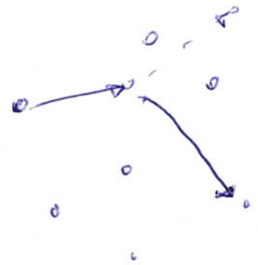
# §1 Introduction

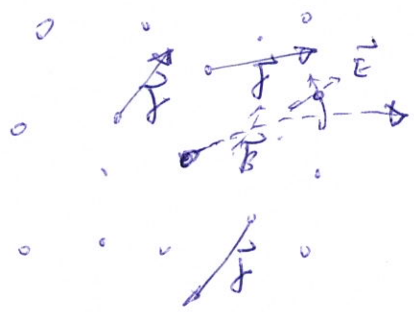
## §1.1 Definition of Plasma

- A plasma is a <u>quasineutral</u> gas of <u>charged and neutral particles</u> which exhibits <u>collective behavior</u>.

* For a molecule, it moves undisturbed until it makes a "collision" with another.



* For ~~charge~~ plasma, which has charged particles, fields effect the motion of other charged particle far away.




$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{E}_{micro} = \vec{E} + \delta\vec{E}$$

$$\vec{E} \equiv \langle \vec{E}_{micro} \rangle$$

$$\vec{B}_{micro} = \vec{B} + \delta\vec{B}$$

$$\vec{B} = \langle \vec{B}_{micro} \rangle$$

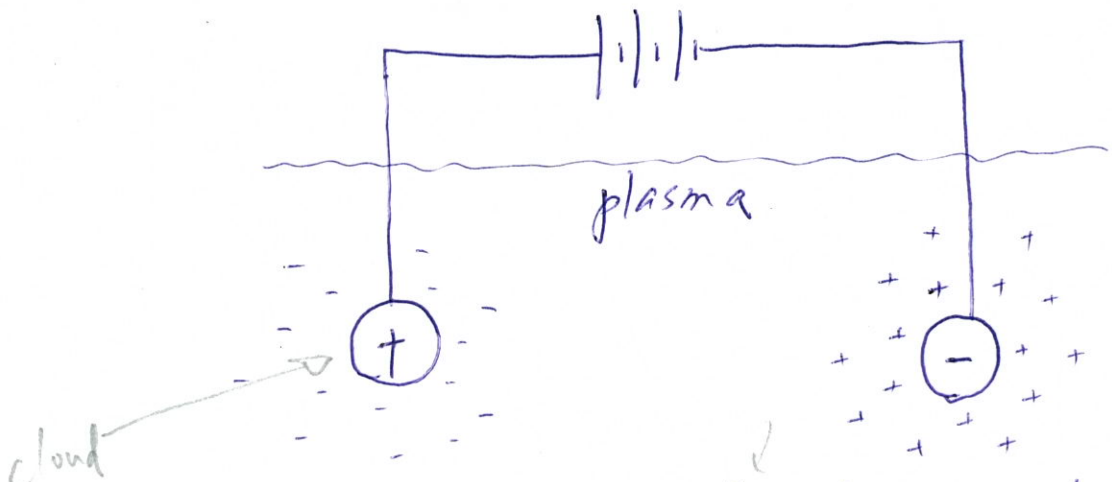
collision term

* "Collisionless" plasmas: the long-range electromagnetic forces are so much larger than the forces due to ordinary local collisions that the latter can be neglected altogether.

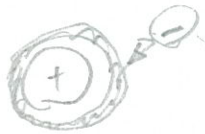
* "Collective behavior" means motion that depend not only on local conditions but on the state of the plasma in remote regions as well.

## §1.2. Debye Shielding

- A plasma is able to shield out electric potentials that are applied to it.



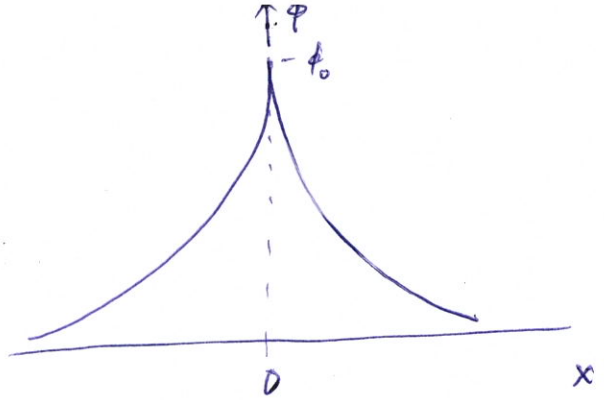

* If the plasma were "cold" (T=0) and there were no thermal motions, there would be just as many charges in the cloud as in the ball; the shielding would be perfect, and no electric field would be present in the body of the plasma outside of the clouds.



