Reference:

1. Introduction to plasma physics and controlled fusion, by Francis F. Chen

2. Principles of Plasma discharges and Material Processing, by M. A. Lieberman and A. J. Lichtenberg.

3. Plasma diagnostics by electrical probes - Practicum manual and documentation by L. Sirghi, G. Popa, D. Alexandroaiei, C. Costin.

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§ Sheaths.

- In all pratical plasma devices, plasma → in a vacuum chamber w/ finite size.
- What happens to the plasma "at the wall"?
- Suppose → NO electric field in the plasma (∵ debye shielding)

$\phi = 0$

- electrons & ions → hit the wall
  → recombine & are lost.

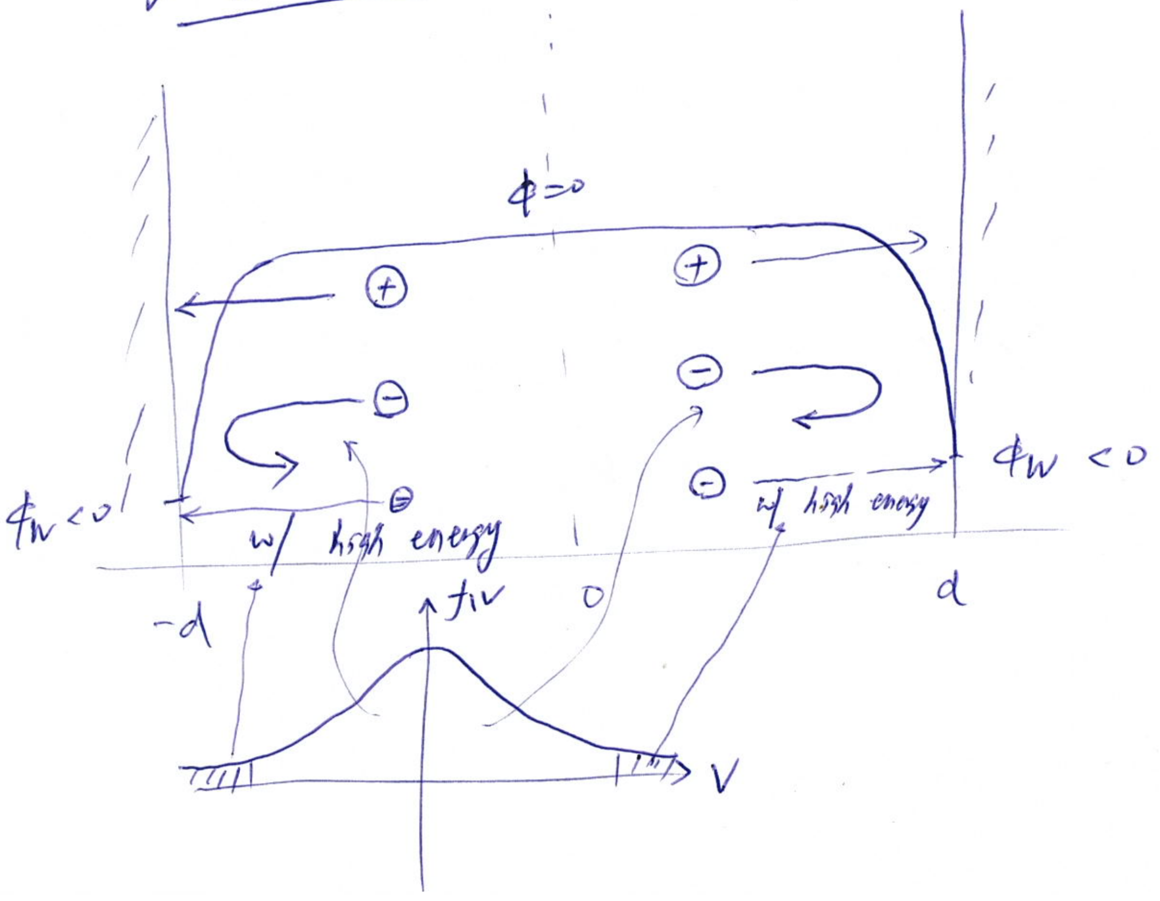
$$E = \frac{1}{2} m v^2 \Rightarrow v \propto \frac{E}{\sqrt{m}} \sim \frac{KT}{\sqrt{m}}$$

$\Rightarrow v_{th,e} \gg v_{th,i}$ ($\because$ $m_e \ll m_i$, $m_i = 1836\, m_e$)

$\Rightarrow$ e^- are lost faster and leave the plasma w/ a net positive charge.

$\Rightarrow \phi_W < 0$

- However, w/ debye shielding ⇒ the potential can NOT be distributed over the entire plasma.
  In fact, potential variation to a layer of the order of several debye lengths in thickness
  → **Sheath**



* The function of a sheath is to form a potential barrier so that the more mobile species, usually electrons, are confined electrostatically.
* The height of the barrier adjusts itself so that the flux of electrons that have enough energy to go over the barrier to the wall is just equal to the flux of ions reaching the wall.

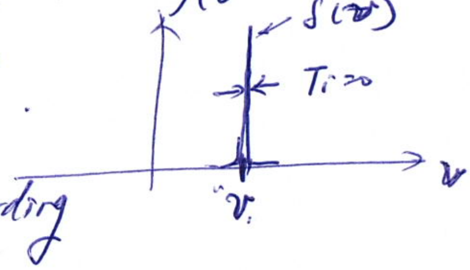
The planar sheath equation.

* In a weakly ionized plasma, electrons are heated by the source while $T_{ion} \approx T_{gas}$. $T_e \approx$ few eV

$T_e \gg T_{ion} \approx T_{gas}$

ions are assumed to be cold. let $T_i \approx 0$

$\Rightarrow$ Monoenergetic ions are accelerated through the sheath potential.



Electron density decreases according to a boltzmann factor.

$$mn\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u}\right] = qn\vec{E} - \nabla p - mn\nu_m\vec{u}$$

collision

For steady state & no drift velocity of electrons

$\frac{\partial \vec{u}}{\partial t} = 0$, $(\vec{u}\cdot\nabla)\vec{u} = 0$ $\Rightarrow$ $qn\vec{E} - \nabla p = 0$

$$+en_e\vec{E} + \nabla p = 0$$

$\vec{E} = -\nabla\phi$, $\quad p_e = n_e k T_e$.

