$$

- **Since the temperature gradients are small in the corona, the heat flux is small**

$$\kappa \partial_x T(x \geq x_{cr}) << \kappa \partial_x T(x \leq x_{cr}) \qquad \left(\kappa \partial_x T(x \geq x_{cr}) \approx \frac{1}{3} \kappa \partial_x T(x \leq x_{cr})\right)$$

- **Integrate around critical surface x_c**

$$\int_{x_{cr}^-}^{x_{cr}^+} \{\partial_t \epsilon + \partial_x[\vec{v}\,(\epsilon + p) - \kappa \partial_x T]\}\, \mathrm{d}x = \int_{x_{cr}^-}^{x_{cr}^+} \{\mathrm{I}\delta(x - x_{cr})\}\, \mathrm{d}x$$

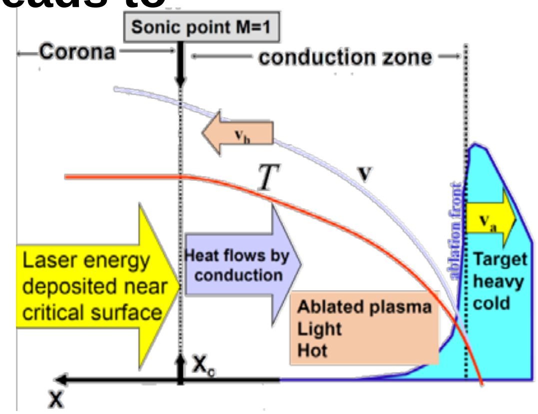
$$\partial_t \epsilon \mathrm{x}\big|_{x_{cr}^-}^{x_{cr}^+} + [\vec{v}\,(\epsilon + p)]_{x_{cr}^-}^{x_{cr}^+} - [\kappa \partial_x T]_{x_{cr}^-}^{x_{cr}^+} = I$$

$$-[\kappa \partial_x T]_{x_{cr}^-}^{x_{cr}^+} = I$$

Laser produced ablation pressure

$$\partial_t \epsilon + \partial_x[\vec{v}(\epsilon + p) - \kappa \partial_x T] = I\delta(x - x_{cr})$$

- Solving at steady state in the conduction zone ($x<x_c$) leads to



$$v(\varepsilon + p) \sim \kappa \partial_x T \qquad \text{for } x \leq x_{cr}^{-}$$

- At the sonic point (i.e., critical surface) $C_s \sim \sqrt{p/\rho}$

$$I = [v(\varepsilon + p)]_{x_{cr}^{-}} = C_s\left(\frac{5}{2}p_{cr} + \rho_{cr}\frac{C_s^2}{2}\right) \sim \frac{p_{cr}^{3/2}}{\rho_{cr}^{1/2}}$$

- The total pressure (static+dynamic) is the ablation pressure

$$p_A = [p + \rho v^2]_{x=x_{cr}} = 2p_{cr} \sim \left(I\rho_{cr}^{1/2}\right)^{2/3} \sim \left(\frac{I}{\lambda_L}\right)^{2/3}$$

- The laser-produced total (ablation) pressure on target:

$$p_A(\text{Mbar}) \approx 83\left(\frac{I_{15}}{\lambda_{L,\mu m}/0.35}\right)^{2/3}$$

I_{15}: laser intensity in 10^{15}w/cm^2
$\lambda_{L,\mu m}$: laser wavelength in μm

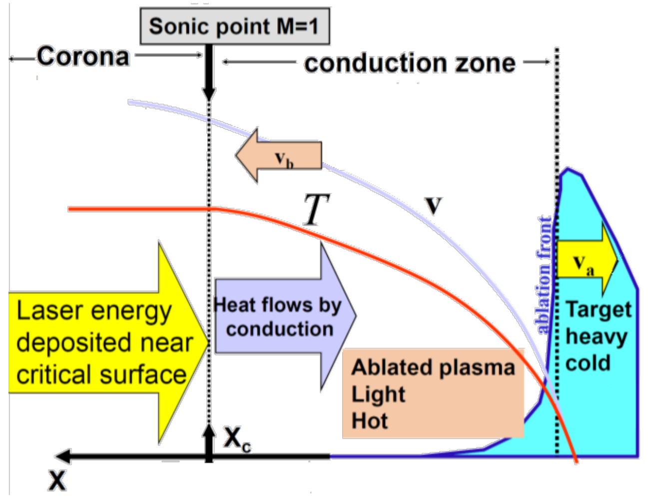
Mass ablation rate induced by the laser

- **At steady state, the mass flow across the critical surface must equal the mass flow off the shell (i.e., the mass ablation rate)**

$$\dot{m}_{\mathrm{a}} = \rho v = \rho_{\mathrm{cr}} v_{\mathrm{cr}} = \rho_{\mathrm{cr}} C_{\mathrm{s}}^{\mathrm{cr}} = \rho_{\mathrm{cr}} \sqrt{\frac{p_{\mathrm{cr}}}{\rho_{\mathrm{cr}}}} = \sqrt{\rho_{\mathrm{cr}} p_{\mathrm{cr}}}$$

$$\rho_{\mathrm{cr}} \sim \frac{1}{\lambda_{\mathrm{L}}^2} \qquad p_{\mathrm{cr}} \sim \left(\frac{I}{\lambda}\right)^{2/3} \qquad \Rightarrow \dot{m}_{\mathrm{a}} = \frac{I^{1/3}}{\lambda_{\mathrm{L}}^{4/3}}$$

$$\dot{m}_{\mathrm{A}} = 3.3 \times 10^5 \frac{I_{15}{}^{1/3}}{\lambda_{\mathrm{L}}{}^{4/3}}\ g/\mathrm{cm}^2\, s$$



Entropy of an ideal gas/plasma

- The entropy S is a property of a gas just like p, T, and ρ

$$S = c_{\rm v} \ln \left[\frac{p}{\rho^{5/3}} \text{const} \right] = c_{\rm v} \ln \alpha \qquad\qquad \alpha = \text{const} \frac{p}{\rho^{5/3}}$$

- α is called the “adiabat”
- The entropy/adiabat S/α changes through dissipation or heat sources/sinks

$$\rho \left(\frac{\partial S}{\partial t} + \vec{u} \cdot \nabla S \right) = \frac{DS}{Dt} = \mu \frac{|\nabla \vec{u}|^2}{T} + \frac{\nabla \cdot \kappa \nabla T}{T} + \text{sources/sinks}$$

- In an ideal gas (no dissipation) and without sources and sinks, the entropy/adiabat is a constant of motion of each fluid element

$$\frac{DS}{dt} = 0 \Rightarrow S\ ,\ \alpha = \text{const} \Rightarrow p \sim \alpha \rho^{5/3}$$

It is easier to compress a low adiabat (entropy) gas

- **Smaller α -> less work to compress from low to high density**

$$W_{1\rightarrow 2} = -\int p dV \sim -\int_{\rho_1}^{\rho_2} \alpha \rho^{5/3} d\left(\frac{M}{\rho}\right) \sim \alpha M \left(\rho_2^{2/3} - \rho_1^{2/3}\right)$$

- **Smaller α -> higher density for the same pressure**

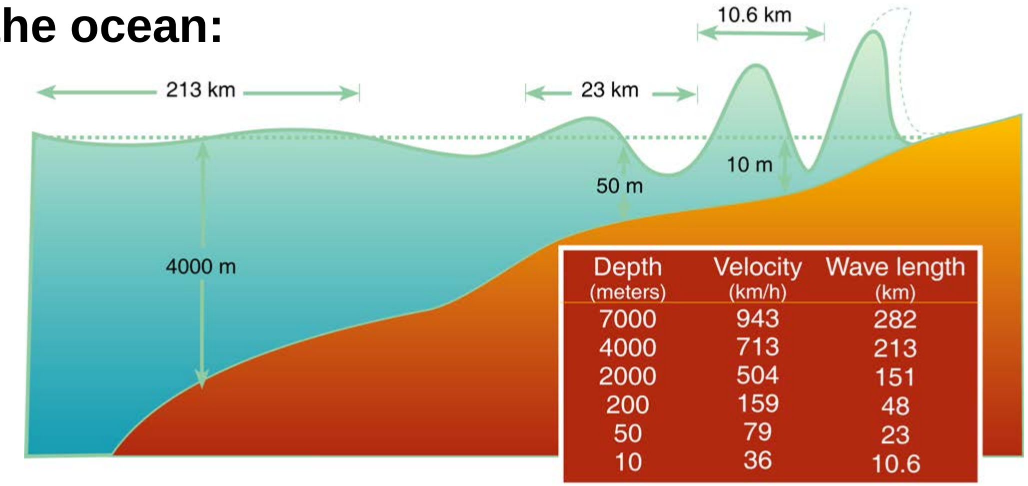
$$\alpha \sim \frac{p}{\rho^{5/3}} \Rightarrow \rho \sim \left(\frac{p}{\alpha}\right)^{3/5}$$

- **In HEDP, the constant in adiabat definition comes from the normalization of the pressure against the Fermi pressure.**
- **When thermal effects are negligible at very high densities, the pressure is proportional to ρ5/3 due to the quantum mechanical effects (degenerate electron gas) just like isentropic flow**

$$\alpha \equiv \frac{p}{p_F} \quad \Rightarrow \alpha_{DT} = \frac{p_{\mathrm{Mbar}}}{2.2\rho_{\mathrm{g/cc}}^{5/3}}$$

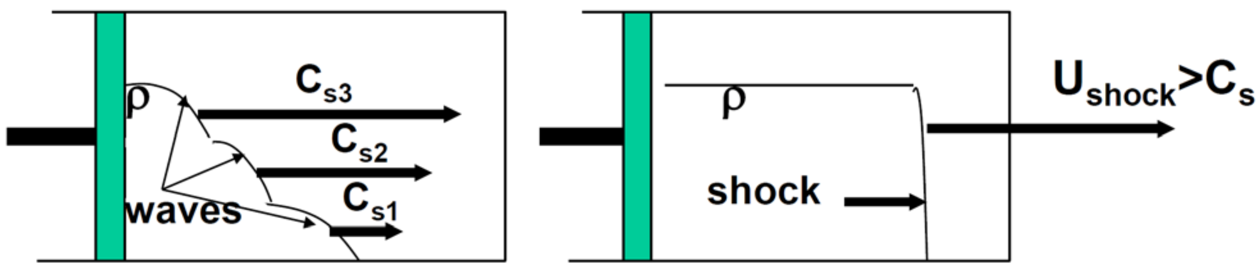
A shock is formed due to the increasing sound speed of a compressed gas/plasma

- **Wave in the ocean:**



Depth (meters)	Velocity (km/h)	Wave length (km)
7000	943	282
4000	713	213
2000	504	151
200	159	48
50	79	23
10	36	10.6

- **Acoustic/compression wave driven by a piston:**



$$C_\mathrm{s} \sim \sqrt{\frac{p}{\rho}} \sim \sqrt{\frac{\alpha\rho^{5/3}}{\rho}} \sim \sqrt{\alpha}\rho^{1/3}$$

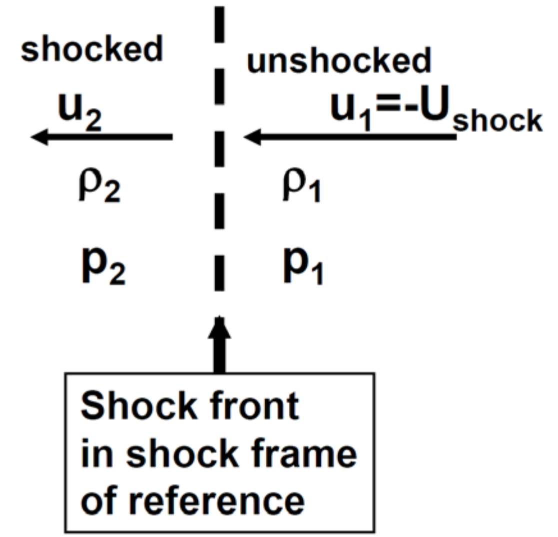
http://neamtic.ioc-unesco.org/tsunami-info/the-cause-of-tsunamis