* If the temperature is "finite", those particles that are at the edge of the cloud, where the electric field is weak, have enough thermal energy to escape from the electrostatic potential well.
The "edge" of the cloud then occurs at the radius where the potential energy is approximately equal to the thermal energy $KT$ of the particles, and the shielding is not complete. Potentials of the order of $KT/e$ can ~~l~~ leak into the plasma and cause finite electric fields to exist here.


$$\left.\begin{aligned} V_+ &= \frac{kq}{r^2} \\ V_- &= \frac{k(-q)}{r^2} \end{aligned}\right\} \Rightarrow \underline{V_+ + V_- = 0}$$

$$V_- \neq \frac{k(-q)}{r^2} \Rightarrow \underline{V_+ + V_- \neq 0}$$




Assuming $\frac{M}{m} \to \infty$    M: ion mass
m: electron mass

$\Rightarrow$ ions do not move but form a uniform background of positive charge.

Poisson's eq:

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0} \Rightarrow \epsilon_0 \frac{d^2\phi}{dx^2} = -e(n_i - n_e) \quad \text{for } Z=1$$

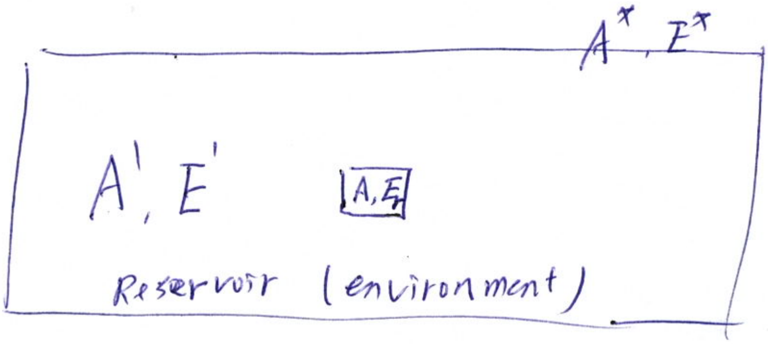
* Since ion doesn't move, $n_i = n_\infty$

* For electron, the electron distribution function:

$$f(u) = A \cdot \exp\left[-\frac{(\frac{1}{2}mu^2 + e\phi)}{kT_e}\right]$$

$\to$ There are fewer particles at places where the potential energy is large, since not all particles have enough energy to get there.

* Boltzman distribution (canonical distribution) P12




$A^*$ — total system with energy $E^*$

$A$ - small system with energy $E_r$ at state r

$A'$ - environment (reservoir) with energy $E'$

$$E^* = E_r + E', \qquad E' \gg E_r$$

$\Omega(E) \to$ # of states with energy $E$

$\because$ the small system is at state r w/ energy $E_r$,

$\therefore$ ~~the only one~~ $\Omega(E_r) = 1$

$\Omega'(E') = \Omega'(E^* - E_r)$ = # of state of the environment.

$\because$ the isolated system $A^*$ is equally likely to be found in each one of its accessible states.

$$P_r \propto \Omega'(E^* - E_r) \qquad \Omega'(E')$$

Note that $E^* \ggg E_r$

$$\ln P_r \propto \ln[\Omega'(E^* - E_r)]$$

$$\simeq \ln[\Omega'(E^*)] - \underbrace{\frac{\partial \ln \Omega'}{\partial E'}}_{\beta} E_r \equiv \ln[\Omega'(E^*)] - \beta E_r$$

$$\beta \equiv \frac{1}{kT}$$

$$\Rightarrow P_r \propto \Omega'(E^* - E_r) \simeq \underbrace{\Omega'(E^*)}_{const} e^{-\beta E_r} \Rightarrow P_r = C e^{-\beta E_r}$$

$\ln(\Omega(E^* + \Delta E)) = \ln(\Omega(E^*)) + \left.\frac{\partial \ln(\Omega(E))}{\partial E}\right|_{E^*} \Delta E$

$\frac{\partial \ln(\Omega(E))}{\partial \Omega(E)} \left.\frac{\partial \Omega(E)}{\partial E}\right|_{E^*} = \frac{1}{\Omega(E)} \frac{\partial \Omega(E)}{\partial E}$

$-E_r = \left.\frac{\partial \ln \Omega(E)}{\partial E}\right|_{E^*} \equiv \beta \equiv \frac{1}{kT}$

