

Application of Plasma Phenomena



Po-Yu Chang

Institute of Space and Plasma Sciences, National Cheng Kung University

Lecture 2

2026 fall semester

Thursday 9:00-12:00

Materials:

<https://capst.ncku.edu.tw/PGS/index.php/teaching/>

Online link:

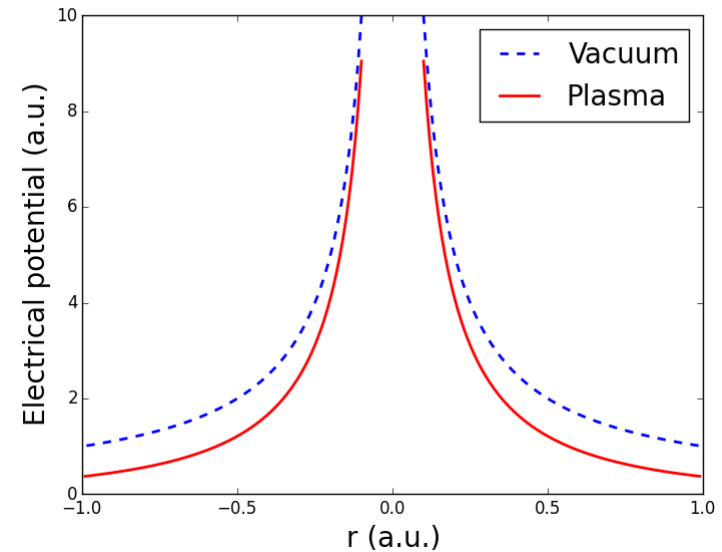
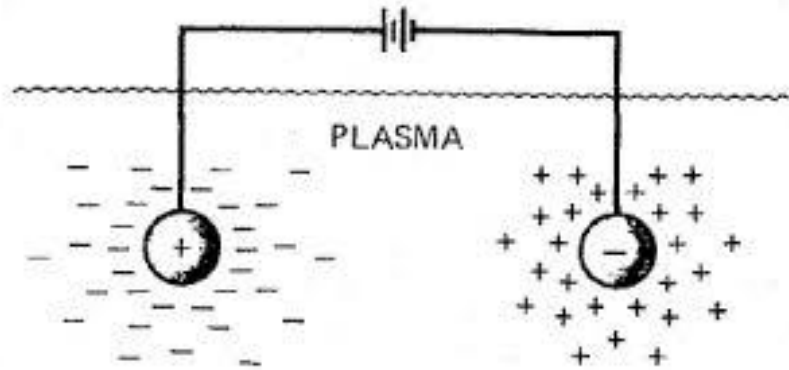
<https://meet.google.com/nac-fqvr-ifj>

There are several Important plasma parameters that need to be considered



- Debye length $\lambda_D \equiv \left(\frac{KT_e}{4\pi n e^2} \right)^{1/2}$
- Plasma parameter $\Lambda \equiv n \frac{4\pi}{3} \lambda_D^3$
- Plasma frequency $\omega_{pe} \equiv \left(\frac{4\pi n_e e^2}{m_e} \right)^{1/2}$
- Collision time $\tau_e \equiv \frac{3\sqrt{m_e}(KT_e)^{3/2}}{4\sqrt{2\pi} n e^4 \ln \Lambda}$
- Hall parameter $\chi \equiv \omega_{ce} \tau_e$, where $\omega_{ce} \equiv \frac{eB}{m_e c}$ is the electron gyrofrequency
- Plasma beta $\beta \equiv \frac{P}{P_B}$, where $P_B \equiv \frac{B^2}{8\pi}$ is the magnetic pressure

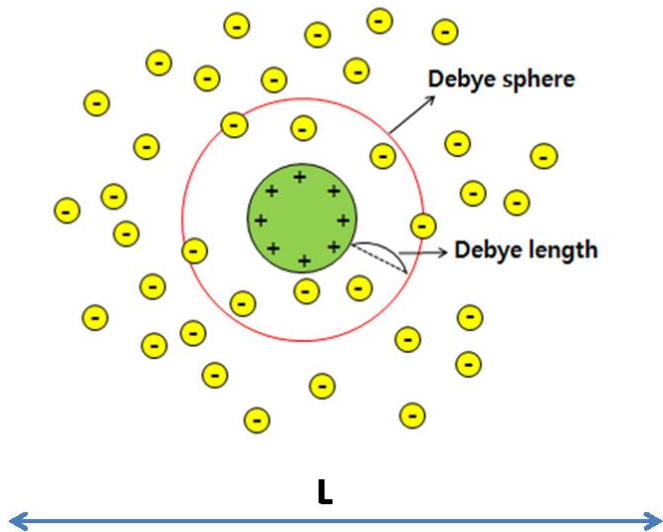
A test ion in the plasma gathers a shielding cloud that tends to cancel its own charge



$$\phi_V = \frac{\phi_0}{r}$$

$$\phi_P = \frac{\phi_0}{r} \exp\left(-\frac{r}{\lambda_D}\right) \quad \lambda_D \approx \left(\frac{KT_e}{4\pi n e^2}\right)^{1/2}$$

Plasma parameter Λ is the number of particles in a sphere with radius of λ_D



- Plasma parameter:

$$\Lambda \equiv n \frac{4\pi}{3} \lambda_D^3$$

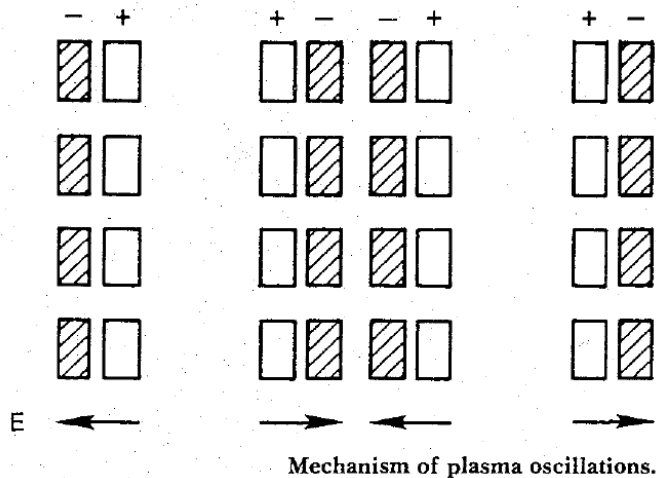
- Criterion for an ionized gas to be plasma:

$$\lambda_D \ll L$$

- Requirement of “collective behavior”:

$$\Lambda \gg 1$$

Electron plasma frequency is the characteristic frequency such that electrons oscillate around their equilibrium positions



- Assumption:

$$\vec{\nabla} \equiv \hat{x} \frac{\partial}{\partial x}, \quad \vec{E} = \hat{x} E, \quad \vec{v}_e = \hat{x} v, \quad ,$$

$$\vec{\nabla} \times \vec{E} = 0, \quad \vec{E} = -\vec{\nabla} \phi$$

- Continuity and momentum equation for electron:

$$\frac{\partial n_e}{\partial t} + \vec{\nabla} \cdot (n_e \vec{v}_e) = 0$$

$$m_e n_e \left[\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e \cdot \vec{\nabla}) \vec{v}_e \right] = -e n_e \vec{E}$$

- Gauss' law:

$$\frac{\partial E}{\partial x} = 4\pi e (n_i - n_e)$$

Electron plasma frequency is obtained by linearizing the hydrodynamic equations



- The oscillation is assumed to be small:

$$n_e = n_0 + n_1, \vec{E} = \vec{E}_0 + \vec{E}_1, v_e = v_0 + v_1$$

where

$$\frac{\partial n_0}{\partial x} = v_0 = \vec{E}_0 = 0$$

$$\frac{\partial n_0}{\partial t} = \frac{\partial v_0}{\partial t} = \frac{\partial \vec{E}_0}{\partial t} = 0$$

- Linearization:

$$m_e \frac{\partial v_1}{\partial t} = -eE_1$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} = 0$$

$$\frac{\partial E_1}{\partial x} = -4\pi en_1$$

- Plane wave solution:

$$\eta_1 = \hat{\eta} \exp[i(kx - \omega t)]$$

$$\eta_1 = v_1, n_1, E_1$$

- Substitute into the previous equations:

$$-im_e \omega v_1 = -eE_1$$

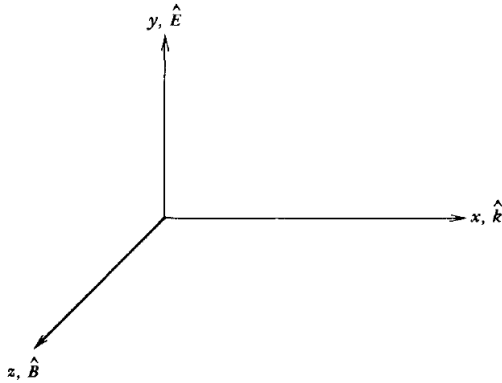
$$-i\omega n_1 = -n_0 ikv_1$$

$$ikE_1 = -4\pi en_1$$

- Electron plasma frequency is obtained by eliminating n_1 and E_1 :

$$\omega_{pe} \equiv \omega = \left(\frac{4\pi n_e e^2}{m_e} \right)^{1/2}$$

Electromagnetic wave can not propagate in a plasma with density higher than critical density



$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \partial_t \vec{B}$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{j} + \frac{1}{c} \partial_t \vec{E}$$

$$\vec{j} = -en_e \vec{v}_e$$

$$\begin{aligned} m_e n_e \partial_t \vec{v}_e + m_e n_e (\vec{v}_e \cdot \vec{\nabla}) \vec{v}_e \\ = -\vec{\nabla} p_e - en_e \vec{E} - \frac{en_e \vec{v}_e}{c} \times \vec{B} \end{aligned}$$

$$\partial_t n_e + \vec{\nabla} \cdot (n_e \vec{v}_e) = 0$$

- Linearization:

$$\vec{\nabla} \times \vec{E}_1 = -\frac{1}{c} \partial_t \vec{B}_1$$

$$\vec{\nabla} \times \vec{B}_1 = \frac{4\pi e}{c} n_0 \vec{v}_e + \frac{1}{c} \partial_t \vec{E}_1$$

$$m_e n_0 \partial_t \vec{v}_1 = -en_0 \vec{E}_1$$

- Eliminate $\partial_t \vec{v}_1$ and $\vec{B}_1$

$$-c \vec{\nabla} \times (\vec{\nabla} \times \vec{E}_1) = \frac{1}{c} \frac{4\pi n_0 e^2}{m_e} \vec{E}_1 + \frac{1}{c} \frac{\partial^2 \vec{E}_1}{\partial t^2}$$

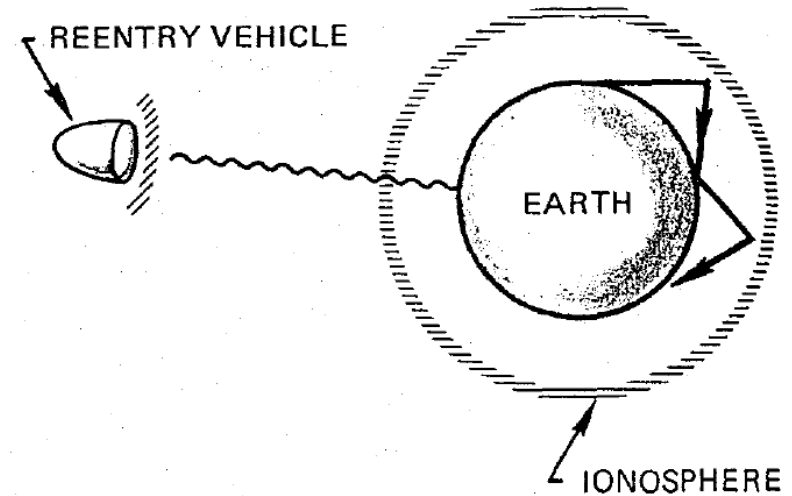
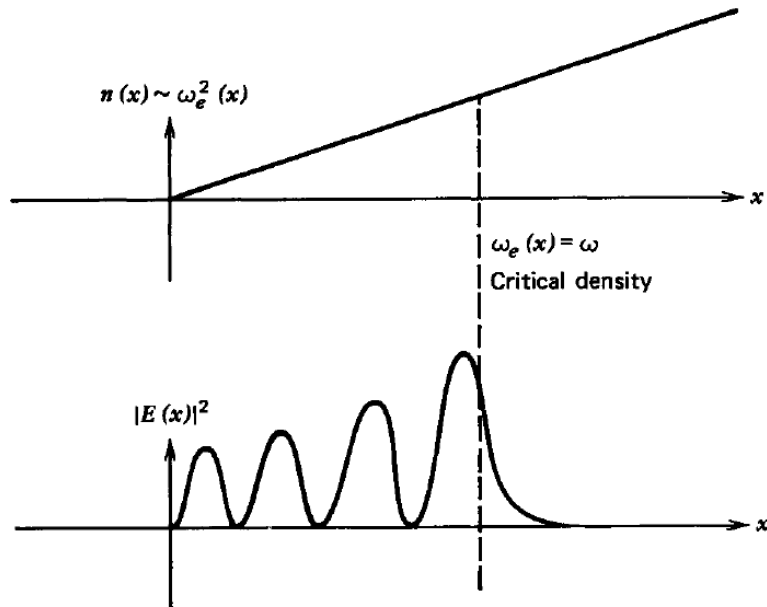
- Take $\vec{\nabla} \times (\vec{\nabla} \times) \rightarrow k^2$ and $\partial_t \rightarrow \omega$

$$(\omega^2 - k^2 c^2 - \omega_e^2) \vec{E}_1 = 0$$

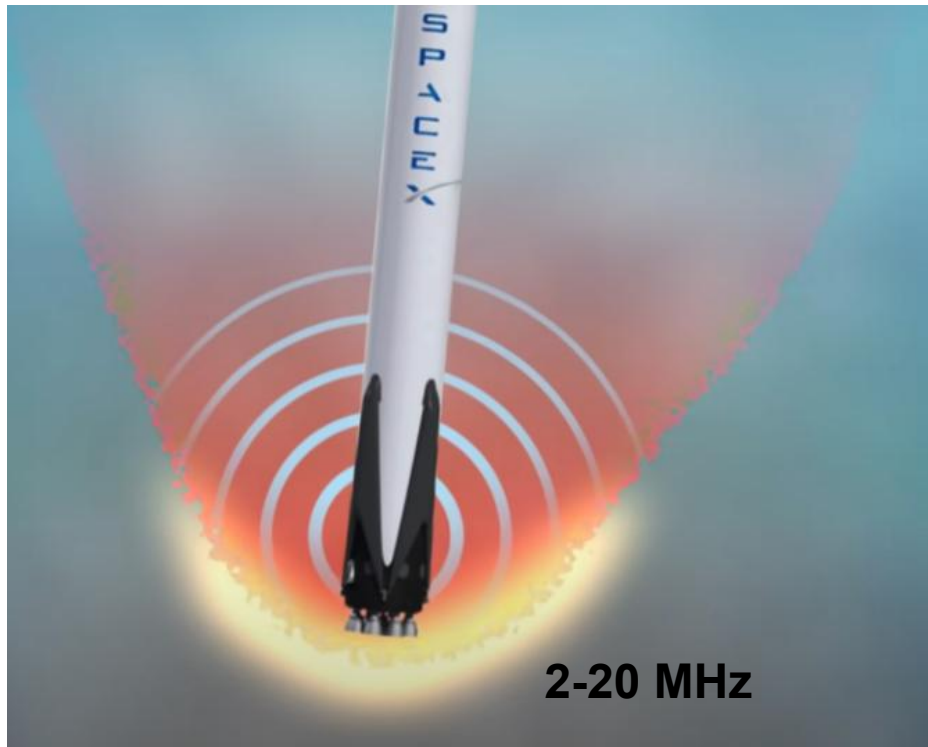
$$\omega^2 - k^2 c^2 - \omega_e^2 = 0$$

- Wave number k becomes imaginary when $\omega < \omega_e$.