Rankine-Hugoniot conditions are obtained using conservation of mass, momentum and energy across the shock front

$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$u_1 \left(\varepsilon_1 + p_1\right) = u_2 \left(\varepsilon_2 + p_2\right)$$



- **Ideal gas/plasma:**

$$\varepsilon = \frac{3}{2}p + \rho\frac{u^2}{2}$$

- **With assigned ρ_1, p_1, and p_2, ρ_2, u_2, and u_1=-U_{shock} can be obtained using Rankine-Hugoniont conditions**

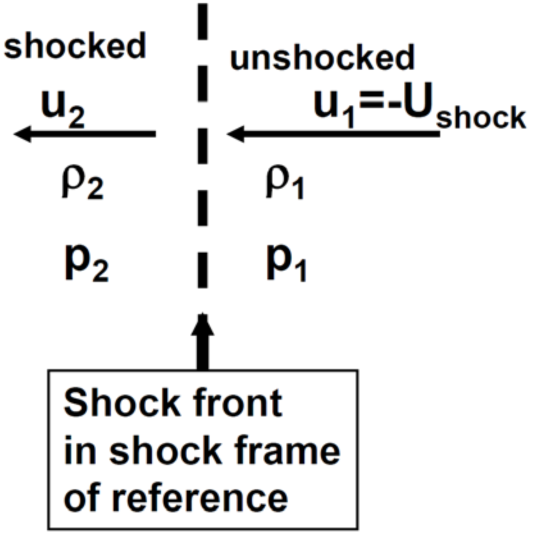
For a strong shock where $p_2 >> p_1$, the R-H conditions are simplified

$$\frac{\rho_2}{\rho_1} \approx 4$$

$$U_{\text{shock}} = -u_1 \approx \sqrt{\frac{4p_2}{3\rho_1}}$$

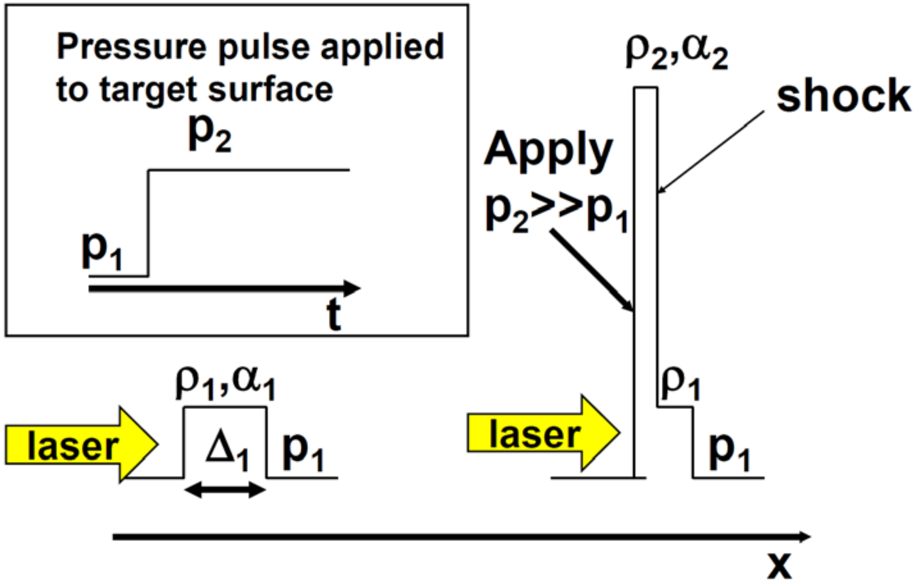
$$u_2 \approx \sqrt{\frac{p_2}{12\rho_1}}$$

$$\frac{\alpha_2}{\alpha_1} = \frac{p_2/\rho_2^{5/3}}{p_1/\rho_1^{5/3}} \approx \frac{1}{4^{5/4}}\frac{p_2}{p_1} >> 1$$



- **The adiabat increases through the shock.**

In an ideal gas/plasma, the adiabat α only raises when a shock is present



- **Post-shock density**

$$\rho_2 \approx 4\rho_1$$

- **Adiabat set by the shock for DT:**

$$\alpha_2 \approx \frac{p_{2,\mathrm{Mbar}}}{2.2\left(4\rho_{1,\mathrm{g/cc}}\right)^{5/3}}$$

- **Time required for the shock to reach the rear target surface (shock break-out time, t_{sb})**

$$t_{\mathrm{sb}} = \frac{\Delta_1}{u_{\mathrm{shock}}} = \Delta_1\sqrt{\frac{3\rho_1}{4p_2}} \propto \sqrt{\frac{1}{\alpha_2\rho_1^{2/3}}}$$

Higher laser intensity leads to higher adiabat

- **For a cryogenic solid DT target at 18 k:**

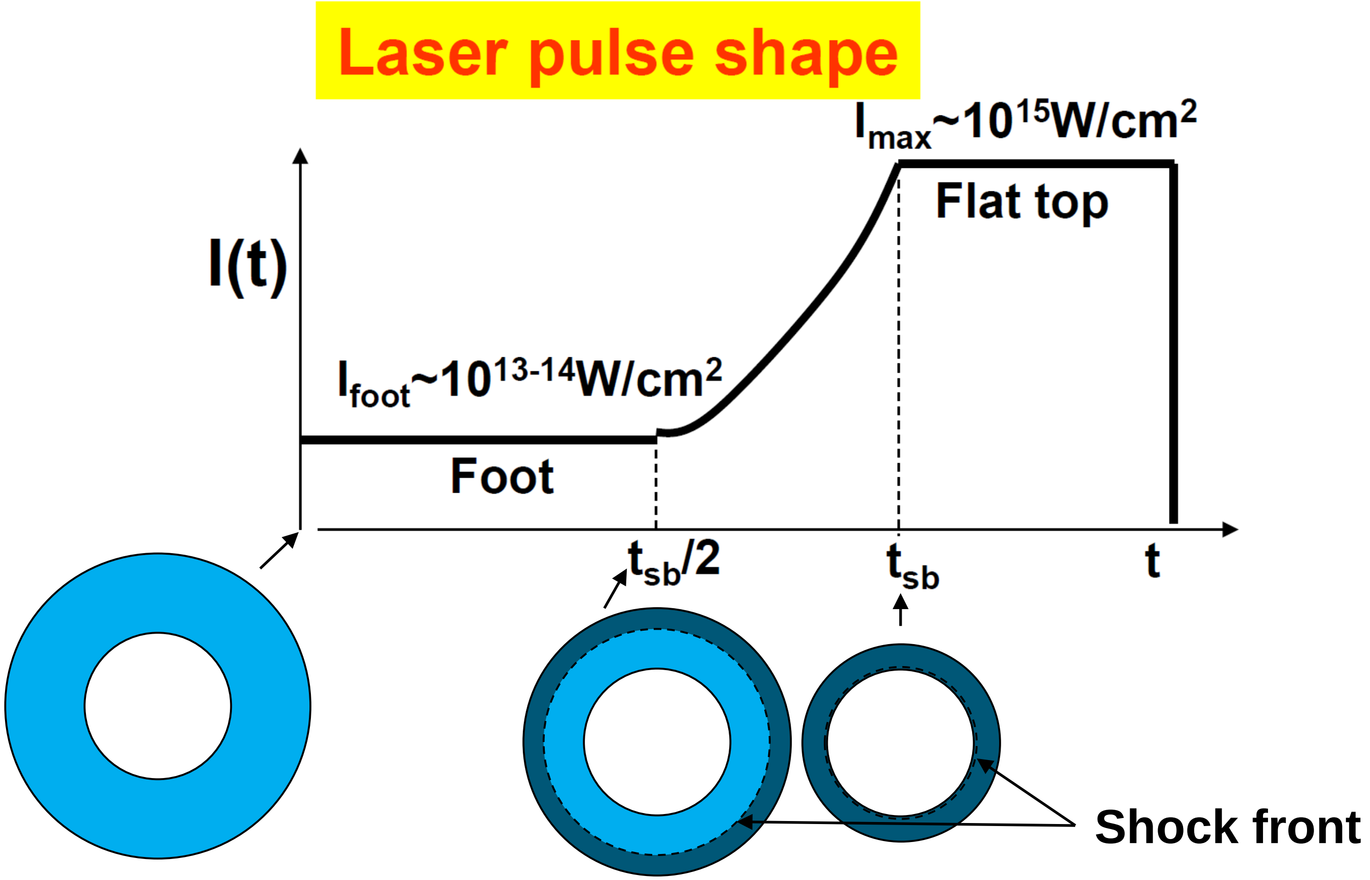
$$\rho_1 = 0.25 \text{ g/cc} \qquad \alpha = \frac{p_{\text{Mbar}}}{2.2} \qquad p \approx 83 \left(\frac{I_{15}}{\lambda_{\mu\text{m}}/0.35} \right)^{2/3}$$

$$I \approx 4.3 \times 10^{12} \text{ w/cm}^2 \quad \Rightarrow \quad p = 2.2 \text{ Mbar} \quad \Rightarrow \quad \alpha = 1$$

$$I \approx 1.2 \times 10^{13} \text{ w/cm}^2 \quad \Rightarrow \quad p = 4.4 \text{ Mbar} \quad \Rightarrow \quad \alpha = 2$$

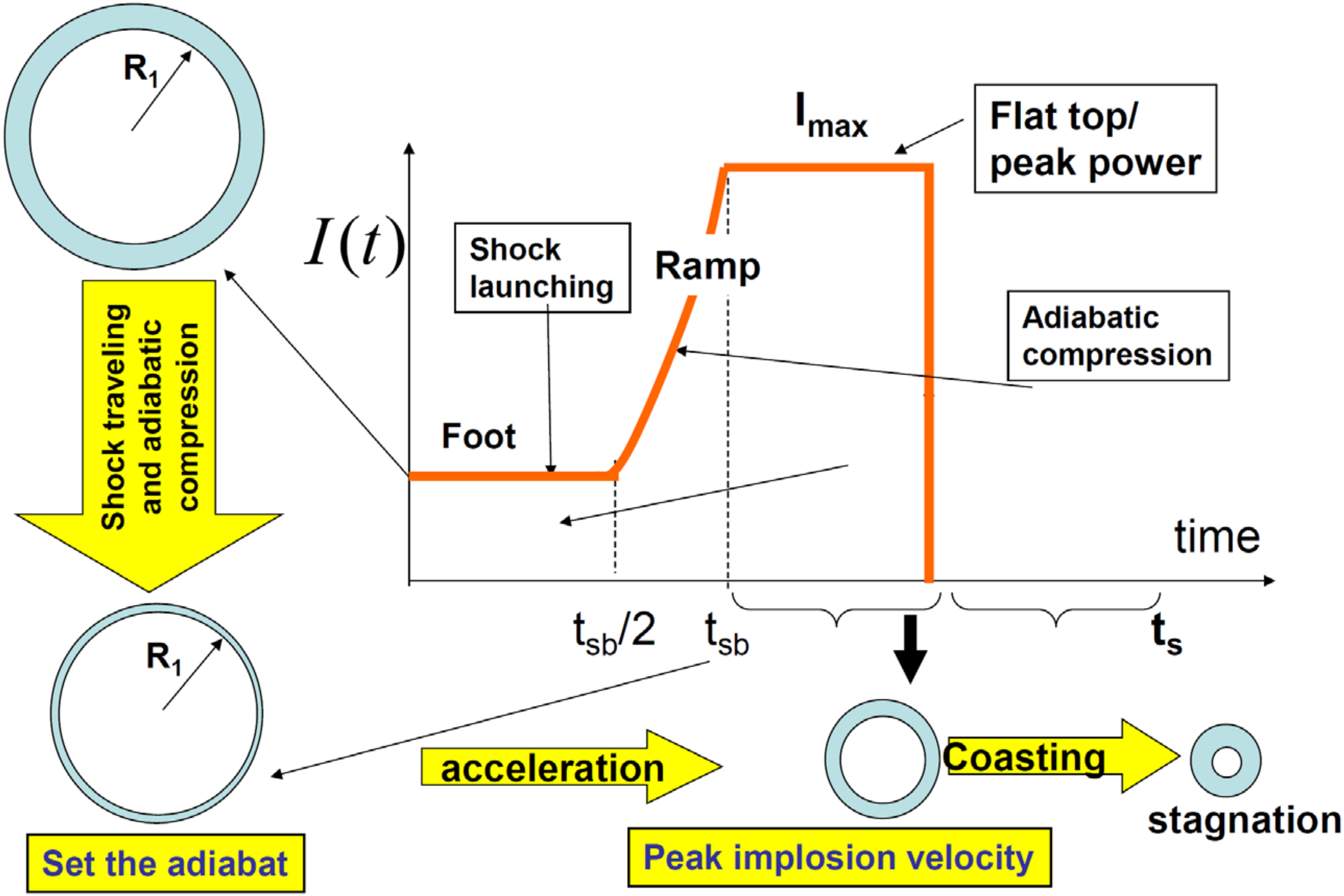
$$I \approx 2.2 \times 10^{13} \text{ w/cm}^2 \quad \Rightarrow \quad p = 6.6 \text{ Mbar} \quad \Rightarrow \quad \alpha = 3$$

The pressure must be "slowly" increased after the first shock to avoid raising the adiabat



- After the foot of the laser pulse, the laser intensity must be raised starting at about $0.5t_{sb}$ and reach its peak at about t_{sb}
- Reaching I_{max} at t_{sb} prevents a rarefaction/decompression wave to propagate back from the rear target surface and decompress the target.

