PULSED POWER SYSTEM
脈衝功率系統

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2023 Fall Semester

Tuesday 9:10-12:00

Lecture 10

http://capst.ncku.edu.tw/PGS/index.php/teaching/

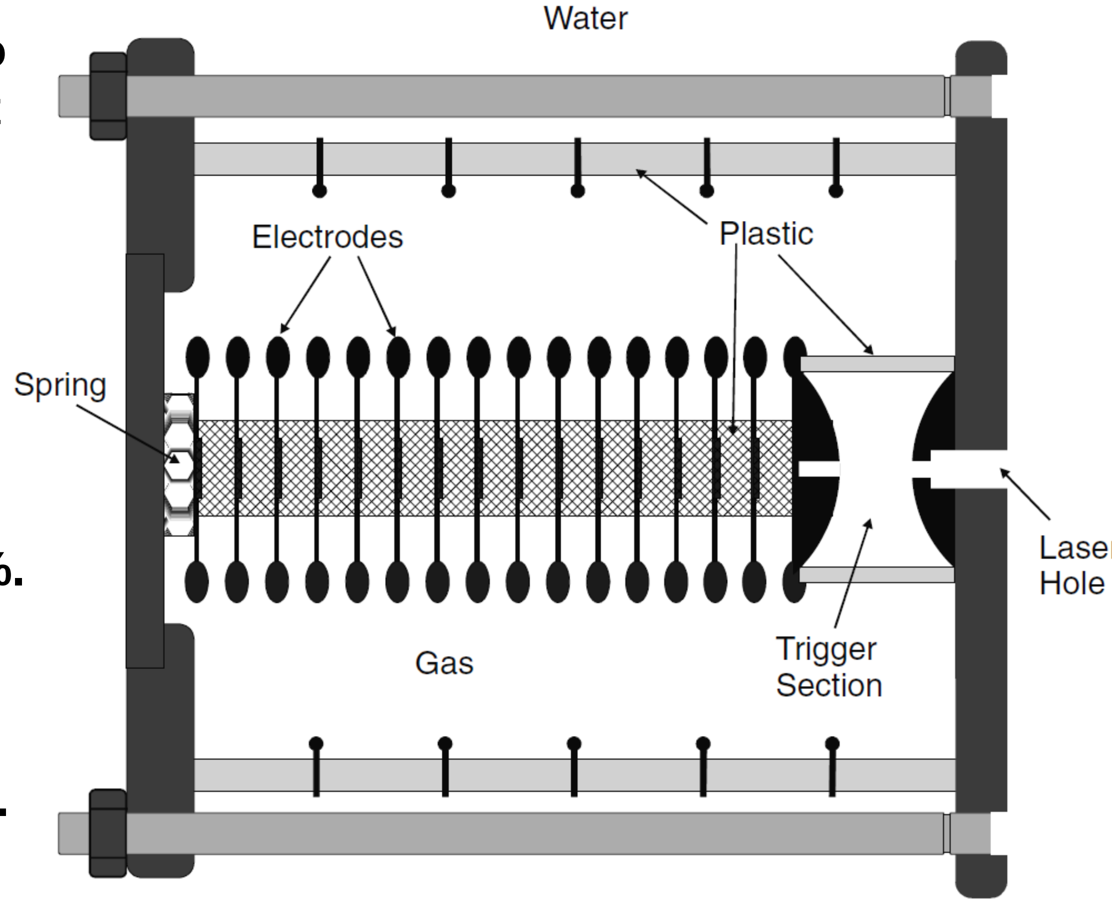
Online courses:

https://nckucc.webex.com/nckucc/j.php?MTID=md577c3633c5970f80cbc9e821927e016

2023/11/21 updated

Multistage spark-gap switch with laser triggering

- Simply scaling a three-electrode spark gap to multimegavolt operating voltages would lead to large gaps, making the jitter and inductance unacceptably high.
- Operating voltage of up to 6 MV and a switch current of 0.5 MA.
- It consists of 15 equal spark gaps and a trigger section.
- The operating voltage is around 90% of the self-breakdown value with a prefire probability of 0.1 %.
- The gap capacitances are small, 20 % of the operating voltage occurs across the trigger section.



Multistage spark-gap switch with laser triggering

- **The switch is 68 cm long and 61 m in diameter.**
- **The 1st gap is 5.7 cm and a UV laser pulse (KrF) with a 25 mJ pulse energy is necessary.**
- **~1 ns after the laser pulse, a breakdown occurs in thetrigger gap and the voltage increases across the remaining gaps rapidly. An ignition wave propagates to the other gaps and ignites them sequentially.**
- **Total inductance: 400 nH; Trigger delay: 20 ns; jitter <0.4 ns.**

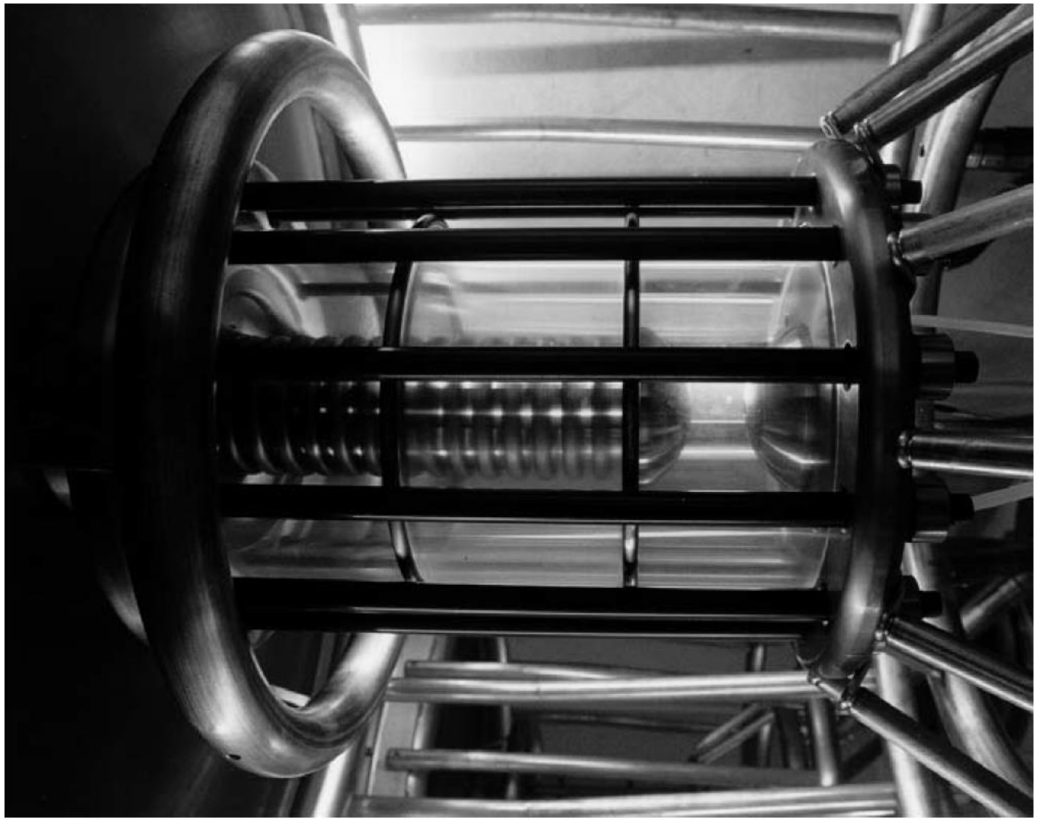
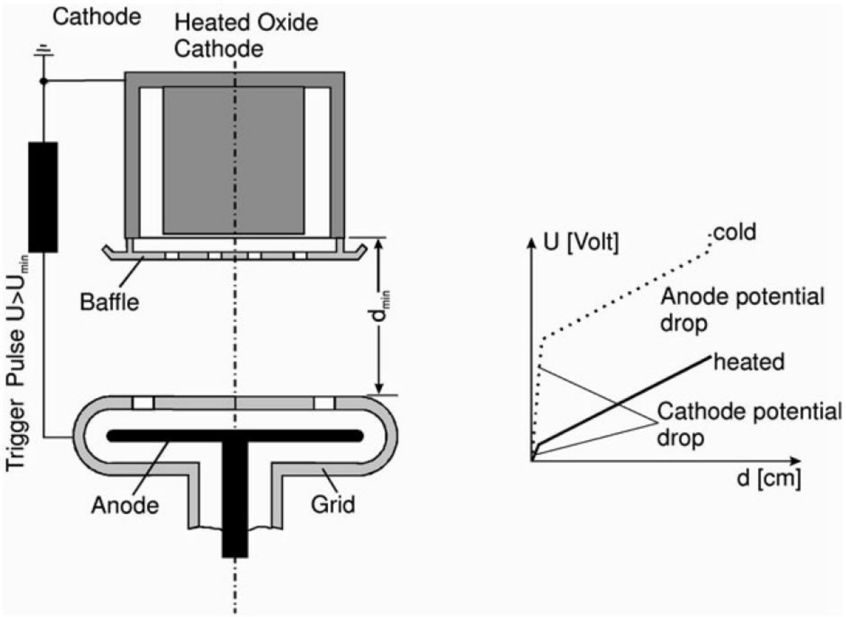
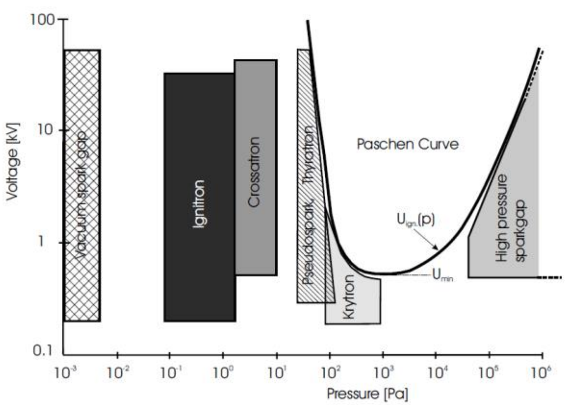


Fig. 4.13. A 4 MV version of a multigap spark switch

Thyratrons

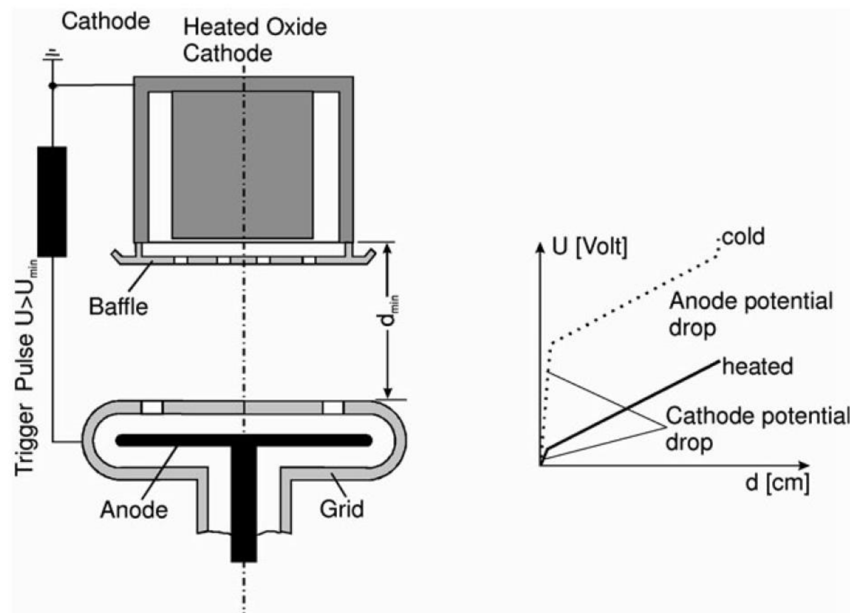
- Thyratrons are gas-filled switching devices with a gas pressure (30-80 Pa/3x10^{-4} – 8x10^{-4} atm) much lower than a spark-gap switches.
- A triode configuration is used.
- The thyratron is characterized by the presence of a plasma, which allows the passage of large currents without significant electrode erosion.
- The hold-off voltage is limited by field emission, > 10^5 V/cm.
- The anode-grid distance is 2-3 mm, ~40 kV hold-off voltage.

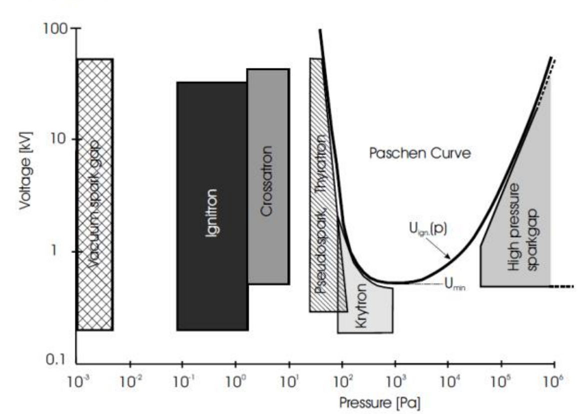




Thyratrons

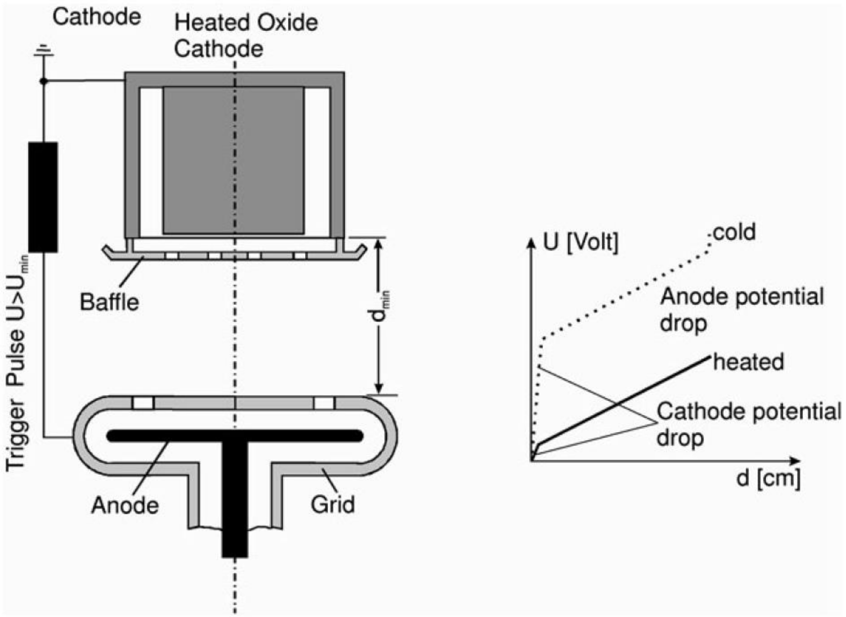
- **The cathode-grid distance corresponds to the Paschen minimum U_{min}.**
- **If $U > U_{min}$, a glow discharge is initiated between the cathode and the grid. => electrons from the glow discharge plasma can migrate rapidly through the openings in the grid to the main discharge region between the grid and the anode. => thyratron closes.**

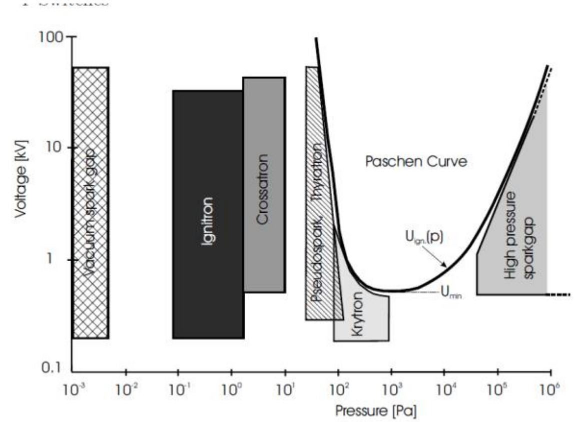




Thyratrons

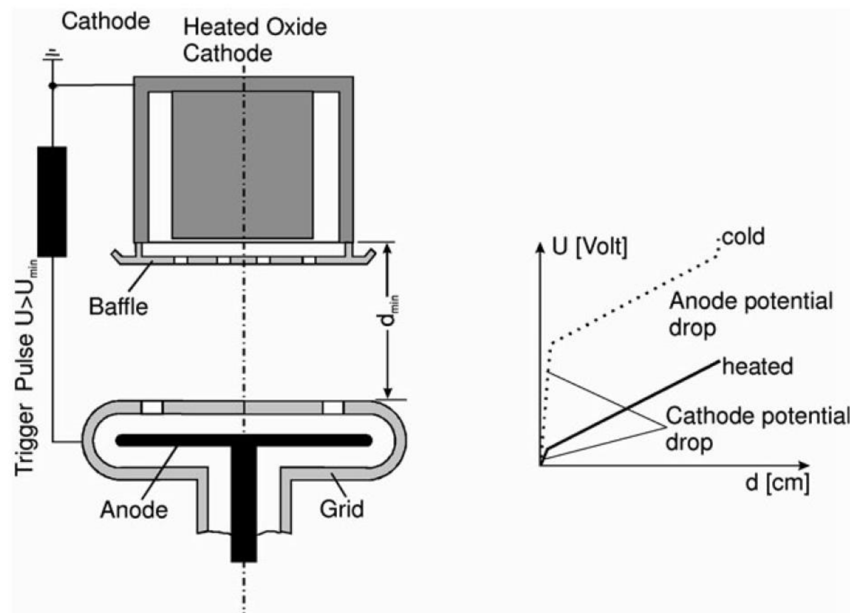
- **Operating voltage: several times 10 kV. After ignition: ~100 V => an appreciable power loss occurs and need to be dealt with by cooling.**
- **Delay: ~200 ns; jitter: ~ns.**
- **Operating times: 10^5 hours; Repetition rates: few kHz; Operating power: MW.**
- **To regain the initial hold-off voltage: anode voltage must become slightly negative for 25-75 us for plasma to decay.**

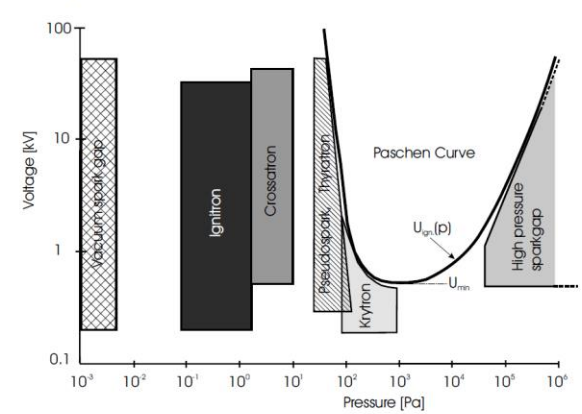




Thyratrons

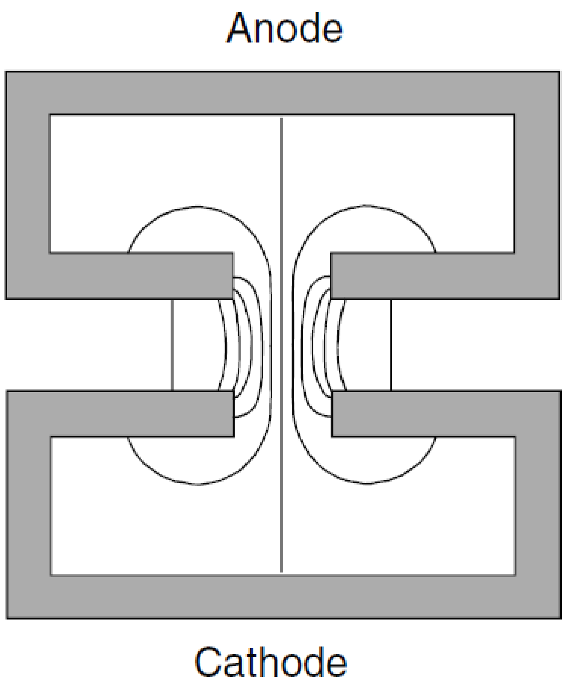
- **A thermionic cathode is used in a thyratron.**
- **Advantage: absence of a marked cathode potential drop using hot cathode.**
- **If cold cathode is used, potential drop is needed to accelerate the ions for secondary-electron production => lead to erosion of the cathode and thus the lifetime.**
- **A baffle is used as a screening element to avoide electron directly reaching the anode and causing the damage. It is shifted relatively to the grid to prevent a direct line of sight between cathode and anode.**

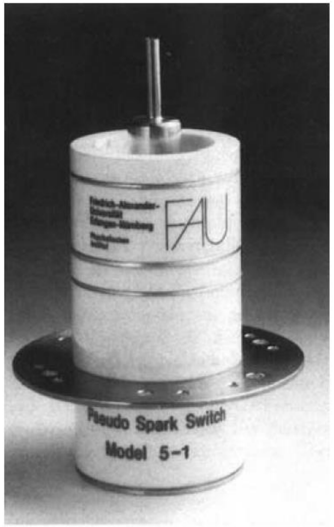


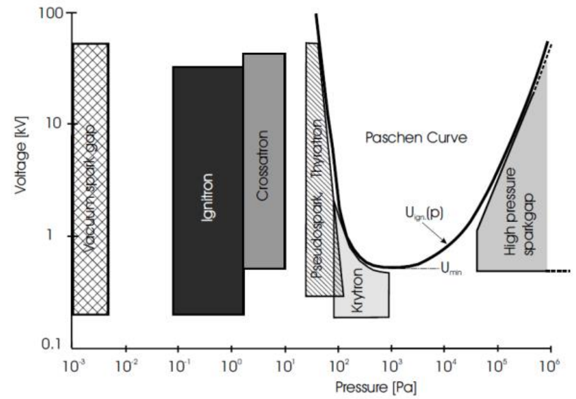


The pseudospark switch

- **The pseudospark switch operates in a low-pressure regime, where the mean free path of electrons and ions become comparable to the electrode spacing. Most electrons reach the anode without any ionizing collisions in the gas.**
- **Hollow cathode: increases the possible discharge path lengths.**
- **The diameter of the aperture determines the field penetration into the hollow cathode.**

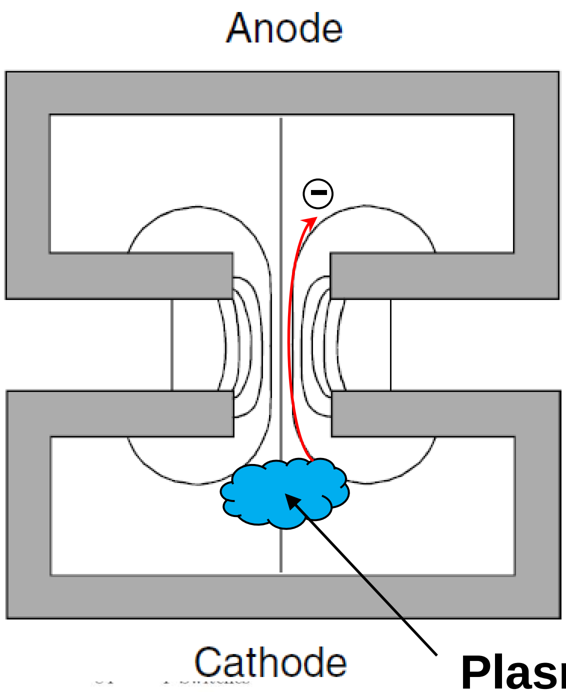


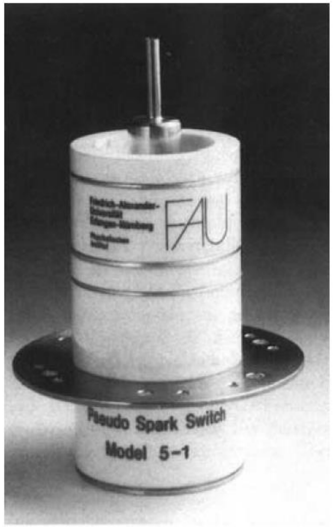


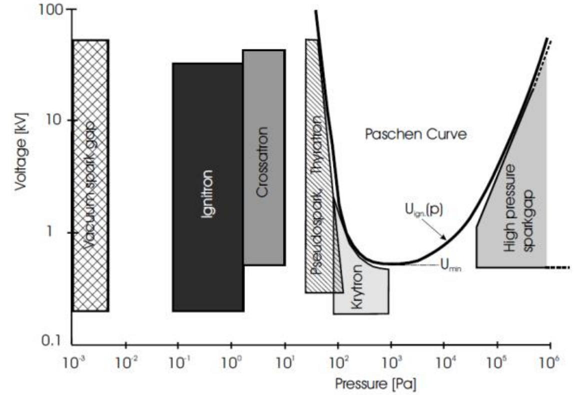


The pseudospark switch

- A small number of initial electrons, triggered discharge in the hollow cathode can initiate the pseudospark discharge.
- The switching mechanism is based on the build-up of a highly ionized plasma.
- plasma build-up occurs first inside the hollow cathode where E/P is low.

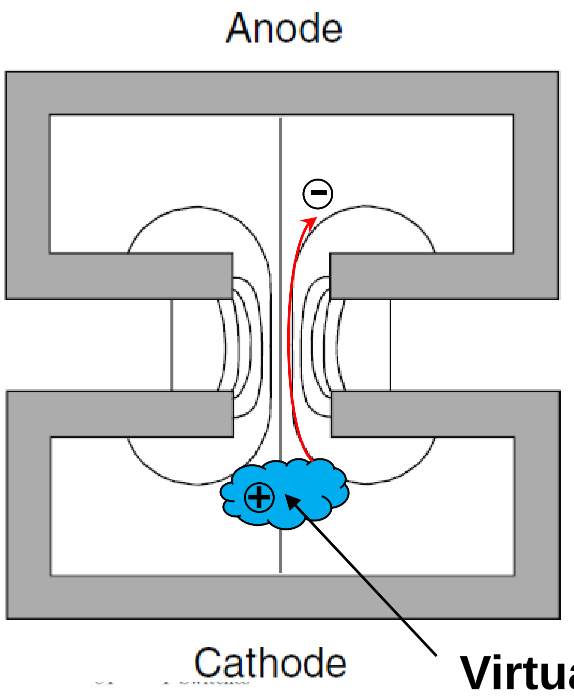


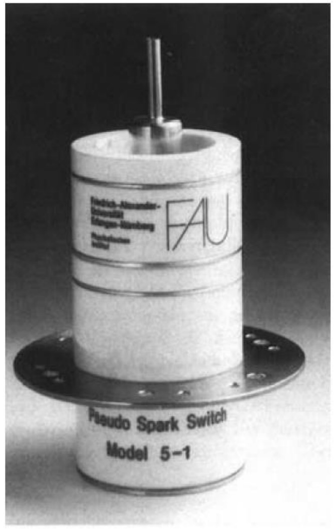


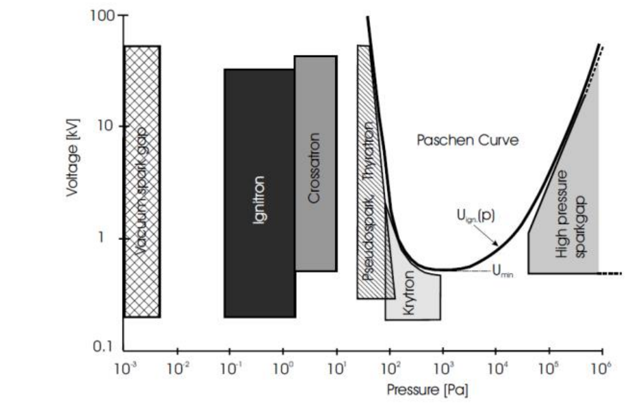


The pseudospark switch

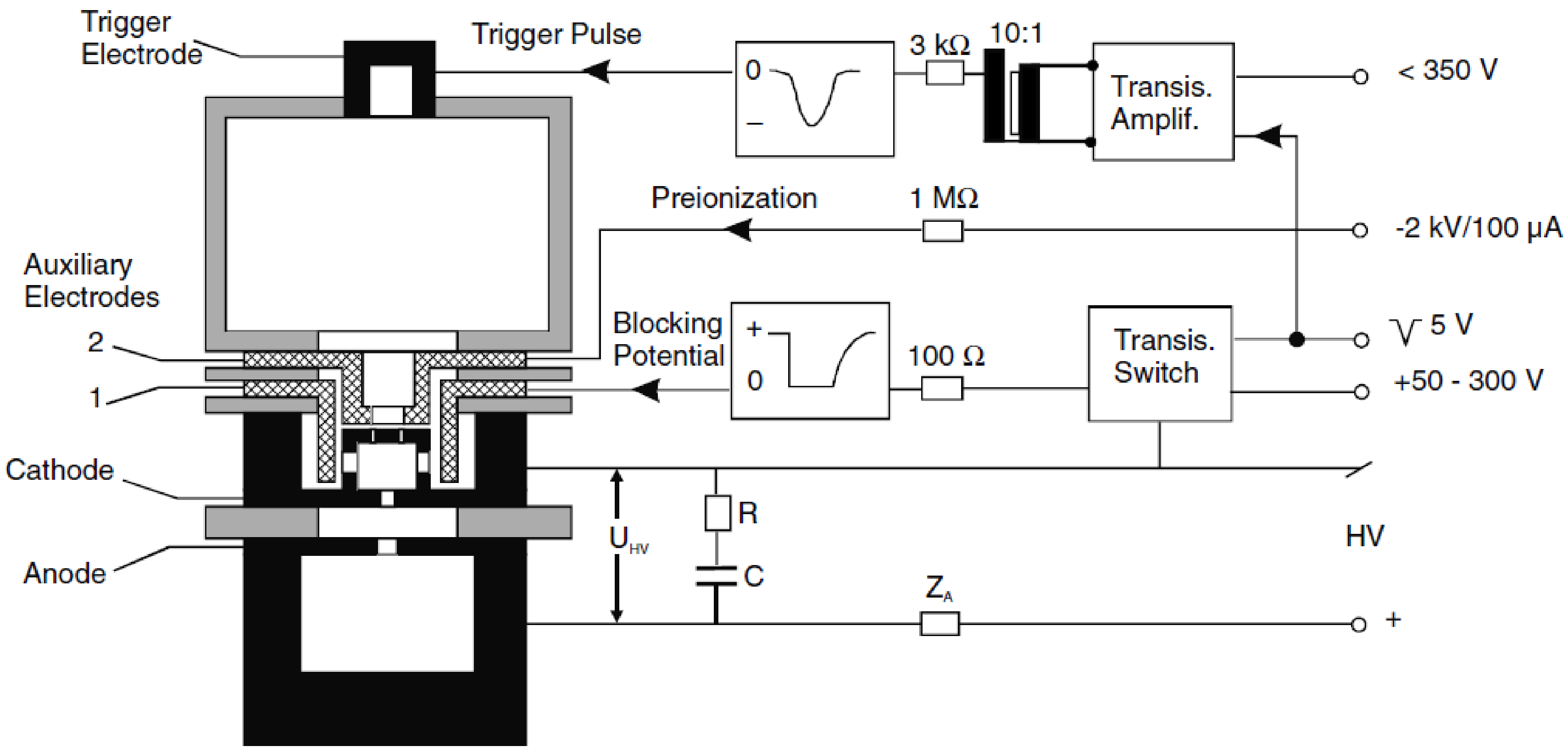
- Ions drift back into the hollow cathode => forming a positive space charge (virtual anode).
- Static electric field inside the hollow cathode is distorted.
- Electron production rate > loss rate in the hollow cathode and subsequently in the anode-cathode gap.
- A low-resistivity plasma is estabilished, and breakdown of the gap occurs.
- Jitter: 10 ns; Delay: 0.5 us.
- Advantage: high dI/dt, reverse current, long lifetime, low jitter.





