

Pulsed Power system

PI

* Prerequisite courses:

Electric Circuits.

Engineering Mathematics / Phys Mathematics (ODE)

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No office hour, please feel free to stop by my office whenever my door is open & nobody is in my office. Or you may email me to schedule a meeting.

* Class time : 9:10 ~ 12:00

→ 9:10 ~ 11:30

11:30 ~ 12:00

English ~~class~~ with 10 mins break

~~class~~ Mandarin for PQA

* Assignments : 70% → No roll call.

Presentations : 30%

* References: Pulsed power systems by H. Bluhm.
Circuits analysis by Cunningham and Stuller

* Additional References:

Pulsed Power by Gennady A. Mesyats

J. C. Martin on Pulsed Power

* Class material: myweb.ncku.edu.tw/~pchang

* Course outline:

- Introduction to pulsed-power system. - 9/14 → 9/21. p2
- Review of circuit analysis - 9/21. ← ^{4 hr. of rec.} _{contents}

72. Static and dynamic breakdown strength of dielectric material. - 9/28, 10/5

- [Gas. → avalanche, Townsend condition, Paschen Law. ← ^{4 hr?}
- Liquid
- Solid.

73 Energy storage. 10/12 - 10/19.

- [pulsed discharge capacitors ← ^{4 hr. of exploring Marx}
- Marx Generators.
- [Inductive energy storage.

74 Switches . 10/26 - ~~11/2~~. 11/9

← ~~calorimetry experiments~~
~~solving Poisson's eq.~~

- [Closing switches → gas switches
- [Opening switches →

75 Pulse-forming networks

~~10/9~~; ~~10/16~~ 11/23
11/16

- [Transmission lines
- [RLC Networks

76 Pulse transmission and transformation

~~11/2~~ - ~~11/3~~ 11/17

- [self-magnetic insulation in vacuum lines
- [pulse transformers
- [High voltage power supply
- [Transformation lines

37. Power and voltage adding ~~12/14~~ 12/14 P3

- └─ Addng of power
- └─ voltage adding

39. Diagnostics. 12/1 - 12/28.

- └─ Electromagnetic - Field Sensors
 - └─ Capacitive sensor
 - └─ Inductive
- └─ Current - viewing resistors (CVRs)
- └─ Current measurements based on Faraday Effect.
- └─ Z - field - - - - - Electro-optic effects.
- └─ Magnetic ion energy analysers
- └─ Vacuum voltage monitors

310. Applications of pulsed-power system 1/4

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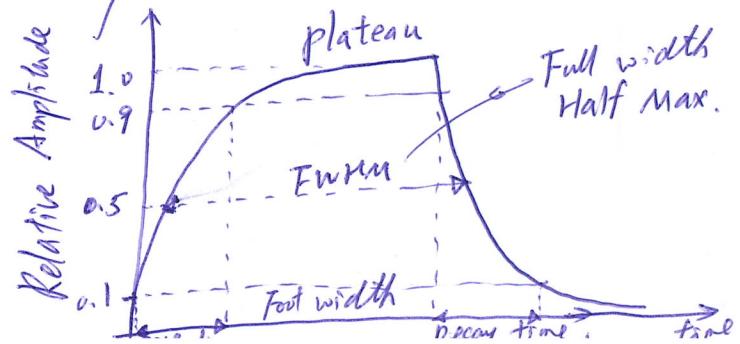
3 Introduction to pulsed-power system

P4.

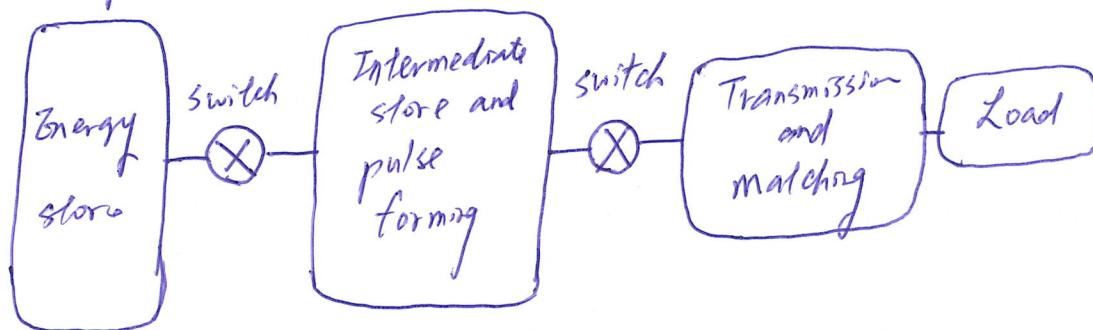
- * Pulsed power is a scheme where stored energy is discharged as electrical energy into a load in a short pulse or as short pulses with a controllable repetition rate.
- * Examples of pulsed power in daily life:
 - Driven piles
 - Hammer.
 - making ~~steamy~~ rice cake. → great example of short pulses with a controllable repetition rate.
- * Pulsed power in general:
 - $P \sim 10^9 \text{ W}$ (1 GW)
 - $E \gtrsim \text{kJ}$