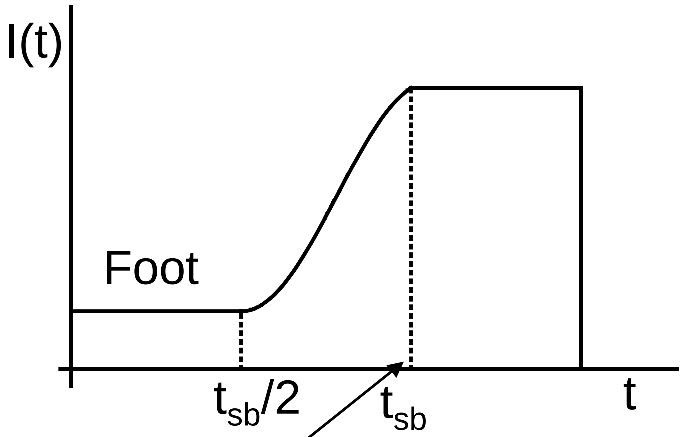
There are three stages in the laser pulse: foot, ramp, and flat top



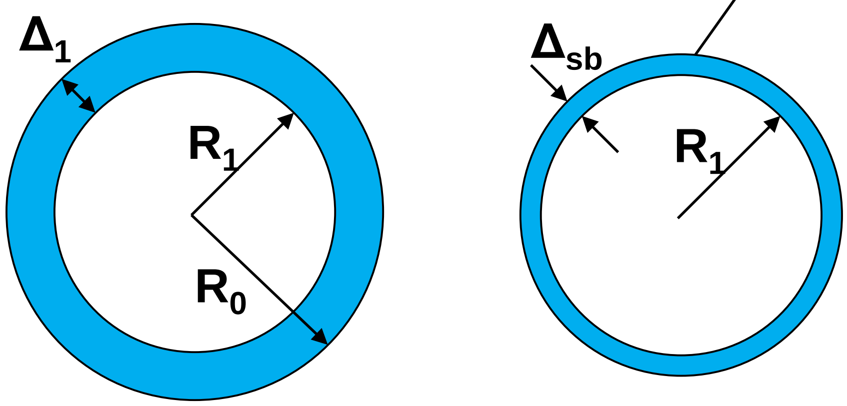
The adiabat is set by the shock launched by the foot of the laser pulse

$$\alpha \sim \frac{p_{\text{foot}}}{(4\rho_1)^{5/3}}$$

$$\rho_1 = \text{initial density}$$



Shock break-out

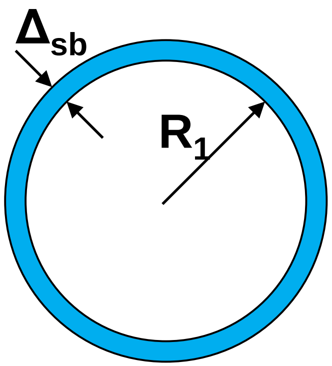


$$\rho_{sb} \sim \left(\frac{p_{\max}}{\alpha}\right)^{5/3} = 4\rho_1 \left(\frac{p_{\max}}{p_{\text{foot}}}\right)^{3/5}$$

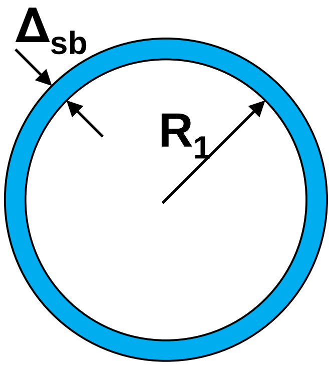
$$\Delta_{sb} \sim \frac{\Delta_1}{4}\left(\frac{p_{\text{foot}}}{p_{\max}}\right)^{3/5}$$

Density and thickness at shock break out time are expressed in laser intensity

- **Use** $p \sim I^{2/3}$
- **Shell density** $$\rho_{\mathrm{sb}} \sim 4\rho_1 \left(\frac{p_{\mathrm{max}}}{p_{\mathrm{foot}}}\right)^{3/5} = 4\rho_1 \left(\frac{I_{\mathrm{max}}}{I_{\mathrm{foot}}}\right)^{2/5}$$
- **Shell thickness** $$\Delta_{\mathrm{sb}} \sim \frac{\Delta_1}{4} \left(\frac{p_{\mathrm{foot}}}{p_{\mathrm{max}}}\right)^{3/5} = \frac{\Delta_1}{4} \left(\frac{I_{\mathrm{foot}}}{I_{\mathrm{max}}}\right)^{2/5}$$
- **Shell radius** $R \approx R_1$



The aspect ratio is maximum at shock break out



$$A_1 = \frac{R_1}{\Delta_1} = \text{initial aspect ratio}$$

$$A_{\rm sb} = IFAR = \frac{R_1}{\Delta_{\rm sb}} = 4A_1\left(\frac{I_{\rm max}}{I_{\rm foot}}\right)^{2/5}$$

$$A_{\rm sb} = A_{\rm max}$$

IRAR ≡ Maximum In-Flight-Aspect-Ratio = aspect ratio at shock break-out

The IFAR scales with the Mach number

- **The shell kinetic energy = the work done on the shell**

$$M u_{\text{max}}^2 \sim \int_R^{R_1} p r^2 dr \sim p\left(R_1^3 - R^3\right) \approx p R_1^3$$

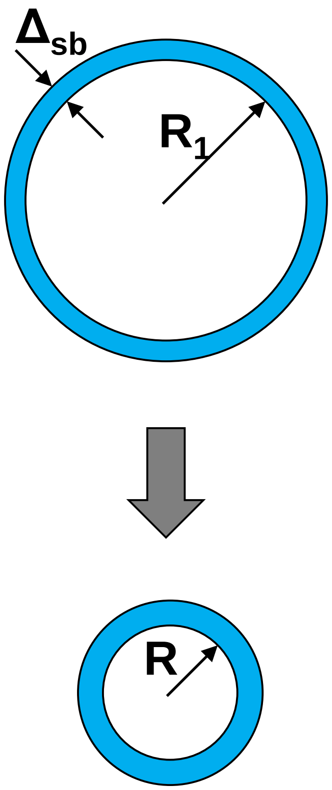
$$M \sim \rho_{\text{sb}} \Delta_{\text{sb}} R_1^2$$

$$R_1 >> R$$

$$IFAR = A_{\text{sb}} = \frac{R_1}{\Delta_{\text{sb}}} \sim \frac{u_{\text{max}}^2}{p/\rho_{\text{sb}}} \sim Mach_{\text{max}}^2$$

$$\rho \sim (p/\alpha)^{3/5} \qquad p \sim I^{2/3}$$

$$\boldsymbol{IFAR \sim \frac{u_{\text{max}}{}^2}{\alpha^{3/5} I^{4/15}}}$$



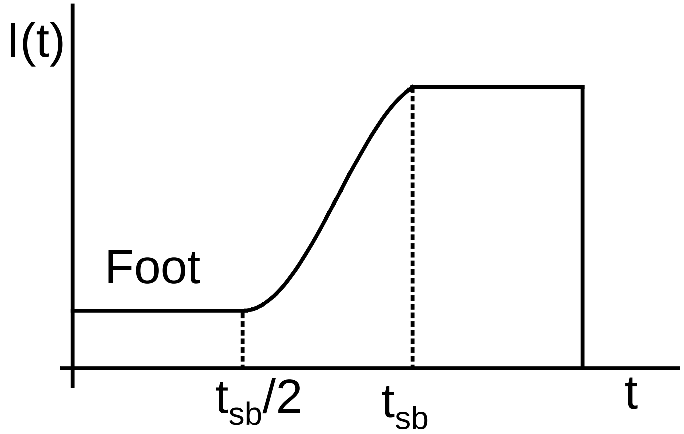
The final implosion velocity can be found using IFAR

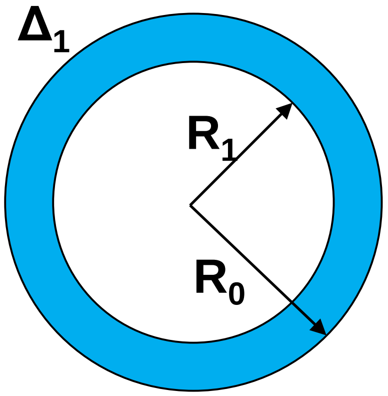
$$u_{\max}^2 \sim IFAR \times \alpha^{3/5} I^{4/15}$$

$$IFAR = 4A_1 \left(\frac{I_{\max}}{I_{\text{foot}}}\right)^{2/5}$$

$$A_1 = \frac{R_1}{\Delta_1}$$

$$\boldsymbol{u_{\max,\mathrm{cm}/s} \approx 10^7 \sqrt{0.7 A_1 \alpha^{3/5} I_{15,max}^{4/15} \left(\frac{I_{\max}}{I_{\text{foot}}}\right)^{2/5}}}$$

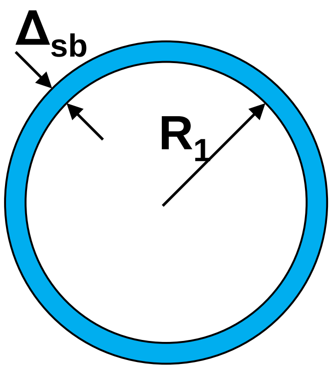




A simple implosion theory can be derived in the limit of infinite initial aspect ratio