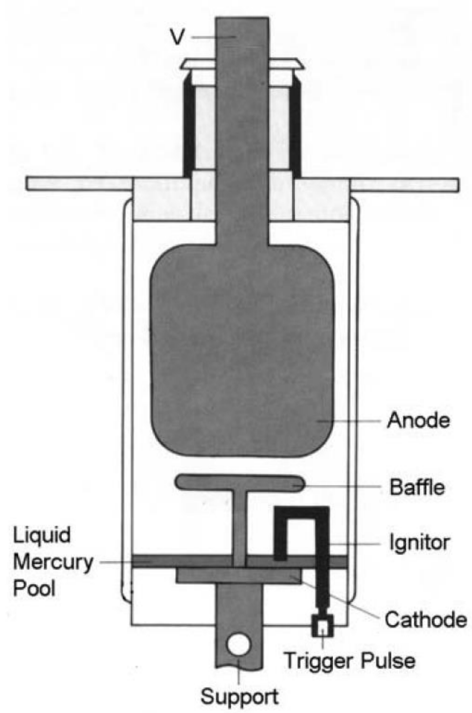


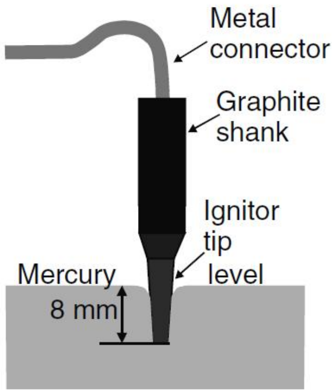
The pseudospark switch with triggering system



Ignitrons

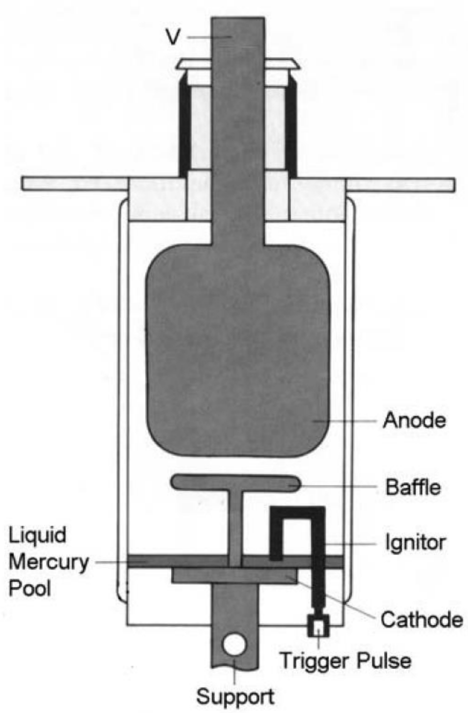
- Ignitron is a very high-current, high-voltage switch with
  - a liquid mercury pool cathode
  - an ignitor pin dipping into the liquid-metal reservoir.
- Internal mercury pressure: ~5 Pa
- Can switch a pulse charge of up to 2000 Colum.
- Air/water cooled may be needed.
- Internal splash and deionization baffles may be contained in some devices.
- Anode:
  - Anode is massive to prevent an impulsive temperature rise during conduction.
  - Anode is cooled through
    (1) anode stem;
    (2) radiation to the cooled walls.

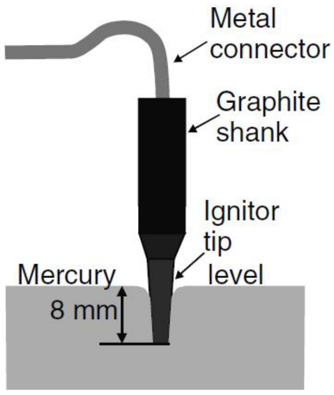




Ignitrons

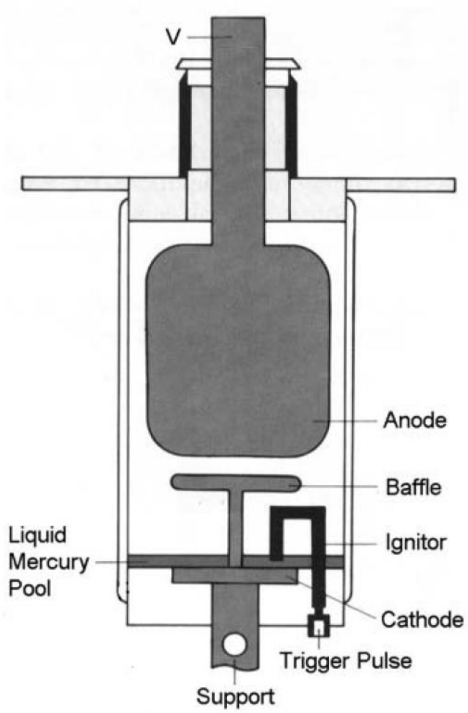
- **Cathode: a mercury pool at the bottom of the stainless steel tube.**
- **Ignitor: serves as the triggering electrode and is insulated from the cathode by a glass feedthrough.**
- **A high resistance forms at the mercury-ignitor interface.**
- **The electrical power in the ignitor circuit is mainly dissipated in a small volume near this interface. => creates an intense source of mercury vapor and free electrons => cathode hot spots spread over the entire mercury surface in ~50-100 ns. => mercury vapor pressure rises rapidly in the cathode-anode space. => pd value approaches the Paschen minimum => with free electrons, ionization avalanche develops.**

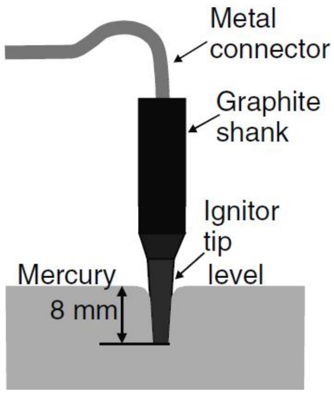




Ignitrons

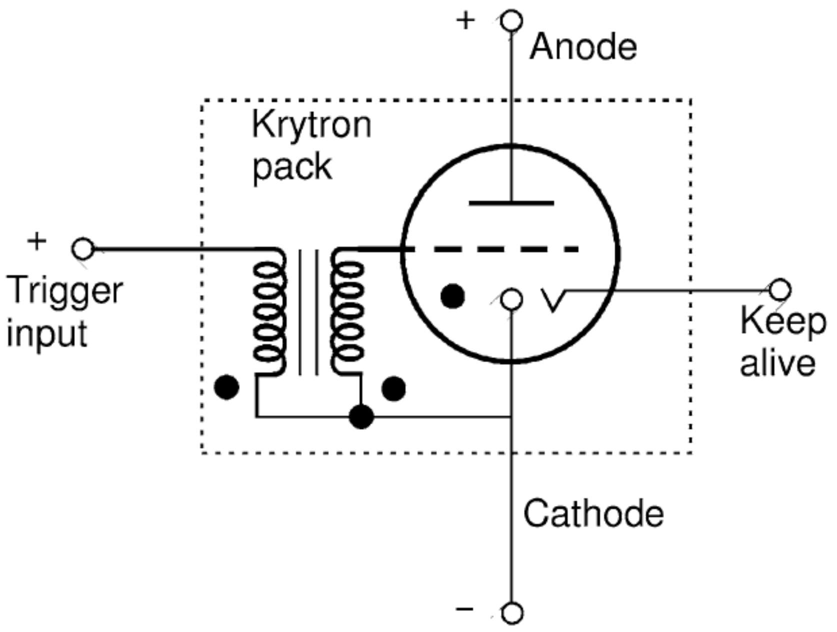
- **Rise time ~ 300-500 ns.**
- **After current drops below a critical value => no more additional vapor is produced => with additional time to allow recombination and recondensation of mercury.**
- **The mercury vapor must be forced to recondense back into the pool.**
- **Repetition rate ~1 Hz**
- **Progressively eliminated due to the mercury-containing waste.**





Krytrons

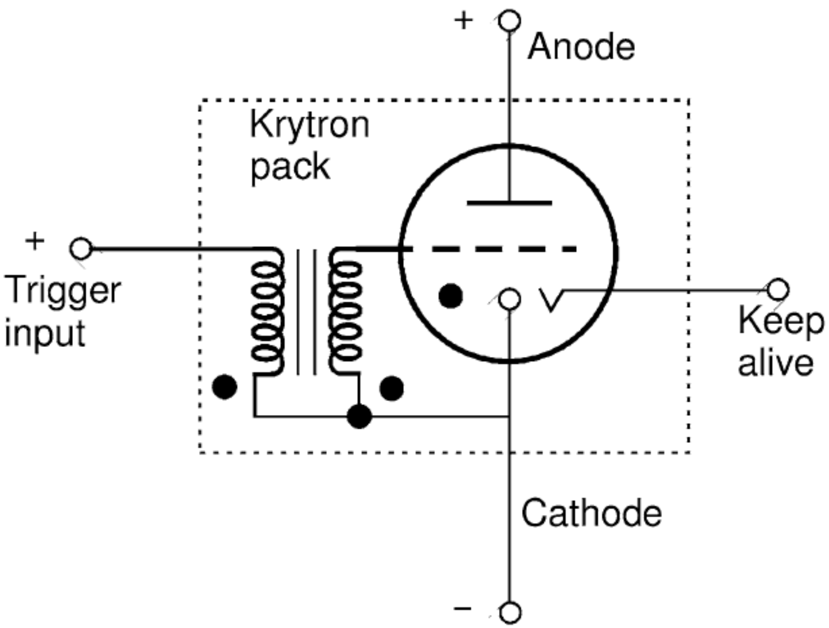
- **Low-pressure gas discharge device with a tetrode configuration, sealed in a glass tube with a cold cathode.**
- **1.3 kPa (9.75 torr) of helium gas.**
- **A special design of the anode-grid area + applied gas pressure**
  => large hold-off voltage.
- **An already existing plasma is created by a glow discharge between the special keep-alive electrode and the cathode.**
  => short trigger delay: ~30 ns.
- **Rise time: ~1 ns, Vmax: 8kV,**
  Imax: 3 kA.
- **Pulse length~10 us,**
  repetition rate ~1 kHz
- **A positive pulse at the control grid initiate the switch.**



Krytrons

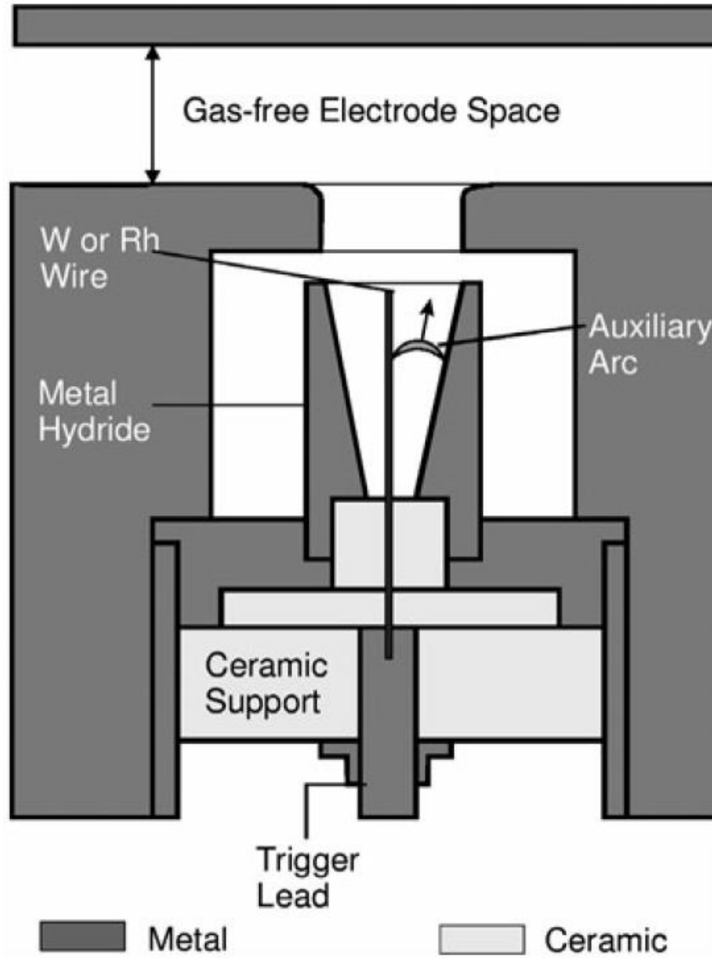
- A ^{63}Ni β-emitter may be enclosed to create a weak permanent pre-ionization.

- It is widely used in fast trigger generators and Pockels cell driver and also ideal for use in the detonating circuitry of bombs.



Triggered Vacuum Gap (TVG)

- A three-electrode system with P=0.001 Pa (7.5 x 10^{-6} Torr).
- Closed by injection of a plasma cloud.
- Hold-off voltage depends on the properties of the electrode surfaces.
- I up to 10 kA, V up to 100 kV. Repetition rates of several kHz are possible if cooled.
- The gas-plasma mixture is created with the help of an auxiliary arc, burning between two electrodes inserted into one of the main electrodes.
- Jitter ~ 30 ns; switching time ~100 ns.



Semiconductor closing switches

- The limiting switching characteristics of semiconductor devices are:
  - Relatively low mobility
  - Low density of charge carries
  - Comparatively low operating temperature

=> Large volume of the conducting region is required to conduct large currents.

Thyristors

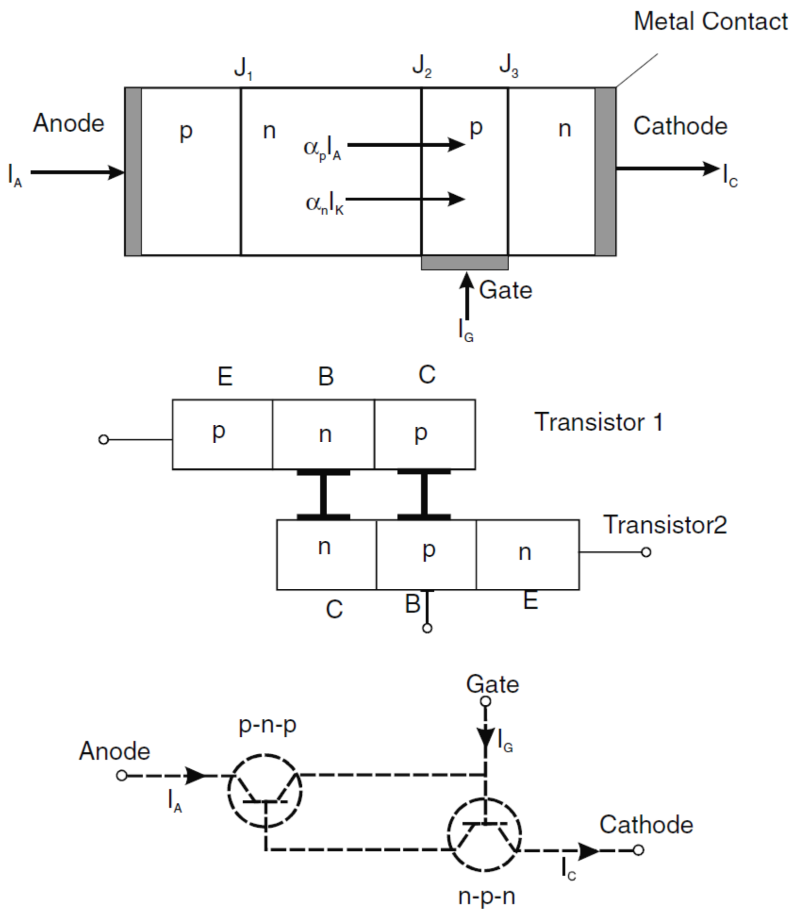
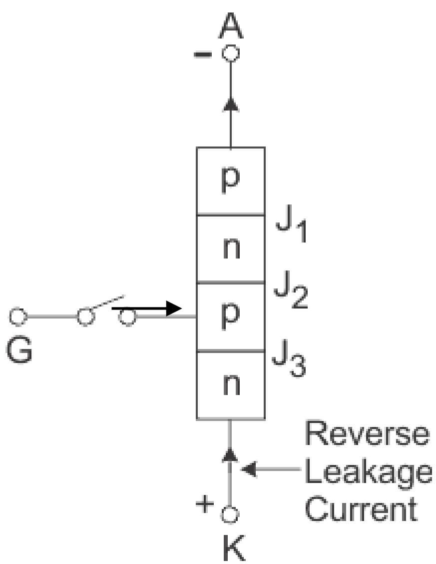


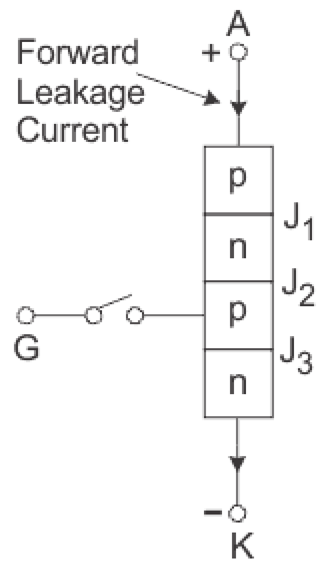
Fig. 4.22. Structure of thyristor, and two-transistor equivalent circuit

Thyristors

- Three modes of operation:
  - Reverse blocking state
  - Forward blocking state
  - Conduction or on state



Reverse Blocking Mode



Forward Biased Condition

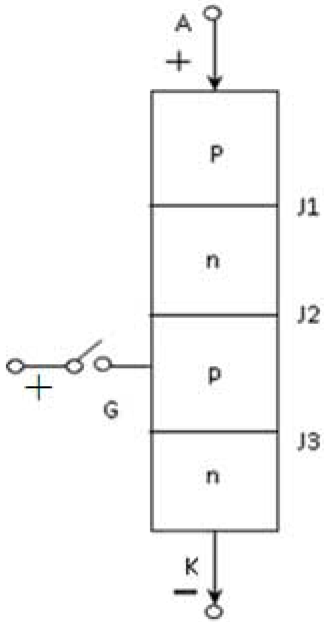


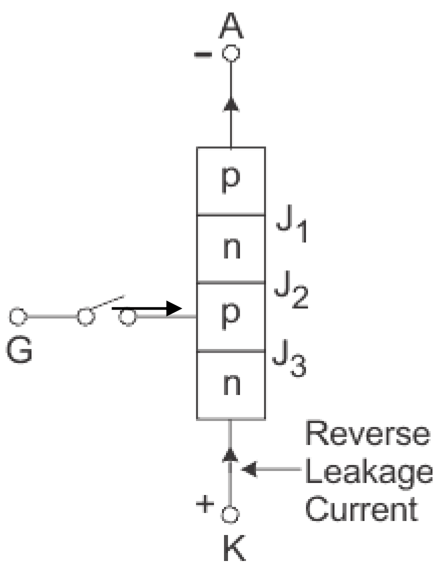
Fig 2: Forward Conduction

Most of the voltage is held by J_1.

Most of the voltage is held by J_2.

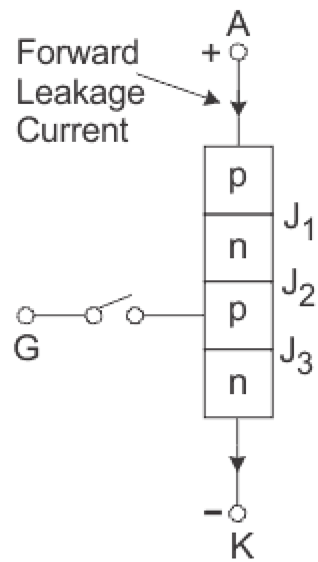
Thyristors

- **Three modes of operation:**
  - **Reverse blocking state**
  - **Forward blocking state**
  - **Conduction or on state**



Reverse Blocking Mode

Most of the voltage is held by J_1.



Forward Biased Condition

Most of the voltage is held by J_2.

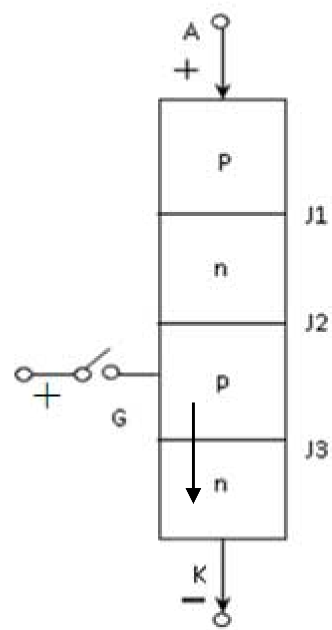


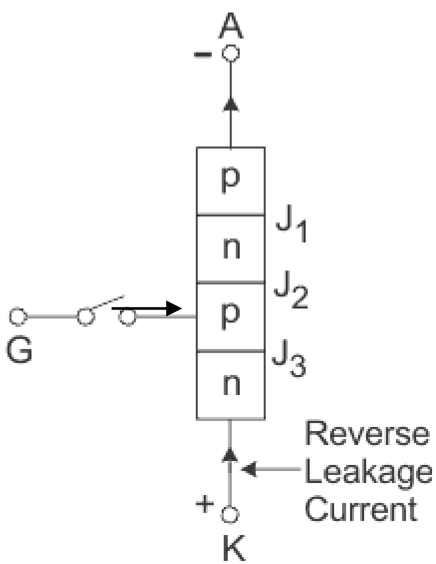
Fig 2: Forward Conduction

e^- will fill up the holes

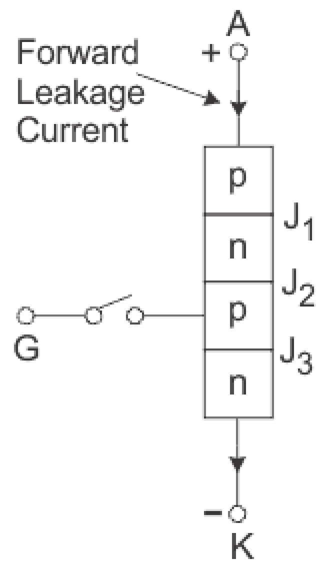
w/ more e^- => p type becomes N type

Thyristors

- Three modes of operation:
  - Reverse blocking state
  - Forward blocking state
  - Conduction or on state



Reverse Blocking Mode



Forward Biased Condition

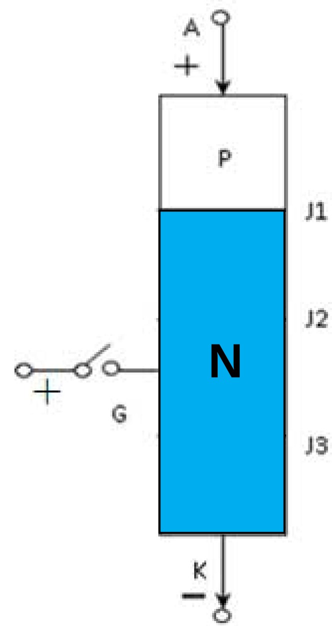


Fig 2: Forward Conduction

e$^-$ will fill up the holes

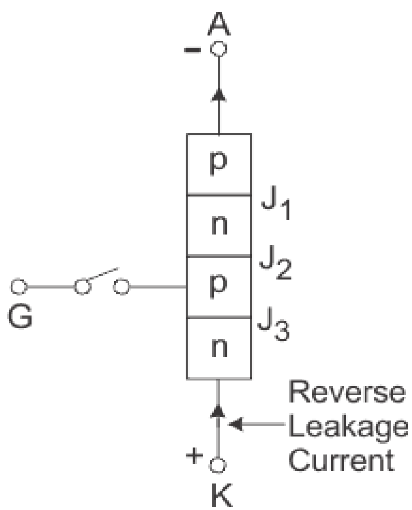
w/ more e$^-$ => p type becomes N type

Most of the voltage is held by J_1.

Most of the voltage is held by J_2.

Thyristors

- **Without any external action, the thyristor cannot come back from the conducting to the blocking state.**
- **Two methods are generally applied:**
  - **Commutation of the current by polarity inversion.**
  - **Commutation of the current, supported by gate-assisted turn-off.**



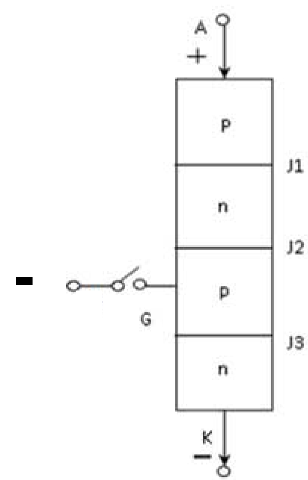
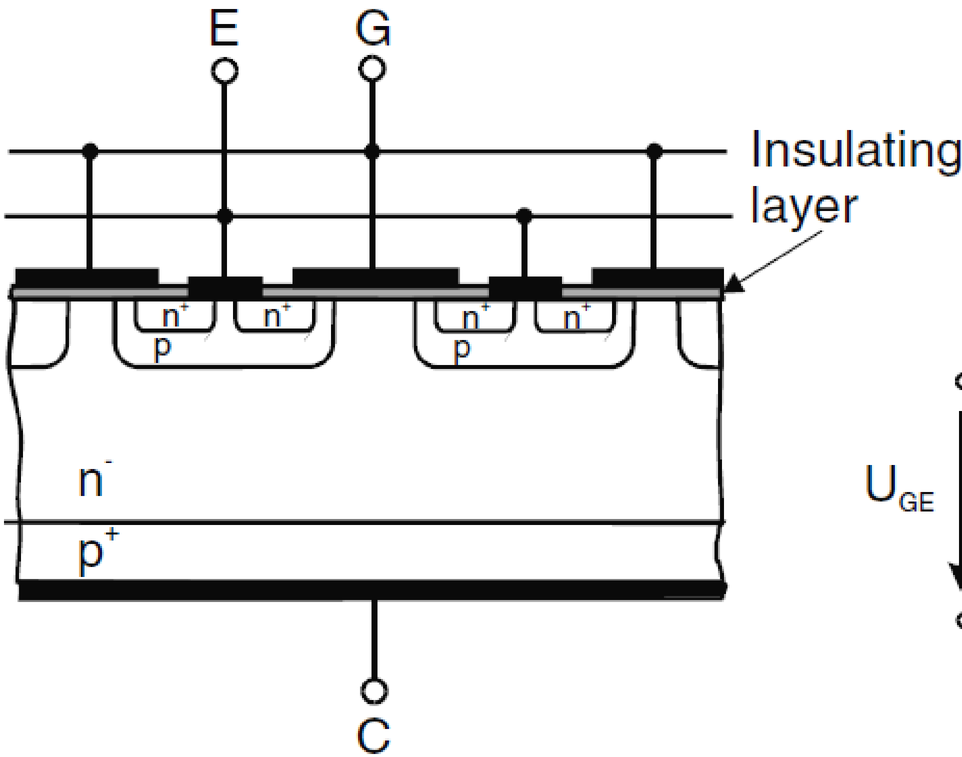
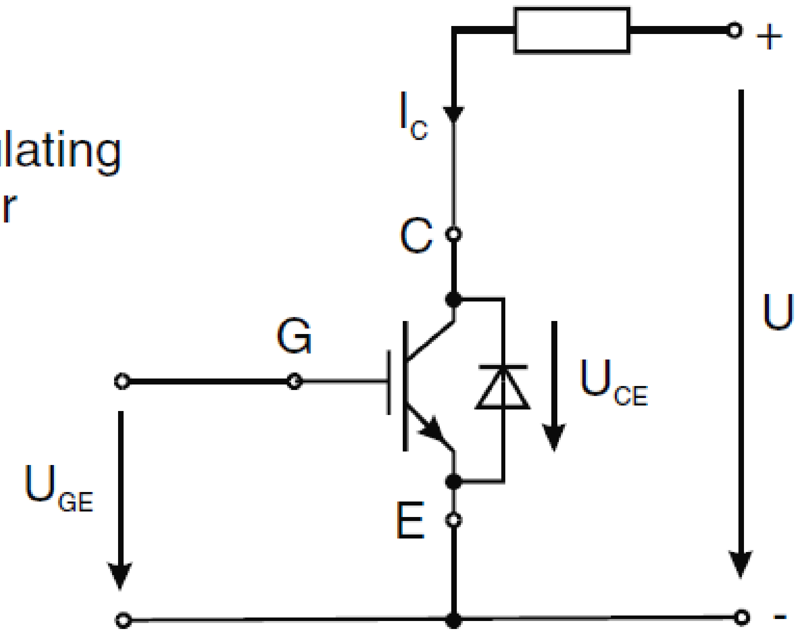


Fig 2: Forward Conduction

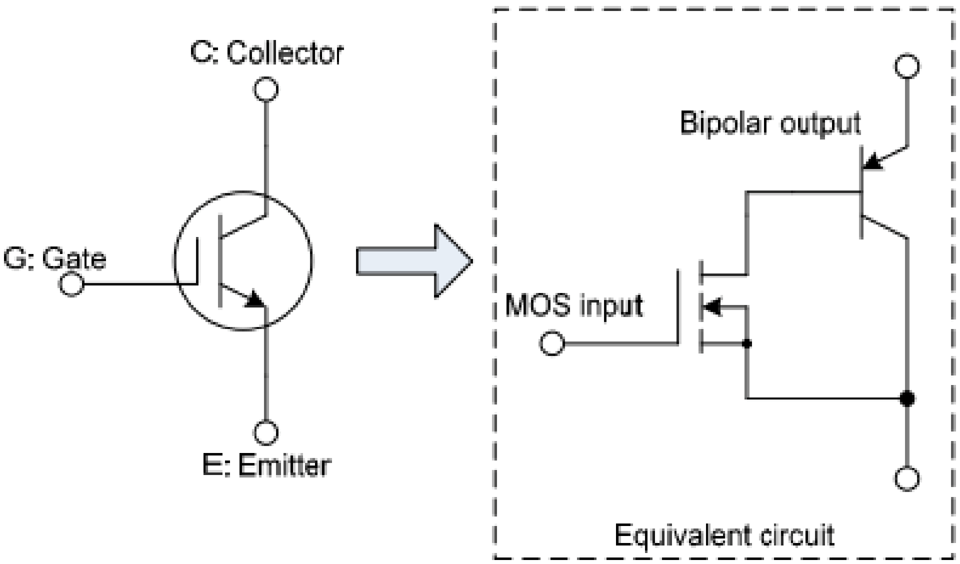
IGBT





IGBT

- **Advantage:**
  - **Bipolar transistors (BJT) – low resistance in the switched-on state**
  - **Field effect transistors (FET) – loss-free gate control**
- **Switch-on times:**
  ~ several times 10 ns.
- **It has a limited reverse-blocking capability => an external diode is sometimes used in parallel.**
- **High-power IGBT: blocking voltages V~4 kV, on state I ~3kA**



Optically activated semiconductor switches

$$\nabla j_n = e(R_\mathrm{n} - G_\mathrm{n}) + e\frac{\partial n}{\partial t}$$

$$\nabla j_p = -e(R_\mathrm{p} - G_\mathrm{p}) - e\frac{\partial p}{\partial t}$$

$$\mathrm{eG_{av}} = \alpha_n|j_n| + \alpha_p|j_p|$$

R_n: recombination rate.
G_n: generation rate.