The highest energy and power that have been achieved in a single pulse are in the order of 100 MJ & few hundred TW, respectively.

 - $V: 10 \text{ kV} \sim 50 \text{ MV}$
 - $I: 1 \text{ kA} \sim 10 \text{ MA}$.
- * A pulse is characterised by its shape, i.e., by its rise time & fall times and by the duration and flatness of its plateau region



- P5
- Pulse rise time - the time it takes the voltage to rise from 10% to 90%.
 - Pulse fall time - - - - - fall
decay 90% to 10%.
 - Both the fall and the rise time of a pulse depend on the evolution of the "Load impedance," which in most cases varies with time.
 - Pulse duration - no unique definition.
 - ↳ FWHM
 - ② ~~some~~ It is defined as the duration at 90% of the peak amplitude.
 - ↳ Flatness of the plateau region is an important requirement for driving some ~~loads~~ such as Pockels cells.
 - A ~~good~~ generator scheme for the production of high-power electrical pulses is always based on an energy store that is charged slowly at a relatively low charging power and is discharged rapidly by acting a switch.
 - To achieve the desired power magnification factor and to shape the pulse, the above process can be repeated several times.

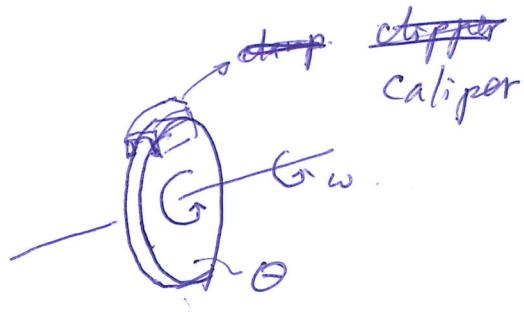


- * The energy can be stored either chemically, mechanically, pb. or electrically.

Ex: Mechanical energy:

$$W_{kin} = \frac{1}{2} \Theta \omega^2$$

↑ ← angular frequency
moment of inertia



For a massive cylinder: $\Theta = \frac{1}{2} M r^2$

↑ ↑
mass radius

$$\Rightarrow W_{kin} = \frac{1}{2} \cdot \frac{1}{2} M r^2 \cdot \omega^2$$

$$\Rightarrow \text{Stored energy density: } \underbrace{\omega}_{\text{tension}} = \frac{W_{kin}}{M} = \frac{1}{4} r^2 \omega^2$$

- The ultimate energy density is limited by the tensile strength of the material used to construct the rotor.



$$\begin{aligned} d\Sigma &= \frac{dF}{A} \\ &= \frac{(r d\theta) s dr \delta}{r d\theta \cdot \delta} \cdot r \omega^2 \\ &= \rho \omega^3 r dr \\ \Sigma &= \int_0^R \rho \omega^3 r dr \end{aligned}$$

$$\Sigma = \int \omega_{max}^2 \frac{r^2}{2} \quad \begin{array}{l} \text{for a stainless steel cylinder} \\ \text{or radius 1m.} \end{array}$$

↑ ↓
tensile strength tensile strength

$$\omega_{max} = 400 \text{ / sec.} \quad \begin{array}{l} \text{AISI 302} \\ \text{stainless steel} \end{array}$$

$$\Sigma = 520 / 860 \text{ MPa}$$

↑ ↑
yield strength ultimate tensile strength.