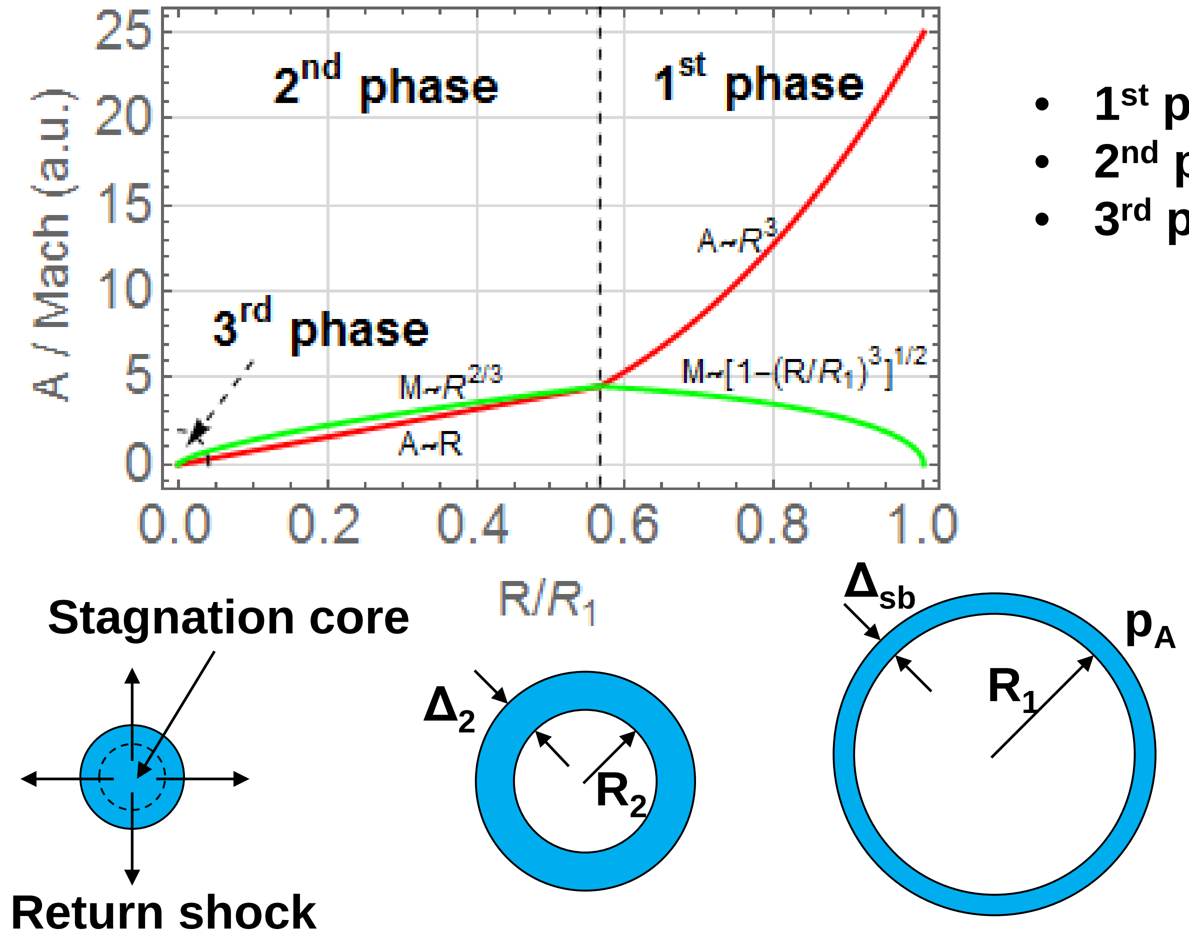
- **Start from a high aspect ratio shell (thin shell) at the beginning of the acceleration phase**

$$IFAR = A_{\rm sb} = \frac{R_1}{\Delta_{\rm sb}} >> 1$$



- **Ref: Basko and Meyer-ter-vehn, Phys. Rev. Lett. *88*, 244502-1, 2002**

The implosion are divided in 3 phases after the shock break out



- 1st phase: acceleration
- 2nd phase: coasting
- 3rd phase: stagnation

The shell density is constant

- **Shell expansion/contraction:** $t_{\text{ex}} \sim \dfrac{\Delta}{C_{\text{s}}}$
- **Implosion time:** $t_{\text{i}} \sim \dfrac{R}{u_{\text{i}}}$

$$\frac{t_{\text{i}}}{t_{\text{ex}}} \sim \frac{R}{\Delta}\frac{C_{\text{s}}}{u_{\text{i}}} = \frac{A}{Mach} \qquad A = \frac{R}{\Delta} \qquad Mach = \frac{u}{C_{\text{s}}}$$

- **In the acceleration phase** $A \sim Mach^2$

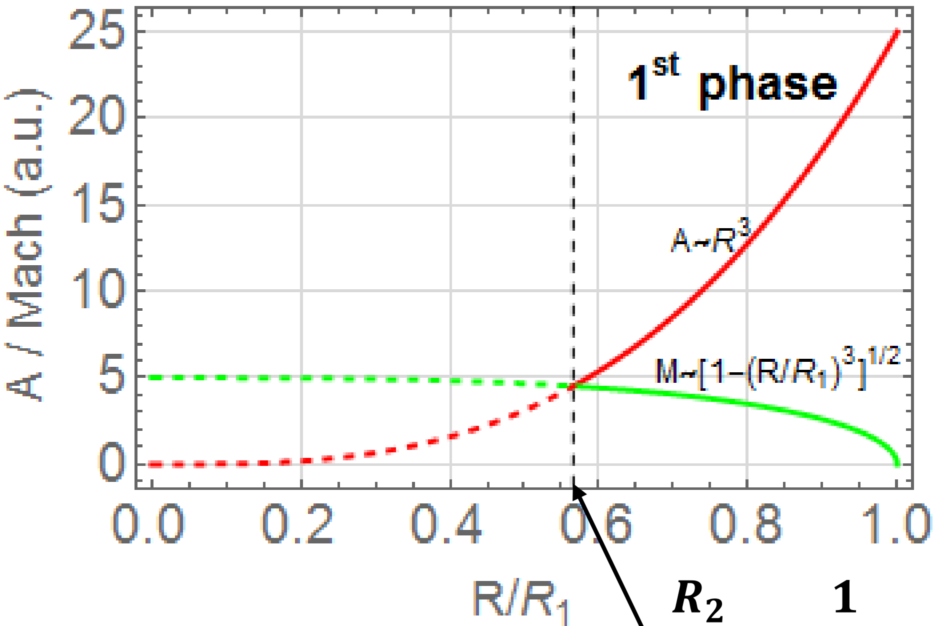
$$\frac{t_{\text{i}}}{t_{\text{ex}}} \sim \frac{A}{Mach} \sim Mach \sim \sqrt{A} >> 1 \;\Rightarrow\; \rho \approx \text{const}$$

- **From mass conservation:**

$$M \sim 4\pi R^2 \Delta\rho \;\Rightarrow\; \Delta \sim R^{-2}$$

$$\boldsymbol{A = \frac{R}{\Delta} \sim R^3 \Rightarrow A = A_{\text{sb}}\left(\frac{R}{R_1}\right)^3}$$

Summary of phase 1 (acceleration phase)



$$\frac{R_2}{R_1} \sim \frac{1}{{A_{sb}}^{1/6}}$$

$$\frac{1}{{A_{sb}}^{1/6}} < \frac{R}{R_1} \leq 1$$

$$A = A_{sb}\left(\frac{R}{R_1}\right)^3 = \mathrm{IFAR}\left(\frac{R}{R_1}\right)^3$$

$$\mathrm{Mach} = \mathrm{Mach}_{max}\left[1 - \left(\frac{R}{R_1}\right)^3\right]^{1/2}$$

$$\mathrm{Mach}_2 \simeq \mathrm{Mach}_{max}\left(1 - \frac{1}{\sqrt{A_{sb}}}\right)^{1/2} \simeq \mathrm{Mach}_{max} = \sqrt{A_{sb}} \qquad A_2 \sim \sqrt{A_{sb}}$$

The 2nd phase starts when R < R_2

- A decreases as R decreases. Eventually, A < Mach
- A >> 1 is required for thin shell model
- Assuming that the laser is off (coasting phase) when $R/R_1 \sim {A_{sb}}^{1/6}$

$$\rho(\partial_t \vec{w} + \vec{w} \cdot \nabla \vec{w}) = -\nabla p + \cancel{\rho g} \hat{r} \qquad \vec{w} \sim \dot{\Delta} \qquad \partial_t \sim 1/t_{\text{imp2}} \qquad \nabla \sim 1/\Delta$$

$$\Rightarrow \frac{\dot{\Delta}}{t_{\text{imp2}}} + \frac{\dot{\Delta}^2}{\Delta} \sim -\frac{p/\rho}{\Delta} \qquad \frac{\dot{\Delta}}{t_{\text{imp2}}} = \frac{\dot{\Delta}}{\Delta}\frac{u_i}{R_2/\Delta} = \frac{\dot{\Delta}}{\Delta}\frac{u_i}{A} \qquad t_{\text{imp2}} \sim \frac{R_2}{u_i}$$

$$\underbrace{\frac{\dot{\Delta} u_i}{A}}_{(1)} + \underbrace{\dot{\Delta}^2}_{(2)} \sim \underbrace{{C_s}^2}_{(3)}$$

- There are two cases:
  - Case 1: (3) << (1) and/or (2)
  - Case 2: (3) ~ (1) and/or (2)

The shell thickness does not change in the 2nd phase (coasting phase)

$$\underbrace{\frac{\dot{\Delta} u_i}{A}}_{(1)} + \underbrace{\dot{\Delta}^2}_{(2)} \sim \underbrace{C_s^{\;2}}_{(3)}$$

- **Case 1: (3) << (1) and/or (2)**

$$\dot{\Delta}\left(\frac{u_i}{A} + \dot{\Delta}\right) \sim 0 \qquad \dot{\Delta} \sim 0 \;\; \text{or} \;\; \Delta \equiv \Delta_2 = \text{constant}$$

- **Case 2: (3) ~ (1) and/or (2) and** $A \ll \text{Mach}$
  - **(3) ~ (1)**

$$\frac{\dot{\Delta} u_i}{A} \sim C_s^{\;2} \;\Rightarrow\; \dot{\Delta} \sim \frac{C_s A}{u_i / C_s} = \frac{C_s A}{\text{Mach}}$$

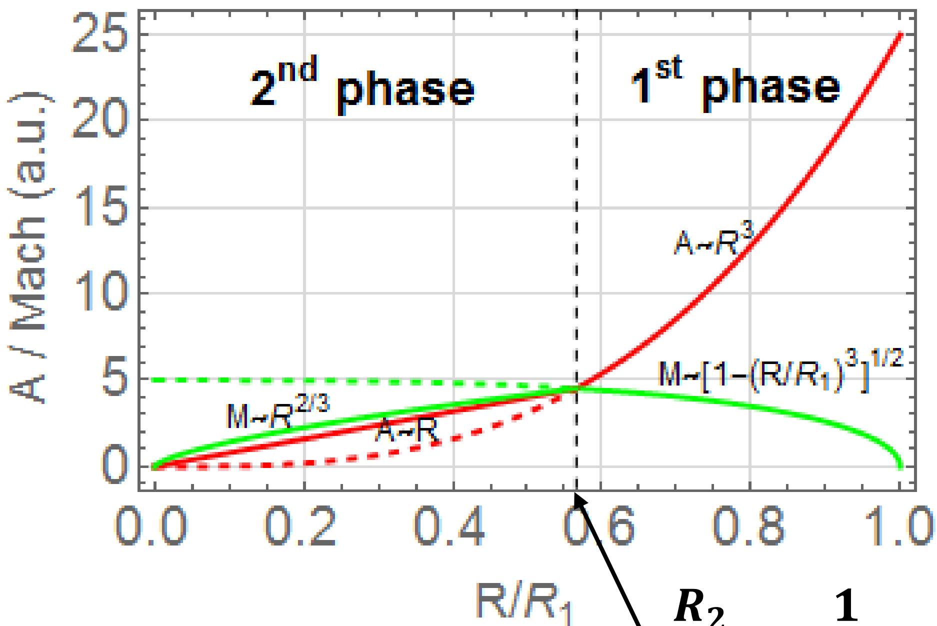
$$\frac{\delta \Delta}{\Delta} \sim \frac{\dot{\Delta} t_{\text{imp2}}}{\Delta} \sim \frac{1}{\Delta} \frac{C_s A}{\text{Mach}} \frac{R_2}{u_i} \sim \frac{A^2}{\text{Mach}^2} \ll 1$$