- **Electron and hole generation is caused either by optical excitation or by avalanche ionization at sufficiently high electric fields.**

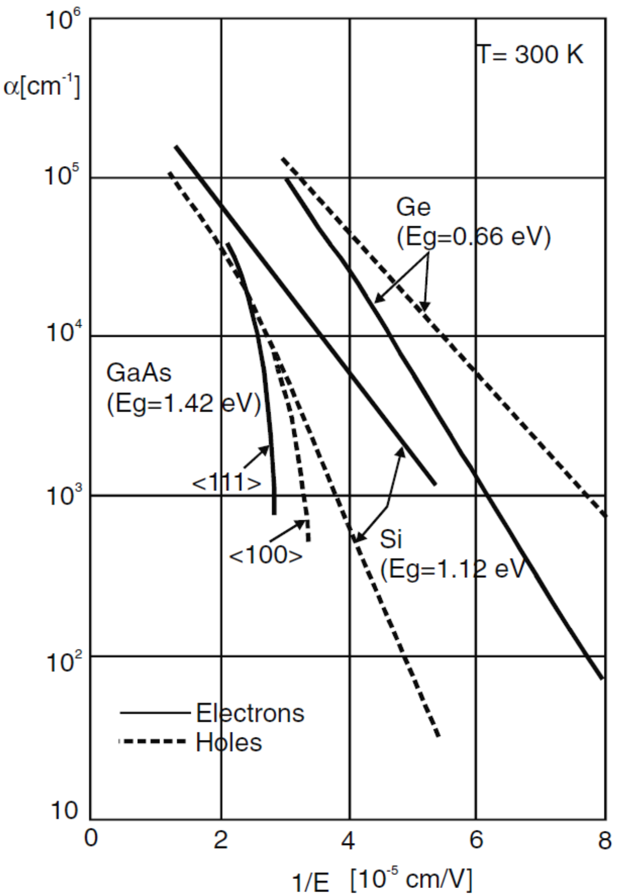


Fig. 4.31. Ionisation rate coefficients α_n and α_p

Optically activated semiconductor switches

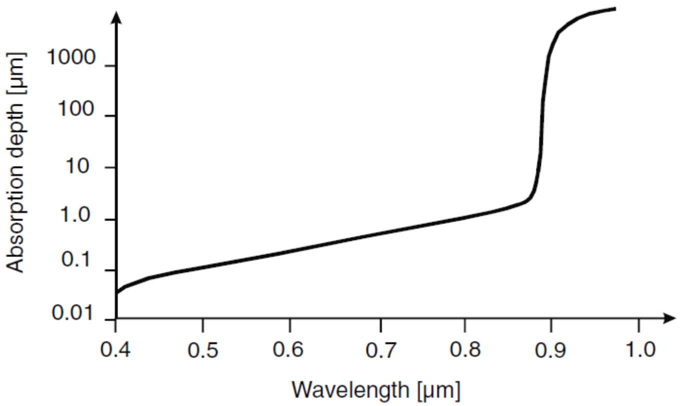
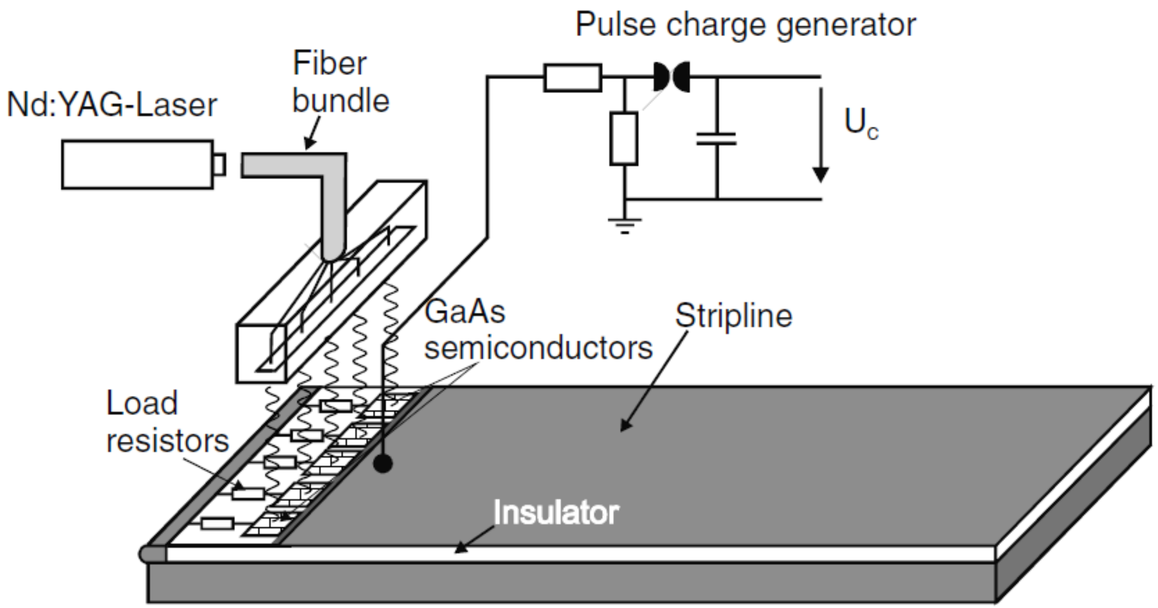
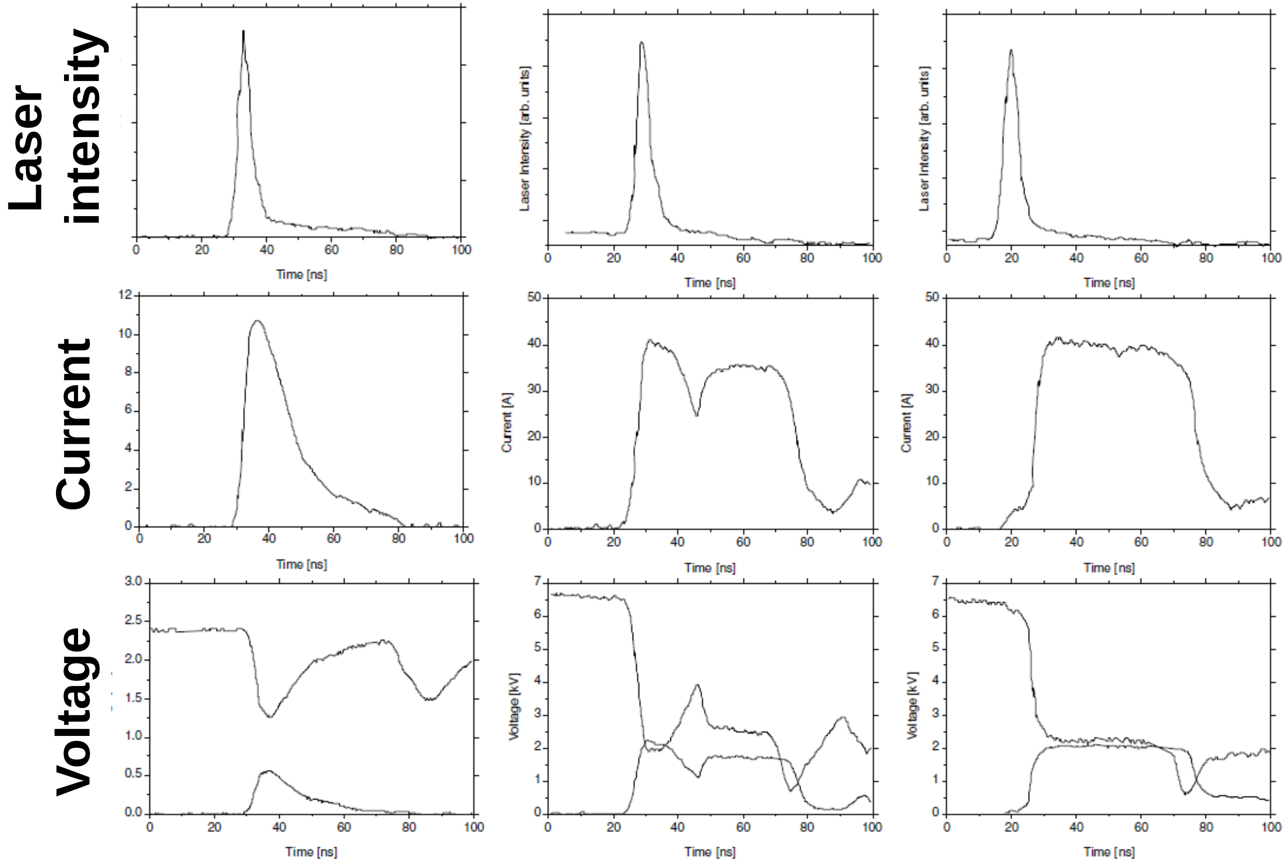


Fig. 4.32. Optical absorption depth in GaAs as a function of wavelength

- **The wavelength should be larger than 0.9 um. Therefore a Nd:YAG laser, wavelength = 1.06 um, is an appropriate light source.**

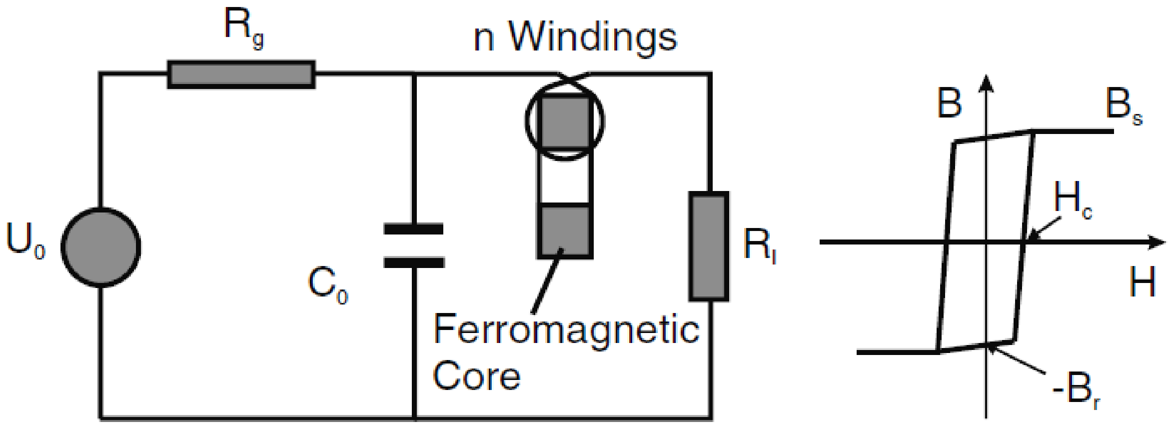


Optically activated semiconductor switches



- **Linear photoconducting regime: the available number of charge carriers is determined only by the laser intensity.**
- **Nonlinear regime: the number of charge carriers is increased by collisional ionization and as in a gas switch increases exponentially.**

Magnetic switches



- **Relatively small losses and without wear.**
- **While the capacitor is being charged: the coil has a ferromagnetic core with high inductance at the beginning: V=LdI/dt => like an open switch.**
- **When saturation of the core is reached by the leakage current flowing through the coil => L drops abruptly by a factor of μ => switch is closed.**
- **μ=B/H ->0 when saturated.**
- **The hysteresis loop should approximate a rectangular form, with an abrupt change of the permeability over several orders of magnitude when the saturation point is reached.**

Summary

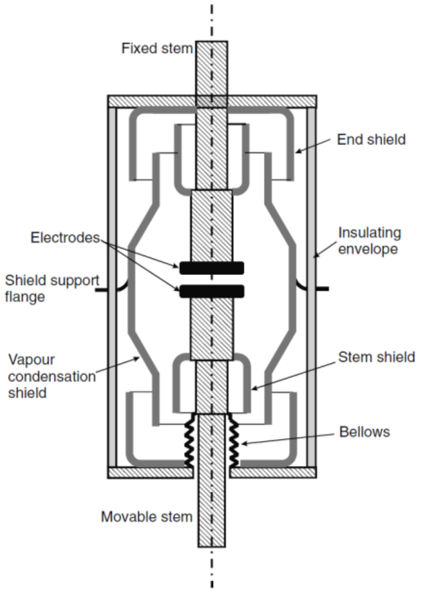
Type	Hold-off potential (kV)	Peak current (kA)	Cumulative charge (A s)	Repetition rate (Hz) [commutation time (ns)]	Lifetime (number of pulses)	Remarks
Spark gap	1–6000	10^{-3}–1000	0.1–50	1–10 [1–1000]	10^3–10^7	Lifetime is determined by electrode erosion
Thyratron	5–50	0.1–10	10^{-3}	1000 [5–100]	10^7–10^8	Applied in lasers and accelerators
Ignitron	> 10	> 100	2000	1 [1000]	10^5–10^6	Applied in lasers and accelerators
TVG	0.5–50	1–10	40	1 [10–100]	$> 10^4$	
Pseudo-spark	1–50	1–20	1	1–1000 [> 10]	10^6–10^8	Similar to Thyratron
Krytron	8	3	0.01–0.1	< 1000 [1–10]	10^7	Very short delay and commutation time
Magnetic Switch	1000	100–1000		10 [5–10000]	10^8–10^9	Cannot be triggered; one operating point only
Thyristor	< 5	< 5	10^{-2}	10 [> 1000]	10^8	Can be stacked; expensive; complex
IGBT	< 4	3		100	10^8	Can be switched off
GaAs photoactivated switch	< 20	1–10	$< 10^{-4}$	< 10 [1–10]	10^2–10^3	Needs intense light source

Outlines

- Switches
  - Closing switches
  - Opening switches
- Pulse-forming lines
  - Blumlein line
  - Pulse-forming network
  - Pulse compressor
- Pulse transmission and transformation
  - Self-magnetic insulation
  - Pulse transformer
  - Voltage multiplier
  - H-bridge pulse generator
  - Fast high-voltage pulse generator

Opening switches

- An opening switch is characterised by “a sudden growth of its impedance” by
  - External actuator
  - Internal process – depend on the amount of the charge conducted through the switch
- The mechanism can be
  - Resistive nature: common fuse
  - Inductive nature: flux compression, L(t) >> L(0)
  - Capactive nature, C(t) << C(0)

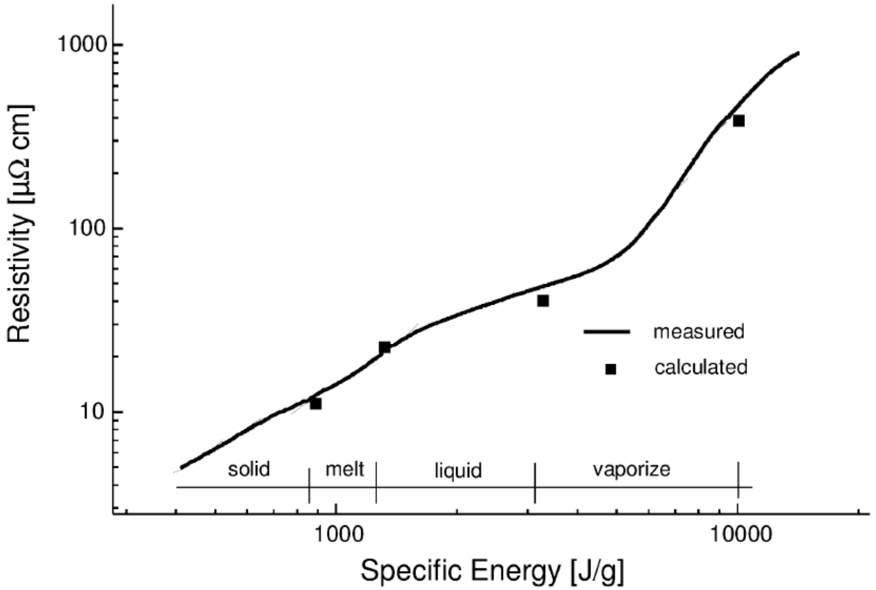


Opening switches

- **Requirement:**
  - **Long current conduction time.**
  - **Large current and small losses during conduction.**
  - **Fast impedance rise during opening.**
  - **High impedance after opening & large voltage hold-off during current interruption.**
  - **Short recovery time, i.e., high repetition rate capability.**
  - **Long lifetime, i.e., small wear.**

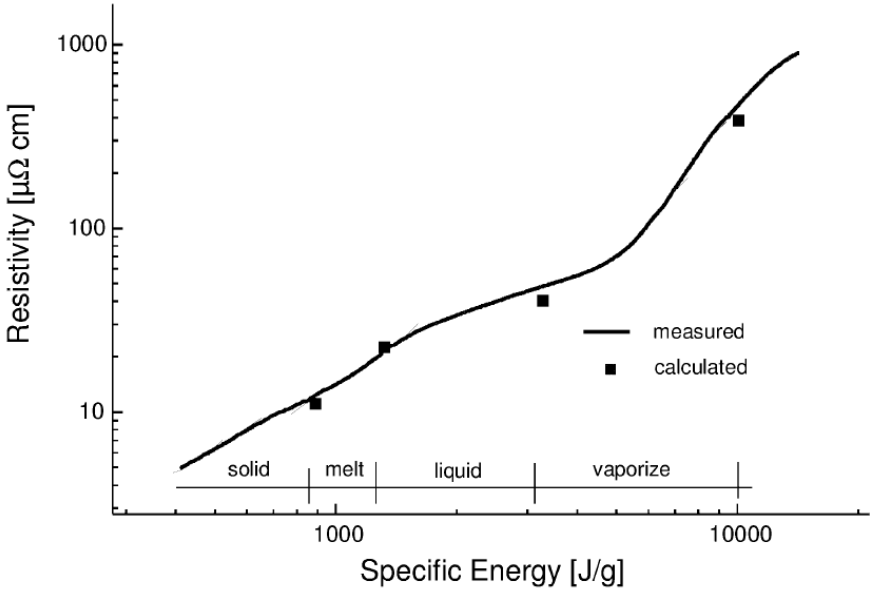
Fuses

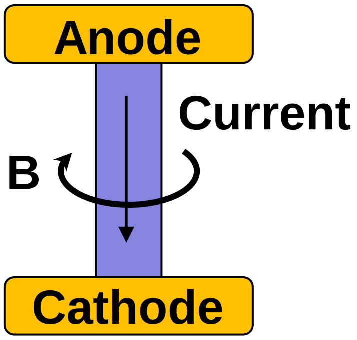
- **Melting fuse – the most widely known opening switch.**
- **A thin wire / a foil embedded in a gaseous, liquid or granular medium.**
- **Based on Melting, Boiling, or Vaporization of a conductor,**
- **Fast opening is possible: <50 ns**
- **Conduction time can be determined by the type of material and its geometry.**



Fuses

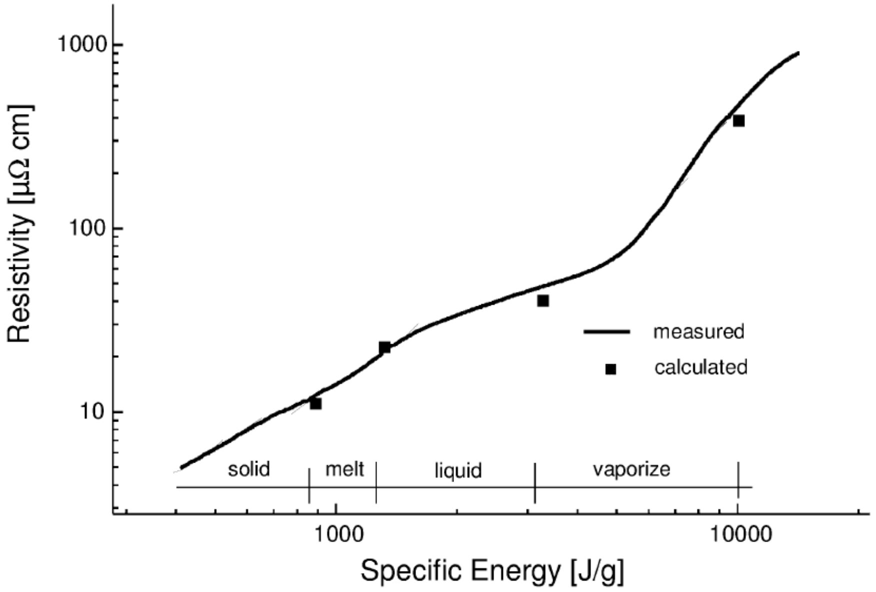
- The resistivity of most metals rises continuously with T both in the solid and in the liquid phase.
- The high magnetic pressure associated with the current flowing through the fuse can maintain a high density and therefore metallic conductivity beyond the critical temperature.
- Only after the onset of expansion does the metallic conductivity disappear.

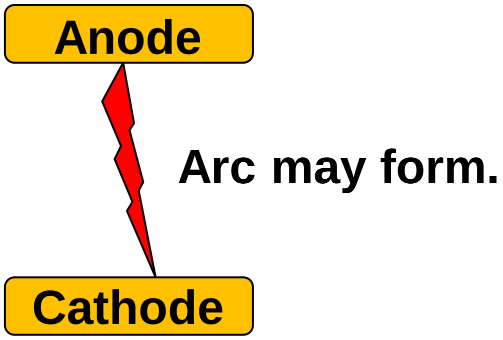




Fuses

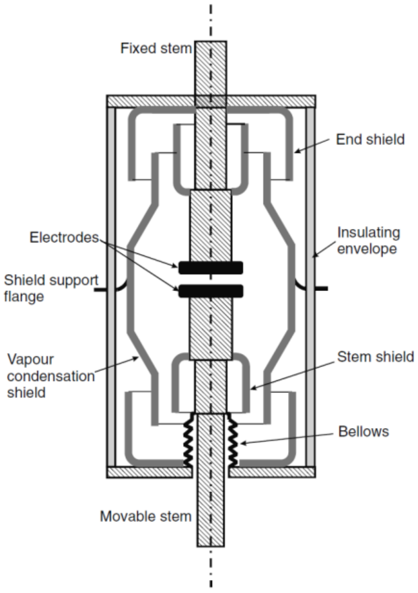
- If the density of the metal vapor becomes sufficiently small -> electron avalanche processes can lead to the initiation of arcs in the vapor.
- The purpose of the surrounding medium is therefore to quench or prevent arc formation.
- Advantage – simplicity, adapt their parameters to the experimental conditions by choosing the appropriate cross-section, length, and #/ of elements.



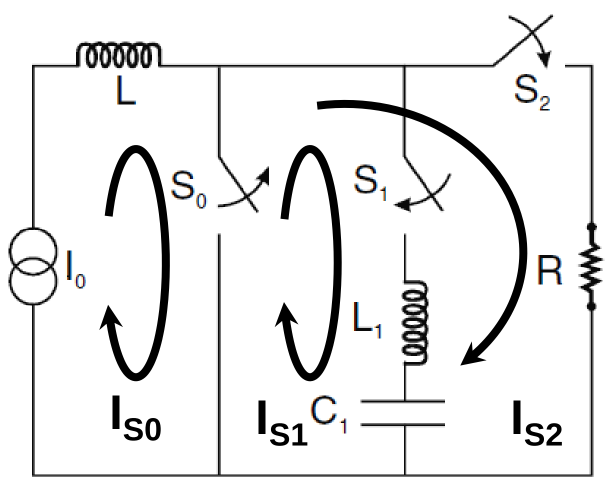


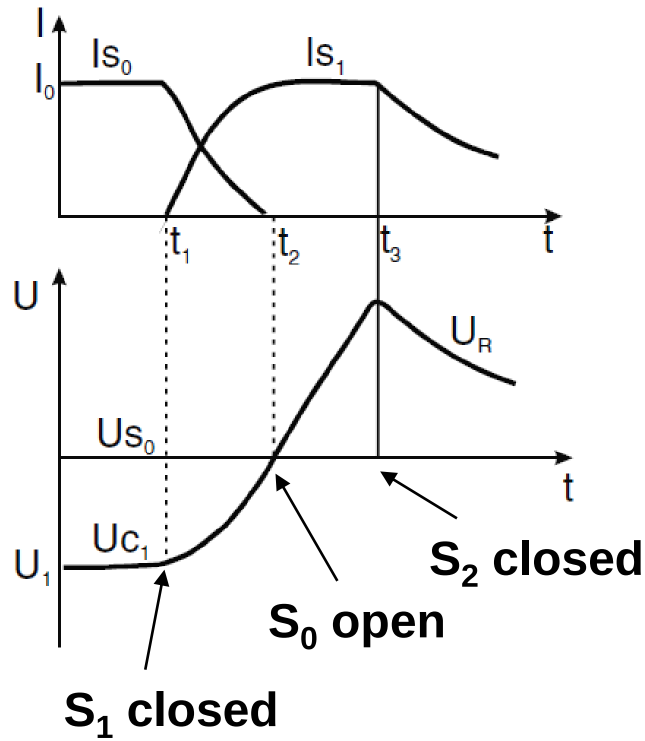
Mechanical Interrupters

- **Vacuum interrupter switch: 2 planar/disc electrodes (1 fixed the other movable) in a vacuum envelope (0.1 Pa (7.5 x 10^{-4} Torr) or less).**
- **Closed position – low resistance (10-50 uΩ) from a tight metal-to-metal contact**
- **Open position – separated by an actuator (致動器).**
- **During the process of switch breaking – an arc is likely to be drawn and sustained by metal vapor evaporated from the electrodes.**
- **In unipolar system, a current counter-pulse is needed to reduce the power input to the arc to allow the residual arc plasma to recombine.**
- **After I=0, dU/dt=24 kV/us is possible.**
- **Repetitive frequency – few tens of hertz.**
- **Opening speed – tens of us.**



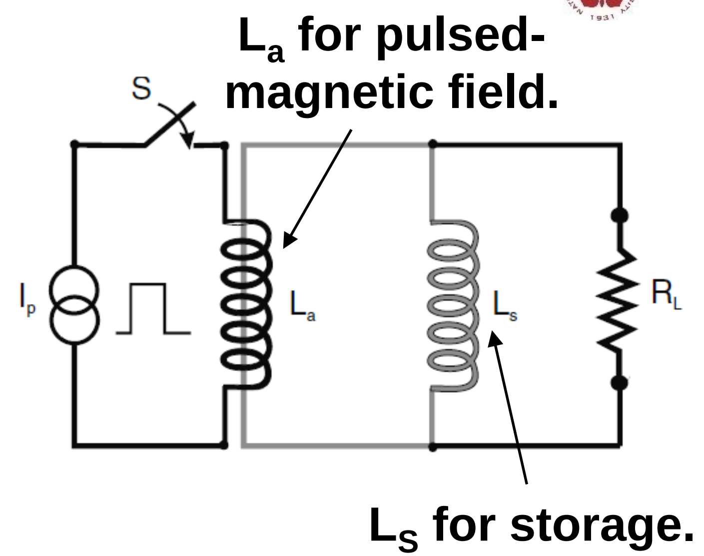
Counter-pulse arrangement





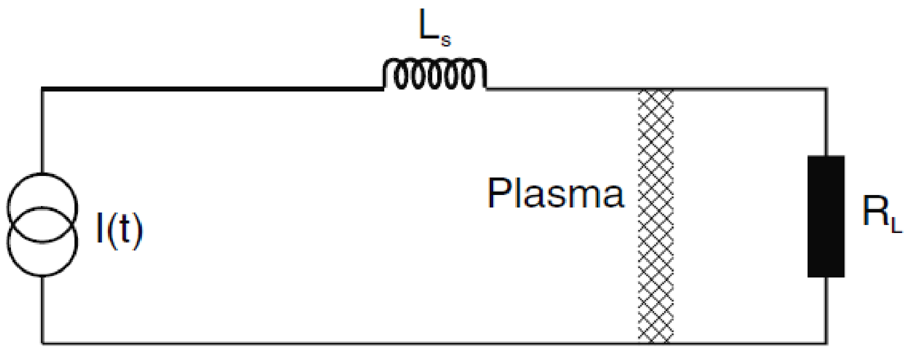
Superconducting opening switches

- **Superconducting state -> normal conduction**
- **Three ways to trigger:**
  - **The current itself**
  - **An external pulsed magnetic field**
  - **pulse heating**
- **The repetition rate depends on the speed of recovery to the superconducting state.**
- **Problem: consists of the additional cooling necessary to remove the heat flowing into the cryogenic coolant during opening.**

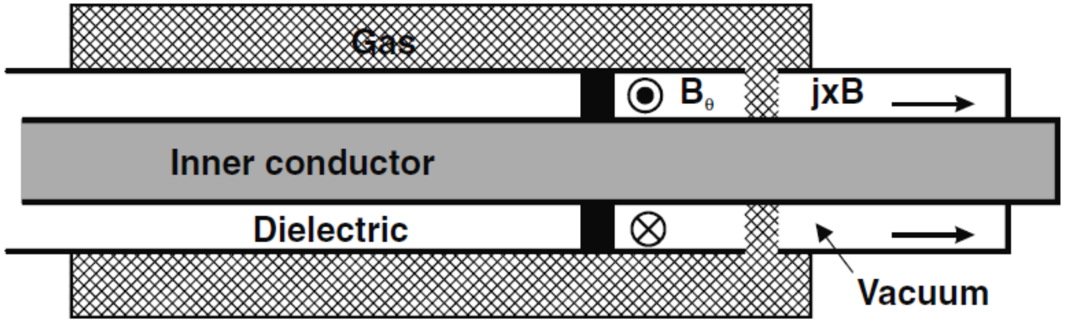


Plasma opening switches

- Suitable for high currents and short switching times.
- Plasma bridge of low density (10^{13}-10^{15} cm^{-3}).
- 10^{15}-10^{16} cm^{-3} for several hundred kA or MA.
- 10^{13} cm^{-3} is needed to conduct currents for less than 100 ns and opening in less than 10 ns.