$$\rho = 8.19 \text{ g/cm}^3 = 8190 \text{ kg/m}^3$$

$$\Rightarrow \omega = \cancel{400} \cdot \sqrt{\frac{2 \Sigma}{\rho r^2}} = \sqrt{\frac{2 \cdot 860 \times 10^6}{8190 \times 1}} = 458 \sim 400$$

$$W_{kin} = \frac{1}{4} r^2 \omega^2 = \frac{1}{4} \cdot 1^2 \cdot 400^2 = 4 \times 10^4 \text{ J/kg}$$

high strength alloy ASTM A514 steel

$$\rho = 7.8 \text{ g/cm}^3. \quad \Sigma = 690 / 760 \text{ MPa}$$

↑ ↑
yield strength ultimate tensile strength.

$$\omega = \sqrt{\frac{2 \times 690 \times 10^6}{7800 \times 1^2}} = 420 \sim 400 \text{ / sec}$$

$$W_{kin} = \frac{1}{4} r \omega^2 = \frac{1}{4} \times 1 \times 400^2 = 4 \times 10^4 \text{ J/kg} = 3.1 \times 10^8 \text{ J/m}^3$$

- The problem with mechanical storage is to release the energy in a sufficiently short time.
- Several electrical compression stages are needed in combination with the mechanical storage to achieve the desired power level.

Sx.: Electrical energy can be stored either capacitively in an electric field or inductively in a magnetic field.

Sx1: Electric field:

$$W_e = \frac{1}{2} \epsilon_0 E^2$$

for oil/impregnated paper: $\epsilon = 6$, breakdown strength $E = 0.98 \times 10^8 \text{ V/m}$

$$\Rightarrow W_e = \frac{1}{2} \times 6 \times 8.85 \times 10^{-12} \cdot (0.98 \times 10^8)^2 = 161 \text{ kJ/m}^3$$

With the finite packing density:

$$E \approx E_{\text{re}} = \frac{1}{2} C V^2, C = \epsilon_0 \cdot \frac{A}{d}$$

$\Rightarrow d \rightarrow, E_{\text{re}} \rightarrow \rightarrow$ space is occupied by the electrode.

\Rightarrow to estimate the energy storage in space

$$W'_e \approx \frac{1}{2} \times W_e \approx 80 \text{ kJ/m}^3$$



Sx2: Magnetic field:

$$W_B = \frac{1}{2} \mu_0 D M_0 B^2 \frac{B^2}{2M_0} = \frac{1}{2} \mu_0 B^2$$



The maximum energy density is limited by the onset of melting at the conductor surface or by the mechanical strength of the storage inductor.

$$C_0 \cdot g \cdot T = \frac{B^2}{2\mu_0} \cdot \frac{1}{2M_0} B^2 \vartheta \cdot$$

↑ ↓ ↓ ↓
 heat capacity mass density surface temperature
 per unit mass

a factor of order unity depending on the form of the pulse

$$B \cdot l = \mu_0 N I$$

$$\Rightarrow B = \mu_0 N I \propto I$$

$$R E = P \cdot t = I^2 \cdot R \cdot t \propto B^2 \cdot R \cdot t$$

$$\Rightarrow C_0 \propto T = \frac{1}{2\mu_0} B^2 \propto \frac{B^2}{2\mu_0} \quad \text{take } \theta = 1$$

$$B^2 \approx \sqrt{2\mu_0 \cdot C_0 \cdot P \cdot T} = \sqrt{2 \times 4\pi \times 10^{-7} \times \frac{0.385 \text{ J}}{10^3 \text{ kg} \cdot \text{K}}} \times 8960 \times (1085 - 25)$$

Copper: $C_0 = 0.385 \text{ J/g} \cdot \text{K}$ $= 96 \text{ T} \approx 100 \text{ T}$

Melting T: 1085°C .

$$\rho = 8.96 \text{ g/cm}^3 = 8960 \text{ kg/m}^3$$

$$B \leq \Sigma \rightarrow \text{yield stress} \quad \Sigma = 70 \text{ MPa for Cu.}$$

$$\Rightarrow \frac{B^2}{2\mu_0} \leq \Sigma \rightarrow B \leq \sqrt{2\mu_0 \Sigma} = \sqrt{2 \times 4\pi \times 10^{-7} \times 70 \times 10^6} = 13 \text{ T}$$