  - **(3) ~ (2)**

$$\dot{\Delta}^2 \sim C_s^{\;2} \qquad \frac{\delta \Delta}{\Delta} \sim \frac{\dot{\Delta} t_{\text{imp2}}}{\Delta} \sim \frac{C_s}{\Delta} \frac{R_2}{u_i} \sim \frac{A}{\text{Mach}} \ll 1$$

$$\Delta \equiv \Delta_2 = \text{constant} = \frac{\Delta_2}{R_2} R_2 = \frac{R_2}{A_2} \frac{R_1}{R_1} = \frac{1}{A_{sb}^{\;1/6}} \frac{R_1}{\sqrt{A_{sb}}} \sim \frac{R_1}{A_{sb}^{\;2/3}}$$

Summary of phase 2 (coasting phase)



$$\frac{R_2}{R_1} \sim \frac{1}{{A_{sb}}^{1/6}}$$

$$1 < A < \sqrt{A_{sb}} \qquad A < \mathrm{Mach}$$

$$\frac{1}{\sqrt{A_{sb}}} \sim \frac{1}{A_2} < \frac{R}{R_2} < 1$$

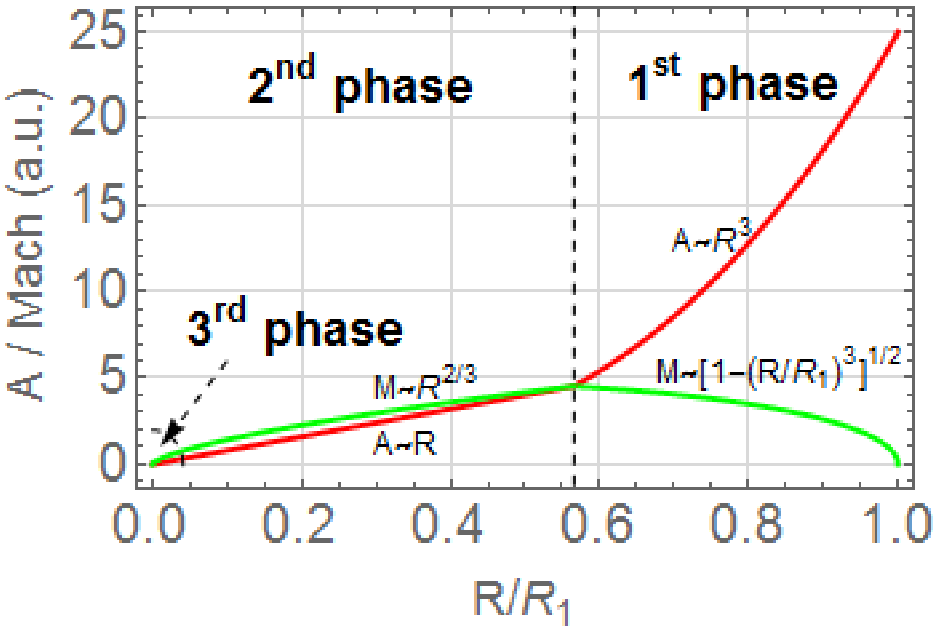
$$A = A_2\left(\frac{R}{R_2}\right) \sim \sqrt{A_{sb}}\left(\frac{R}{R_2}\right)$$

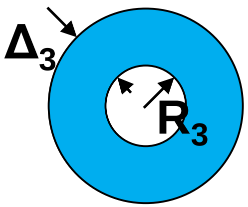
$$\mathrm{Mach} \sim \mathrm{Mach}_2\left(\frac{R}{R_2}\right)^{2/3} \sim \sqrt{A_{sb}}\left(\frac{R}{R_2}\right)^{2/3}$$

$$\mathrm{Mach}_2 = \mathrm{Mach}_{max} \simeq A_2 = \sqrt{A_{sb}}$$

$$\Delta \simeq \text{constant} = \Delta_2 \sim \frac{R_1}{{A_{sb}}^{2/3}} \qquad \bar{\rho} \simeq \rho_2\left(\frac{R_2}{R}\right)^2 \sim \rho_{sb}\left(\frac{R_2}{R}\right)^2$$

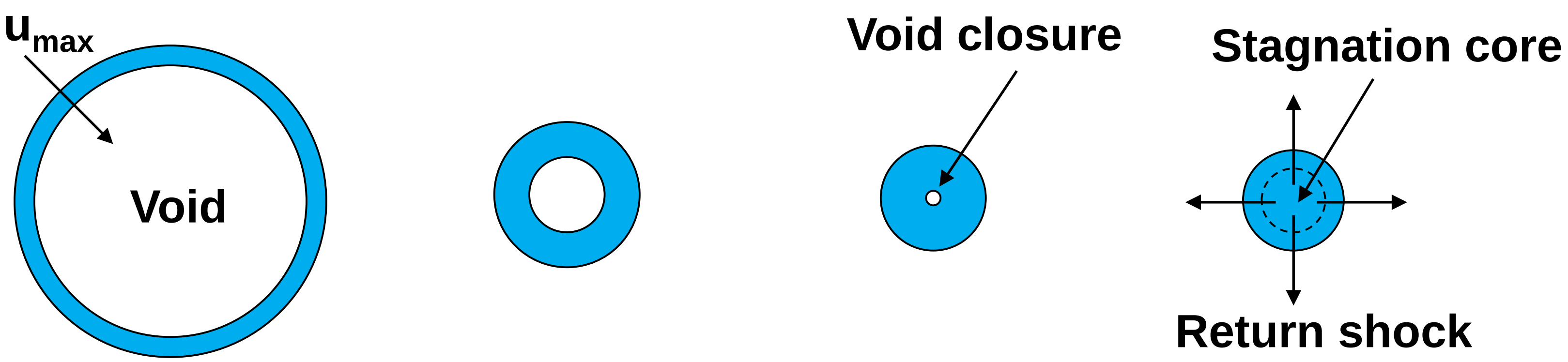
How about the 3rd phase where A~1 ?





- 1st phase: acceleration
- 2nd phase: coasting
- 3rd phase: stagnation

The thin shell model breaks down when A~1



- When A~1 => Δ~R, the "void" inside the shell closes and a "return shock" propagating outward is generated due to the collision of the shell with itself
- The density is compressed by a factor no more than 4 even if the strong shock is generated

$\rho_{st} \sim \rho_3$ where ρ_3 is the density right before the void closure

The stagnated density scales with square of the maximum Mach number

$$\rho_3 \sim \rho_2 \left(\frac{R_2}{R_3}\right)^2 \sim \rho_{\mathrm{sb}} \left(\frac{R_2}{R_3}\right)^2$$

(ρ is constant in phase 1.)

$$A = A_3 \sim 1 \Rightarrow \frac{R_3}{\Delta_3} \sim \frac{R_3}{\Delta_2} \sim 1 \Rightarrow R_3 \sim \Delta_2$$

(Δ is constant in phase 2.)

$$\rho_{\mathrm{st}} \sim \rho_3 \sim \rho_{\mathrm{sb}} \left(\frac{R_2}{\Delta_2}\right)^2 \sim \rho_{\mathrm{sb}} {A_2}^2 \sim \rho_{\mathrm{sb}} {\mathrm{Mach}_2}^2 \sim \rho_{\mathrm{sb}} {\mathrm{Mach}_{\mathrm{max}}}^2$$

$$\frac{\rho_{\mathrm{st}}}{\rho_{\mathrm{sb}}} \sim {\mathrm{Mach}_{\mathrm{max}}}^2$$

The stagnated pressure scales to the 4th power of the maximum Mach number

- **Conservation of energy at stagnation:**

$$p_{\mathrm{st}} {R_{\mathrm{st}}}^3 \sim m {u_{\max}}^2 \qquad R_{\mathrm{st}} \sim R_3 \sim \Delta_3 \sim \Delta_2 \quad \Rightarrow \quad p_{\mathrm{st}} {\Delta_2}^3 \sim m {u_{\max}}^2 \sim \rho_2 {R_2}^2 \Delta_2 {u_{\max}}^2$$

$$p_{\mathrm{st}} \sim \rho_2 \left(\frac{R_2}{\Delta_2}\right)^2 {u_{\max}}^2 = \rho_2 {A_2}^2 {u_{\max}}^2 \sim p_2 \frac{{\mathrm{Mach}_2}^2 {u_{\max}}^2}{p_2/\rho_2} \sim p_A {\mathrm{Mach}_2}^4 \sim p_A {\mathrm{Mach}_{\max}}^4$$

$$\boxed{\frac{p_{\mathrm{st}}}{p_A} \sim {\mathrm{Mach}_{\max}}^4}$$

$$\alpha_{\mathrm{st}} \sim \frac{p_{\mathrm{st}}}{{\rho_{\mathrm{st}}}^{5/3}} \sim \frac{p_A {\mathrm{Mach}_{\max}}^4}{{\rho_{\mathrm{sb}}}^{5/3} {\mathrm{Mach}_{\max}}^{10/3}} = \alpha_{\mathrm{sb}} {\mathrm{Mach}_{\max}}^{2/3}$$

$$\boxed{\frac{\alpha_{\mathrm{st}}}{\alpha_{\mathrm{sb}}} \sim {\mathrm{Mach}_{\max}}^{2/3}}$$

Scaling of the areal density of the compressed core

$$\rho_{st}R_{st} \sim \rho_{st}\Delta_2 \sim \left(\frac{p_{st}}{\alpha_{st}}\right)^{3/5} \frac{\Delta_2}{R_2}\frac{R_2}{R_1}R_1 \sim \left(\frac{p_A\,\mathrm{Mach_{max}}^4}{\alpha_{sb}\,\mathrm{Mach_{max}}^{2/3}}\right)^{3/5} \frac{1}{A_2}\frac{1}{A_{sb}{}^{1/6}}R_1$$

$$A_2 \sim \mathrm{Mach_{max}} \qquad A_{sb} \sim \mathrm{Mach_{max}}^2$$

$$\rho_{st}R_{st} \sim \left(\frac{p_A}{\alpha_{sb}}\right)^{3/5} \mathrm{Mach_{max}}^2 \frac{1}{\mathrm{Mach_{max}}}\frac{1}{\mathrm{Mach_{max}}^{1/2}}R_1$$

$$\sim \left(\frac{p_A}{\alpha_{sb}}\right)^{3/5} \mathrm{Mach_{max}}^{2/3}R_1 \sim \left(\frac{p_A}{\alpha_{sb}}\right)^{3/5} \frac{u_{max}{}^{2/3}}{(p_A/\rho_{sb})^{1/3}}\frac{p_A{}^{1/3}R_1}{p_A{}^{1/3}}$$

$$\sim \left(\frac{p_A}{\alpha_{sb}}\right)^{3/5} \frac{u_{max}{}^{2/3}}{(p_A{}^{2/5}\alpha_{sb}{}^{3/5})^{1/3}}\frac{\left(p_A R_1{}^3\right)^{1/3}}{p_A{}^{1/3}} \sim \frac{p_A{}^{2/15}}{\alpha_{sb}{}^{4/5}}u_{max}{}^{2/3}E_k{}^{1/3}$$

$$E_k \sim E_{las} \;\Rightarrow\qquad \rho_{st}R_{st} \sim \frac{p_A{}^{2/15}u_{max}{}^{2/3}E_{las}{}^{1/3}}{\alpha_{sb}{}^{4/5}}$$