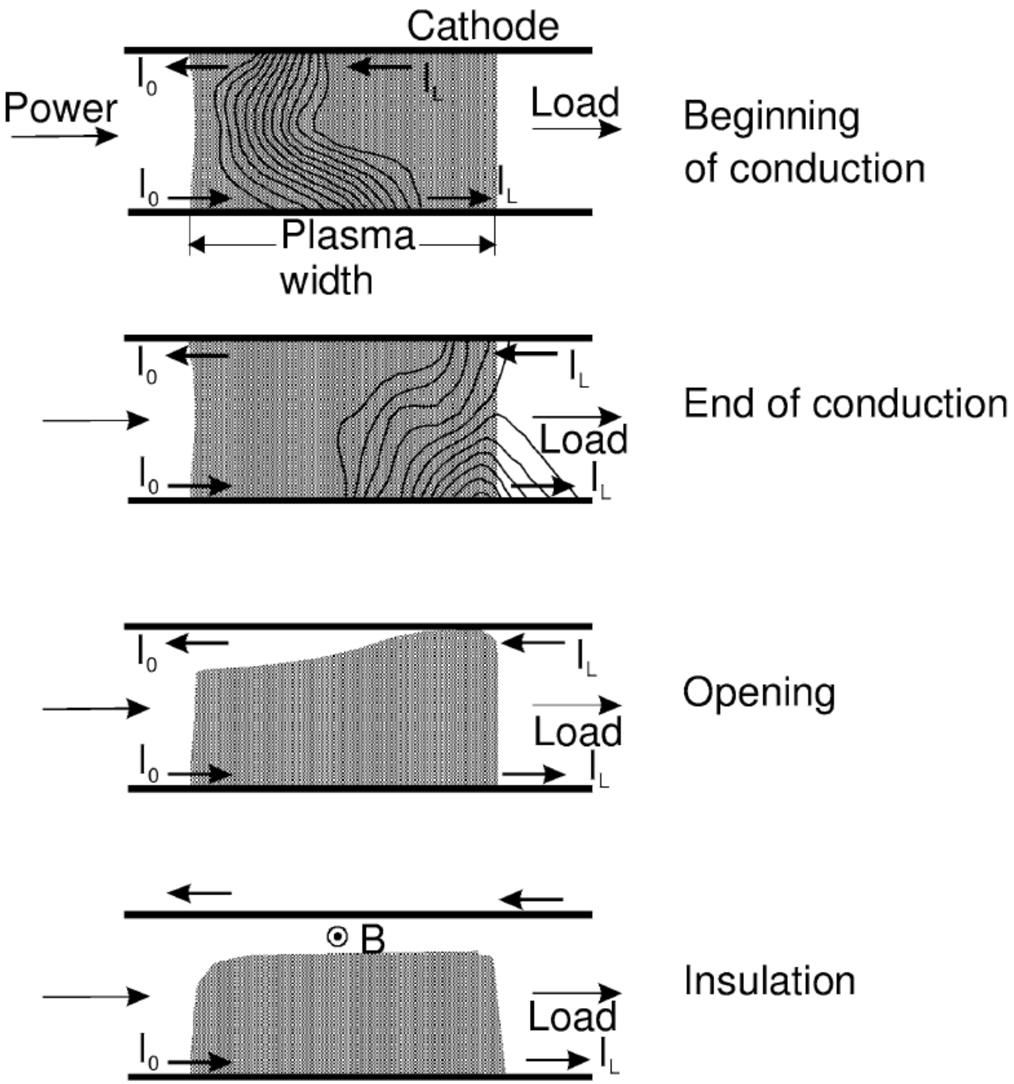


Coaxial system with an injected pulsed gas column



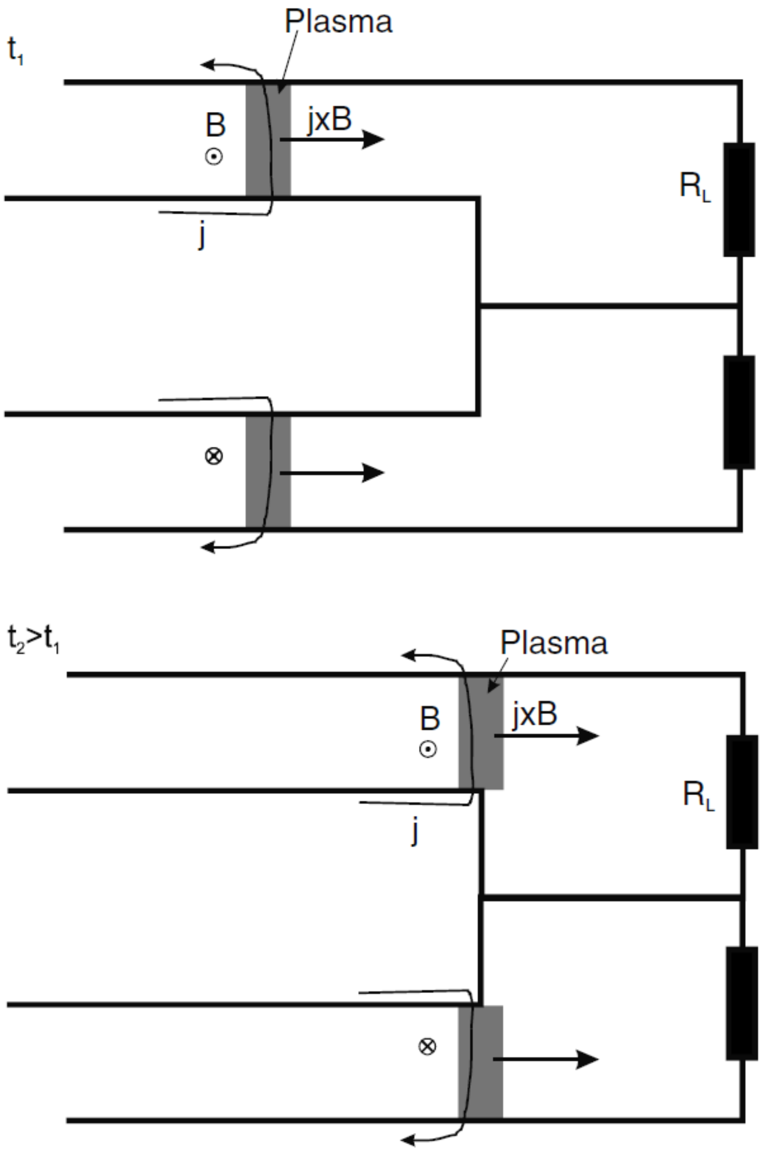
- **The gas is made into a plasma by an auxiliary electric pulse before the coaxial inductor is charged.**
- **Conduction phase – the current, the magnetic field penetrates into the plasma**
- **Opening – occurs if the plasma becomes pushed out determined by self-magnetic insulation**

Self-magnetic insulation process



Plasma Flow Switches

- **Higher plasma densities (10^{15} cm^{-3}).**
- **Conduction times – μs.**



Outlines

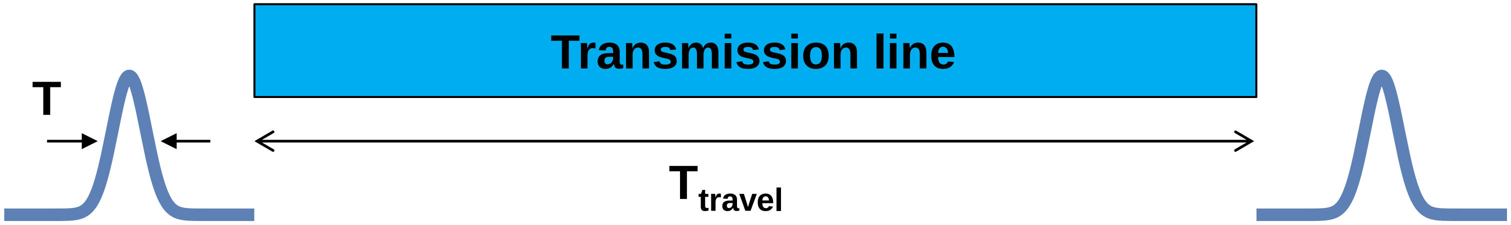
- Switches
  - Closing switches: the switching process is associated with voltage breakdown across an initially insulant element.
  - Opening switches: the switching process is associated with a sudden growth of its impedance.
- **Pulse-forming lines**
  - **Blumlein line**
  - Pulse-forming network
  - Pulse compressor
- Pulse transmission and transformation

Pulse-forming lines

- **A constant-voltage plateau is needed for many pulsed-power applications.**
- **Various arrangements of LC elements are necessary. It is called “Pulse-Forming Networks (PFN)”.**

Transmission lines

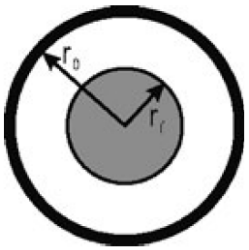
- **Transmission lines are the continuous borderline case of a network consisting of discrete LC elements.**
- **Depending on the time T during which energy is extracted from or supplied to the element, a transmission line can be described as lumped circuit element or an extended object.**
  - **$T > T_{travel}$, the time it takes for an EM wave to move from one terminal of the element to the next → lumped circuit element.**
  - **$T < T_{travel}$, the time it takes for an EM wave to move from one terminal of the element to the next → Transmission line.**



Different kinds of transmission line and the inductance and the capacitance per unit

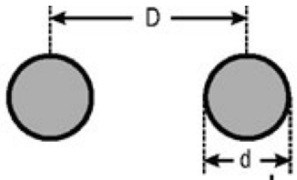
1. Coaxial transmission line:

$$C' = 2\pi\varepsilon / \ln(r_\mathrm{o}/r_\mathrm{i})$$
$$L' = (\mu/2\pi)\ln(r_\mathrm{o}/r_\mathrm{i})$$
$$Z_0 = \left((\mu/\varepsilon)^{1/2}/2\pi\right)\ln(r_\mathrm{o}/r_\mathrm{i}) = 60(\mu_\mathrm{r}/\varepsilon_\mathrm{r})^{1/2}\ln(r_\mathrm{o}/r_\mathrm{i})$$



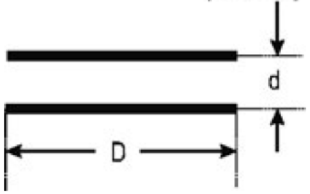
2. Double-wire line:

$$C' = \pi\varepsilon / \operatorname{arcosh}(D/d)$$
$$L' = (\mu/\pi)\operatorname{arcosh}(D/d)$$
$$Z_0 = \left((\mu/\varepsilon)^{1/2}/\pi\right)\operatorname{arcosh}(D/d)$$



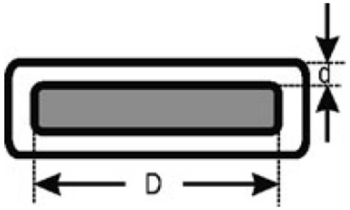
3. Parallel-plate line:

$$C' = \varepsilon D/d$$
$$L' = \mu d/D$$
$$Z_0 = (\mu/\varepsilon)^{1/2}(d/D)$$

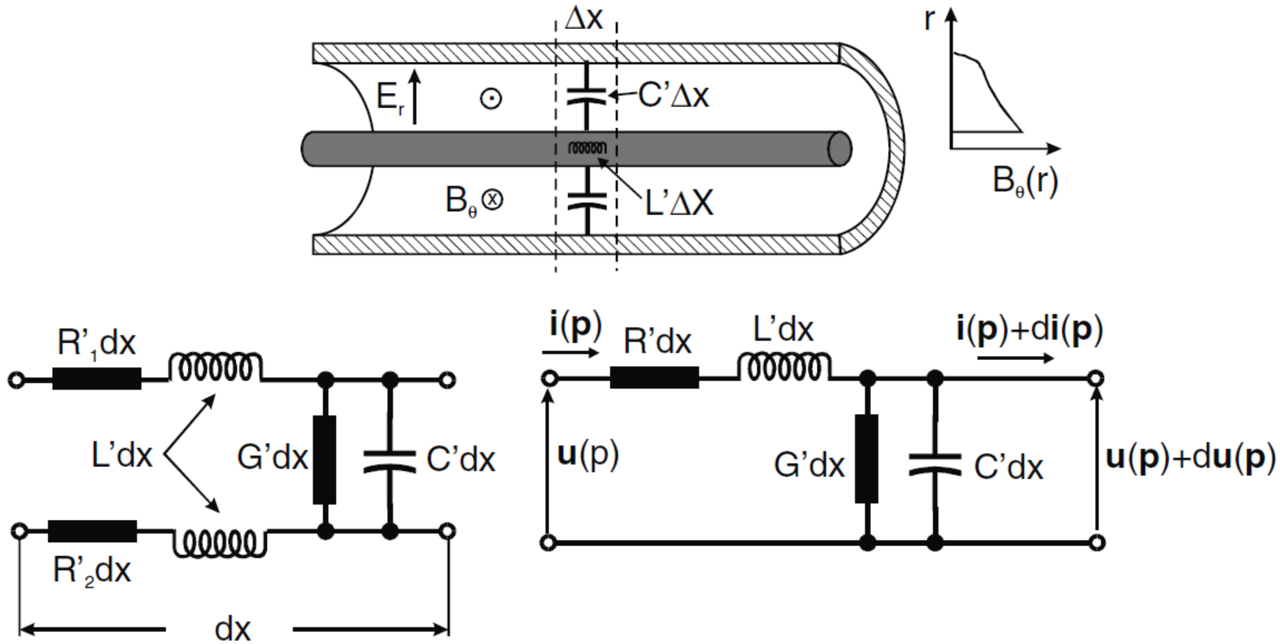


4. Stripline:

$$C' = 2\varepsilon D/d$$
$$L' = \mu d/2D$$
$$Z_0 = (\mu/\varepsilon)^{1/2}(d/2D)$$

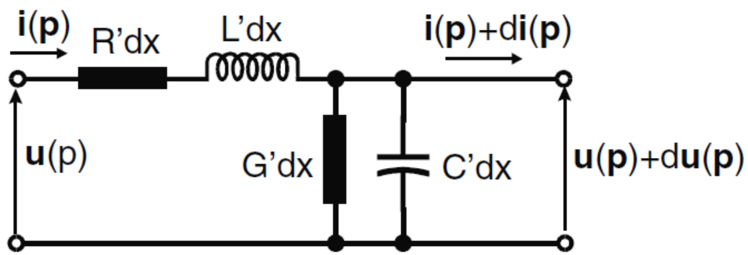


An infinitesimal section of a homogeneous coaxial transmission line



- Resistances per unit length: $R'_1 + R'_2 \rightarrow R' \rightarrow R$.
- Conductance per unit length: $G' \rightarrow G$.
- All quantities are frequency-dependent because of the skin effect and because the dielectric constant depends on the frequency.
- Assume that they are independent of the position x, the voltage V and current I.

An infinitesimal section of a homogeneous coaxial transmission line



$$U - I\mathrm{R}\mathrm{d}x - \mathrm{L}\mathrm{d}x\frac{\mathrm{d}I}{\mathrm{d}t} - U' = 0$$

$$I - U'\mathrm{G}\mathrm{d}x - \mathrm{C}\mathrm{d}x\dot{U}' - I' = 0 \qquad U' = U + \frac{\partial U}{\partial x}\mathrm{d}x \qquad \dot{U}' = \dot{U} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial U}{\partial x}\right)\mathrm{d}x$$

$$I - \left(U + \frac{\partial U}{\partial x}\mathrm{d}x\right)\mathrm{G}\mathrm{d}x - \mathrm{C}\mathrm{d}x\left[\dot{U} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial U}{\partial x}\right)\mathrm{d}x\right] - I' = 0$$

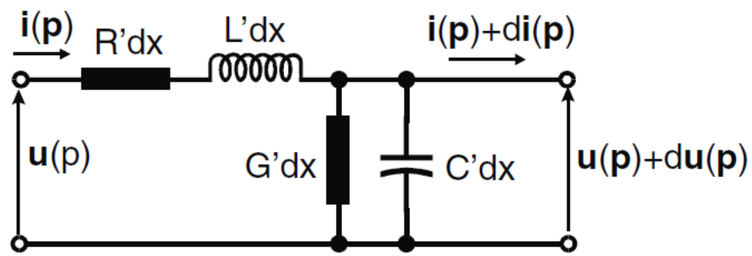
$$I - U\mathrm{G}\mathrm{d}x - \frac{\partial U}{\partial x}\mathrm{G}\mathrm{d}x^2 - \dot{U}\mathrm{C}\mathrm{d}x - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial U}{\partial x}\right)\mathrm{C}\mathrm{d}x^2 - I' = 0$$

$$I - U\mathrm{G}\mathrm{dx} - \mathrm{C}\mathrm{d}x\dot{U} - I' = 0$$

$$\begin{cases} \dfrac{U' - U}{\mathrm{dx}} \equiv \dfrac{\partial U}{\partial x} = -\mathrm{R}I - L\dot{I} \\[2ex] \dfrac{I' - I}{\mathrm{dx}} \equiv \dfrac{\partial I}{\partial x} = -\mathrm{G}U - C\dot{U} \end{cases}$$

An infinitesimal section of a homogeneous coaxial transmission line

$$\begin{cases} \dfrac{U'-U}{\mathrm{dx}} \equiv \dfrac{\partial U}{\partial x} = -\mathrm{R}I - L\dot{I} \\[2ex] \dfrac{I'-I}{\mathrm{dx}} \equiv \dfrac{\partial I}{\partial x} = -\mathrm{G}U - C\dot{U} \end{cases}$$



- **Laplace transform:** $x = \tilde{x}e^{\mathrm{pt}}$, $\dfrac{d}{\mathrm{dt}} \to p$

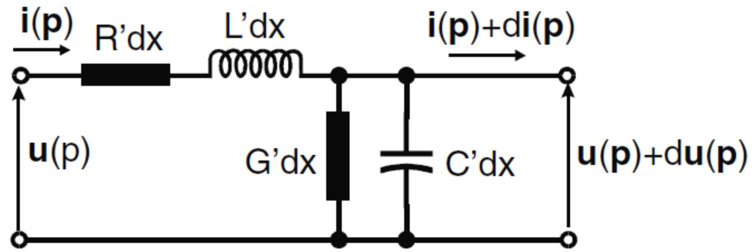
$$\frac{\mathrm{d}\tilde{U}}{\mathrm{d}x} = -R\tilde{I} - \mathrm{pL}\tilde{I} = -(R+\mathrm{pL})\tilde{I} \qquad \frac{\mathrm{d}^2\tilde{U}}{\mathrm{d}x^2} = -(R+\mathrm{pL})\frac{\mathrm{d}\tilde{I}}{\mathrm{d}x} = (R+\mathrm{pL})(G+\mathrm{pC})\tilde{U}$$

$$\frac{\mathrm{d}\tilde{I}}{\mathrm{d}x} = -G\tilde{U} - \mathrm{pC}\tilde{U} = -(G+\mathrm{pC})\tilde{U} \qquad \frac{\mathrm{d}^2\tilde{I}}{\mathrm{d}x^2} = -(G+\mathrm{pC})\frac{\mathrm{d}\tilde{U}}{\mathrm{d}x} = (G+\mathrm{pC})(R+\mathrm{pL})\tilde{I}$$

- **Lossless line where R=0, G=0:**

$$\boxed{\begin{array}{ll} \dfrac{\mathrm{d}\tilde{U}}{\mathrm{d}x} = -p\mathrm{L}\tilde{I} & \dfrac{\mathrm{d}^2\tilde{U}}{\mathrm{d}x^2} = p^2\mathrm{LC}\tilde{U} \\[2ex] \dfrac{\mathrm{d}\tilde{I}}{\mathrm{d}x} = -p\mathrm{C}\tilde{U} & \dfrac{\mathrm{d}^2\tilde{I}}{\mathrm{d}x^2} = p^2\mathrm{LC}\tilde{I} \end{array}}$$

An infinitesimal section of a homogeneous coaxial transmission line



- **Lossless line where R=0, G=0:**

$$\frac{\mathrm{d}^2\tilde{U}}{\mathrm{d}x^2} = p^2\mathrm{LC}\tilde{U}$$

$$\frac{\mathrm{d}^2\tilde{I}}{\mathrm{d}x^2} = p^2\mathrm{LC}\tilde{I}$$

$$U(x,p) = U_x(p) = \begin{cases} \tilde{U}_+ e^{-p\sqrt{\mathrm{LC}}x} \\ \tilde{U}_- e^{p\sqrt{\mathrm{LC}}x} \end{cases}$$

- **Inverse Laplace transform:** $L\{U_x(t-\tau)\} = \tilde{U}e^{-\mathrm{pt}}$

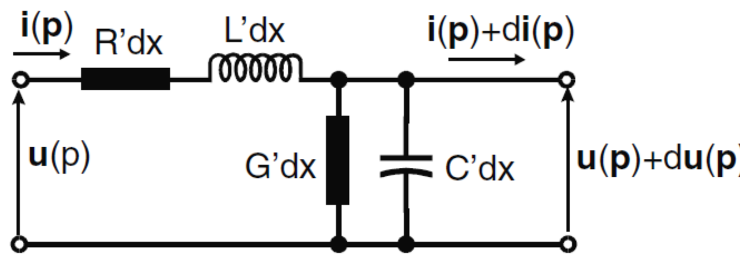
$$U(x,t) = U_x(t) = \begin{cases} U_+\left(t - x\sqrt{\mathrm{LC}}\right) \\ U_-\left(t + x\sqrt{\mathrm{LC}}\right) \end{cases}$$

or Linear combination:

$$\boxed{U_x(t) = U_+\left(t - x\sqrt{\mathrm{LC}}\right) + U_-\left(t + x\sqrt{\mathrm{LC}}\right)}$$

An infinitesimal section of a homogeneous coaxial transmission line

$$U(x,t) = U_x(t) = \begin{cases} U_+\left(t - x\sqrt{\mathrm{LC}}\right) \\ U_-\left(t + x\sqrt{\mathrm{LC}}\right) \end{cases}$$

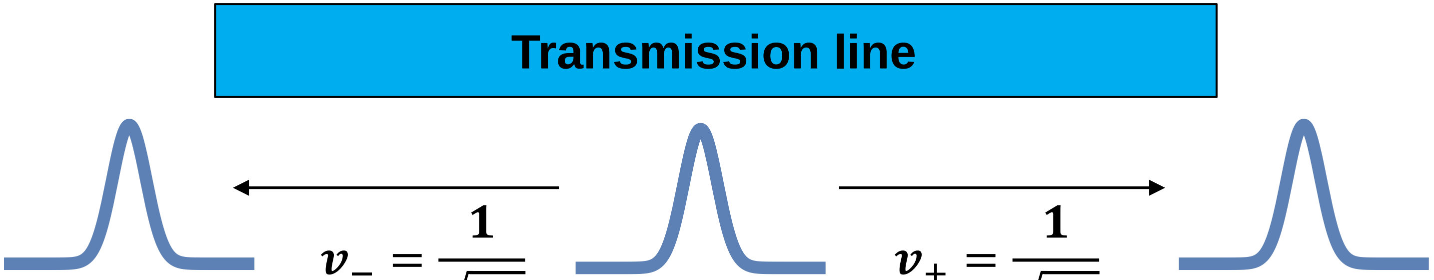


or Linear combination: $U_x(t) = U_+\left(t - x\sqrt{\mathrm{LC}}\right) + U_-\left(t + x\sqrt{\mathrm{LC}}\right)$

$$\frac{\mathrm{d}\tilde{U}}{\mathrm{d}x} = -p\mathrm{L}\tilde{I} \qquad \mp p\sqrt{\mathrm{LC}}\,\tilde{U}_\pm = -\mathrm{pL}\tilde{I}_\pm$$

$$\boxed{\frac{\tilde{U}_+}{\tilde{I}_+} = -\frac{\tilde{U}_-}{\tilde{I}_-} = \sqrt{\frac{L}{C}} \equiv Z_0}$$

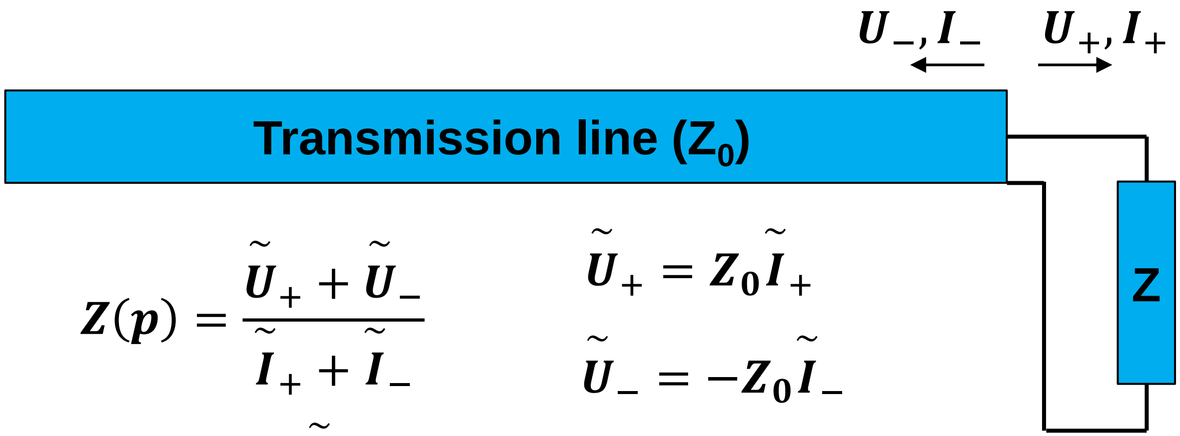
$$Z_0 = \frac{U_+\left(t - x\sqrt{\mathrm{LC}}\right)}{I_+\left(t - x\sqrt{\mathrm{LC}}\right)} = -\frac{U_-\left(t + x\sqrt{\mathrm{LC}}\right)}{I_-\left(t + x\sqrt{\mathrm{LC}}\right)}$$



$$v_- = \frac{1}{\sqrt{\mathrm{LC}}} \qquad v_+ = \frac{1}{\sqrt{\mathrm{LC}}}$$

Termination of arbitrary freq-dependent impedance Z(p)

- **Lossless transmission line terminates with an arbitrary freq-dependent impedance Z(p) in Laplace space.**



Ohm's law: $$Z(p) = \frac{\tilde{U}_+ + \tilde{U}_-}{\tilde{I}_+ + \tilde{I}_-}$$

$$\tilde{U}_+ = Z_0 \tilde{I}_+$$

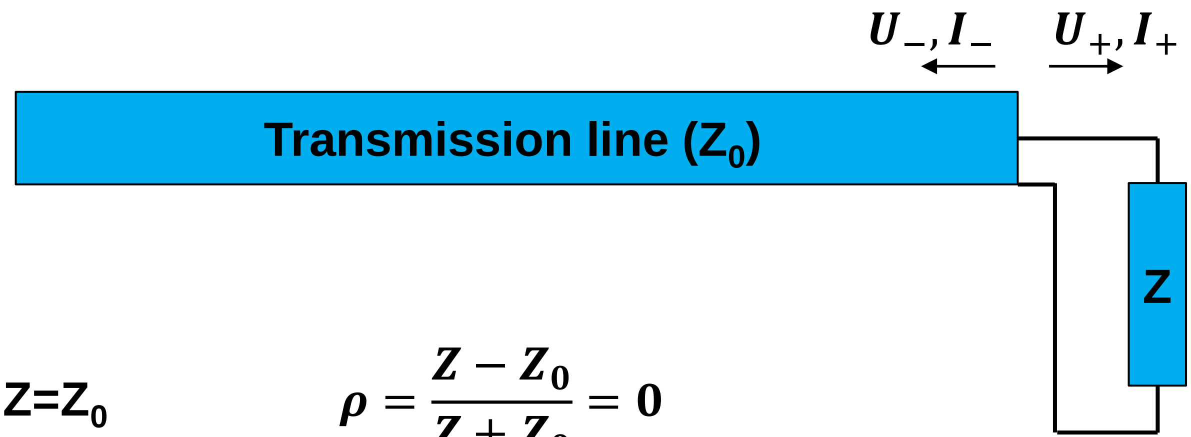
$$\tilde{U}_- = -Z_0 \tilde{I}_-$$

$$Z = \frac{\tilde{U}_+ + \tilde{U}_-}{\frac{\tilde{U}_+}{Z_0} - \frac{\tilde{U}_-}{Z_0}} = Z_0 \frac{1 + \frac{\tilde{U}_-}{\tilde{U}_+}}{1 - \frac{\tilde{U}_-}{\tilde{U}_+}} = Z_0 \frac{1+\rho}{1-\rho}$$

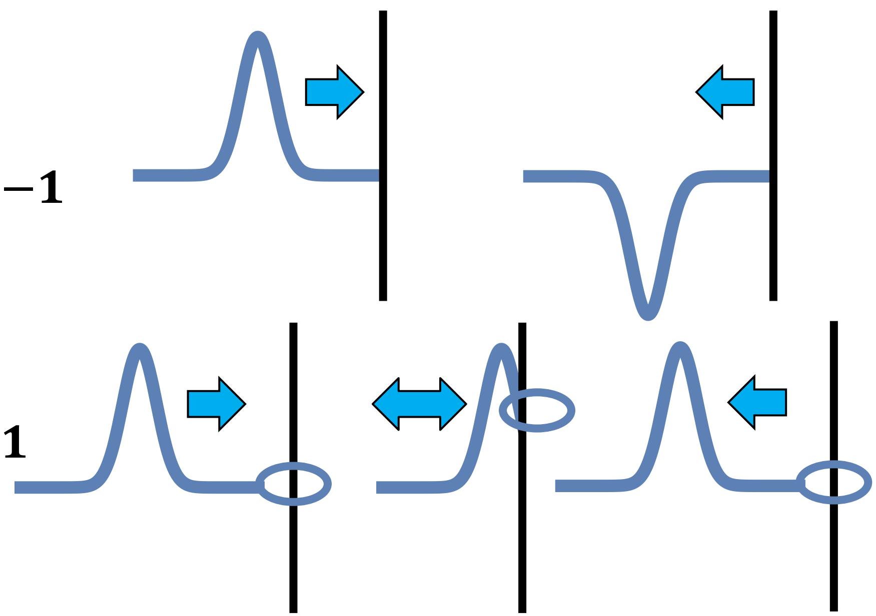
$$Z - Z\rho = Z_0 + Z_0\rho$$

Reflection: $$\rho \equiv \frac{\tilde{U}_-}{\tilde{U}_+} = \frac{Z - Z_0}{Z + Z_0}$$

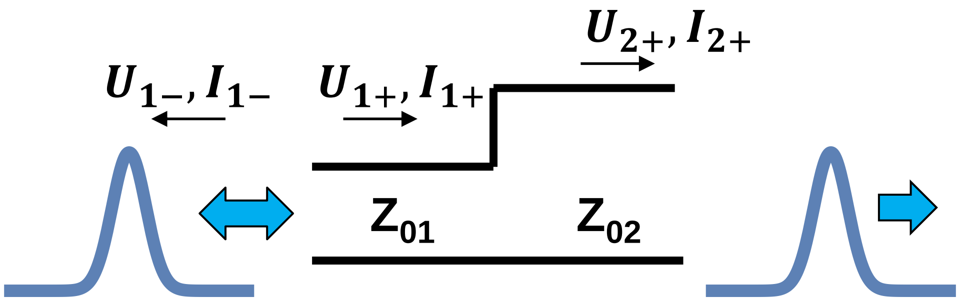
Termination of arbitrary freq-dependent impedance Z(p)



- **Match load: Z=Z_0** $\quad \rho = \dfrac{Z - Z_0}{Z + Z_0} = 0$
  - **No reflection.**
- **Short-circuit case: Z=0** $\quad \rho = \dfrac{Z - Z_0}{Z + Z_0} = -1$
  - **Completely reflective with inverted voltage amplitude.**
- **Open-circuit case: Z=∞** $\quad \rho = \dfrac{Z - Z_0}{Z + Z_0} = 1$
  - **Completely reflective with the same polarity.**



Pulse is reflected when there is an impedance mismatch



Reflection: $$\rho = \frac{U_{1-}}{U_{1+}} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$ $$\rho \equiv \frac{\tilde{U}_-}{\tilde{U}_+} = \frac{Z - Z_0}{Z + Z_0}$$