The cutoff of the electromagnetic wave is important in laser fusion and in the interaction of radio waves with the ionosphere



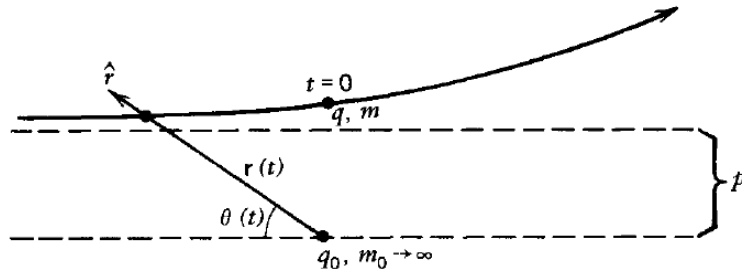
SpaceX moves their S-band transmitter to the top of their rocket to avoid communication blackout



Plasma was generated during the starship re-entry on March 14, 2024



Charged particles collide with each other through coulomb collisions



$$mv_{\perp} = \int_{-\infty}^{\infty} dt F_{\perp}(t)$$

- Coulomb force:

$$m \ddot{\vec{r}} = \frac{qq_0}{r^2} \hat{r}$$

$$F_{\perp} = \frac{qq_0}{p^2} \sin^3 \theta$$

- Relation between θ and t is

$$x = -r \cos \theta = -\frac{p \cos \theta}{\sin \theta} = v_0 t$$

- Therefore,

$$v_{\perp} = \frac{qq_0}{mv_0 p} \int_0^{\pi} d\theta \sin \theta = \frac{2qq_0}{mv_0 p} \equiv \frac{v_0 p_0}{p}$$

where $p_0 \equiv \frac{2qq_0}{mv_0^2}$

- Note that this is valid only when $v_{\perp} \ll v_0$, i.e., $p \gg p_0$.

Cumulative effect of many small angle collisions is more important than large angle collisions



- Consider a variable Δx that is the sum of many small random variables Δx_i , $i=1,2,3,\dots,N$,

$$\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 + \dots + \Delta x_N = \sum_{i=1}^N \Delta x_i$$

- Suppose $\langle \Delta x_i \rangle = \langle \Delta x_i \Delta x_j \rangle_{i \neq j} = 0$

$$\langle (\Delta x)^2 \rangle = \left\langle \left(\sum_{i=1}^N \Delta x_i \right)^2 \right\rangle = \sum_{i=1}^N \langle (\Delta x_i)^2 \rangle = N \langle (\Delta x_i)^2 \rangle$$

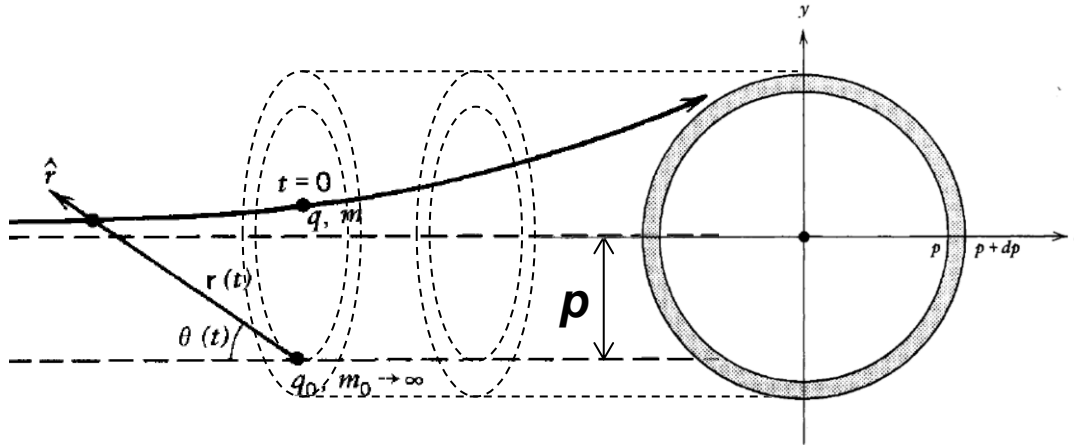
- For one collision:

$$\langle v_{\perp}^2 \rangle = \langle (\Delta v_x)^2 \rangle + \langle (\Delta v_y)^2 \rangle = \frac{v_0^2 p_0^2}{p^2} \quad \langle (\Delta v_x)^2 \rangle = \langle (\Delta v_y)^2 \rangle = \frac{1}{2} \frac{v_0^2 p_0^2}{p^2}$$

- The total velocity in $\hat{x}$

$$\langle (\Delta v_x^{\text{tot}})^2 \rangle = N \langle (\Delta v_x)^2 \rangle = \frac{N}{2} \frac{v_0^2 p_0^2}{p^2}$$

The collision frequency can be obtained by integrating all the possible impact parameter



- Number of collisions in a time interval:

$$dN = n_0 2\pi p dp v_0 dt$$

i.e., $\frac{dN}{dt} = 2\pi p dp n_0 v_0$

- Therefore

$$\begin{aligned} \frac{d}{dt} \langle (\Delta v_x^{\text{tot}})^2 \rangle &= \frac{1}{2} \frac{v_0^2 p_0^2}{p^2} \frac{dN}{dt} \\ &= \pi n_0 v_0^3 p_0^2 \frac{dp}{p} \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} \langle (\Delta_{\perp}^{\text{tot}})^2 \rangle &= 2 \frac{d}{dt} \langle (\Delta v_x^{\text{tot}})^2 \rangle \\ &= 2\pi n_0 v_0^3 p_0^2 \int_{p_{\min}}^{p_{\max}} \frac{dp}{p} \\ &= 2\pi n_0 v_0^3 p_0^2 \ln \left(\frac{p_{\max}}{p_{\min}} \right) \\ &\approx 2\pi n_0 v_0^3 p_0^2 \ln \left(\frac{\lambda_D}{|p_0|} \right) \\ &\approx 2\pi n_0 v_0^3 p_0^2 \ln \Lambda \end{aligned}$$

- Note that

$$\lambda_D \approx \left(\frac{KT_e}{4\pi n_0 e^2} \right)^{1/2}$$

$$\begin{aligned} \frac{\lambda_D}{|p_0|} &\approx \frac{\lambda_D m_e v_e^2}{2e^2} \approx \frac{\lambda_D KT_e}{e^2} \approx 4\pi n_0 \lambda_D^3 \\ &\approx \Lambda \end{aligned}$$

Comparison between the mean free path and the system size L determines the regime of the plasma



- A reasonable definition for the scattering time due to small angle collisions is the time it takes $\langle (\Delta v_{\perp}^{\text{tot}})^2 \rangle$ to equal v_0^2 . The collision frequency ν_c due to small-angle collisions:

$$\frac{d}{dt} \langle (\Delta v_{\perp}^{\text{tot}})^2 \rangle \approx 2\pi n_0 v_0^3 p_0^2 \ln \Lambda \approx v_0^2 \nu_c, \quad p_0 \equiv \frac{2qq_0}{m_e v_0^2} \Rightarrow \nu_c = \frac{8\pi n_0 e^4 \ln \Lambda}{m_e^2 v_0^3}$$

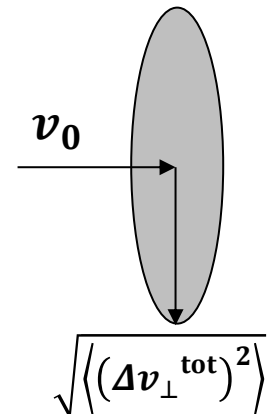
- With more careful derivation, the collisional time is obtained:

$$\tau_e^{-1} = \nu_c = \frac{4\sqrt{2\pi} n e^4 \ln \Lambda}{3\sqrt{m_e} (KT_e)^{3/2}}$$

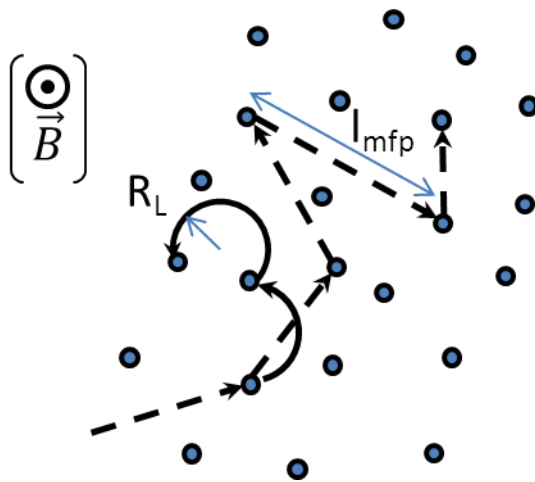
- Mean free path:

$$l_{\text{mfp}} = v_e \tau_e$$

$$\begin{cases} l_{\text{mfp}} < L & \text{Fluid Theory} \\ l_{\text{mfp}} > L & \text{Kinetic Theory} \end{cases}$$



Thermal conduction perpendicular to the magnetic field can be suppressed when the plasma is magnetized



$$m_e \frac{d\vec{v}}{dt} = -\frac{e}{c} \vec{v} \times \vec{B}$$

- Assuming $\vec{B} = B\hat{z}$ and the electron oscillates in x-y plane

$$\begin{aligned} m_e \dot{v}_x &= -\frac{e}{c} B v_y \\ m_e \dot{v}_y &= \frac{e}{c} B v_x \end{aligned} \quad m_e \dot{v}_z = 0$$

- Plasma is magnetized when

$$\frac{R_L}{l_{\text{mfp}}} = \frac{v_e}{\omega_{ce}} \frac{1}{v_e \tau_e} < 1$$

i.e., the hall parameter

$$\chi \equiv \omega_{ce} \tau_e > 1$$

$$\begin{aligned} \ddot{v}_x &= -\frac{eB}{m_e c} \dot{v}_y = -\left(\frac{eB}{m_e c}\right)^2 v_x \\ \ddot{v}_y &= -\frac{eB}{m_e c} \dot{v}_x = -\left(\frac{eB}{m_e c}\right)^2 v_y \end{aligned}$$

- Therefore

$$\omega_{ce} = \frac{eB}{m_e c}$$

Plasma β is the ratio between hydro pressure and magnetic pressure



- Momentum equation in Magnetohydrodynamics (MHD) approach:

$$\rho \frac{d \vec{v}}{dt} + \rho (\vec{v} \cdot \vec{\nabla}) \vec{v} = - \vec{\nabla} p + \frac{1}{c} \vec{j} \times \vec{B}$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{j}$$

$$\vec{j} \times \vec{B} = \frac{c}{4\pi} (\vec{\nabla} \times \vec{B}) \times \vec{B} = \frac{c}{4\pi} \left[(\vec{B} \cdot \vec{\nabla}) \vec{B} - \frac{1}{2} \vec{\nabla} B^2 \right]$$

$$\rho \frac{d \vec{v}}{dt} + \rho (\vec{v} \cdot \vec{\nabla}) \vec{v} = - \vec{\nabla} \left(p + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} (\vec{B} \cdot \vec{\nabla}) \vec{B}$$

- Magnetic pressure: $\frac{B^2}{8\pi}$

$$\beta \equiv \frac{p}{B^2/8\pi}$$

- Magnetic tension: $\frac{1}{4\pi} (\vec{B} \cdot \vec{\nabla}) \vec{B}$

There are several Important plasma parameters that need to be considered

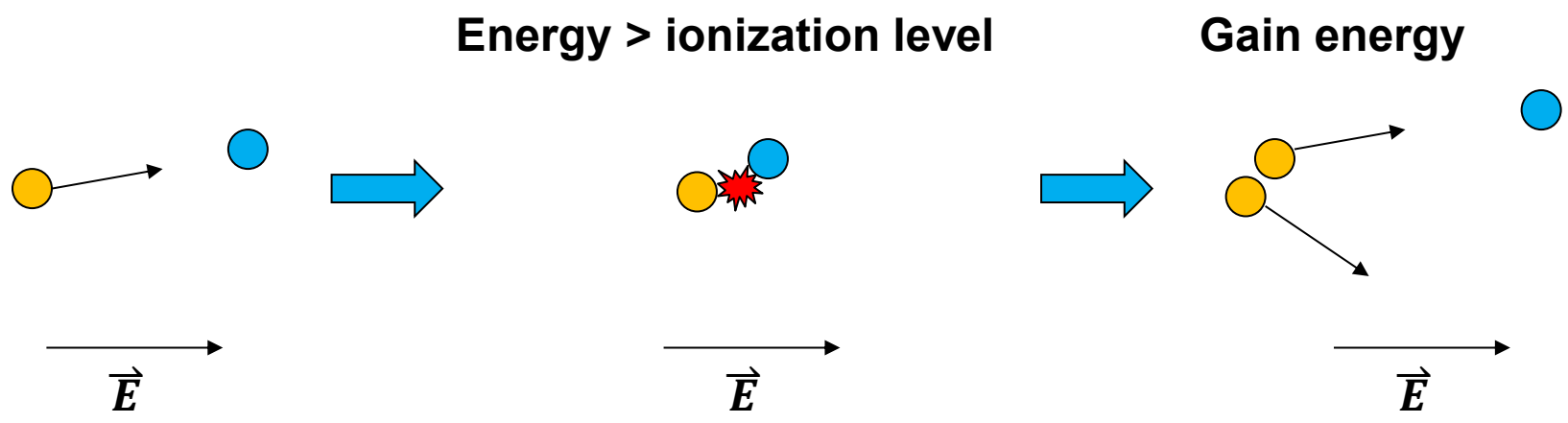


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- Hall parameter $\chi \equiv \omega_{ce} \tau_e$, where $\omega_{ce} \equiv \frac{eB}{m_e c}$ is the electron gyrofrequency
- Plasma beta $\beta \equiv \frac{P}{P_B}$, where $P_B \equiv \frac{B^2}{8\pi}$ is the magnetic pressure

Collisions play an important role in ionization process



- At the microscopic level, breakdown requires the presence of sufficiently energy charge particles that have acquired enough energy from the applied electric field between two energy-dissipating collisions to ionize the material and to create more charge particles.