Amplification of areal density

$$\rho_{st}R_{st} \sim \rho_{st}^{2/3}\left(\rho_{st}R_{st}^{3}\right)^{1/3} \sim \rho_{sb}^{2/3}\mathrm{Mach}_{max}^{4/3}\mathrm{Mass}^{1/3}$$

$$\sim \frac{\rho_{sb}^{2/3}}{\rho_1^{2/3}}\mathrm{Mach}_{max}^{4/3}\rho_1^{2/3}\left(\rho_1 R_1^{2}\Delta_1\right)^{1/3}$$

$$\rho_{st}R_{st} \sim (\rho_1\Delta_1)\mathrm{Mach}_{max}^{4/3}A_1^{2/3}\left(\frac{\rho_{sb}}{\rho_1}\right)^{2/3}$$

$$\frac{\rho_{sb}}{\rho_1} = 4\left(\frac{I_{max}}{I_{foot}}\right)^{2/5}$$

$$(\rho R)_{st} \sim (\rho_1\Delta_1)\mathrm{IFAR}^{2/3}A_1^{2/3}\left(\frac{I_{max}}{I_{foot}}\right)^{4/15}$$

$$E_{las} = 4\pi R_1^{2} I_{max} t_{imp} \approx 4\pi R_1^{2} I_{max}\frac{R_1}{u_{max}}$$

$$E_{las} \approx \frac{4\pi R_1^{3} I_{max}}{u_{max}}$$

Summary

$$A_{\mathrm{sb}} = \mathrm{IFAR} = 4A_1\left(\frac{I_{\max}}{I_{\mathrm{foot}}}\right)^{2/5} \qquad u_{\max,\mathrm{cm}/s} \approx 10^7\sqrt{0.7A_1\alpha^{3/5}{I_{15,\max}}^{4/15}\left(\frac{I_{\max}}{I_{\mathrm{foot}}}\right)^{2/5}}$$

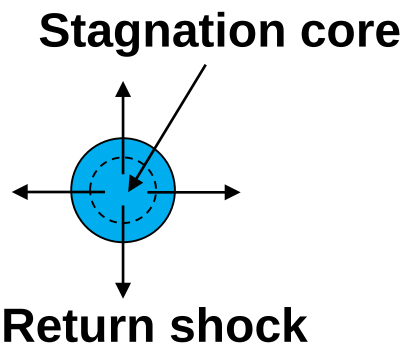
$$\rho_{\mathrm{st}} \sim \rho_{\mathrm{sb}}{\mathrm{Mach}_{\max}}^2 \sim \rho_1\mathrm{IFAR}\left(\frac{I_{\max}}{I_{\mathrm{foot}}}\right)^{2/5}$$

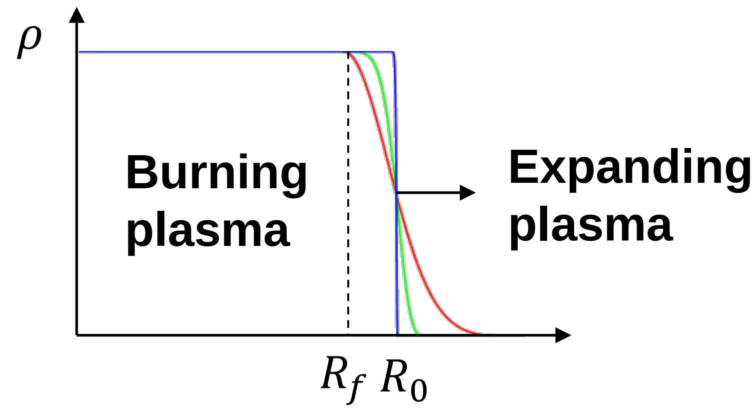
$$p_{\mathrm{st}} \sim p_A{\mathrm{Mach}_{\max}}^4 \sim p_A\mathrm{IFAR}^2$$

$$\alpha_{\mathrm{st}} \sim \alpha_{\mathrm{sb}}{\mathrm{Mach}_{\max}}^{2/3} \sim \alpha_{\mathrm{sb}}\mathrm{IFAR}^{1/3}$$

$$(\rho R)_{\mathrm{st}} \sim (\rho_1 \Delta_1)\mathrm{IFAR}^{2/3}{A_1}^{2/3}\left(\frac{I_{\max}}{I_{\mathrm{foot}}}\right)^{4/15}$$

Calculation of the burn-up fraction





$$R_f = R_0 - C_s t$$

$$\frac{\partial n_i}{\partial t} = -\nabla \cdot (n_i v) - \frac{{n_i}^2}{4} < \sigma v > \times 2 \qquad 4\pi \int_0^{R_f} r^2 \mathrm{dr} \left(\frac{\partial n_i}{\partial t} = -\nabla \cdot (n_i v) - \frac{{n_i}^2}{2} \langle \sigma v \rangle \right)$$

Burn-up fraction:

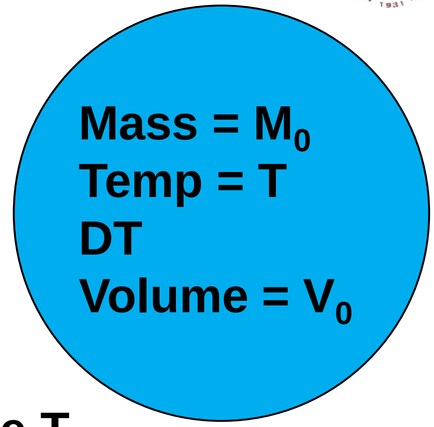
$$\Theta(\xi) = \frac{\xi[6 + \xi(9 + 2\xi)] - 6(1+\xi)^2 \mathrm{Ln}[1+\xi]}{2\xi^3} \qquad \xi \equiv \frac{\langle \sigma v \rangle}{2C_s} n_0 R_0$$

$$\Theta(\xi) \approx \frac{(\rho R)_{g/\mathrm{cm}^2}}{7 + (\rho R)_{g/\mathrm{cm}^2}}$$

For energy applications:

$$\Theta \gtrsim 0.3 \qquad \rho R \geq 3\ g/\mathrm{cm}^2$$

Energy gain



Fusion energy $= \dfrac{M_0}{2m_i}\epsilon_f \Theta$

$\epsilon_f = 17.6\,\mathrm{MeV}$

Energy gain $= \dfrac{\text{Fusion Energy}}{\text{Input Energy}}$

- Input energy: the sphere is heated to the temperature T

Thermal energy in sphere: $\dfrac{3}{2}(n_{i0}T_i + n_{e0}T_e)V_0$

$n_{i0} = n_{e0} \equiv n_0 \qquad T_e = T_i \Longrightarrow 3n_0\mathrm{T}\mathrm{V}_0 = 3\dfrac{M_0}{m_i}T$

Set heating efficiency: $\eta = \dfrac{\text{Thermal Energy}}{\text{Input Energy}}$

$$\text{Gain} = \eta \frac{M_0}{2m_i}\frac{\epsilon_f \Theta}{3\frac{M_0}{m_i}T} = \frac{\eta}{6}\frac{\epsilon_f}{T}\Theta$$

$$\text{Gain} = \eta\, 293\left(\frac{10}{T_{\text{keV}}}\right)\Theta$$

The power to heat the plasma is enormous

- Consider the small T limit:

$$\boldsymbol{\Theta}(\xi) \approx \frac{\xi}{4+\xi} \qquad \xi \equiv \frac{\langle \sigma v \rangle}{2C_s} n_0 R_0 = \frac{\langle \sigma v \rangle}{\sqrt{T}} (\rho R_0) \frac{1}{2\sqrt{2m_i}}$$

$$\langle \sigma v \rangle \sim T^4 \text{ for } T \to 0 \text{ , then} \quad \xi \sim T^{7/2} \quad \text{and} \quad \text{Gain} \sim T^{5/2} \to 0$$

$$P_w = \frac{E_{\text{input}}}{\tau_{\text{input}}} \qquad \tau_{\text{input}} \ll \tau_{\text{burn}} = \frac{R}{C_s}$$

(Heat out before it runs away)

$$P_w = \frac{E_{\text{input}}}{\mu R / C_s} = \frac{E_{\text{thermal}}}{\eta \mu R / C_s} = 3 \frac{M_0}{m_i} \frac{T}{R} \frac{C_s}{\eta \mu} \qquad \tau_{\text{input}} = \mu \frac{R}{C_s}$$

Ex: $\mu \sim 0.1$

$$\frac{P_w}{M_0} = \frac{3}{m_i} \frac{T}{R} \frac{C_s}{\eta \mu} = \frac{3}{m_i} \frac{T}{R} \sqrt{\frac{2T}{m_i}} \frac{1}{\eta \mu}$$

$$\frac{P_w}{M_0} = 10^{18} \left(\frac{T_{\text{keV}}}{10} \right)^{3/2} \frac{0.1}{\mu} \frac{1}{R_{\text{cm}}} \frac{1}{\eta} \text{ Watts}/g$$

A clever way is needed to ignite a target

- **For T = 10 keV**

$$\xi \approx 0.18(\rho R) \qquad \mathrm{Gain}|_{10\mathrm{keV}} \approx 293\eta \frac{0.18\rho R}{4 + 0.18\rho R} \approx 293\eta \frac{\rho R_{g/\mathrm{cm}^2}}{22 + \rho R_{g/\mathrm{cm}^2}}$$

- **For T=40 keV**

$$\xi \approx 0.54(\rho R) \qquad \mathrm{Gain}|_{40\mathrm{keV}} \approx 73\eta \frac{\rho R_{g/\mathrm{cm}^2}}{7 + \rho R_{g/\mathrm{cm}^2}}$$

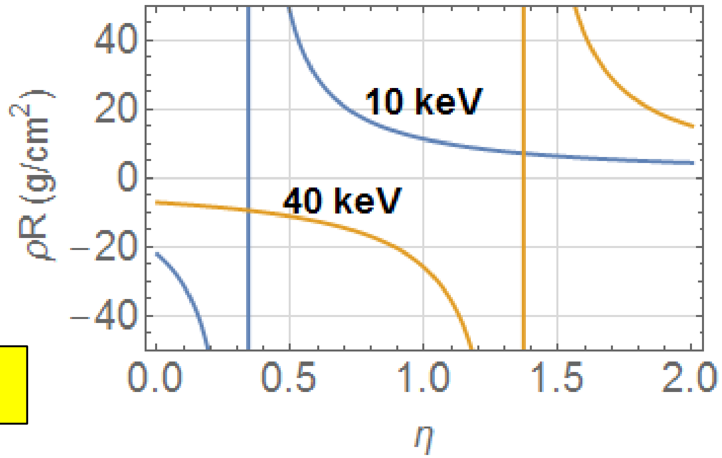
- **For Gains ≳ 100**
  - **T = 10 keV**

    $\rho R \gtrsim 22\, g/\mathrm{cm}^2 \quad \eta > 1$
  - **T = 40 keV**

    $\eta > 1.5$

How do we get $\eta > 1$?

Required ρR for Gain=300



Requirement to ignite a target

- **For T=10keV and ρR≳22 g/cm²**

$$\rho R = \frac{4\pi}{3}\frac{\rho R^3}{4\,\pi R^2/3} = \frac{M_0}{\frac{4\pi}{3}R^2} = \frac{3}{4\pi}\frac{M_0}{R^2} \gtrsim 22\, g/\text{cm}^2$$

$$\frac{M_0}{R^2} \gtrsim 92\, g/\text{cm}^2 \qquad P_w\Big|_{10\text{keV}} = 10^{18}\left(\frac{T_{\text{keV}}}{10}\right)^{3/2}\frac{0.1}{\mu}\frac{M_0}{R_{\text{cm}}}\frac{1}{\eta}$$

$$= 10^{18}\frac{0.1}{\mu}\frac{1}{\eta}92R_{\text{cm}} \quad \text{Watts}$$

$$P_w\Big|_{10\text{keV}} \approx 10^{20}\frac{0.1}{\mu}\frac{R_{\text{cm}}}{\eta}\text{Watts}$$

- **For T=40keV**

$$\rho R \gtrsim 7 \quad \Longrightarrow \quad \frac{M_0}{R^2} \gtrsim 30\, g/\text{cm}^2$$

$$P_w\Big|_{40\text{keV}} \approx 2.4\times 10^{20}\frac{0.1}{\mu}\frac{R_{\text{cm}}}{\eta}\text{Watts}$$

- **Needed:**

  $R_{\text{cm}} \ll 1$

  $\eta \gg 1$

  $\mu \gg 0.1$

Requirements to ignite a target

$$P_w\Big|_{10\text{keV}} \approx 10^{20}\,\frac{0.1}{\mu}\,\frac{R_{\text{cm}}}{\eta}\,\text{Watts}$$