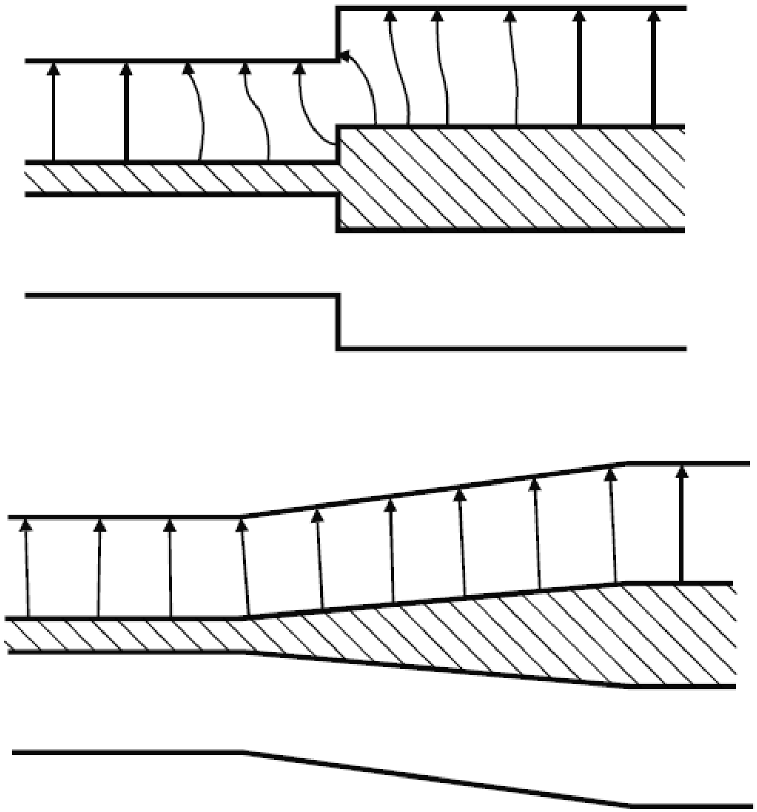
$$U_{1+} + U_{1-} = U_{2+}$$

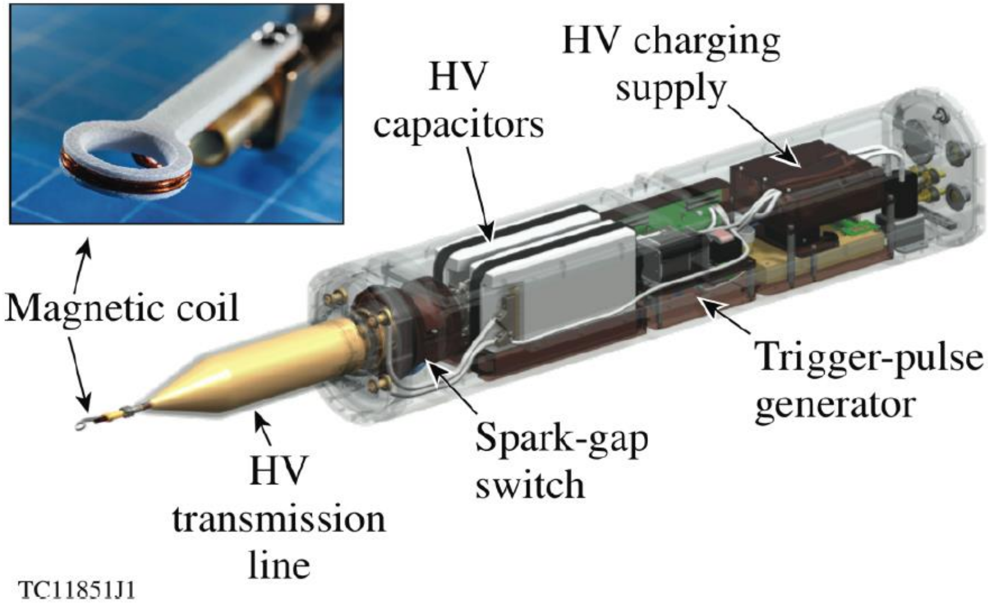
Transmission: $$T \equiv \frac{U_{2+}}{U_{1+}} = 1 + \frac{U_{1-}}{U_{1+}} = 1 + \rho$$

- **For impedance match, i.e., Z_{01}=Z_{02}:** $\rho = 0$ $\quad$ $T = 1$
  - **Reflection-free junction: Z_{01}=Z_{02} is necessary but not sufficient.**
- **If the geometry of a line changes arbitrary, it becomes impossible to satisfy Maxell's equation just by superposing the fundamental waves.**

Smooth transition is required

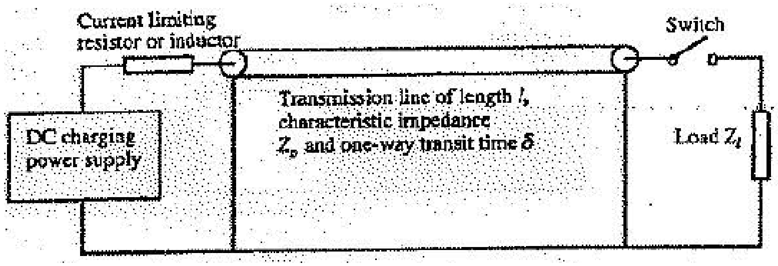
- **Only by a smooth transition can we achieve the condition that the fields are not disturbed too much so that the reflections can be avoid for high frequency.**





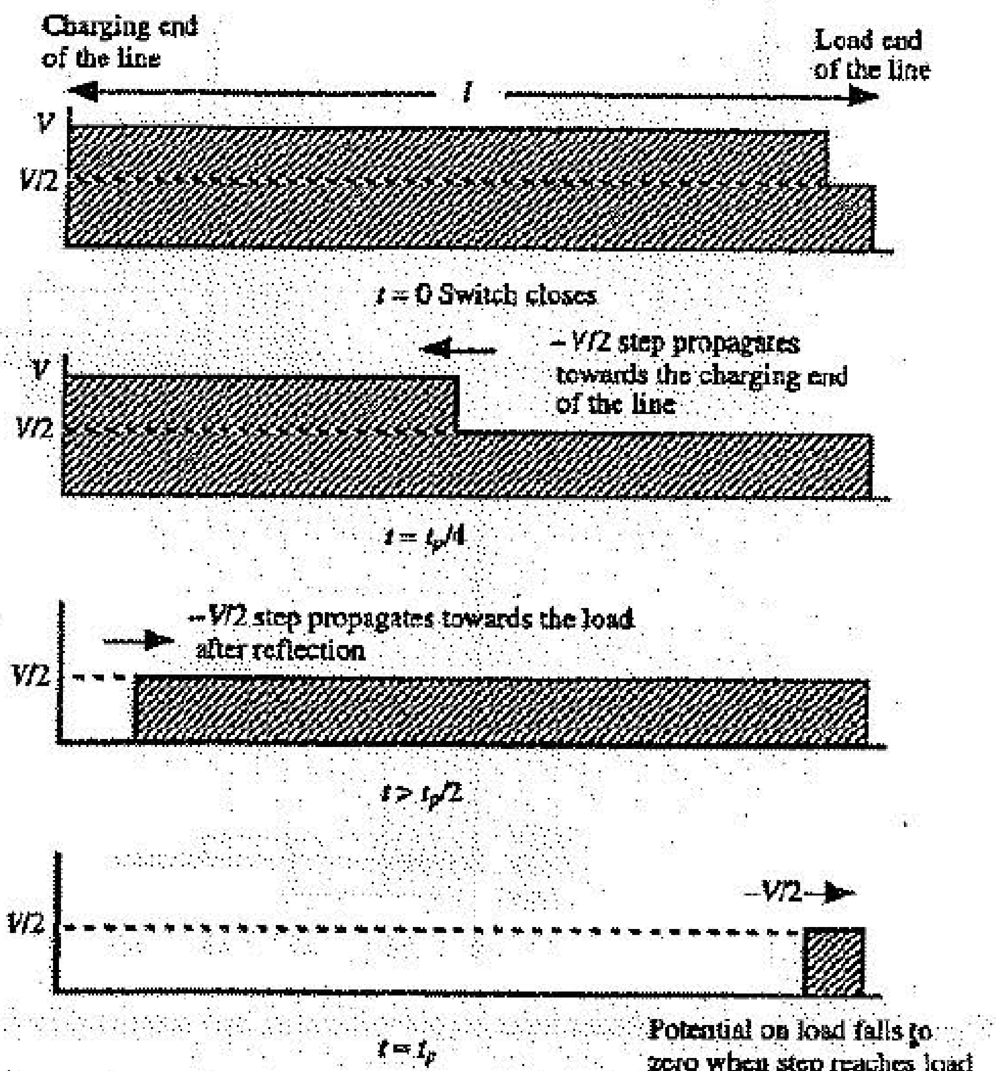
G. Fiksel, etc., Rev. Sci. Instrum. 86, 016105 (2015)

Only half of charged voltage is provided in a basic pulse forming line (PFL)

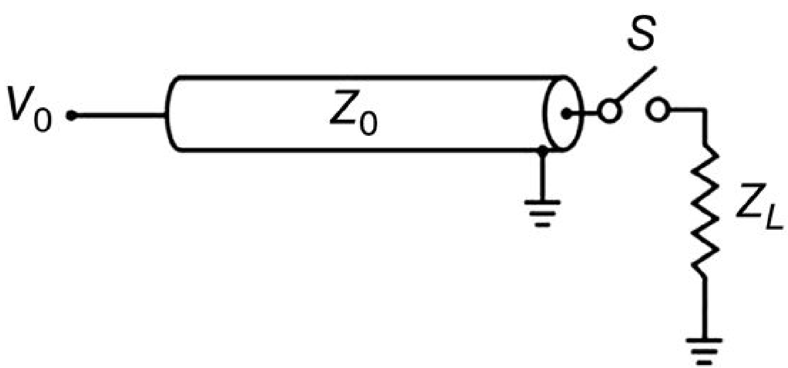


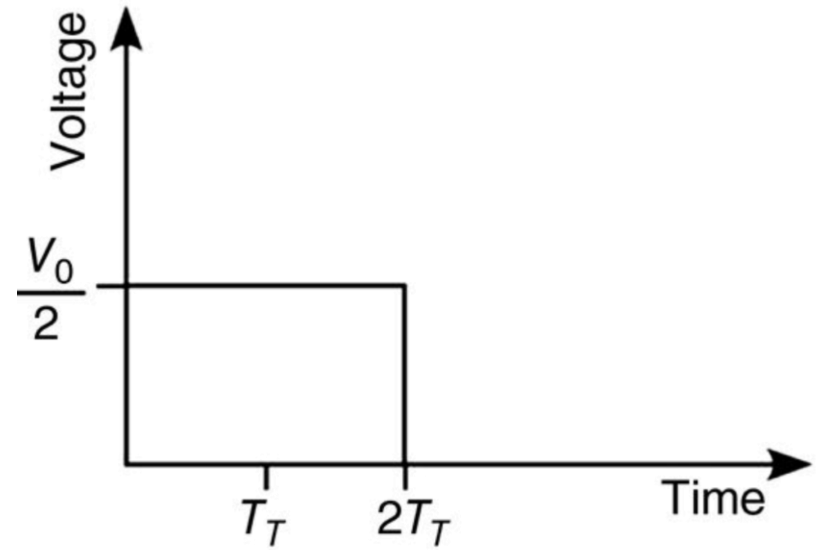
$$t_p = \frac{2l}{v_p} = 2\delta$$

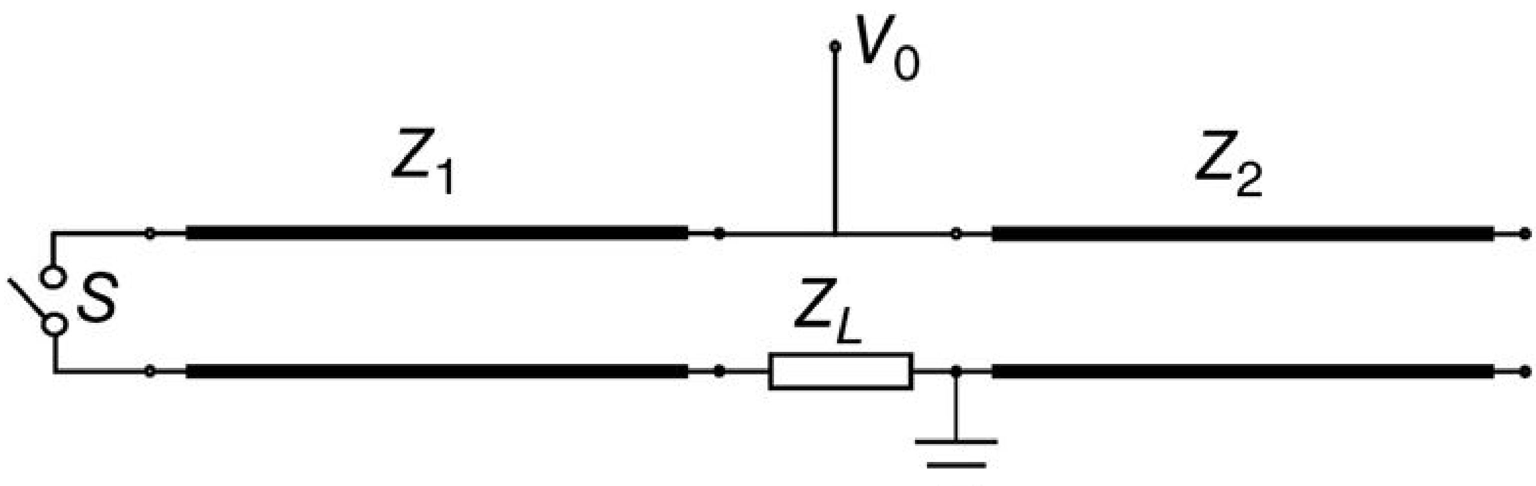
$$V_L = V\frac{Z_L}{Z_0 + Z_L}$$



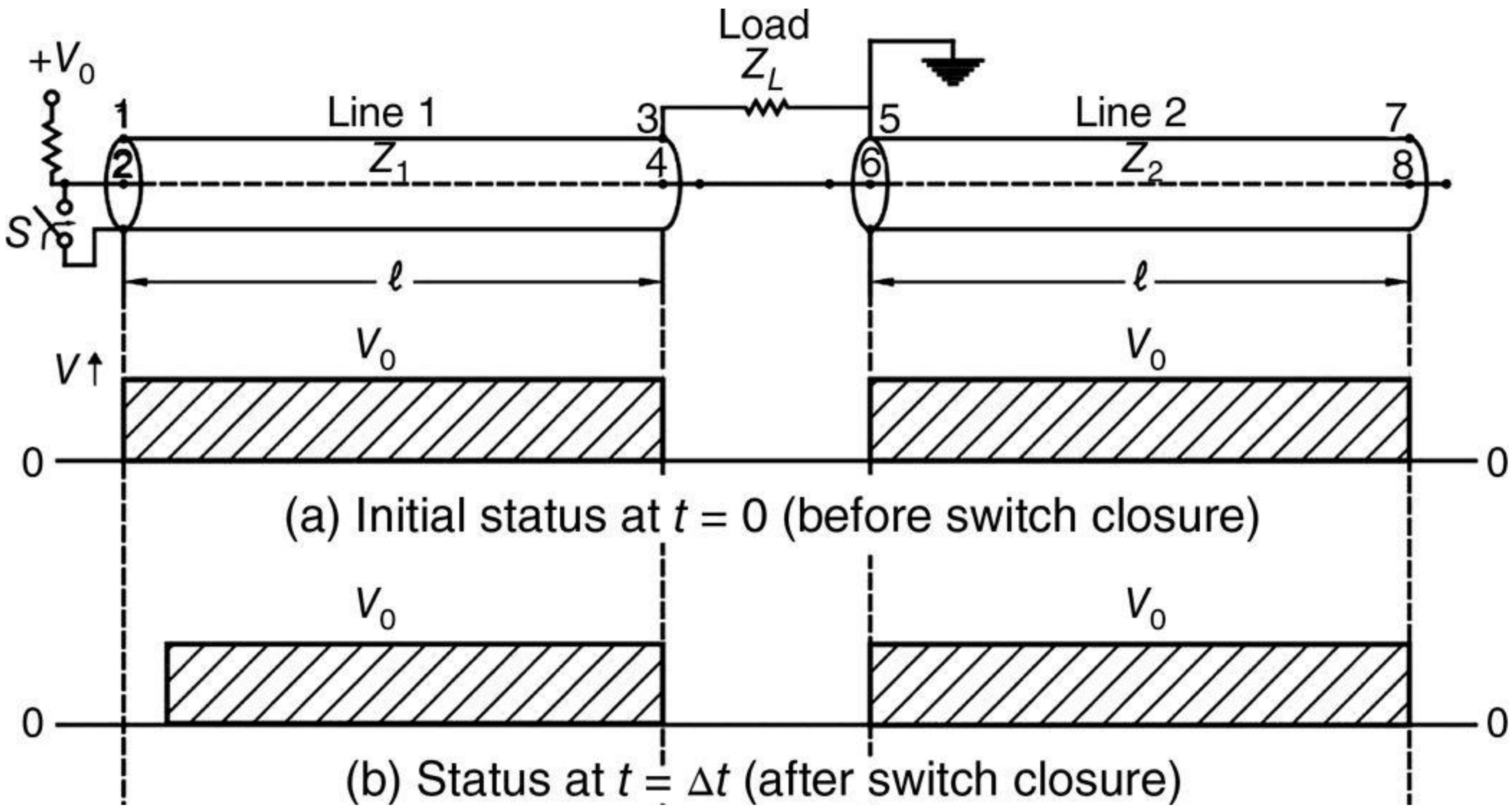
Blumlein pulse forming line (PFL)





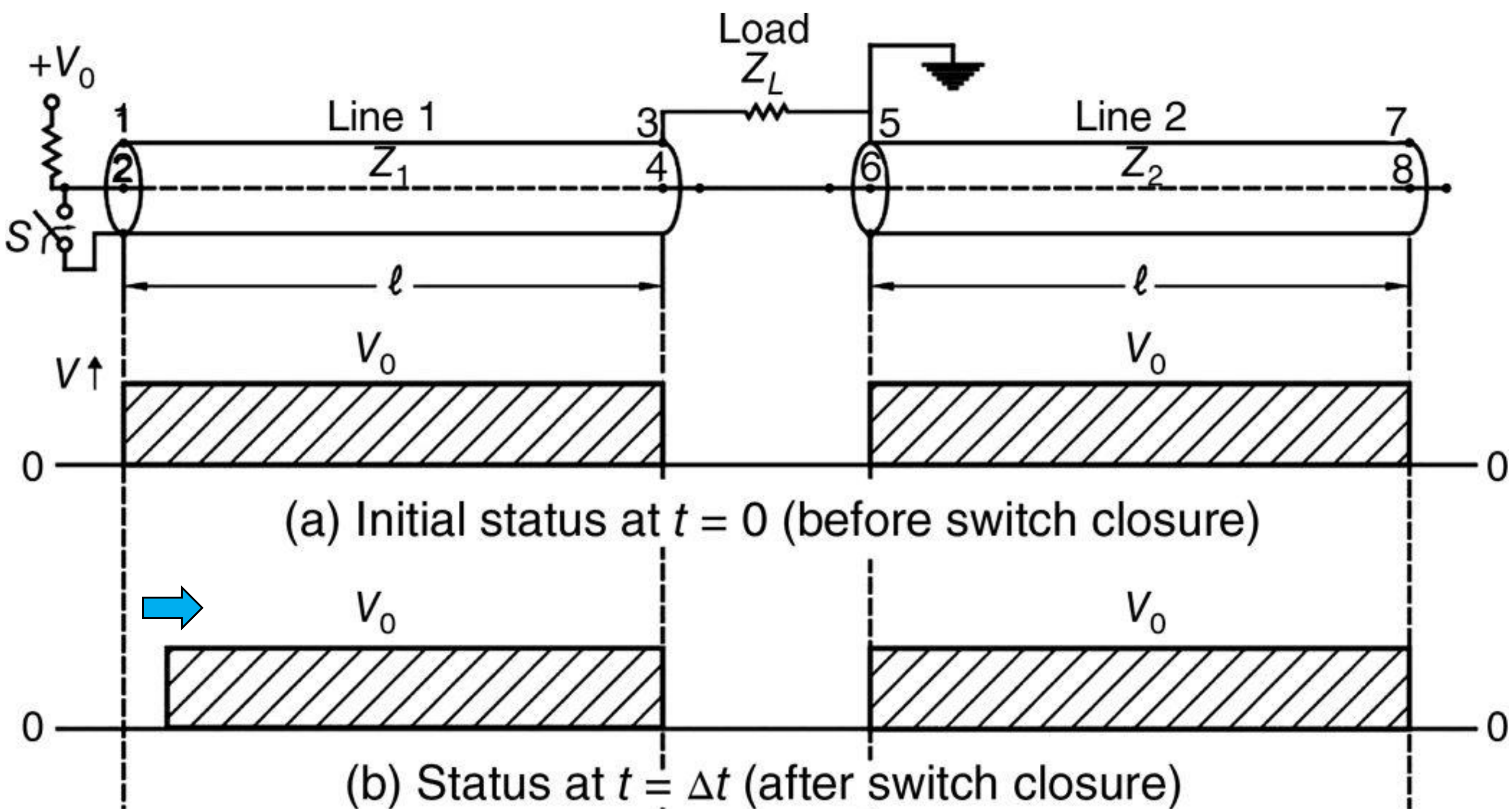


Sequence of Blumlein line



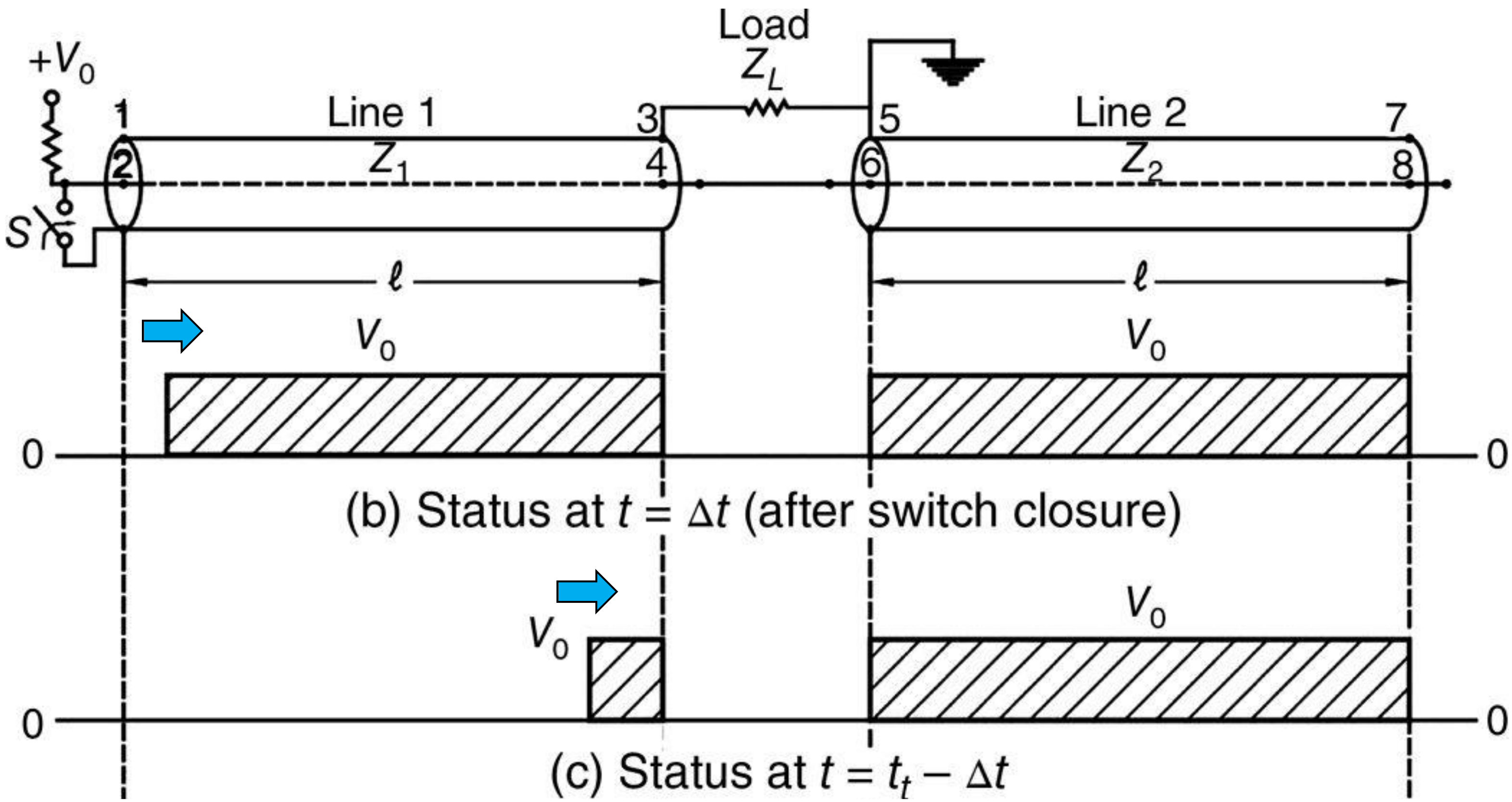
(a)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	V_0	V_0	0	V_0	V_0	0	V_0	V_0	0	0	V_0	V_0

Sequence of Blumlein line



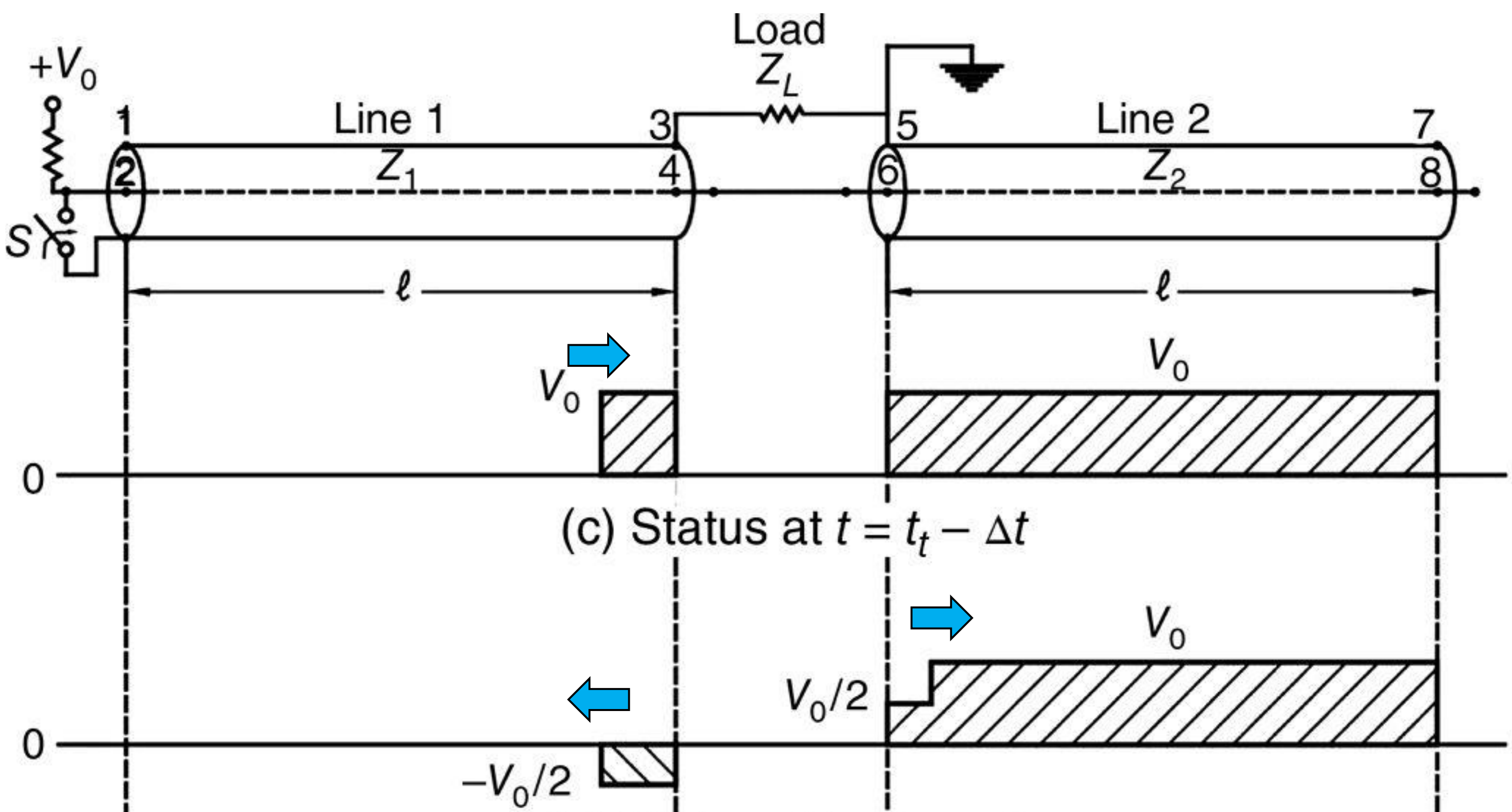
(b)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	0	V_0	V_0	0	V_0	V_0	0	0	V_0	V_0

Sequence of Blumlein line



(c)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	0	V_0	V_0	0	V_0	V_0	0	0	V_0	V_0

Sequence of Blumlein line

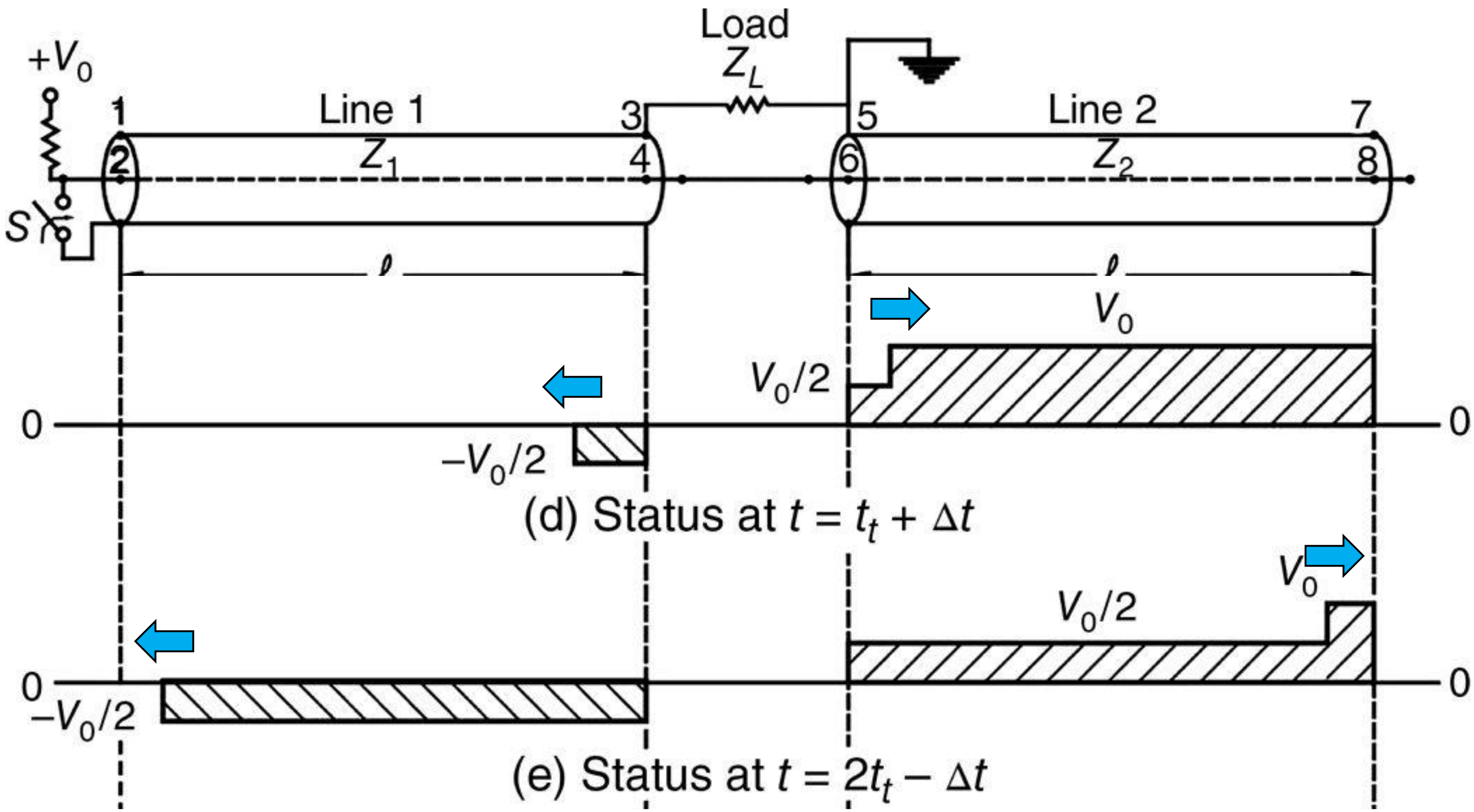


(c) Status at $t = t_t - \Delta t$

(d) Status at $t = t_t + \Delta t$

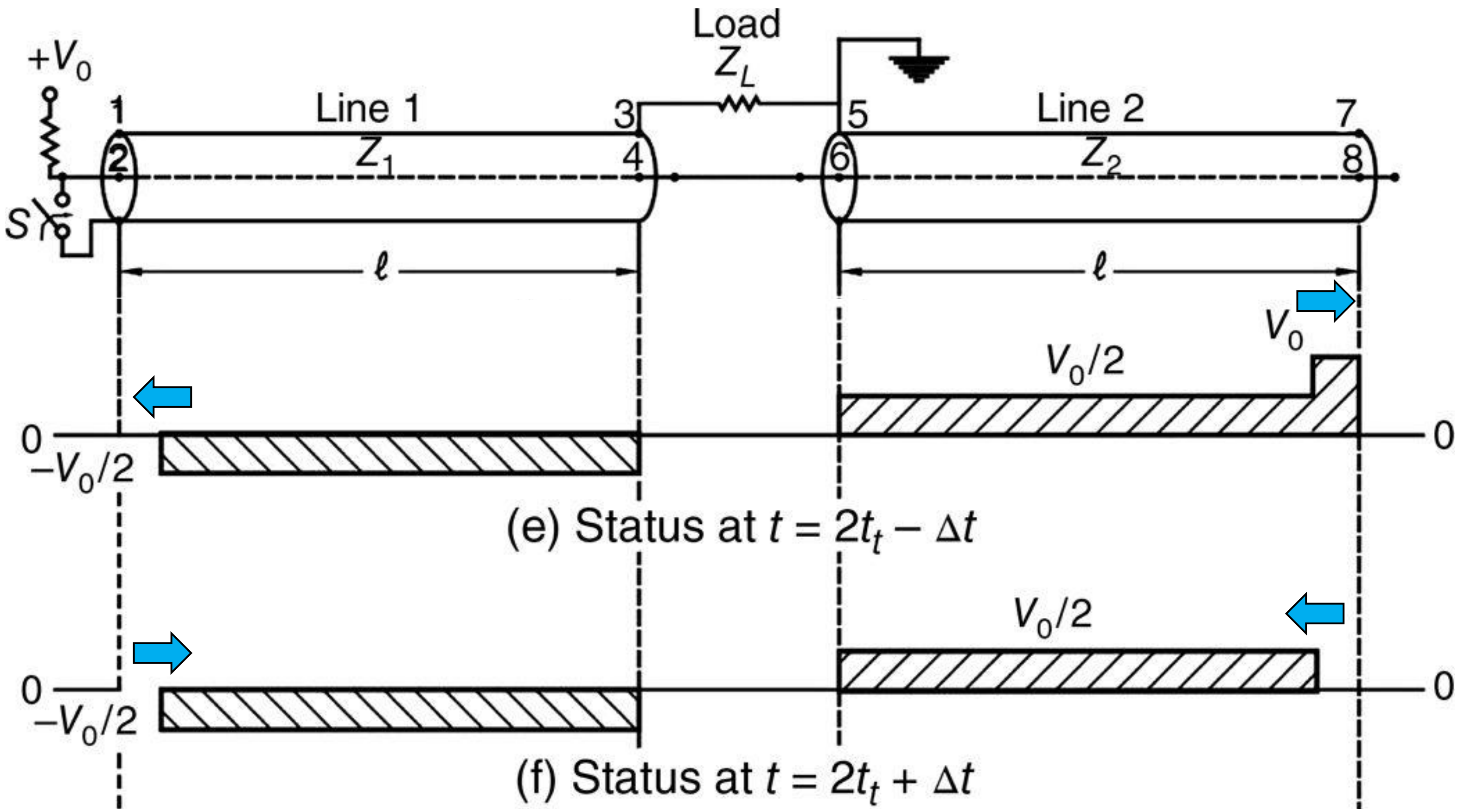
(d)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	V_0	$V_0/2$	$-V_0/2$	0	$V_0/2$	$V_0/2$	V_0	0	V_0	V_0