In most cases, electrons dominate the breakdown process since its mobility is much larger than that of ions



$$E_k = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$E_k \sim kT$$

Collision time: $t = \frac{s}{\sqrt{\frac{2E_k}{m}}} \sim \frac{n^{-1/3}}{\sqrt{T}} \sqrt{m}$

$$n = \frac{\#}{V} \sim \frac{\#//}{S^3}$$

$$s \sim n^{-1/3}$$

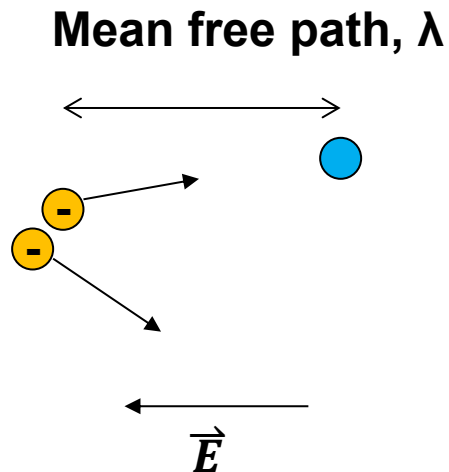
$$\frac{m_i}{m_e} \sim 2000 \times \text{Atomic mass}$$

$$\frac{t_i}{t_e} \sim 45 \times \sqrt{A}$$

Mean free path is important in ionization process



- For an electron to acquire enough energy between collisions, its mean free path in the material must be sufficiently long.



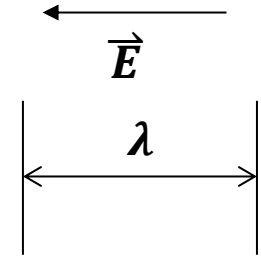
$$E_k = e \times E \times \lambda = eV$$

Kinetic energy needs to be greater than the ionization energy to ionize the gas



- Between each collision, the kinetic energy increases.

$$\lambda eE = \frac{1}{2}mv^2 \quad v = \sqrt{\frac{2eE\lambda}{m}}$$

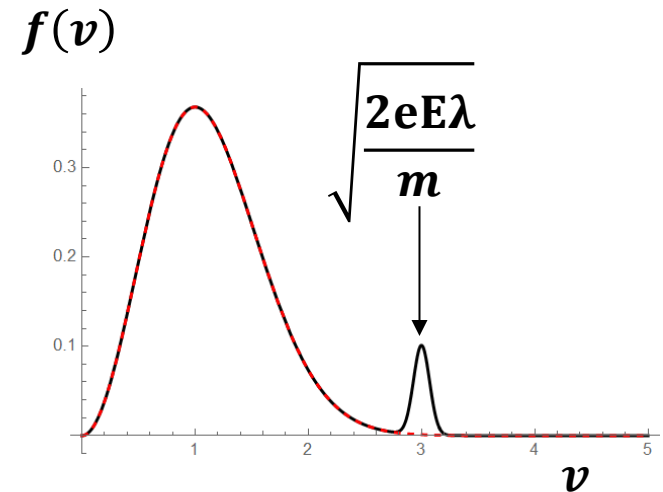


- Mean time between ionization collisions for electron with velocity v :

$$\tau = \frac{\lambda}{v}$$

- The rate of ionization is:

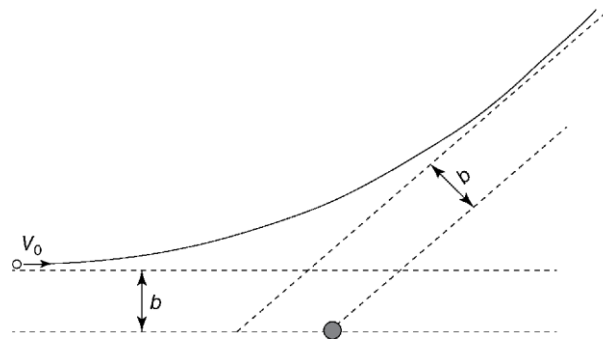
$$\frac{1}{\tau} = \frac{v}{\lambda} = \Sigma v$$



Collisions can be elastic or inelastic



- **Elastic collisions – NO energy exchanges. Momentum is redistributed.**



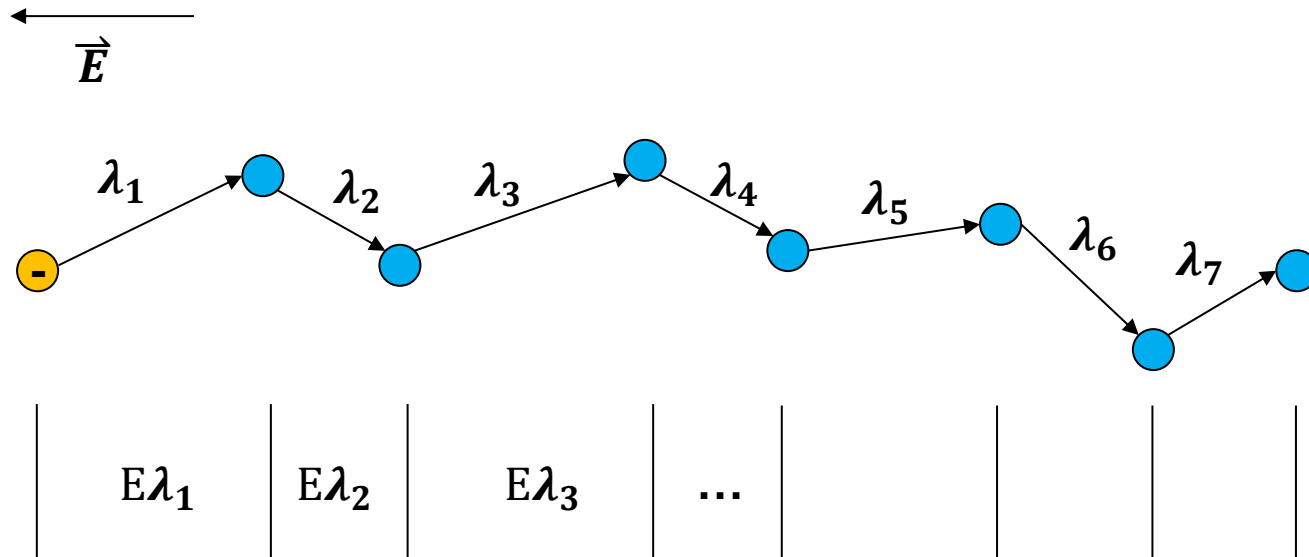
- **Inelastic collisions – energy is exchanged between the collision partners
→ production of molecules & particles.**
 - **A portion of the kinetic energy before collision is converted to potential energy of one of the particles in the system.**
 - **Ionization: $A + B \rightarrow A + B^+ + e^-$**
 - **The process of ionization is dominated by e^- acceleration in an electric field and is greatly aided by the appearance of initiatory electrons: (1) ionization in the gas; (2) emission from the cathode.**

Electron impact ionization is the most important process in a breakdown of gases



- Electron impact ionization: $A + e^- \rightarrow A^+ + e^- + e^-$
 - The most important process in the breakdown of gases but is not sufficient alone to result in the breakdown.

$$eE\lambda_{e,i} \geq eV_i \quad V_i: \text{ionization potential}$$



Photoionization & collisions with excited molecules

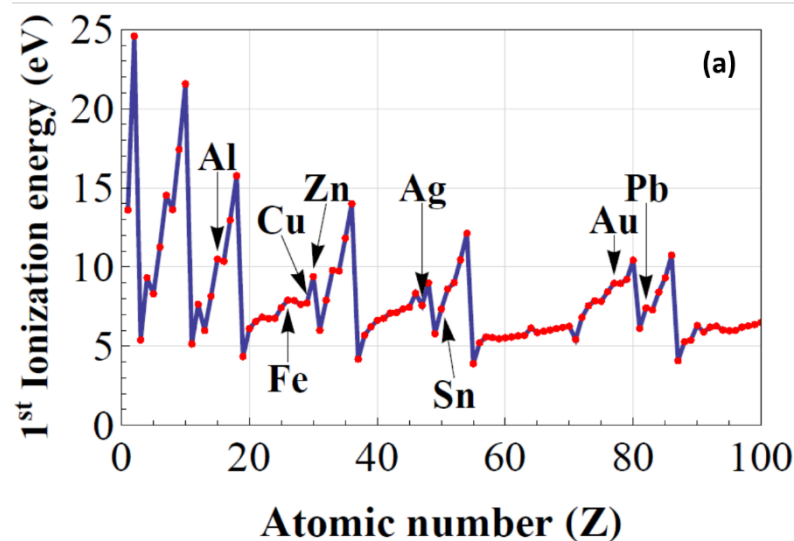


• Metastable production (1~10 ms life time):	$A + B \rightarrow A^* + B$
• Electron impact excitation:	$A + e^- \rightarrow A^* + e^-$
• Step ionization:	$A^* + e^- \rightarrow A^+ + e^- + e^-$
• De-excitation:	$A^* + e^- \rightarrow A + e^- + h\nu$
• Radiative recombination:	$A^+ + e^- \rightarrow A + h\nu$
• Dielectronic excitation:	$A^* + e^- \rightarrow A^{**} + e^-$
• Autoionization:	$A^{**} \rightarrow A^+ + e^-$
• Dielectronic recombination:	$A^{**} \rightarrow A + h\nu$
• Step photoionization:	$A^* + h\nu \rightarrow A^+ + e^-$
• Photoionization:	$A + h\nu \rightarrow A^+ + e^-$

Photoionization is very complex



- Photons with $\lambda=125$ nm (UV) @ 9.9 eV can ionize almost all gases despite that almost all molecules and atom have ionization energy > 9.9 eV!
- Dust or water vapor can emit electrons through photon absorption.
- All photoionization occurs between 6~ 50 eV.



A. Kramida, Yu. Ralchenko, J. Reader, and NIST ASD Team.
NIST Atomic Spectra Database (ver. 5.5.1), [Online]. Available:
<https://physics.nist.gov/asd> [2017, December 24]. National
Institute of Standards and Technology, Gaithersburg, MD., 2017.

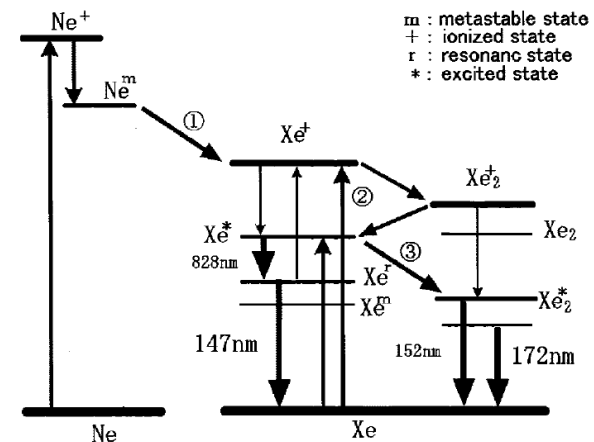
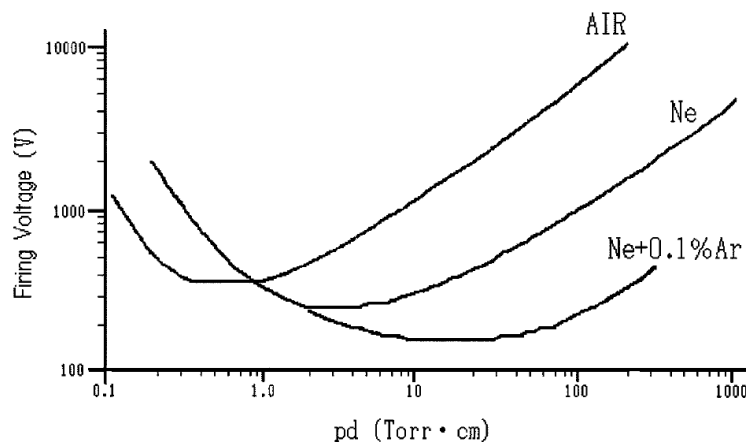
Penning ionization – breakdown voltage may reduce with mixture of inert gas



- $A^* + B^* \rightarrow A^+ + B + e^-$
- May be from impurities or engineered mixture called penning mixture.
- A penning mixture is a mixture of an inert gas with a small amount of a quench gas, which has lower ionization potential than the 1st excited state of the inert gas.
- Ex: neon lamp: Ne + Ar (<2%)

plasma display: He/Ne + Xe

Gas ionization detector: Ar/Xe, Ne/Ar, Ar/acetylene(乙炔)