- $R_{\text{cm}} \ll 1$: sphere size in the order of 100's um
- $\eta \gg 1$: input energy amplification
- $\mu \gg 0.1$: energy delivery time decoupled from burn time. Need longer energy delivery time. Need to bring down power to ~10^{15} W

The energy from the fusion reaction can be used to heat the plasma

$$P_w = 10^{18}\frac{M_{0,g}}{\eta}\left(\frac{T_{\text{keV}}}{10}\right)^{3/2}\frac{0.1}{\mu}\frac{1}{R_{\text{cm}}}\ \text{Watts}/g$$

$$\tau_{\text{input}} = \mu\frac{R}{C_s} \quad \text{Ex: } \mu\sim 0.1 \qquad \eta = \frac{\text{Thermal Energy}}{\text{Input Energy}}$$

$$\text{Gain} = \eta\, 293\left(\frac{10}{T_{\text{keV}}}\right)\Theta(\xi) \qquad \Theta(\xi) \approx \frac{\xi}{4+\xi} \qquad \xi = \frac{\langle\sigma v\rangle}{2m_i C_s}(\rho R_0)$$

$$G_{\max} \equiv 293\eta\left(\frac{10}{T_{\text{keV}}}\right) \qquad G = G_{\max}\frac{\xi}{4+\xi} \Longrightarrow \xi = \frac{4G}{G_{\max}-G}$$

$$P_w = \frac{10^{18}}{\eta}\left(\frac{T_{\text{kev}}}{10}\right)^{3/2}\frac{0.1}{\mu}\frac{4\pi}{3}\frac{\rho {R_0}^3}{R_0} = \frac{10^{18}}{\eta}\left(\frac{T_{\text{kev}}}{10}\right)^{3/2}\frac{0.1}{\mu}\frac{4\pi}{3}(\rho R_0)R_0$$

Need to lower the power by 5 orders of magnitude

$$P_w \approx \frac{7\times 10^{19}}{\eta}\frac{0.1}{\mu}R_{0,\mathrm{cm}}\frac{G}{G_{\max}}\,\mathrm{Watts}$$

- $\mu \uparrow$:
- $\eta \uparrow$: require the fuel ignition from a "spark." Ignite only a small portion of the DT plasma, i.e., $M_h << M_0$
- $R_0 \downarrow$: smaller system size

$$P_w = P_w(M_0)\frac{M_h}{M_0}$$

$$P_w^{\min} = \frac{7\times 10^{15}}{\eta_h}\left(\frac{M_h/M_0}{0.01}\right)\left(\frac{R_{0,\mu\mathrm{m}}}{100}\right)\left(\frac{0.1}{\mu}\right)\left(\frac{G}{G_{\max}}\right)\mathrm{Watts}$$

Effective increase in η

Target design using an 1MJ laser

$$P_w^{\min} = \frac{7\times 10^{15}}{\eta_h}\left(\frac{M_h/M_0}{0.01}\right)\left(\frac{R_{0,\mu\mathrm{m}}}{100}\right)\left(\frac{0.1}{\mu}\right)\left(\frac{G}{G_{\max}}\right)\mathrm{Watts}$$

- **For the case of using a huge laser, ex: 1MJ.**
- **The ignition requires temperatures** $T \gtrsim 5\mathrm{keV}$ **, then**

$$E_{\mathrm{ign}} \approx 3\frac{M_h}{m_i}\frac{T}{\eta_h}$$

$$M_h \approx \frac{m_i}{3}\frac{\eta_h E_{\mathrm{ign}}}{T}$$

$$M_{h,\mu\mathrm{g}} \approx 17\left(\frac{5}{T_{\mathrm{keV}}}\right)E_{\mathrm{igm,MJ}}\left(\frac{\eta_h}{0.01}\right)$$

$$M_h \approx 20\mu\mathrm{g}$$

Target design using an 1MJ laser - continue

- For "inefficient" heating mechanism $(\eta_h \approx 1\%)$, the mass that can be heated to T≈5keV is in the order of M_h≈20μg
- If M_h/M_0≈0.01, then M_0≈2mg .
- Assuming that the burned-up fraction $\Theta \approx \frac{\rho R}{7+\rho R}$

for $\Theta \approx 30\% \rightarrow \rho R \approx 3\ g/\mathrm{cm}^2$

$$M_0 = \frac{4\pi}{3}\rho R^3 = \frac{4\pi}{3}R^2(\rho R) \qquad R = \sqrt{\frac{4\pi}{3}\frac{M_0}{\rho R}} = 126\sqrt{\frac{M_{0,\mathrm{mg}}}{2}}\sqrt{\frac{3}{\rho R}}\ \mu\mathrm{m}$$

$$\rho = \frac{3M_0}{4\pi R^3} = 240\sqrt{\frac{M_{0,\mathrm{mg}}}{2}}\left(\frac{126}{R_{\mu\mathrm{m}}}\right)^3\ g/\mathrm{cm}^3 \qquad \rho_{\mathrm{DT}} = 0.25\ g/\mathrm{cm}^3$$

- DT must be compressed ~1000 times
- The initial radius of a 2mg sphere of DT is $R_{\mathrm{init}} \simeq 2.6\mathrm{mm}$ while the final radius $R_{\mathrm{final}} \simeq 100\mu\mathrm{m}$, the convergence ratios of 30~40 are required.

Requirements of the density and size of the ignition mass

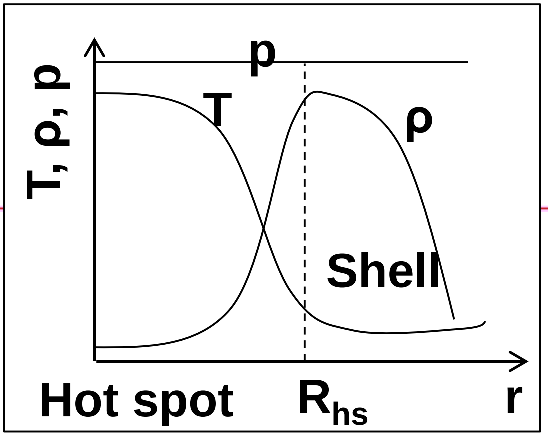
$$M_h \approx 20\mu g$$

$$\rho_h R_h \approx 0.3\ g/cm^2$$ ← **To stop 3.5 MeV α particles**

$$R_h \simeq \sqrt{\frac{3}{4\pi}\frac{M_h}{\rho_h R_h}} \approx 40\mu m$$

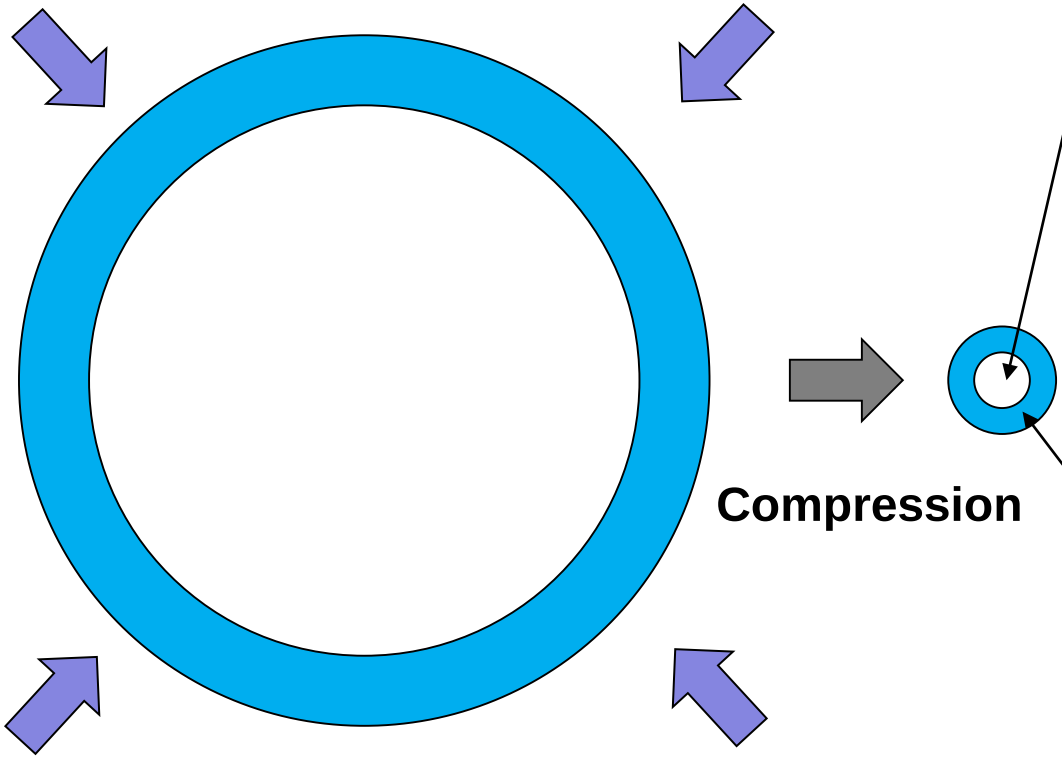
$$\rho_h \approx \frac{(\rho_h R_h)}{R_h} = \frac{0.3}{40 * 10^{-4}} = 75\ g/cm^3$$

Summary



- **Possible fuel assembly for 1MJ ICF driver**

$E_{\text{laser}} = 1\text{MJ}$ $\quad \eta_h \approx 1\%$



Hot spot

$M_h \approx 20\mu\text{g}$ $\quad (\rho R)_h \approx 0.3\ g/\text{cm}^2$

$\rho_h \approx 70\ g/\text{cm}^3$ $\quad R_h \approx 40\mu\text{m}$

$\frac{M_h}{M_0} \sim 0.01$

Dense fuel

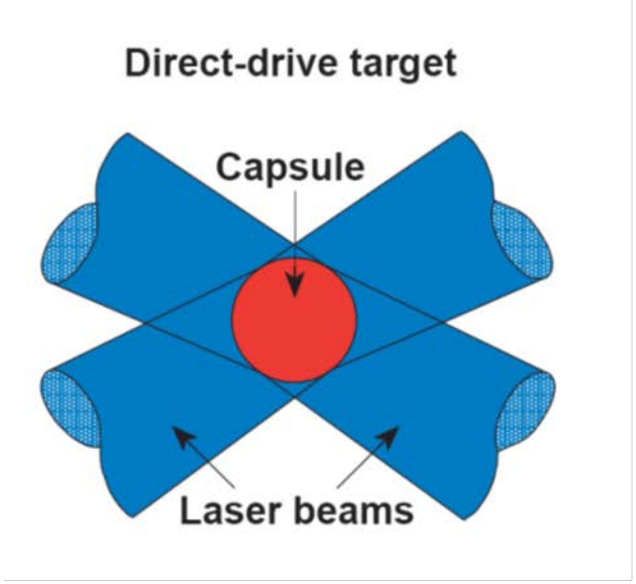
$M_0 \approx 2\text{mg}$ $\quad (\rho R) \approx 3\ g/\text{cm}^2$

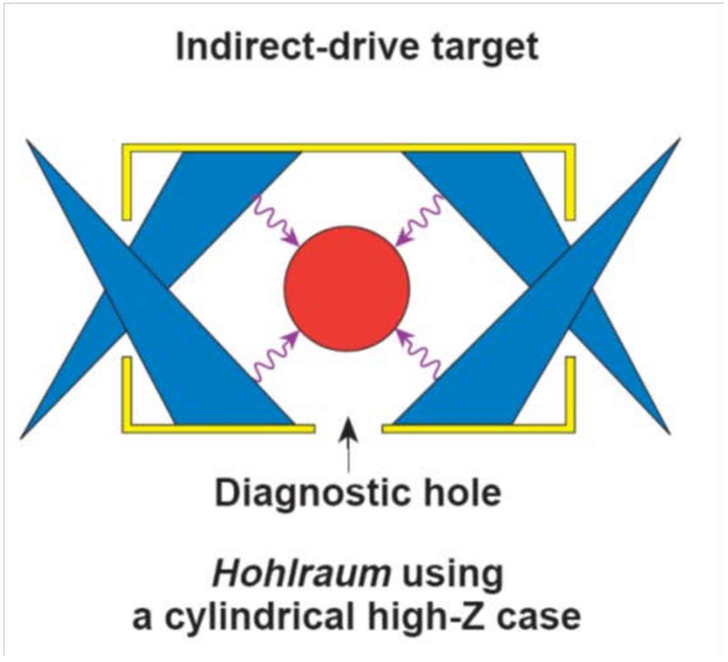
$\rho \approx 250\ g/\text{cm}^3$ $\quad R \approx 120\mu\text{m}$