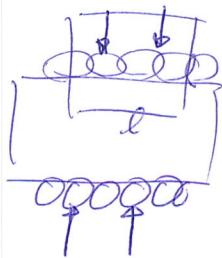
$$\Rightarrow W_B = \frac{1}{2} \mu_0 B^2 = \frac{1}{2} \mu_0 \times 13^2 = \frac{1}{2} \times 4\pi \times 10^{-7} \times 169 = 6.7 \times 10^7 \text{ J/m}^3$$

$$\Rightarrow W_B = \frac{1}{2} \mu_0 B^2 = \frac{1}{2} \times 4\pi \times 10^{-7} \times 169 = 6.7 \times 10^7 \text{ J/m}^3$$

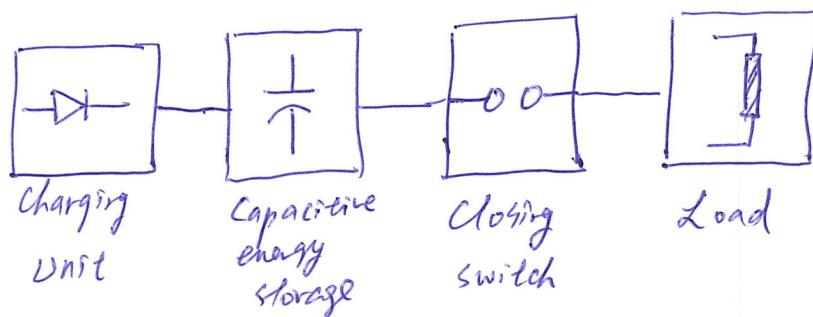
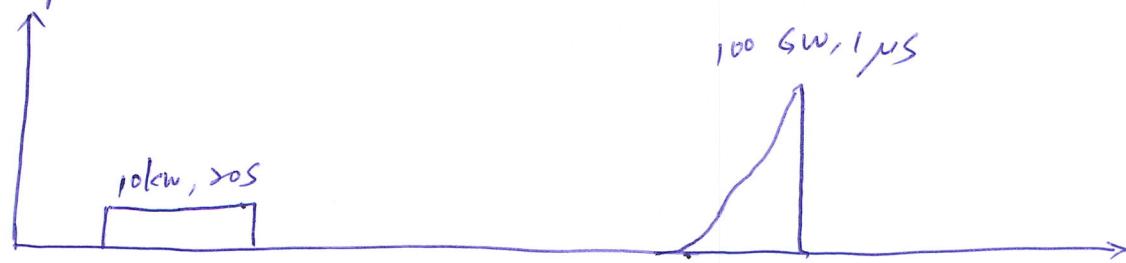
\Rightarrow The energy density stored in a magnetic field can be about 2 orders of magnitude higher than that storables in an electric field!!

* Capacitive storage - requires one or more closing switches which remain open during charging and hold the charging voltage.

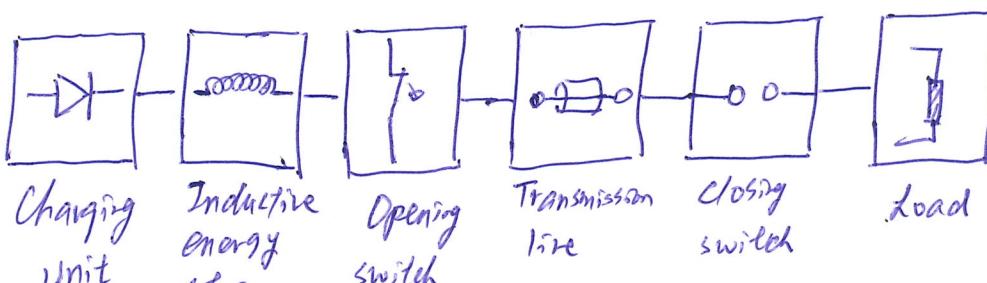
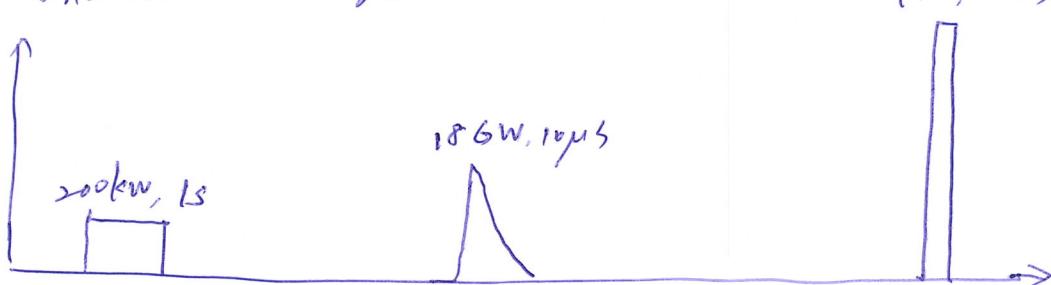
- power multiplication is done by current amplification.
* Inductive storage - requires an opening switch which is closed during charge-up, carrying a large current at this stage.
- power multiplication is done by voltage amplification.