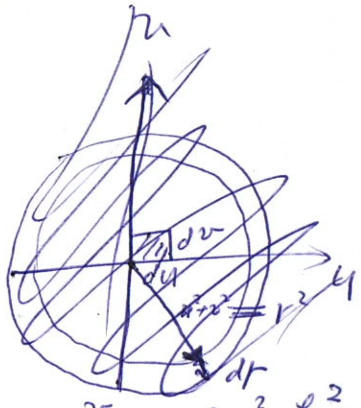
To obtain $n_e(\phi)$

$$n_e(\phi) = \int_{-\infty}^{\infty} A\, e^{-\frac{\left(\frac{1}{2}mu^2 + q\phi\right)}{kT}}\, du$$

$q = -e$

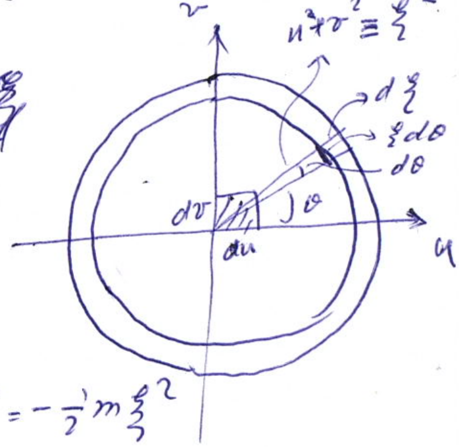
$$= A\, e^{\frac{e\phi}{kT}} \underbrace{\int_{-\infty}^{\infty} e^{-\frac{1}{2}mu^2}\, du}_{\equiv I}$$

$$I^2 = I \cdot I = \int_{-\infty}^{\infty} e^{-\frac{1}{2}mu^2}\, du \int_{-\infty}^{\infty} e^{-\frac{1}{2}mv^2}\, dv$$




$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{1}{2}m(u^2+v^2)}\, du\, dv$$

$$= \cancel{\int_0 d\theta \int_0^{\infty} d\xi} = \iint \xi\, d\theta \cdot d\xi\; e^{-\frac{1}{2}m\xi^2}$$




$$= \int_0^{2\pi} d\theta \int_0^{\infty} \xi\, d\xi\; e^{-\frac{1}{2}m\xi^2}$$

let $X = -\frac{1}{2}m\xi^2$

$dX = -m\xi\, d\xi$

$$= 2\pi \int \frac{-dX}{m}\, e^{X} = -\frac{2\pi}{m}\, e^{X}\Big] = -\frac{2\pi}{m}\, e^{-\frac{1}{2}m\xi^2}\Big]_0^{\infty}$$

$$= \frac{2\pi}{m}\, e^{-\frac{1}{2}m\xi^2}\Big|_{\infty}^{0} = \frac{2\pi}{m} \Rightarrow I = \sqrt{\frac{2\pi}{m}}$$

$$\Rightarrow n_e(\phi) = A \cdot e^{\frac{e\phi}{kT}} \cdot \sqrt{\frac{2\pi}{m}}$$

For $\phi \to 0$, i.e., $x \to \infty$, $n_e \to n_\infty$

$$\Rightarrow n_e(0) = A \cdot \sqrt{\frac{2\pi}{m}} = n_\infty$$

$$\Rightarrow \underline{n_e = n_\infty e^{\frac{e\phi}{kT}}}$$

$$mn\left[\frac{\partial \vec{v}}{\partial t} + \cancel{(\vec{v}\cdot\nabla)\vec{v}}\right] = qn\vec{E} - \nabla p$$

$$\frac{\partial \vec{v}}{\partial t} = \frac{q}{m}\vec{E} - \frac{\nabla p}{mn}$$

$p = \gamma nkT$, $\vec{E} = -\nabla\phi$

$$\Rightarrow \underset{①}{\frac{\partial \vec{v}}{\partial t}} = -\frac{q}{m}\underset{②}{\nabla\phi} - \frac{\gamma kT}{mn}\underset{③}{\nabla n}$$

