

PULSED POWER SYSTEM

脈衝功率系統



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2023 Fall Semester

Tuesday 9:10-12:00

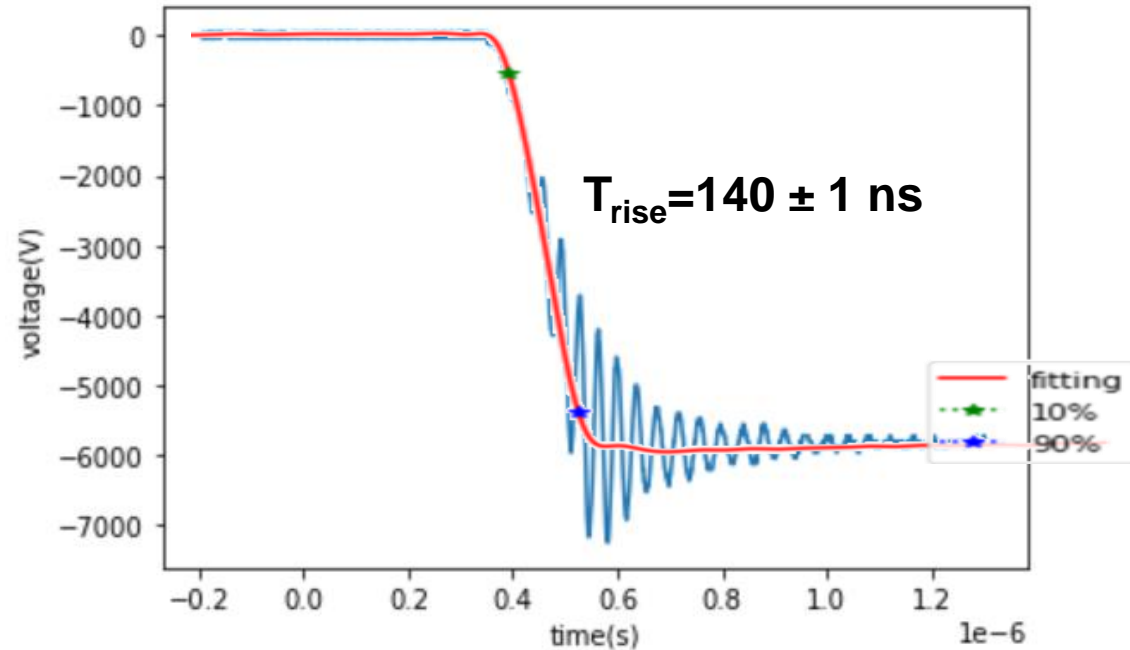
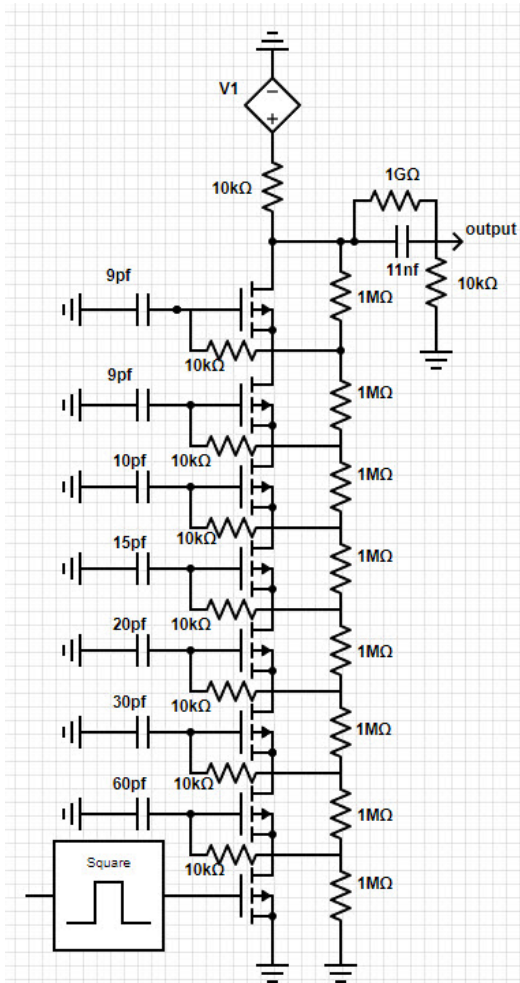
Lecture 11

<http://capst.ncku.edu.tw/PGS/index.php/teaching/>

Online courses:

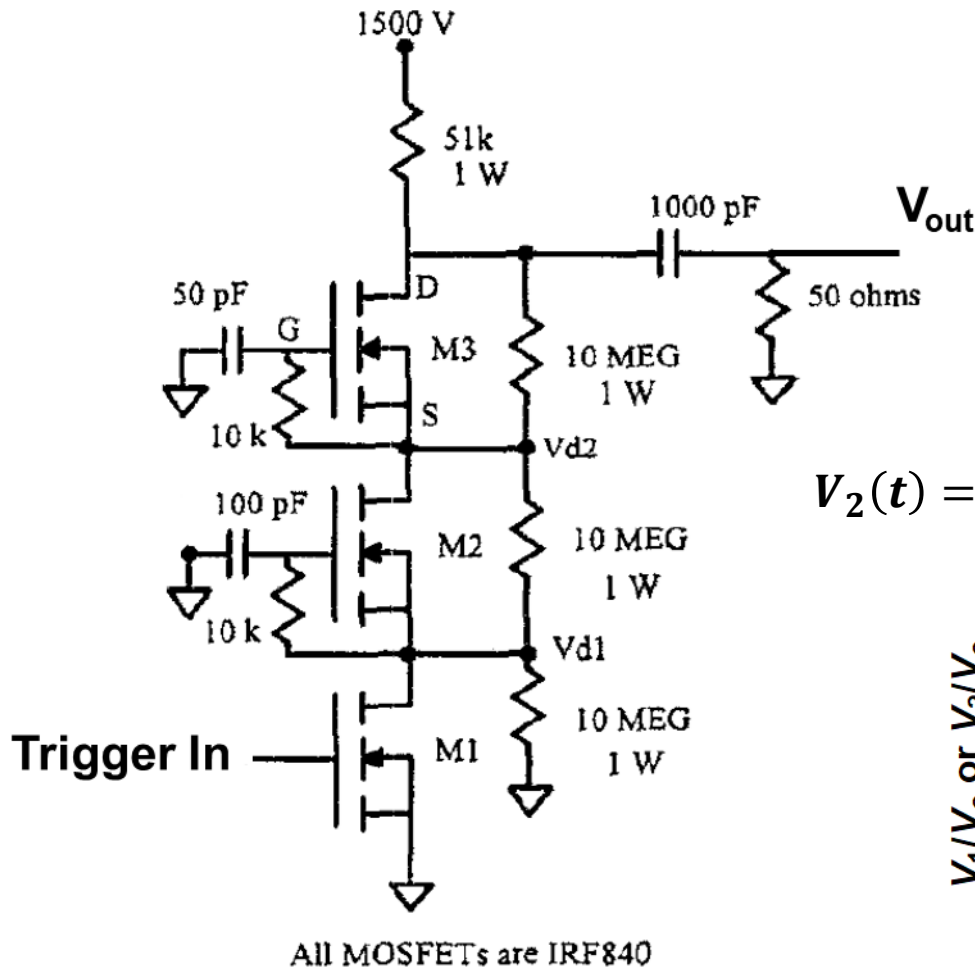
<https://nckucc.webex.com/nckucc/j.php?MTID=md577c3633c5970f80cbc9e821927e016>

Many MOSFET connected in series can be used to provide a fast high-voltage triggering pulse



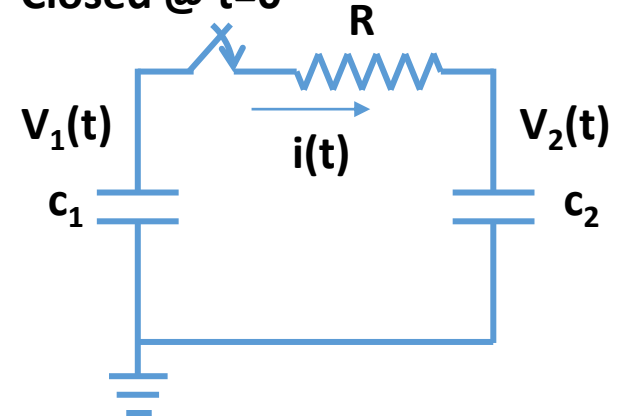
Old one:
 $T_{\text{rise}} = 55.5 \text{ us}$
Jitter: $\pm 0.4 \mu\text{s}$

Pulse generator by stacking power MOSFETs

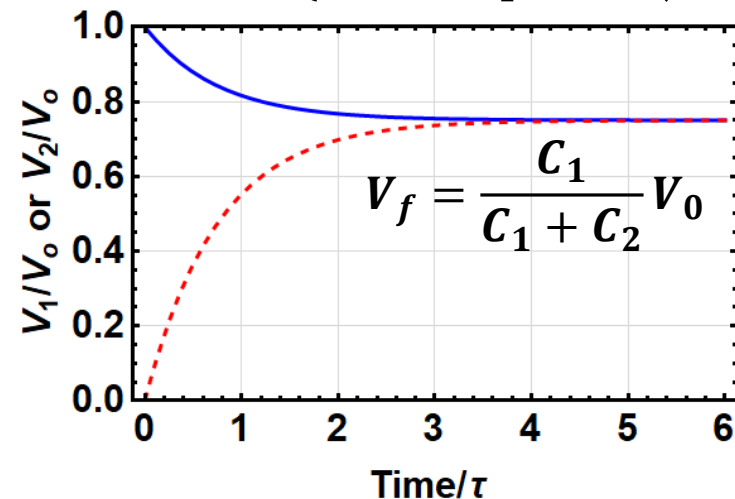


$C_{gs} = 2500 \text{ pF}$

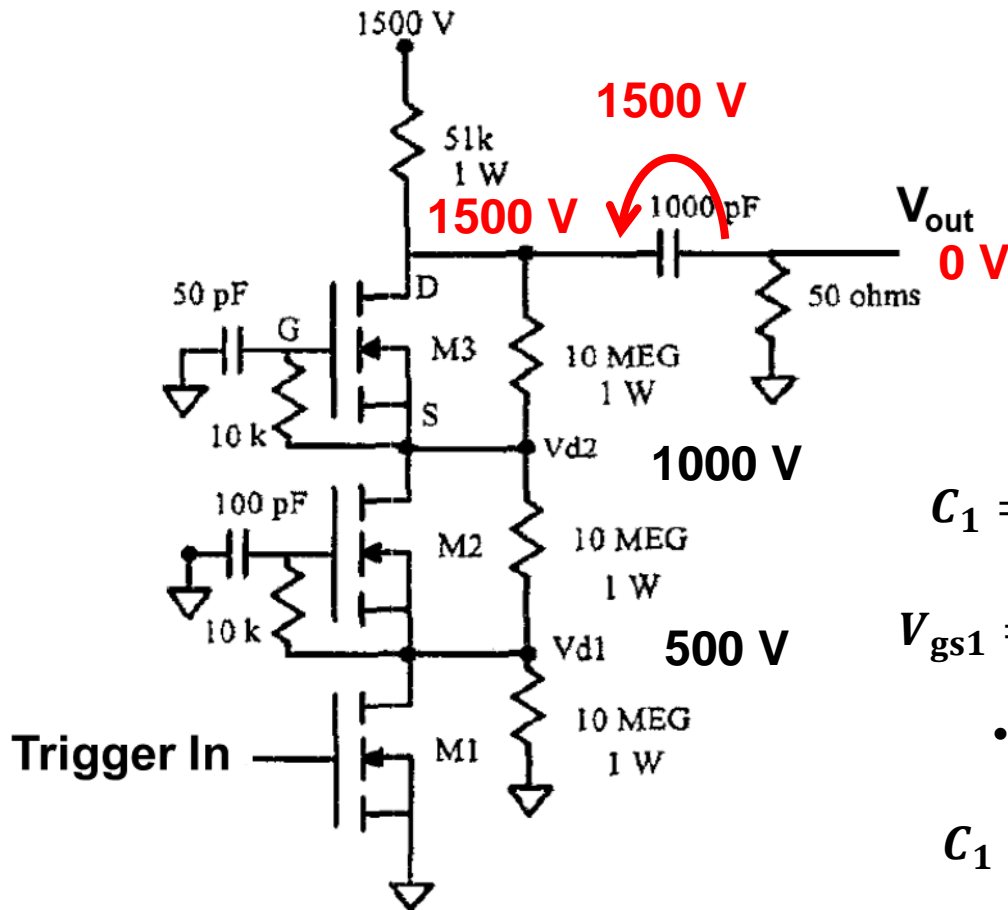
Closed @ $t=0$



$$V_2(t) = \frac{C_1}{C_1 + C_2} V_0 \left\{ 1 - \exp \left[- \frac{t}{R} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \right] \right\}$$



Pulse generator by stacking power MOSFETs



All MOSFETs are IRF840

$C_{gs} = 2500 \text{ pF}$

$$V_f = \frac{C_1}{C_1 + C_2} V_0 = \frac{C_i}{C_i + C_{gs}} V_0$$

$$C_i = \frac{V_f}{V_0 - V_f} C_{gs}$$

• i=1:

$$C_1 = \frac{20}{500 - 20} \times 2500 = 104 \approx 100 \text{ pF}$$

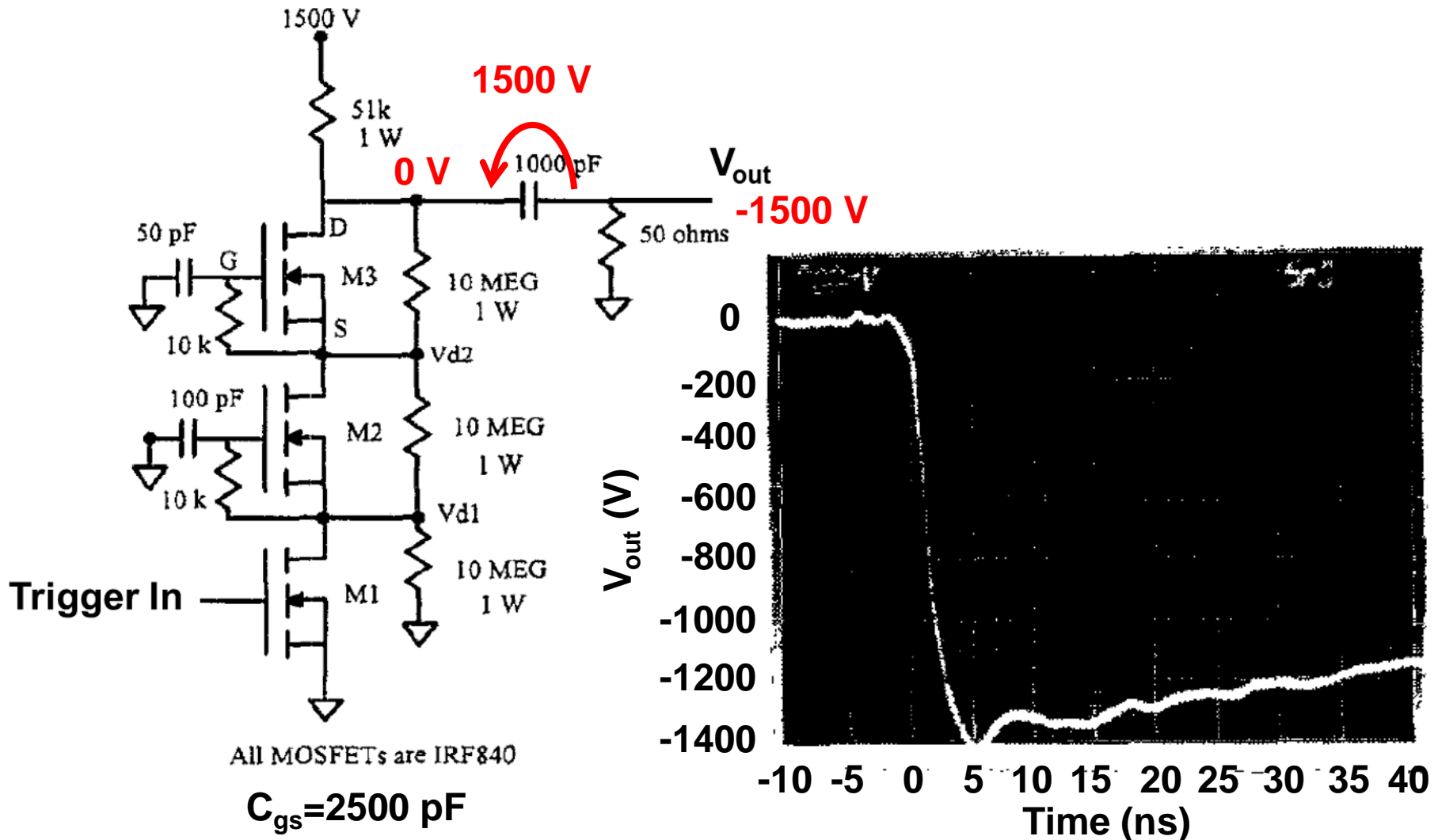
$$V_{gs1} = \frac{100}{2500 + 100} \times 500 = 19.2 \text{ V}$$

• i=2:

$$C_1 = \frac{20}{1000 - 20} \times 2500 = 51 \approx 50 \text{ pF}$$

$$V_{gs2} = \frac{50}{2500 + 50} \times 1000 = 19.6 \text{ V}$$

Pulse generator by stacking power MOSFETs



Outlines



- **Switches**
 - Closing switches: the switching process is associated with voltage breakdown across an initially insulant element.
 - Opening switches: the switching process is associated with a sudden growth of its impedance.
- **Pulse-forming lines**
 - **Blumlein line**
 - Pulse-forming network
 - Pulse compressor
- **Pulse transmission and transformation**

Pulse-forming lines

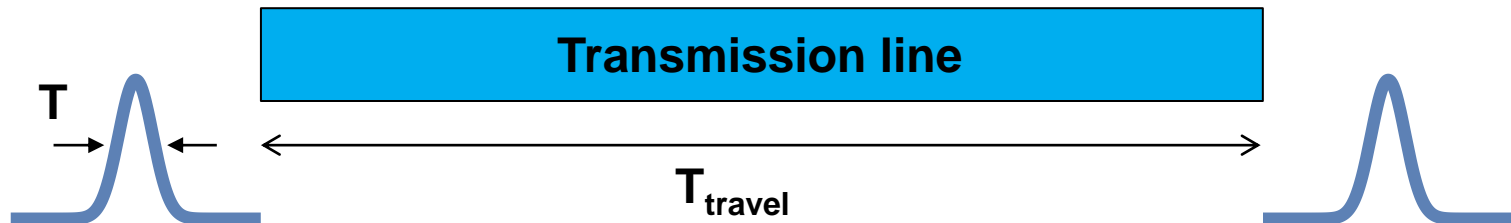


- **A constant-voltage plateau is needed for many pulsed-power applications.**
- **Various arrangements of LC elements are necessary. It is called “Pulse-Forming Networks (PFN)”.**

Transmission lines



- Transmission lines are the continuous borderline case of a network consisting of discrete LC elements.
- Depending on the time T during which energy is extracted from or supplied to the element, a transmission line can be described as lumped circuit element or an extended object.
 - $T > T_{\text{travel}}$, the time it takes for an EM wave to move from one terminal of the element to the next \rightarrow lumped circuit element.
 - $T < T_{\text{travel}}$, the time it takes for an EM wave to move from one terminal of the element to the next \rightarrow Transmission line.



Different kinds of transmission line and the inductance and the capacitance per unit

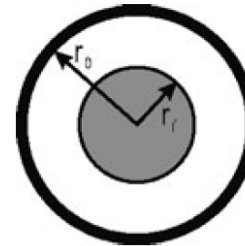


1. Coaxial transmission line:

$$C' = 2\pi\epsilon / \ln(r_o/r_i)$$

$$L' = (\mu/2\pi) \ln(r_o/r_i)$$

$$Z_0 = \left((\mu/\epsilon)^{1/2} / 2\pi \right) \ln(r_o/r_i)$$
$$= 60(\mu_r/\epsilon_r)^{1/2} \ln(r_o/r_i)$$

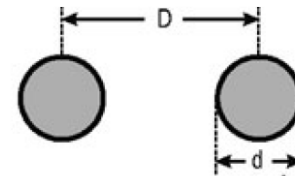


2. Double-wire line:

$$C' = \pi\epsilon / \operatorname{arcosh}(D/d)$$

$$L' = (\mu/\pi) \operatorname{arcosh}(D/d)$$

$$Z_0 = \left((\mu/\epsilon)^{1/2} / \pi \right) \operatorname{arcosh}(D/d)$$

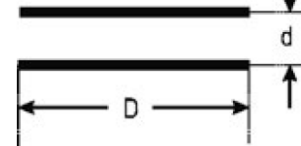


3. Parallel-plate line:

$$C' = \epsilon D/d$$

$$L' = \mu d/D$$

$$Z_0 = (\mu/\epsilon)^{1/2} (d/D)$$

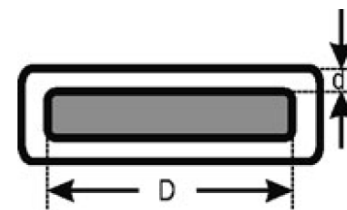


4. Stripline:

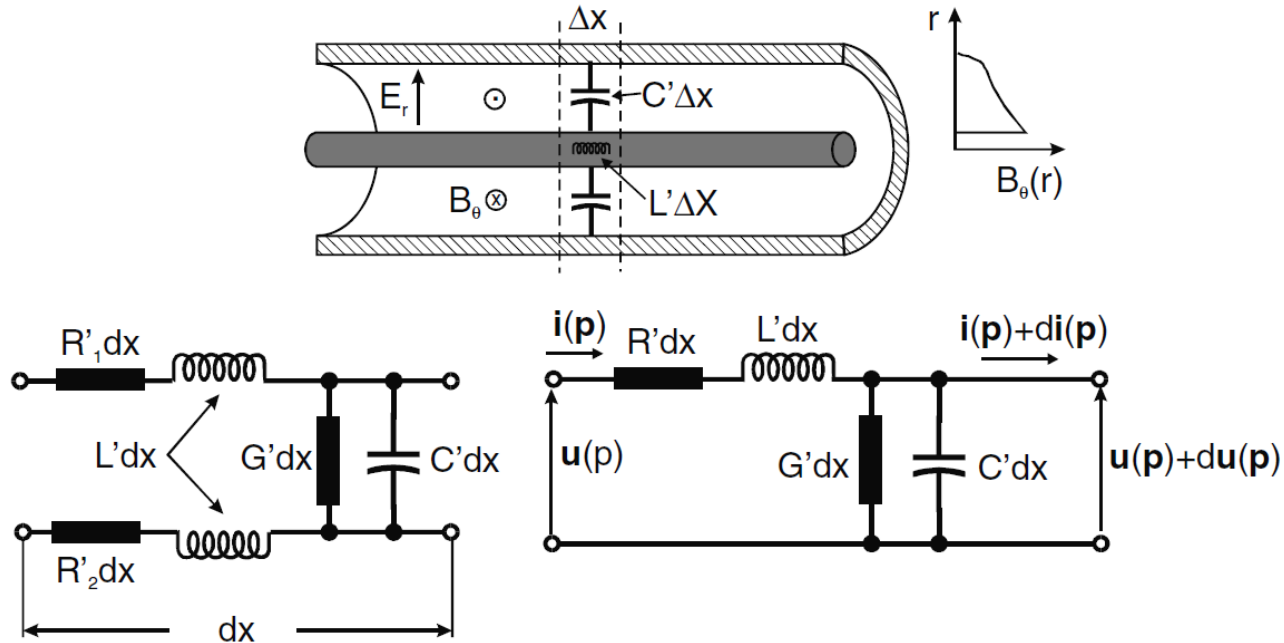
$$C' = 2\epsilon D/d$$

$$L' = \mu d/2D$$

$$Z_0 = (\mu/\epsilon)^{1/2} (d/2D)$$

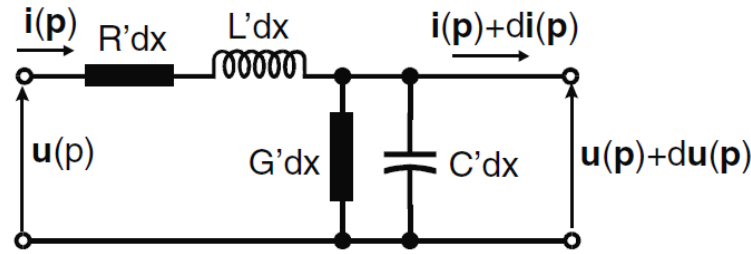


An infinitesimal section of a homogeneous coaxial transmission line



- Resistances per unit length: $R'_1 + R'_2 \rightarrow R' \rightarrow R$.
- Conductance per unit length: $G' \rightarrow G$.
- All quantities are frequency-dependent because of the skin effect and because the dielectric constant depends on the frequency.
- Assume that they are independent of the position x , the voltage V and current I .

An infinitesimal section of a homogeneous coaxial transmission line



$$U - IRdx - Ldx \frac{dI}{dt} - U' = 0$$

$$I - U'Gdx - Cdx \dot{U}' - I' = 0 \quad U' = U + \frac{\partial U}{\partial x} dx \quad \dot{U}' = \dot{U} + \frac{d}{dt} \left(\frac{\partial U}{\partial x} \right) dx$$

$$I - \left(U + \frac{\partial U}{\partial x} dx \right) Gdx - Cdx \left[\dot{U} + \frac{d}{dt} \left(\frac{\partial U}{\partial x} \right) dx \right] - I' = 0$$

$$I - UGdx - \frac{\partial U}{\partial x} Gdx^2 - \dot{U}Cdx - \frac{d}{dt} \left(\frac{\partial U}{\partial x} \right) Cdx^2 - I' = 0$$

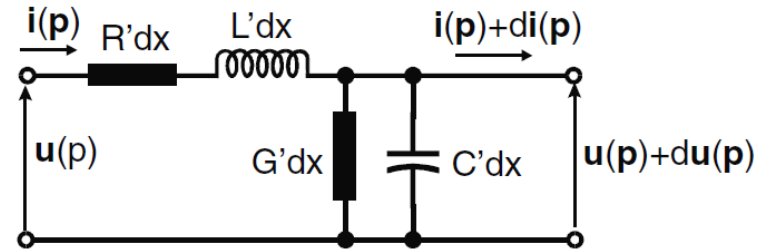
$$I - UGdx - Cdx \dot{U} - I' = 0$$

$$\begin{cases} \frac{U' - U}{dx} \equiv \frac{\partial U}{\partial x} = -RI - LI \\ \frac{I' - I}{dx} \equiv \frac{\partial I}{\partial x} = -GU - C\dot{U} \end{cases}$$

An infinitesimal section of a homogeneous coaxial transmission line



$$\left\{ \begin{array}{l} \frac{U' - U}{dx} \equiv \frac{\partial U}{\partial x} = -RI - LI \\ \frac{I' - I}{dx} \equiv \frac{\partial I}{\partial x} = -GU - CU \end{array} \right.$$



- Laplace transform: $x = \tilde{x}e^{pt}$, $\frac{d}{dt} \rightarrow p$

$$\frac{d\tilde{U}}{dx} = -R\tilde{I} - pL\tilde{I} = -(R + pL)\tilde{I}$$

$$\frac{d^2\tilde{U}}{dx^2} = -(R + pL)\frac{d\tilde{I}}{dx} = (R + pL)(G + pC)\tilde{U}$$

$$\frac{d\tilde{I}}{dx} = -G\tilde{U} - pC\tilde{U} = -(G + pC)\tilde{U}$$

$$\frac{d^2\tilde{I}}{dx^2} = -(G + pC)\frac{d\tilde{U}}{dx} = (G + pC)(R + pL)\tilde{I}$$

- Lossless line where $R=0$, $G=0$:

$\frac{d\tilde{U}}{dx} = -pL\tilde{I}$	$\frac{d^2\tilde{U}}{dx^2} = p^2LC\tilde{U}$
$\frac{d\tilde{I}}{dx} = -pC\tilde{U}$	$\frac{d^2\tilde{I}}{dx^2} = p^2LC\tilde{I}$

An infinitesimal section of a homogeneous coaxial transmission line

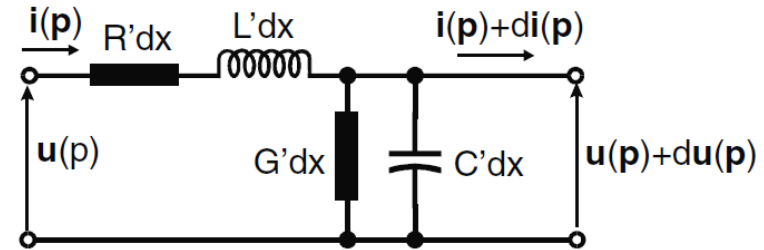


- Lossless line where $R=0$, $G=0$:

$$\frac{d^2 \tilde{U}}{dx^2} = p^2 LC \tilde{U}$$

$$\frac{d^2 \tilde{I}}{dx^2} = p^2 LC \tilde{I}$$

$$U(x, p) = U_x(p) = \begin{cases} \tilde{U}_+ e^{-p\sqrt{LC}x} \\ \tilde{U}_- e^{p\sqrt{LC}x} \end{cases}$$



- Inverse Laplace transform: $L\{U_x(t - \tau)\} = \tilde{U} e^{-pt}$

$$U(x, t) = U_x(t) = \begin{cases} U_+(t - x\sqrt{LC}) \\ U_-(t + x\sqrt{LC}) \end{cases}$$

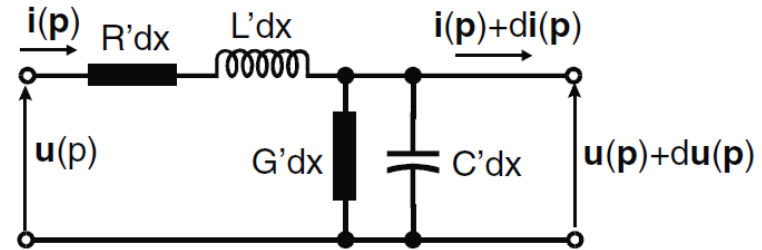
or Linear combination:

$$U_x(t) = U_+(t - x\sqrt{LC}) + U_-(t + x\sqrt{LC})$$

An infinitesimal section of a homogeneous coaxial transmission line



$$U(x, t) = U_x(t) = \begin{cases} U_+(t - x\sqrt{LC}) \\ U_-(t + x\sqrt{LC}) \end{cases}$$



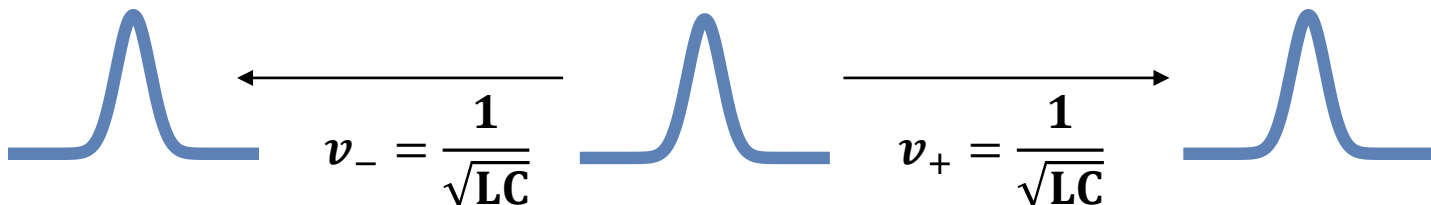
or Linear combination: $U_x(t) = U_+(t - x\sqrt{LC}) + U_-(t + x\sqrt{LC})$

$$\frac{d\tilde{U}}{dx} = -pL\tilde{I} \quad \mp p\sqrt{LC}\tilde{U}_{\pm} = -pL\tilde{I}_{\pm}$$

$$\frac{\tilde{U}_+}{\tilde{I}_+} = -\frac{\tilde{U}_-}{\tilde{I}_-} = \sqrt{\frac{L}{C}} \equiv Z_0$$

$$Z_0 = \frac{U_+(t - x\sqrt{LC})}{I_+(t - x\sqrt{LC})} = -\frac{U_-(t + x\sqrt{LC})}{I_-(t + x\sqrt{LC})}$$

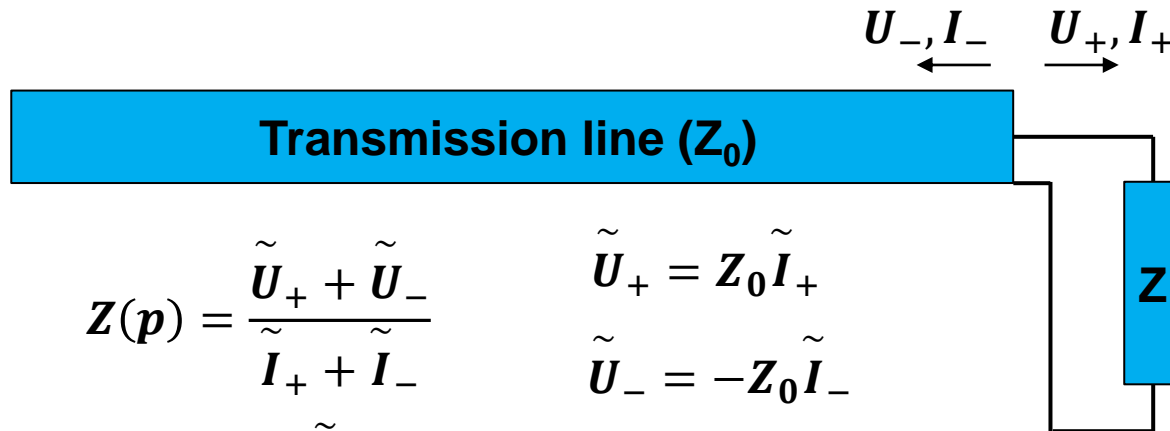
Transmission line



Termination of arbitrary freq-dependent impedance $Z(p)$



- Lossless transmission line terminates with an arbitrary freq-dependent impedance $Z(p)$ in Laplace space.