$$\Rightarrow \; -e n_e \nabla\phi + kT_e \nabla n_e = 0$$ for isothermal

$$\Rightarrow \; \nabla\phi - \underbrace{\frac{kT_e}{e}}_{const} \underbrace{\frac{1}{n_e}\nabla n_e}_{\nabla \ln n_e} = 0$$

$$\Rightarrow \; \nabla\left(e\phi - kT_e \ln n_e\right) = 0$$

$$\Rightarrow \; e\phi - kT_e \ln n_e = const \equiv n_0$$

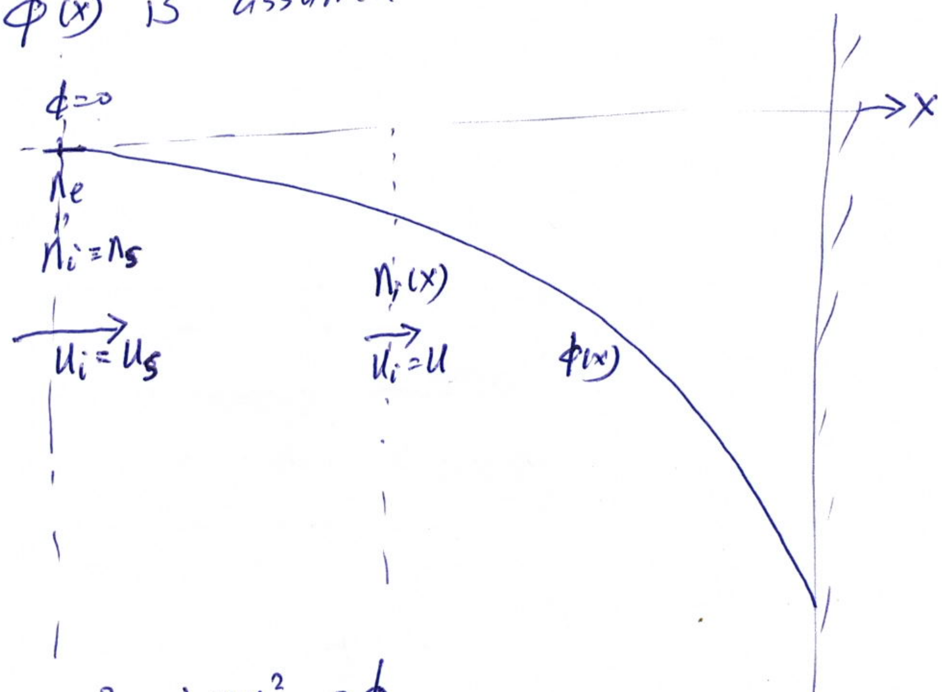
$$n_e(\vec{r}) = n_0 \, e^{e\phi(\vec{r})/kT_e}$$

for ~~one~~ 1D, $\vec{r} \equiv x$

$$\underline{n_e(x) = n_0 \, e^{e\phi(x)/kT_e}}$$

* Consider <u>steady state</u> in a collisionless sheath.

$\phi(x)$ is assumed to decrease monotically w/ x.



$$\frac{1}{2} M u^2 = \frac{1}{2} M u_s^2 - e\phi$$

$$u = \left(u_s^2 - \frac{2e\phi}{M}\right)^{1/2}$$

<u>M : ion mass</u>

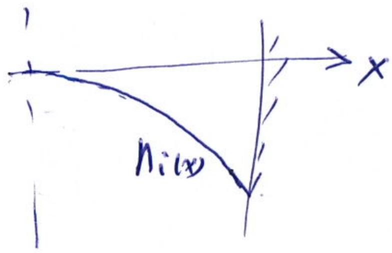
Continuity of ions.

$$n_s u_s = n_i u$$

$$n_i(x) = n_s u_s u^{-1} = n_s u_s \left(u_s^2 - \frac{2e\phi}{M} \right)^{-1/2}$$

$$= n_s \left(1 - \frac{2e\phi}{M u_s^2} \right)^{-1/2}$$

$$|\phi| \uparrow \Rightarrow \underbrace{\left(1 - \frac{2e\phi}{M u_s^2} \right)^{1/2}}_{>0} \uparrow \Rightarrow n_i \downarrow$$



For electrons in steady state:

$$n_e(x) = n_s \exp\left(\frac{e\phi}{KT_e} \right)$$

Poisson's Eq: $\nabla \cdot \vec{E} = \frac{\sigma}{\epsilon_0}$ $\qquad \vec{E} = -\nabla\phi$

$$\Rightarrow \epsilon_0 \nabla^2 \phi = -\sigma$$

$$\Rightarrow \epsilon_0 \frac{d^2\phi}{dx^2} = -(e n_i - e n_e)$$

$$= e(n_e - n_i)$$

$$\epsilon_0 \frac{d^2\phi}{dx^2} = e n_s \left[\exp\left(\frac{e\phi}{KT_e} \right) - \left(1 - \frac{2e\phi}{M u_s^2} \right)^{-1/2} \right]$$

Let $e \mathcal{E}_s \equiv \frac{1}{2} M u_s^2$

$$\frac{d^2\phi}{dx^2} = \frac{e n_s}{\epsilon_0} \left[\exp\left(\frac{e\phi}{KT_e} \right) - \left(1 - \frac{\phi}{\mathcal{E}_s} \right)^{-1/2} \right]$$

§ The Bohm sheath criterion.

$\times$ "$\frac{d\phi}{dx}$" on both side.

$$\underbrace{\frac{d\phi}{dx}\cdot\frac{d}{dx}\left(\frac{d\phi}{dx}\right)} = \frac{en_s}{\epsilon_0}\left[\exp\left(\frac{e\phi}{kT_e}\right)-\left(1-\frac{\phi}{\varepsilon_s}\right)^{-1/2}\right]\frac{d\phi}{\cancel{dx}}$$

$$\frac{1}{2}\frac{d}{\cancel{dx}}\left[\left(\frac{d\phi}{dx}\right)^2\right]$$

$$\Rightarrow \frac{1}{2}\int d\left[\left(\frac{d\phi}{dx}\right)^2\right] = \frac{en_s}{\epsilon_0}\int_0^{\phi} d\phi\left[\exp\left(\frac{e\phi}{kT_e}\right)-\left(1-\frac{\phi}{\varepsilon_s}\right)^{-1/2}\right]$$

$$LHS: = \frac{1}{2}\left(\frac{d\phi}{dx}\right)^2\Bigg|_{\phi=0}^{\phi=\phi} = \frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 \quad \text{assuming } \underbrace{\phi=0,\ \frac{d\phi}{dx}=0 \ @\ x=0}_{\text{field free plasma.}}$$

$$RHS: = \frac{en_s}{\epsilon_0}\left[\frac{kT_e}{e}\exp\left(\frac{e\phi}{kT_e}\right)\Big|_0^{\phi} + 2\varepsilon_s\left(1-\frac{\phi}{\varepsilon_s}\right)^{1/2}\Big|_0^{\phi}\right]$$

$$= \frac{en_s}{\epsilon_0}\left[\frac{kT_e}{e}\exp\left(\frac{e\phi}{kT_e}\right)-\frac{kT_e}{e}+2\varepsilon_s\left(1-\frac{\phi}{\varepsilon_s}\right)^{1/2}-2\varepsilon_s\right]$$

$$\frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 = \frac{en_s}{\epsilon_0}\left[\frac{kT_e}{e}\exp\left(\frac{e\phi}{kT_e}\right)-\frac{kT_e}{e}+2\varepsilon_s\left(1-\frac{\phi}{\varepsilon_s}\right)^{1/2}-2\varepsilon_s\right] \geq 0 \quad ④$$

④ can be integrated numerically, to obtain $\phi(x)$

Expand RHS to 2nd order of Taylor series.