Capacitive storage $W_e = 10 - 80 \text{ kJ/m}^3$

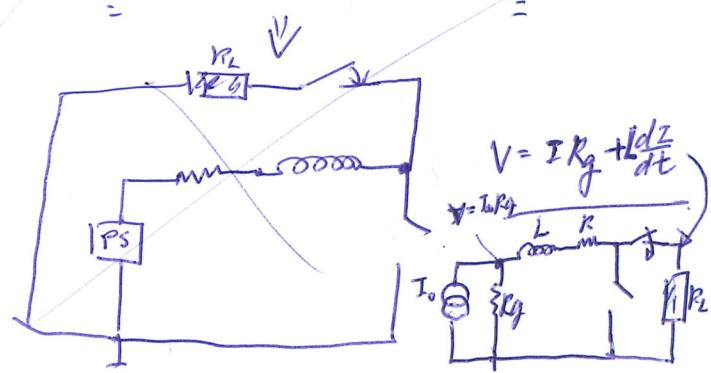
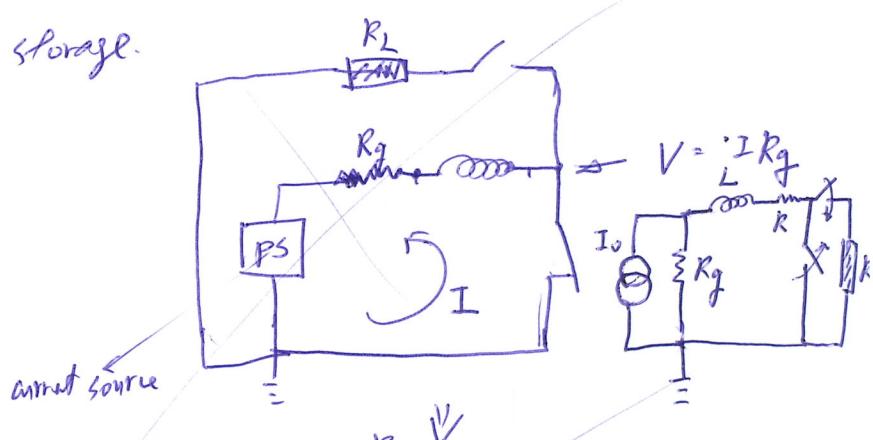
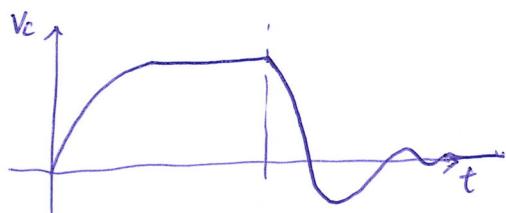
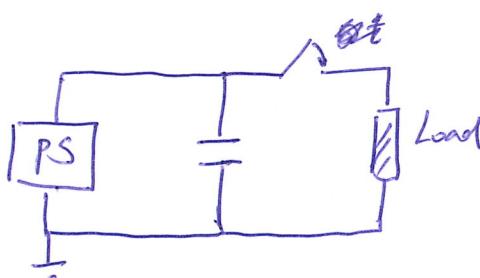


Inductive storage $W_m = 1 - 50 \text{ MJ/m}^3$



1. Compressing stage 2. Compressing stage (or pulse forming line.)

Example of capacitive storage.



3 Review of circuit analysis

* Kirchhoff's Current Law.