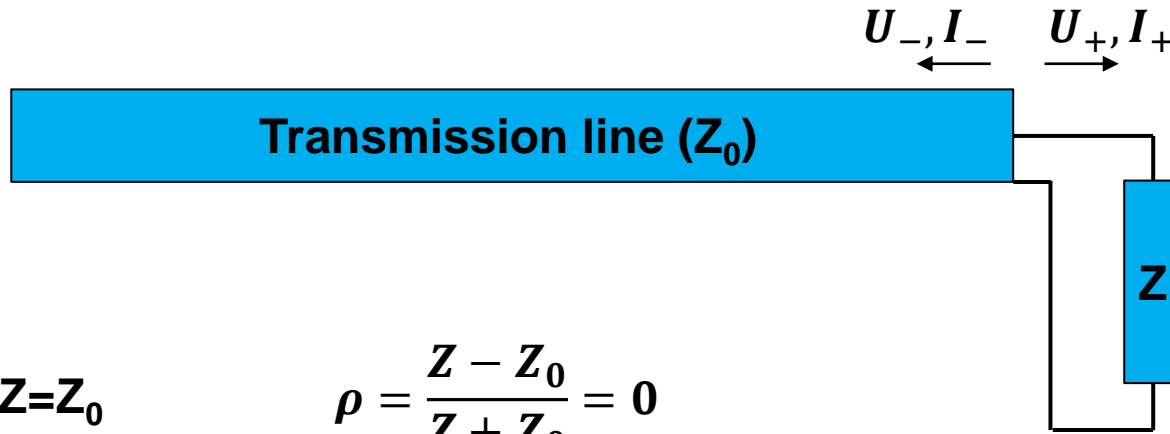
Ohm's law: $Z(p) = \frac{\tilde{U}_+ + \tilde{U}_-}{\tilde{I}_+ + \tilde{I}_-}$ $\tilde{U}_+ = Z_0 \tilde{I}_+$
 $\tilde{U}_- = -Z_0 \tilde{I}_-$

$$Z = \frac{\tilde{U}_+ + \tilde{U}_-}{\frac{\tilde{U}_+}{Z_0} - \frac{\tilde{U}_-}{Z_0}} = Z_0 \frac{1 + \frac{\tilde{U}_-}{\tilde{U}_+}}{1 - \frac{\tilde{U}_-}{\tilde{U}_+}} = Z_0 \frac{1 + \rho}{1 - \rho}$$

$$Z - Z\rho = Z_0 + Z_0\rho$$

Reflection: $\rho \equiv \frac{\tilde{U}_-}{\tilde{U}_+} = \frac{Z - Z_0}{Z + Z_0}$

Termination of arbitrary freq-dependent impedance $Z(p)$



- Match load: $Z=Z_0$
 - No reflection.

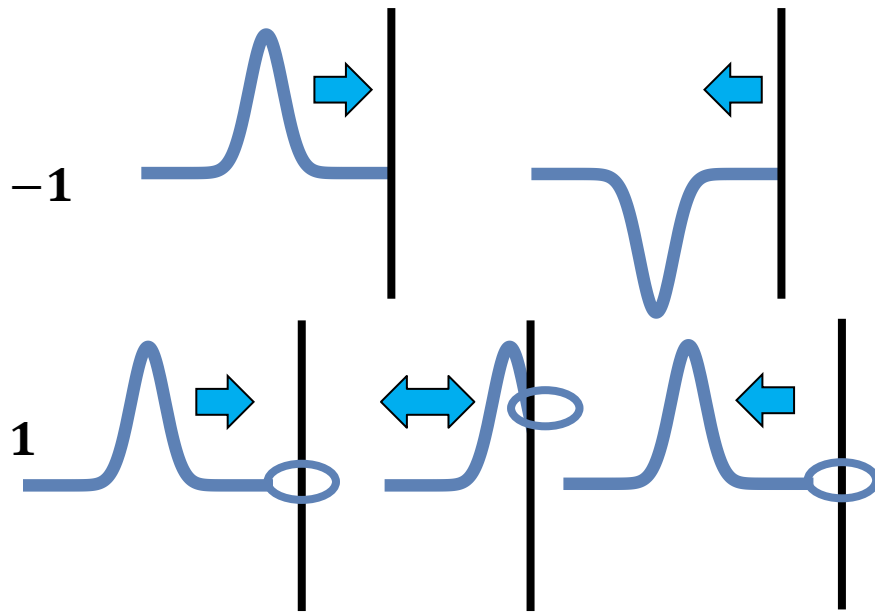
$$\rho = \frac{Z - Z_0}{Z + Z_0} = 0$$

- Short-circuit case: $Z=0$
 - Completely reflective with inverted voltage amplitude.

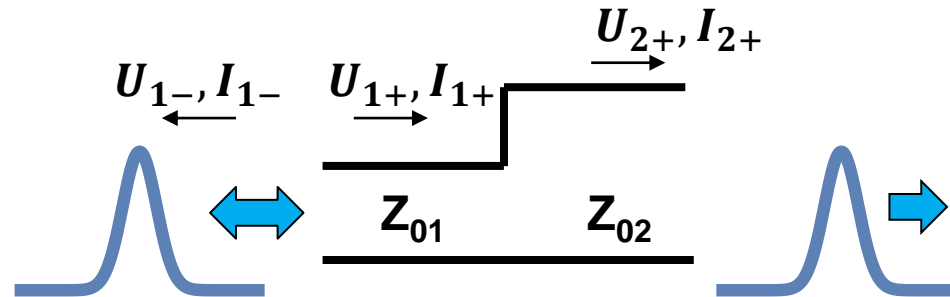
$$\rho = \frac{Z - Z_0}{Z + Z_0} = -1$$

- Open-circuit case: $Z=\infty$
 - Completely reflective with the same polarity.

$$\rho = \frac{Z - Z_0}{Z + Z_0} = 1$$



Pulse is reflected when there is an impedance mismatch



Reflection:

$$\rho = \frac{U_{1-}}{U_{1+}} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$

$$U_{1+} + U_{1-} = U_{2+}$$

$$\rho \equiv \frac{\tilde{U}_-}{\tilde{U}_+} = \frac{Z - Z_0}{Z + Z_0}$$

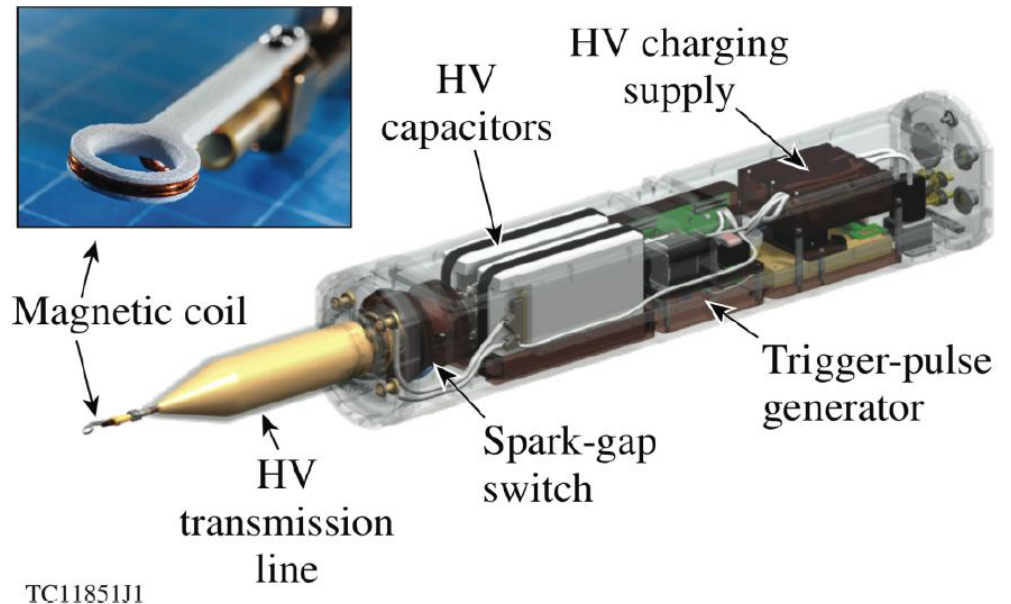
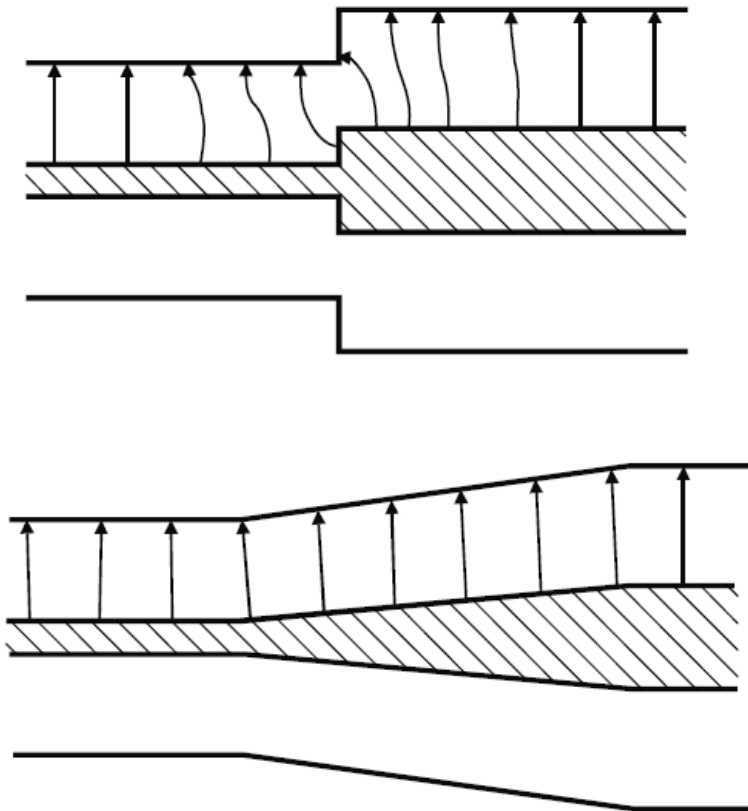
Transmission: $T \equiv \frac{U_{2+}}{U_{1+}} = 1 + \frac{U_{1-}}{U_{1+}} = 1 + \rho$

- For impedance match, i.e., $Z_{01}=Z_{02}$: $\rho = 0$ $T = 1$
 - Reflection-free junction: $Z_{01}=Z_{02}$ is necessary but not sufficient.
- If the geometry of a line changes arbitrary, it becomes impossible to satisfy Maxwell's equation just by superposing the fundamental waves.

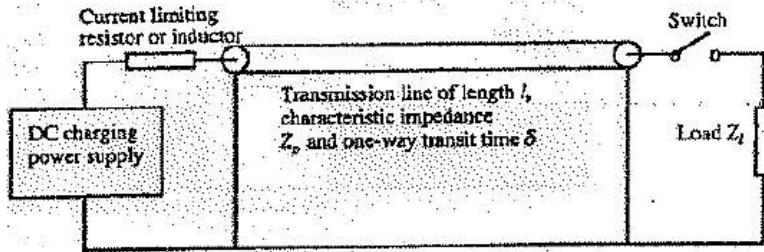
Smooth transition is required



- Only by a smooth transition can we achieve the condition that the fields are not disturbed too much so that the reflections can be avoided for high frequency.

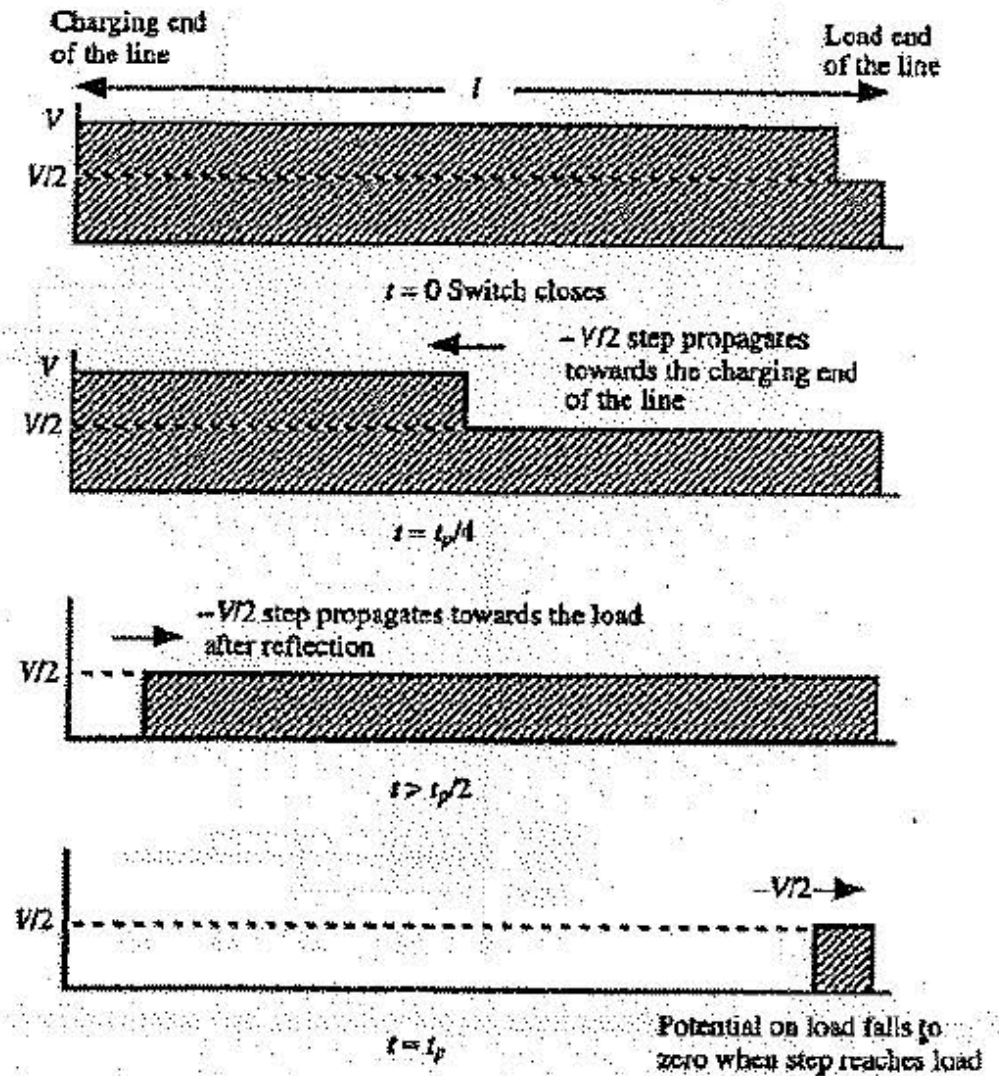


Only half of charged voltage is provided in a basic pulse forming line (PFL)

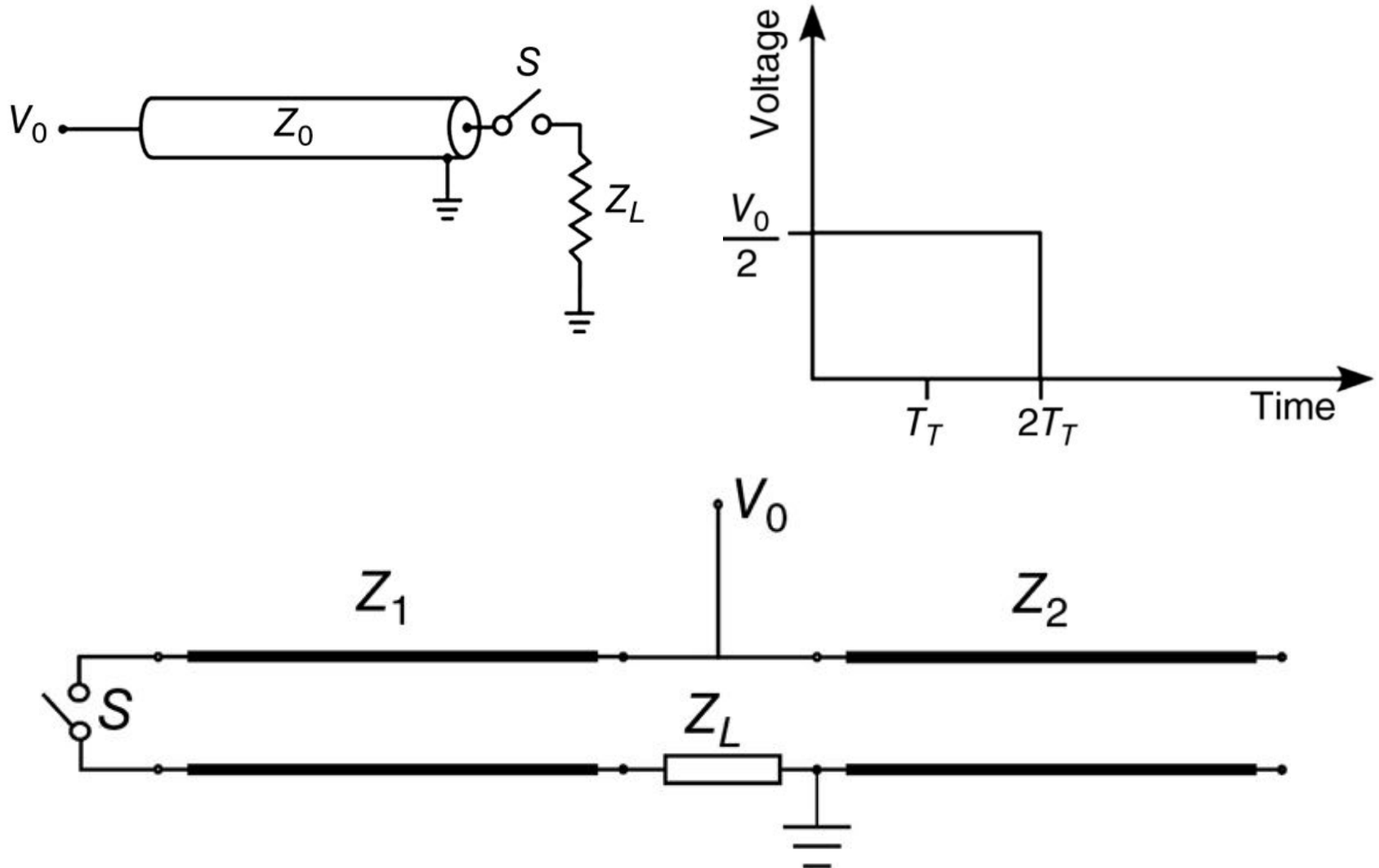


$$t_p = \frac{2l}{v_p} = 2\delta$$

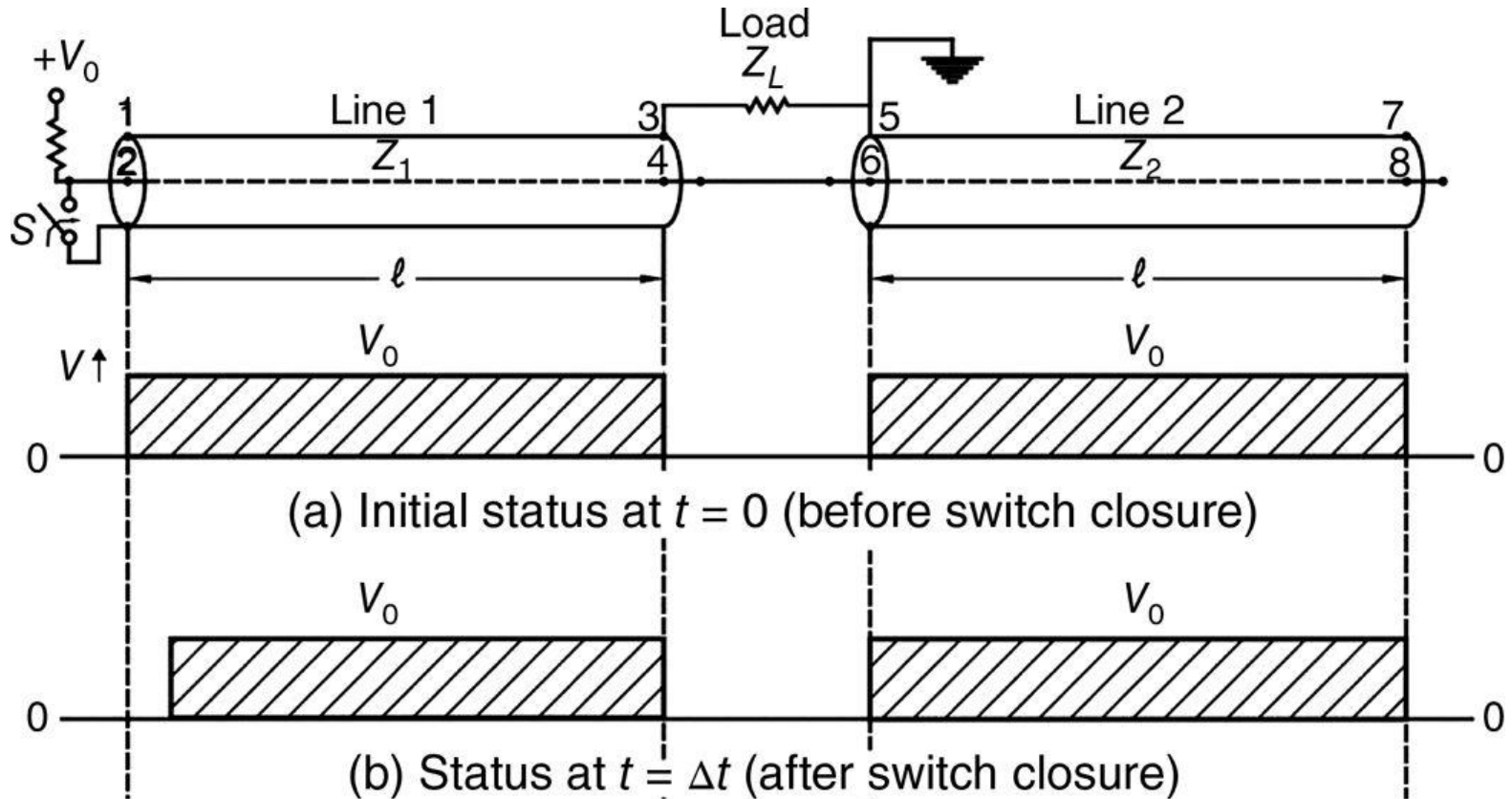
$$V_L = V \frac{Z_L}{Z_0 + Z_L}$$



Blumlein pulse forming line (PFL)

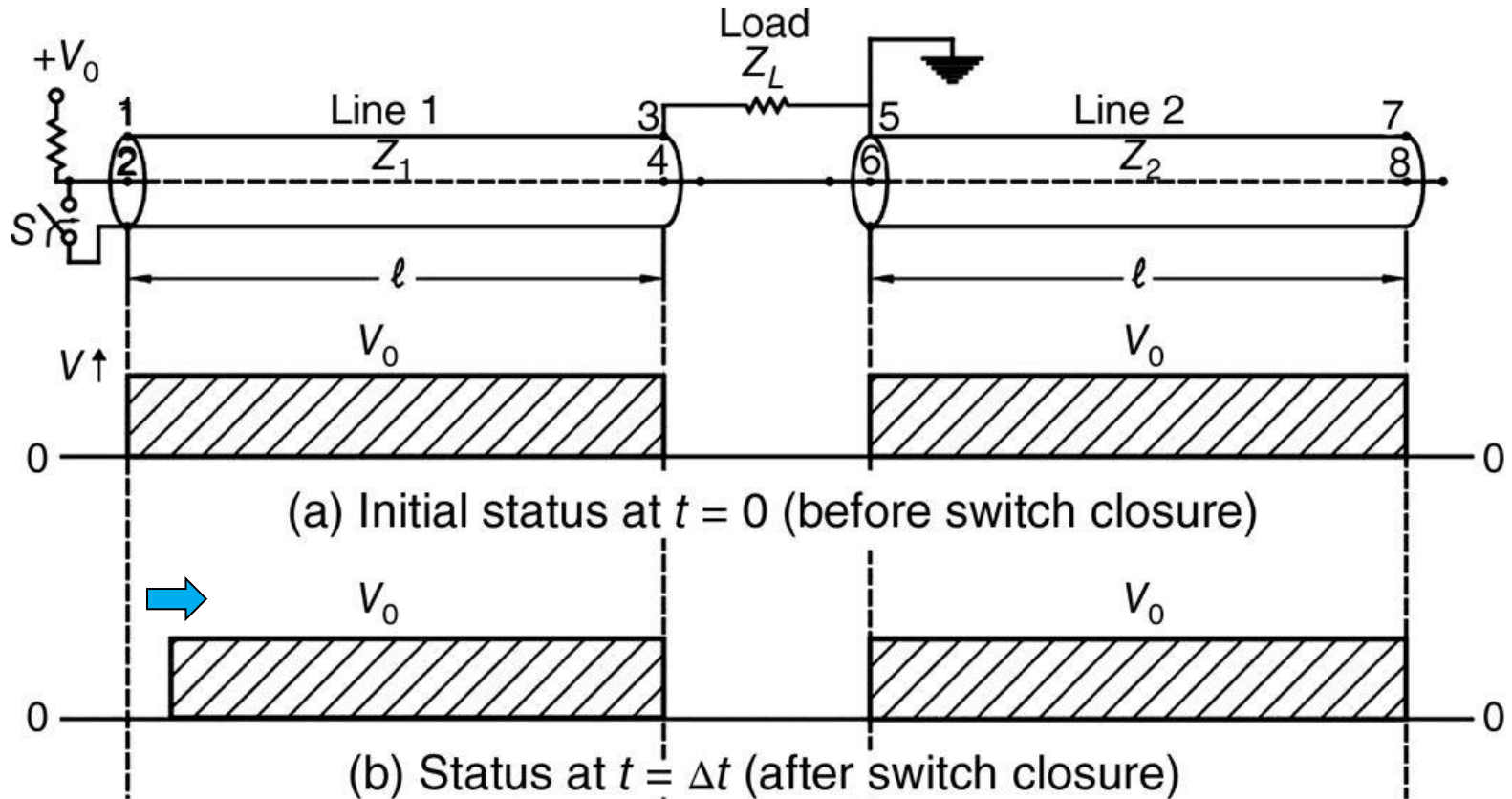


Sequence of Blumlein line



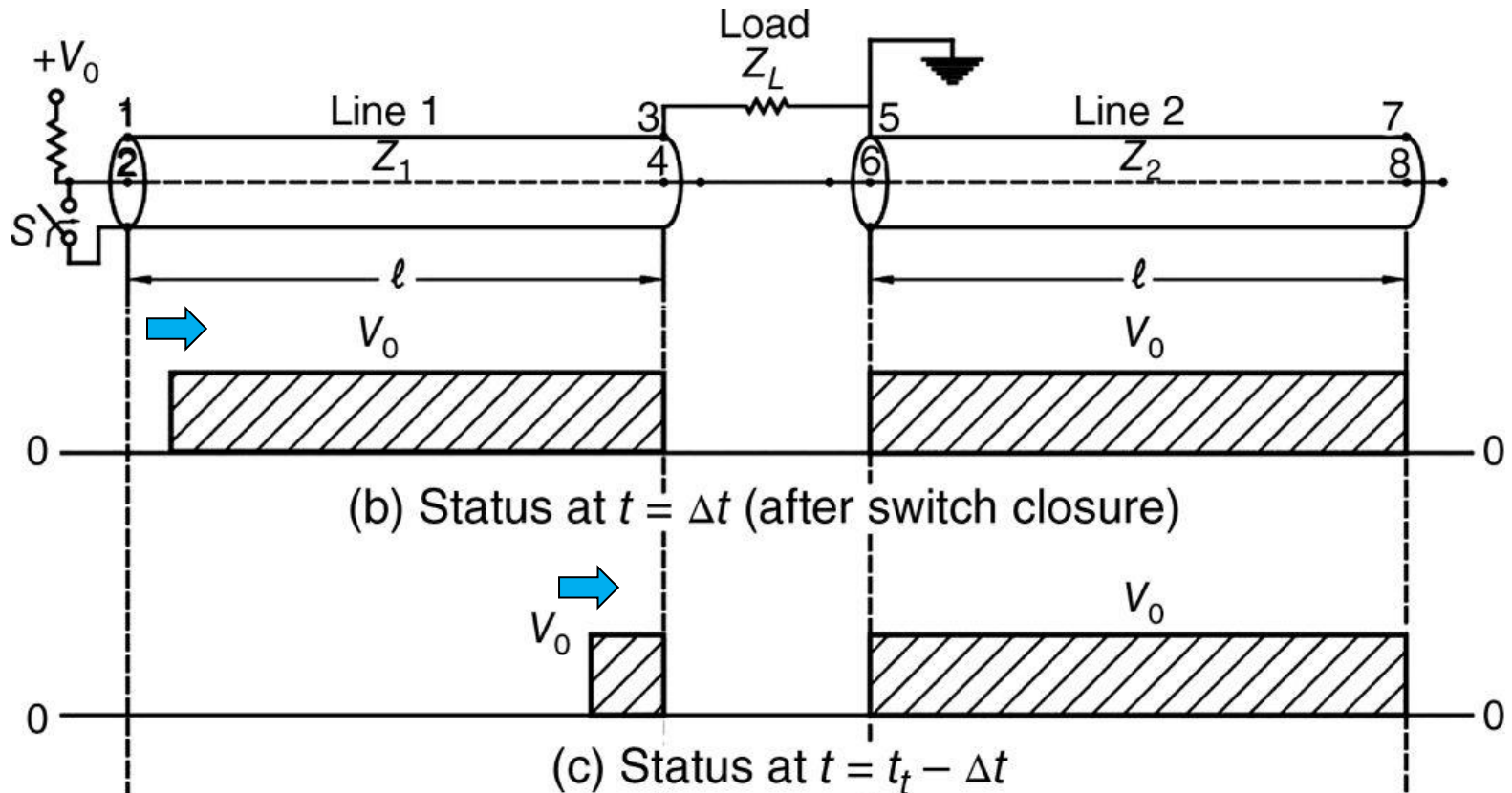
(a)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	V_0	V_0	0	V_0	V_0	0	V_0	V_0	0	0	V_0	V_0

Sequence of Blumlein line



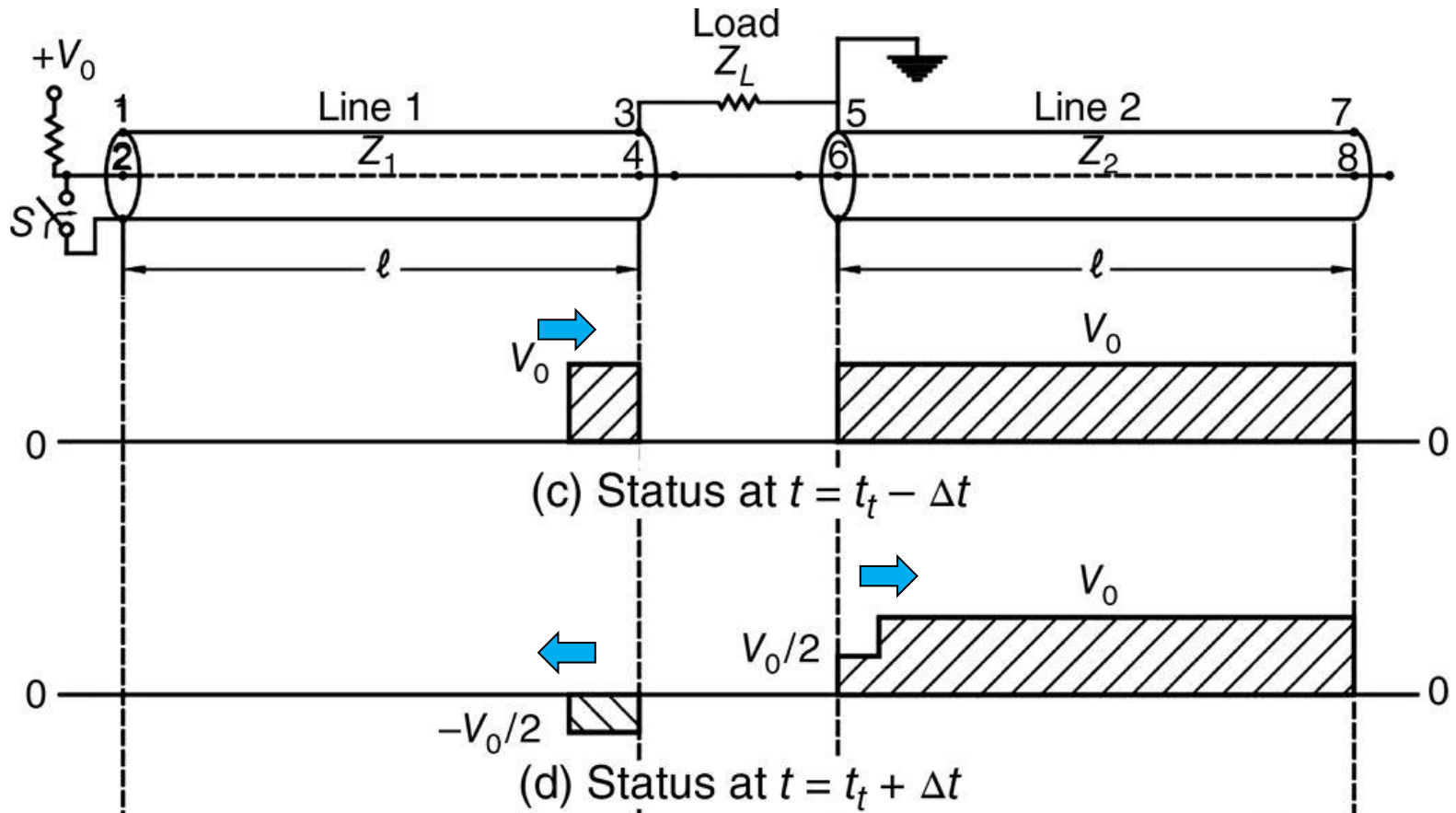
(b)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	0	V_0	V_0	0	V_0	V_0	0	0	V_0	V_0

Sequence of Blumlein line



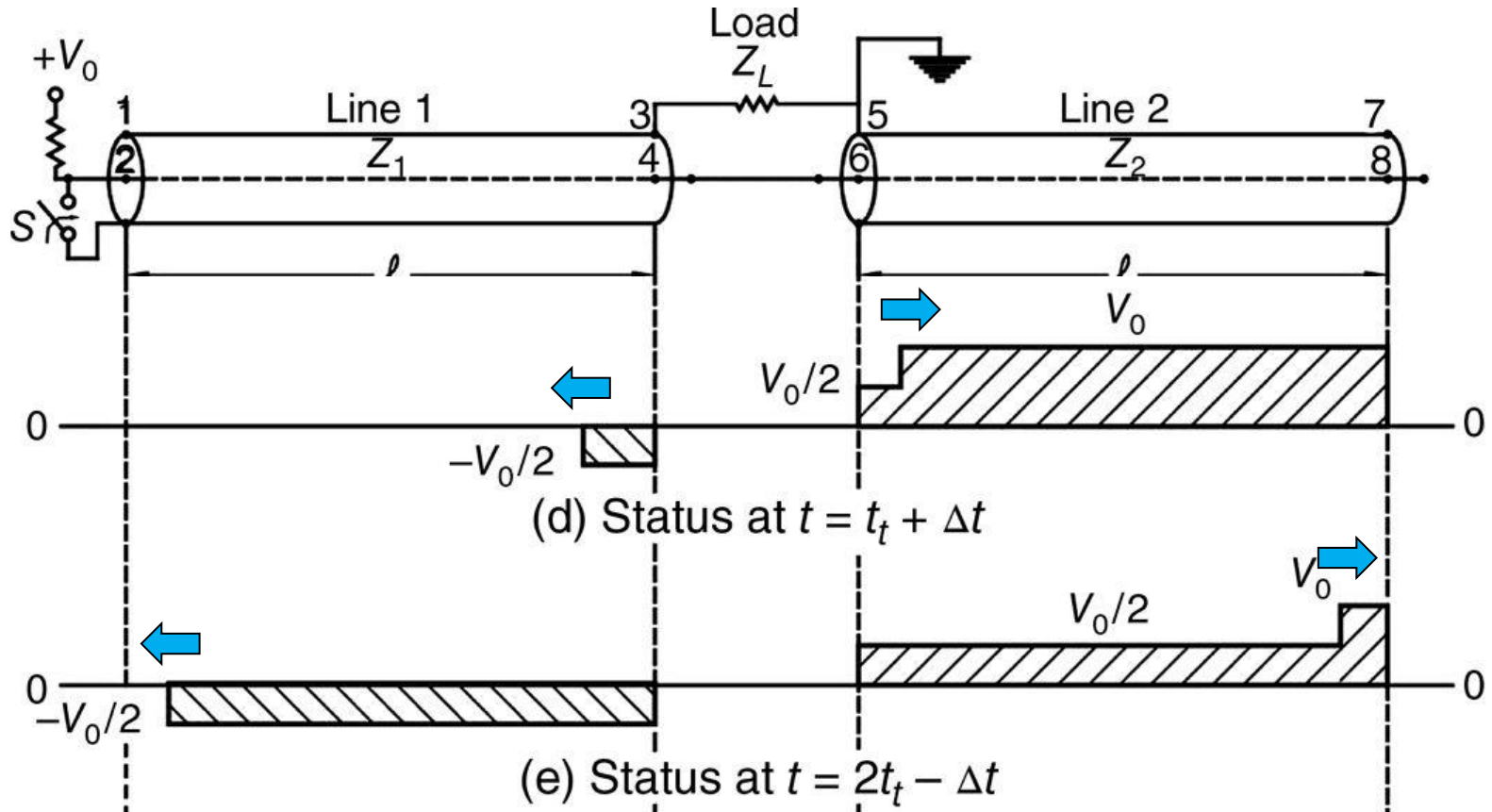
(c)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	0	V_0	V_0	0	V_0	V_0	0	0	V_0	V_0