$f(x) = e^x \quad f(0)=1$

$f'(x) = e^x \quad f'(0)=1$

$f''(x) = e^x \quad f''(0)=1$

$\therefore e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$\approx 1 + x + \frac{1}{2}x^2$

$f(x) = (1-x)^{1/2} \quad f(0) = 1$

$f'(x) = \frac{-1}{2}(1-x)^{-1/2} \quad f'(0) = -1/2$

$f''(x) = \frac{-1}{4}(1-x)^{-3/2} \quad f''(0) = -1/4$

$(1-x)^{1/2} = 1 - \frac{1}{2}x - \frac{1}{4}\frac{x^2}{2!} + \cdots$

$\approx 1 - \frac{x}{2} - \frac{x^2}{8}$

$$\approx \frac{en_s}{\epsilon_0}\left[\frac{kT_e}{e}\left(1+\frac{e\phi}{kT_e}+\frac{1}{2}\frac{e^2\phi^2}{(kT_e)^2}\right)-\frac{kT_e}{e}\right.$$
$$\left.+2\mathcal{E}_s\left(1-\frac{\phi}{2\mathcal{E}_s}-\frac{\phi^2}{8\mathcal{E}_s^2}\right)-2\mathcal{E}_s\right]$$

$$=\frac{en_s}{\epsilon_0}\left[\cancel{\frac{kT_e}{e}}+\cancel{\phi}+\frac{1}{2}\frac{e\phi^2}{kT_e}-\cancel{\frac{kT_e}{e}}+\cancel{2\mathcal{E}_s}-\cancel{\phi}-\frac{\phi^2}{4\mathcal{E}_s}-\cancel{2\mathcal{E}_s}\right]$$

$$=\frac{en_s}{\epsilon_0}\left[\frac{1}{2}\frac{e\phi^2}{kT_e}-\frac{\phi^2}{4\mathcal{E}_s}\right]=\frac{1}{2}\left(\frac{d\phi}{dx}\right)^2\geq 0$$

$$\Rightarrow \frac{1}{2}\frac{e\phi^2}{kT_e}-\frac{\phi^2}{4\mathcal{E}_s}\geq 0$$

$$\Rightarrow e\mathcal{E}_s\geq\frac{kT_e}{2}$$

Note that $e\mathcal{E}_s\equiv\frac{1}{2}Mu_s^2$

$$\frac{1}{2}Mu_s^2\geq\frac{kT_e}{2}$$

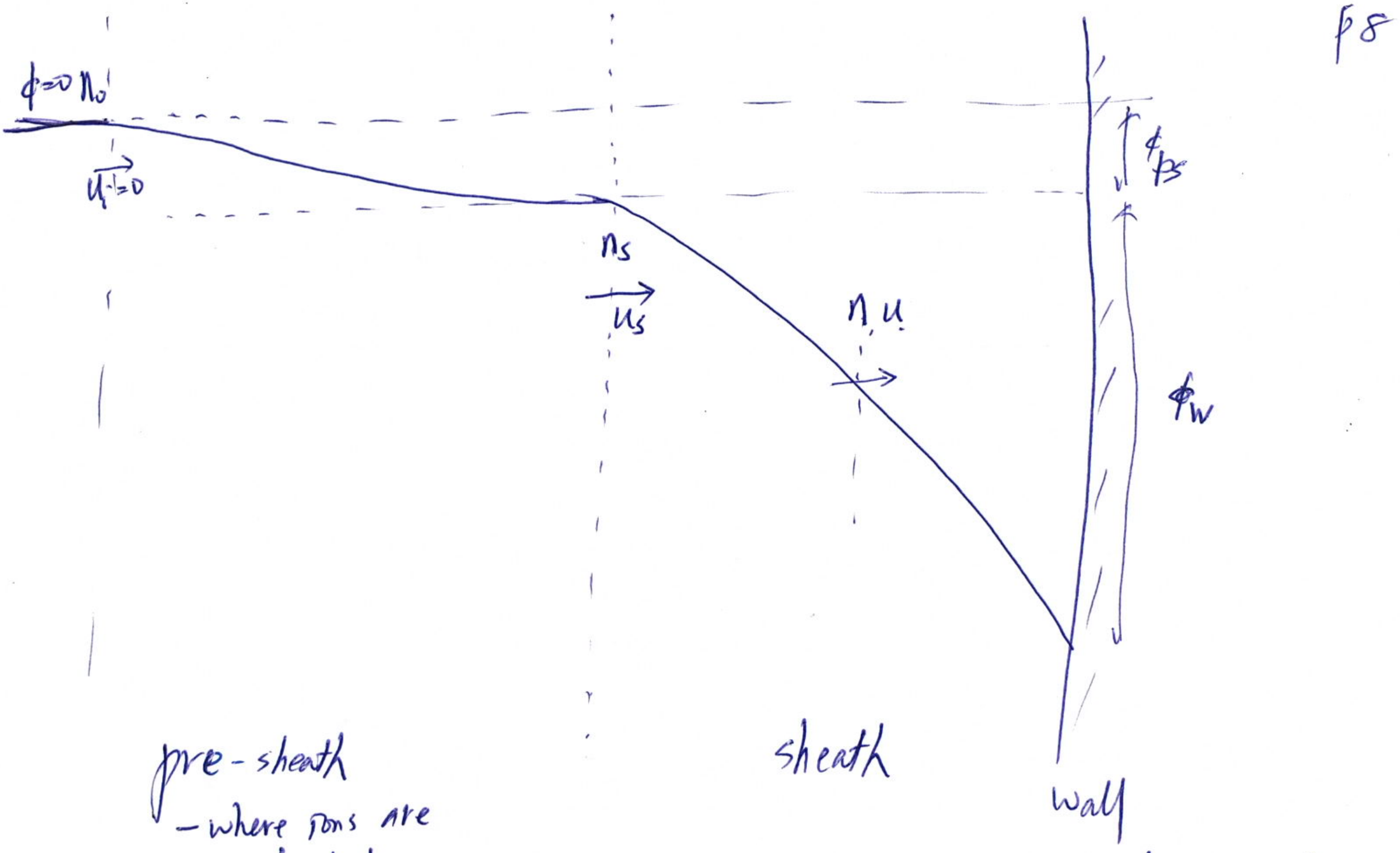
$$u_s\geq\left(\frac{kT_e}{M}\right)^{1/2}\equiv u_B \quad *$$

Bohm sheath criterion

* Ions must enter the sheath region w/ a velocity greater than the acoustic velocity u_B

$\Rightarrow$ There must be a finite electric field in the plasma.

$\Rightarrow \frac{d\phi}{dx}=0$ @ $x=0$ is only an approximation



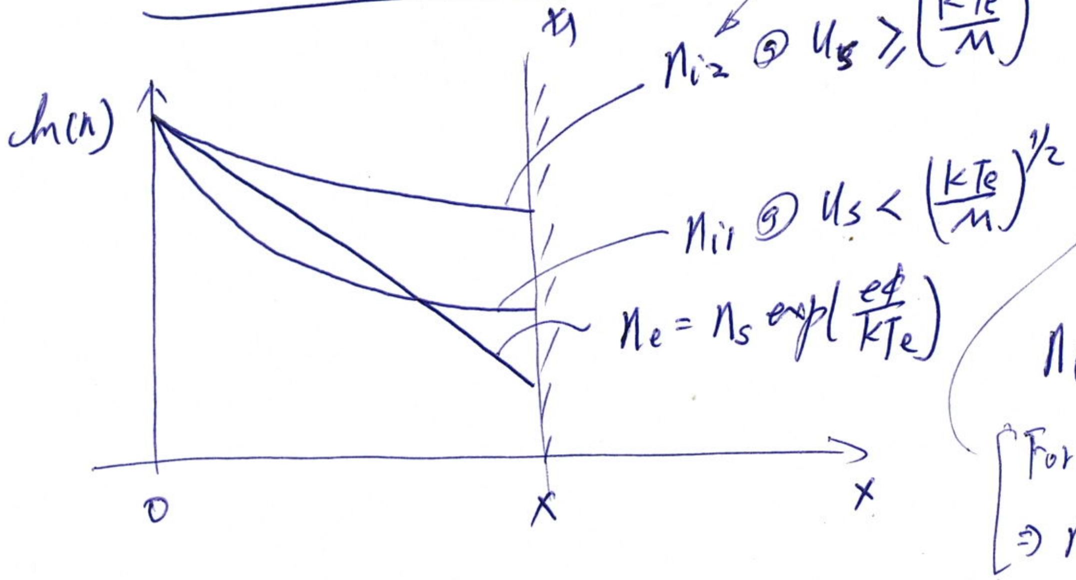
Note that $n_s u_s$ is fixed by the ion production rate.