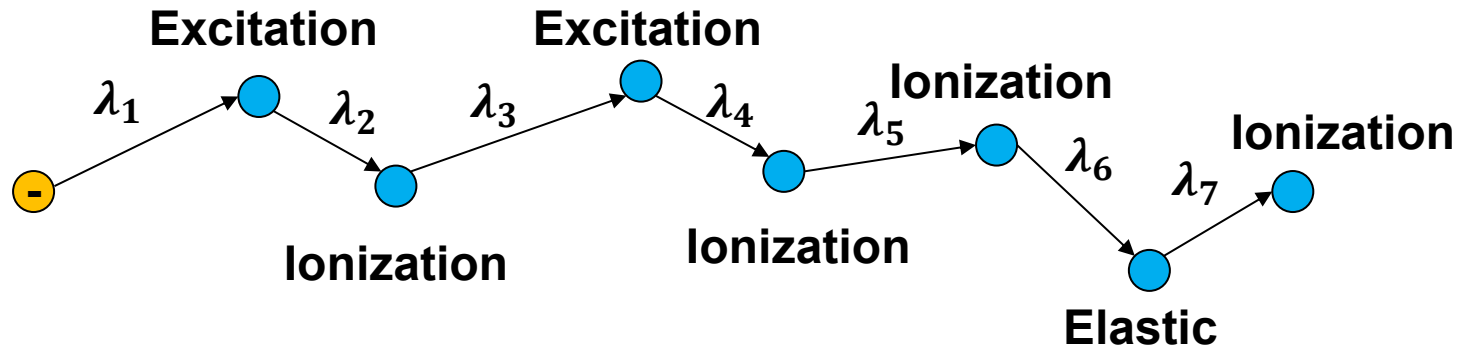


More complex collisions



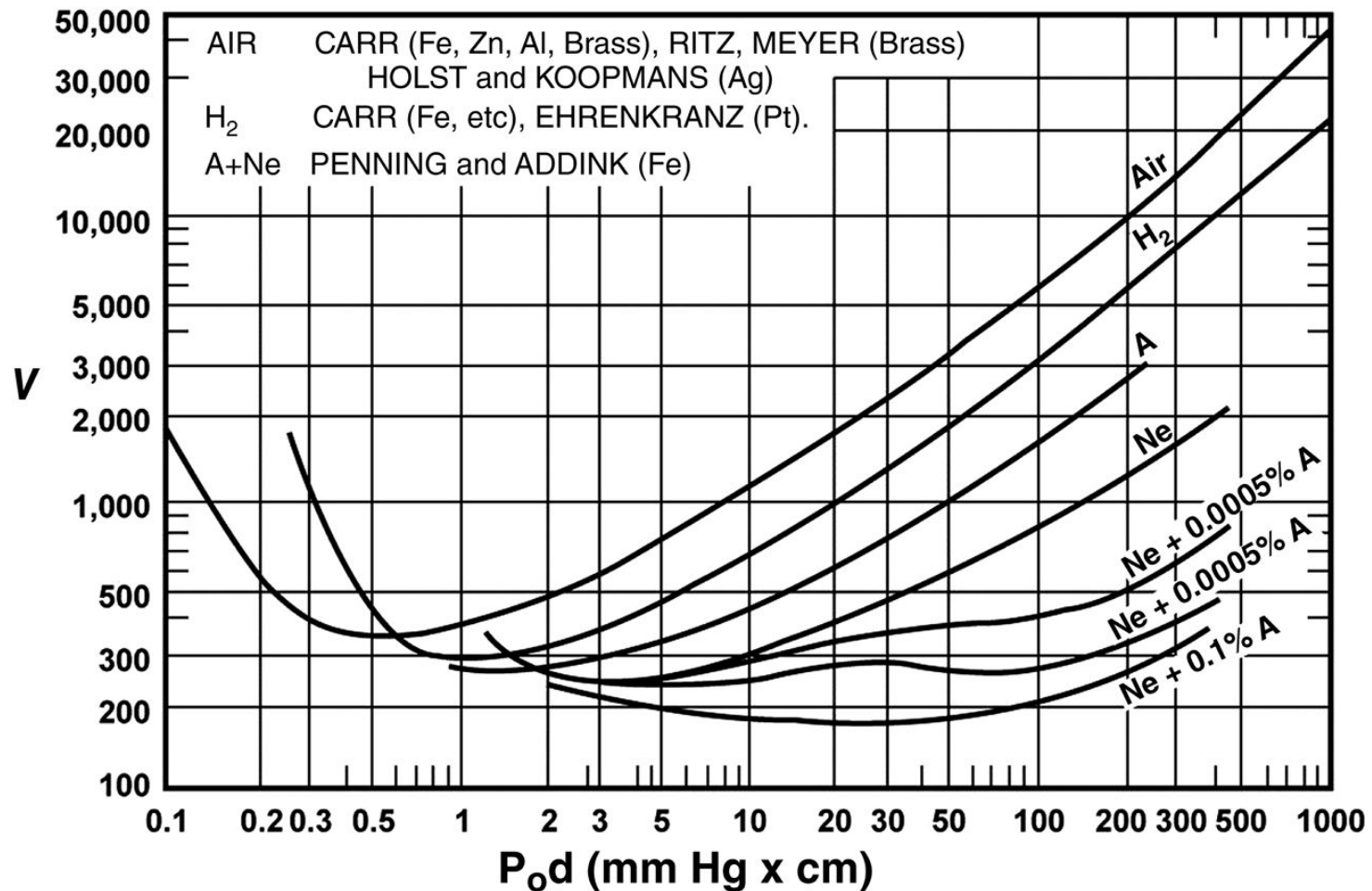
- 3-body collision: $A^+ + e^- + e^- \rightarrow A^* + e^-$
- Ion impact excitation: $A^+ + B \rightarrow A^+ + B^*$
- 3-body collision: $A^+ + B + e^- \rightarrow A^* + B$
- Ion impact ionization: $A^+ + B \rightarrow A^+ + B^+ + e^-$
- Total collisional cross section:

$$\sigma(\nu) = \sigma_{\text{el}} + \sigma_{\text{ex}} + \sigma_{\text{ion}} + \cdots = \sum_i \sigma_i$$



- Inert gas can be ionized easier since there are less exciting state compared to gas molecules.
- Molecules, e.g., SF_6 , dry air (with O_2), that capture electron easier provides a higher breakdown voltage.

Breakdown voltage of different gas



Methods of plasma production



- **DC electrical discharges**
 - Dark electrical discharges in gases
 - DC electrical glow discharges in gases
 - DC electrical arc discharges in gases
- **AC electrical discharges**
 - RF electrical discharges in gases
 - Microwave electrical discharges in gases
 - Dielectric-barrier discharges (DBDs)
- **Other mechanism**
 - Laser produced plasma
 - Pulsed-power generated plasma

Reference



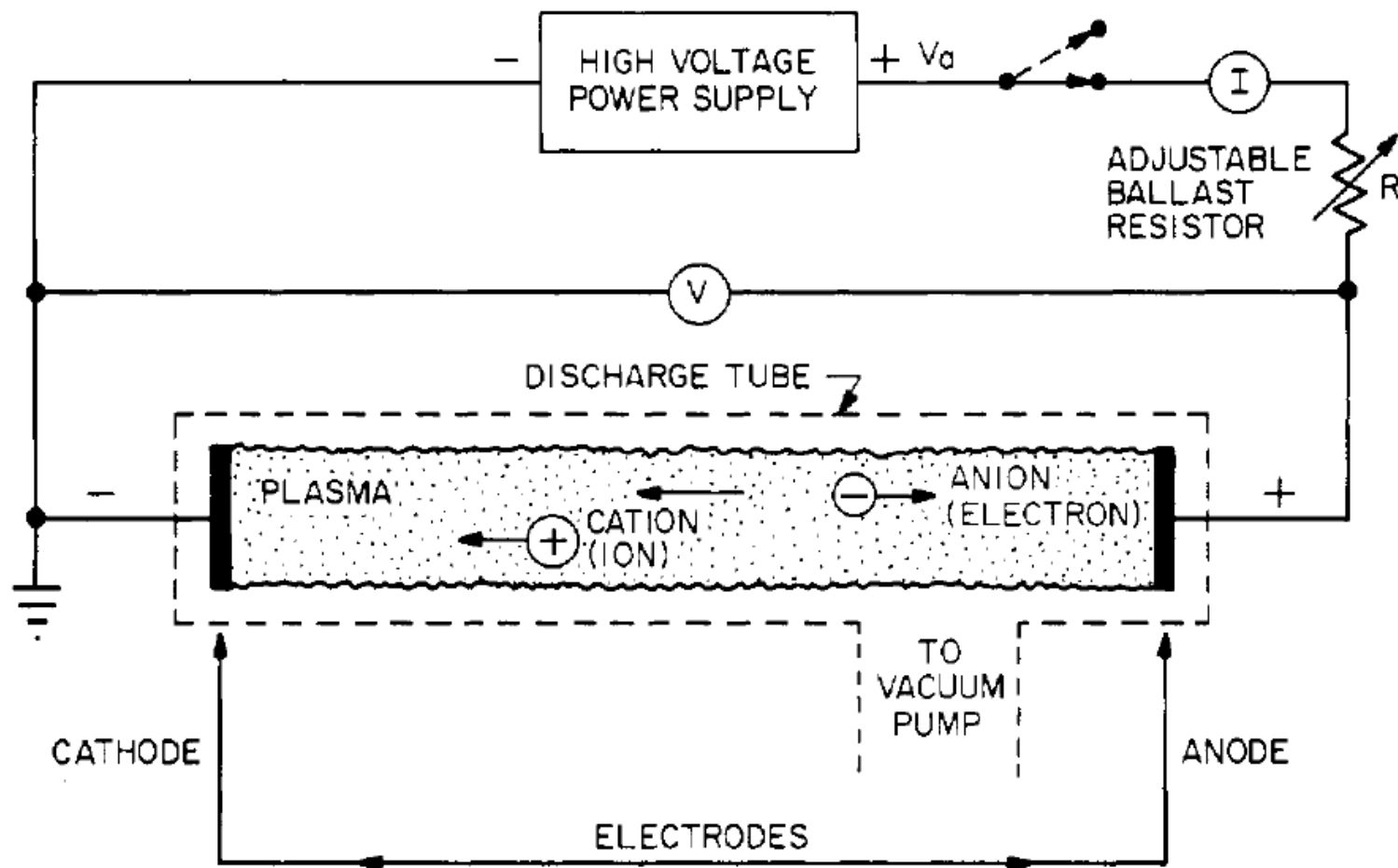
- **Industrial plasma engineering, volume 1, by J. Reece Roth, Chapter 8 - 13.**
- **Plasma physics and engineering, by Alexander Fridman and Lawrence A. Kennedy.**
- **Plasma medicine, by Alexander Fridman and Gary Fridman.**

Methods of plasma production

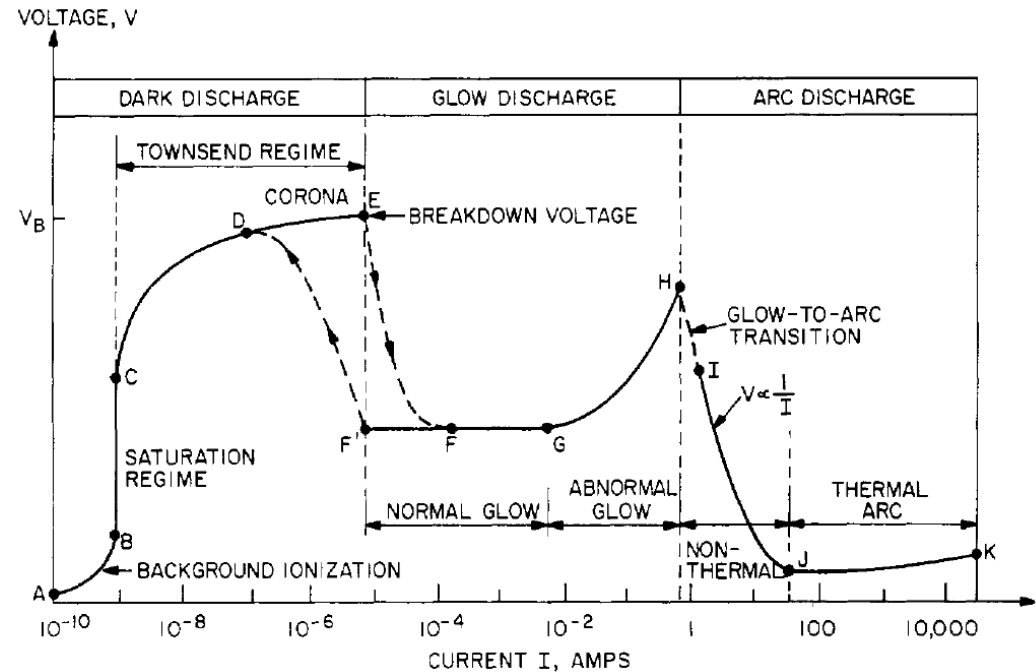
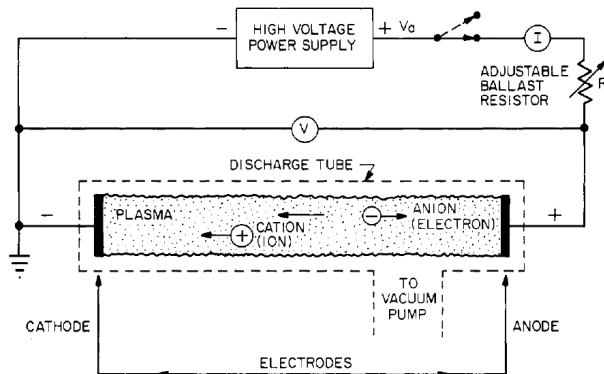


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 - **Pulsed-power generated plasma**

Electrical discharge physics was studied using the classical low pressure electrical discharge tube

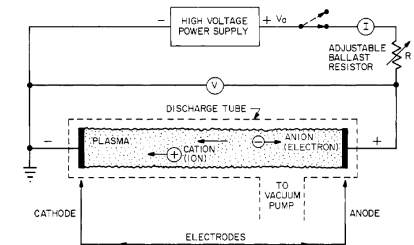
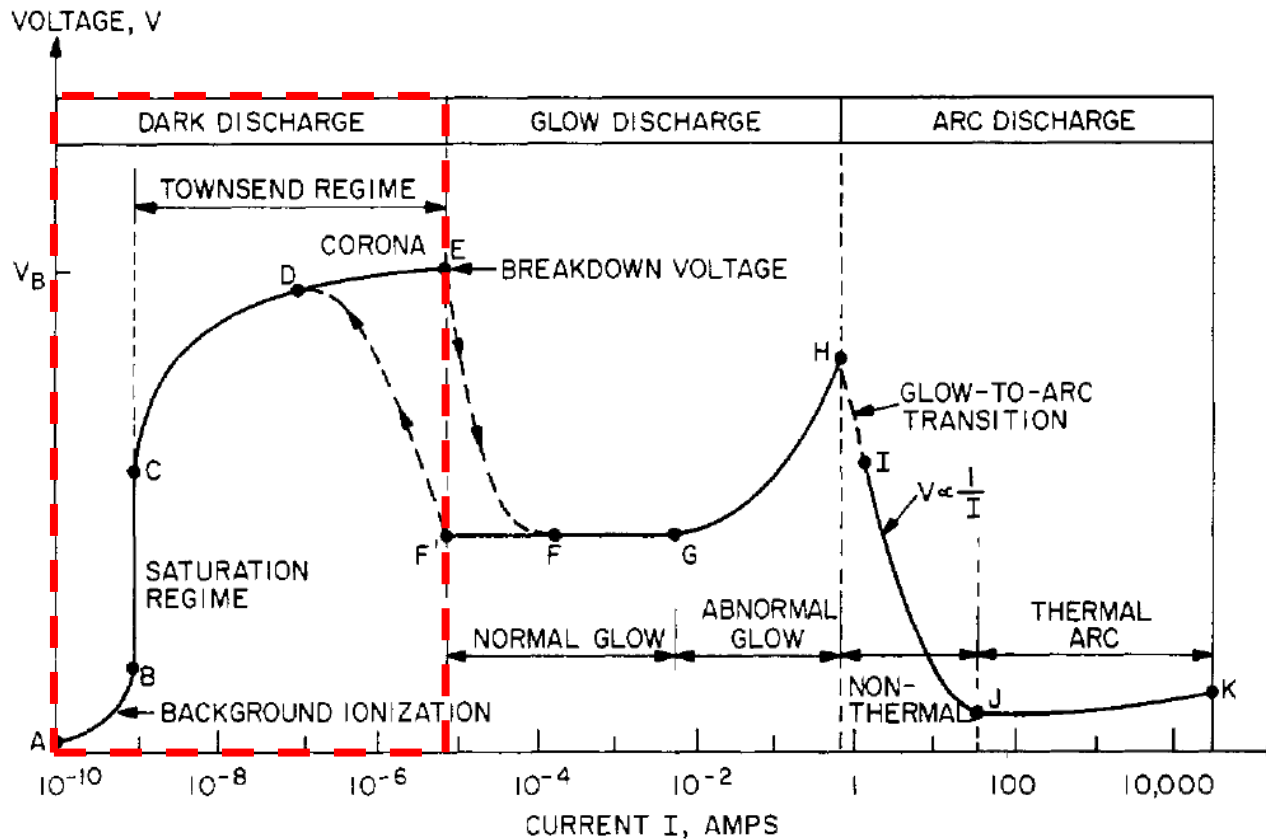