Sequence of Blumlein line



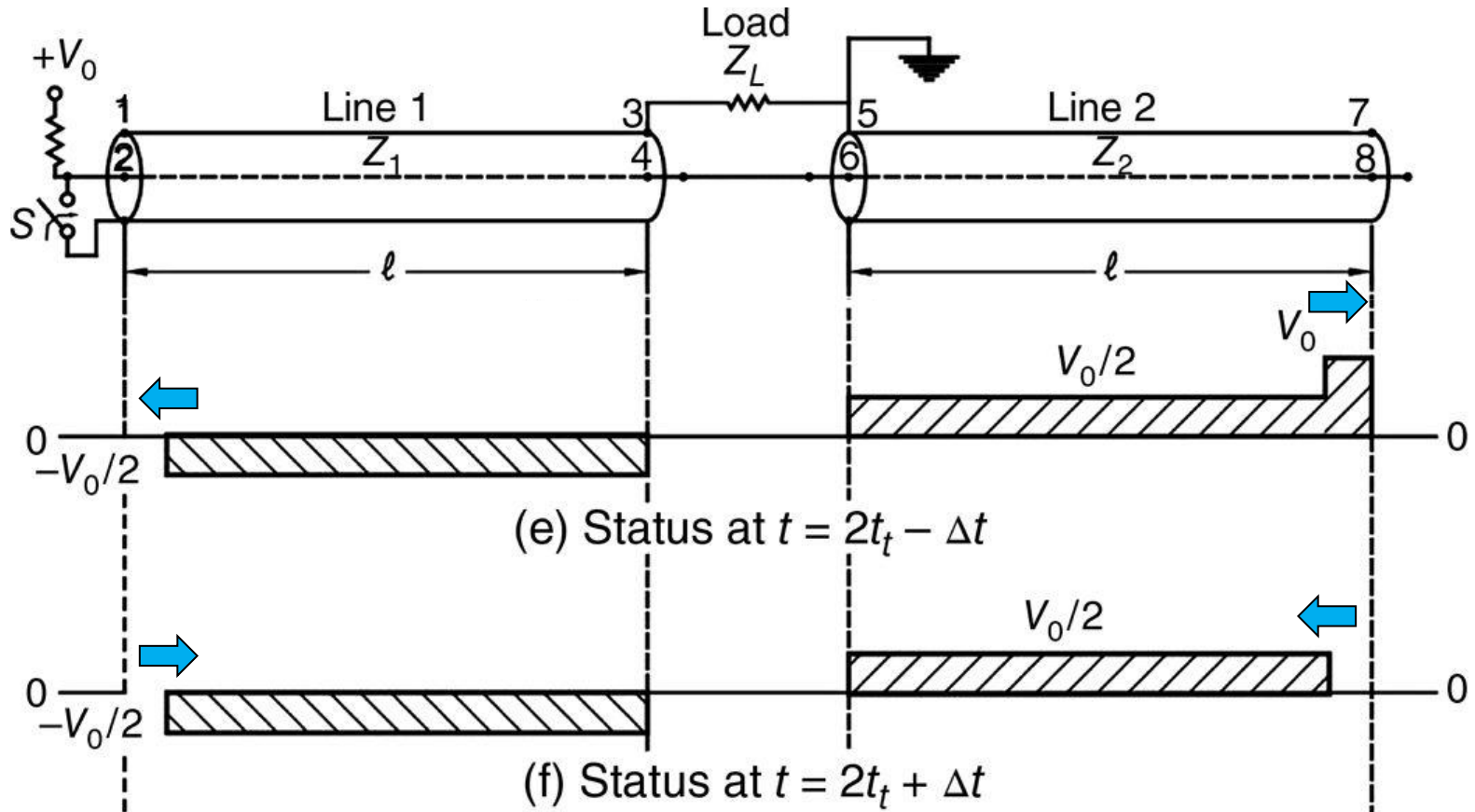
(d)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	V_0	$V_0/2$	$-V_0/2$	0	$V_0/2$	$V_0/2$	V_0	0	V_0	V_0

Sequence of Blumlein line



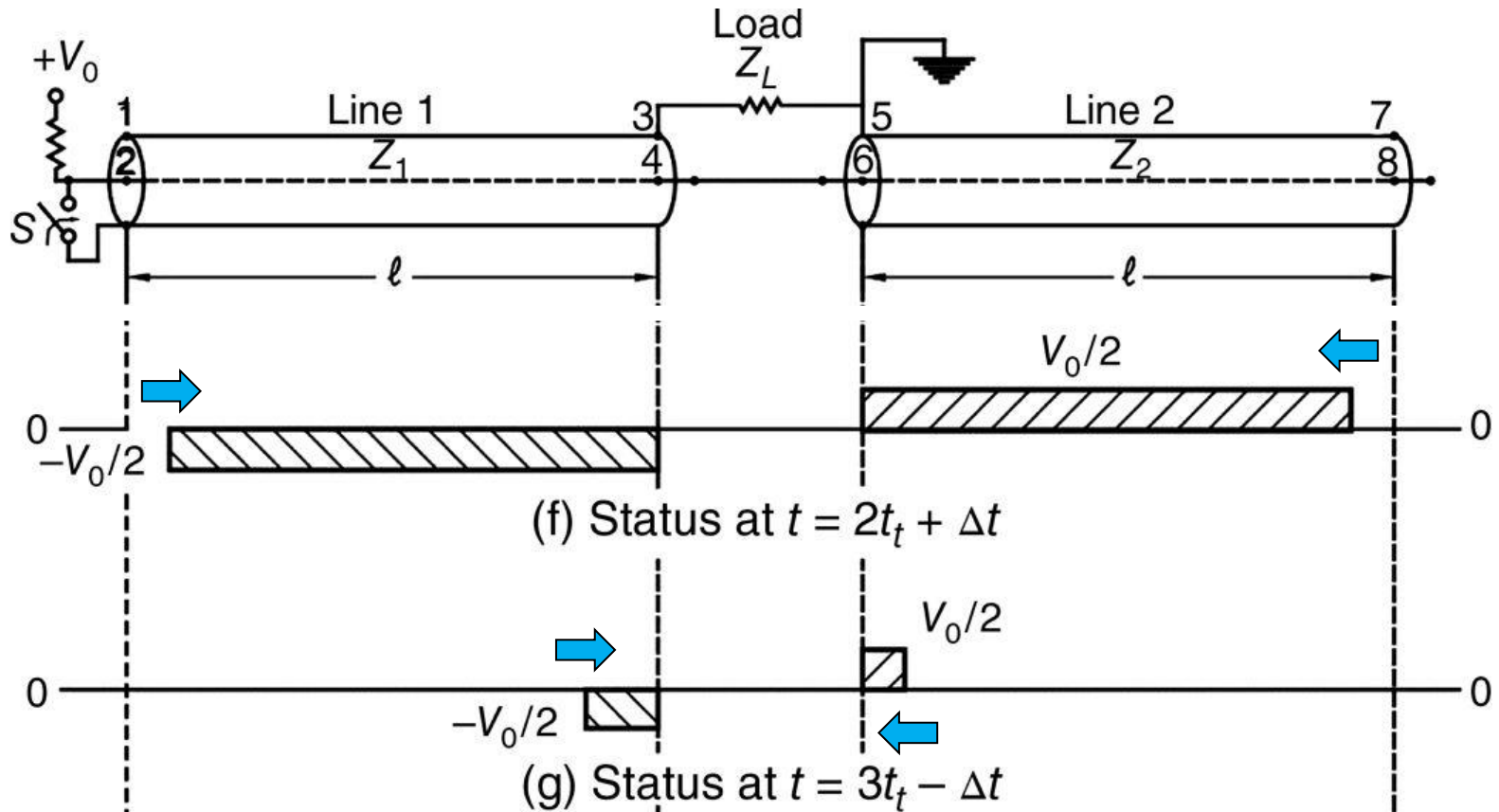
(e)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	V_0	$V_0/2$	$-V_0/2$	0	$V_0/2$	$V_0/2$	V_0	0	V_0	V_0

Sequence of Blumlein line



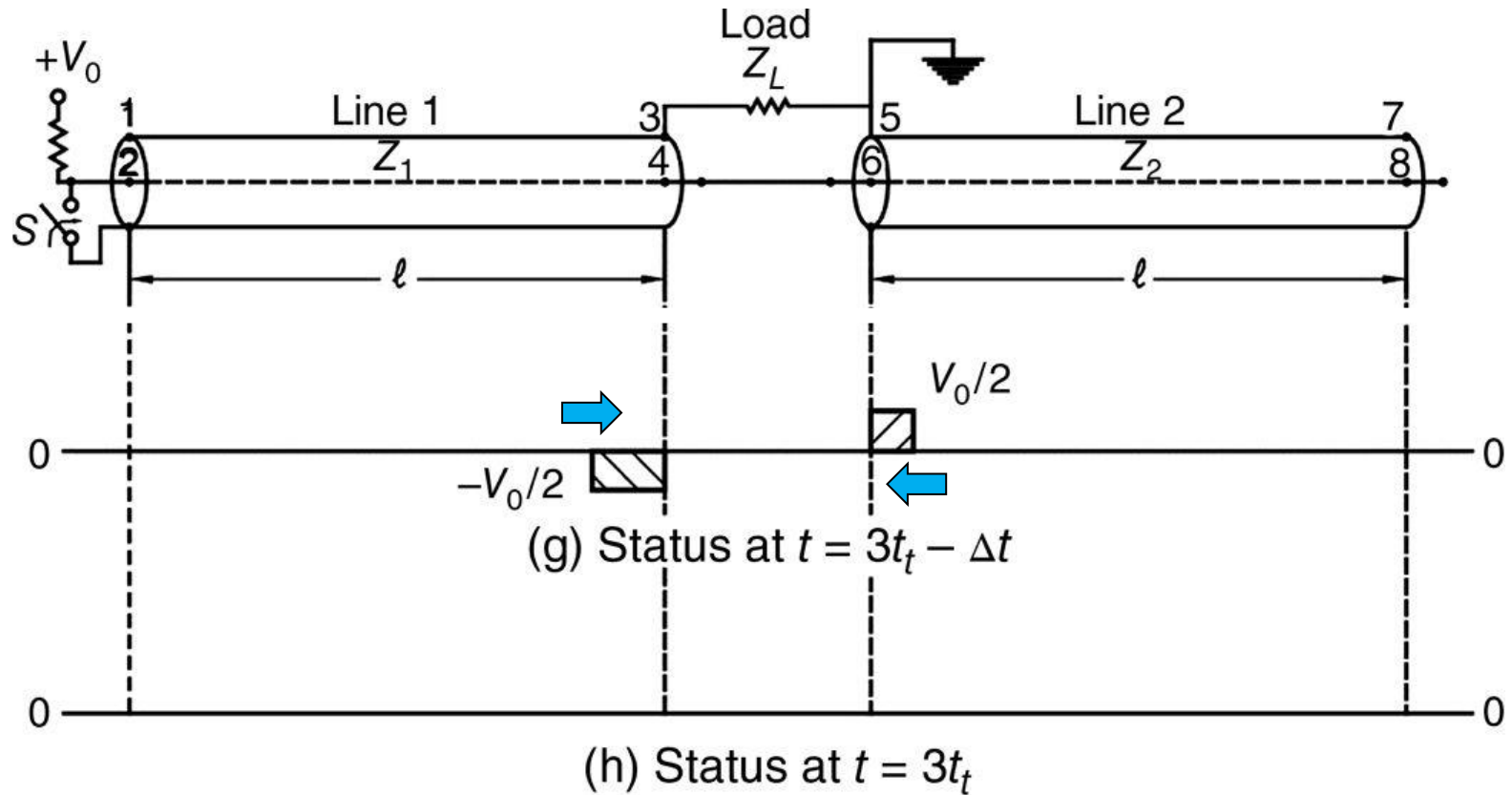
(f)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	V_0	$V_0/2$	$-V_0/2$	0	$V_0/2$	$V_0/2$	V_0	0	0	0

Sequence of Blumlein line



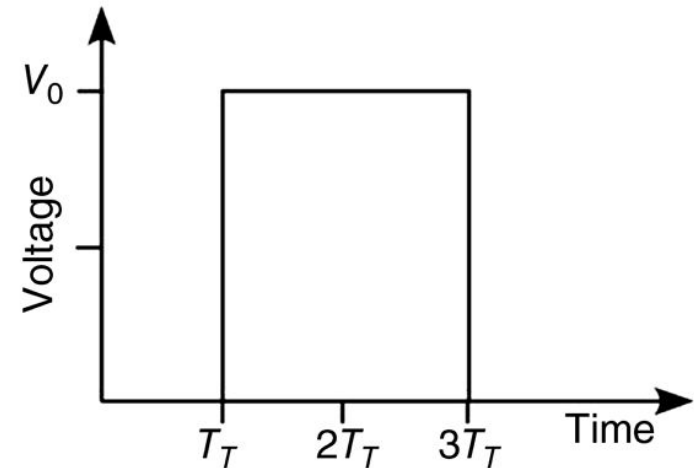
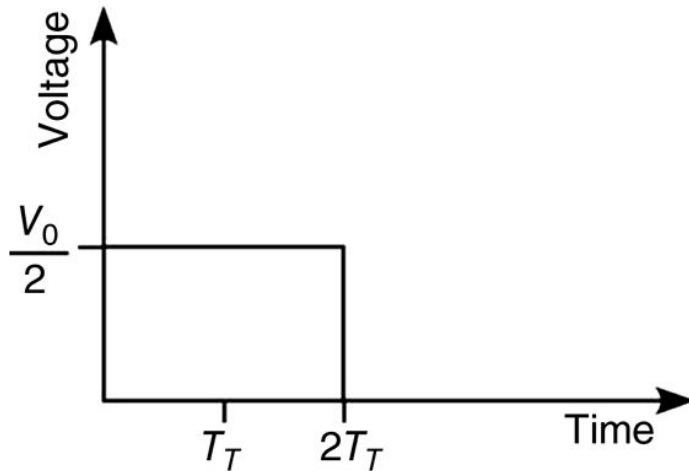
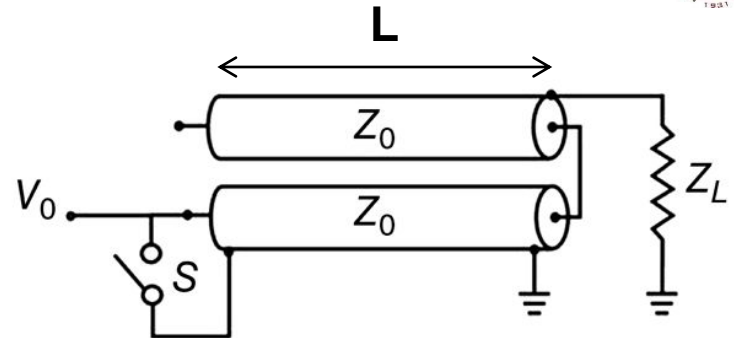
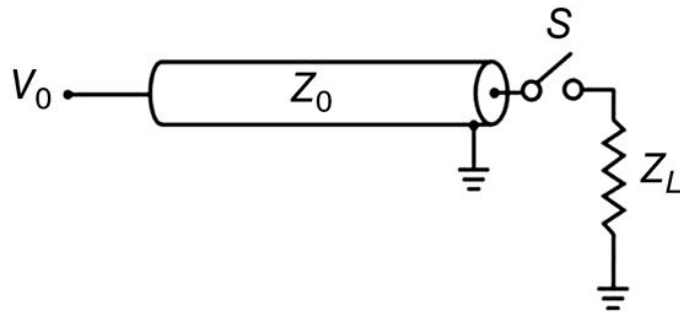
(g)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	V_0	$V_0/2$	$-V_0/2$	0	$V_0/2$	$V_0/2$	V_0	0	0	0

Sequence of Blumlein line



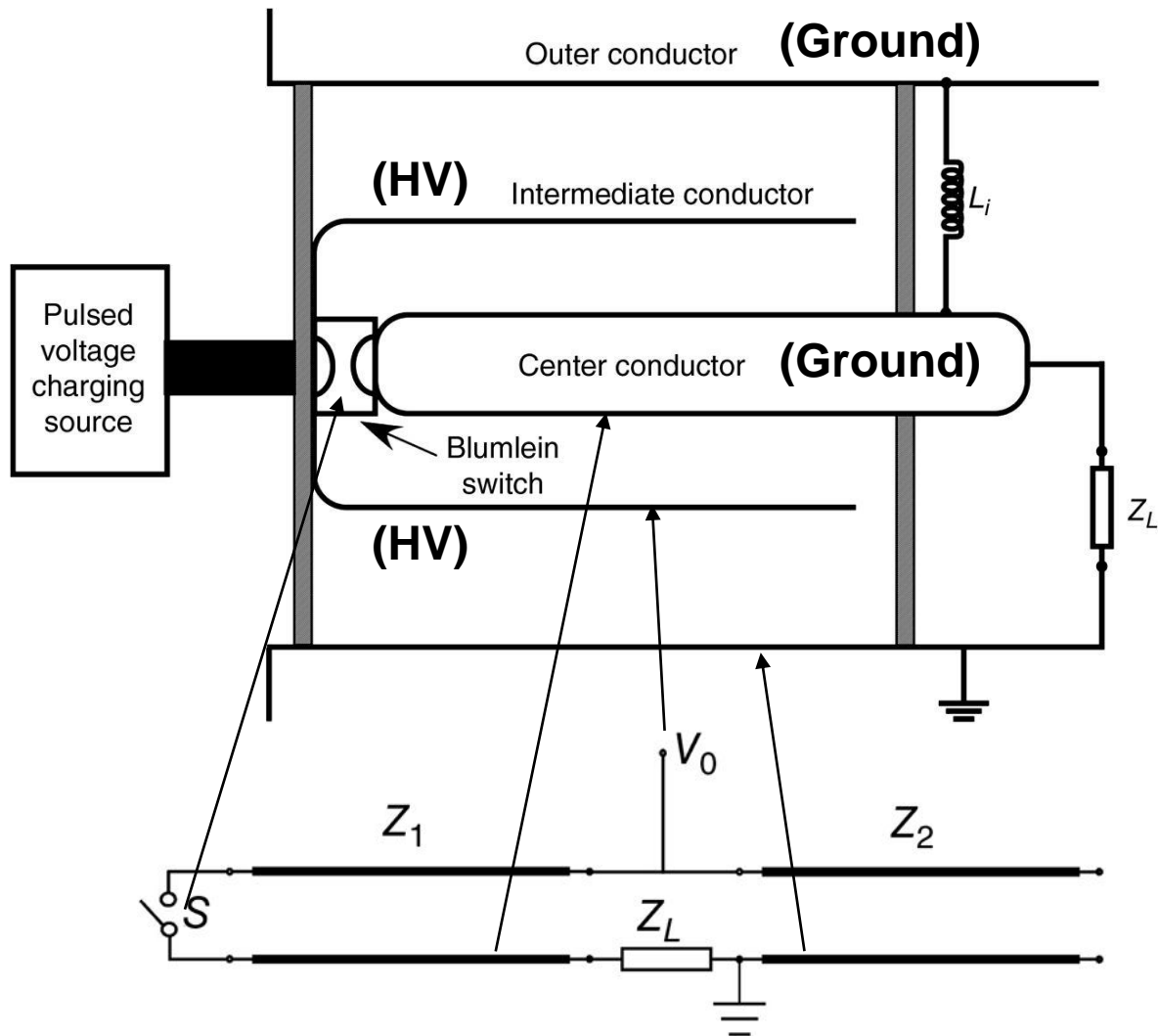
(h)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	0	0	0	0	0	0	0	0	0	0

A Blumlein line can be built by using two coaxial transmission line



- Example: RG58 coaxial cable, 50Ω , $V_{\text{signal}} \sim 2 \times 10^{10} \text{ cm/s}$
 $\Rightarrow L=10 \text{ cm}$, $\Delta t=1 \text{ ns}$.

Coaxial Blumlein line



Outlines

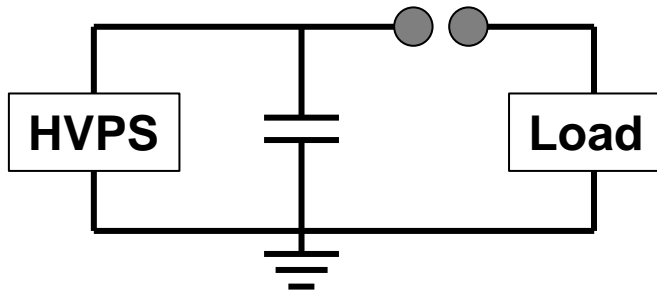


- Switches
 - Closing switches: the switching process is associated with voltage breakdown across an initially insulant element.
 - Opening switches: the switching process is associated with a sudden growth of its impedance.
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 - Blumlein line
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 - Pulse compressor
- Pulse transmission and transformation

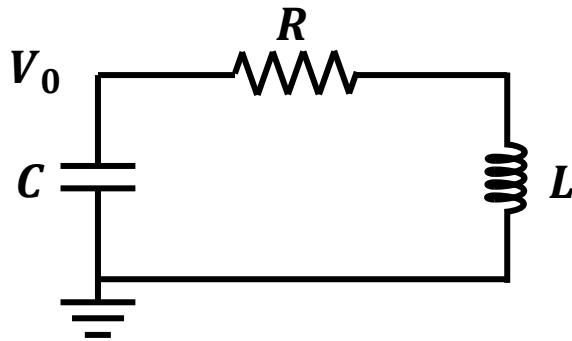
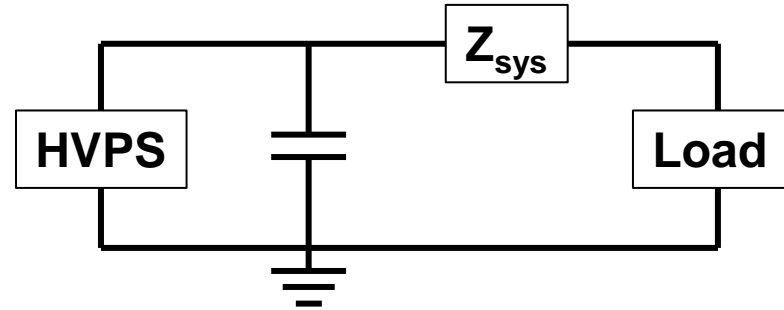
A simple pulsed-power system is a RLC circuit



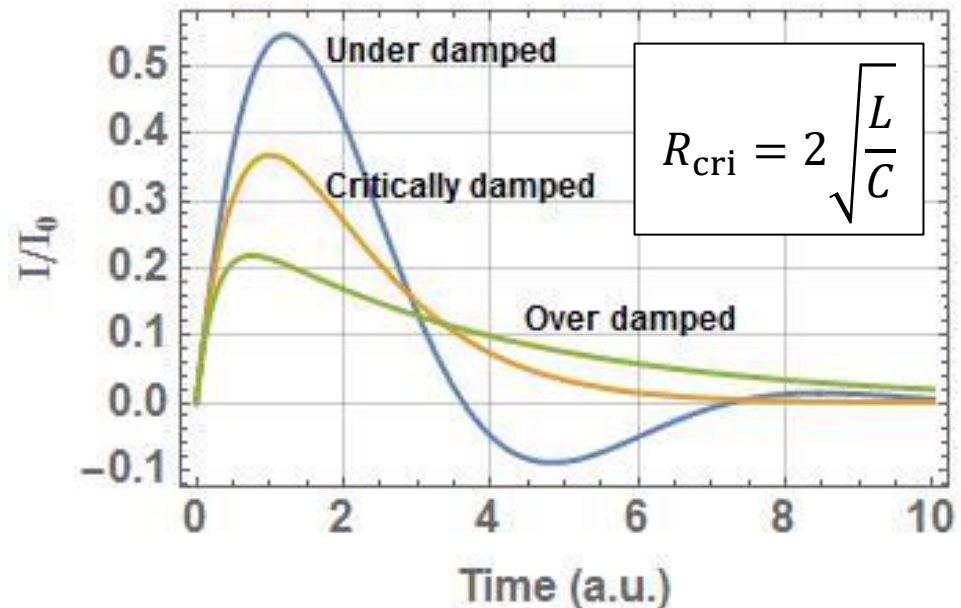
- Before discharge



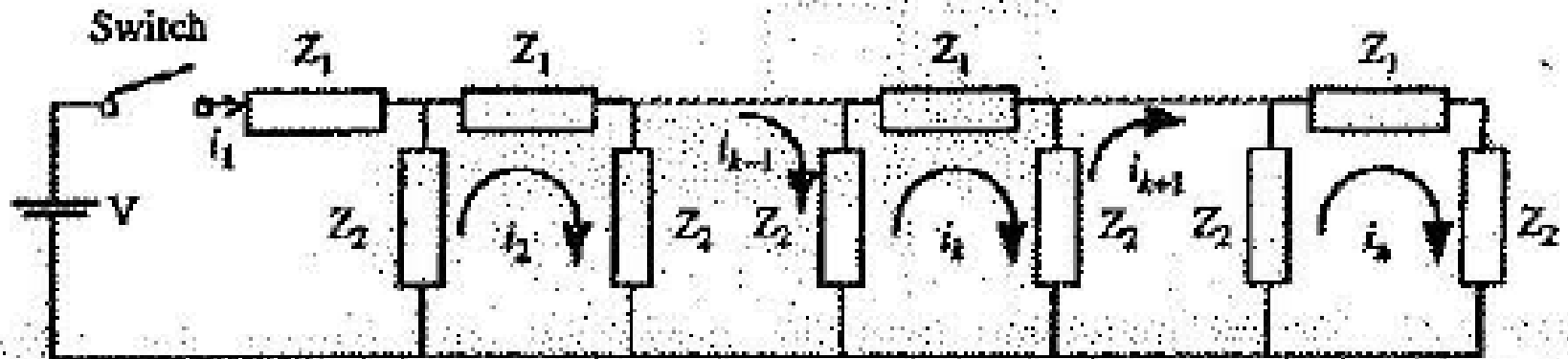
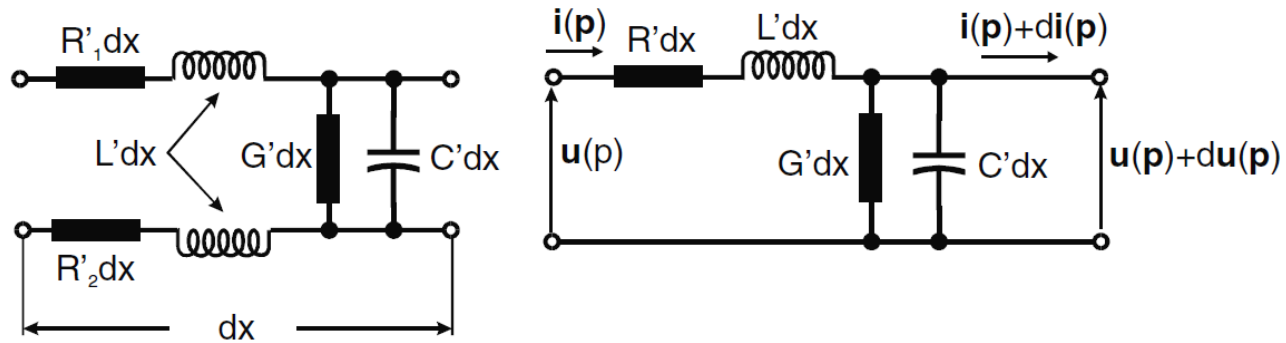
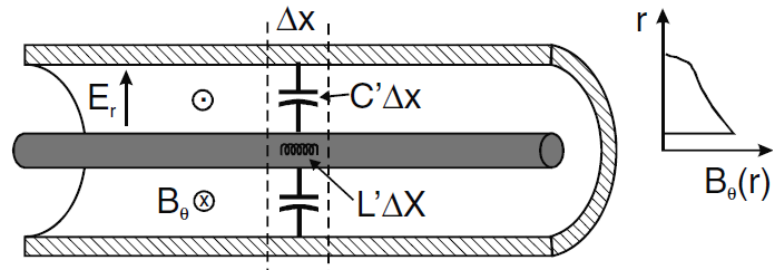
- After discharge



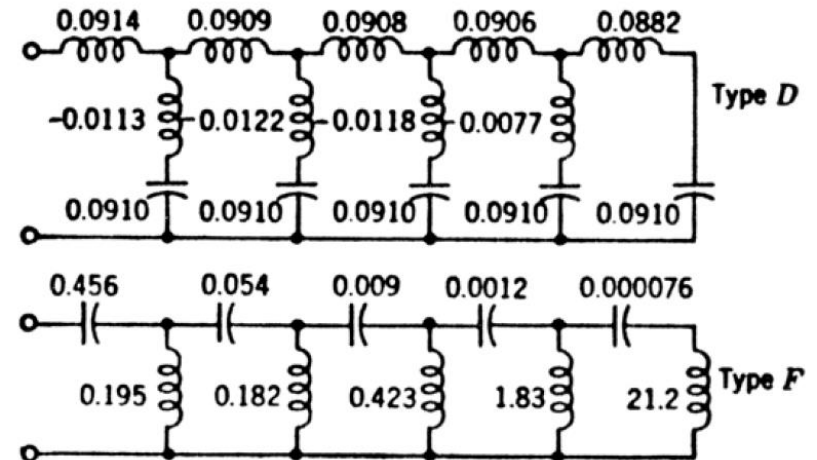
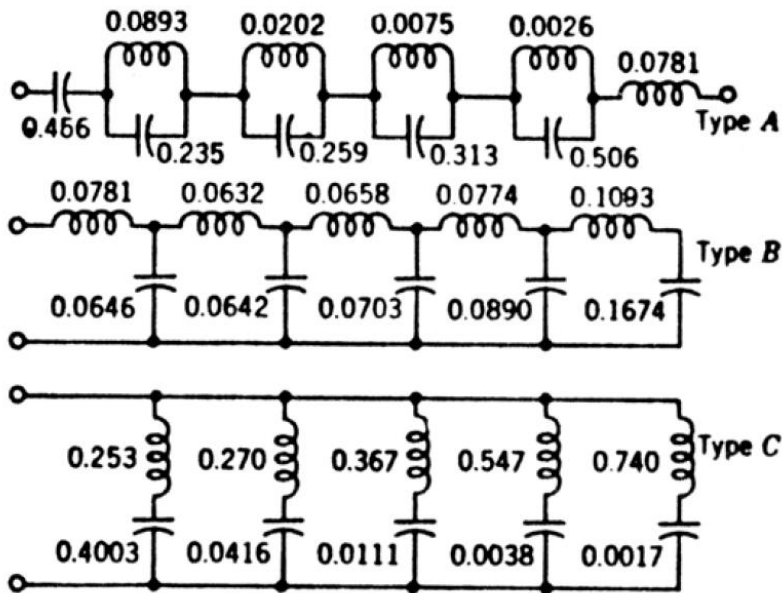
- How can we generate a square current pulse?