Let $u_s = u_B$. $\quad \underbrace{\frac{1}{2} M u_B^2}_{= \frac{kT_e}{2}} = -e\phi_{PS}$

$$\Rightarrow \quad n_s = n_{es} = n_0 \, e^{\frac{e\phi_{PS}}{kT_e}} = n_0 \, e^{-1/2} = 0.61\, n_0$$

$$\frac{e\phi_{PS}}{kT_e} = -\frac{kT_e}{2}\cdot\frac{1}{kT_e}$$

$$\underline{n_s = 0.61\, n_0}$$



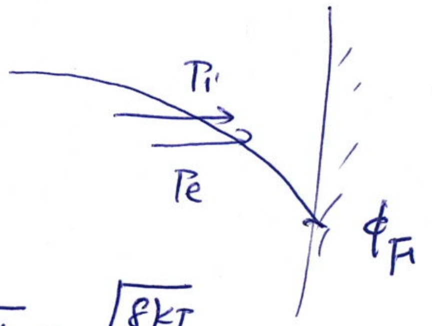
$$n_i = n_s\left(1 - \frac{2e\phi}{M u_s^2}\right)^{-1/2}$$

For larger u_s^2, $\frac{2e\phi}{M u_s^2}$ is small

$\Rightarrow$ n_i changes slower w/ respect to ϕ.

3) sheath potential @ a floating wall. p8a

$\underline{\Gamma_i = \Gamma_e}$ → No current



$\Gamma_i = n_s \cdot u_s = n_s u_B$

$\Gamma_e = \frac{1}{4} n_{ew} \cdot \bar{v}_e$, where $\bar{v}_e = \sqrt{\frac{8kT}{\pi m}}$

For 3D: $\hat{f}_M = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{v^2}{v_{th}^2}\right)$

$v^2 = v_x^2 + v_y^2 + v_z^2$, $v_{th}^2 = \frac{2kT}{m}$

$$\overline{v^2} = \cancel{\left(\frac{m}{2\pi kT}\right)^{3/2}} \int_{-\infty}^{\infty} v^2 \exp\left(-\frac{v^2}{v_{th}^2}\right) \underbrace{d\vec{v}^3}_{4\pi v^2 dv} = \frac{3kT}{m}$$

$d\vec{v}^3 = dv_x dv_y dv_z$

$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$$4\pi \int_0^\infty v^4 \exp\left(-\frac{v^2}{v_{th}^2}\right) dv = \frac{3\sqrt{\pi}}{8} (v_{th}^2)^{5/2}$$

$$= \frac{4\pi}{\pi^{3/2}} \frac{3\sqrt{\pi}}{8} \left(\frac{2kT}{m}\right) = \frac{3}{8/4} \frac{2kT}{m} = \frac{3kT}{m}$$

$$\bar{v} = \int |v| f(v) d\vec{v}^3$$

$$= \cancel{\left(\frac{m}{2\pi kT}\right)^{3/2}} \int_0^\infty v \cdot \exp\left(-\frac{v^2}{v_{th}^2}\right) 4\pi v^2 dv$$

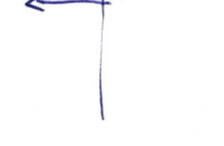
$$4\pi \frac{(v_{th})^4}{2} = \frac{2}{\pi^2 \cdot 8} \left(\frac{2\pi kT}{m}\right)^{1/2} = \frac{2}{\pi}\left(\frac{2\pi kT}{m}\right)^{1/2} = \left(\frac{8kT}{\pi m}\right)^{1/2}$$

$$= 2\sqrt{\frac{2kT}{\pi m}}$$

$$|\bar{v}_x| = \int |v_x| f(v) d\vec{v}^3 = \left(\frac{m}{2\pi kT}\right)^{3/2} \int_{-\infty}^{\infty} dv_y \exp\left(-\frac{v_y^2}{v_{th}^2}\right) \int_{-\infty}^{\infty} dv_z \exp\left(-\frac{v_z^2}{v_{th}^2}\right) \int_0^\infty 2v_x \exp\left(-\frac{v_x^2}{v_{th}^2}\right) dv_x$$

$$= \left(\frac{m}{2\pi kT}\right)^{1/2} \int_0^\infty \exp\left(-\frac{v_x^2}{v_{th}^2}\right) dv_x^2 = \left(\frac{m}{2\pi kT}\right)^{1/2} (-v_{th}^2) \exp\left(-\frac{v_x^2}{v_{th}^2}\right)\Big|_0^\infty$$

$$= \left(\frac{m}{\cancel{2\pi kT}}\right)^{1/2} \left(\frac{2kT}{m}\right)^{1/2} = \left(\frac{2kT}{\pi m}\right)^{1/2} = \frac{1}{2}\bar{v}$$



$\Gamma_+ = \left(\frac{1}{2} n\right) \underset{\frac{1}{2}\bar{v}}{\underset{\parallel}{\bar{v}_x}}$, $\Gamma_- = -\left(\frac{1}{2} n\right) \bar{v}_x$

$$n_{ew} = n_s\, e^{e\phi_{fl}/kT_e}$$

$$\Rightarrow \quad \Gamma_i = \Gamma_e$$

$$n_s \cdot u_s = \cancel{n_s}\left(\frac{kT_e}{M}\right)^{1/2} = \frac{1}{4}\cancel{n_s}\, e^{e\phi_{fl}/kT_e}\,\overbrace{\sqrt{\frac{8kT_e}{\pi m}}}^{\bar{v}_e}$$

$$e^{e\phi_{fl}/kT_e} = 4\left(\frac{\pi m}{8M}\right)^{1/2} = \left(\frac{2\pi m}{M}\right)^{1/2}$$

$$\frac{e\phi_{fl}}{kT_e} = \ln\left(\frac{2\pi m}{M}\right)^{1/2} = -\ln\left(\frac{M}{2\pi m}\right)^{1/2}$$

$$\phi_{fl} = -\frac{kT_e}{e}\ln\left(\frac{M}{2\pi m}\right)^{1/2}$$

$$= -\frac{kT_e}{2e}\ln\left(\frac{M}{2\pi m}\right)$$

M: ion mass
m: e^- mass

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*1

For Argon, $M = 40$ amu.

$\ln\left(\frac{M}{2\pi m}\right) \approx 4.7$

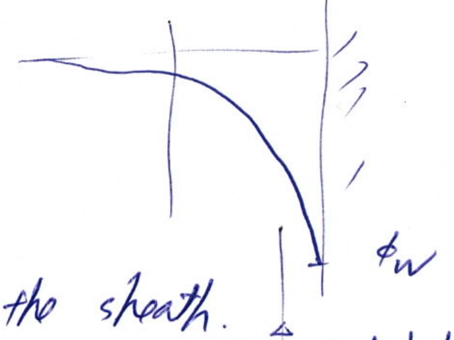
Hydrogen, $M = 1$.