$\rho_{\text{DT}} = 0.25\ g/\text{cm}^3$ $\quad R_{\text{init}} \simeq 2.6\text{mm}$

Convergence ratio ~20

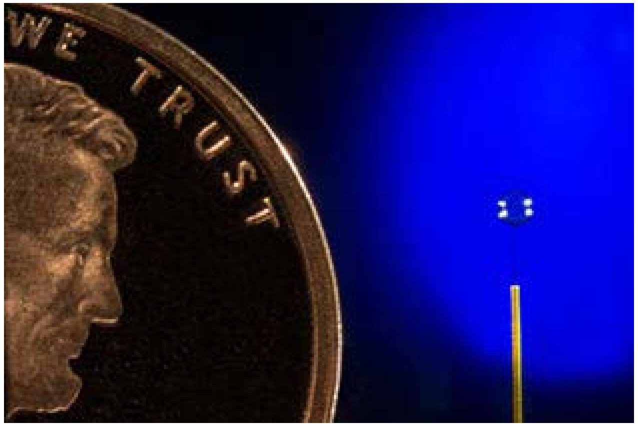
A spherical capsule can be imploded through directly or indirectly laser illumination

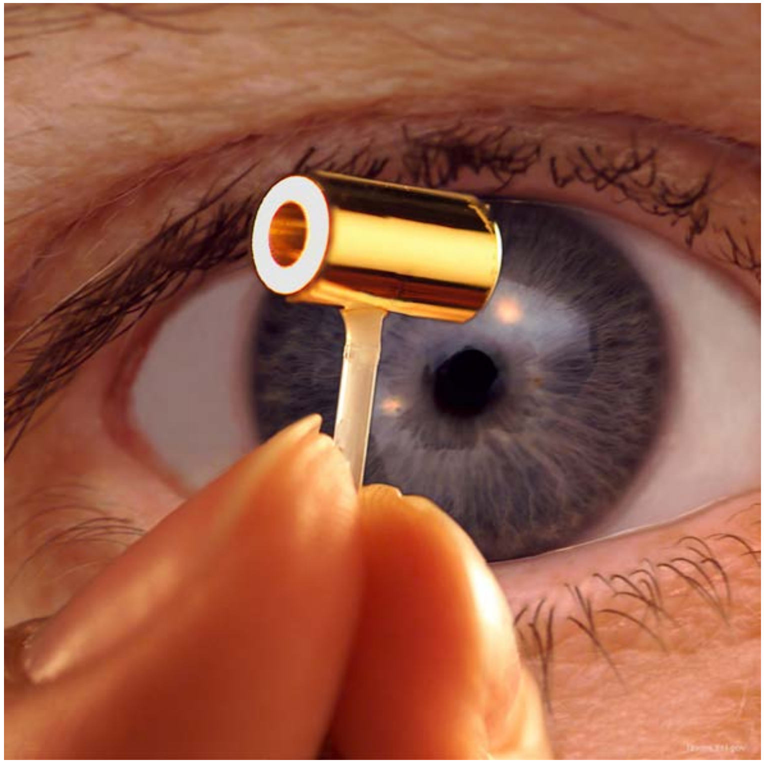




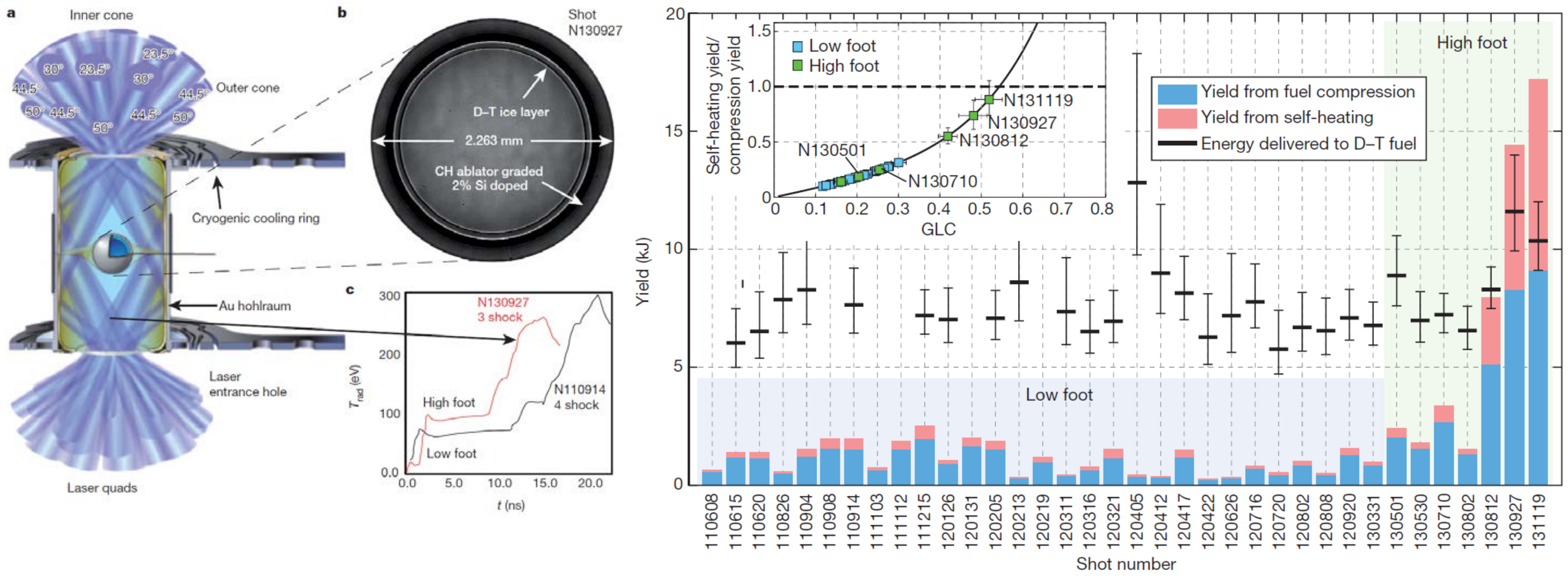
*R. Betti, HEDSA HEDP Summer School, 2015

Targets used in ICF





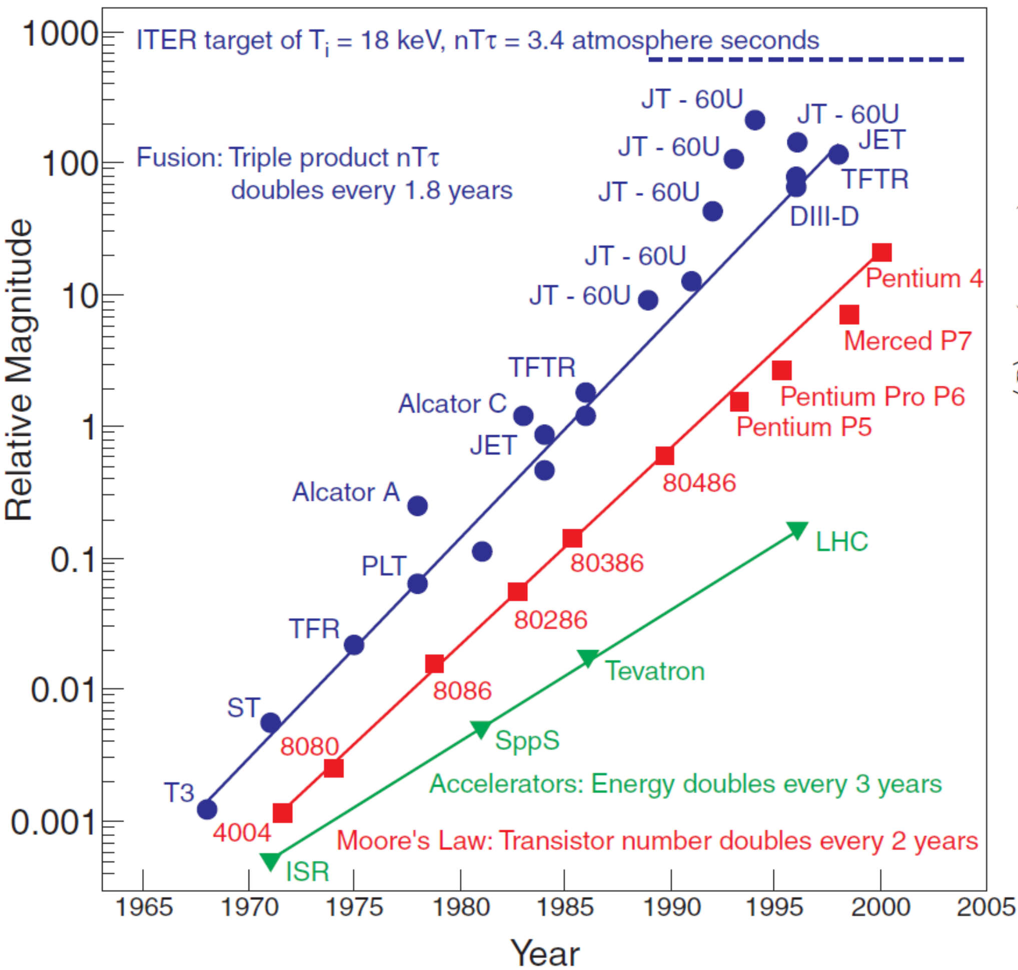
Nature letter “Fuel gain exceeding unity in an inertially confined fusion implosion”

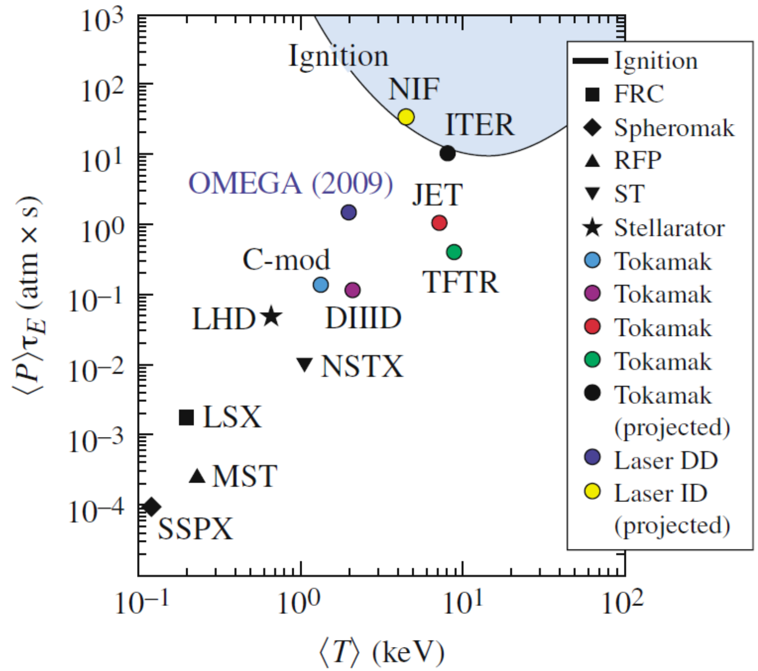


- **Fuel gain exceeding unity was demonstrated for the first time.**

Nature 506, p343, 2014

The performance of a fusion plasma has doubled every 1.8 years like the Moore's law





- **Nuclear fusion is coming in the near future.**

A. J. Webster, Phys. Educ. **38**, 135 (2003)
R. Betti, etc., Phys. Plasmas, **17**, 058102 (2010)