Pulse-forming network (PFN)



Equivalent Guillemin Networks



Pulse-forming LC chain

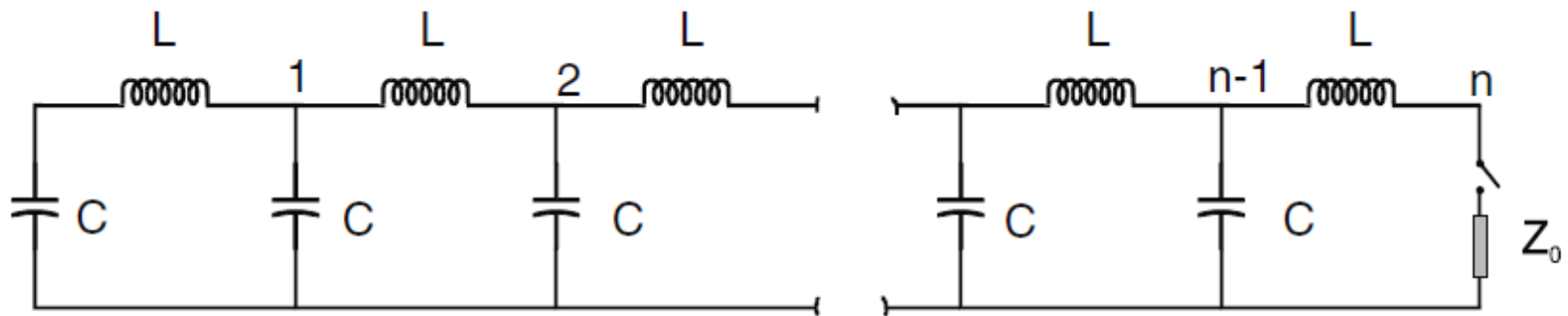
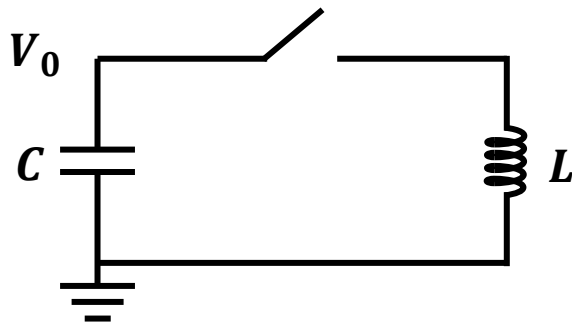


Fig. 5.11. Pulse-forming LC chain

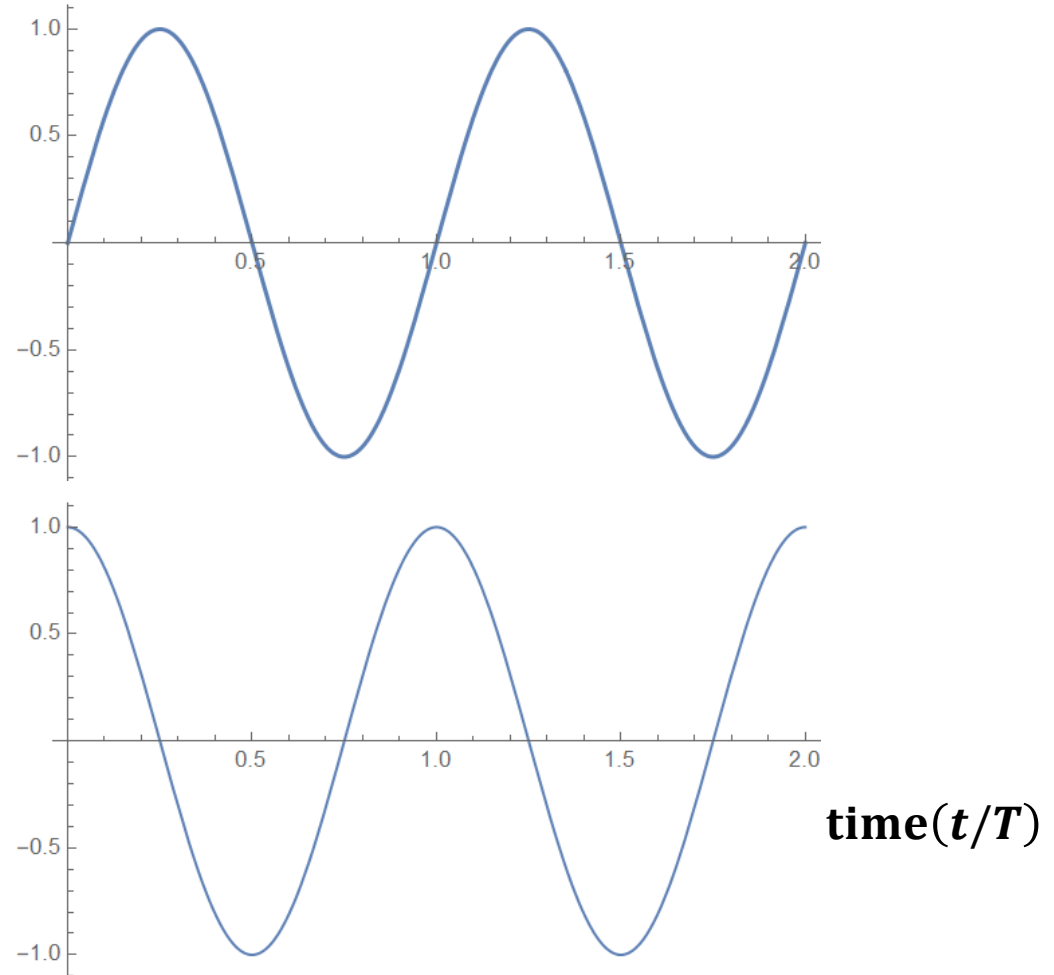
The current output of a LC circuit is a basis of Fourier series



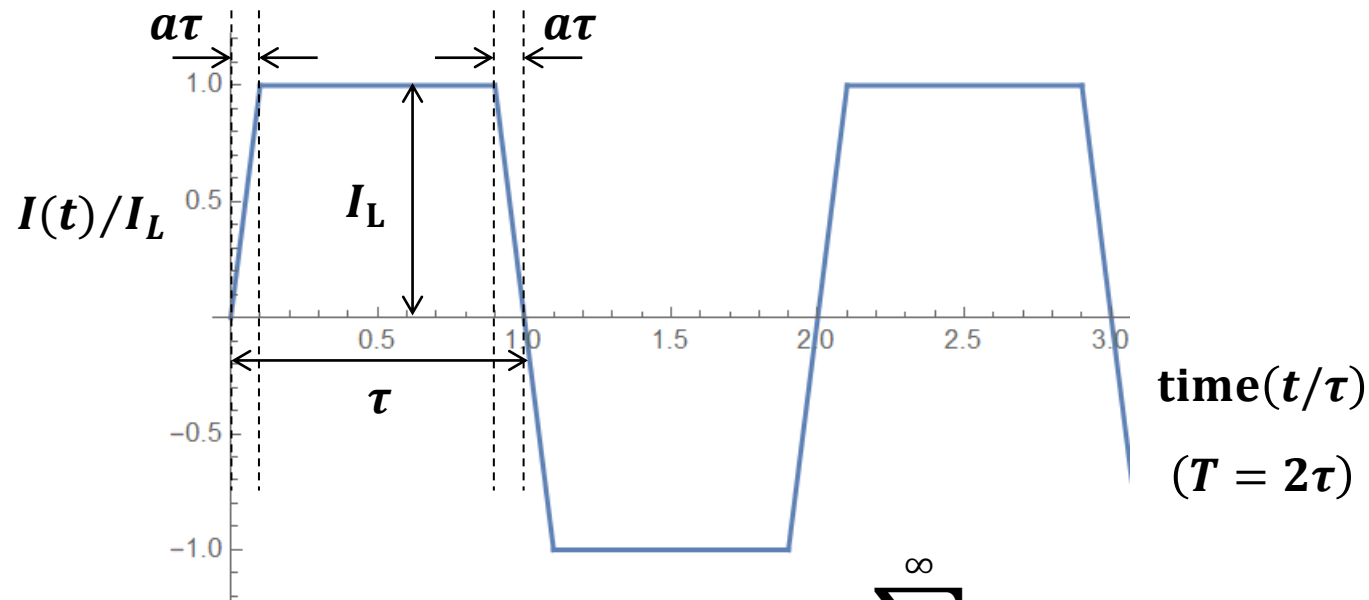
$$I(t) = V_0 \sqrt{\frac{C}{L}} \sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$V(t) = V_0 \cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$Z = \sqrt{\frac{L}{C}} \quad \omega = \frac{1}{\sqrt{LC}}$$



A trapezoidal wave can be expressed by Fourier series (Guillemin's method)



$$\frac{i(t)}{I_L} = \frac{t}{a\tau}, \quad 0 \leq t \leq a\tau$$

$$\frac{i(t)}{I_L} = 1, \quad a\tau \leq t \leq \tau - a\tau$$

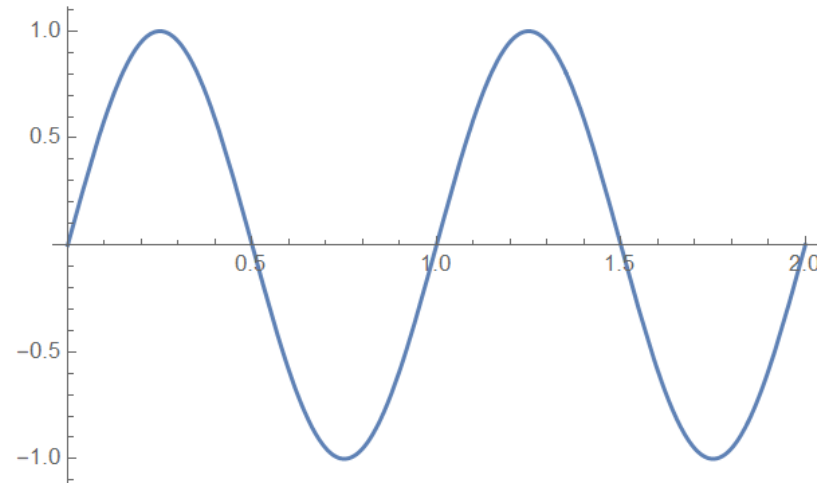
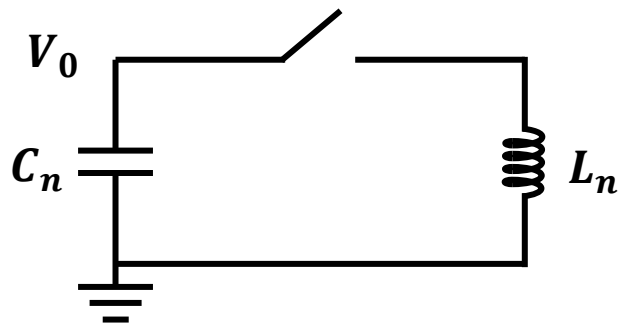
$$\frac{i(t)}{I_L} = \frac{\tau - t}{a\tau}, \quad \tau - a\tau \leq t \leq \tau$$

$$i(t) = I_L \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{\tau}\right)$$

$$\text{where } b_n = \frac{2}{\tau} \int_0^{\tau} \frac{i(t)}{I_L} \sin\left(\frac{n\pi t}{\tau}\right) dt$$

$$b_n = \frac{4}{n\pi} \frac{\sin(n\pi a)}{n\pi a}, \quad \text{where } n = 1, 3, 5, \dots$$

The required inductance and capacitance are obtained by comparing LC output with the Fourier series



$$I_n(t) = V_0 \sqrt{\frac{C_n}{L_n}} \sin\left(\frac{t}{\sqrt{L_n C_n}}\right)$$

$$i(t) = I_L \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{\tau}\right)$$

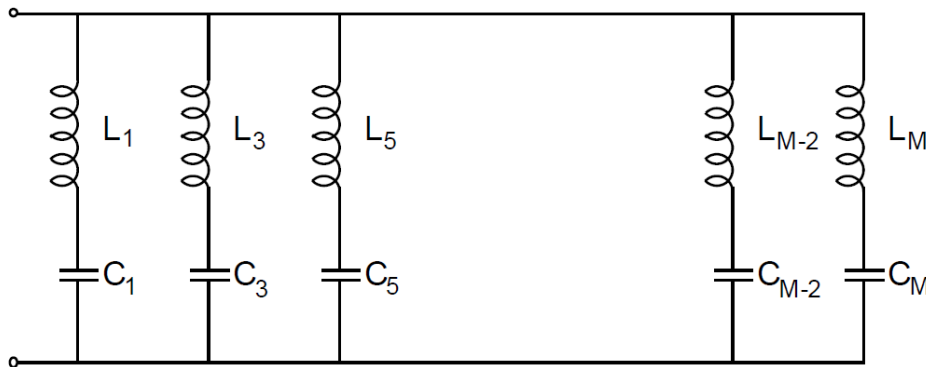
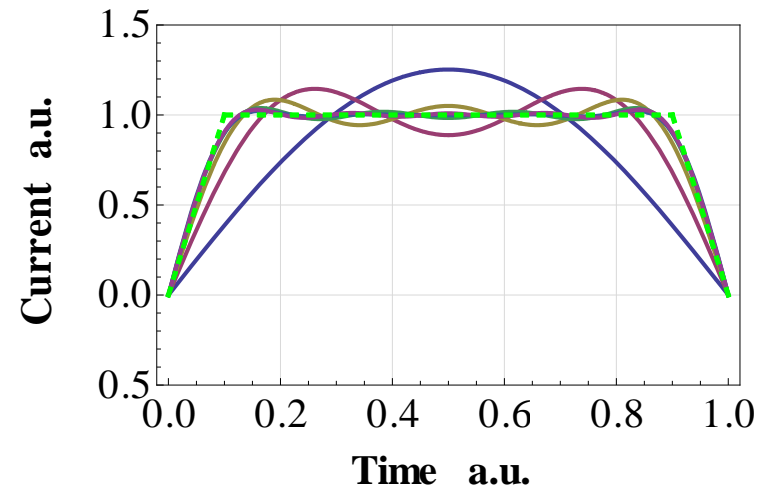
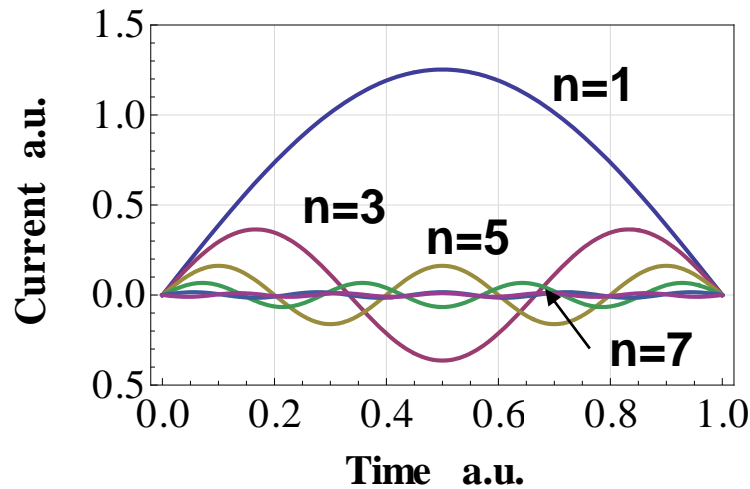
$$b_n = \frac{4}{n\pi} \frac{\sin(n\pi a)}{n\pi a}, \text{ where } n = 1, 3, 5, \dots$$

$$L_n = \frac{Z_n \tau}{n\pi b_n} = \frac{V}{I_L} \frac{\tau}{n\pi b_n}$$

$$C_n = \frac{\tau b_n}{n\pi Z_n} = \frac{I_L}{V} \frac{\tau b_n}{n\pi}$$

$$Z_n = \frac{V}{I_L}$$

A trapezoidal current output can be generated using Guillemin's pulse-forming networks

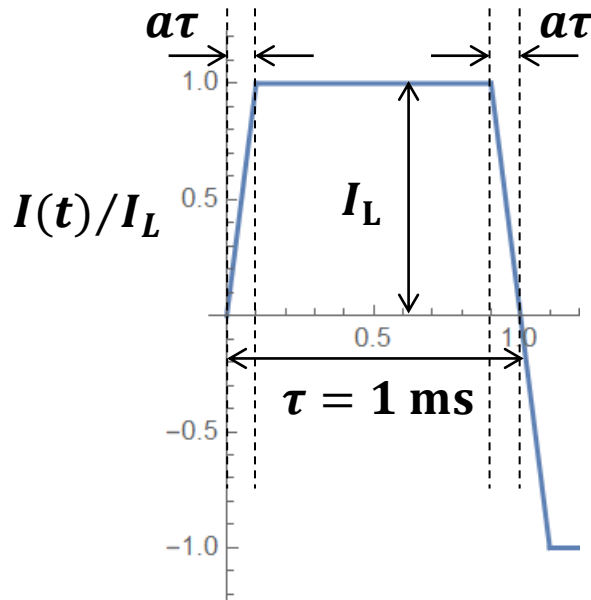


$$I(t) = I_L \sum b_n \sin\left(\frac{n\pi t}{\tau}\right)$$

$$b_n = \frac{4}{n\pi} \frac{\sin(n\pi a)}{n\pi a}$$

$$L_n = \frac{Z\tau}{n\pi b_n} \quad C_n = \frac{\tau b_n}{n\pi Z} \quad Z = \frac{V}{I_L}$$

Fourier components of $\tau=1$ ms, $a=0.1$



time(t/τ)
($T = 2\tau$)

$$i(t) = I_L \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{\tau}\right)$$

$$b_n = \frac{4}{n\pi} \frac{\sin(n\pi a)}{n\pi a}, \text{ where } n = 1, 3, 5, \dots$$

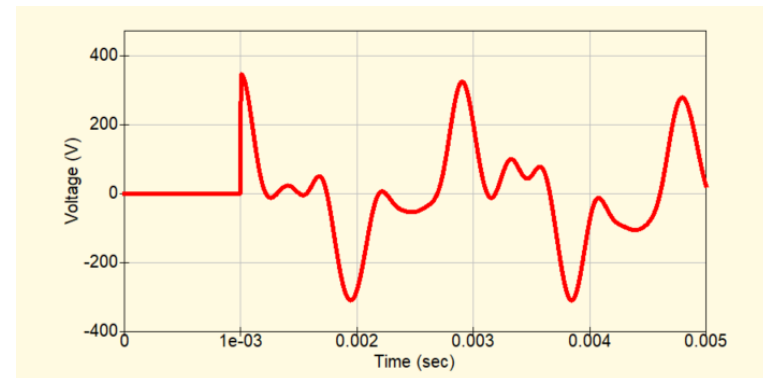
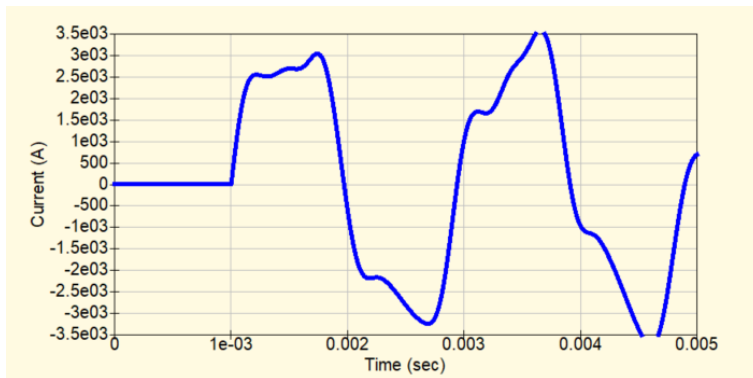
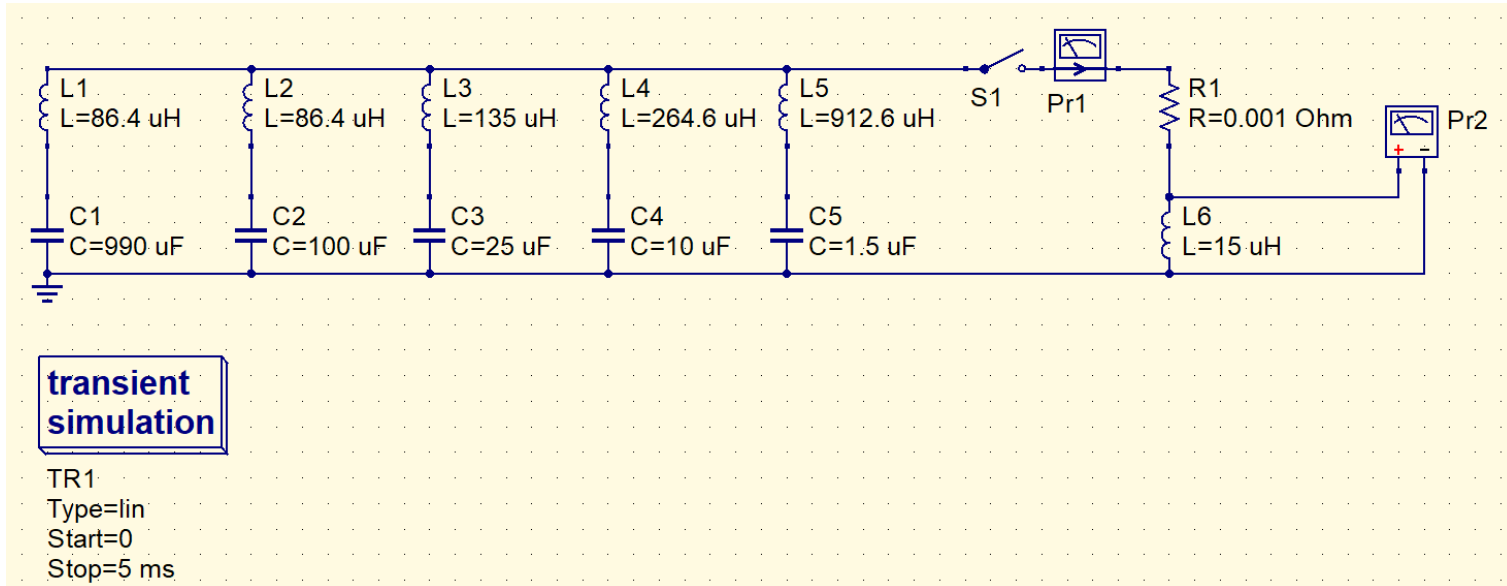
n	#/
b1	1.2524
b3	0.3643
b5	0.1621
b7	0.069
b9	0.0155

Coils with 8 turns and a PFN charged to 1 kV will be used

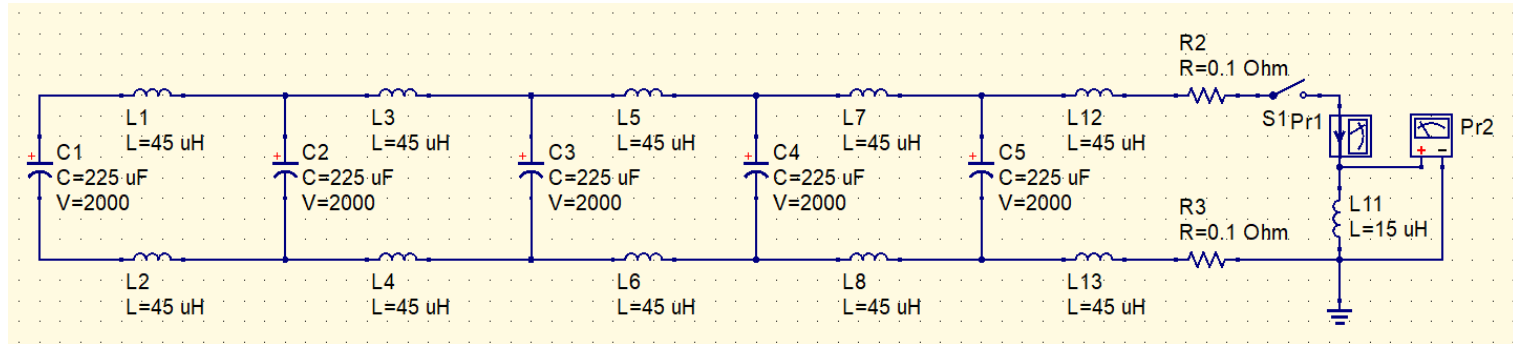


I (kA)	V (kV)		1	2	3	4	5	E (kJ)	% to 100 J
20	2	L(uH)	25.4	26.1	39.3	68.0	228.7	9.0	1.1 %
		C(uF)	3986.5	386.5	103.2	30.4	5.5		
20	1	L(uH)	12.7	14.6	19.6	34.0	114.4	4.5	2.2 %
		C(uF)	7973.0	773.1	206.4	60.9	10.9		
2.5	2	L(uH)	203.3	233.0	314.2	543.7	1830.0	1.1	8.9 %
		C(uF)	498.3	48.3	12.9	3.8	0.7		
2.5	1	L(uH)	101.7	116.5	157.1	271.8	915.0	0.6	17.7 %
		C(uF)	996.6	96.6	25.8	7.6	1.4		

A square pulse with a flat top of 2.5 kA can be generated



A simple PFN with constant C and L in all stages can also be used



$$C \equiv \bar{C} = \frac{1}{N} \sum_{n=1}^N C_n = 225 \mu\text{F}$$

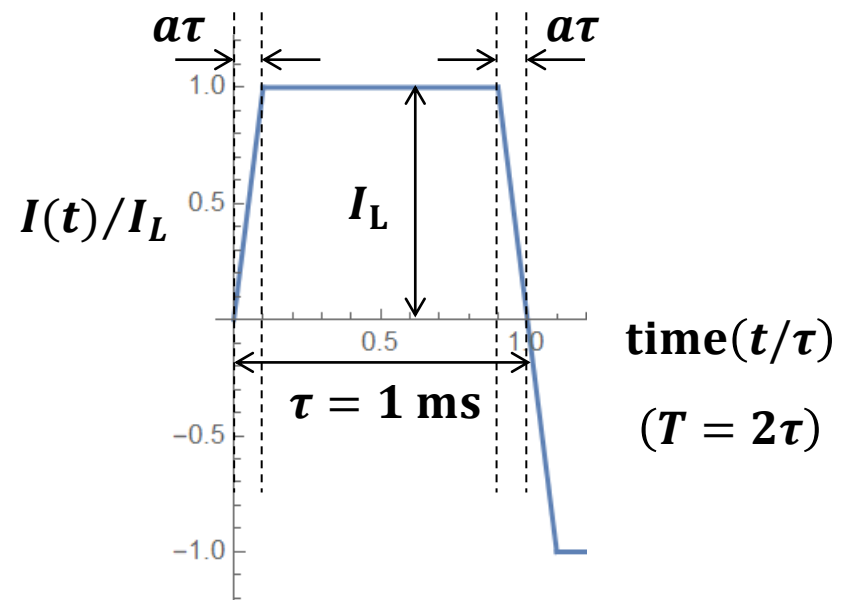
$$L_n = 2nL + L_L \approx 2nL$$

$$\omega_n = \frac{1}{\sqrt{L_n C}} \approx \frac{1}{\sqrt{2nLC}}$$

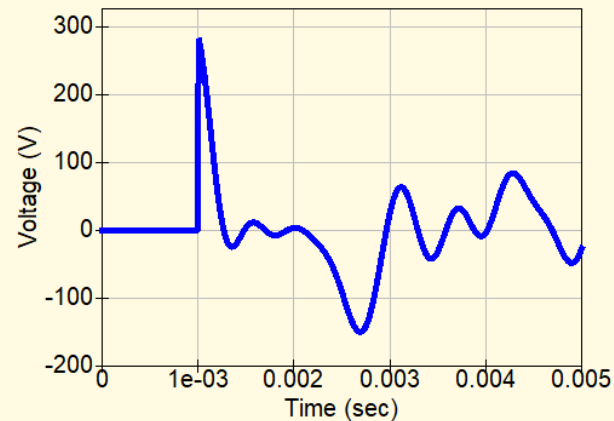
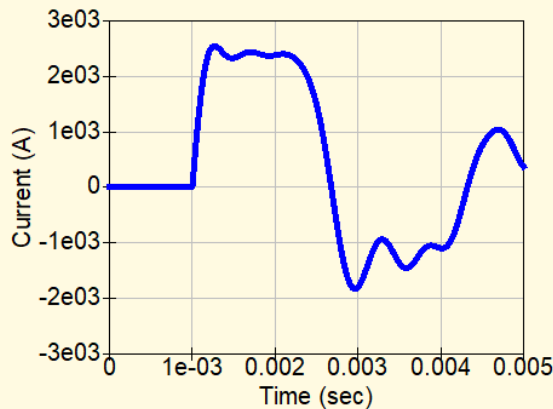
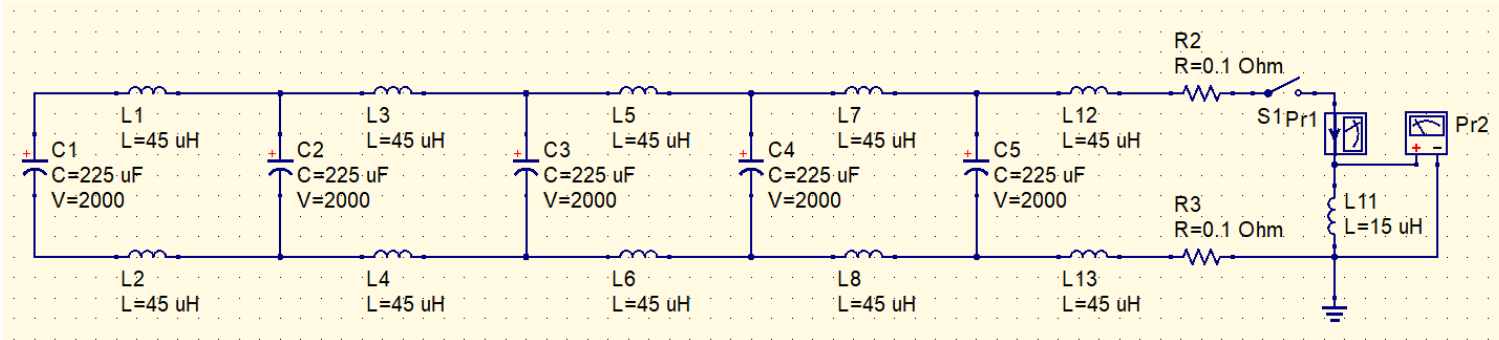
- For 5 stages:

$$\omega_5 = \frac{2\pi}{T} = \frac{\pi}{\tau} = \frac{\pi}{1\text{ms}}$$

$$L = 45 \mu\text{H}$$



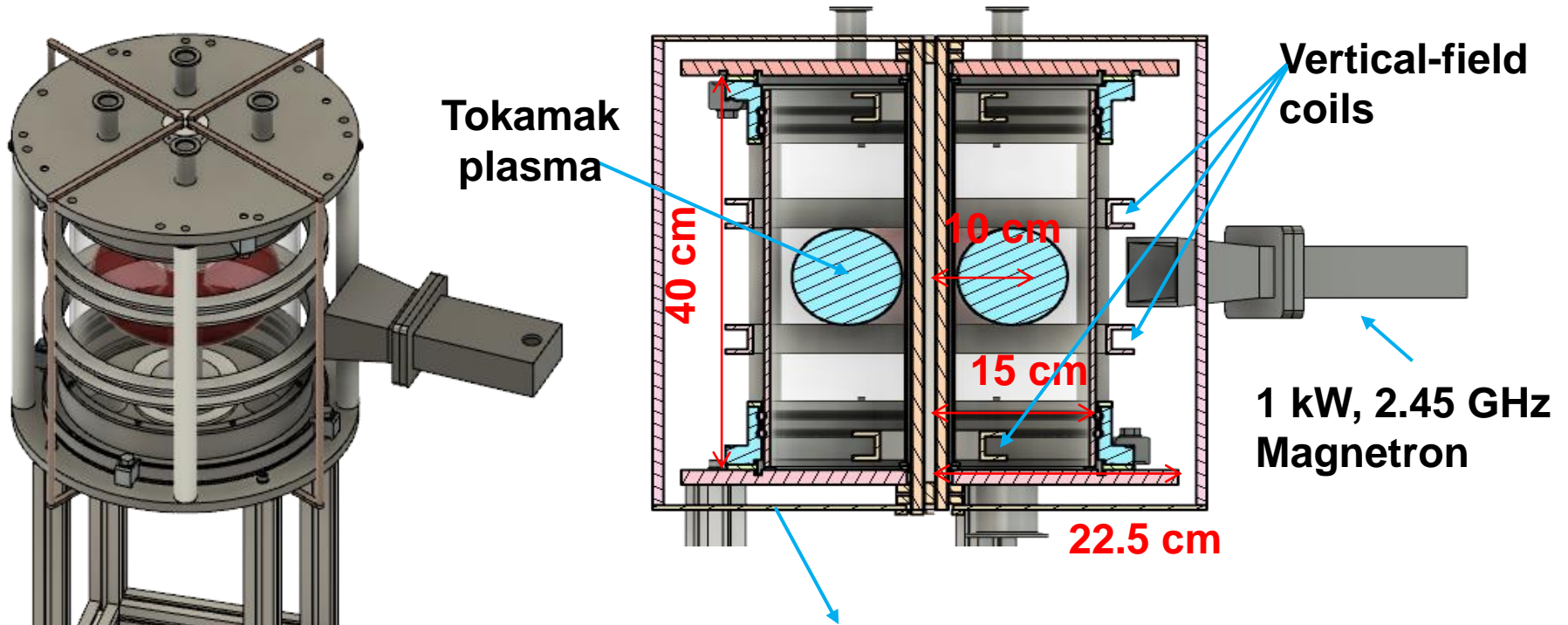
The energy coupling efficiency is lower using the simple PFN



$$E = \frac{1}{2} CV^2 = 2.25 \text{kJ}$$

- Only 4.4 % of the energy is transferred to magnetic energy.

Mini-spherical tokamak



Tokamak plasma

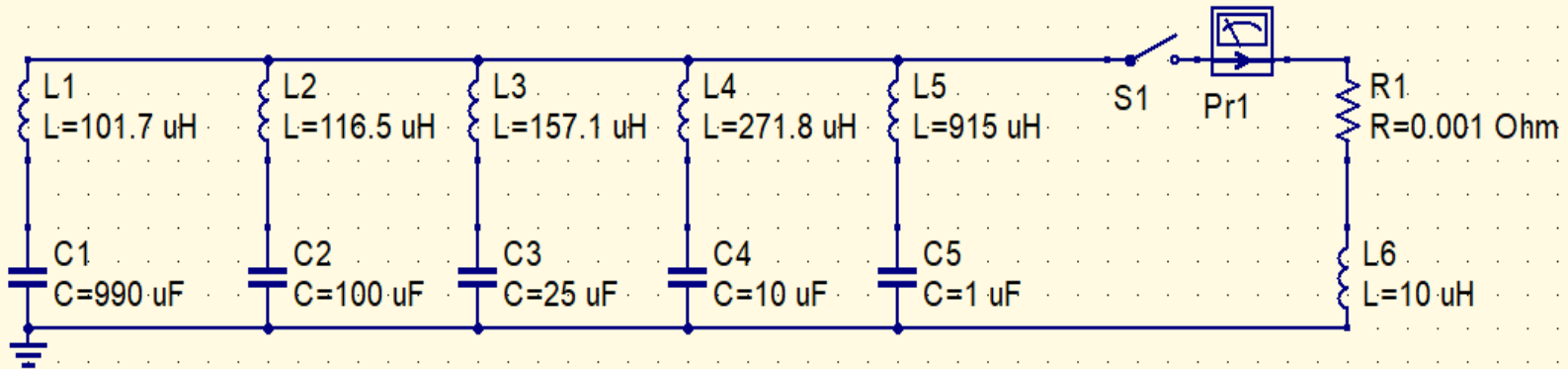
Vertical-field coils

1 kW, 2.45 GHz Magnetron

4 toroidal-field coil connected in series. 1 ms, 2.5 kA pulsed-current.

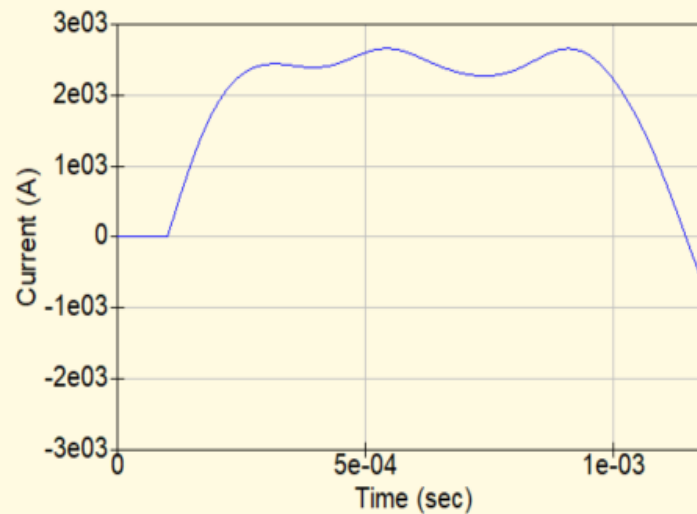
$B=876 \text{ G}$ @ 4.6 cm will be used for ECR heating.