The V-I curve is nonlinear in a DC electrical discharge tube



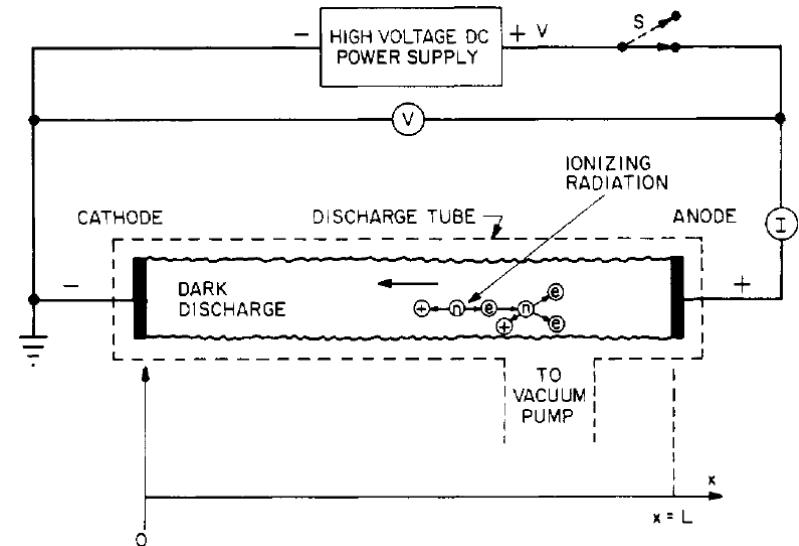
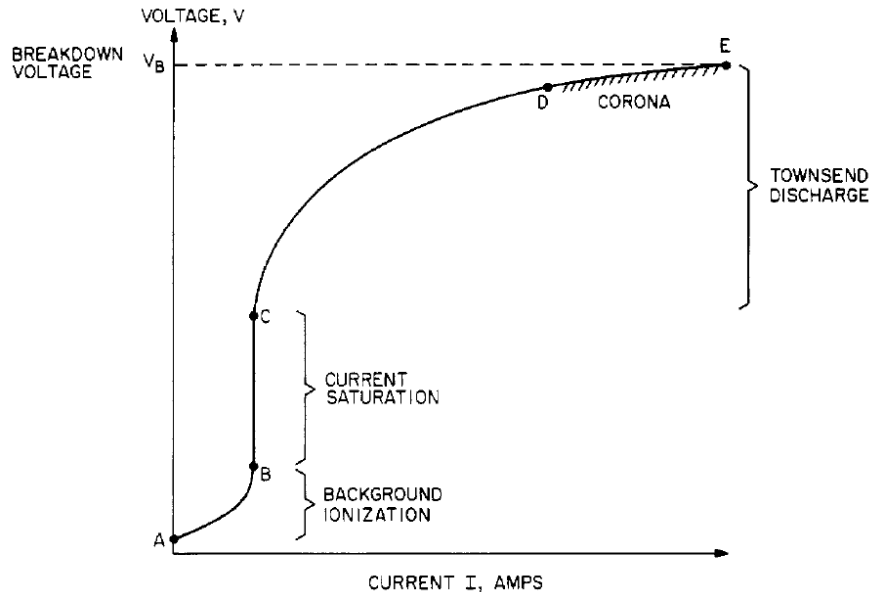
- Depends on the voltage, the adjustable ballast resistor, the voltage-current characteristic behaves differently in different regime.
 - Dark discharge
 - Glow discharge
 - Arc discharge

In a dark discharge, the excitation light is so little and is not visible



- In dark discharge, with the exception of the more energetic corona discharges, the number density of excited species is so small so that it does not emit enough light to be seen by a human observer.

In background ionization, ions and electrons are created by ionization from background radiation

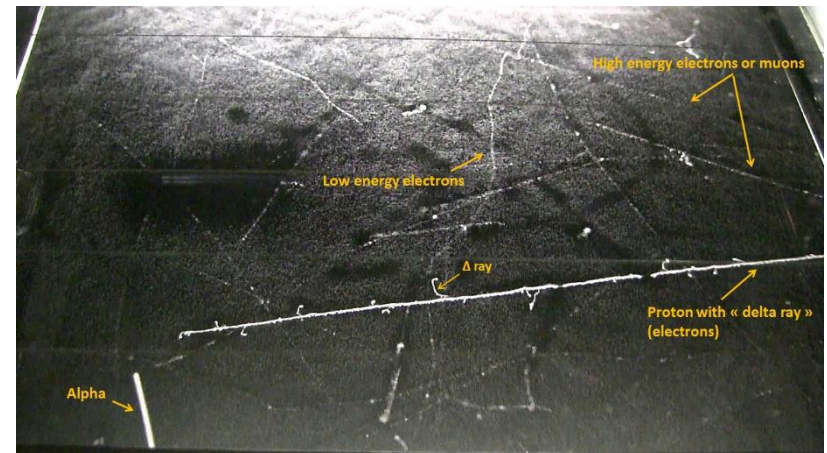
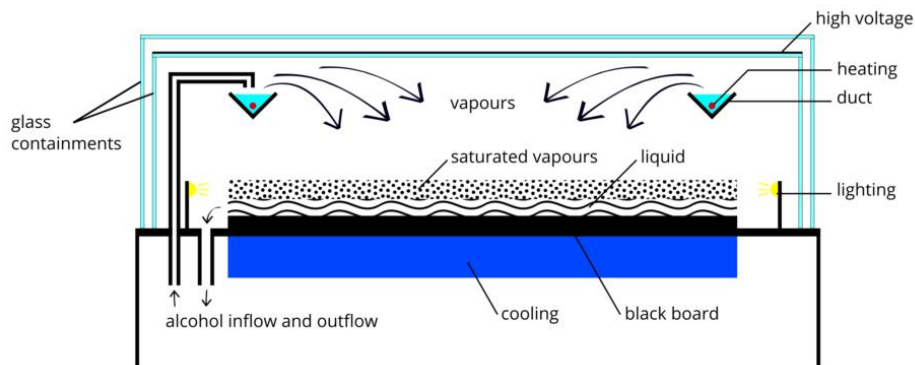


- Sources of background radiation:
 - Cosmic rays
 - Radioactive minerals in the surroundings
 - Electrostatic charge
 - UV light illumination
 - Other sources

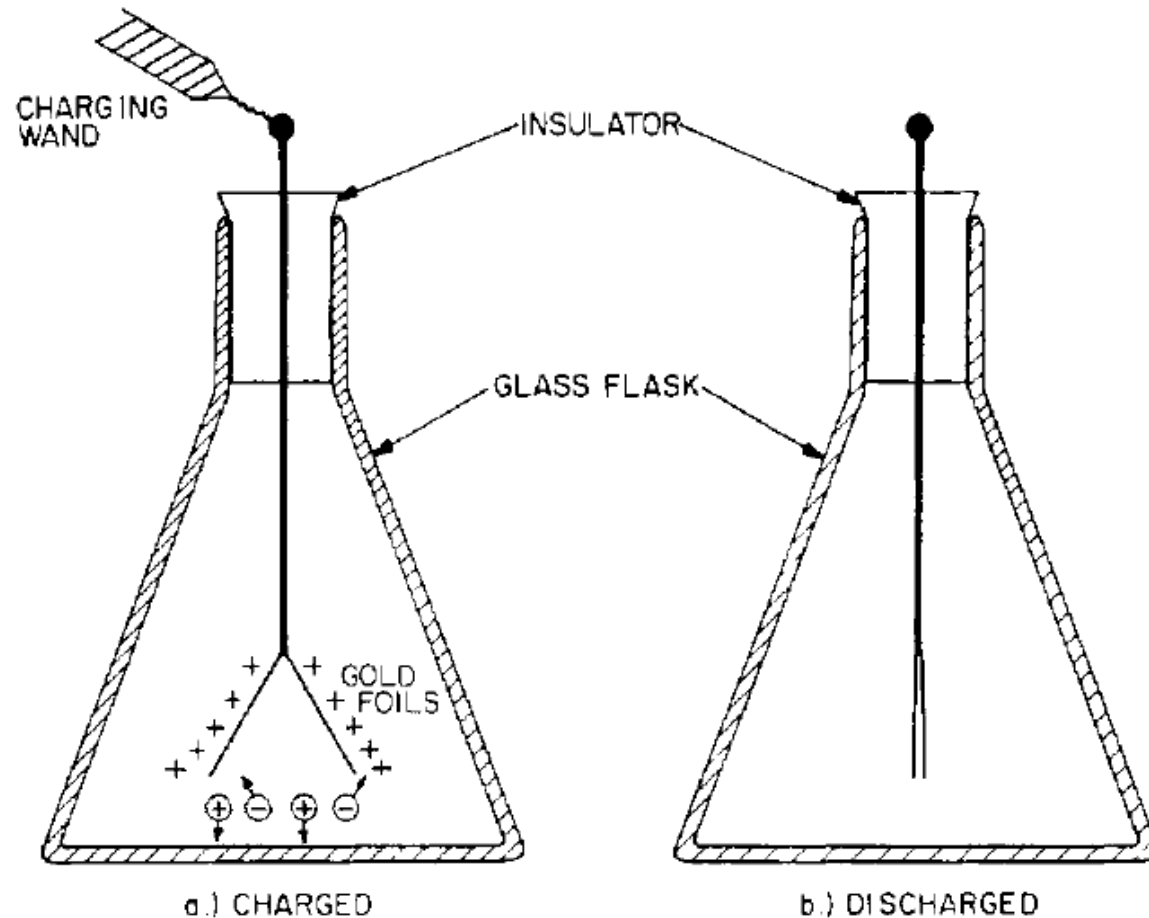
Cosmic rays can be observed by a “cloud chamber”



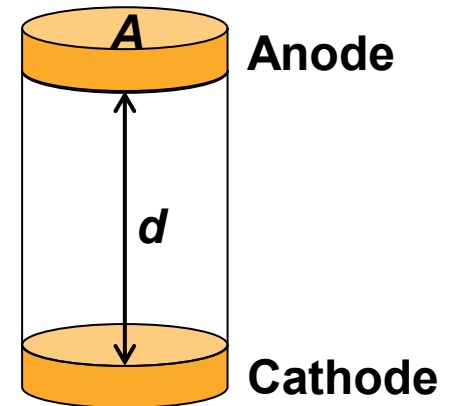
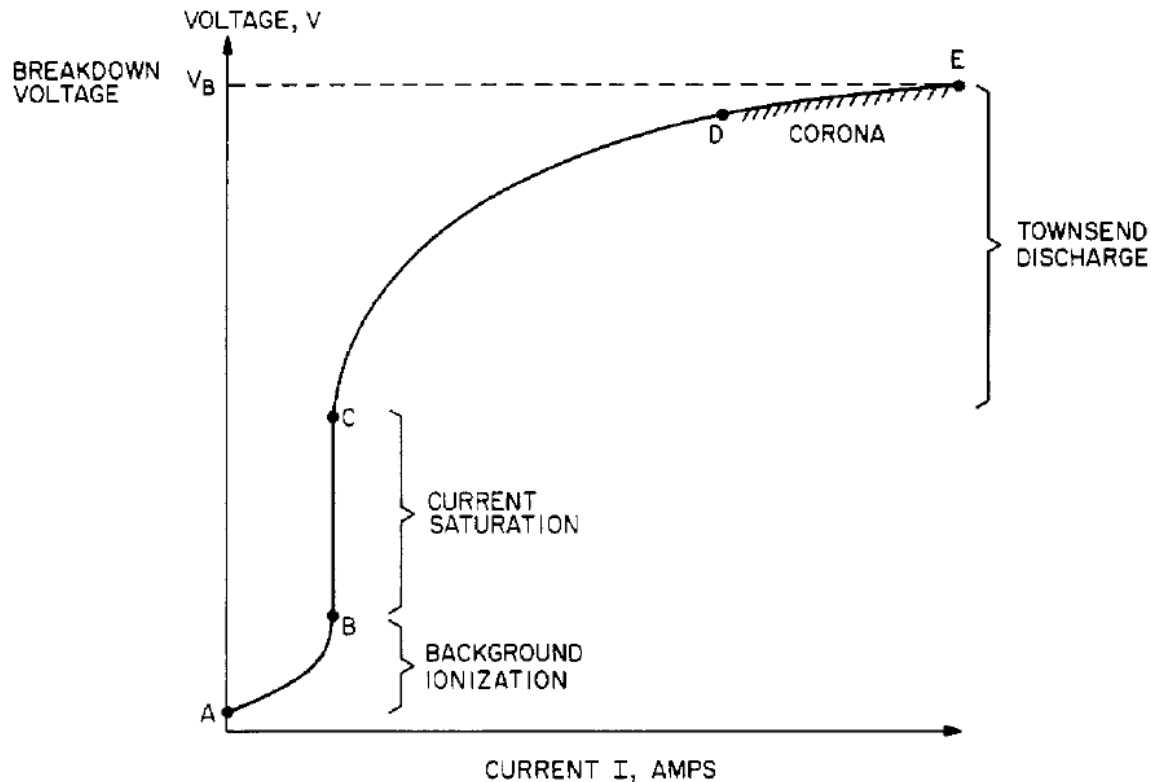
- A cloud chamber consists of a sealed environment containing a supersaturated vapor of water or alcohol. An energetic charged particle interacts with the gaseous mixture by knocking electrons off gas molecules via electrostatic forces during collisions, resulting in a trail of ionized gas particles. The resulting ions act as condensation centers around which a mist-like trail of small droplets form if the gas mixture is at the point of condensation. These droplets are visible as a "cloud" track that persists for several seconds while the droplets fall through the vapor.