$\ln\left(\frac{M}{2\pi m}\right) \approx 2.8$

$kT_e \sim$ few eV,
$\Rightarrow \phi_{fl} \sim$ 5~10 V

3 The high-voltage sheath.

$|e\phi_w| \gg kT_e$ in general



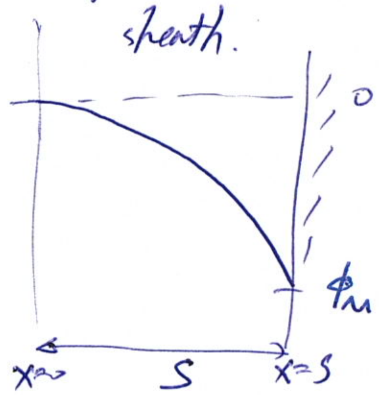
$\Rightarrow n_e = n_s e^{e\phi_w/kT_e} \sim 0$

$\Rightarrow$ only ions are present in the sheath.

3 Matrix sheath.

Assuming a <u>uniform</u> ion density (simplest solution)

Let $n_i = n_s = \text{const.}$



$\epsilon_0 \nabla \cdot \vec{E} = \rho \approx e n_i \quad (n_e \approx 0)$

$$\frac{dE}{dx} = \frac{e n_i}{\epsilon_0} \approx \frac{e n_s}{\epsilon_0}$$

$$E = \frac{e n_s}{\epsilon_0} x \qquad \nabla\phi = -\vec{E} \Rightarrow \frac{d\phi}{dx} = -E$$

$$\frac{d\phi}{dx} = -\frac{e n_s}{\epsilon_0} x \Rightarrow \phi\Big|_0^{\phi_M} = -\frac{1}{2}\frac{e n_s}{\epsilon_0} x^2\Big|_0^S$$

$$+|\phi_M| \equiv \phi_M = +\frac{1}{2}\frac{e n_s}{\epsilon_0} S^2$$

$$\text{or } S = \left(\frac{2\epsilon_0|\phi_M|}{e n_s}\right)^{1/2}$$

Debye length: $\lambda_D = \left(\frac{\epsilon_0 kT_e}{e n_s}\right)^{1/2}$

$$S = \lambda_D \left(\frac{2|\phi_M|}{kT_e}\right)^{1/2}$$ $\approx$ tens of Debye length

7) Child Law Sheath

* Keep the high voltage assumption; i.e., $n_e \approx 0$
* Ion density should decrease as the ions accelerated across the sheath.

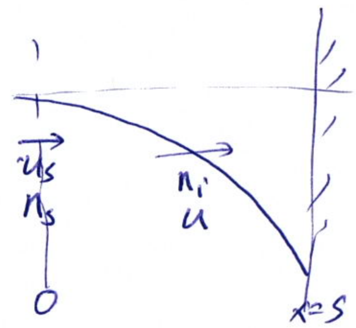
Assuming that the initial ion energy $\frac{1}{2}Mu_s^2 = e\mathcal{E}_s \ll |\phi|$

$\Rightarrow$ neglect $\frac{1}{2}Mu_s^2$ in the following eq.:

$$\frac{1}{2}Mu^2 = \frac{1}{2}Mu_s^2 - e\phi \approx -e\phi$$

$$n_s u_s = n_i u$$

$$u = \left(-\frac{2e\phi}{M}\right)^{1/2}$$



Current density: $J_0 = e\, n_i u = e n_i \left(-\frac{2e\phi}{M}\right)^{1/2}$ ($J_e \to 0$, $\because n_e \to 0$)

$$\Rightarrow n_i = \frac{J_0}{e}\left(-\frac{2e\phi}{M}\right)^{-1/2}$$

Poisson's Eq: $\frac{d^2\phi}{dx^2} = -\frac{e n_i}{\epsilon_0} = -\frac{J_0}{\epsilon_0}\left(-\frac{2e\phi}{M}\right)^{-1/2}$

$\times \frac{d\phi}{dx}$ $\underbrace{\frac{d\phi}{dx}\frac{d}{dx}\left(\frac{d\phi}{dx}\right)}_{\frac{1}{2}\frac{d}{\cancel{dx}}\left[\left(\frac{d\phi}{dx}\right)^2\right]} = -\frac{J_0}{\epsilon_0}\left(-\frac{2e\phi}{M}\right)^{-1/2}\frac{d\phi}{\cancel{dx}}$

$$\Rightarrow \frac{1}{2}\int_0^{\phi} d\left(\frac{d\phi}{dx}\right)^2 = -\frac{J_0}{\epsilon_0}\int_0^{\phi}\left(-\frac{2e\phi}{M}\right)^{-1/2} d\phi$$

Let $\frac{d\phi}{dx} = -E \approx 0$, $\phi \approx 0$ @ $x = 0$

$$\frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 = +\frac{J_0}{\epsilon_0}\left(+\frac{M}{2e}\right)\times 2\left(-\frac{2e\phi}{M}\right)^{1/2}\Bigg]_0^{\phi}$$

$$= 2\,\frac{J_0}{\epsilon_0}\sqrt{\frac{M}{2e}}\,\cancel{\left(\frac{2e}{M}\right)^{1/2}}(-\phi)^{1/2}$$

$$= 2\,\frac{J_0}{\epsilon_0}\left(\frac{2e}{M}\right)^{-1/2}(-\phi)^{1/2}$$

$$\frac{d\phi}{dx} = -\left[4\,\frac{J_0}{\epsilon_0}\left(\frac{2e}{M}\right)^{-1/2}(-\phi)^{1/2}\right]^{1/2} \qquad (\because \frac{d\phi}{dx} < 0)$$

$$= -2\left(\frac{J_0}{\epsilon_0}\right)^{1/2}\left(\frac{2e}{M}\right)^{-1/4}(-\phi)^{1/4}$$

$$\int_{@x=0}^{x=s}(-\phi)^{-1/4}\,d(-\phi) = 2\left(\frac{J_0}{\epsilon_0}\right)^{1/2}\left(\frac{2e}{M}\right)^{-1/4}\int_{x=0}^{s}dx = 2\left(\frac{J_0}{\epsilon_0}\right)^{1/2}\left(\frac{2e}{M}\right)^{-1/4}s$$

$$\parallel$$

$$\frac{4}{3}(-\phi)^{3/4}\Bigg|_{\phi=0}^{\phi=\phi_w} = \frac{4}{3}(-\phi_w)^{3/4} = \frac{4}{3}|\phi_w|^{3/4}$$