A square pulse of 2.5-kA current output with duration of 1 ms can be provided



**transient
simulation**

TR1
Type=lin
Start=0
Stop=5 ms



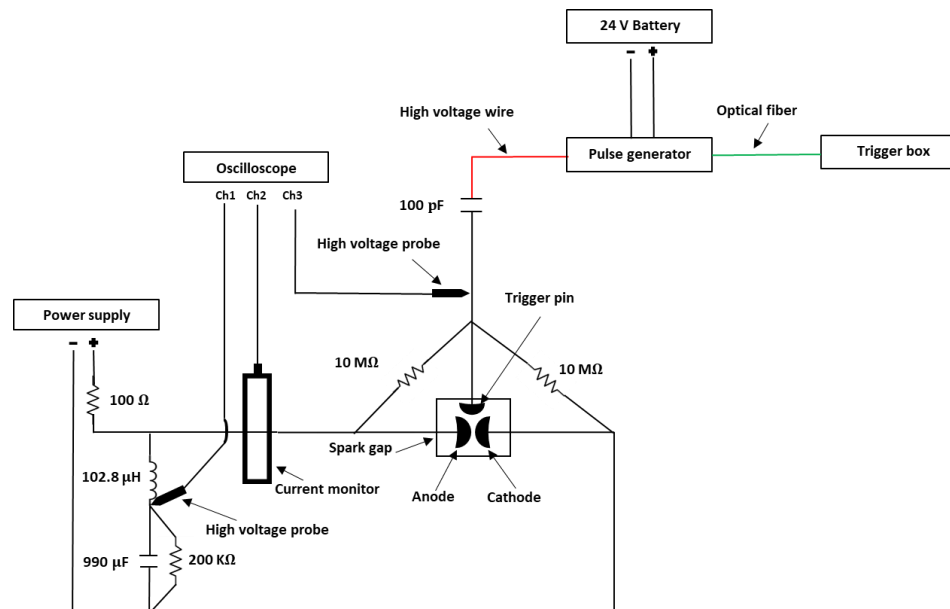
The actual components were determined by what we could get



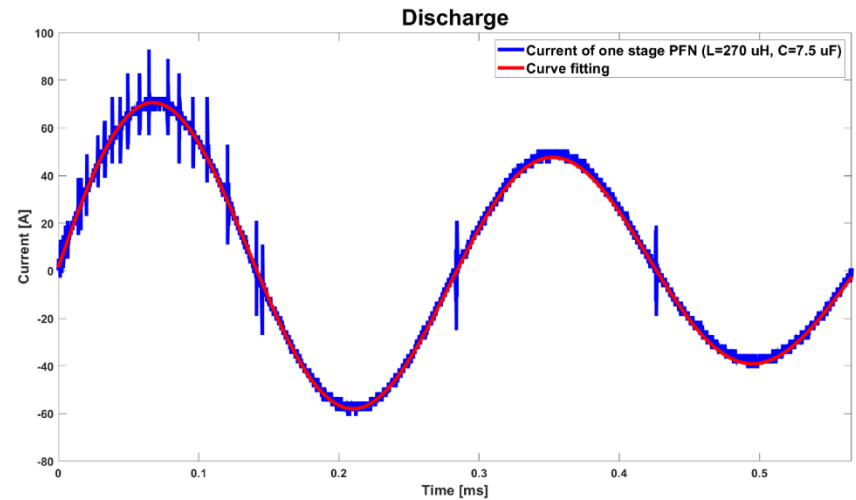
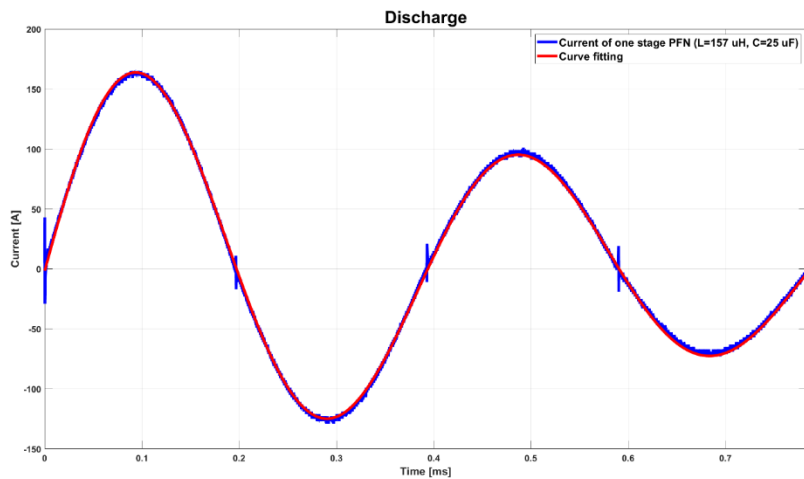
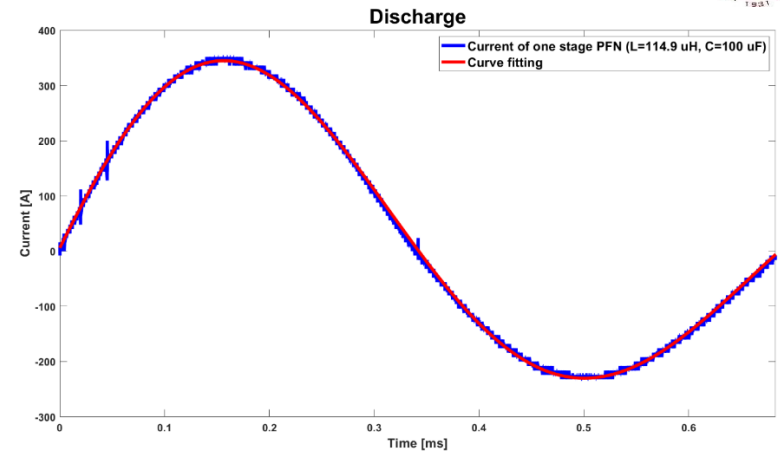
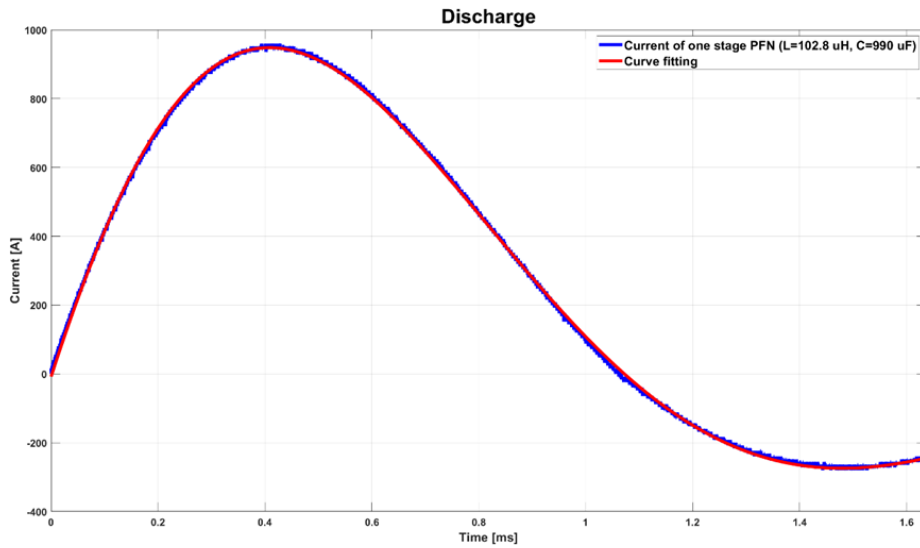
• Design:

I (kA)	V (kV)		1	2	3	4	5
2.5	1	L(uH)	101.7	116.5	157.1	271.8	915.0
		C(uF)	996.6	96.6	25.8	7.6	1.4
2.5	1	L(uH)	102.8	114.9	157	270	-
		C(uF)	990	100	25	10	-

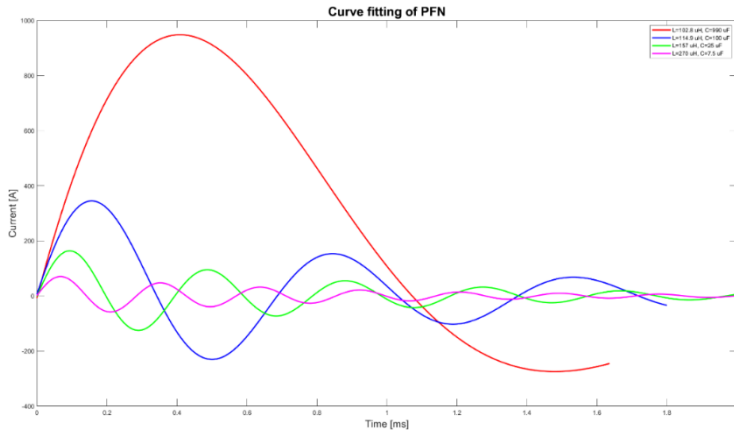
• Built:



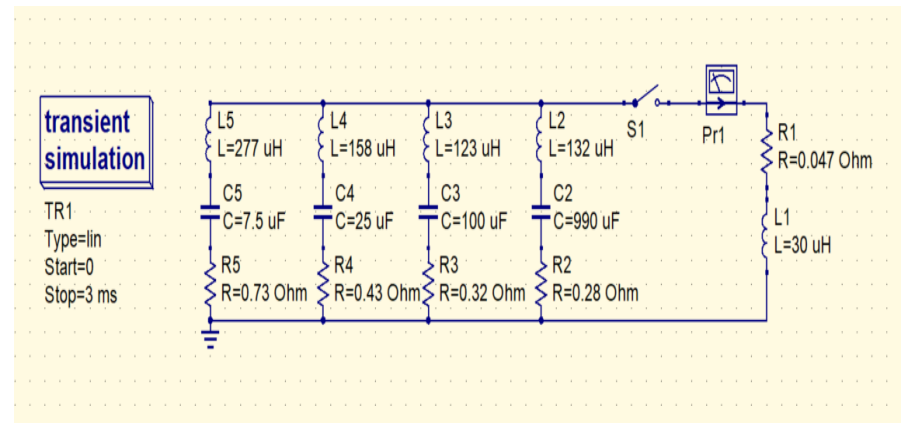
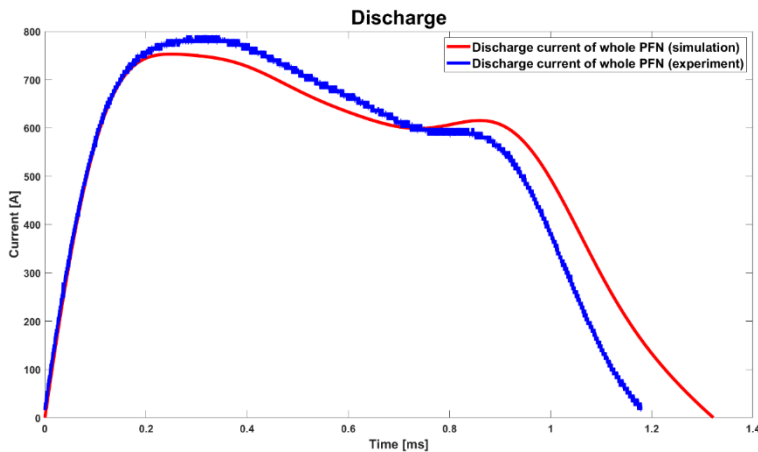
Discharge current measurements



Resistant played an important role



Stage	C (theory)	L (theory)	L (measure)	R (measure)
1	990 (uF)	102.8 (uH)	132±4 (uH)	0.28±0.01 (Ω)
2	100 (uF)	114.9 (uH)	123±0.4 (uH)	0.32±0.02 (Ω)
3	25 (uF)	157 (uH)	158±1 (uH)	0.43±0.01 (Ω)
4	7.5 (uF)	270 (uH)	277±7 (uH)	0.73±0.03 (Ω)



Outlines

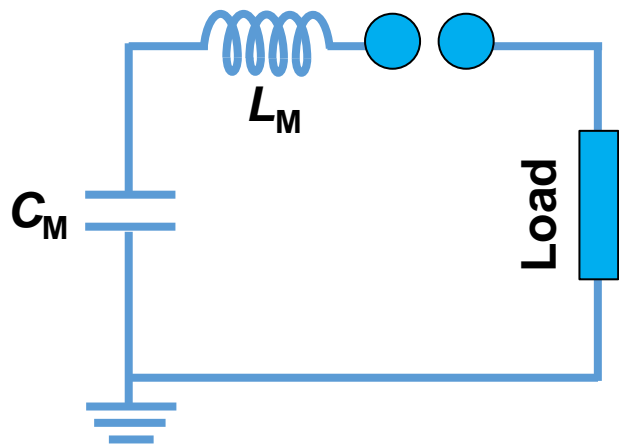


- **Switches**
 - Closing switches: the switching process is associated with voltage breakdown across an initially insulant element.
 - Opening switches: the switching process is associated with a sudden growth of its impedance.
- **Pulse-forming lines**
 - Blumlein line
 - Pulse-forming network
 - **Pulse compressor**
- Pulse transmission and transformation

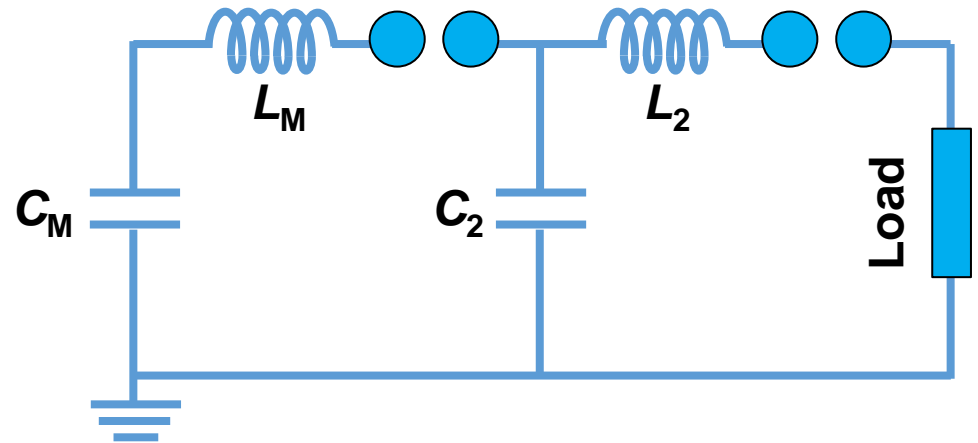
Capacitor load



- **Pulse compression scheme: a charged capacitor can transfer almost all of its energy to an uncharged capacitor if connected through an inductor.**
- **Output voltage can be doubled in a peaking circuit.**



$$I_0 = \frac{V_0}{\sqrt{L_M/C_M}} \quad \omega_0 = \frac{1}{\sqrt{L_M C_M}}$$



$$I_2 = \frac{V_0}{\sqrt{L_2/C_2}} \quad \omega_2 = \frac{1}{\sqrt{L_2 C_2}}$$

$$L_M > L_2 \quad \Rightarrow \quad I_M < I_2 \quad \omega_M < \omega_2 \quad T_M > T_2$$

Capacitor load

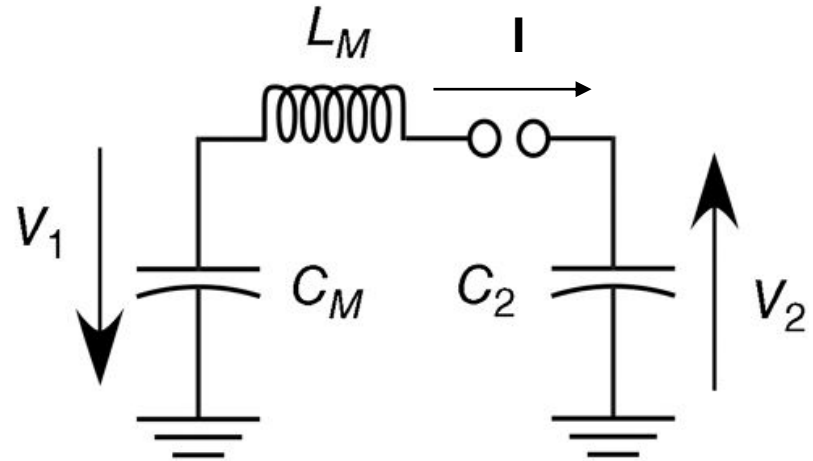


$$V_1 - L_M \frac{dI}{dt} = V_2$$

$$V_1 = V_M - \frac{1}{C_M} \int I dt \quad V_M = NV_0$$

$$V_2 = \frac{1}{C_2} \int I dt$$

$$V_M - \frac{1}{C_M} \int I dt - L_M \frac{dI}{dt} = \frac{1}{C_2} \int I dt$$



$$-\frac{1}{C_M} I - L_M \frac{d^2 I}{dt^2} = \frac{1}{C_2} I \quad L_M \frac{d^2 I}{dt^2} + \left(\frac{1}{C_M} + \frac{1}{C_2} \right) I = 0$$

$$\frac{d^2 I}{dt^2} + \frac{1}{L_M C_{\text{eff}}} I = 0 \quad \frac{1}{C_{\text{eff}}} = \frac{1}{C_M} + \frac{1}{C_2} \quad \omega = \sqrt{\frac{1}{L_M C_{\text{eff}}}}$$

$$I = \alpha \sin(\omega t) + \beta \cos(\omega t)$$

Capacitor load



$$I = \alpha \sin(\omega t) + \beta \cos(\omega t)$$

$$I(t = 0) = 0 \Rightarrow \beta = 0$$

$$I = \alpha \sin(\omega t)$$

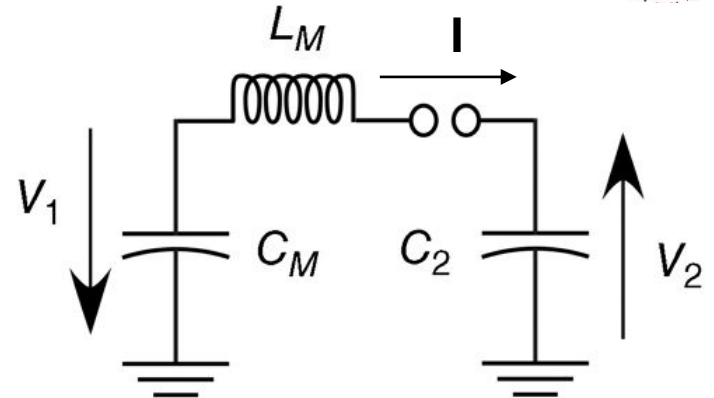
$$\frac{dI}{dt} = \alpha \omega \cos(\omega t)$$

$$L_M \left. \frac{dI}{dt} \right|_{t=0} = L_M \alpha \omega = V_M \quad \alpha = \frac{V_M}{L_M \omega}$$

$$I(t) = \frac{V_M}{L\omega} \sin(\omega t)$$

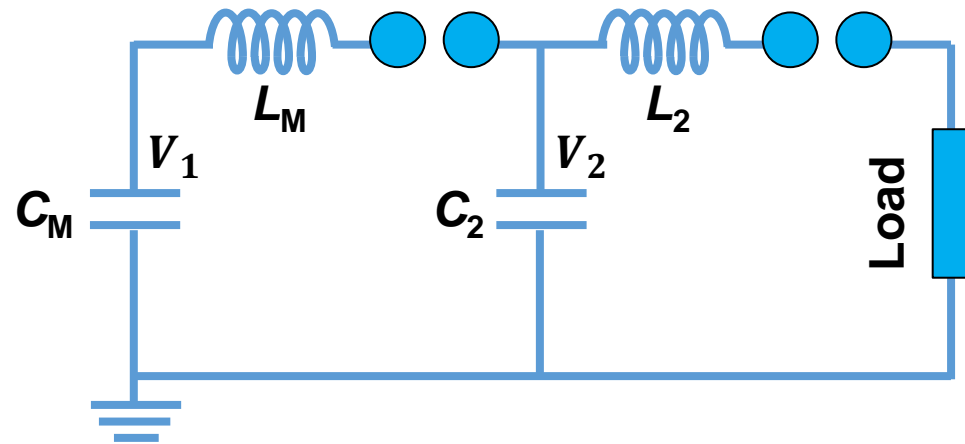
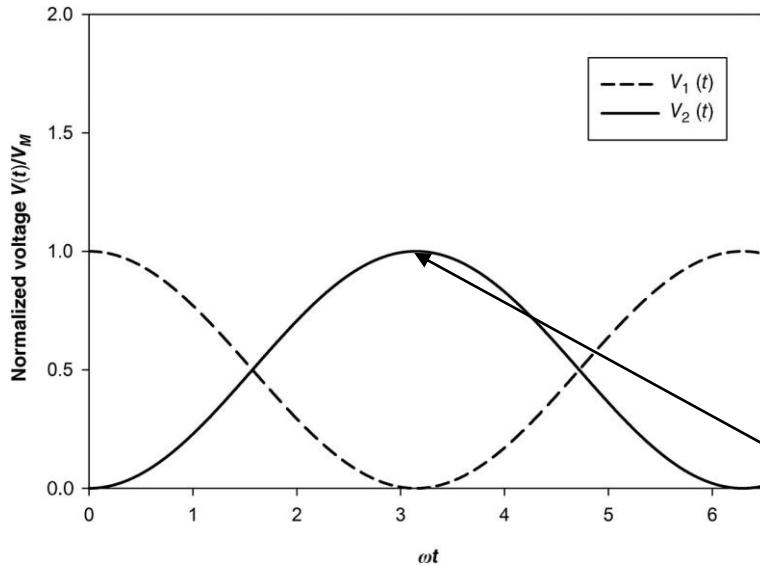
$$V_1 = V_M - \frac{1}{C_M} \int_0^t \frac{V_M}{L\omega} \sin(\omega t) dt = V_M - \frac{V_M C_2}{C_M + C_2} [1 - \cos(\omega t)]$$

$$V_2 = \frac{1}{C_2} \int_0^t \frac{V_M}{L\omega} \sin(\omega t) dt = \frac{V_M C_M}{C_M + C_2} [1 - \cos(\omega t)] \quad \left. \frac{V_2}{V_M} \right|_{\max} = \frac{2C_M}{C_M + C_2}$$



for $C_2 \sim C_M, \frac{V_2}{V_M} \sim 1$

Pulse compression scheme: $C_2 \sim C_M$



Energy is fully transferred to the 2nd cap, i.e., intermediate storage capacitor.

$$V_1 = V_M - \frac{V_M C_2}{C_M + C_2} [1 - \cos(\omega t)] \approx V_M - \frac{V_M}{2} [1 - \cos(\omega t)]$$

$$V_2 = \frac{V_M C_M}{C_M + C_2} [1 - \cos(\omega t)] \approx \frac{V_M}{2} [1 - \cos(\omega t)]$$

$$\text{For } t = \frac{\pi}{\omega}, \quad V_1 \approx 0, \quad V_2 \approx V_M$$

Water is commonly used as the dielectric material for the intermediate capacitor

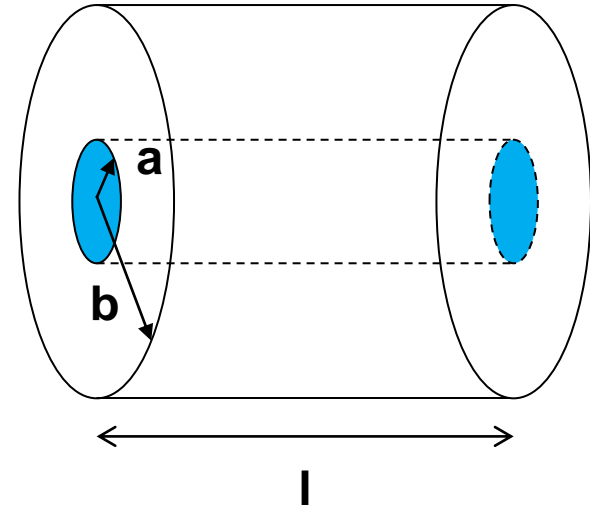


$$C = \frac{2\pi\epsilon_r\epsilon_0}{\ln(b/a)} l \quad \text{For } \frac{b}{a} = \frac{1}{0.9} \approx 1.1$$

- The gap between two cylinders need to be able to handle the high voltage.

$$\text{Air: } \epsilon_r = 1 \Rightarrow \frac{C}{l} = 0.5 \times 10^{-9} \text{ F/m}$$

$$\text{Water: } \epsilon_r = 80 \Rightarrow \frac{C}{l} = 4 \times 10^{-8} \text{ F/m}$$



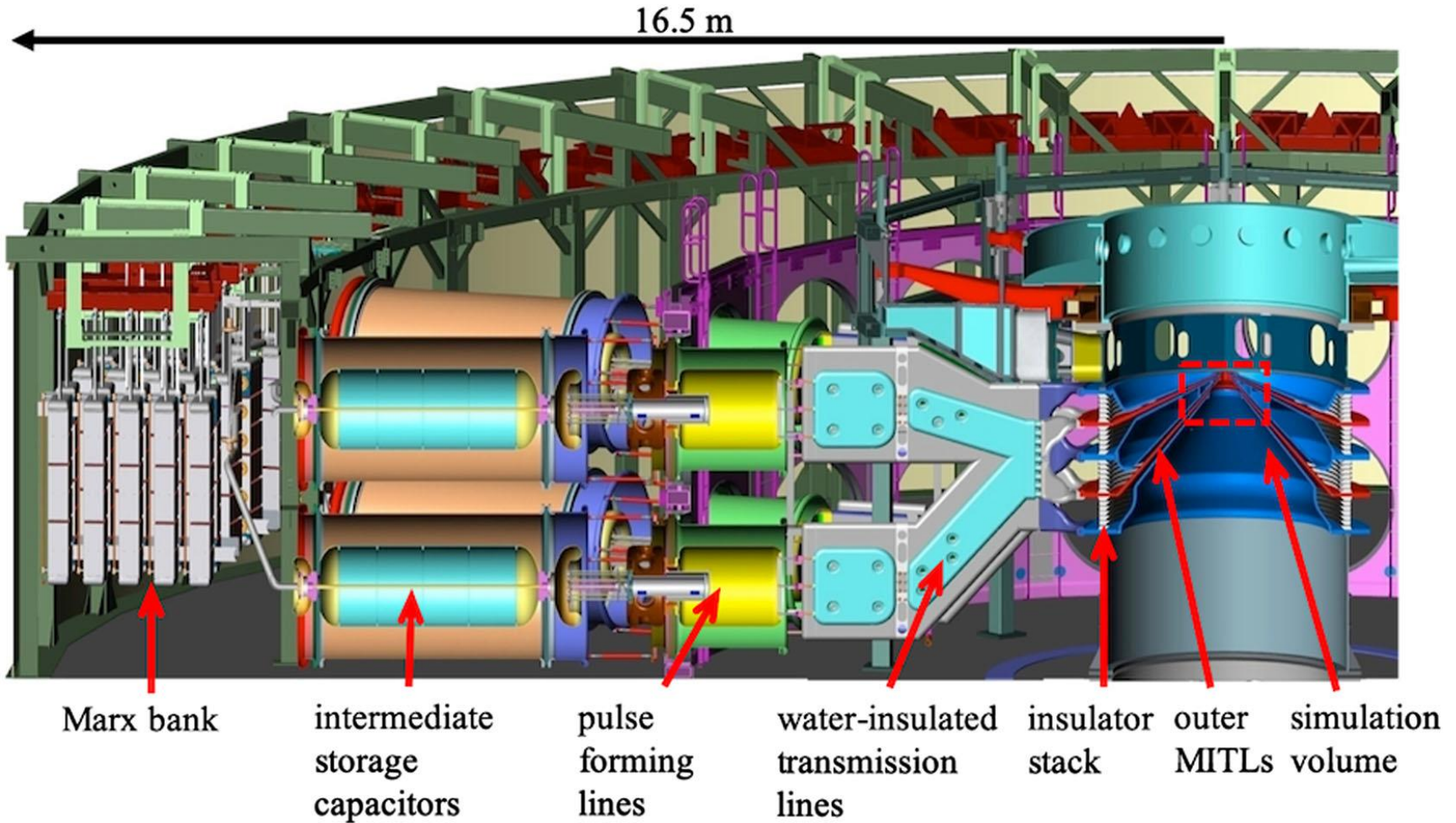
Ex: KALIF, bipolar Marx generator, charged up to ± 100 kV. $V_{M,out} = 5$ MV.

$$C_M = \frac{0.5 \mu\text{F}}{25} = 25 \text{ nF}$$

$$\text{Using air: } l = \frac{25 \times 10^{-9}}{0.5 \times 10^{-9}} = 50 \text{ m}$$

$$\text{Using water: } l = \frac{25 \times 10^{-9}}{4 \times 10^{-8}} = 0.625 \text{ m}$$

Intermediate storage capacitors can be used to compress the pulse



Outlines

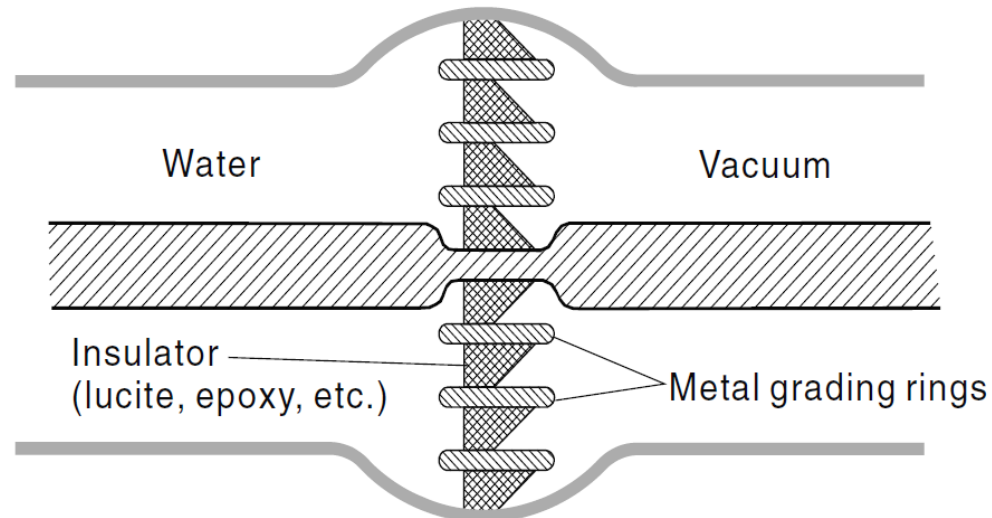


- **Switches**
 - **Closing switches:** the switching process is associated with voltage breakdown across an initially insulant element.
 - **Opening switches:** the switching process is associated with a sudden growth of its impedance.
- **Pulse-forming lines**
 - Blumlein line
 - Pulse-forming network
 - Pulse compressor
- **Pulse transmission and transformation**

Insulating interface separating the vacuum section and the liquid dielectric is needed



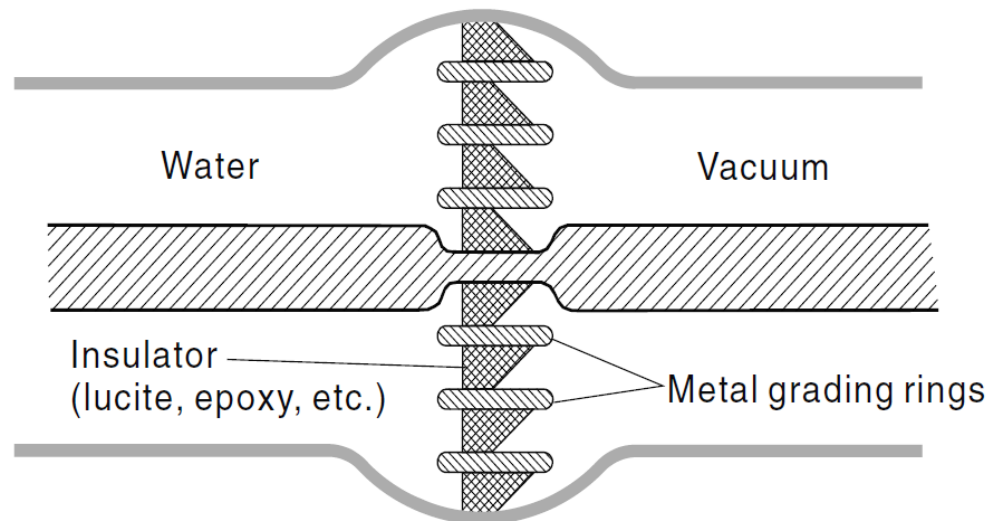
- **Some tasks in science and technology required brightness of intense pulsed radiation $> 100 \text{ TW/cm}^2\text{-Sr}$. With $E > 1 \text{ MJ}$, electric power $> 100 \text{ TW}$, electric power flux density $> 100 \text{ TW/m}^2$ are needed.**
- **Vacuum environment is required.**
- **High-voltage pulse must enter a vacuum vessel hosting the source through an insulating interface separating the liquid dielectric from the vacuum section.**



The interface consists of insulating rings separated by metallic grading rings



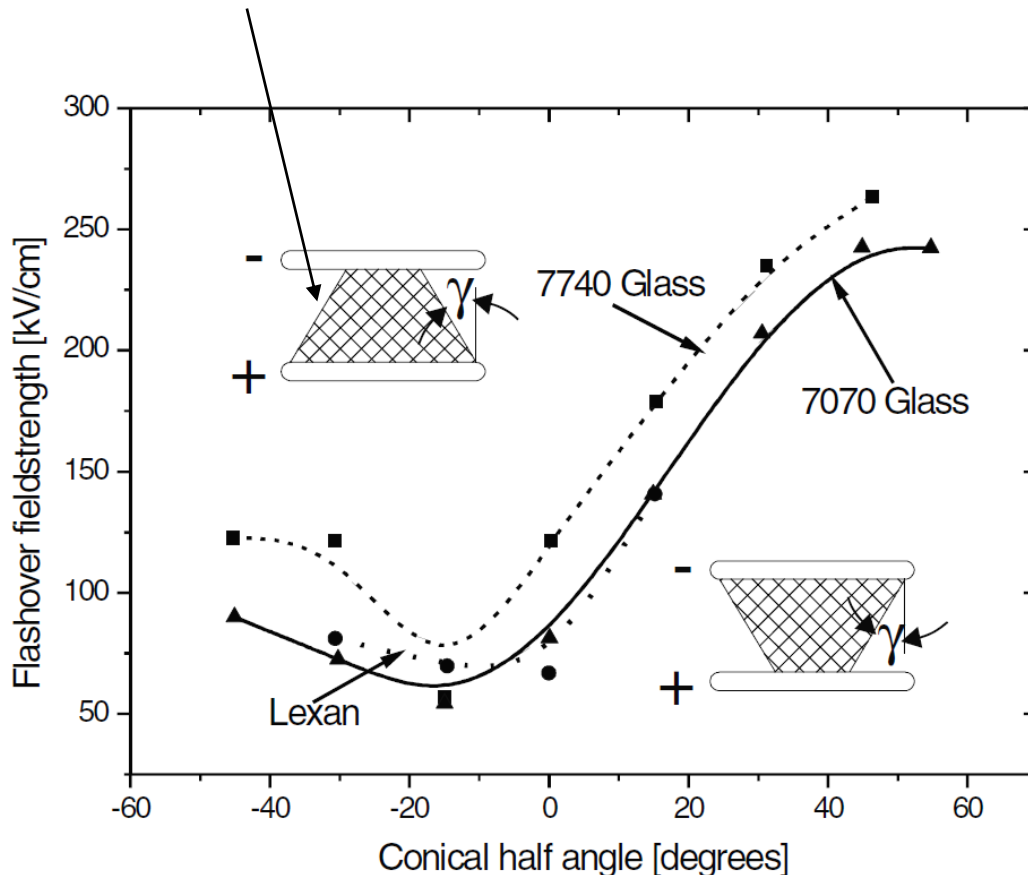
- The metal grading rings are used to distribute the potential homogeneously over the interface on the vacuum surface.
- The metallic and dielectric rings are sealed to hold the high vacuum either by O-rings or by Metal-to-dielectric bond.
- Sparking on the surface on the vacuum side is more important.
- Electrons may be produced by field emission on metallic surfaces.



The side surface of the dielectric material is tilted to prevent flash over



- Out gassing: gas from the “absorbs” released by electron bombardment.
- Electron avalanches may occur with the tangential electric field from the space charge on insulator.