A discharge of a gold-leaf electroscope can illustrate the ionization of air by cosmic rays and background radiation



Current saturation occurs when all ions and electrons produced between the electrodes are collected

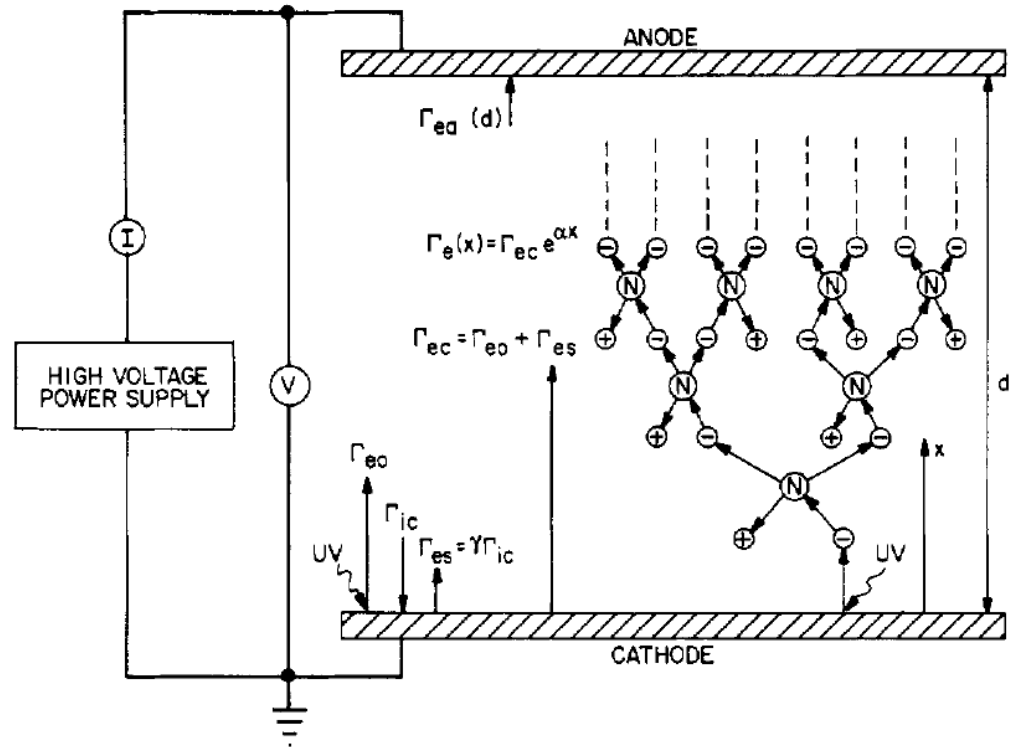
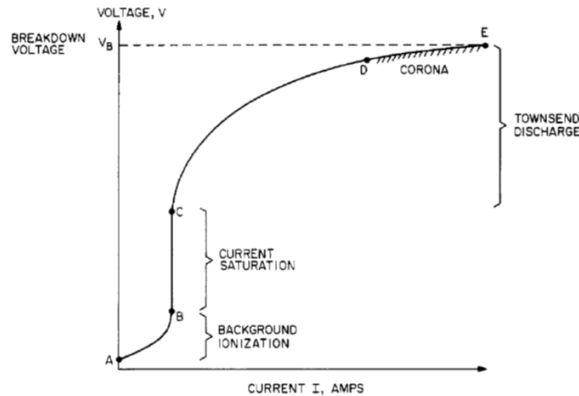


$$S = \frac{dn}{dt} \text{ (electrons or ions/m}^3 \text{ - s)}$$

$$I_s = eAdS$$

$$J_s = edS$$

The region where the current exponentially increases is called the Townsend discharge



- Electrons from photo- or secondary electron emission from the cathode:

$$\Gamma_{ec} = \Gamma_{e0} + \Gamma_{es} (\text{electrons}/m^2 - s)$$

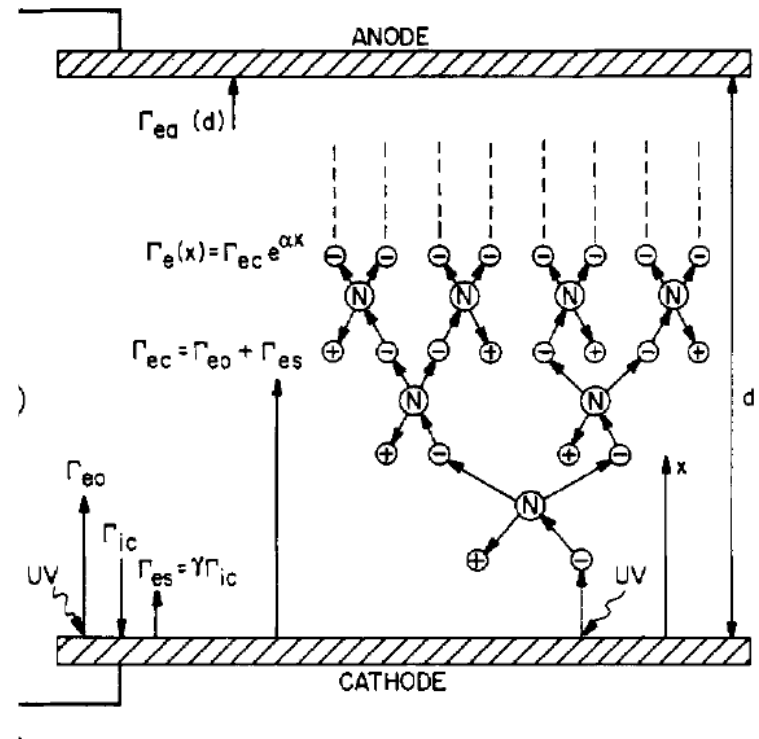
- Volume ionization source from the ionization of the background gas by energetic electrons accelerated in the electric field:

$$S_e = R_e = n_e n_0 \langle \sigma v \rangle_{ne}$$

Chain reaction or avalanche of electron and ion production occurs in a strong electric field



1. The electrons initially produced in the creation of ion-electron pairs by ionizing radiation or from other sources are accelerated in the electric field of the discharge tube.
2. If the electric field is high enough, the electrons can acquire sufficient energy before reaching the anode to ionize another neutral atom.
3. As the electric field becomes stronger, these secondary electrons may themselves ionize a third neutral atom leading to a chain reaction, or avalanche of electron and ion production.



Special case I



- **Assumption:**
 - No recombination or loss of electrons occurs.
 - Initiating electrons are emitted from the cathode, with no contribution by volume ionization.
- Townsend's first ionization coefficient, α : the number of ionizing collisions made on the average by an electron as it travels 1 m along the electric field:

$$\alpha \sim \frac{1}{\lambda_i} = \frac{\nu_{ei}}{\bar{v}_e} = \frac{n_0 \langle \sigma v_e \rangle_{ne}}{\bar{v}_e}$$

- **Differential electron flux:**

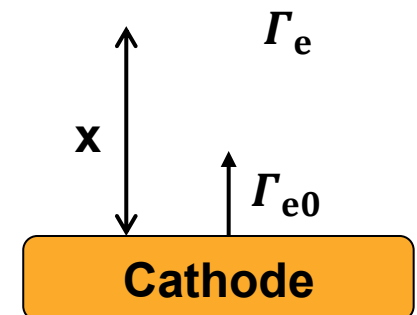
$$d\Gamma_e = \alpha \Gamma_e dx$$

$$\Gamma_e = \Gamma_{e0} e^{\alpha x}$$

$$\int_{\Gamma_{e0}}^{\Gamma_e} \frac{d\Gamma_e}{\Gamma_e} = \int_0^x \alpha dx$$

$$J_e = e\Gamma_e = J_{e0} e^{\alpha x} \quad (\text{A/m}^2)$$

$$I_e = I_{e0} e^{\alpha x} = A J_{e0} e^{\alpha x} \quad (\text{A})$$



Special case II



- **Assumption:**
 - No recombination or loss of electrons occurs.
 - No cathode emission, i.e., $\Gamma_{e0}=0$.
 - Significant volume source of electrons throughout the discharge volume.
- **Differential electron flux:**

$$d\Gamma_e = \alpha\Gamma_e dx + S_e dx$$

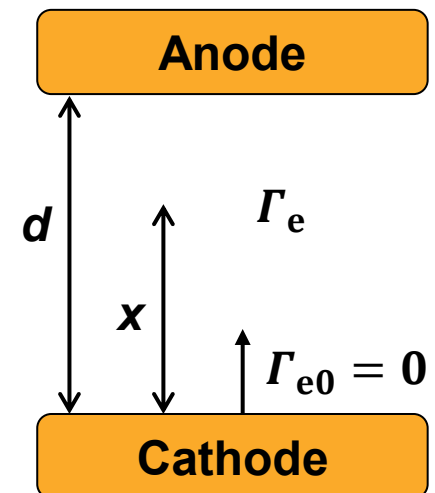
$$\int_0^{\Gamma_e} \frac{1}{\alpha\Gamma_e + S_e} d\Gamma_e = \int_0^x dx = \frac{\ln(\alpha\Gamma_e + S_e)}{\alpha} \Big|_0^{\Gamma_e}$$

$$\Gamma_e = \frac{S_e}{\alpha} (e^{\alpha x} - 1)$$

$$J_e = \frac{J_s}{\alpha d} (e^{\alpha x} - 1)$$

$$J_e = e\Gamma_e = \frac{eS_e}{\alpha} (e^{\alpha x} - 1)$$

$$J_s = edS_e$$



Derivation of Townsend's first ionization coefficient



$$\frac{1}{\lambda_i} = \frac{\nu_{ei}}{\bar{v}_e} = \frac{n_0 \langle \sigma v_e \rangle_{ne}}{\bar{v}_e} = \frac{p}{T} \frac{\langle \sigma v \rangle_{ne}}{\bar{v}_e} \equiv Ap \qquad A \equiv \frac{1}{T} \frac{\langle \sigma v \rangle_{ne}}{\bar{v}_e}$$

- Number of primary electrons with energy higher than the ionization potential:

$$dn_e = -n_e \frac{dx_i}{\lambda_i} \Rightarrow \frac{n_e(x_i)}{n_{e0}} = \exp\left(-\frac{x_i}{\lambda_i}\right)$$

$$\alpha \equiv \frac{\text{\# / ionization collisions}}{\text{per electron}} \times (\text{\# / electron with } E > \text{ionization potential})$$

$$= \frac{1}{\lambda_i} \frac{n_e(x_i)}{n_{e0}} = \frac{1}{\lambda_i} \exp\left(-\frac{x_i}{\lambda_i}\right)$$

$$\alpha = Ap \exp(-Ap x_i)$$

$$\frac{\alpha}{p} = A \exp\left(-\frac{AV^*}{E/p}\right) \equiv A \exp\left(-\frac{C}{E/p}\right) \equiv f\left(\frac{E}{p}\right)$$

$$x_i \approx \frac{V^*}{E} \text{ where } V^* > V_i$$

- The parameters A and C must be experimentally determined.

Phenomenological constants A and C of Townsend's first ionization coefficient for selected gases



Gas	A ion pairs/m-Torr	C V/m-Torr
A	1200	20 000
Air	1220	36 500
CO ₂	2000*	46 600
H ₂	1060	35 000
HCl	2500*	38 000
He	182	5 000
Hg	2000	37 000
H ₂ O	1290*	28 900
Kr	1450	22 000
N ₂	1060	34 200
Ne	400	10 000
Xe	2220	31 000

* These values may be high by as much as a factor of two.

Stoletow point is the pressure for maximum current



- Stoletow experimentally found that for a given electric field between the plates, there is an air pressure in the Townsend discharge where the current is a maximum.

$$p_{\max} = \frac{E}{37200} \text{ (Torr)}$$

$$\frac{\alpha}{p} = A \exp\left(-\frac{C}{E/p}\right)$$

$$\frac{\alpha}{p} = f\left(\frac{E}{p}\right)$$

$$\frac{d\alpha}{dp} = A \left[1 - p \frac{C}{E}\right] \exp\left(-\frac{Cp}{E}\right) = 0$$

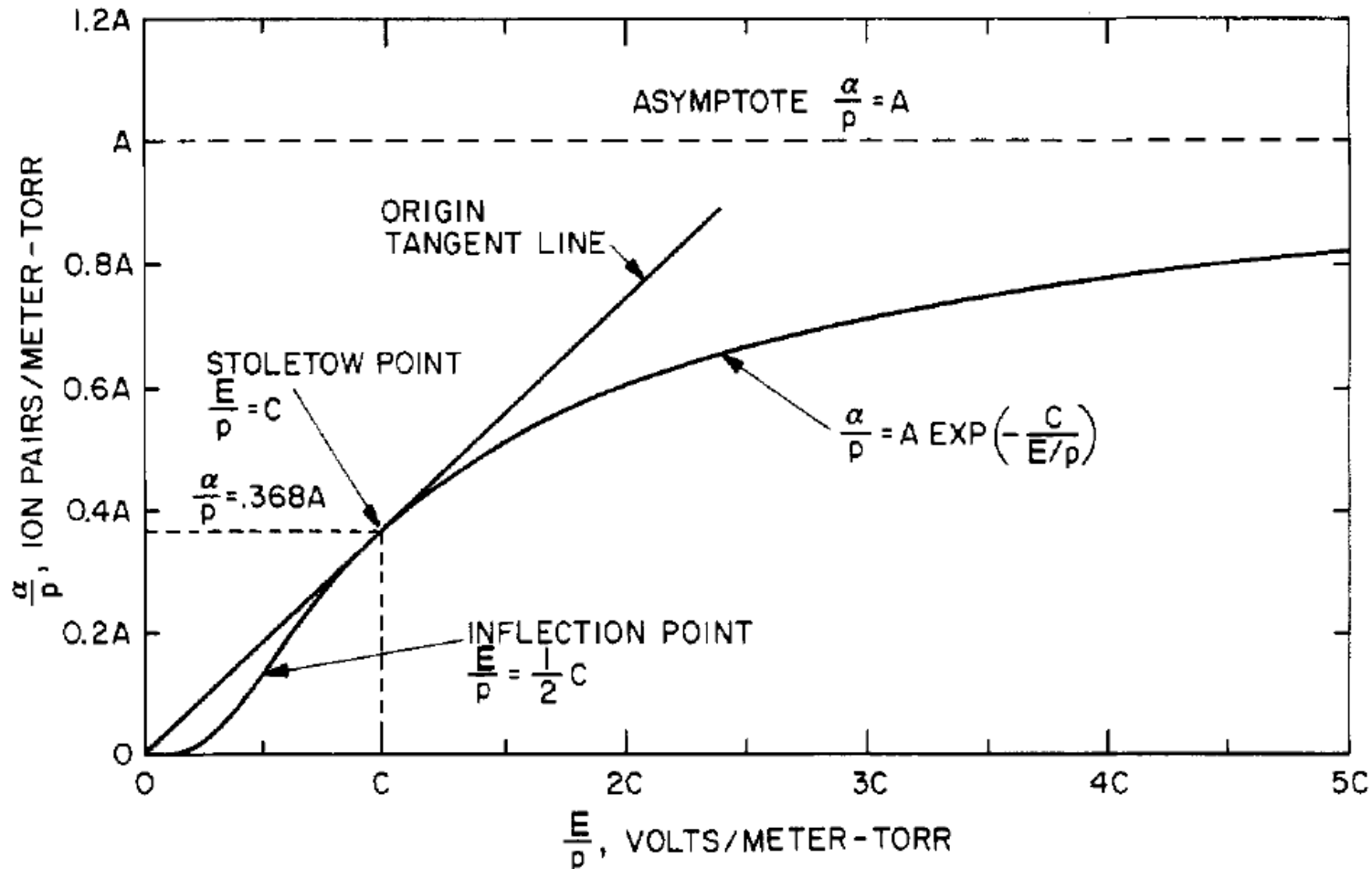
$$\frac{d\alpha}{dp} = \frac{d}{dp} \left[p f\left(\frac{E}{p}\right) \right] \equiv 0$$

$$p_{\max} = \frac{E}{C} = \frac{E}{36500} \text{ for air}$$

$$f\left(\frac{E}{p}\right) - p \frac{E}{p^2} f'\left(\frac{E}{p}\right) = \frac{\alpha}{p} - \frac{E}{p} f'\left(\frac{E}{p}\right) = 0$$