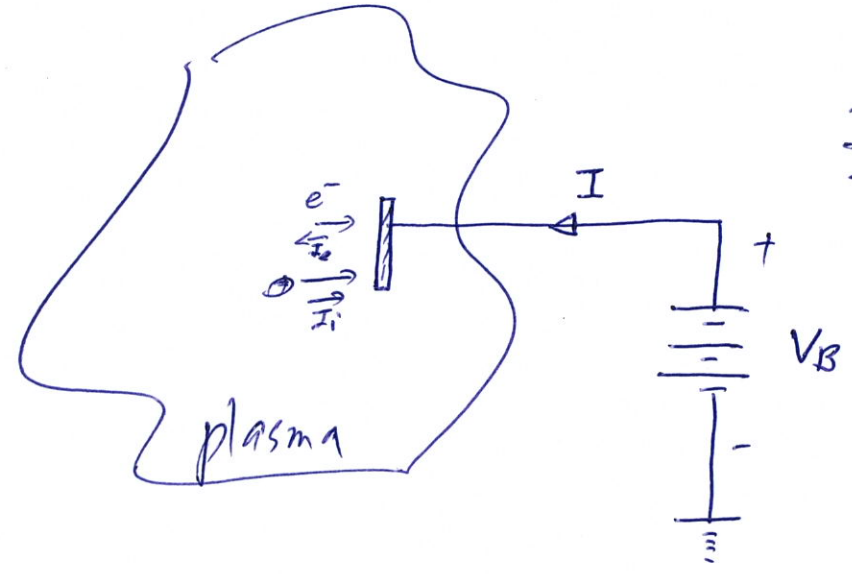
$$\Rightarrow \quad \frac{4\times\cancel{4}}{9}|\phi_w|^{3/2} = \cancel{4}\,\frac{J_0}{\epsilon_0}\left(\frac{2e}{M}\right)^{-1/2}s^2$$

$$J_0 = \frac{4}{9}\epsilon_0\left(\frac{2e}{M}\right)^{1/2}\frac{|\phi_w|^{3/2}}{s^2}$$

Child law of space-charge-limited in a plane diode.

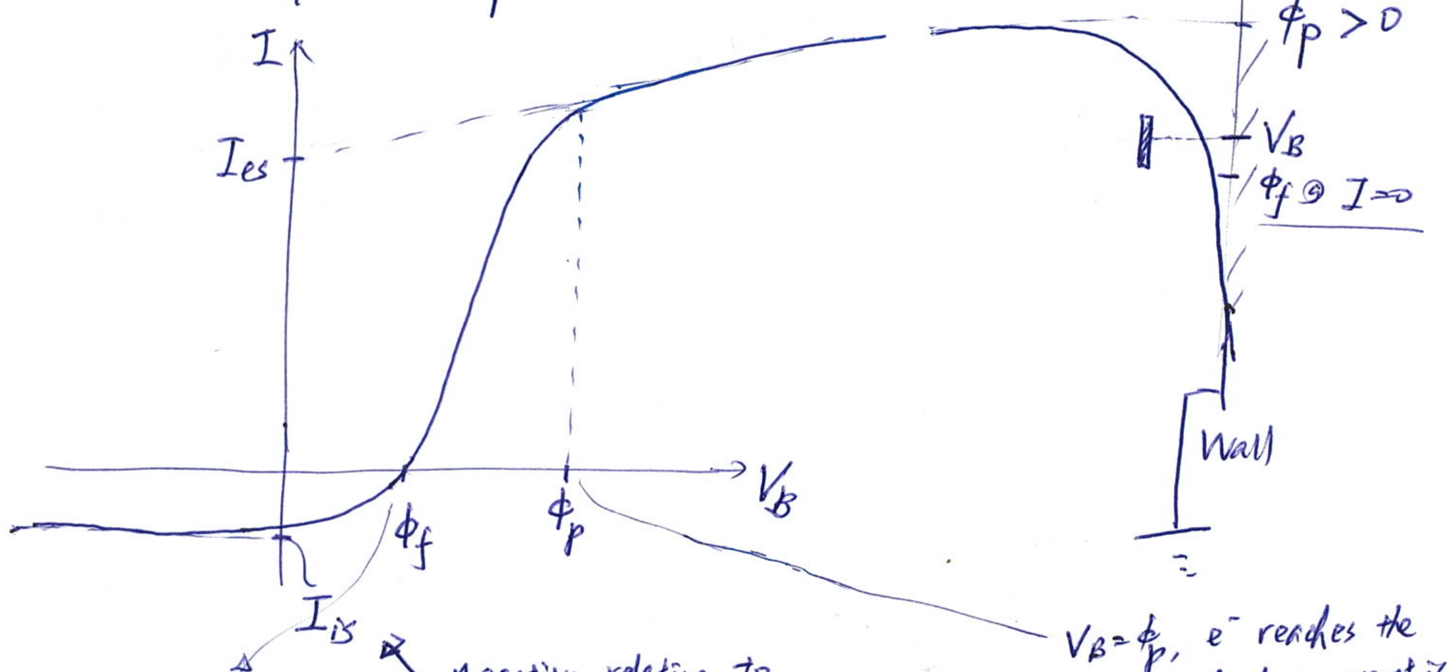
$$\underset{\text{measurable}}{I} = A\cdot J_0 = \frac{4}{9}\epsilon_0\left(\frac{2e}{M}\right)^{1/2}\frac{|\phi_w|^{3/2}}{s^2}\cdot \underset{\text{probe}}{A}$$

7) Electrostatic probe (Langmuir probe)



$I = I_e - I_i$

* A metal probe, inserted in a discharge and biased positively or negatively to draw electron or ion current.
* Probe are usually quite small and under suitable conditions, produce only minor local perturbations of the plasma. ⇒ often thin wires w/ the wire radius $a < \lambda_D$.



Floating potential where e⁻ & ion flux are balanced & no current flow through the probe.

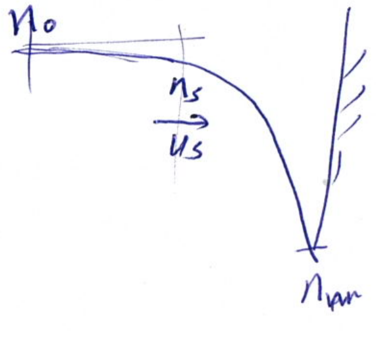
Negative relative to plasma so that all e⁻ are repelled. only ion current, i.e., I_{is}

$V_B = \phi_p$, e⁻ reaches the probe freely, current is mainly from mobile electrons i.e. I_{es}.

⑨ Saturated ion current:

$$I = - I_{is} = - e\, n_s\, u_s \cdot A$$

$$u_s = \left(\frac{kT_e}{M}\right)^{1/2}$$



$$n_0 \approx \frac{n_s}{0.61} \Rightarrow n_s = 0.61\, n_0$$

$$I_i = 0.61\, e\, n_0 \left(\frac{kT_e}{M}\right)^{1/2} A$$

$$n_0 = \frac{1}{0.61} \frac{I_{is}}{e} \frac{1}{A} \left(\frac{M}{kT_e}\right)^{1/2}$$

ϕ

$n_0 = n_{ew}$

~~⑤ Saturated e^- current:~~

~~$I \approx I_{es} - I_{ion} \approx I_{es} = e \cdot \frac{1}{4} n_{ew} \underbrace{\bar{V}_e}_{T_e} \cdot A$~~

~~$n_{ew} = n_0\, e^{e\phi_p / kT_e}$~~

~~$I_{es} = \frac{1}{4} e n_0 A$~~

⑤ Saturated e^- current

all electrons arrive the probe.