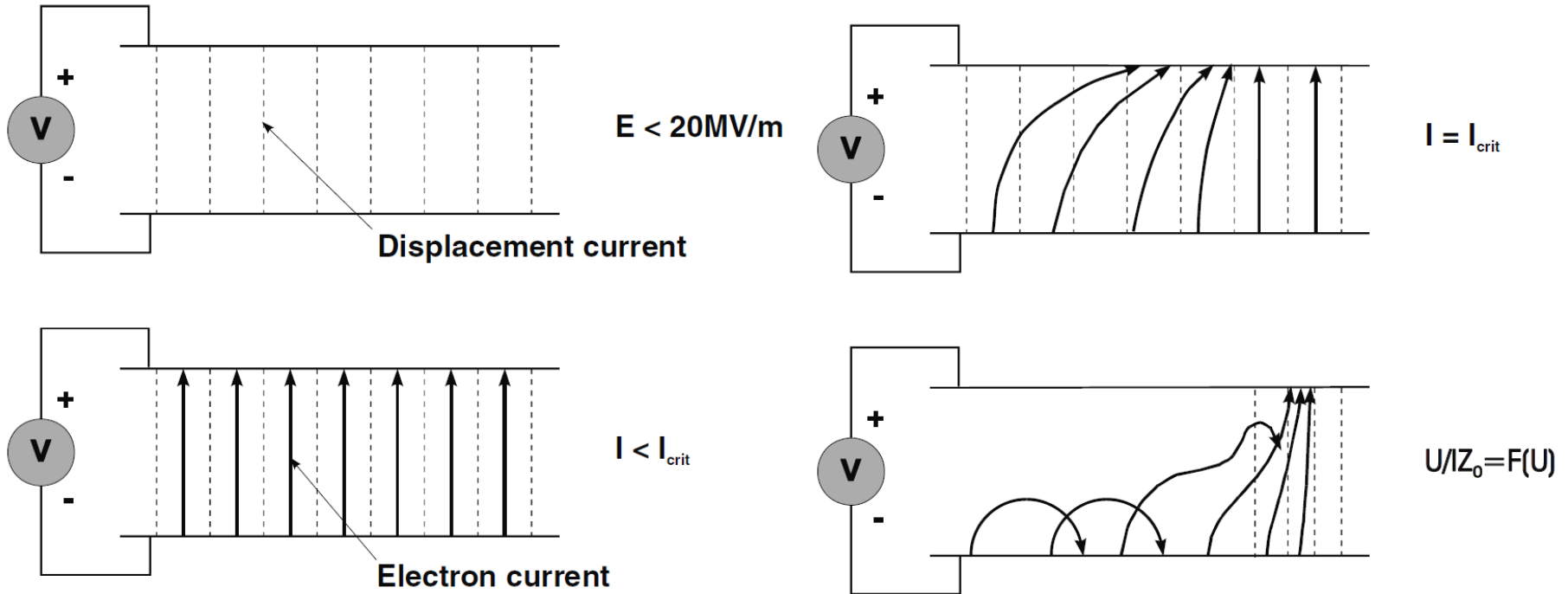


- Dielectric-vacuum interface is the weakest element of a high-voltage pulse line under E-field stress.

$$E_{DB} = \frac{7 \times 10^5}{t^{1/6} A^{1/10}} (V/m)$$

- t: time when $E > 87\% E_{max}$.
- For $t=10$ ns, $E_{max}=20$ MV/m, Max power density that can be delivered is 1 TW/m².

Self-magnetic insulation

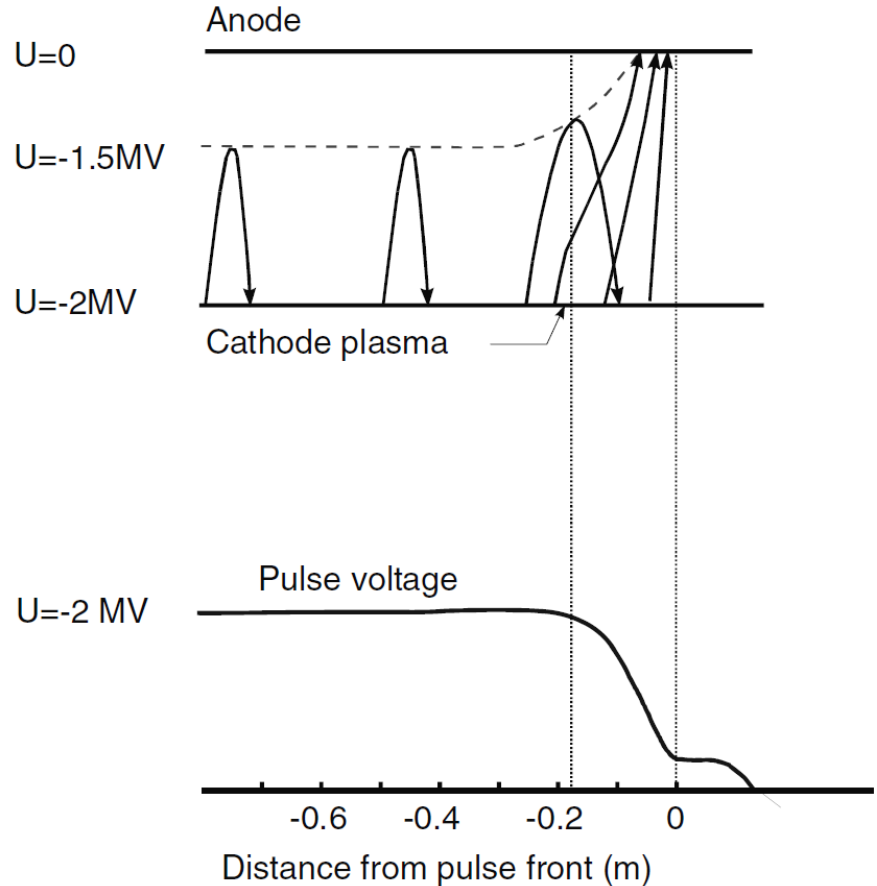


- For $E > 20 \text{ MV/m}$, homogeneous plasma layer is generated within a few nanosecond.
- For $I > I_{\text{crit}}$, electron orbits can no longer reach the anode \Rightarrow more and more sections are insulated. \Rightarrow An electron sheath forms on the negative conductor.

Electromagnetic shock wave is formed



- The propagation velocity of the loss front is less than the speed of light, c .
- As long as the voltage ramp remains below the breakdown threshold, the wave propagates at the speed of light.



Pulse transformers

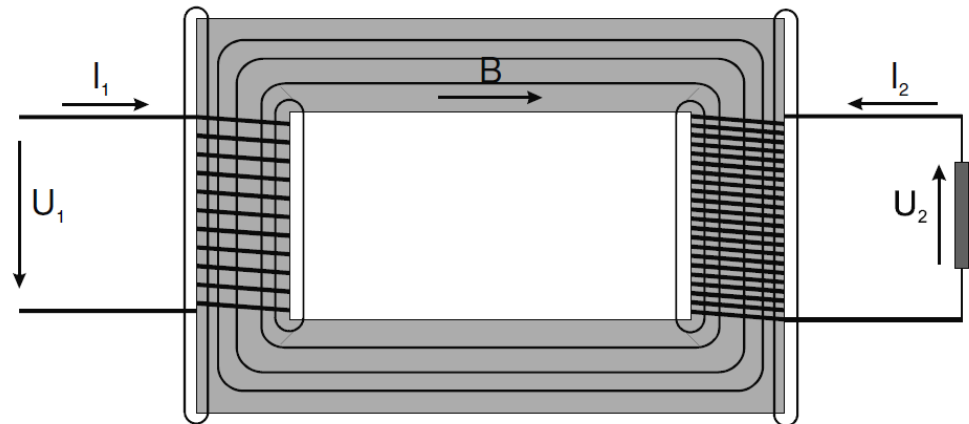


- **High-voltage transformers:** used for transformation of current, voltage, impedance, polarity inversion, insulation and coupling between circuits at different potentials.
- **Based on magnetic coupling between two conducting circuits.**
- **Perfect or ideal transformer:** no ohmic losses, no eddy currents, without hysteresis and stray field => magnetic flux goes completely through both the primary and second coil.
- **Faraday's law:**

$$U_1 = N_1 \frac{d\phi}{dt}$$

$$U_2 = -N_2 \frac{d\phi}{dt}$$

$$\frac{U_2}{U_1} = -\frac{N_2}{N_1}$$



The transformer rise the voltage but reduce the current

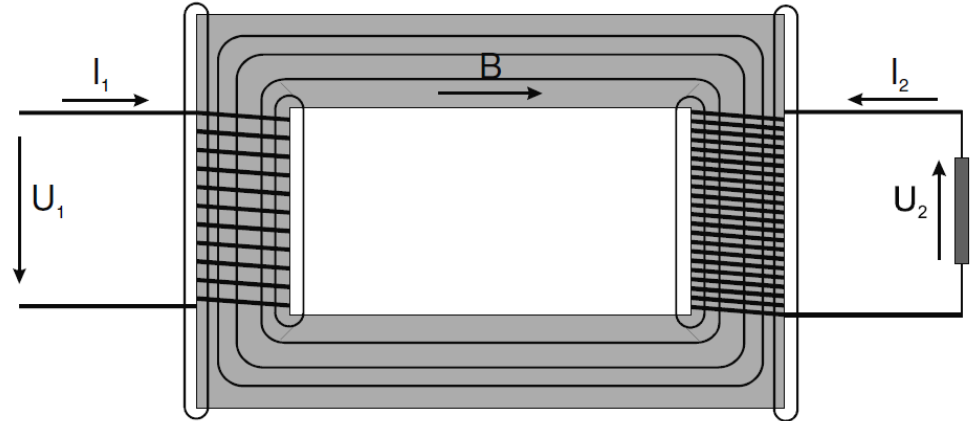


$$U_1 = N_1 \frac{d\phi}{dt} \quad \frac{U_2}{U_1} = -\frac{N_2}{N_1}$$

$$U_2 = -N_2 \frac{d\phi}{dt}$$

- For open circuit, i.e. secondary coil is open $\Rightarrow \phi$ is caused by i_1 only:

$$i_{10} = \frac{U_1}{i\omega L_1}$$



- If a load of complex impedance Z is connected to the secondary coil:

$$i_2 = \frac{U_2}{Z} \quad N_2 i_2 = N_1 i_1' \quad \text{Additional flux from the secondary coil is compensated from primary coil.}$$

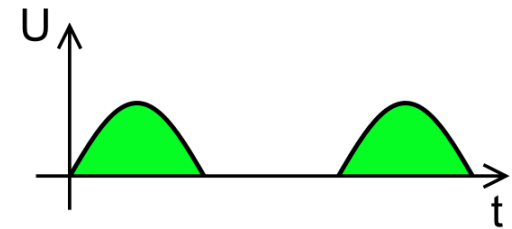
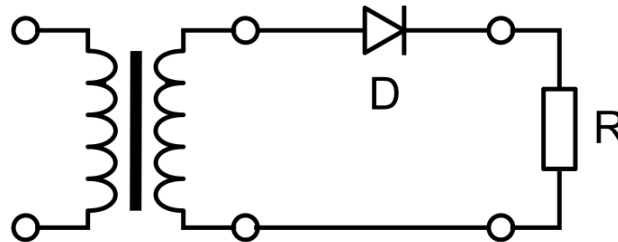
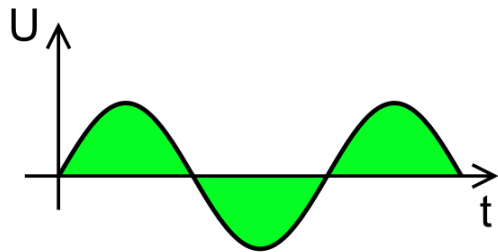
$$i_1' = i_{10} + i_1' = i_{10} - \frac{N_2}{N_1} i_2 \quad \text{Power} = (i_1' - i_{10})U_1 = -\frac{N_2}{N_1} i_2 U_1 = i_2 U_2$$

- If $i_{10} \ll \frac{N_2 i_2}{N_1} \Rightarrow i_1 = -\frac{N_2}{N_1} i_2$

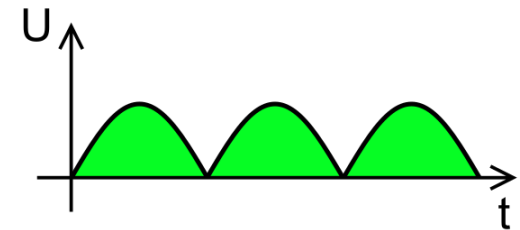
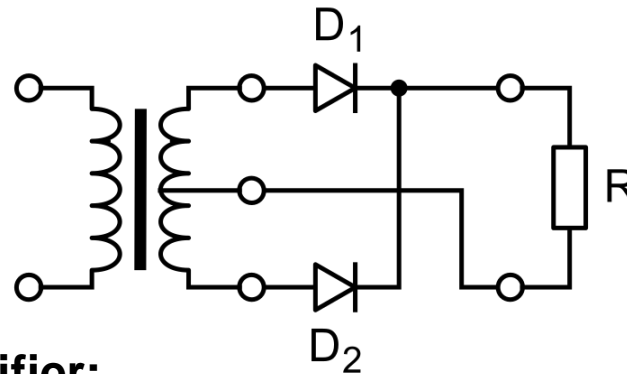
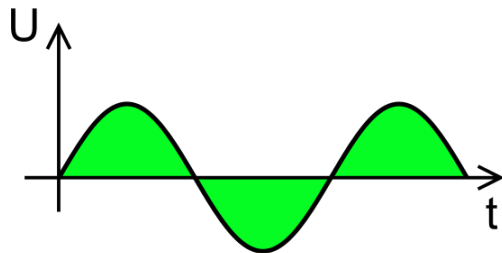
Rectifier



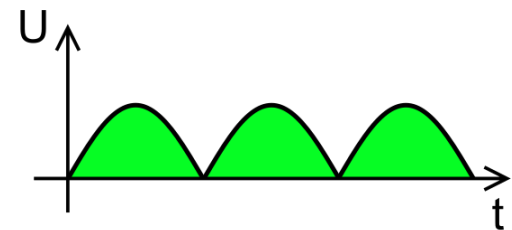
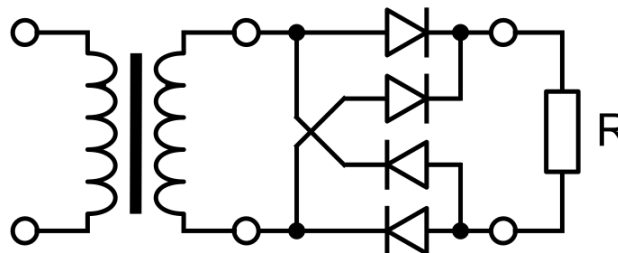
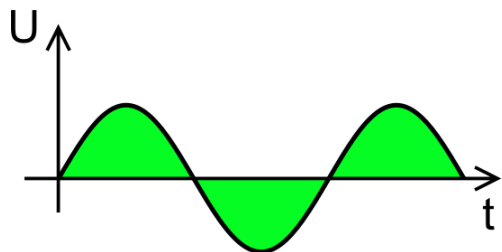
- **Half-wave rectifier:**



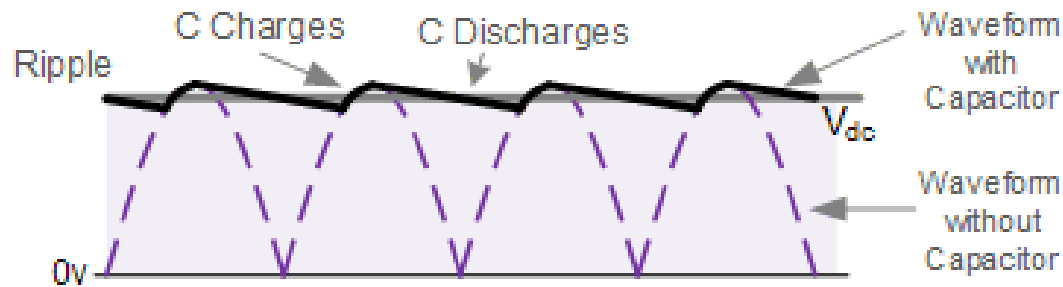
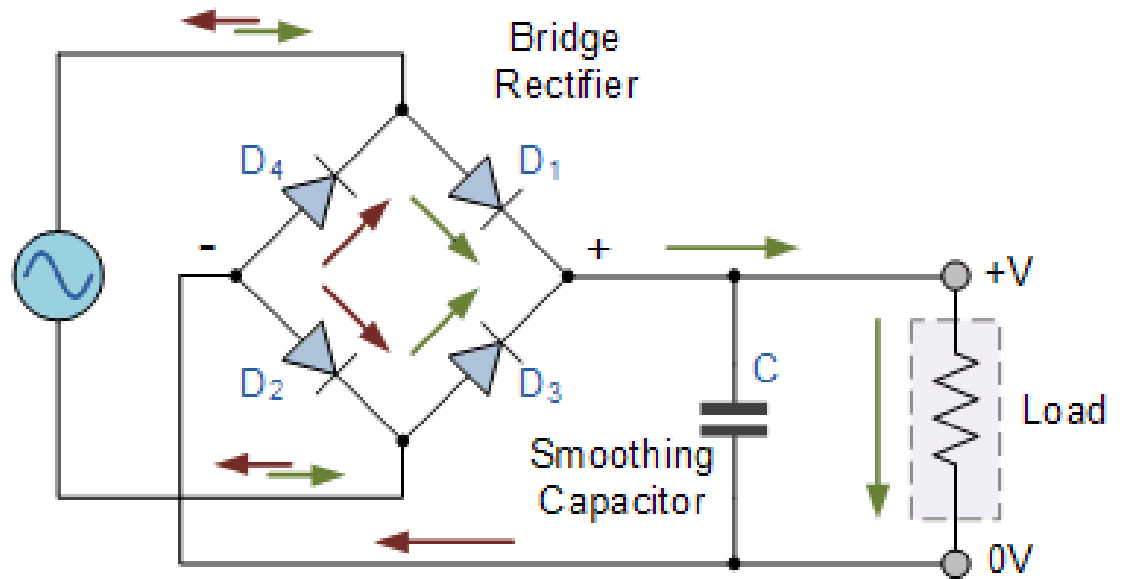
- **Center-tapped full-wave rectifier:**



- **Full-wave bridge rectifier:**



Full-wave rectifier with smoothing capacitor

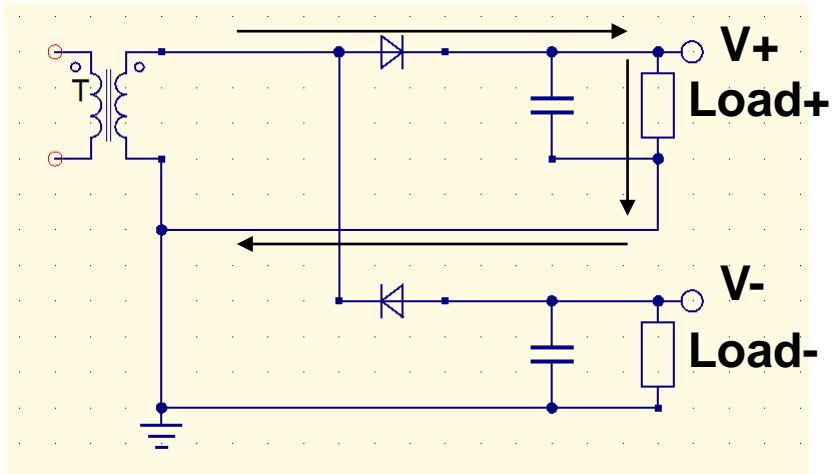


Resultant Output Waveform

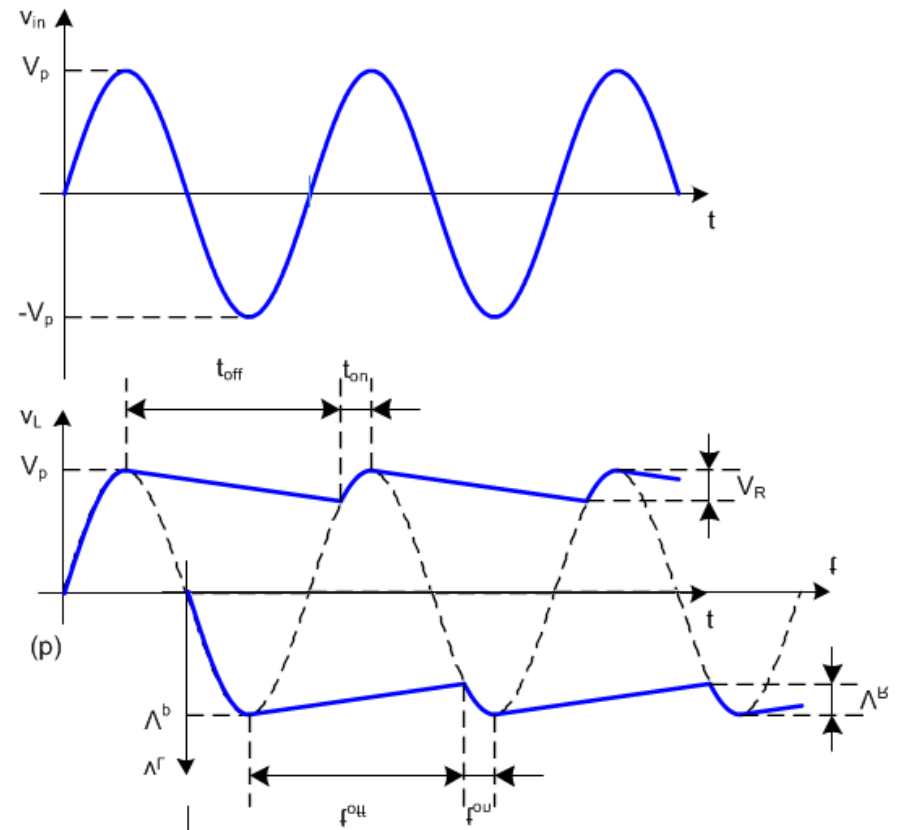
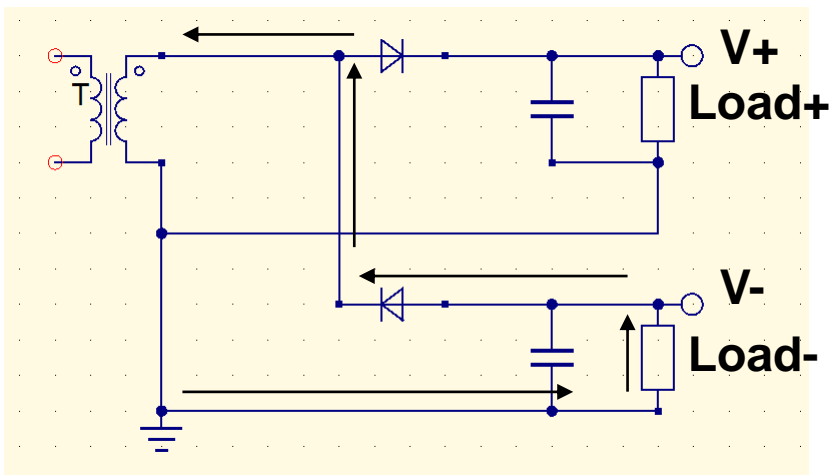
Dual output



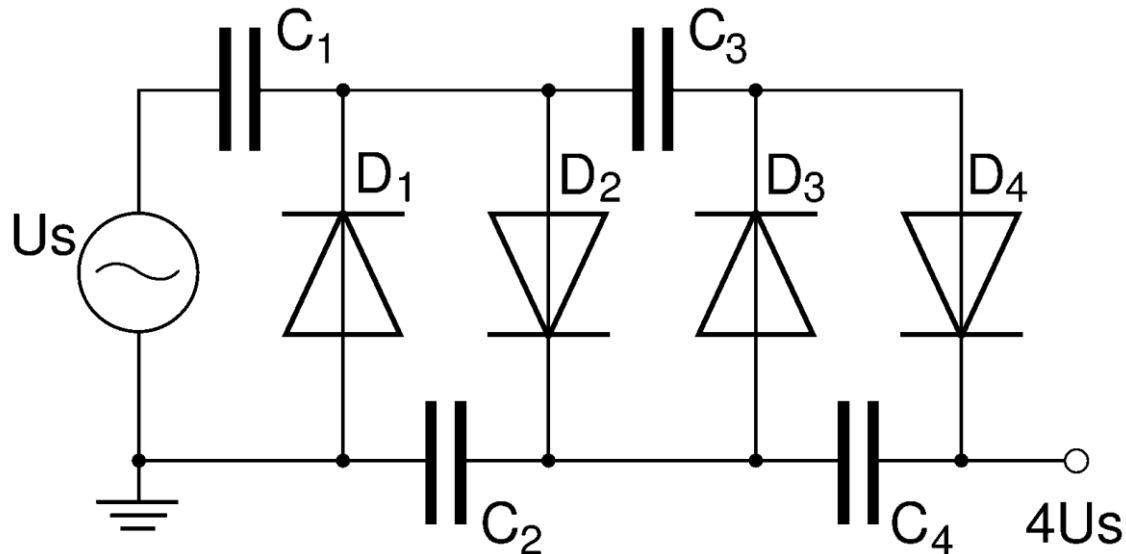
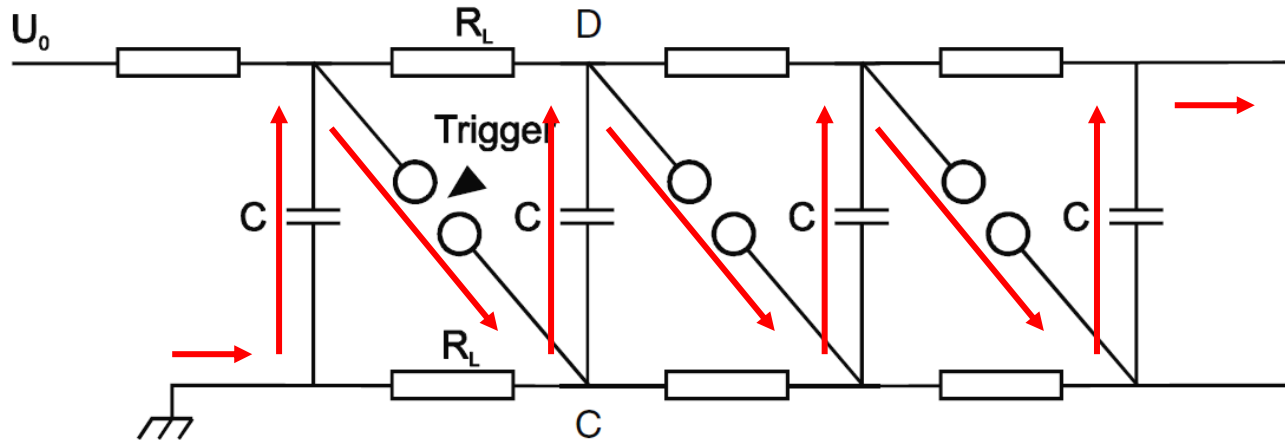
- **Positive cycle:**



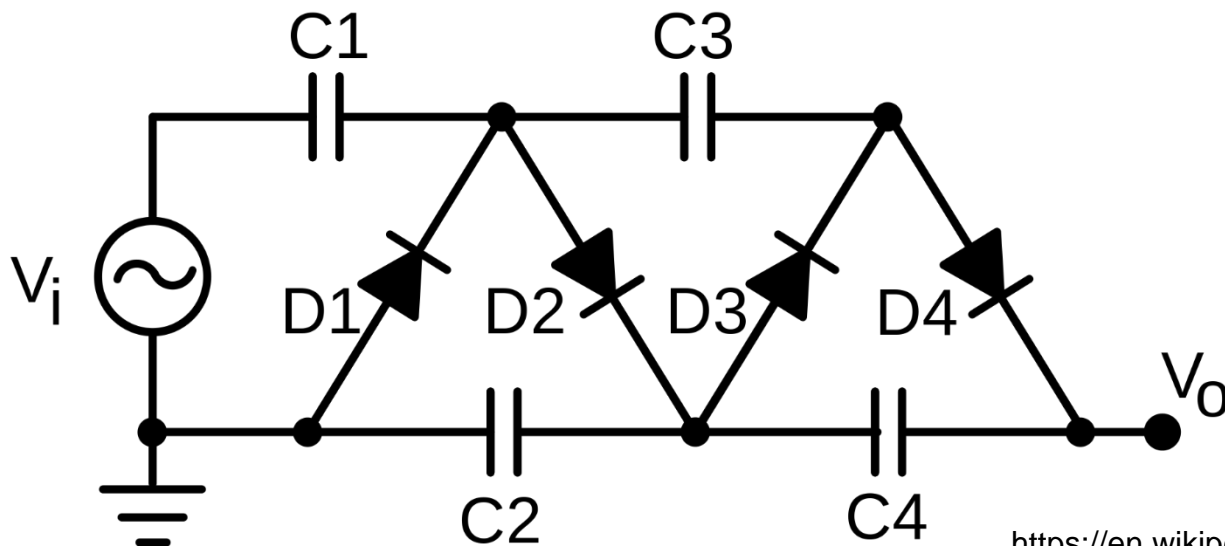
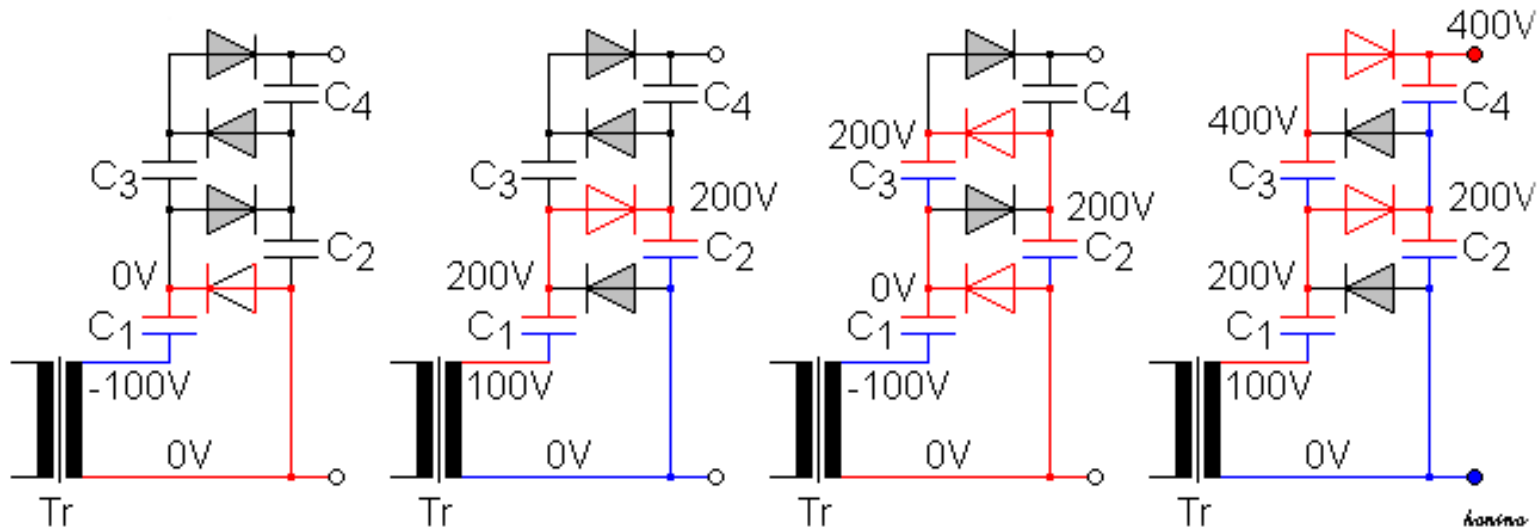
- **Negative cycle:**



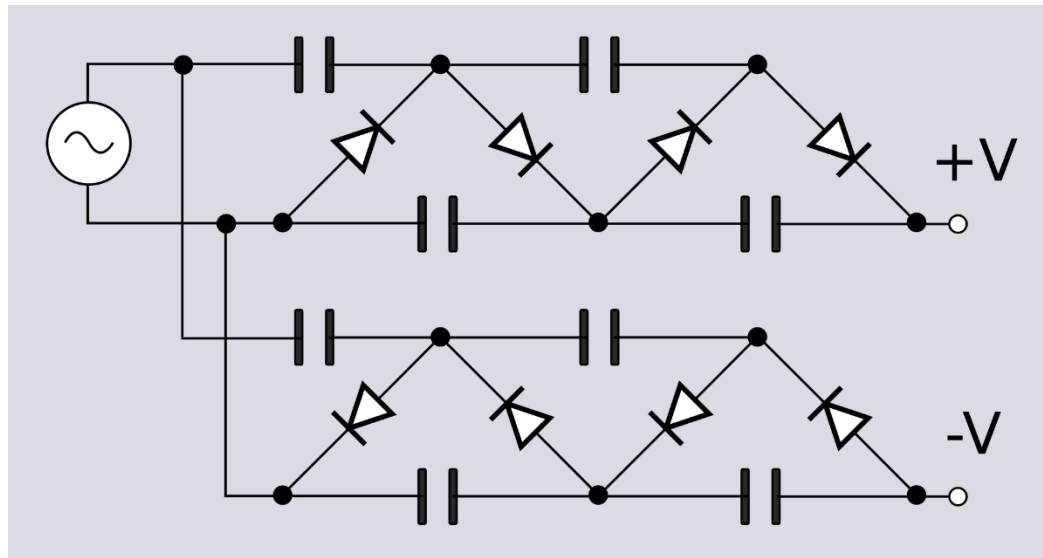
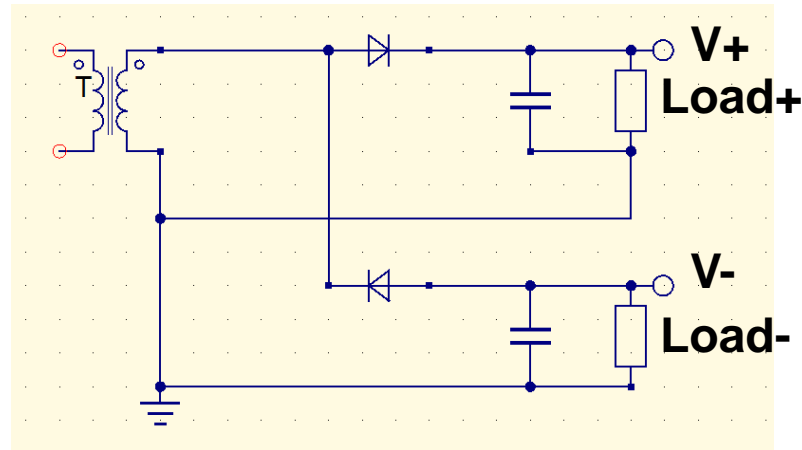
Voltage multiplier (Cockcroft–Walton (CW) generator)



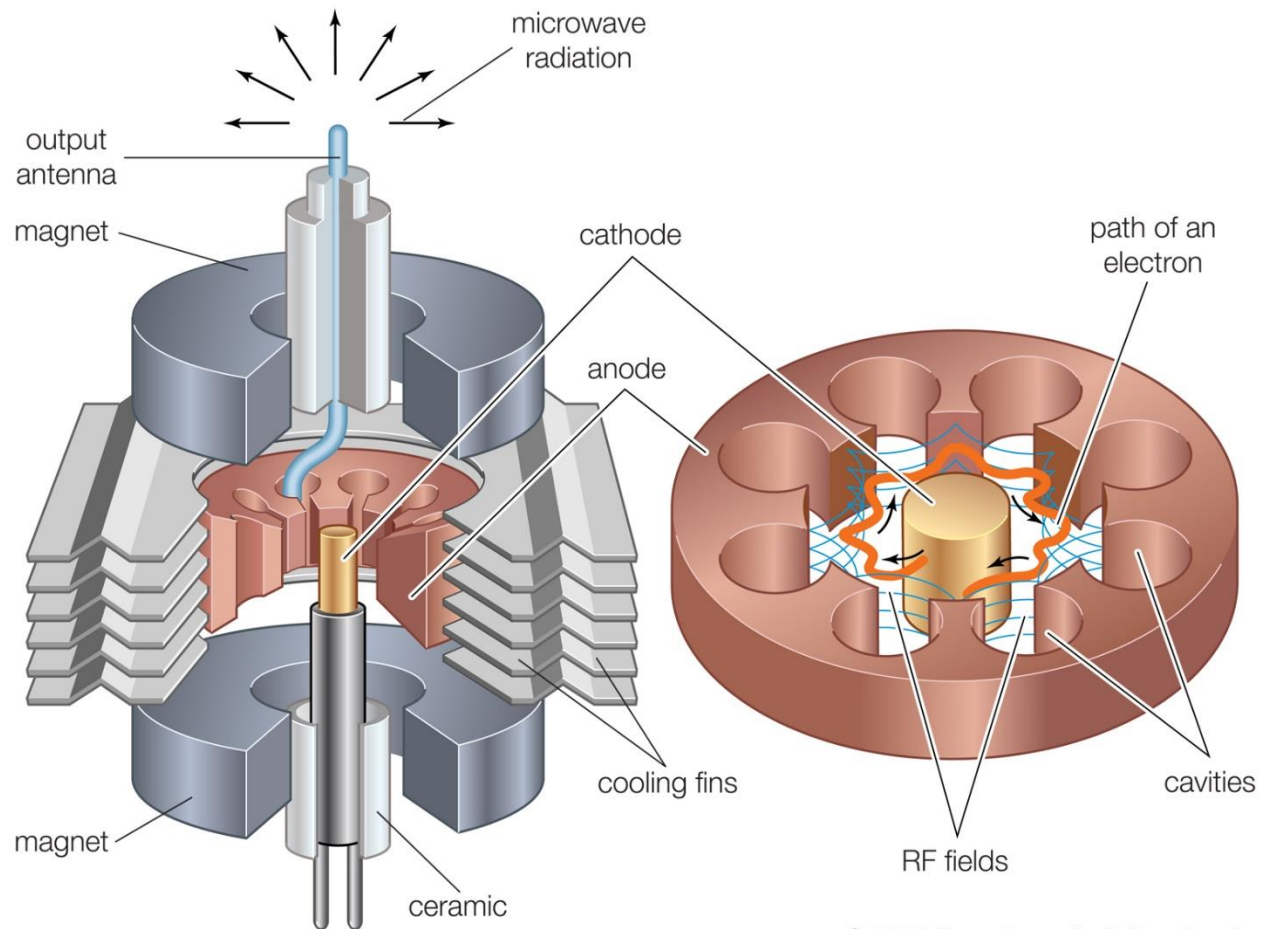
Voltage multiplier (Cockcroft–Walton (CW) generator)



Dual-output



Internal of a magnetron

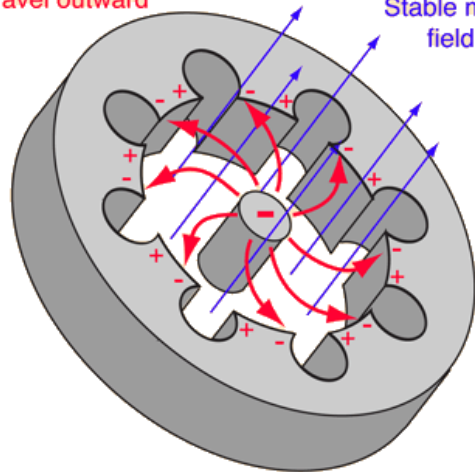


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Magnetron is a forced oscillation driven by electrons between the gap



Hot cathode emits electrons which travel outward

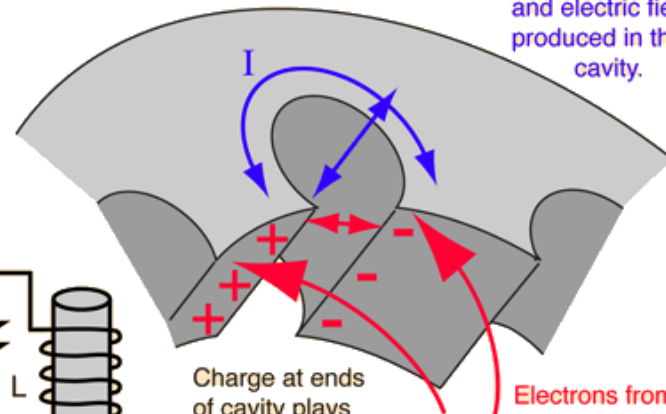


Stable magnetic field B

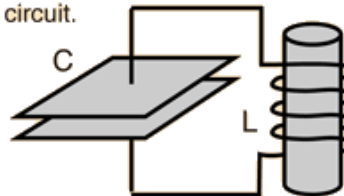
Electrons from a hot filament would travel radially to the outside ring if it were not for the magnetic field. The magnetic force deflects them in the sense shown and they tend to sweep around the circle. In so doing, they "pump" the natural resonant frequency of the cavities. The currents around the resonant cavities cause them to radiate electromagnetic energy at that resonant frequency.