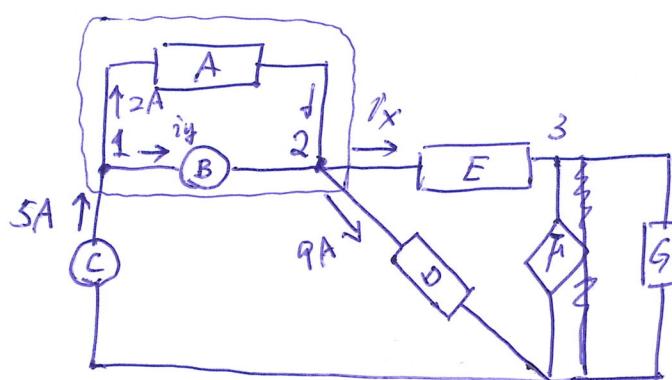
- At any instant in time, the algebraic sum of all currents leaving any closed surface is zero

$$i_1 + i_2 + \dots + i_N = 0,$$

or in abbreviated notation:

$$\sum_{k=1}^N i_k = 0$$

where i_k is the k^{th} current of the N currents leaving the closed surfaces.



$$\begin{aligned}
 i_y + 2 - 5 &= 0 \\
 \Rightarrow i_y &= 3 \text{ (A)} \\
 -5 + i_x + 9 &= 0 \\
 \Rightarrow i_x &= -4 \text{ (A)}
 \end{aligned}$$

* Kirchhoff's Voltage Law.

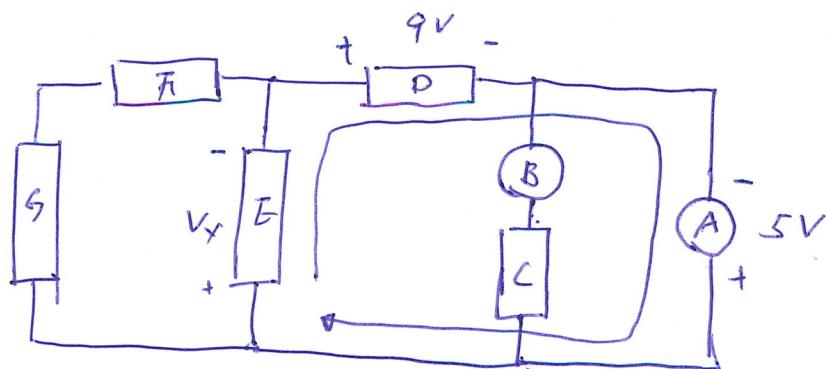
- At any instant in time, the algebraic sum of all voltage drops taken around any closed path is 0:

$$v_1 + v_2 + \dots + v_N = 0.$$

or in abbreviated notation:

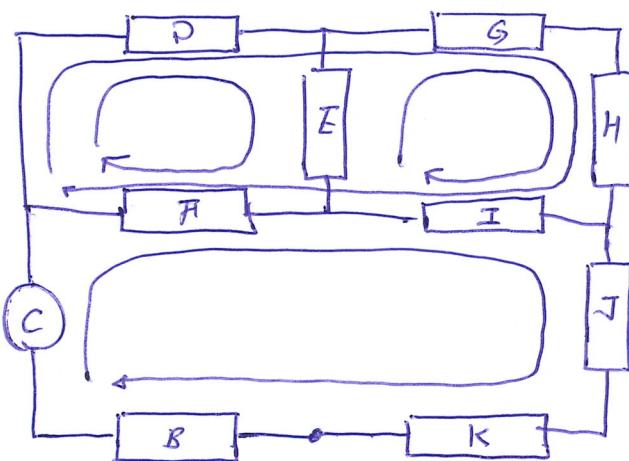
$$\sum_{k=1}^N v_k = 0$$

where v_k is the voltage drop, taken in the direction of the path along the k^{th} segment of the N segments in the closed path.

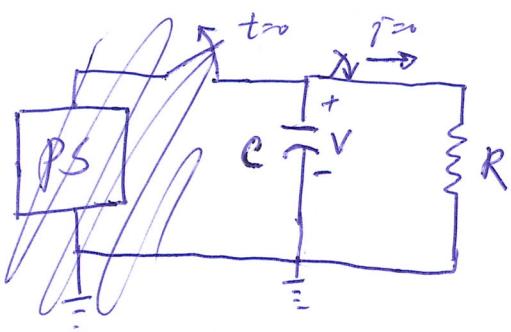


$$-V_x - 9 + 5 = 0 \Rightarrow V_x = -4 \text{ V}$$

* Loops, Meshes, and Planar Networks



* Source-free RC circuit.