Sequence of Blumlein line



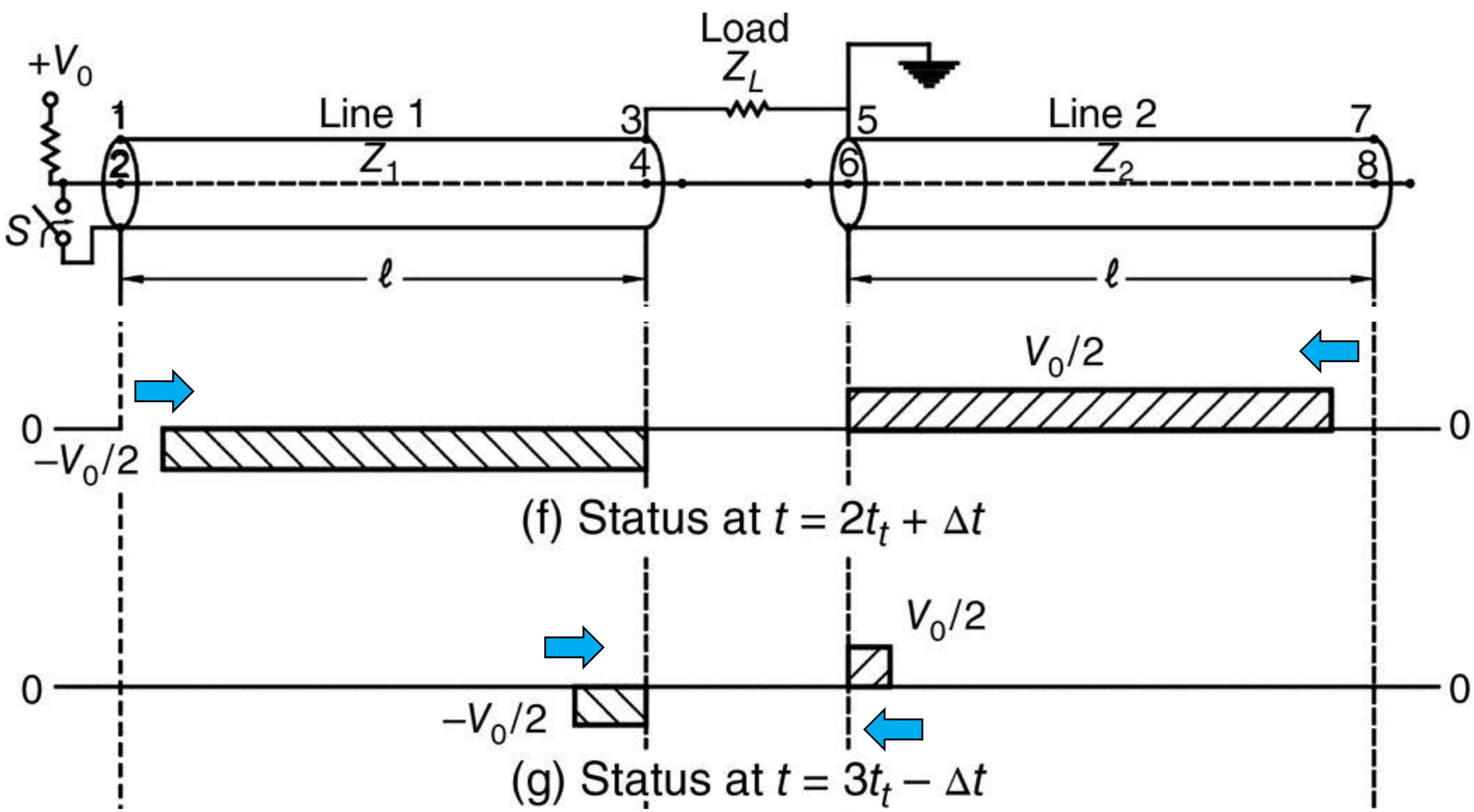
(e)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	V_0	$V_0/2$	$-V_0/2$	0	$V_0/2$	$V_0/2$	V_0	0	V_0	V_0

Sequence of Blumlein line



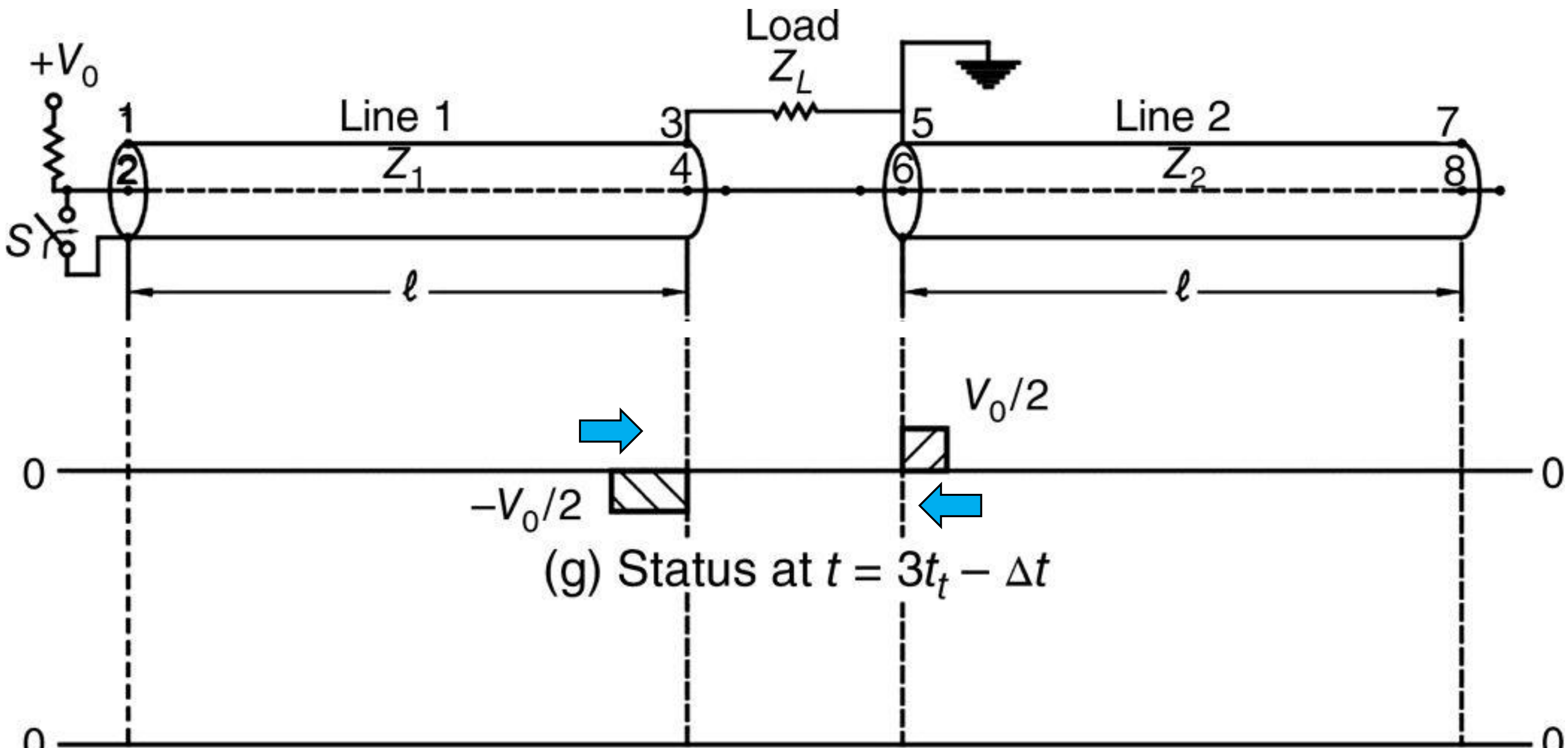
(f)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	V_0	$V_0/2$	$-V_0/2$	0	$V_0/2$	$V_0/2$	V_0	0	0	0

Sequence of Blumlein line



(g)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	V_0	$V_0/2$	$-V_0/2$	0	$V_0/2$	$V_0/2$	V_0	0	0	0

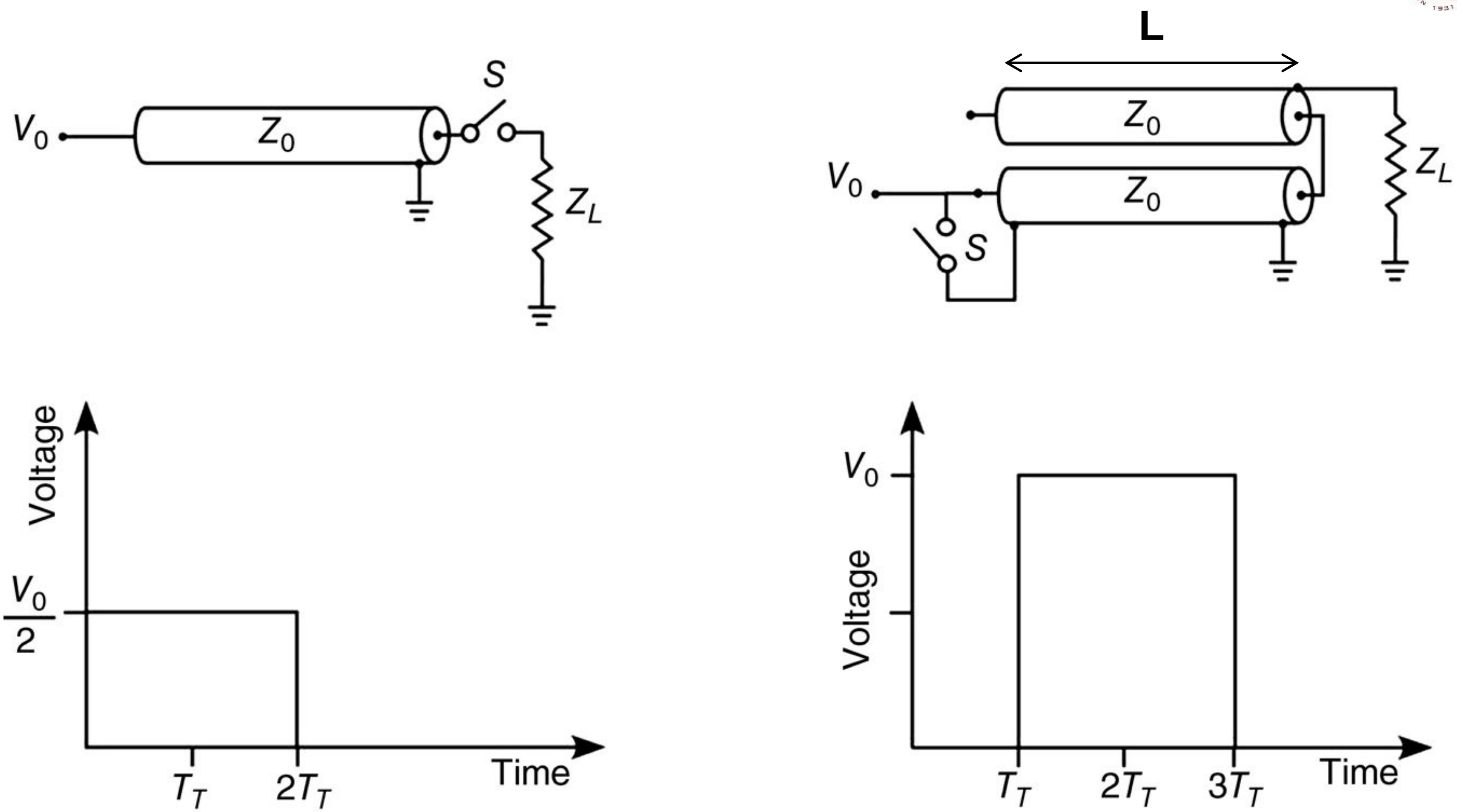
Sequence of Blumlein line



(h) Status at $t = 3t_t$

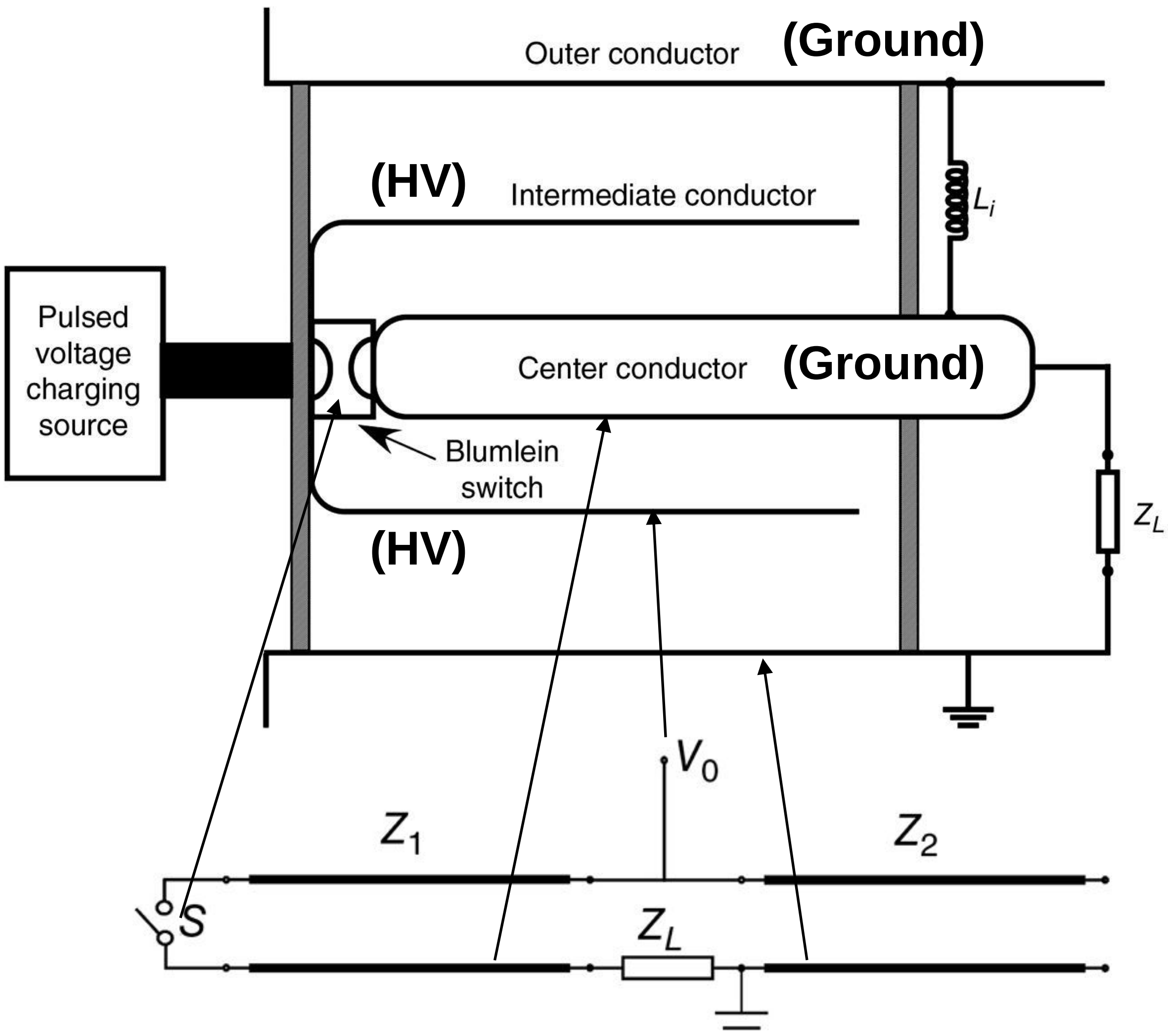
(h)	V_1	V_2	ΔV_{21}	V_3	V_4	ΔV_{43}	V_5	V_6	ΔV_{65}	V_L	V_7	V_8	ΔV_{87}
	0	0	0	0	0	0	0	0	0	0	0	0	0

A Blumlein line can be built by using two coaxial transmission line



- **Example: RG58 coaxial cable, 50 Ω, V_{signal}~2×10^{10} cm/s => L=10 cm, Δt=1 ns.**

Coaxial Blumlein line

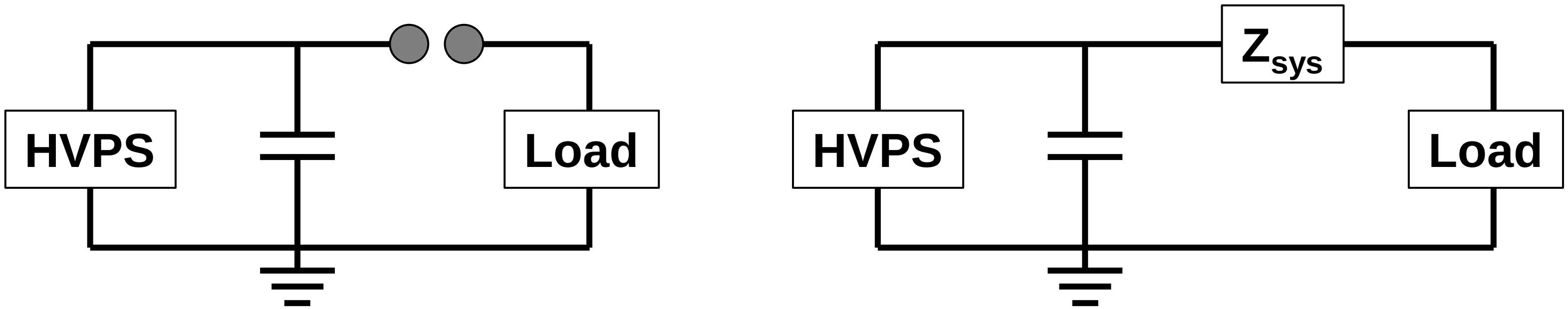


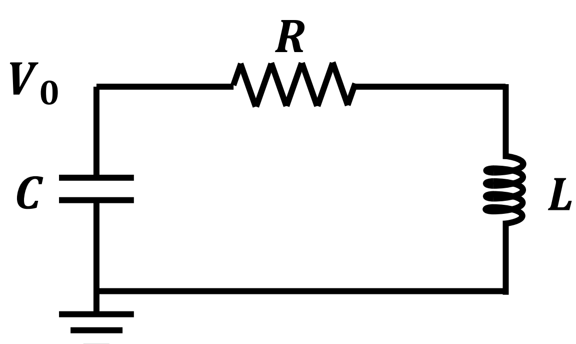
Outlines

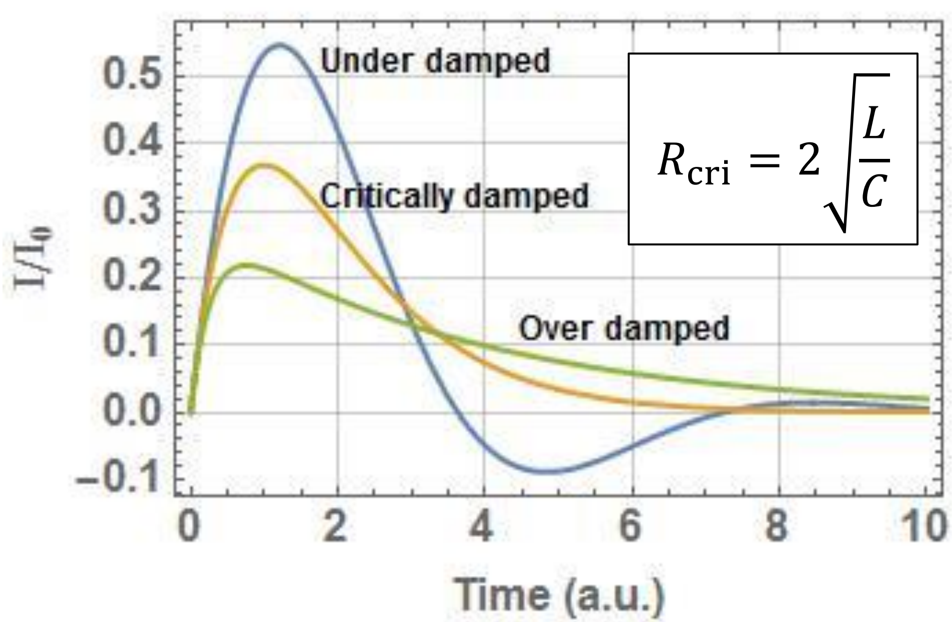
- Switches
  - Closing switches: the switching process is associated with voltage breakdown across an initially insulant element.
  - Opening switches: the switching process is associated with a sudden growth of its impedance.
- **Pulse-forming lines**
  - Blumlein line
  - **Pulse-forming network**
  - Pulse compressor
- Pulse transmission and transformation

A simple pulsed-power system is a RLC circuit

- **Before discharge**
- **After discharge**

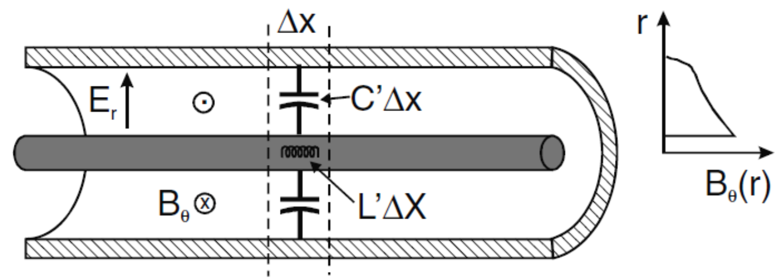


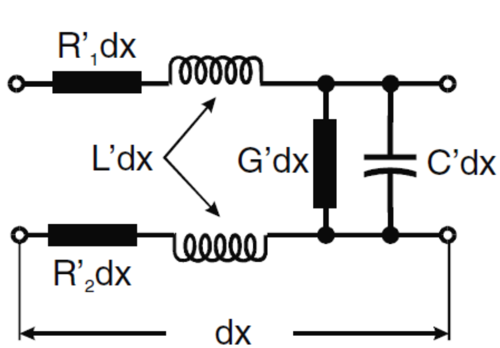


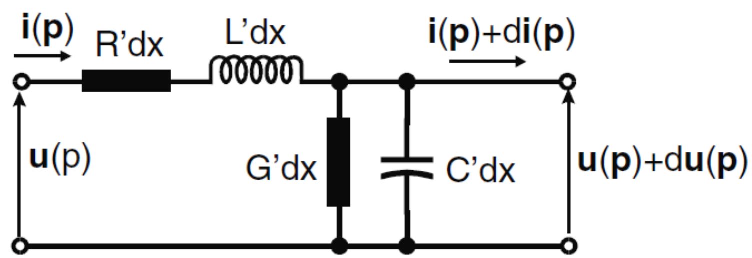


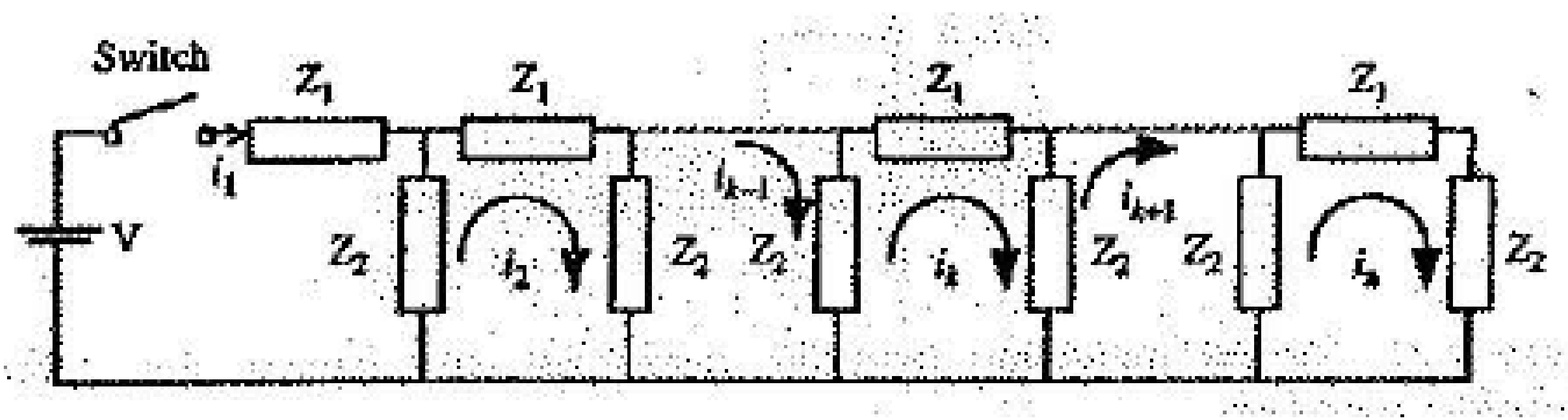
- **How can we generate a square current pulse?**

Pulse-forming network (PFN)

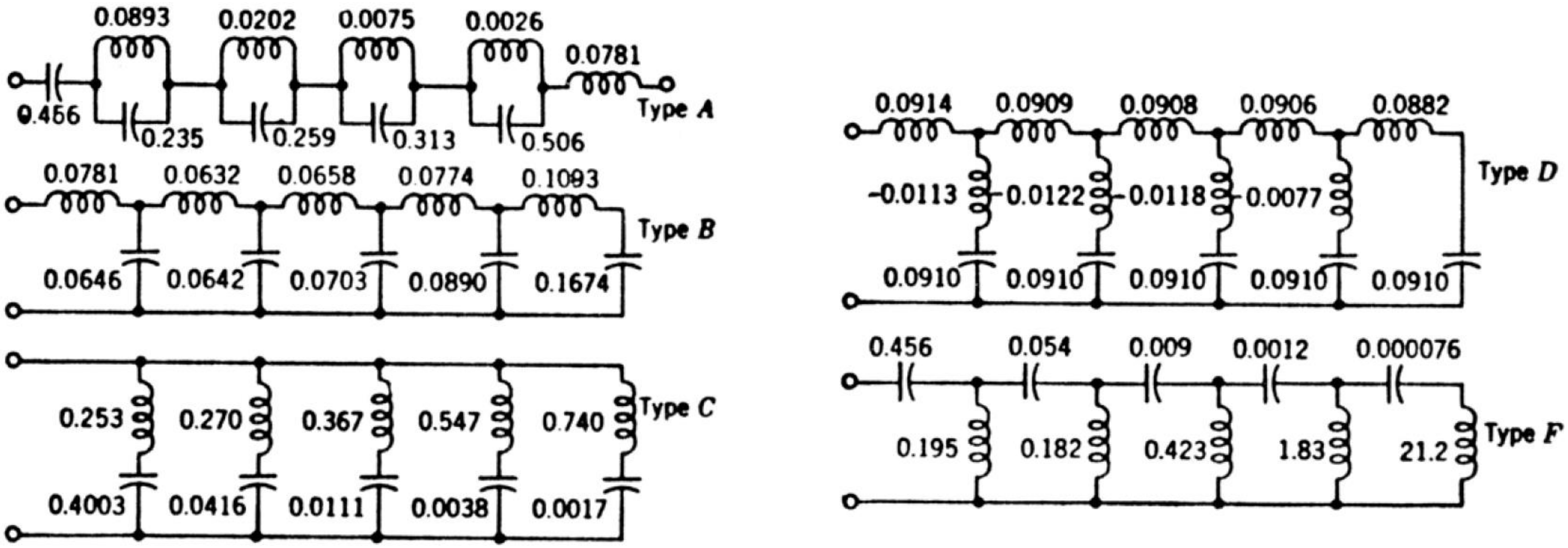








Equivalent Guillemin Networks



Pule-forming LC chain

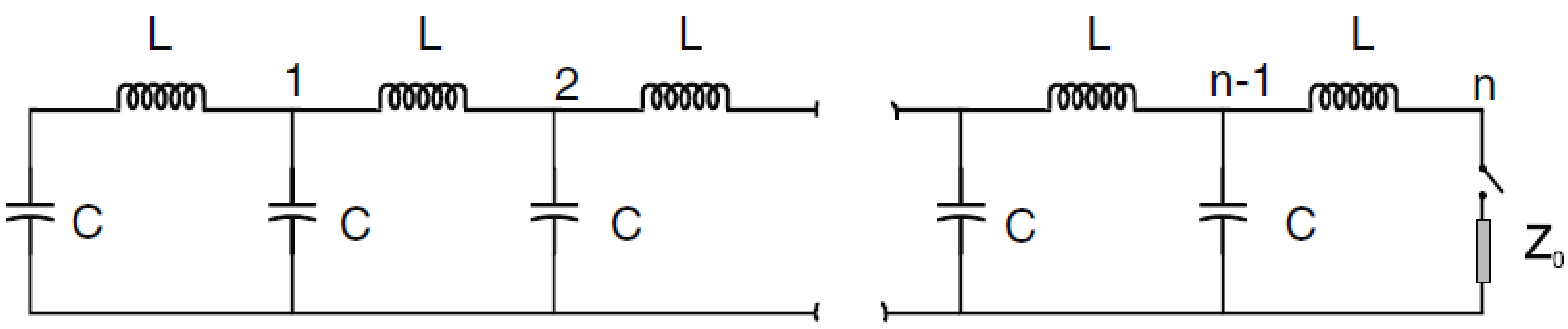
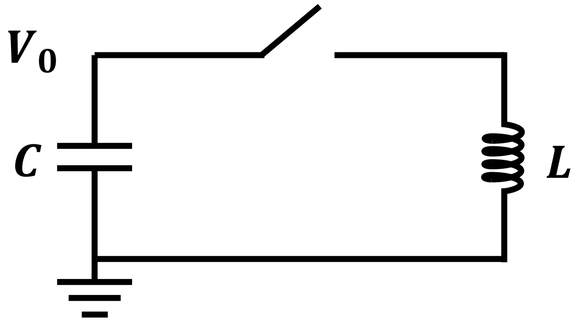