Current around the cavity plays the role of an inductor.

Oscillating magnetic and electric fields produced in the cavity.



The cavity exhibits a resonance analogous to a parallel resonant circuit.

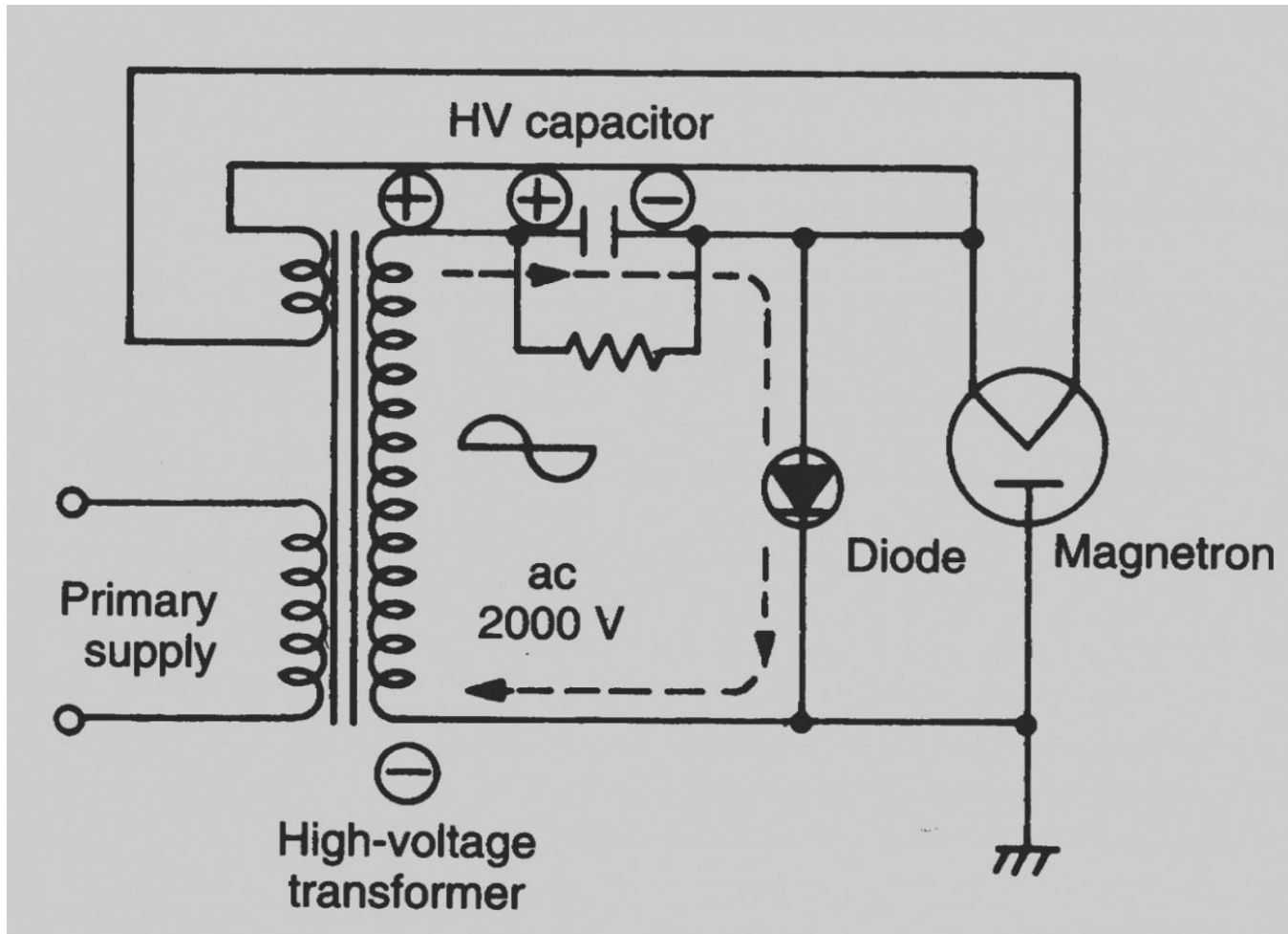


$$f_{resonance} \approx \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

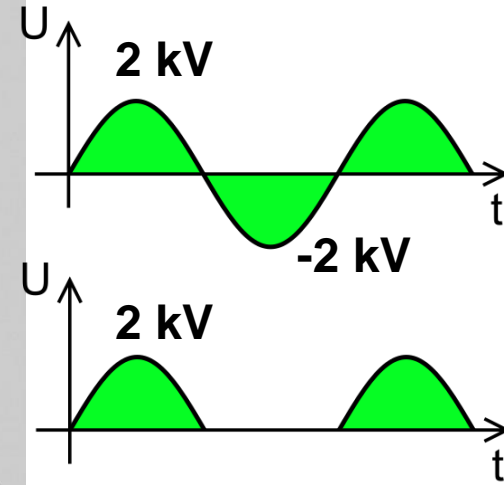
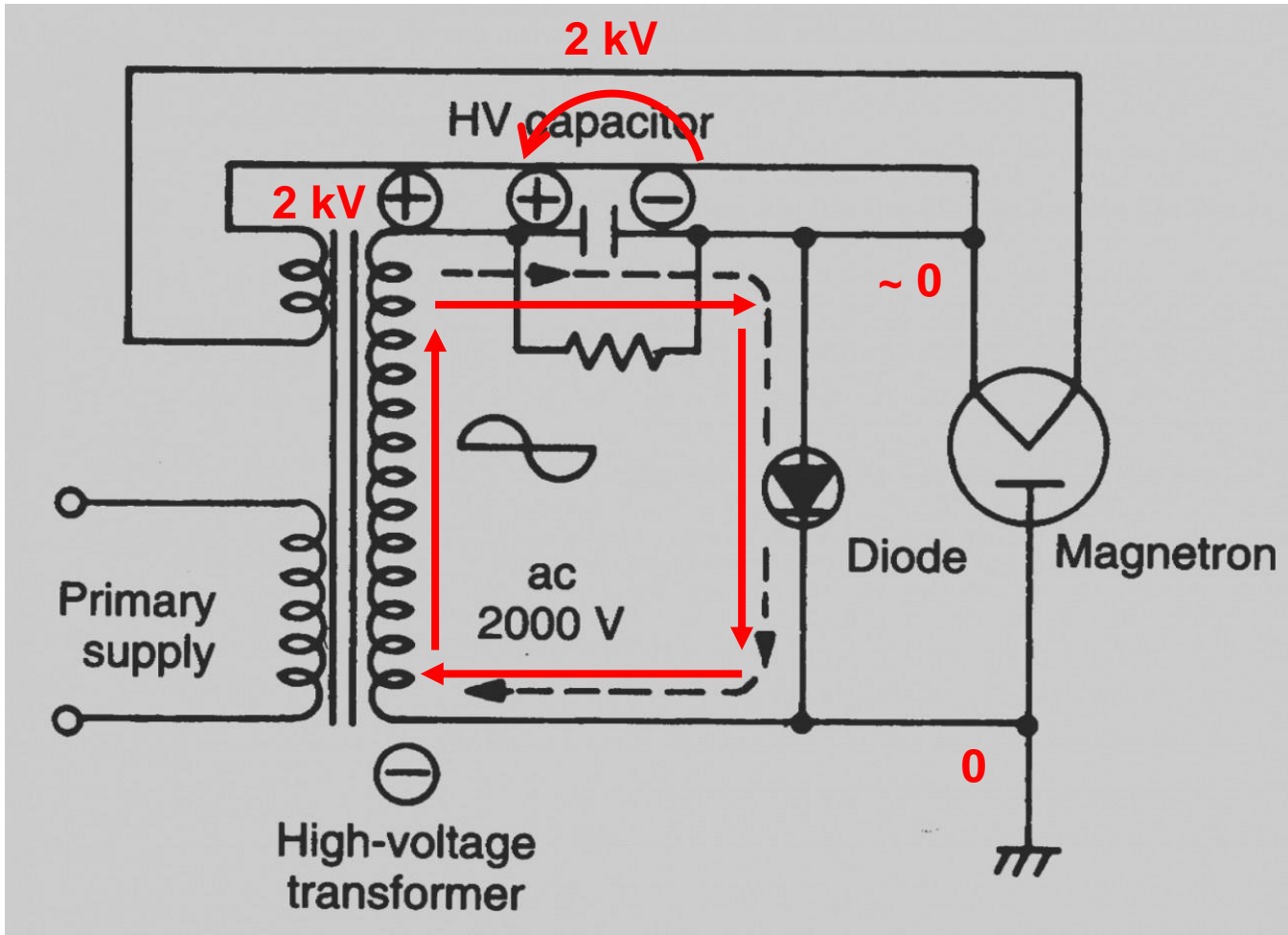
Charge at ends of cavity plays the role of a capacitor.

Electrons from the hot center cathode arriving at a negatively charged region tend to drive it back around the cavity, "pumping" the natural resonant frequency.

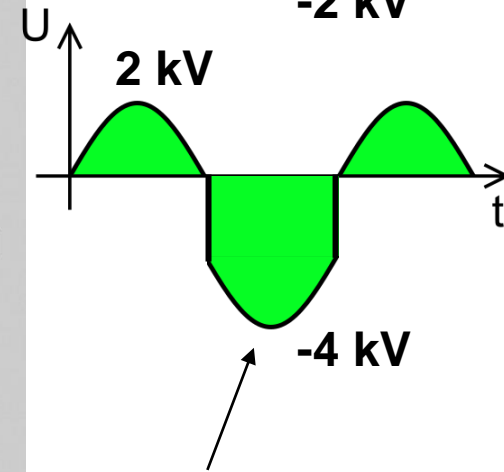
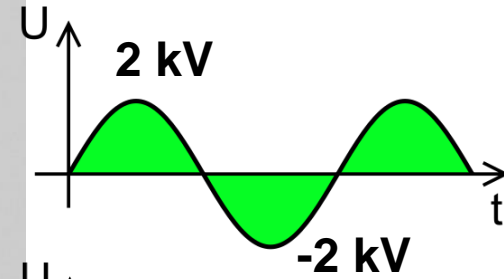
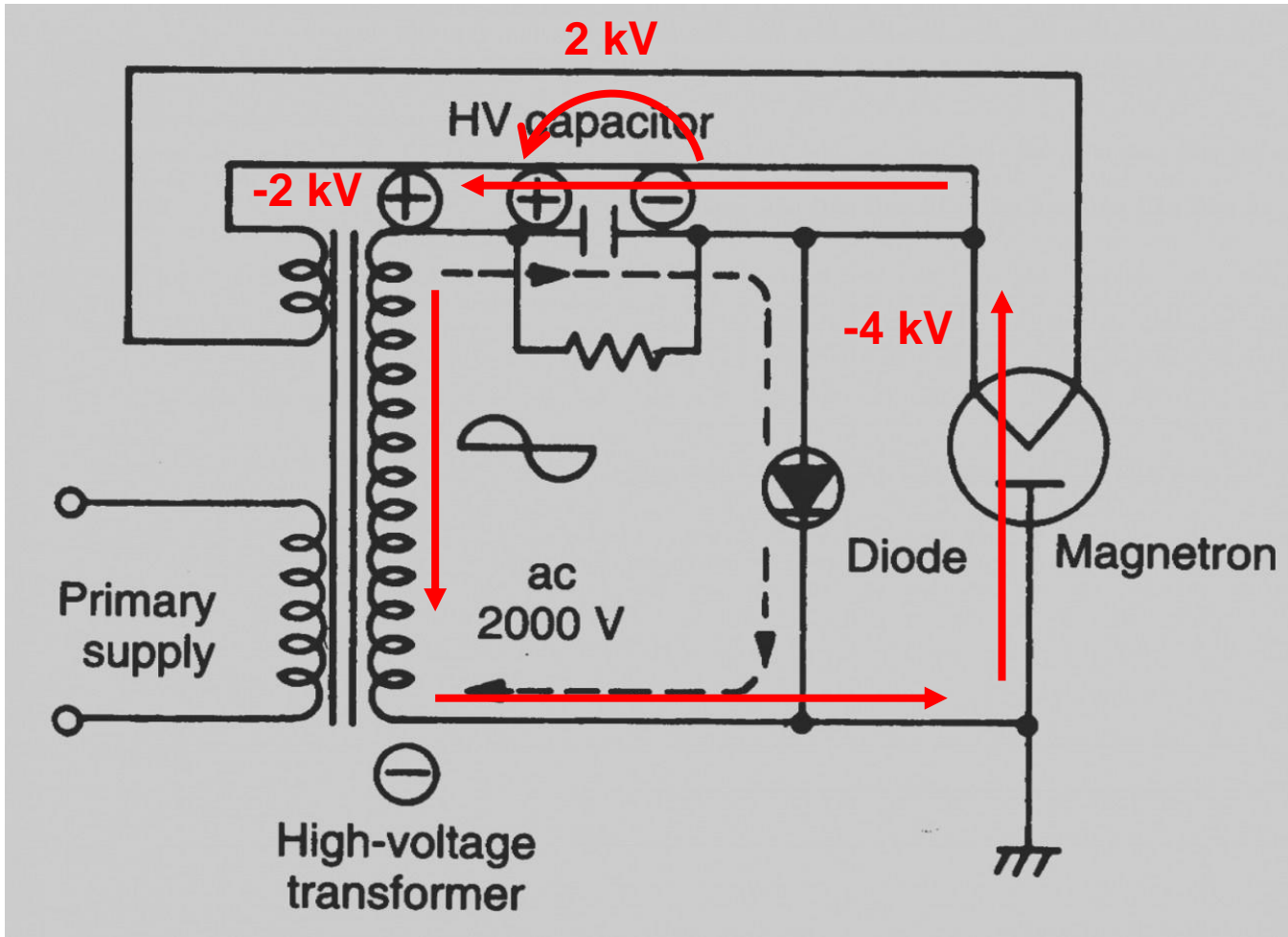
Magnetron schematic diagram



Magnetron schematic diagram

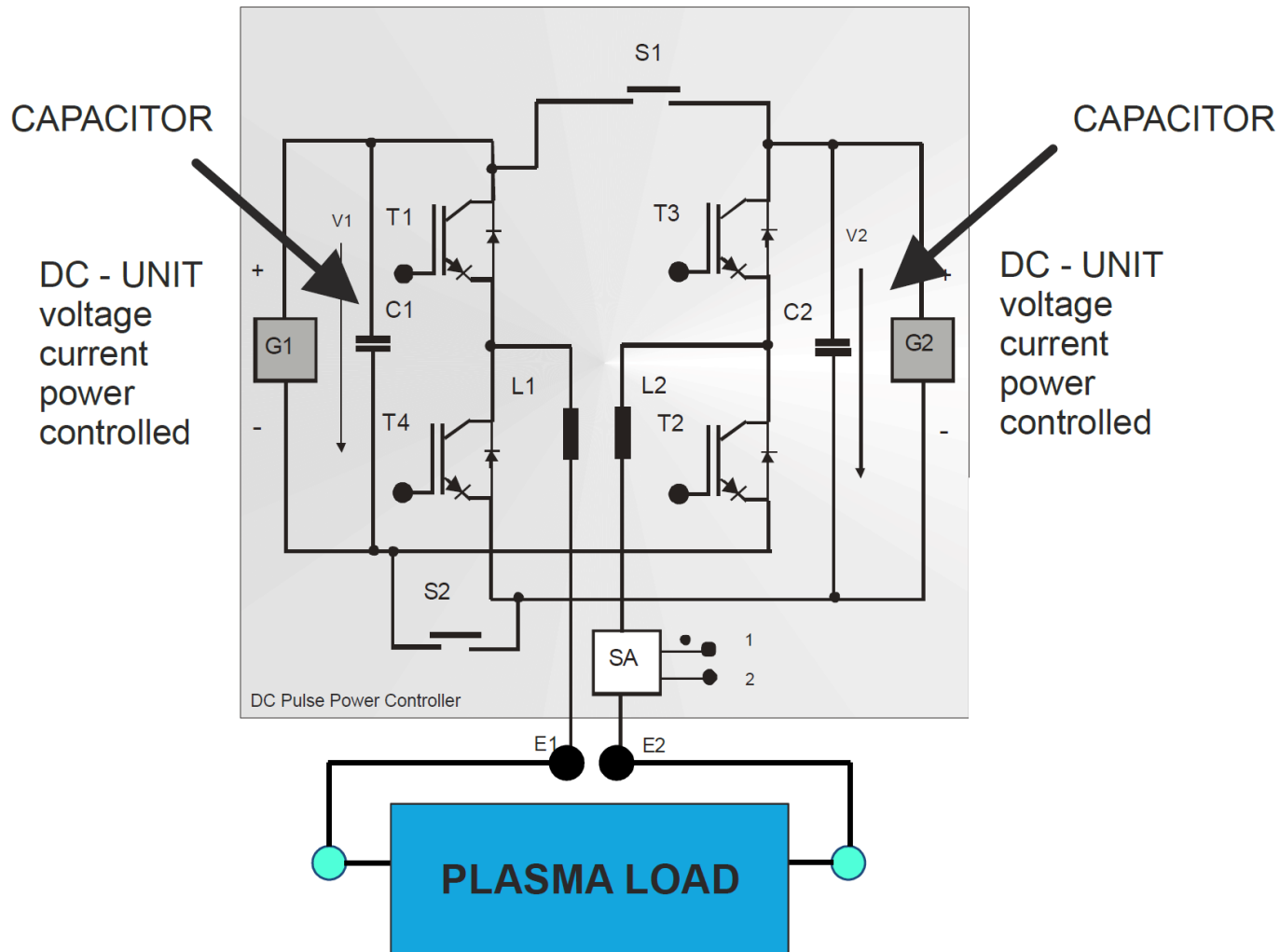


Magnetron schematic diagram

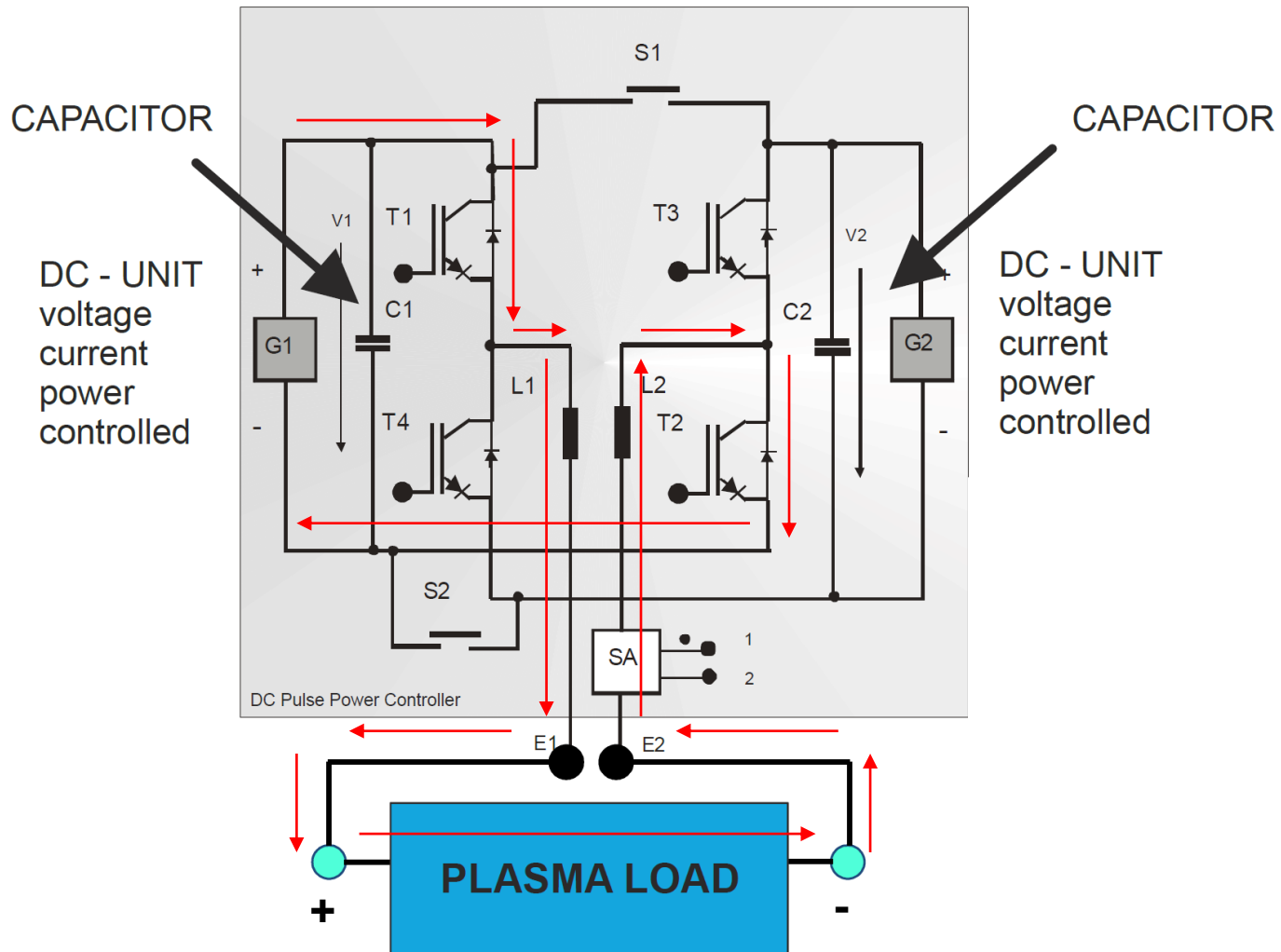


Microwave is generated.