$$I = I_{es} - I_{ion} \approx I_{es} = e \cdot \frac{1}{4} \underbrace{n_0\ \bar{V}_e}_{T_e} \cdot A$$

$$n_{ew} = n_0$$

$$I_{es} = \frac{1}{\cancel{4}} n_0 e A \sqrt{\frac{8kT_e}{2\pi m}}$$

$$n_0 = \frac{I_{es}}{e \cdot A} \sqrt{\frac{2\pi m}{kT_e}}$$

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⑤ Transition ;

$$I = I_e - I_i$$

$$I + I_i = I_e = \frac{1}{4} e n_p \bar{V}_e \cdot A$$

$$n_p = n_o \exp\left(e \frac{V_B - \phi_p}{kT_e}\right) \qquad \underline{V_B - \phi_p < 0}$$

$$\bar{V}_e = \sqrt{\frac{8kT_e}{\pi m}}$$

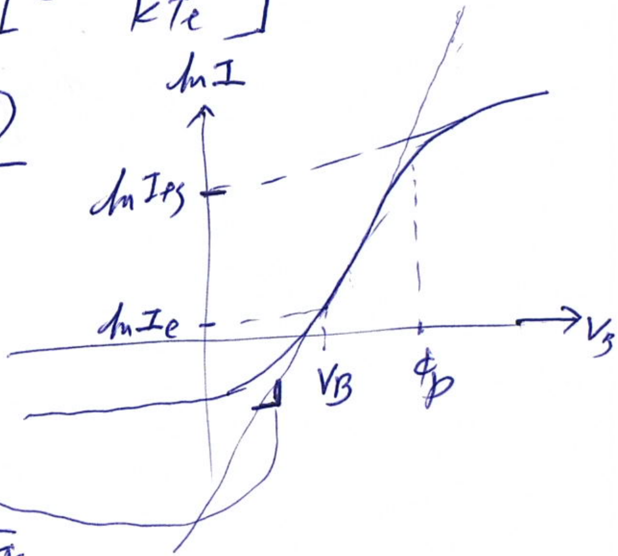
$$I + I_i = I_e = \frac{1}{4} e n_o \exp\left(e \frac{V_B - \phi_p}{kT_e}\right) \cdot \underbrace{\sqrt{\frac{8kT_e}{\pi m}}}_{\bar{V}_e} \cdot A$$

$$I_{es} \equiv \frac{1}{4} e n_o \bar{V}_e \cdot A$$

$$I + I_i = I_e = I_{es} \cdot \exp\left[e \frac{V_B - \phi_p}{kT_e}\right]$$

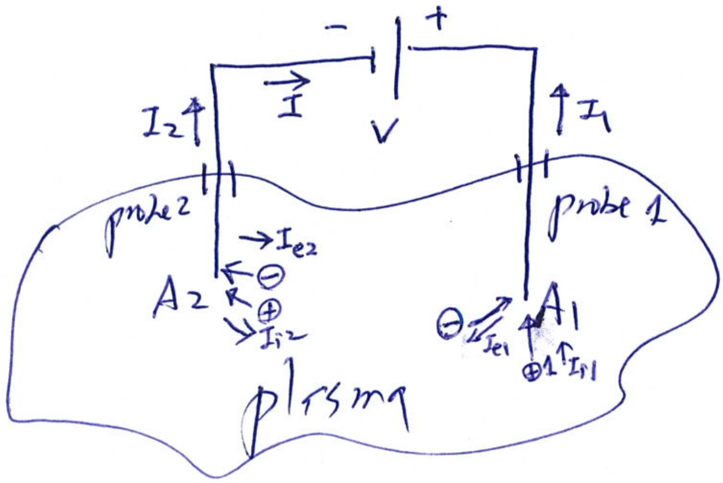
$$\ln\left(\frac{I_e}{I_{es}}\right) = \frac{e(V_B - \phi_p)}{kT_e}$$

$$T_e = \frac{e(V_B - \phi_p)}{k(\ln I_e - \ln I_{es})}$$



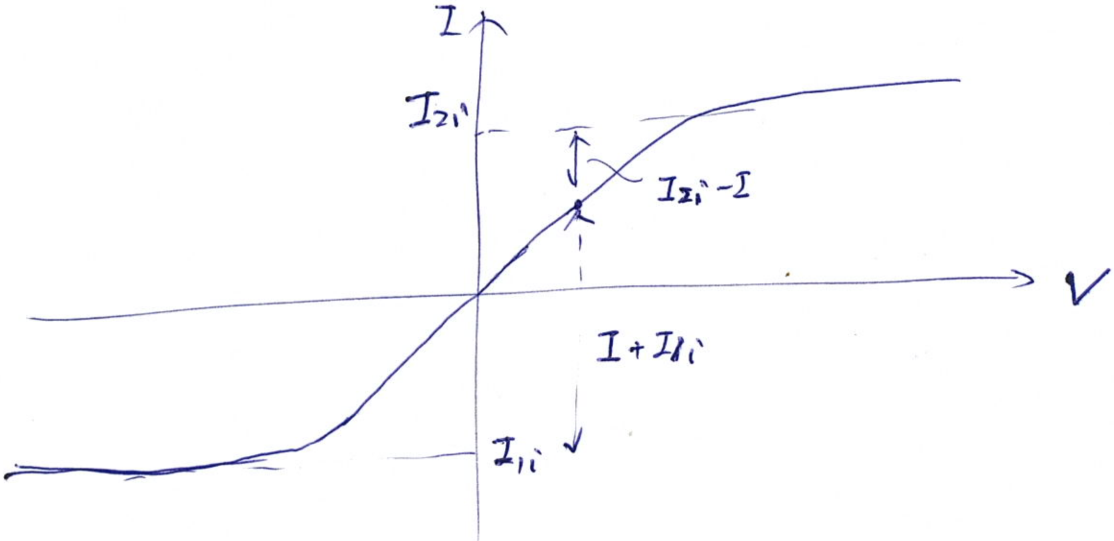
3) Double probes

* When there is no well-defined ground.



* Since the two probes draw no net current, they will both be negative w/ respect to the plasma.
* $I \neq 0$ if $V \neq 0$.
* Advantage: the net current never exceeds the ion saturation current, minimizing the disturbance to the discharge.

Disadvantage: only the high-energy tail of the e^- distribution is collected by either probe. It may not be representative of the distribution of bulk e^- in the discharge



$I_1 = I_{i1} - I_{e1}$, $I_2 = I_{i2} - I_{e2}$

No current (system is floating)

$I_1 + I_2 = 0 \Rightarrow I_{i1} - I_{e1} + I_{i2} - I_{e2} = 0$

$I_{i1} + I_{i2} - I_{e1} - I_{e2} = 0$

$I = -I_1 = I_{e1} - I_{i1} = I_2 = I_{i2} - I_{e2}$

$$\left\{ \begin{array}{l} I_{e1} = A_1 J_{es} \cdot e^{e(V_1 - \phi_p)/kT_e} = I + I_{i1} \\ I_{e2} = A_2 J_{es}\, e^{e(V_2 - \phi_p)/kT_e} = I_{i2} - I \end{array} \right) \div$$

$$\Rightarrow \frac{I + I_{i1}}{I_{i2} - I} = \frac{A_1}{A_2} e^{e(V_1 - V_2)/kT_e} \qquad V \equiv V_1 - V_2$$

$$= \frac{A_1}{A_2} e^{eV/kT_e}$$

For $A_1 = A_2$, $I_{i1} = I_{i2} \equiv I_i$

$$\frac{I + I_i}{I_i - I} = e^{eV/kT_e} \Rightarrow I + I_i = I_i e^{eV/kT_e} - I e^{eV/kT_e}$$

$$I = I_i \frac{e^{eV/kT_e} - 1}{1 + e^{eV/kT_e}} = I_i \frac{e^{eV/2kT_e} - e^{-eV/2kT_e}}{e^{eV/2kT_e} + e^{-eV/2kT_e}}$$

by fitting the curve, both T_e & $I_i \to n_0$ are obtained.