Fig. 5.11. Pulse-forming LC chain

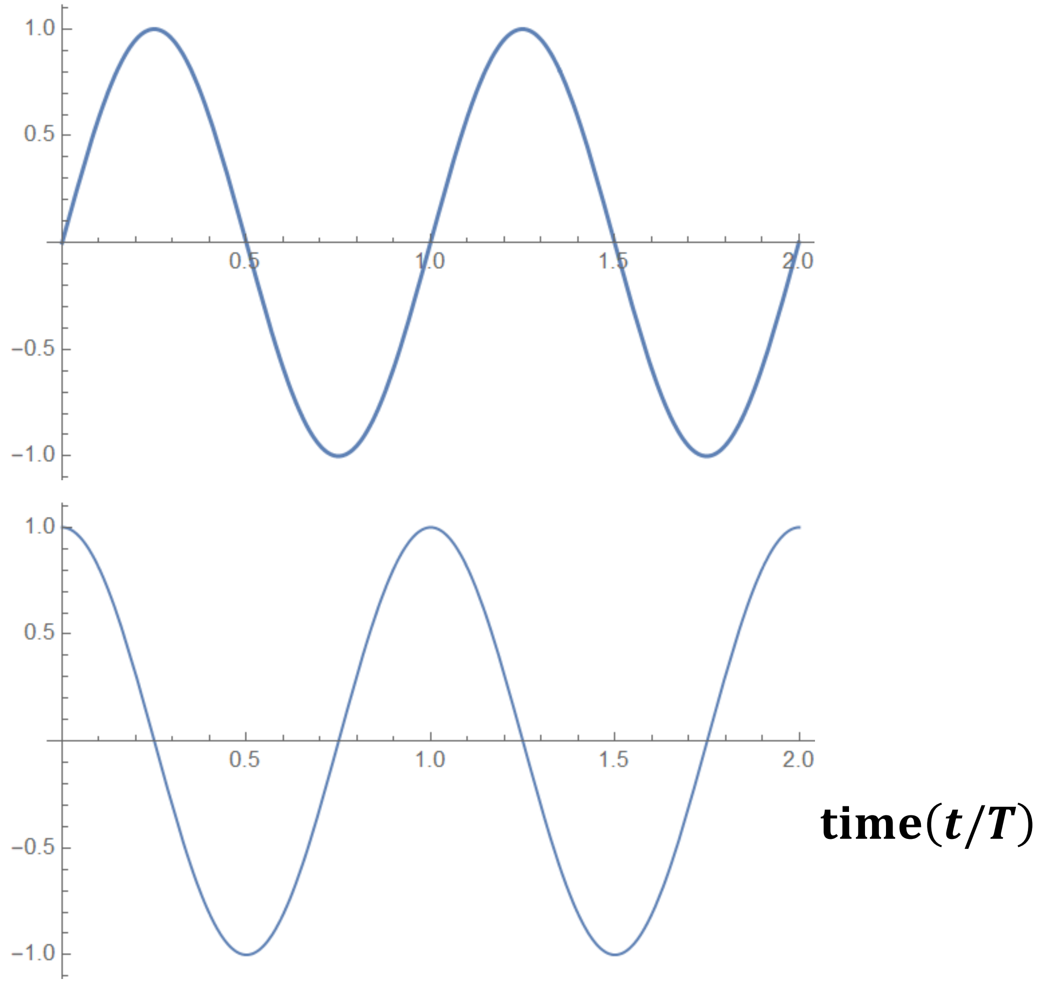
The current output of a LC circuit is a basis of Fourier series



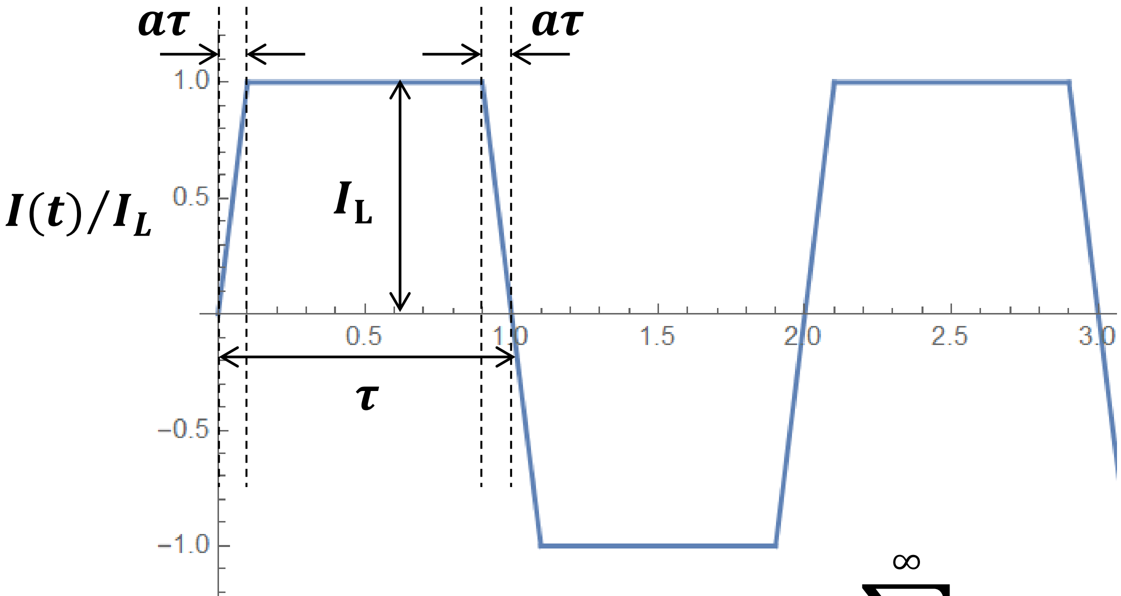
$$I(t) = V_0\sqrt{\frac{C}{L}}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$V(t) = V_0\cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$Z = \sqrt{\frac{L}{C}} \qquad \omega = \frac{1}{\sqrt{LC}}$$



A trapezoidal wave can be expressed by Fourier series (Guillemin's method)



$$i(t) = I_L \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{\tau}\right)$$

$$\frac{i(t)}{I_L} = \frac{t}{a\tau} \qquad , 0 \leq t \leq a\tau$$

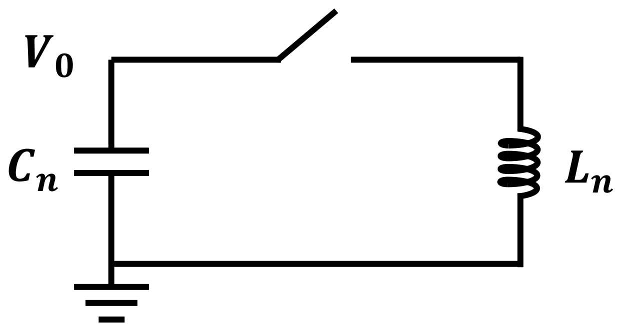
$$\frac{i(t)}{I_L} = 1 \qquad , a\tau \leq t \leq \tau - a\tau$$

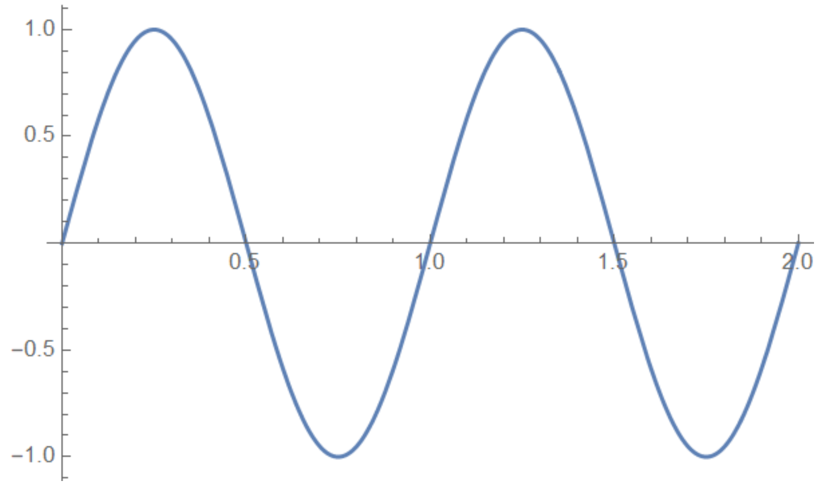
$$\frac{i(t)}{I_L} = \frac{\tau - t}{a\tau} \qquad , \tau - a\tau \leq t \leq \tau$$

$$\text{where } b_n = \frac{2}{\tau}\int_0^{\tau} \frac{i(t)}{I_L} \sin\left(\frac{n\pi t}{\tau}\right) \mathrm{d}t$$

$$b_n = \frac{4}{n\pi}\frac{\sin(n\pi a)}{n\pi a}, \text{where } n = 1, 3, 5 \ldots$$

The required inductance and capacitance are obtained by comparing LC output with the Fourier series





$$I_n(t) = V_0\sqrt{\frac{C_n}{L_n}}\sin\left(\frac{t}{\sqrt{L_nC_n}}\right)$$

$$L_n = \frac{Z_n\tau}{n\pi b_n} = \frac{V}{I_L}\frac{\tau}{n\pi b_n}$$

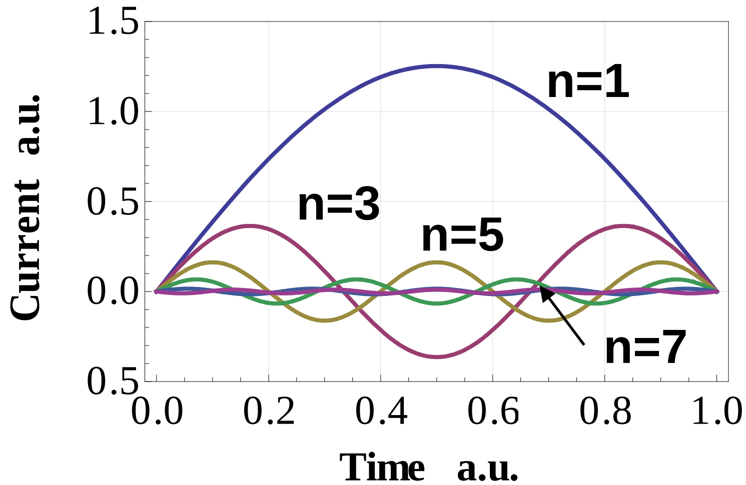
$$i(t) = I_L\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{\tau}\right)$$

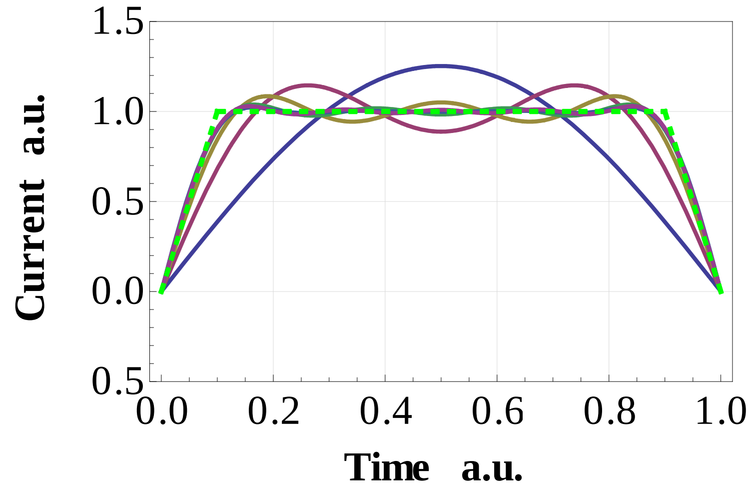
$$C_n = \frac{\tau b_n}{n\pi Z_n} = \frac{I_L}{V}\frac{\tau b_n}{n\pi}$$

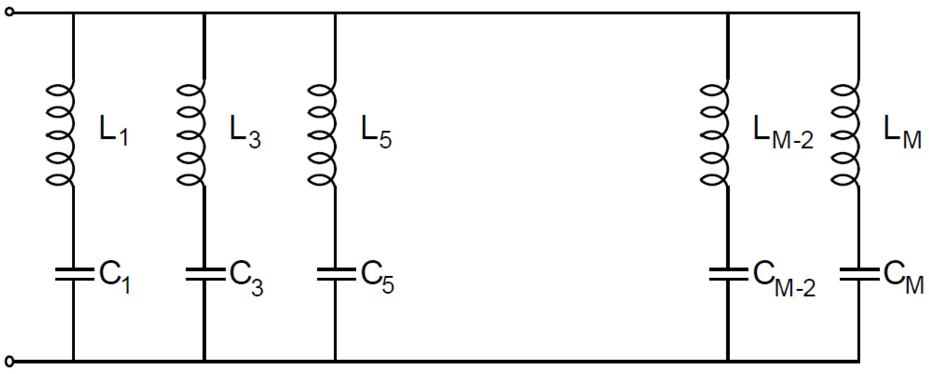
$$Z_n = \frac{V}{I_L}$$

$$b_n = \frac{4}{n\pi}\frac{\sin(n\pi a)}{n\pi a}, \text{where } n = 1, 3, 5\ldots$$

A trapezoidal current output can be generated using Guillemin's pulse-forming networks





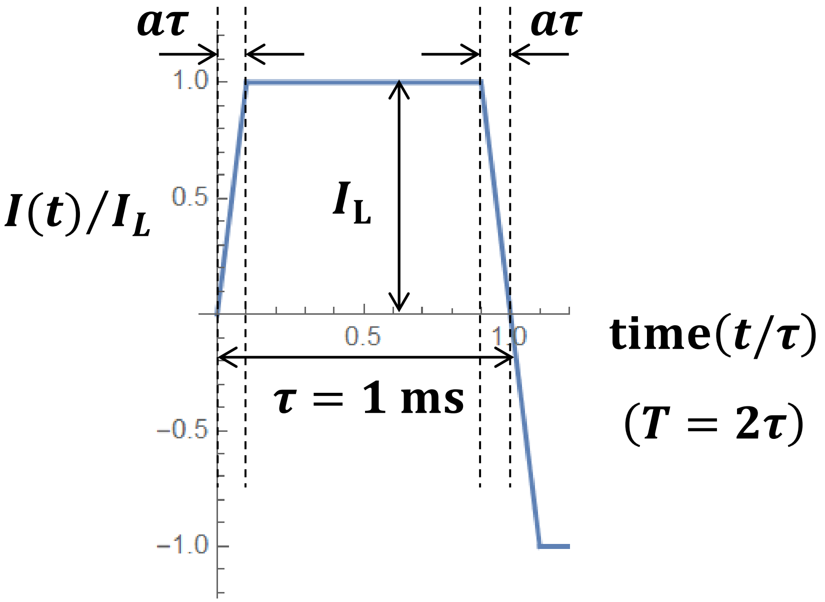


$$I(t) = I_L \Sigma b_n \mathrm{Sin}\left(\frac{n\pi t}{\tau}\right)$$

$$b_n = \frac{4}{n\pi}\frac{\mathrm{Sin}(n\pi a)}{n\pi a}$$

$$L_n = \frac{Z\tau}{n\pi b_n} \quad C_n = \frac{\tau b_n}{n\pi Z} \quad Z = \frac{V}{I_L}$$

Fourier components of τ=1 ms, a=0.1



$$i(t) = I_L \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{\tau}\right)$$

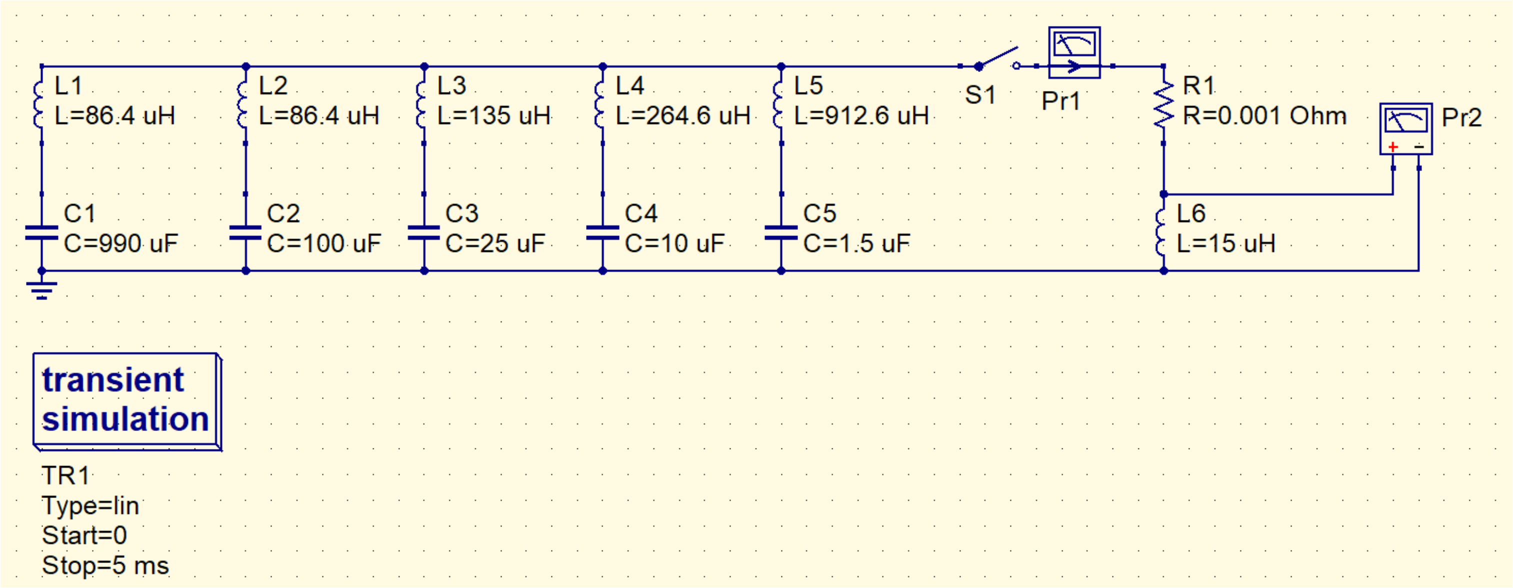
$$b_n = \frac{4}{n\pi}\,\frac{\sin(n\pi a)}{n\pi a}, \text{where } n = 1, 3, 5\ldots$$

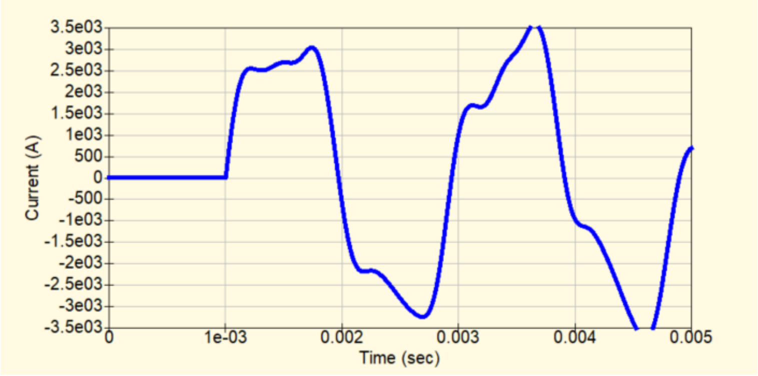
n	#I
b1	1.2524
b3	0.3643
b5	0.1621
b7	0.069
b9	0.0155

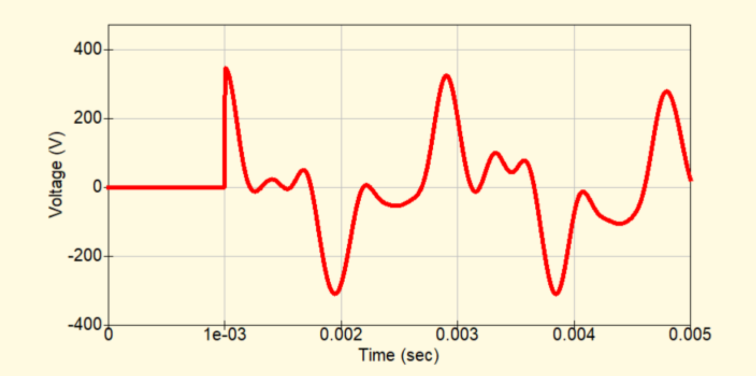
Coils with 8 turns and a PFN charged to 1 kV will be used

I (kA)	V (kV)		1	2	3	4	5	E (kJ)	% to 100 J
20	2	L(uH)	25.4	26.1	39.3	68.0	228.7	9.0	1.1 %
		C(uF)	3986.5	386.5	103.2	30.4	5.5		
20	1	L(uH)	12.7	14.6	19.6	34.0	114.4	4.5	2.2 %
		C(uF)	7973.0	773.1	206.4	60.9	10.9		
2.5	2	L(uH)	203.3	233.0	314.2	543.7	1830.0	1.1	8.9 %
		C(uF)	498.3	48.3	12.9	3.8	0.7		
2.5	1	L(uH)	101.7	116.5	157.1	271.8	915.0	0.6	17.7 %
		C(uF)	996.6	96.6	25.8	7.6	1.4		

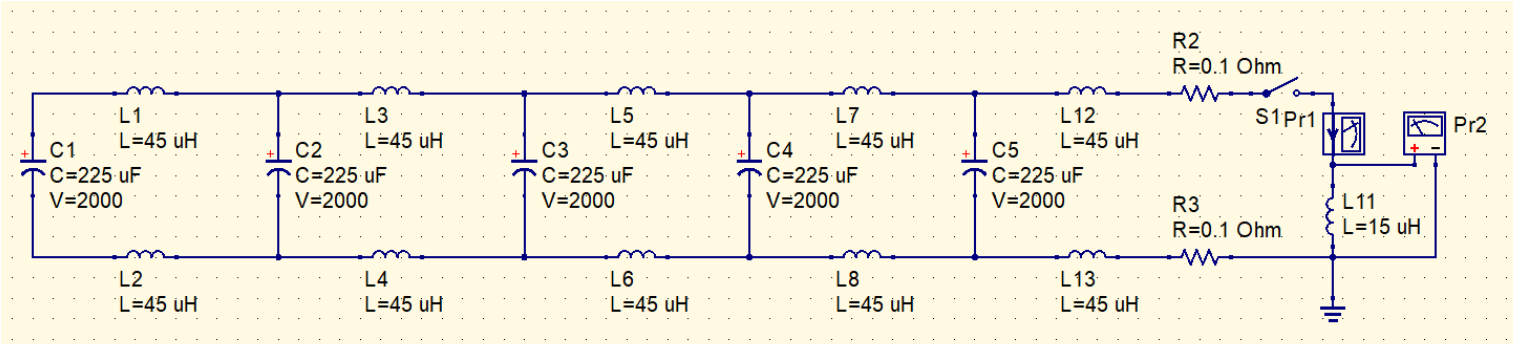
A square pulse with a flat top of 2.5 kA can be generated







A simple PFN with constant C and L in all stages can also be used



$$C \equiv \bar{C} = \frac{1}{N}\sum_{n=1}^{N} C_n = 225\mu\text{F}$$

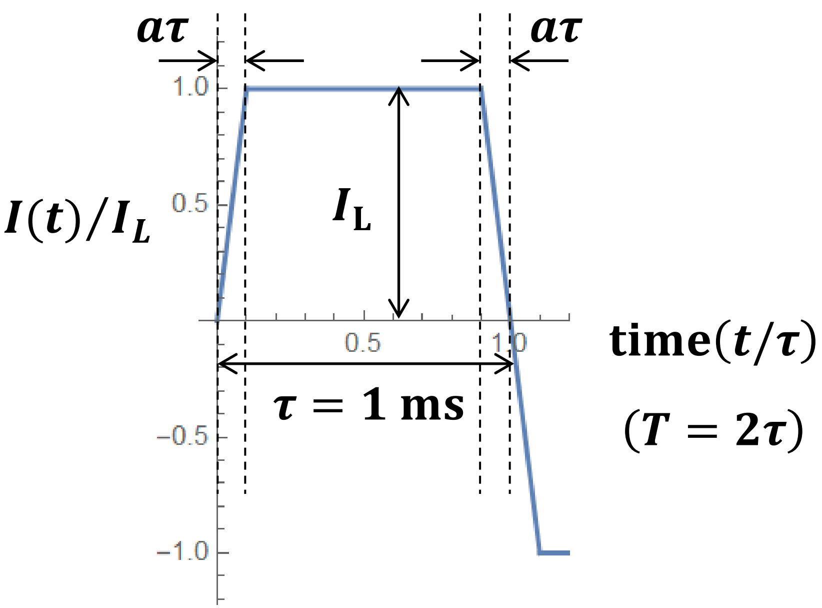
$$L_n = 2nL + L_L \approx 2nL$$

$$\omega_n = \frac{1}{\sqrt{L_n C}} \approx \frac{1}{\sqrt{2nLC}}$$

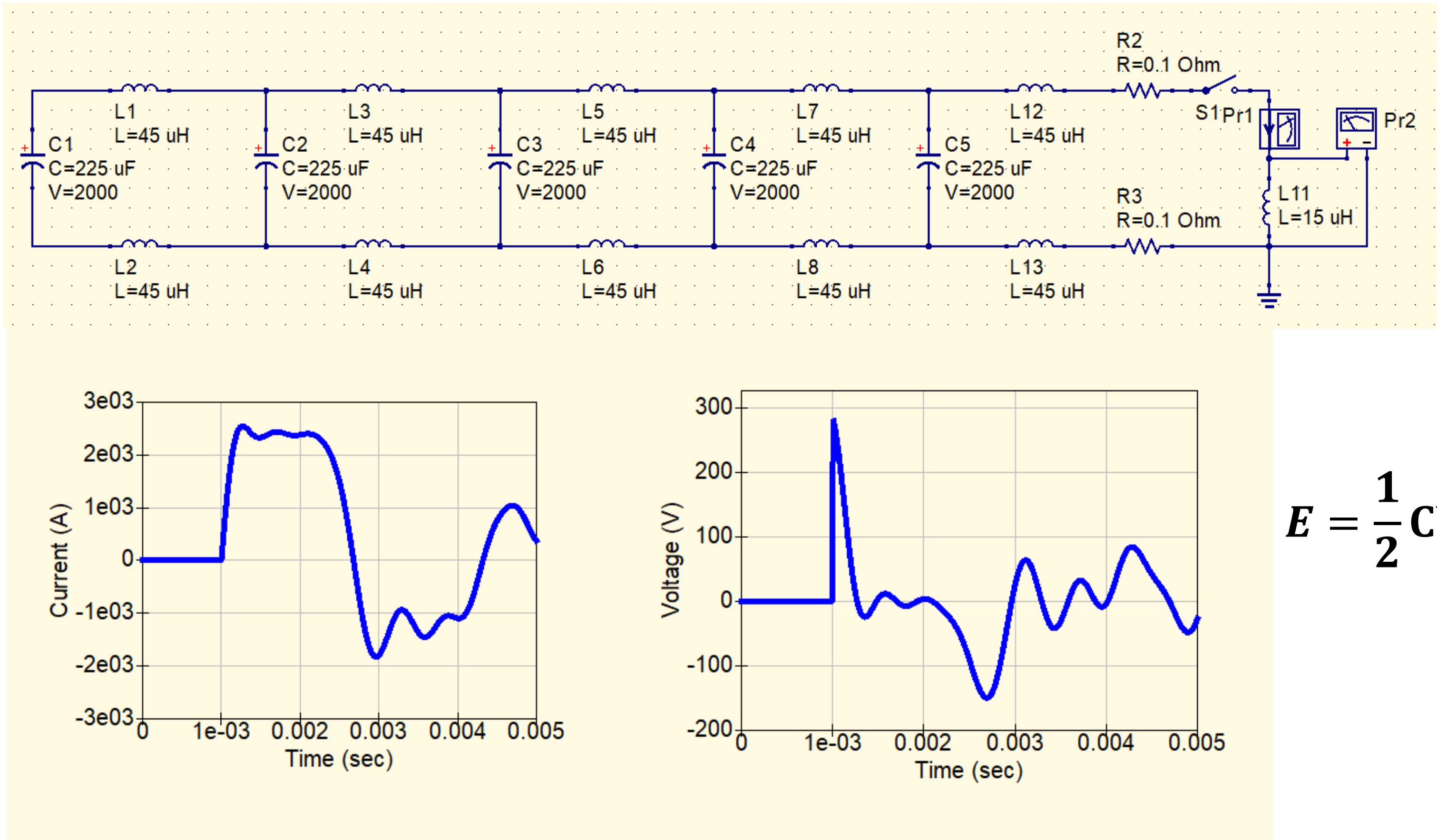
- **For 5 stages:**

$$\omega_5 = \frac{2\pi}{T} = \frac{\pi}{\tau} = \frac{\pi}{1\text{ms}}$$

$$L = 45\mu\text{H}$$



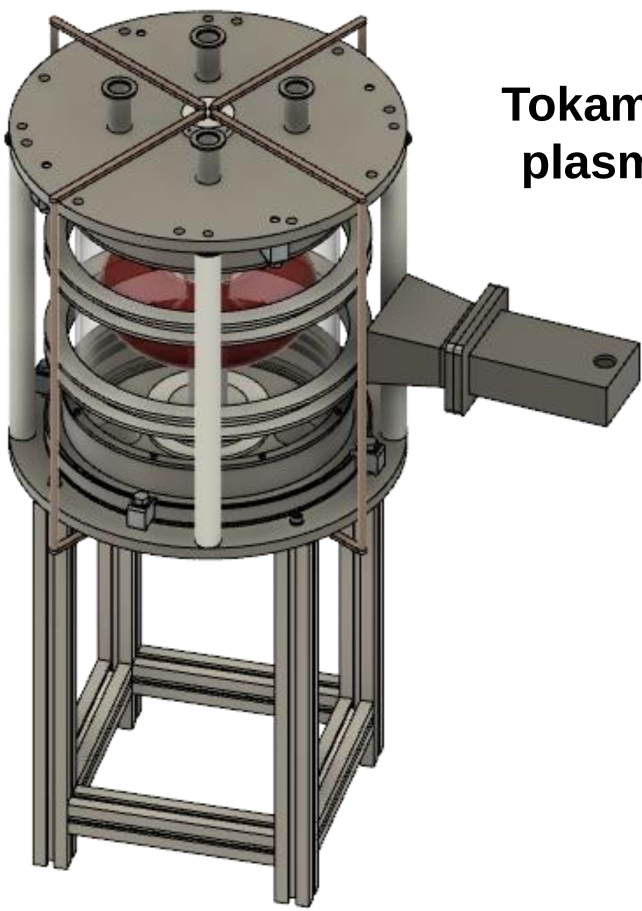
The energy coupling efficiency is lower using the simple PFN