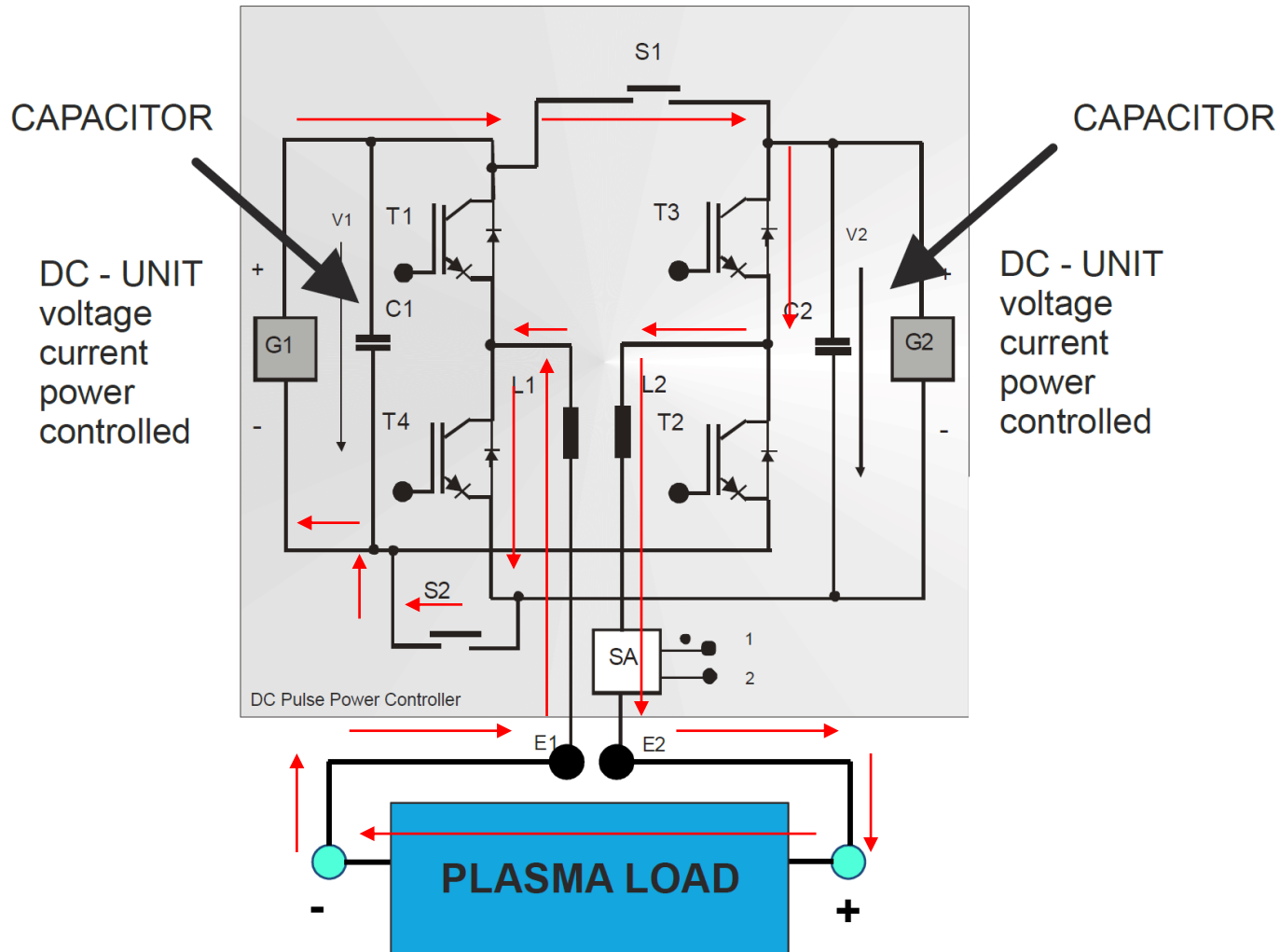
Pulse generator using H-bridge inverter



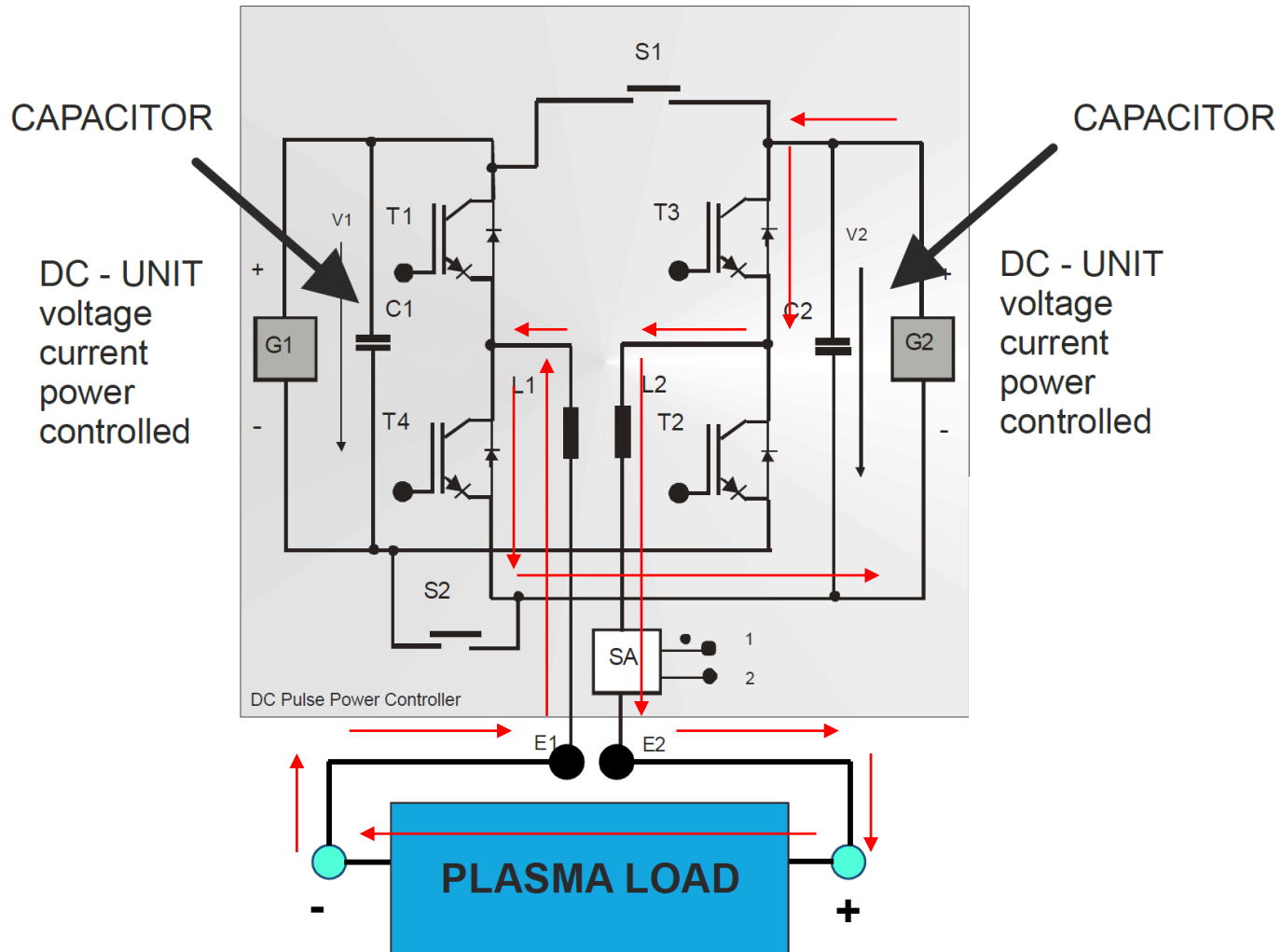
Pulse generator using H-bridge inverter



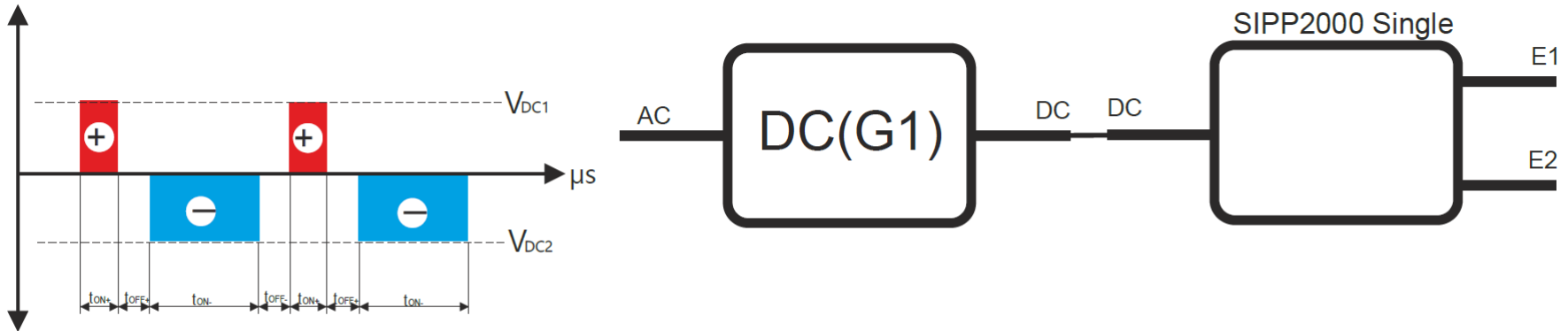
Pulse generator using H-bridge inverter



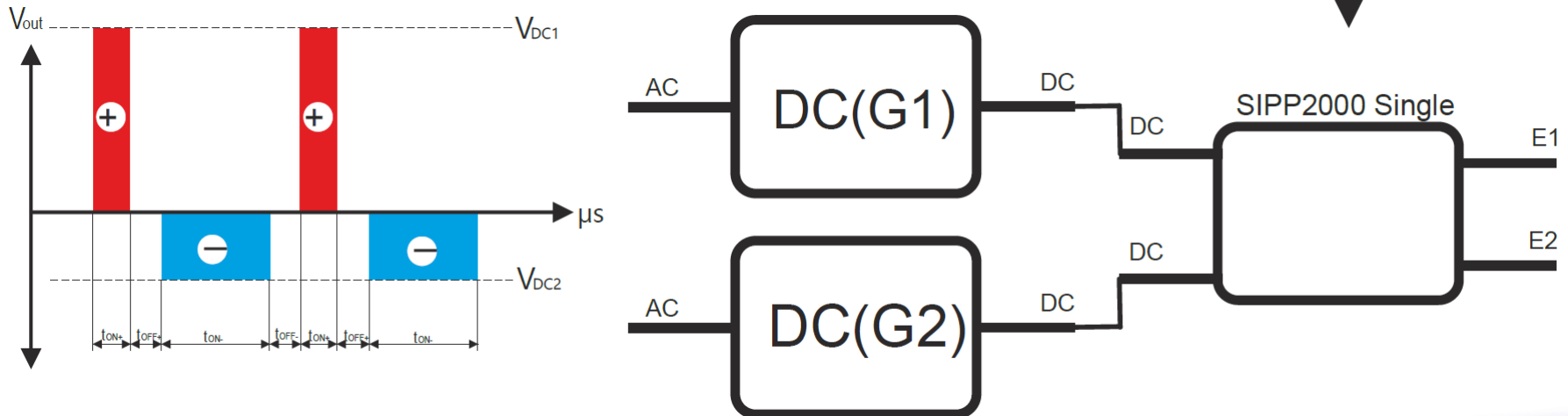
Pulse generator using DC power supply



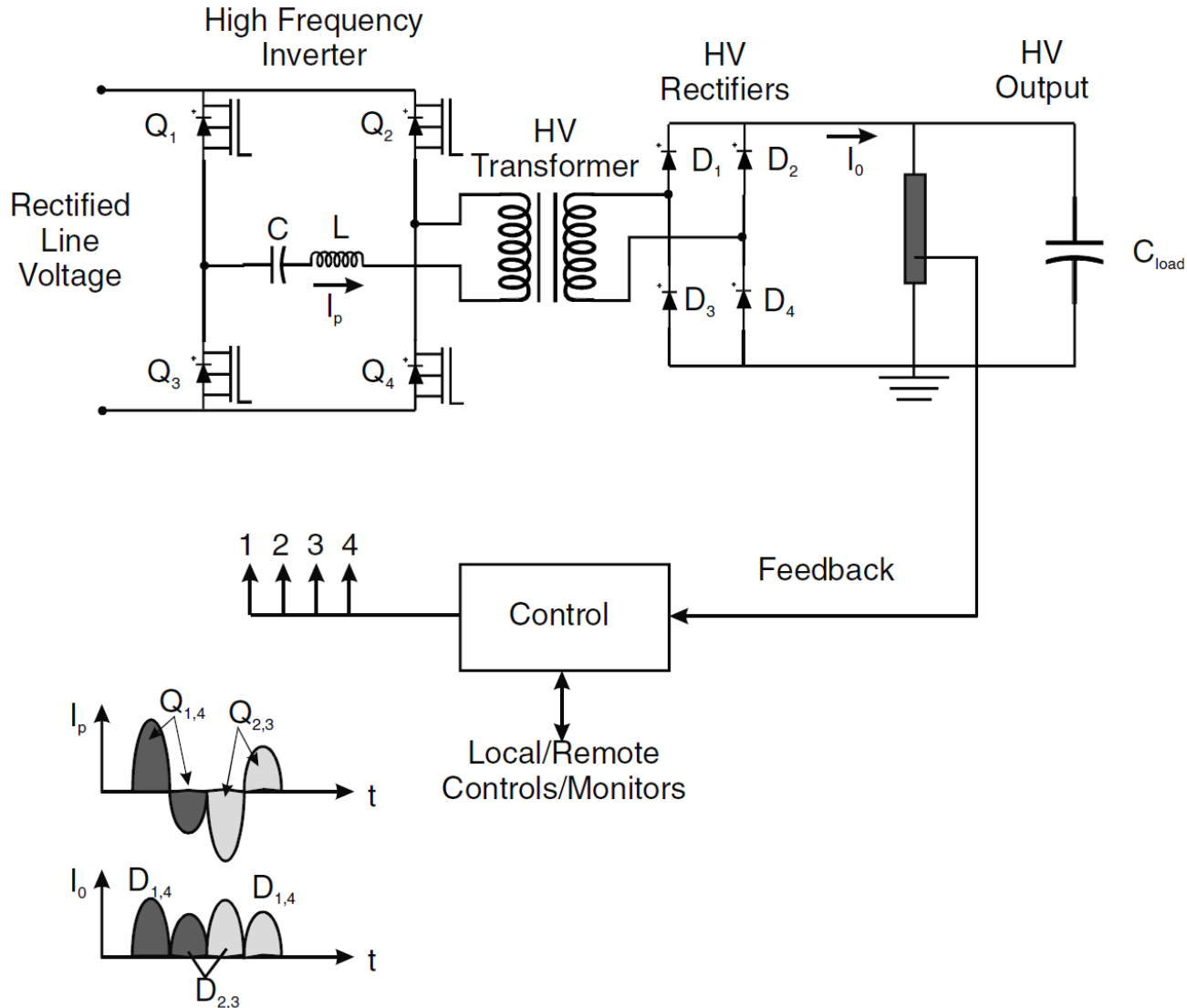
Pulse generator



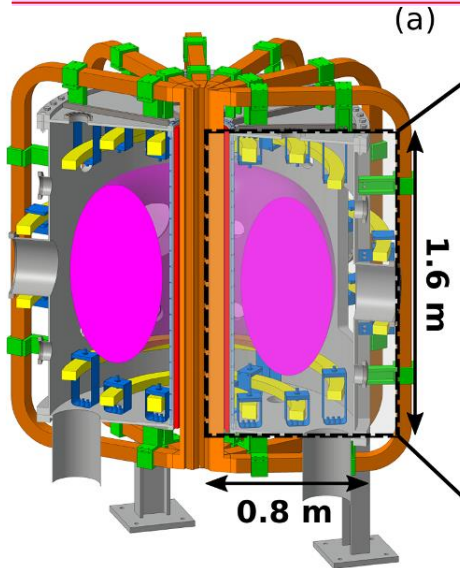
ASYMMETRIC : S1/S2 open- G1/G2



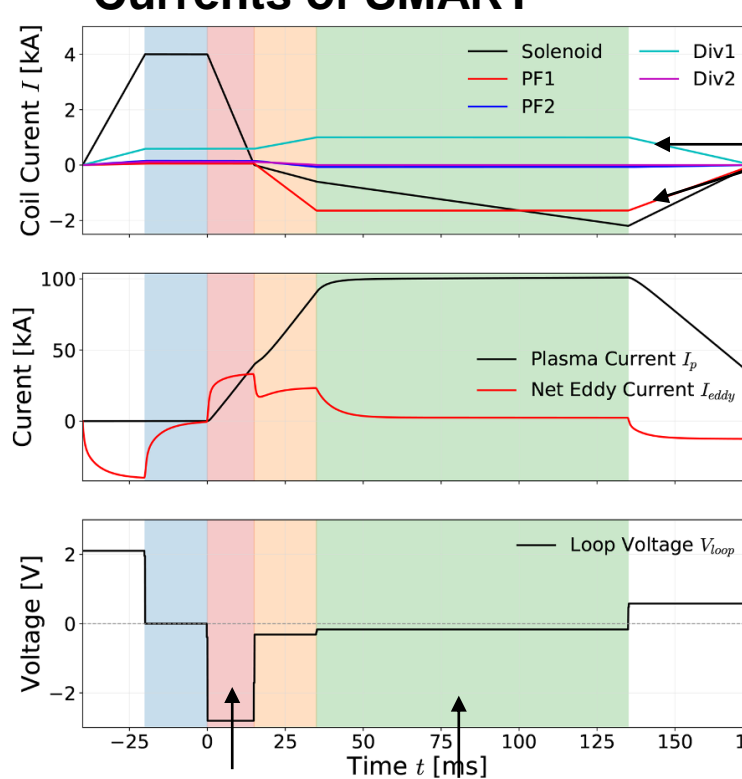
High-frequency switch mode power supply



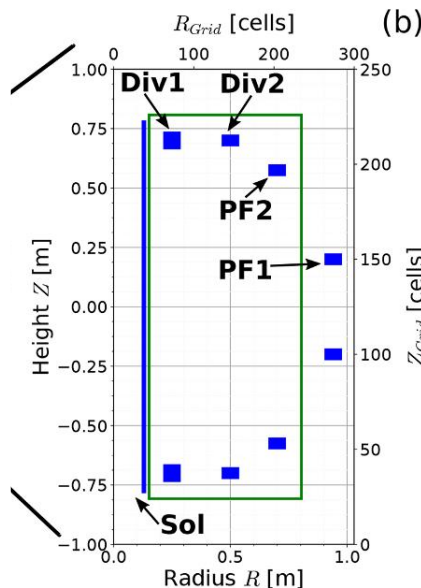
Currents with specific profiles needed to be provided to drive coils in Tokamaks to confine the plasma



• Currents of SMART



Equilibrium state can be achieved with poloidal field coils.



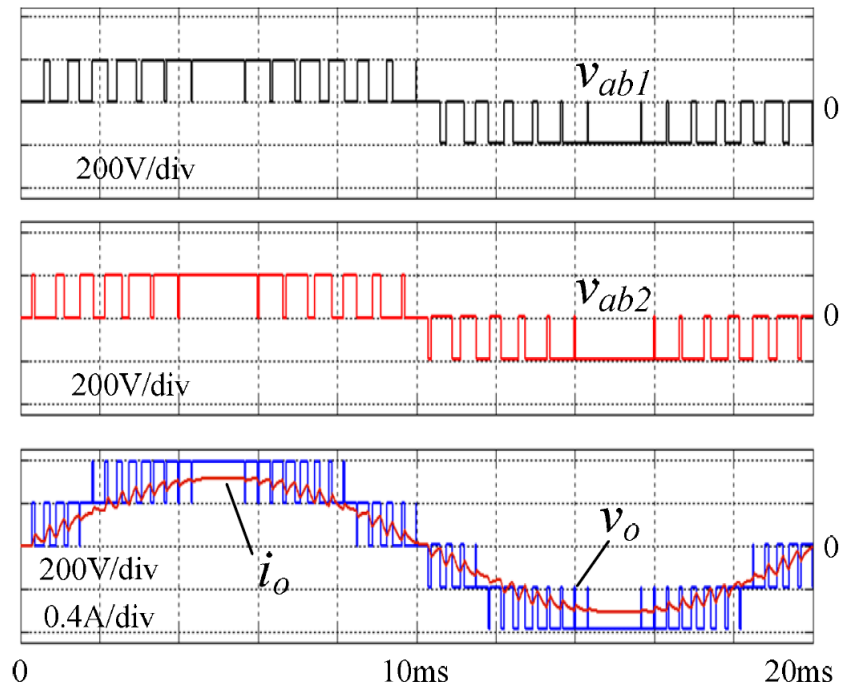
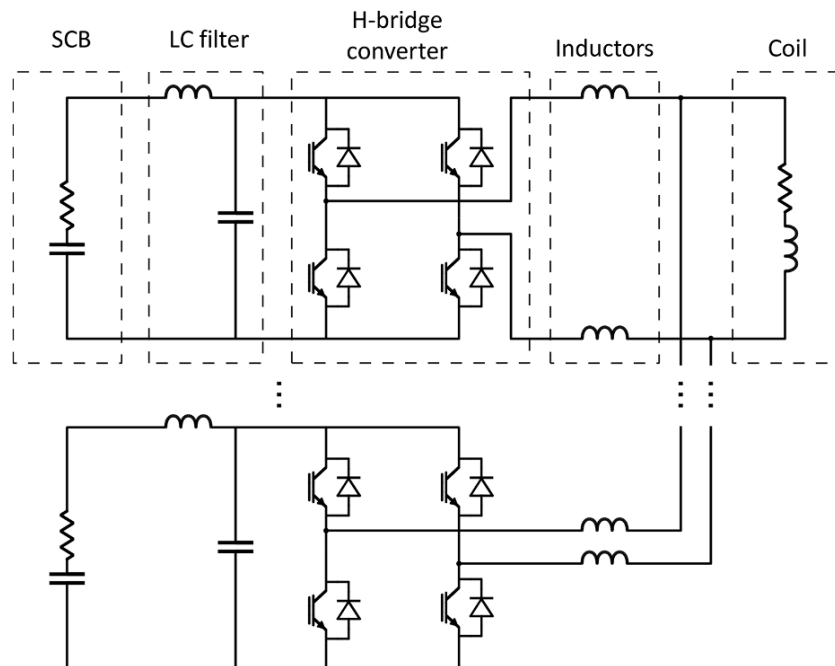
Breakdown Plasma current is driven.

- The current of the CS will be determined by the required breakdown voltage and plasma current.

An H-bridge combining pulse width modulation technique will be used to provide the controllable currents



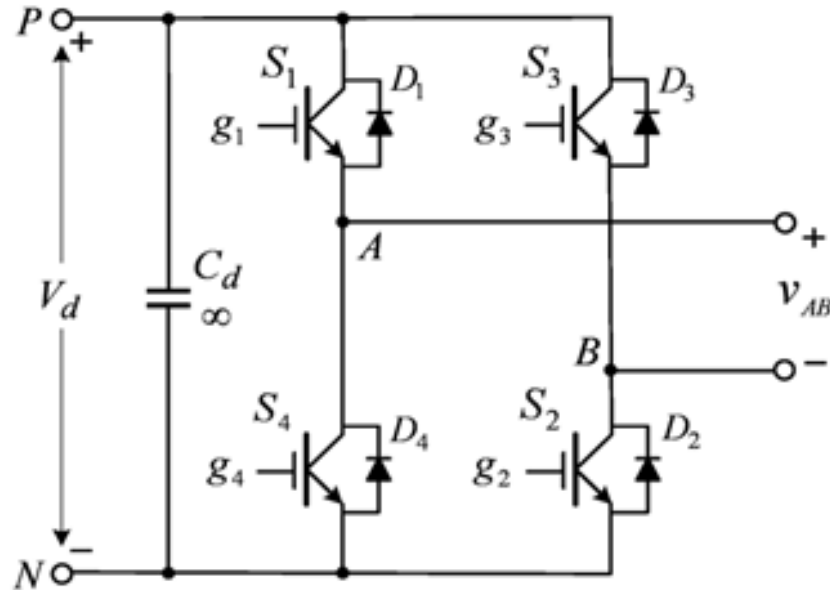
- H-bridge configuration provides the capability of reversing the current direction:
- Pulse width modulation provides the capability of controllable currents



M. Agredano-Torres, etc., Fusion Eng. Des. **168**, 112683 (2021)

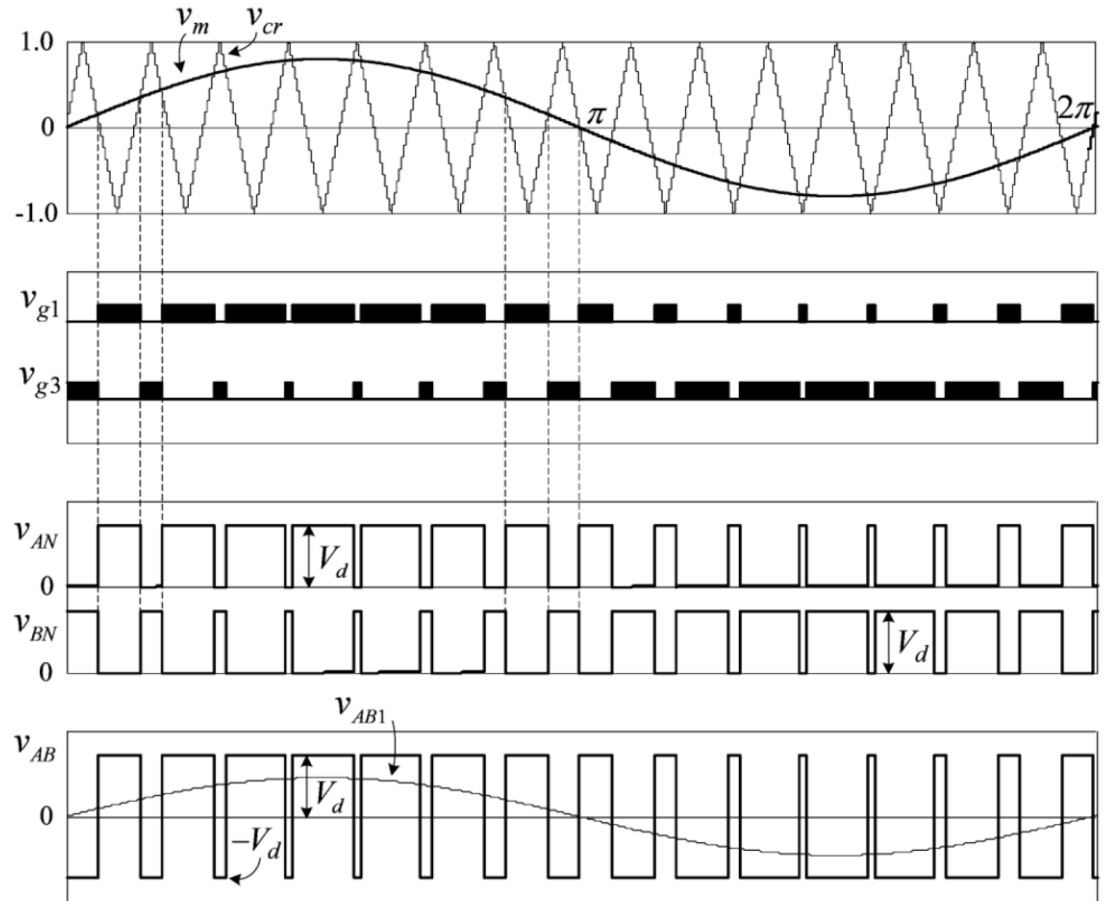
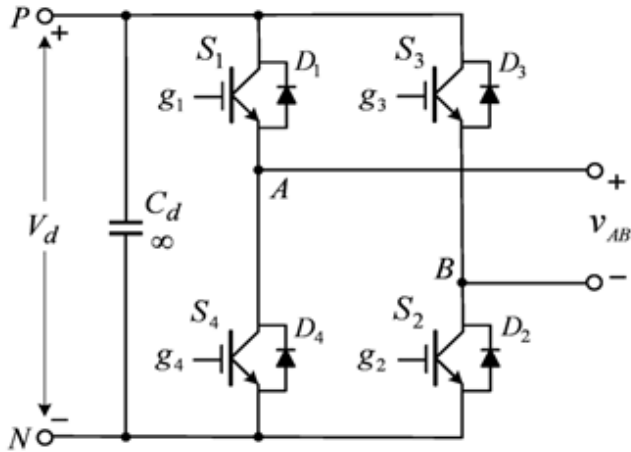
C. Boonmee and Y. Kumsuwan, 2012 15th International Power Electronics and Motion Control Conference, Novi Sad, Serbia, 2012, pp. LS8c.3-1

The output voltage is controlled by the status of switches S1~S4



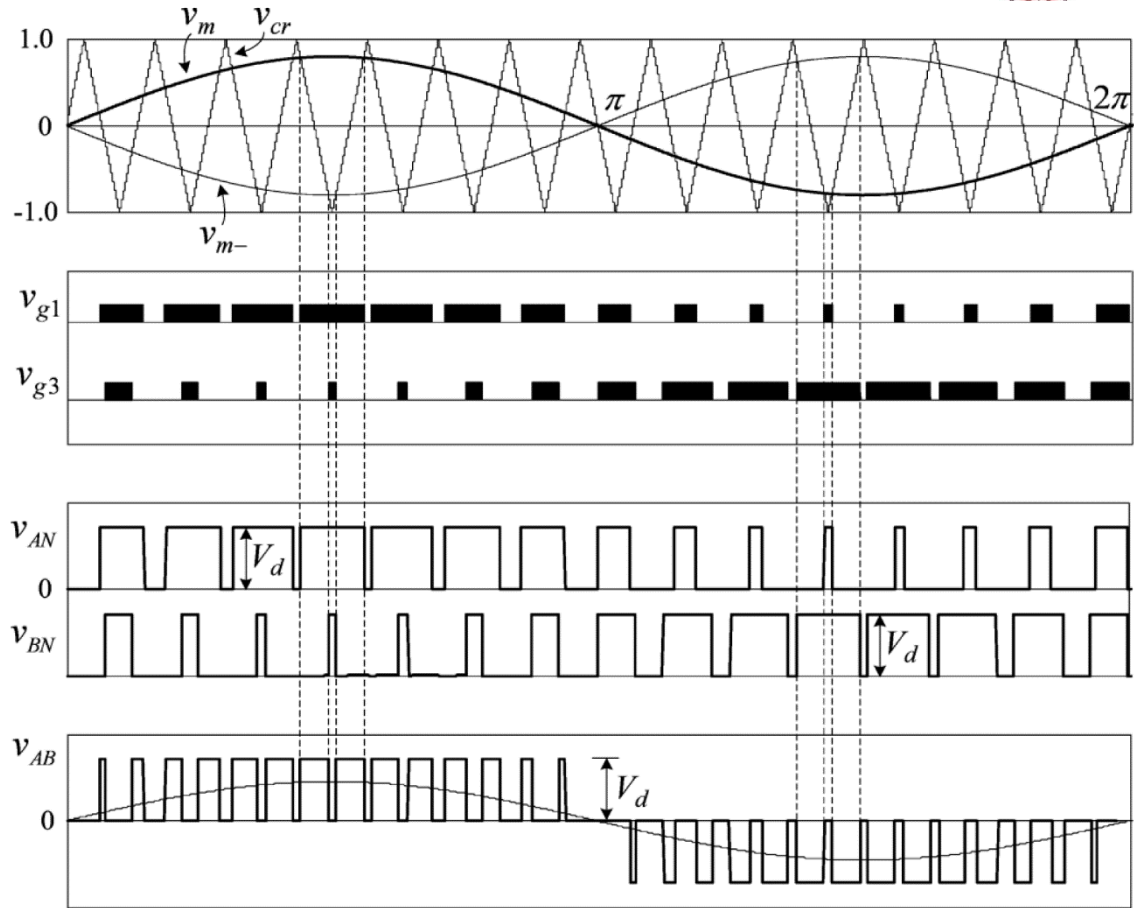
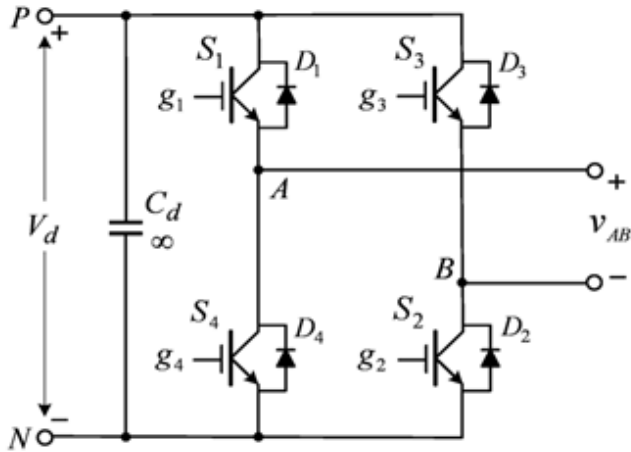
- **S₁/S₂ ON; S₃/S₄ Off: $V_{AB} = V_d$.**
- **S₁/S₂ Off; S₃/S₄ ON: $V_{AB} = -V_d$.**
- **S₁/S₂ ON; S₃/S₄ ON: $V_{AB} = 0$.**

Bipolar Modulation Scheme



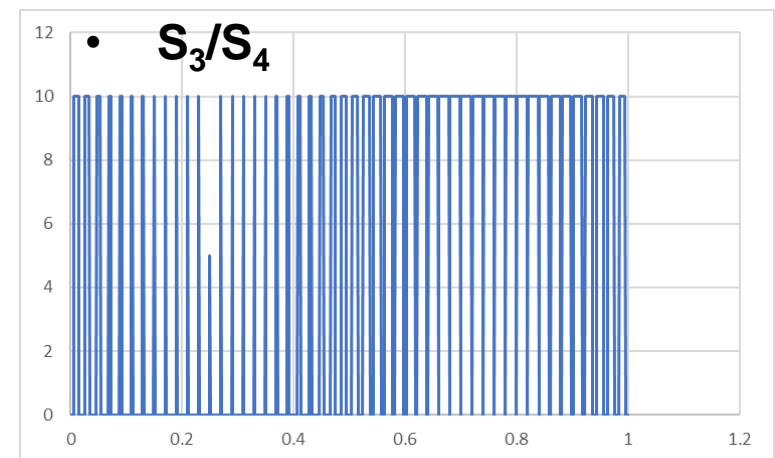
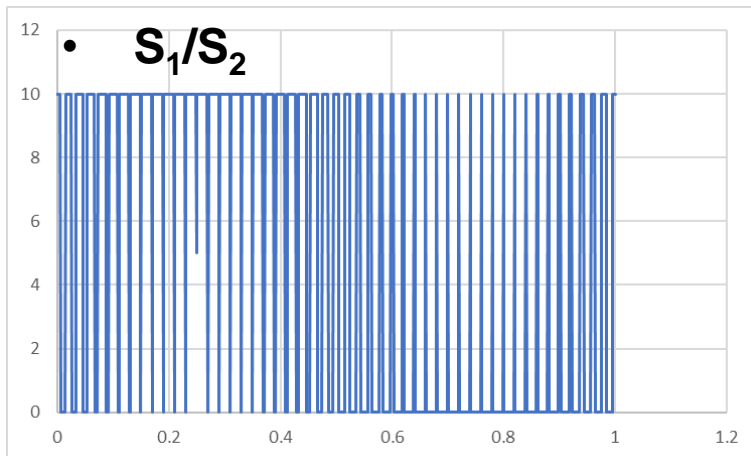
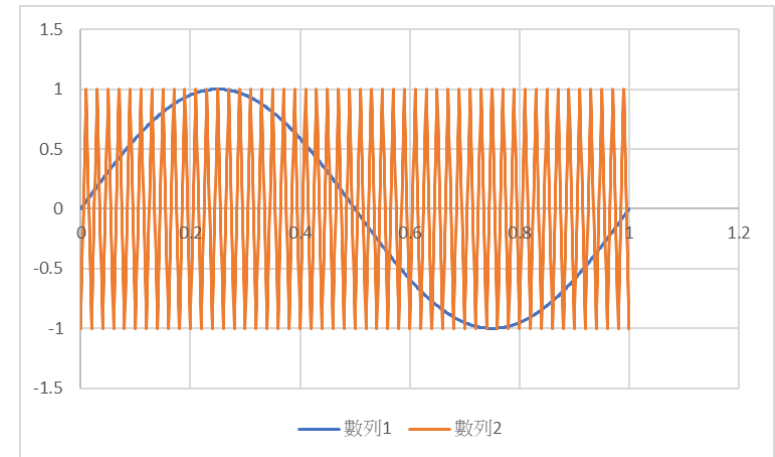
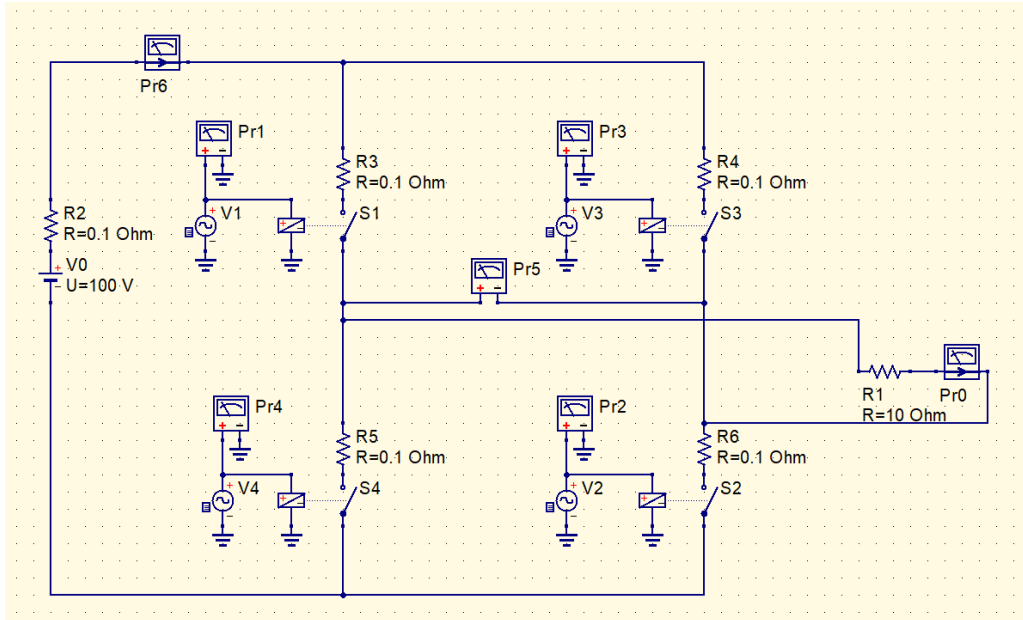
- **S_1/S_2 ON; S_3/S_4 Off: $V_{AB} = V_d$**
- **S_1/S_2 Off; S_3/S_4 ON: $V_{AB} = -V_d$**
- **S_1/S_2 ON; S_3/S_4 ON: $V_{AB} = 0$**

Unipolar Modulation Scheme

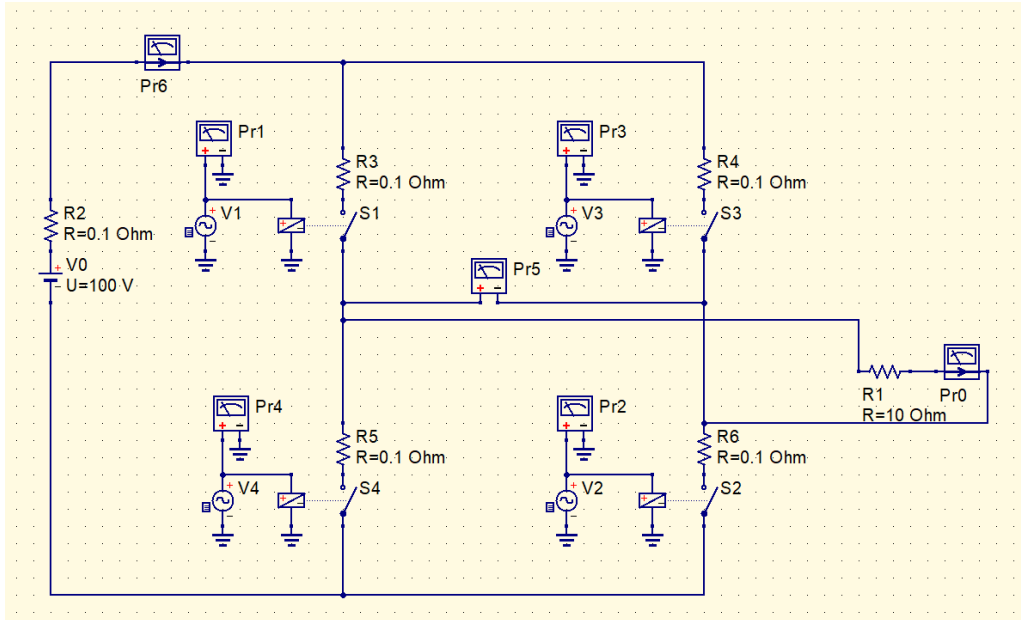


- S_1/S_2 ON; S_3/S_4 Off: $V_{AB} = V_d$
- S_1/S_2 Off; S_3/S_4 ON: $V_{AB} = -V_d$
- S_1/S_2 ON; S_3/S_4 ON: $V_{AB} = 0$.

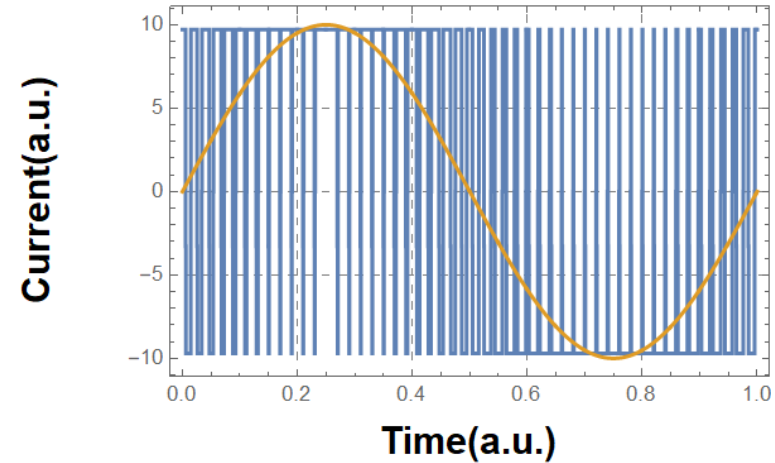
Simulation using bipolar modulation scheme



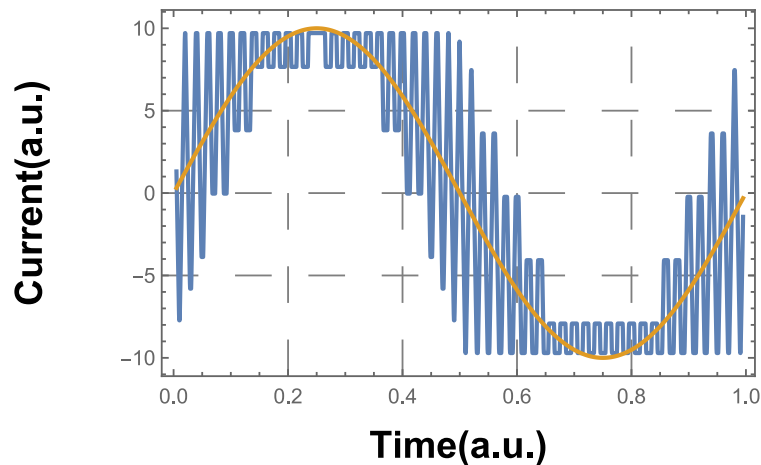
Simulation using bipolar modulation scheme



- Raw data



- Moving average = 100



- Moving average = 1000