$$\longrightarrow I = I_i \tanh\left(\frac{eV}{2kT_e}\right)$$

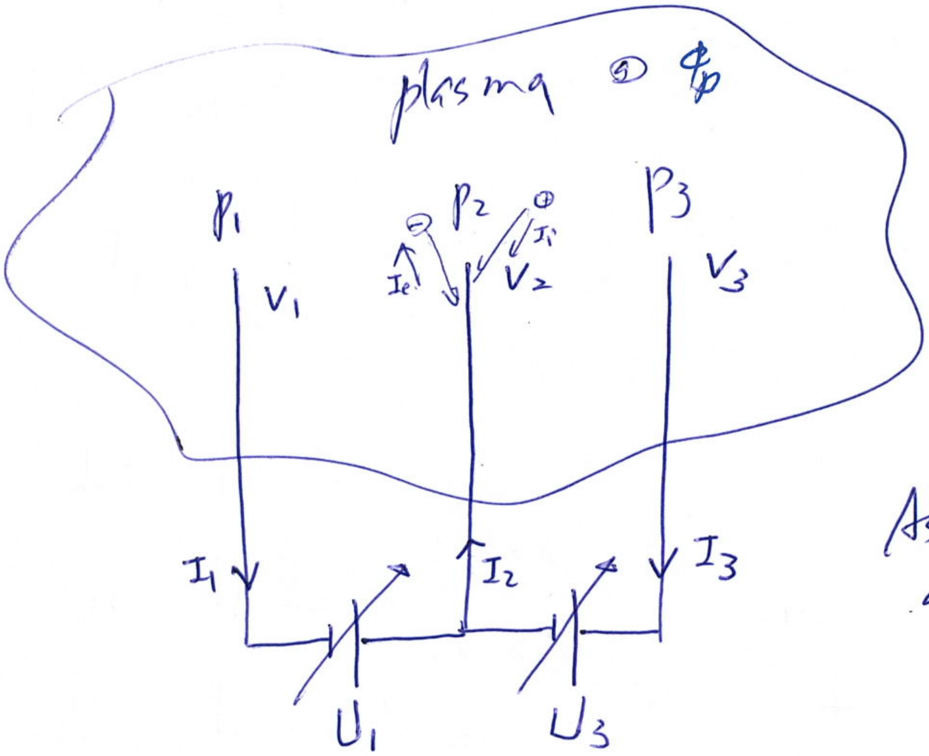
③ $V = 0$

$$\left.\frac{dI}{dV}\right|_{V=0} = I_i \frac{e}{2kT_e} \frac{d}{dX}\Big[\tanh(X)\Big]_{X = \frac{eV}{2kT_e}} = I_i \frac{e}{2kT_e} \operatorname{sech}\left(\frac{eV}{2kT_e}\right)\Big|_{V}$$

$$= I_i \frac{e}{2kT_e} \frac{2}{\underbrace{e^{eV/2kT_e} + e^{-eV/2kT_e}}_{=1}}\Bigg|_{V=0}$$

$$= \frac{e}{2kT_e} I_i$$

3) Triple probe.



Assumy three probes are identical.

$$I_1 = I_{i0} - I_{e0} \exp\left(e \frac{V_1 - \phi_p}{kT_e}\right)$$

$$I_2 = -I_{i0} + I_{e0} \exp\left(e \frac{V_2 - \phi_p}{kT_e}\right)$$

$$I_3 = I_{i0} - I_{e0} \exp\left(e \frac{V_3 - \phi_p}{kT_e}\right)$$

$$I_{i0} = \frac{1}{4} n_0 e \sqrt{\frac{8kT_i}{\pi M}} \cdot A$$

$$I_{e0} = \frac{1}{4} n_0 e \sqrt{\frac{8kT_e}{\pi m}} \cdot A$$

$$U_1 = V_2 - V_1, \quad U_3 = V_3 - V_2$$

$$\frac{I_1 + I_2 = \cancel{I_{e0}} \left[\exp\left[e \frac{V_2 - \cancel{\phi_p}}{kT_e}\right] - \exp\left[e \frac{V_1 - \cancel{\phi_p}}{kT_e}\right]\right]}{I_3 + I_2 = \cancel{I_{e0}} \left[\exp\left(e \frac{V_2 - \cancel{\phi_p}}{kT_e}\right) - \exp\left(e \frac{V_3 - \cancel{\phi_p}}{kT_e}\right)\right]}$$

$$= \frac{1 - \exp\left(-e \frac{V_2 - V_1}{kT_e}\right)}{1 - \exp\left(e \frac{V_3 - V_2}{kT_e}\right)} = \frac{1 - \exp\left(-\frac{eU_1}{kT_e}\right)}{1 - \exp\left(\frac{eU_3}{kT_e}\right)}$$

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$$I_1 \exp\left(e\frac{V_3}{kT_e}\right) = I_{i0} \exp\left(e\frac{V_3}{kT_e}\right) - \cancel{I_{e0} \exp\left(e\frac{V_1+V_3-\phi_p}{kT_e}\right)}$$

$$-)\ I_3 \exp\left(e\frac{V_1}{kT_e}\right) = I_{i0} \exp\left(e\frac{V_1}{kT_e}\right) - \cancel{I_{e0} \exp\left(e\frac{V_1+V_3-\phi_p}{kT_e}\right)}$$

$$I_1 \exp\left(e\frac{V_3}{kT_e}\right) - I_3 \exp\left(e\frac{V_1}{kT_e}\right) = I_{i0}\left[\exp\left(\frac{eV_3}{kT_e}\right) - \exp\left(e\frac{V_1}{kT_e}\right)\right]$$

$$I_1 \exp\left[e\frac{V_3-V_1}{kT_e}\right] - I_3 = I_{i0}\left[\exp\left(\frac{e(V_3-V_1)}{kT_e}\right) - 1\right]$$

$$V_3 = V_2 + U_3 \quad \Rightarrow \quad V_3 - V_1 = U_1 + U_3 \equiv \Delta U$$

$$V_1 = V_2 - U_1$$

$$I_{i0} = \frac{I_1 \exp\left(e\frac{\Delta U}{kT_e}\right) - I_3}{\exp\left(e\frac{\Delta U}{kT_e}\right) - 1} = \frac{I_1 - I_3 \exp\left(-\frac{e\Delta U}{kT_e}\right)}{1 - \exp\left(-\frac{e\Delta U}{kT_e}\right)}$$

By measuring I_1, I_2, I_3, with given U_1 & U_3

T_e, & $I_{i0} \Rightarrow n_0$ can be obtained directly.

$$\frac{I_1+I_2}{I_3+I_2} \equiv \Pi = \frac{1-\exp\left(-\frac{eU_1}{kT_e}\right)}{1-\exp\left(\frac{eU_3}{kT_e}\right)} \Rightarrow \text{solve for } T_e$$

$$\cancel{\Pi - \Pi \exp\left(\frac{eU_3}{kT_e}\right) = 1 - \exp\left(-\frac{eU_1}{kT_e}\right)}$$

solve for I_{i0}

$\Downarrow$

n_0