* Assuming
Assuming that the capacitor is fully charged to V_0 .
At $t=0$, the switch is opened

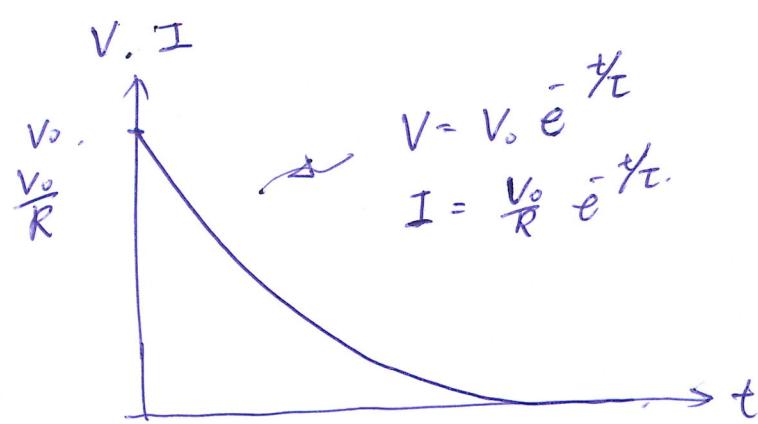
$$V_C - IR = 0 \quad I = \frac{dQ}{dt} = -C \frac{dV}{dt} \quad C = \frac{Q}{V}$$

$$\Rightarrow V_C + RC \frac{dV}{dt} = 0 \quad \text{or} \quad \frac{dV}{dt} + \frac{1}{RC} V = 0$$

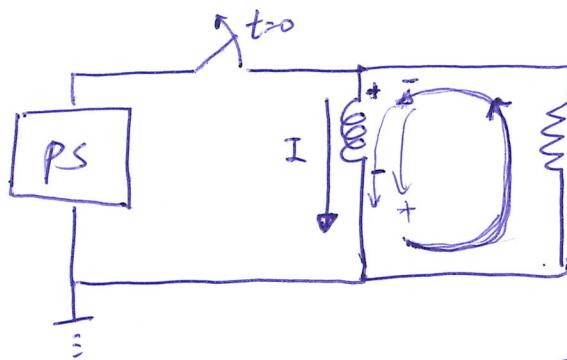
$$\Rightarrow + \int \frac{dV}{V} = - \frac{1}{RC} \int dt \quad \Rightarrow \ln \frac{V(t)}{V_0} = - \frac{t}{RC}$$

$$\Rightarrow V(t) = V_0 e^{-\frac{t}{RC}}$$

$$I = -C \frac{dV}{dt} = +V_0 C \left(+\frac{1}{RC} \right) e^{-\frac{t}{RC}} = \frac{V_0}{R} e^{-\frac{t}{RC}}$$



* Source - free RL circuit



* Assume the current is at steady state for $t < 0$, $I(t=0) = I_0$
At $t=0$, the switch is open

~~$$V_L = IR + V_B \Rightarrow V_L = L \frac{dI}{dt}$$~~

$$-IR - V_L = 0 \quad V_L = L \frac{dI}{dt}$$

$$\Rightarrow IR + L \frac{dI}{dt} = 0 \quad \Rightarrow \frac{dI}{dt} + \frac{R}{L} I = 0$$

$$\Rightarrow \int \frac{dI}{I} = -\frac{R}{L} dt \quad \Rightarrow \ln \frac{I(t)}{I(0)} = -\frac{R}{L} t = -\frac{\tau_L}{L} t \quad \tau_L = \frac{L}{R}$$

$$\Rightarrow I(t) = I_0 e^{-\frac{R}{L} t} = I_0 e^{-\frac{t}{\tau_L}}$$

$$V(t) = L \frac{dI}{dt} = L \cdot I_0 \cdot \left(-\frac{1}{\tau_L}\right) e^{-\frac{t}{\tau_L}} = -\frac{R I_0}{\tau_L} e^{-\frac{t}{\tau_L}}$$