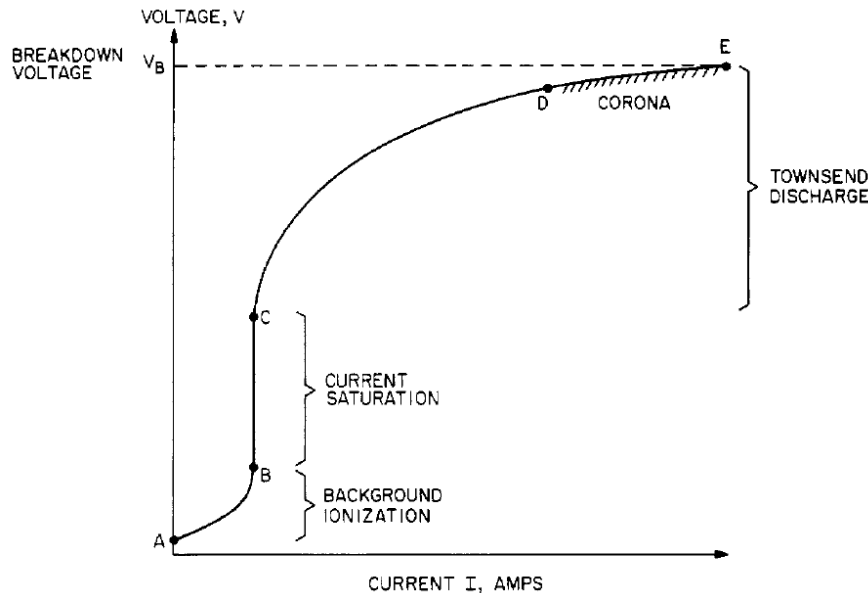
$$\left(\frac{\alpha/p}{E/p}\right) = f'\left(\frac{E}{p}\right) = \tan\theta$$

The current will be a maximum when the tangent to the α/p versus E/p curve intersects the origin



- Stoletow point is the minimum of the Paschen breakdown curve for gases.

Corona discharge (unipolar discharge) is a very low current, continuous phenomenon



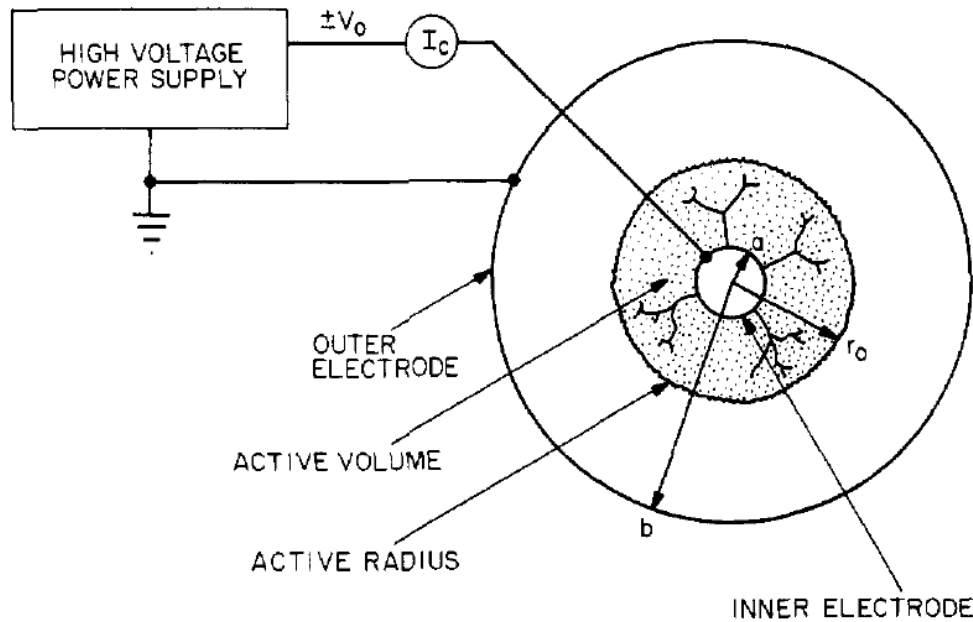
- Break down condition for dry air:

$$E_B = 3000 + \frac{1.35}{d} \text{ kV/m}$$

$$V_B = 3000d + 1.35 \text{ kV}$$

- Corona can initiate on sharp points at potentials as low as 5 kV.
- It can initiate from sharp points, fine wires, sharp edges, asperities, scratches or anything which creates a localized electric field greater than the breakdown electric field of the medium surrounding it.
- It can be a “glow discharge”, i.e., visible to eyes. For low currents, the entire corona is dark.

Phenomenology of corona generated by a fine wire

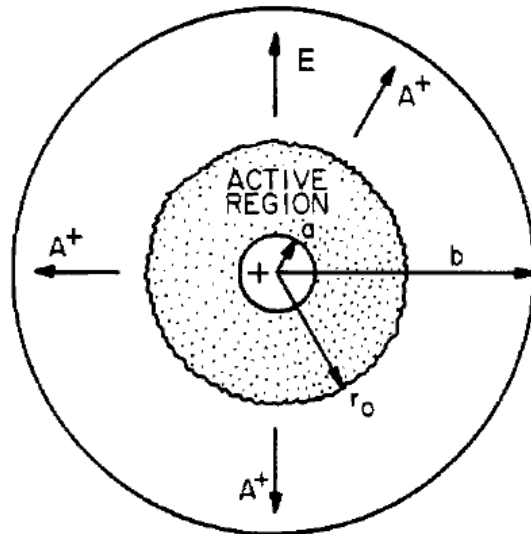


- The point of corona initiation is that point at which the voltage on the inner conductor of radius a is high enough that corona is just detectable.
- The electric field will drop off to the breakdown value at a radius r_0 called the active radius.

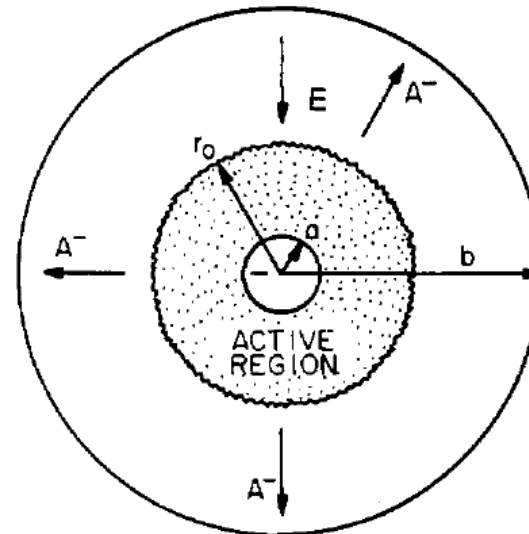
Corona can occur for both positive and negative polarity



- **Positive polarity**

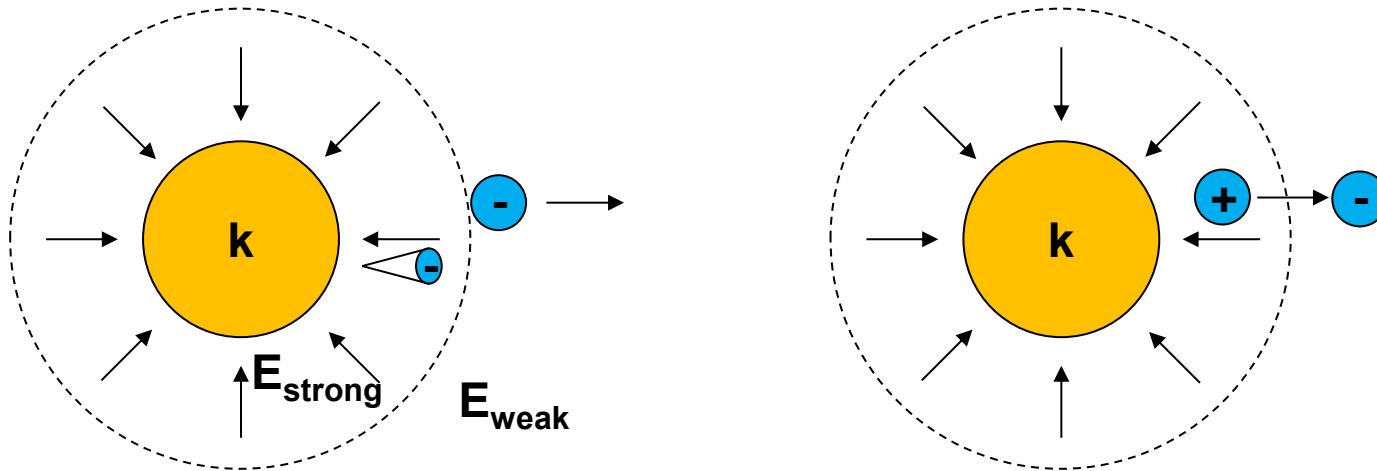


- **Negative polarity**



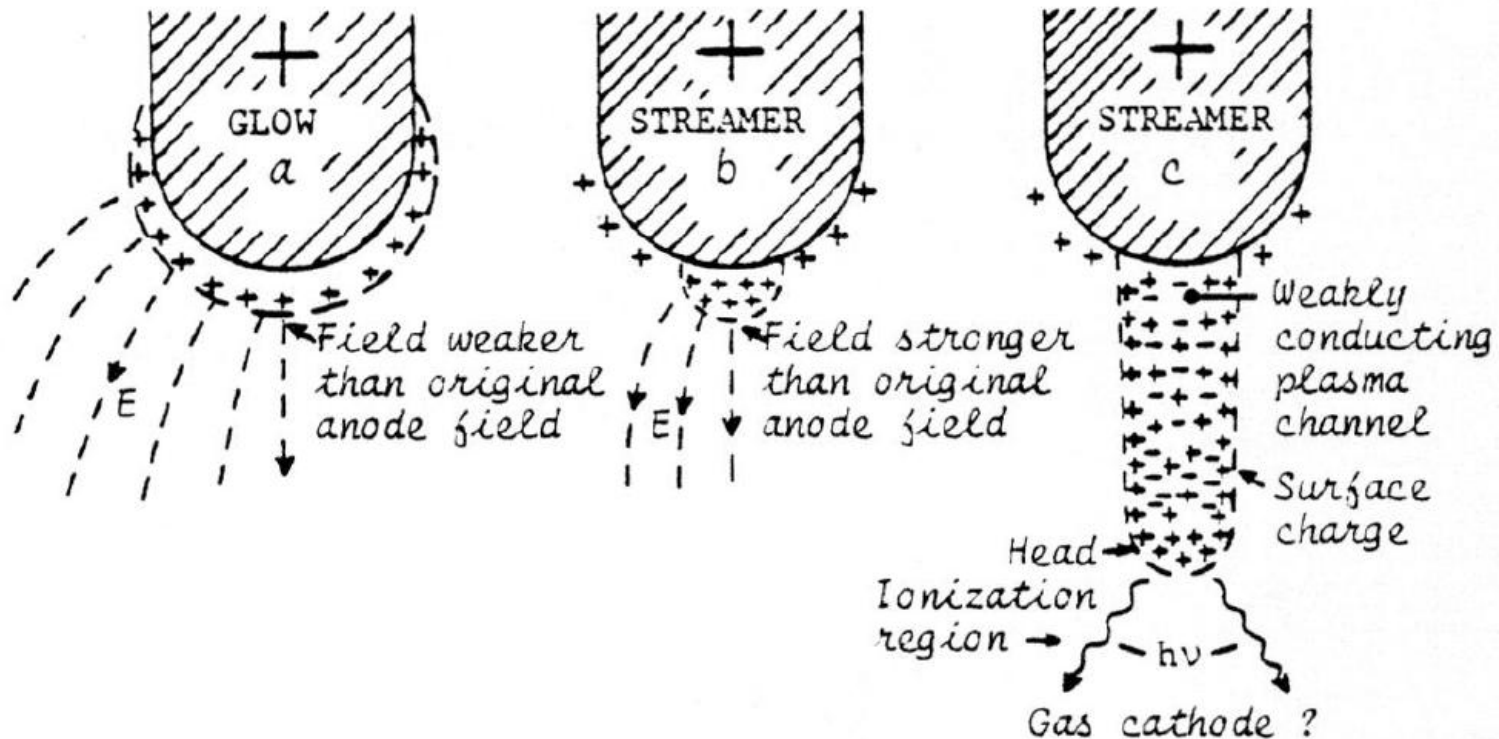
- The initiation voltages or coronal current are slightly different between positive and negative polarity.
- A continuous (positive polarity, DC) or intermittent (negative polarity, usually) current, usually in the order of $\mu\text{A} \sim \text{mA}$ per decimeter of length will flow to the power supply.