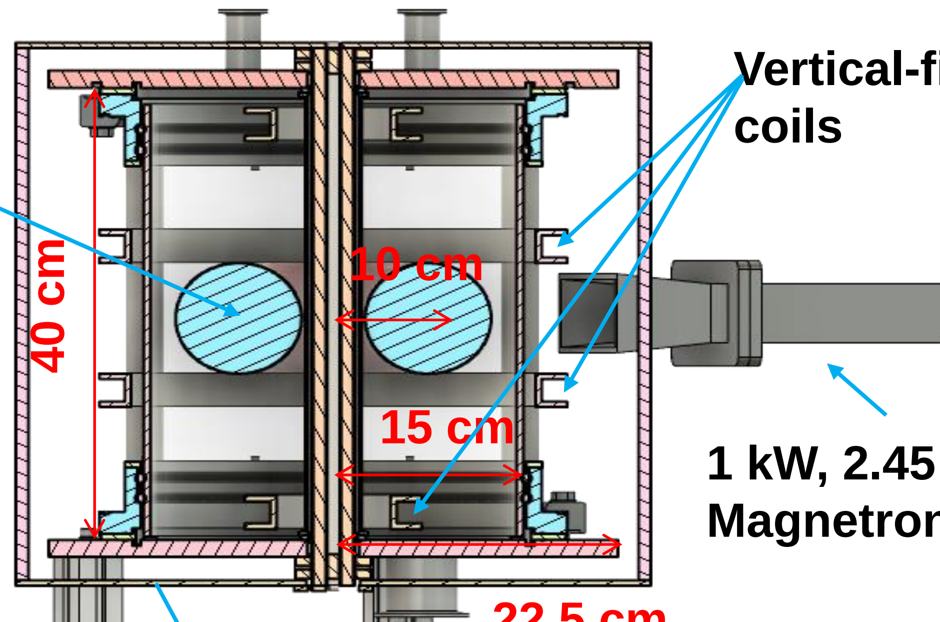
$$E = \frac{1}{2}CV^2 = 2.25\text{kJ}$$

- **Only 4.4 % of the energy is transferred to magnetic energy.**

Mini-spherical tokamak



Tokamak plasma



22.5 cm

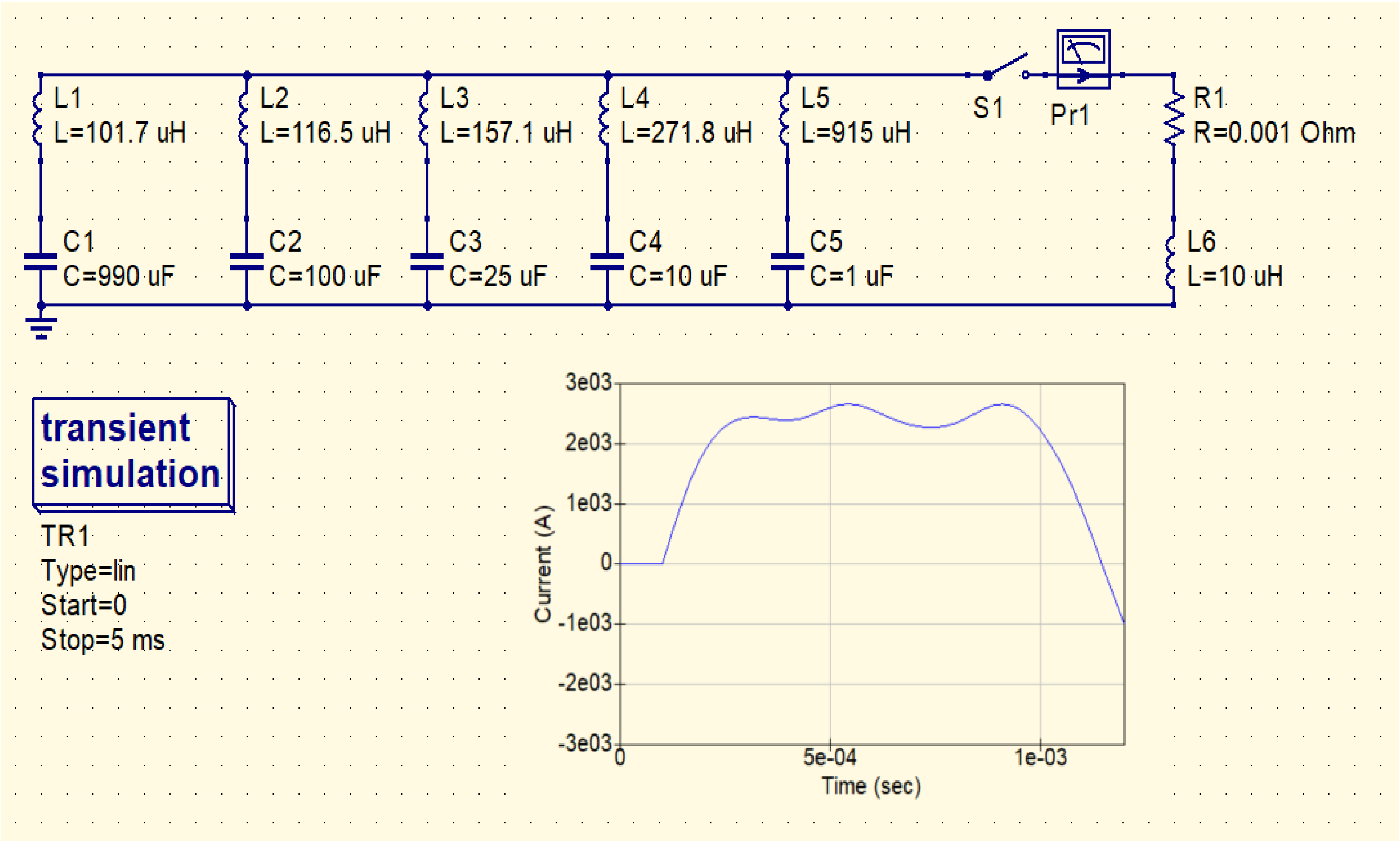
Vertical-field coils

1 kW, 2.45 GHz Magnetron

4 toroidal-field coil connected in series. 1 ms, 2.5 kA pulsed-current.

B=876 G @ 4.6 cm will be used for ECR heating.

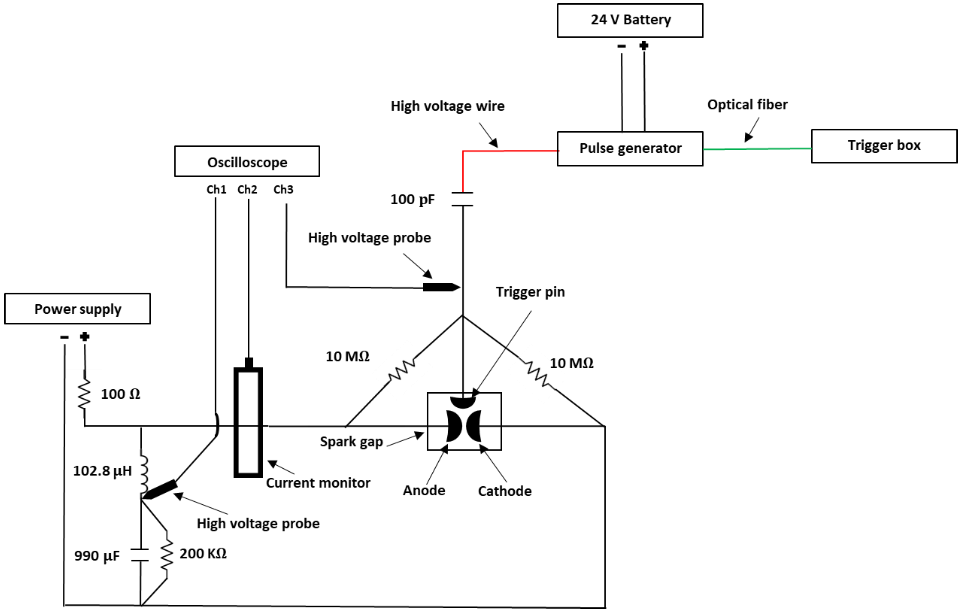
A square pulse of 2.5-kA current output with duration of 1 ms can be provided



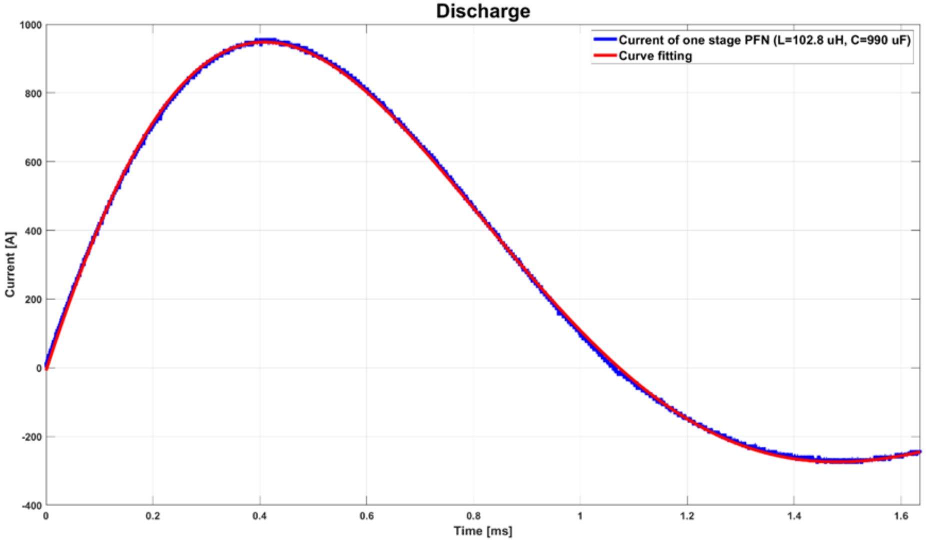
The actual components were determined by what we could get

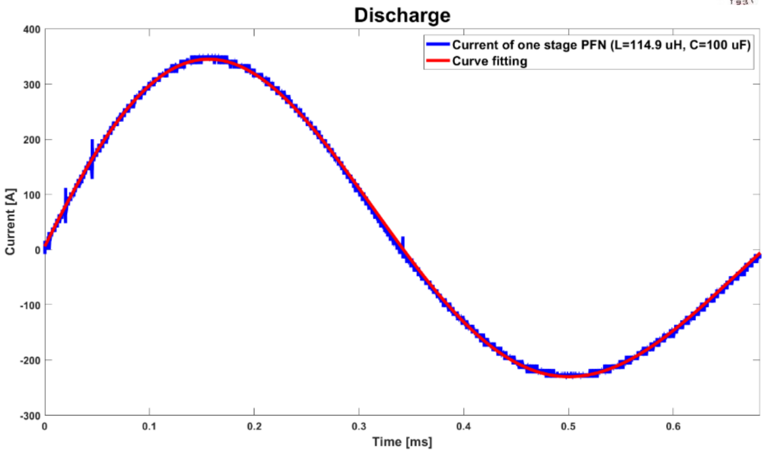
- **Design:**
- **Built:**

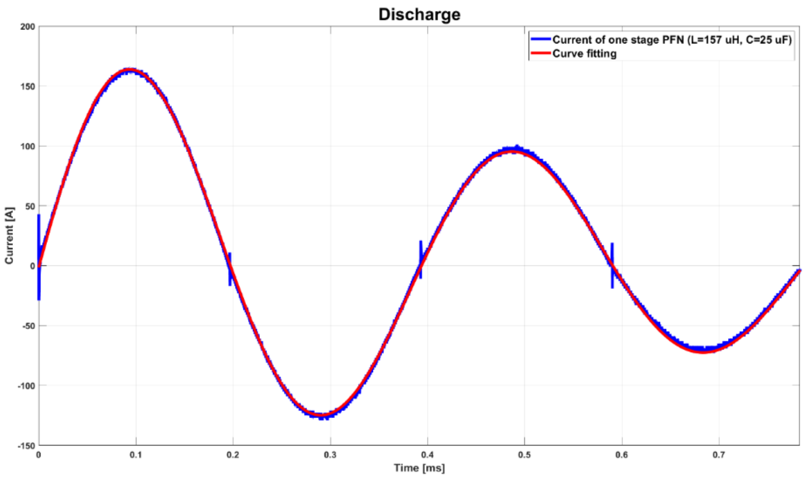
	I (kA)	V (kV)		1	2	3	4	5
Design	2.5	1	L(uH)	101.7	116.5	157.1	271.8	915.0
			C(uF)	996.6	96.6	25.8	7.6	1.4
Built	2.5	1	L(uH)	102.8	114.9	157	270	-
			C(uF)	990	100	25	10	-

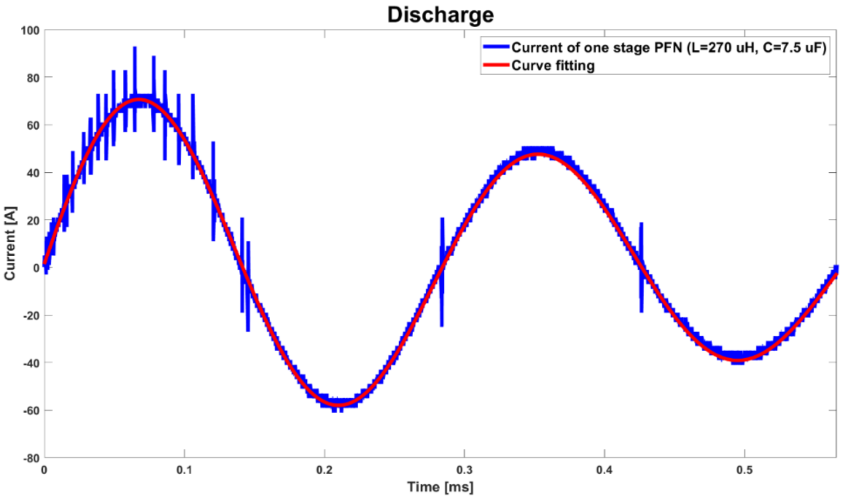


Discharge current measurements

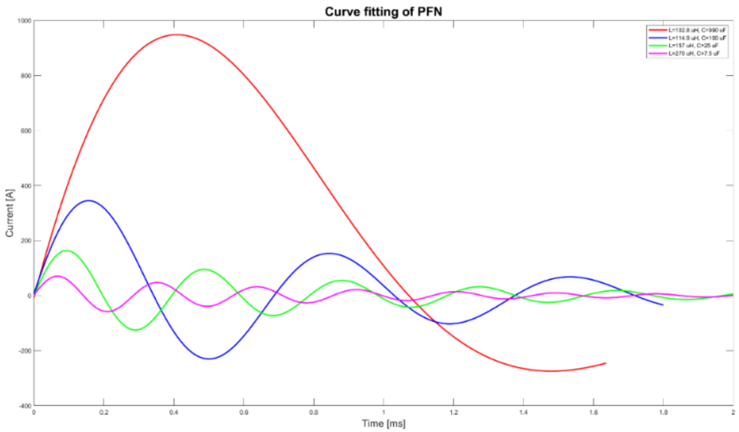




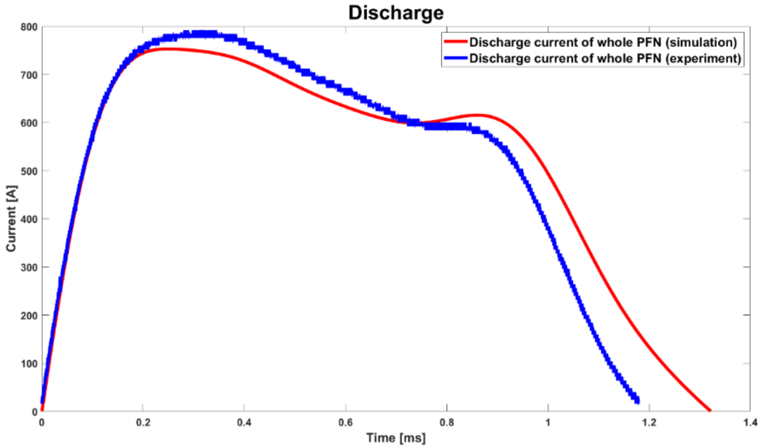


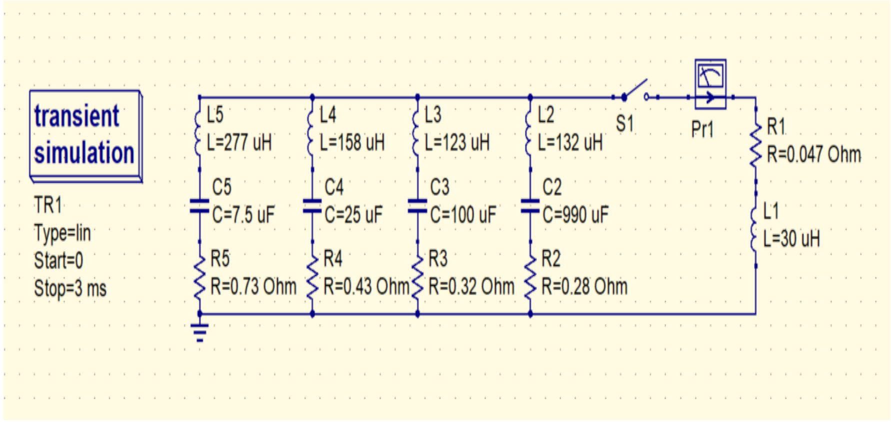


Resistant played an important role



Stage	C (theory)	L (theory)	L (measure)	R (measure)
1	990 (uF)	102.8 (uH)	132±4 (uH)	0.28±0.01 (Ω)
2	100 (uF)	114.9 (uH)	123±0.4 (uH)	0.32±0.02 (Ω)
3	25 (uF)	157 (uH)	158±1 (uH)	0.43±0.01 (Ω)
4	7.5 (uF)	270 (uH)	277±7 (uH)	0.73±0.03 (Ω)



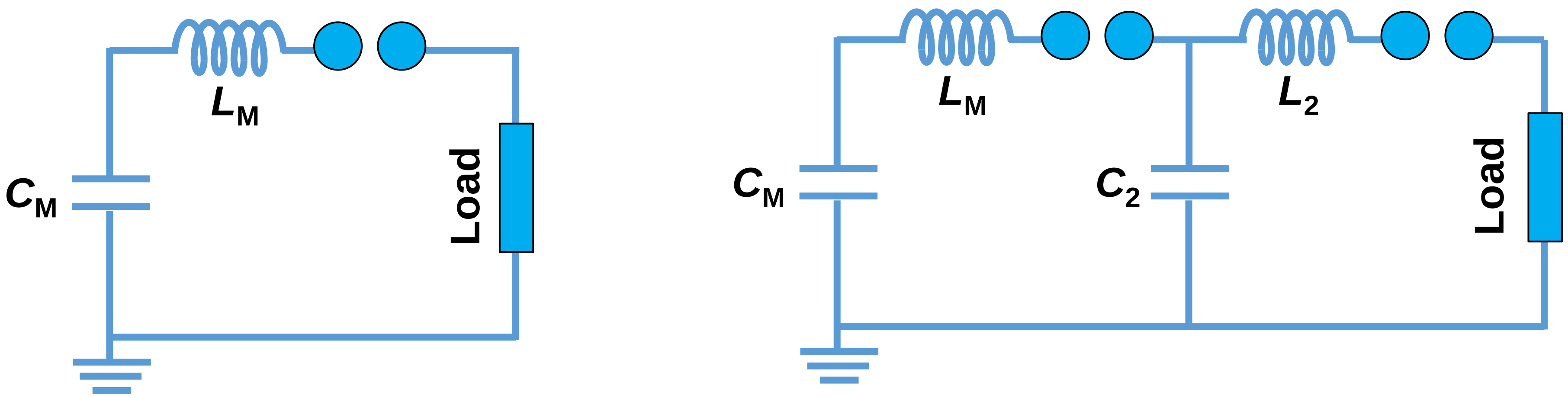


Outlines

- Switches
  - Closing switches: the switching process is associated with voltage breakdown across an initially insulant element.
  - Opening switches: the switching process is associated with a sudden growth of its impedance.
- **Pulse-forming lines**
  - Blumlein line
  - Pulse-forming network
  - **Pulse compressor**
- Pulse transmission and transformation

Capacitor load

- **Pulse compression scheme: a charged capacitor can transfer almost all of its energy to an uncharged capacitor if connected through an inductor.**
- **Output voltage can be doubled in a peaking circuit.**

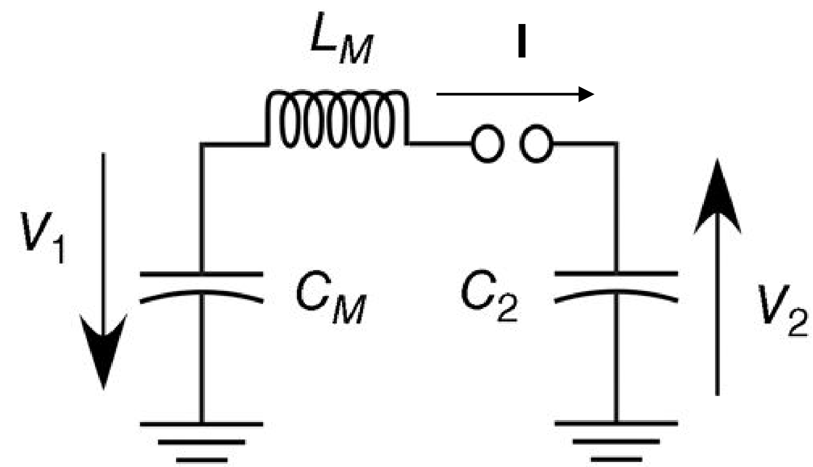


$$I_0 = \frac{V_0}{\sqrt{L_M/C_M}} \quad \omega_0 = \frac{1}{\sqrt{L_M C_M}}$$

$$I_2 = \frac{V_0}{\sqrt{L_2/C_2}} \quad \omega_2 = \frac{1}{\sqrt{L_2 C_2}}$$

$L_M > L_2$ => $I_M < I_2$ $\omega_M < \omega_2$ $T_M > T_2$

Capacitor load



$$V_1 - L_M \frac{\mathrm{dI}}{\mathrm{dt}} = V_2$$

$$V_1 = V_M - \frac{1}{C_M} \int I \,\mathrm{d}t \qquad V_M = \mathrm{NV_0}$$

$$V_2 = \frac{1}{C_2} \int I \,\mathrm{d}t$$

$$V_M - \frac{1}{C_M} \int I \,\mathrm{d}t - L_M \frac{\mathrm{dI}}{\mathrm{dt}} = \frac{1}{C_2} \int I \,\mathrm{d}t$$

$$-\frac{1}{C_M} I - L_M \frac{d^2 I}{\mathrm{dt}^2} = \frac{1}{C_2} I \qquad L_M \frac{d^2 I}{\mathrm{dt}^2} + \left(\frac{1}{C_M} + \frac{1}{C_2}\right) I = 0$$

$$\frac{d^2 I}{\mathrm{dt}^2} + \frac{1}{L_M C_{\mathrm{eff}}} I = 0 \qquad \frac{1}{C_{\mathrm{eff}}} = \frac{1}{C_M} + \frac{1}{C_2} \qquad \omega = \sqrt{\frac{1}{L_M C_{\mathrm{eff}}}}$$

$$I = \alpha \sin(\omega t) + \beta \cos(\omega t)$$

Capacitor load

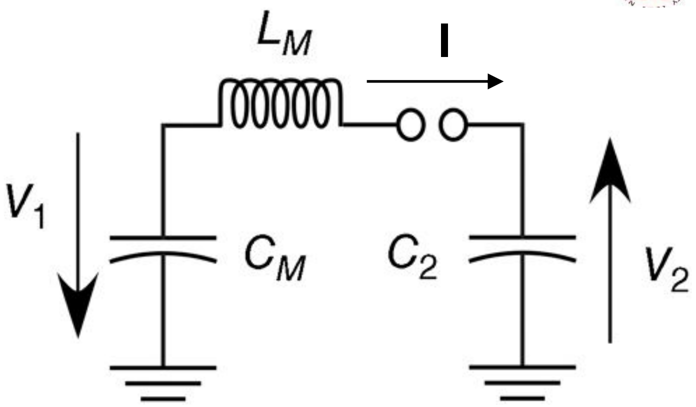
$$I = \alpha\sin(\omega t) + \beta\cos(\omega t)$$

$$I(t=0) = 0 \Rightarrow \beta = 0$$

$$I = \alpha\sin(\omega t)$$

$$\frac{dI}{dt} = \alpha\omega\cos(\omega t)$$

$$L_M \left.\frac{dI}{dt}\right|_{t=0} = L_M\alpha\omega = V_M \qquad \alpha = \frac{V_M}{L_M\omega}$$



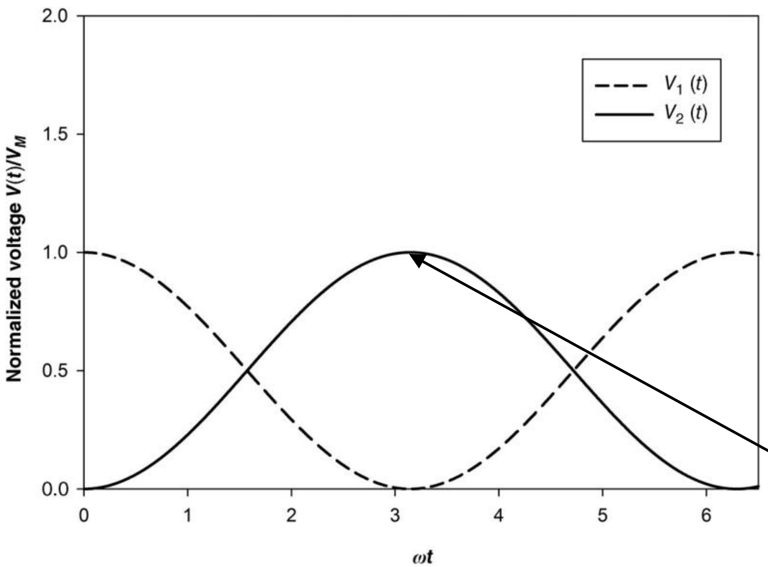
$$I(t) = \frac{V_M}{L\omega}\sin(\omega t)$$

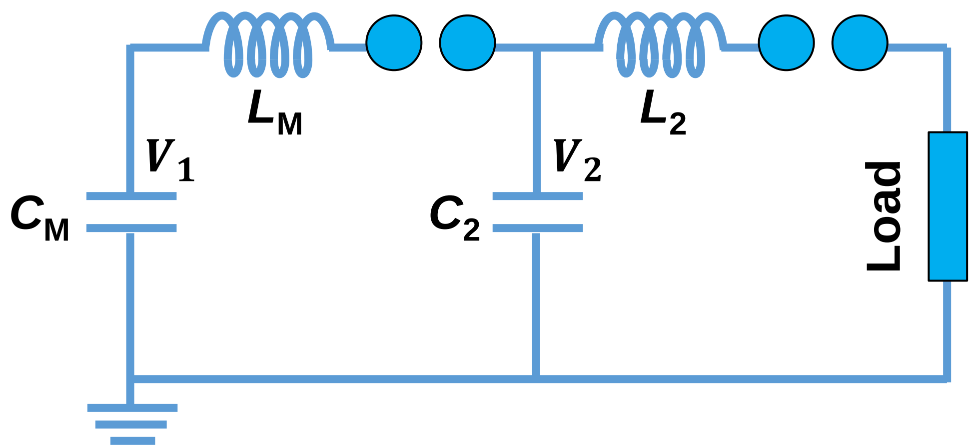
$$V_1 = V_M - \frac{1}{C_M}\int_0^t \frac{V_M}{L\omega}\sin(\omega t)\,dt = V_M - \frac{V_M C_2}{C_M + C_2}[1 - \cos(\omega t)]$$

$$V_2 = \frac{1}{C_2}\int_0^t \frac{V_M}{L\omega}\sin(\omega t)\,dt = \frac{V_M C_M}{C_M + C_2}[1 - \cos(\omega t)] \qquad \left.\frac{V_2}{V_M}\right|_{\max} = \frac{2C_M}{C_M + C_2}$$

$$\boxed{\text{for } C_2 \sim C_M, \frac{V_2}{V_M} \sim 1}$$

Pulse compression scheme: $C_2 \sim C_M$





Energy is fully transferred to the 2nd cap, i.e., intermediate storage capacitor.

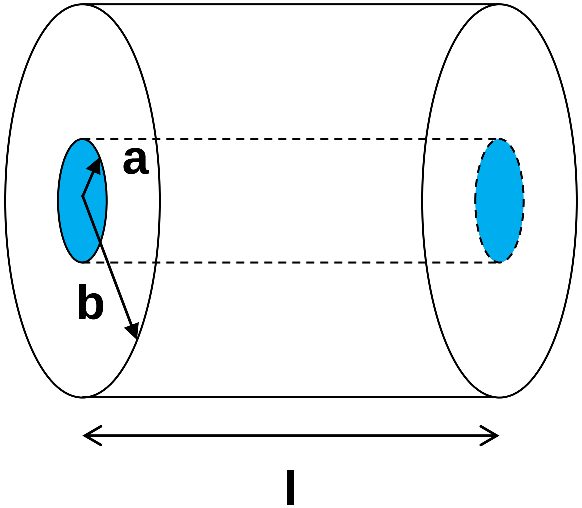
$$V_1 = V_M - \frac{V_M C_2}{C_M + C_2}[1-\cos(\omega t)] \approx V_M - \frac{V_M}{2}[1-\cos(\omega t)]$$

$$V_2 = \frac{V_M C_M}{C_M + C_2}[1-\cos(\omega t)] \approx \frac{V_M}{2}[1-\cos(\omega t)]$$

$$\text{For } t = \frac{\pi}{\omega}, \qquad V_1 \approx 0, \qquad V_2 \approx V_M$$

Water is commonly used as the dielectric material for the intermediate capacitor

$$C = \frac{2\pi\epsilon_r\epsilon_0}{\ln(b/a)} l \qquad \text{For } \frac{b}{a} = \frac{1}{0.9} \approx 1.1$$



- **The gap between two cylinders need to be able to handle the high voltage.**

$$\text{Air: } \epsilon_r = 1 \Rightarrow \frac{C}{l} = 0.5 \times 10^{-9}\, F/m$$

$$\text{Water: } \epsilon_r = 80 \Rightarrow \frac{C}{l} = 4 \times 10^{-8}\, F/m$$

Ex: KALIF, bipolar Marx generator, charged up to ±100 kV. $V_{M,out}$=5 MV.

$$C_M = \frac{0.5\mu\text{F}}{25} = 25\text{nF}$$

$$\text{Using air: } \quad \text{l} = \frac{25 \times 10^{-9}}{0.5 \times 10^{-9}} = 50 \text{ m}$$

$$\text{Using water: } \quad \text{l} = \frac{25 \times 10^{-9}}{4 \times 10^{-8}} = 0.625 \text{ m}$$

Intermediate storage capacitors can be used to compress the pulse