① for $m \to 0$, ②, ③ $\gg$ ①, ② $\approx$ ③

$q = -e$

$$\frac{e}{m}\nabla\phi = \frac{\gamma kT}{mn}\nabla n = \frac{\gamma kT}{m}\nabla[\ln n]$$

$$\Rightarrow \phi = \frac{\gamma kT}{em}\ln n$$

or $n = n_0 \exp\left[\frac{e\phi}{\gamma kT}\right]$ ($\gamma = 1$) for isothermal

$$\Rightarrow \epsilon_0 \frac{d^2\phi}{dx^2} = -e(n_i - n_e)$$

$$= -e\left(n_\infty - n_\infty e^{\frac{e\phi}{kT_e}}\right)$$

$$= e n_\infty \left[e^{\frac{e\phi}{kT_e}} - 1\right]$$

For $\frac{e\phi}{kT_e} \ll 1$, i.e. far away from the charge.

$$\epsilon_0 \frac{d^2\phi}{dx^2} \simeq e n_\infty \left[\cancel{1} + \left(\frac{e\phi}{kT_e}\right) + \frac{1}{2}\left(\frac{e\phi}{kT_e}\right)^2 + \cdots - \cancel{1}\right]$$

$$\simeq \frac{n_\infty e^2}{kT_e}\phi \quad \Rightarrow \quad \left(\frac{\epsilon_0 kT_e}{ne^2}\right)\frac{d^2\phi}{dx^2} = \phi$$

where $n \equiv n_\infty$

$$\Rightarrow \underline{\lambda_D \equiv \sqrt{\frac{\epsilon_0 kT_e}{ne^2}}} \quad \Rightarrow \lambda_D^2 \frac{d^2\phi}{dx^2} = \phi$$

$$\Rightarrow \underline{\phi = \phi_0 \exp\left(-\frac{|x|}{\lambda_D}\right)}$$

$\lambda_D$ is called the "Debye length" which is a measure of the shielding distance or thickness of the sheath.

* Useful forms:

$$\lambda_D = 69\left(\frac{T}{n}\right)^{1/2} \ (m), \quad T \text{ in kelvin}$$

$$= 7430\left(\frac{KT}{n}\right)^{1/2} \ (m), \quad KT \text{ in "eV"}$$

* "quasineutrality": the dimensions of a system $L$ are much ~~than~~ larger than $\lambda_D$, local charges are always shielded out in a distance short compared w/ $L$. ⇒ free of large electric potentials or fields.

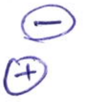
⇒ $n_i \simeq n_e \simeq n$ → plasma density.

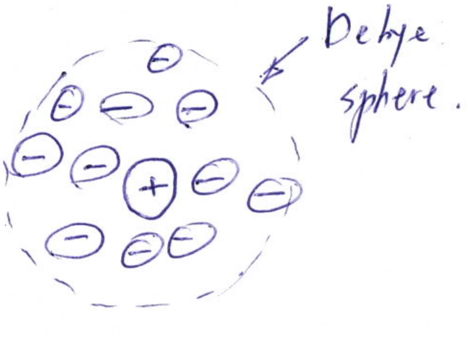
* A criterion for an ionized gas to be a plasma is that it be dense enough that (or cold enough)

$\lambda_D \ll L$.

## §1.3 The plasma parameter.

* Debye shielding is valid only if there are enough ~~charged~~ particles in the charge cloud.





$$N_D = n \cdot \frac{4}{3}\pi\lambda_D^3$$
$$= \frac{4\pi}{3} n \cdot \left(\frac{\epsilon_0 k T_e}{n e^2}\right)^{3/2}$$
$$= 1.38 \times 10^6 \frac{T_e^{3/2}}{n^{1/2}}$$

$T_e$ in kelvin.
$n$ in $1/m^3$

"Collective behavior" requires.

$$N_D \gg 1$$

## § 1.4 Criteria for Plasma.

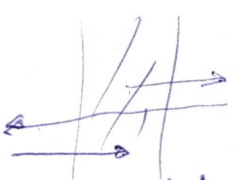
$\omega$: frequency of typical plasma oscillations.

$\tau$: mean time between collisions w/ ~~ne~~ neutral atoms.

→ The weakly ionized gas in a jet exhaust, for example ~~does~~ is NOT ~~not~~ ~~qualify~~ qualified as a plasma because the charged particles collide so frequently w/ neutral atoms that their motion is controlled by ordinary hydrodynamic forces rather than by electromagnetic forces.

$\omega\tau > 1$ is required to behave like a plasma

$\hookrightarrow 2\pi \frac{\tau}{T} > 1 \Rightarrow \tau > T \Rightarrow$ Not much collision w/ neutral gas within one oscillation.



oscillation.

Criteria for Plasma:

$$\begin{cases} 1. \quad \lambda_D \ll L. \\ 2. \quad N_D \gg 1 \\ 3. \quad \omega\tau > 1. \end{cases}$$