Negative point corona, also known as Trichel pulses



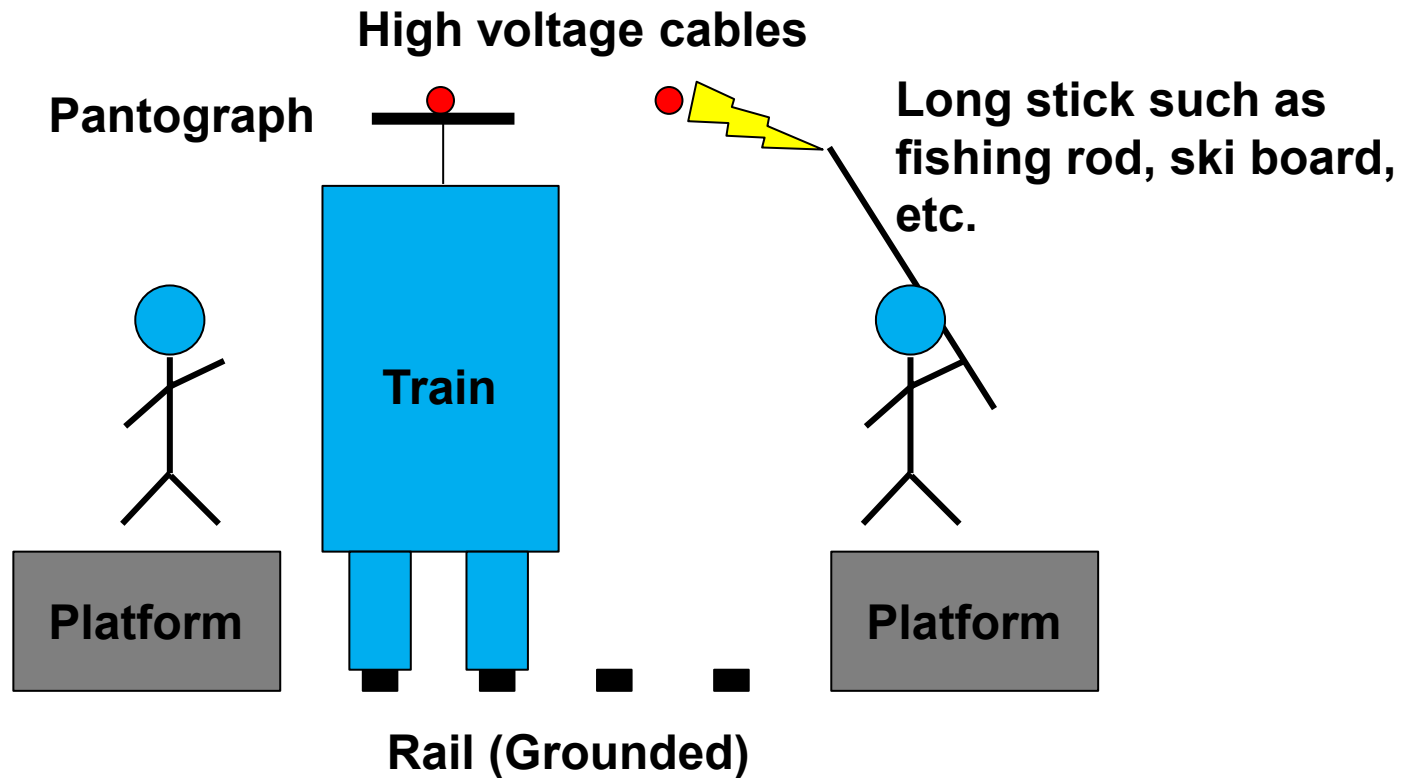
- Avalanche toward anode occurs in the strong electric field region.
- No further ionization occurs in the weak field region.
- Electrons are slow down by positively charged ions (ion+) behind.
- Electrons attach to gas molecules forming negatively charged ions (ion-).
- The presence of the negative ions reduces the electric field at the point electrode and the discharge extinguishes.
- When positively/negatively charged ions drifted away, the original high-field conditions are re-established

Positive point corona



- Electron avalanche initiated near the high-field region propagating toward anode.
- Streamer is developed.
- Lateral avalanches feed into the streamer core.

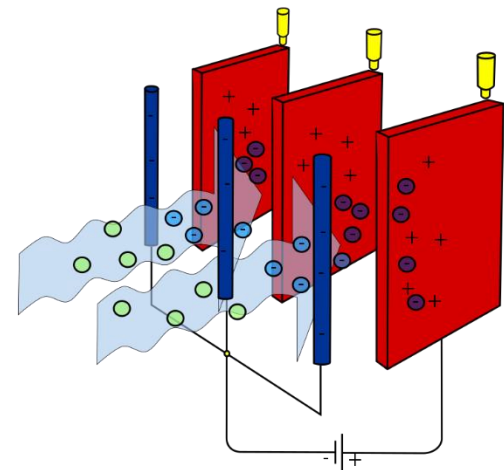
Don't bring a long stick to a train station



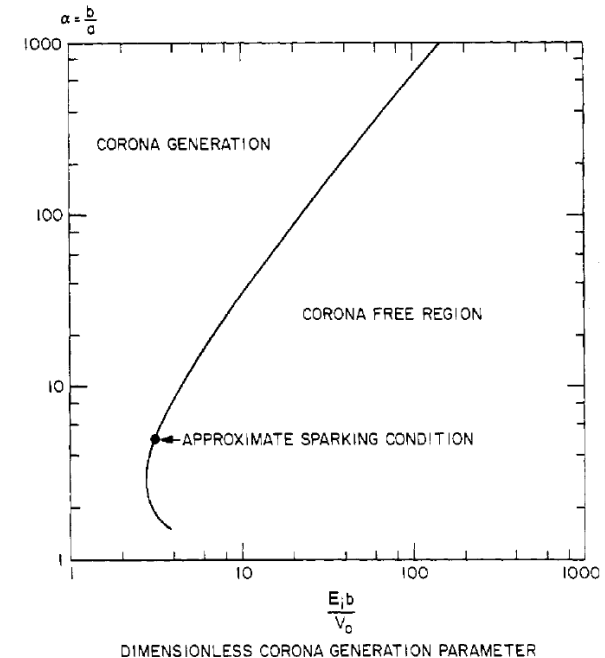
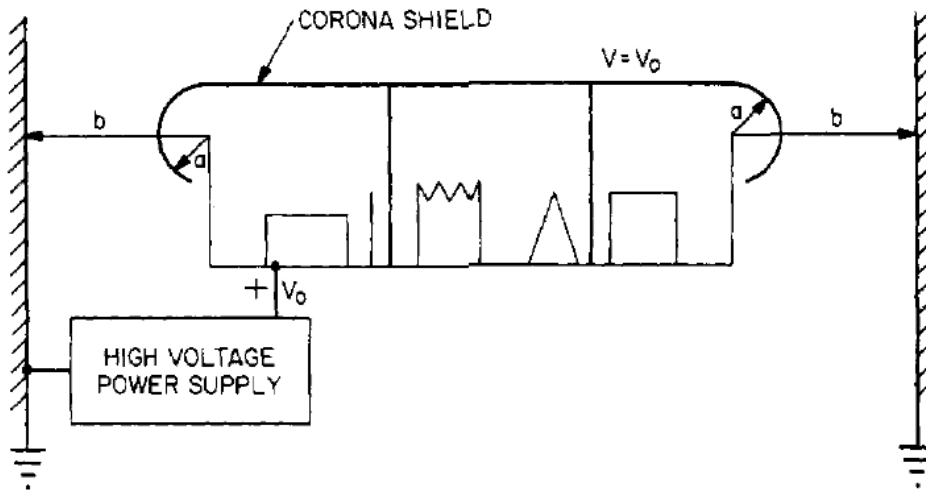
A corona discharge causes some problems even no breakdown occurs



- Ozone (O_3) is generated.
- Rubber is destroyed by O_3 .
- NO_3^+ is generated with moisture.
- Disadvantage:
 - Power losses.
 - Radio frequency (RF) interference.
 - Reduce the service life of solid and liquid insulation via initiating partial discharge.
 - Chemical decomposition.
- Advantage:
 - Pseudospark discharge – fast switch.
 - Electrostatic precipitator (dust remover) using corona discharge.
 - Hair dryer



A corona shield is used to suppress corona



- **Cylindrical approximation:**

$$E_s = \frac{V_0}{a \ln(b/a)} = \frac{bV_0}{a b \ln(b/a)} \quad \chi \equiv \frac{b}{a}$$

$$E_s \equiv E_i \text{ (} E \text{ @ surface for corona initiation)}$$

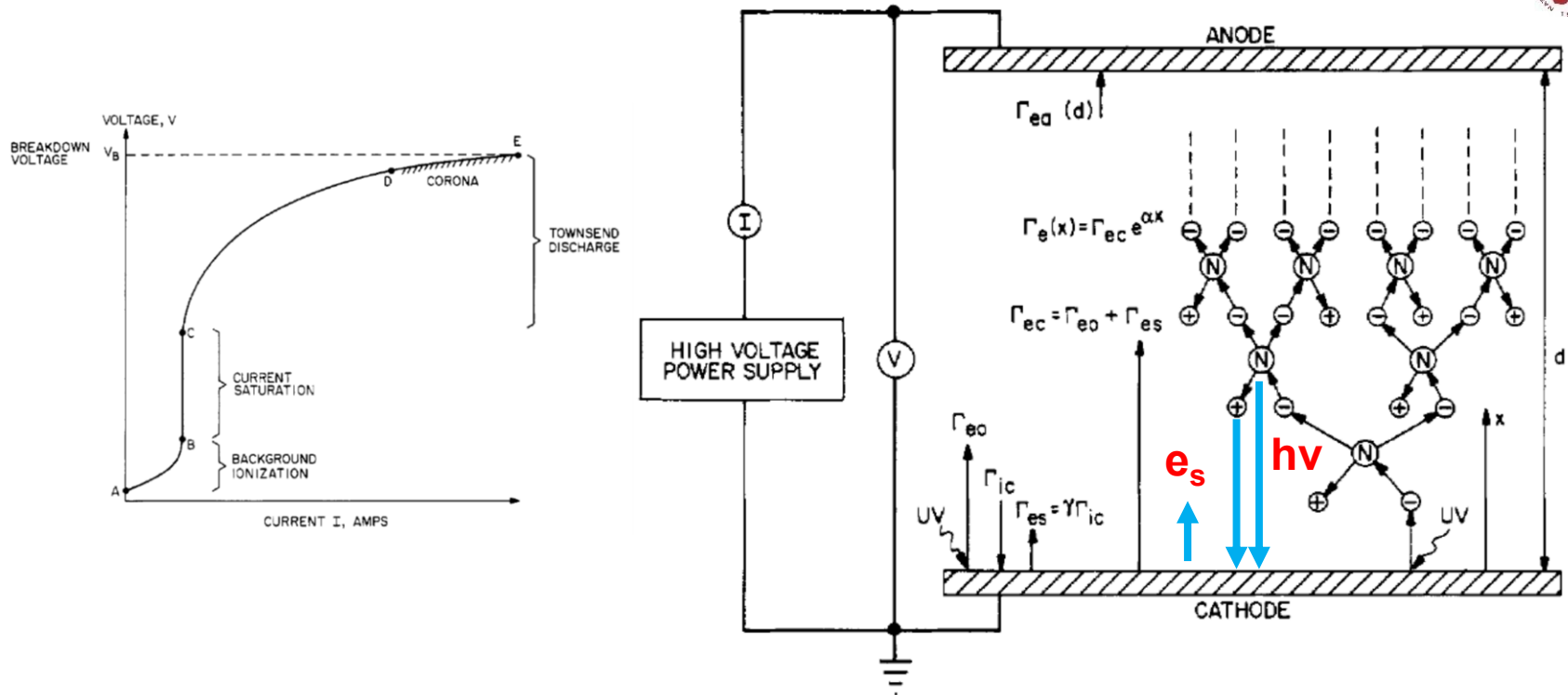
$$\Rightarrow \frac{E_i b}{V_0} = \frac{\chi}{\ln \chi}$$

- For $b=0.5 \text{ m}$, $V_0=50 \text{ kV}$, $E_i \sim E_B \sim 3 \text{ MV/m}$

$$\frac{E_B b}{V_0} = \frac{3 \times 10^6 \times 0.5}{5 \times 10^4} = 30 = \frac{\chi}{\ln \chi}$$

$$\chi \approx 150, \text{ i.e., } a = 0.33 \text{ mm}$$

Electrical breakdown occurs when applied voltage is greater than the breakdown voltage



- **Primary electrons:** electrons from the cathode due to photoemission, background radiation, or other processes.
- **Secondary electrons:** electrons emitted from the cathode per incident ion or photon created from ionization in gas.

Derivation of electrical breakdown



- Secondary electron emission coefficient:

$$\gamma \equiv \frac{\text{\# / of electrons emitted}}{\text{\# / of incident ions or photons}}$$

$$\Gamma_{es} = \gamma \Gamma_{ic}$$

$$\Gamma_{ec} = \Gamma_{e0} + \Gamma_{es}$$

$$I_{ea} = I_{ec} + I_{ic} \Rightarrow \Gamma_{ea} = \Gamma_{ec} + \Gamma_{ic}$$

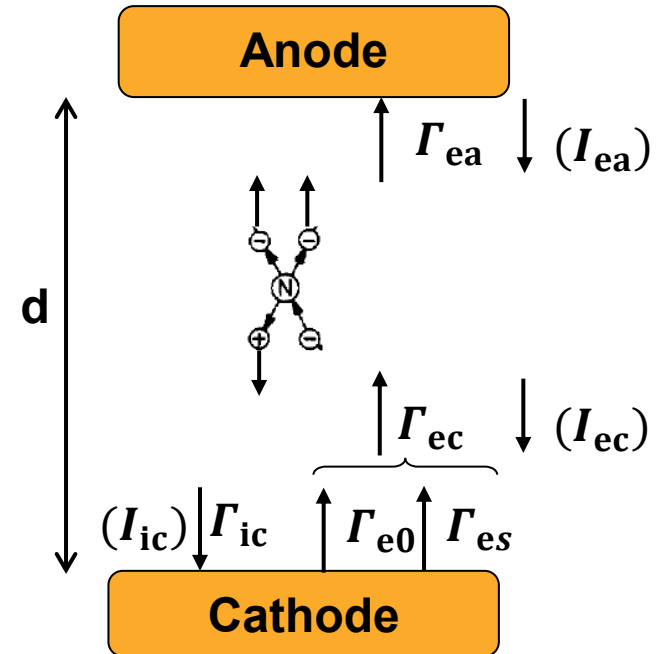
$$\Gamma_{ea} - \Gamma_{ec} = \Gamma_{ic} = \frac{\Gamma_{es}}{\gamma} \quad (\Gamma_{ea} = \Gamma_{ec} e^{\alpha d})$$

$$\Gamma_{es} = \gamma(\Gamma_{ea} - \Gamma_{ec}) = \gamma \Gamma_{ec}(e^{\alpha d} - 1)$$

$$\Gamma_{ec} = \Gamma_{es} + \Gamma_{e0} = \gamma \Gamma_{ec}(e^{\alpha d} - 1) + \Gamma_{e0}$$

$$\Gamma_{ec} = \frac{\Gamma_{e0}}{1 - \gamma(e^{\alpha d} - 1)}$$

$$\Gamma_{ea} = \Gamma_{e0} \frac{e^{\alpha d}}{1 - \gamma(e^{\alpha d} - 1)} \text{ (electrons/m}^2\text{ - s)}$$



$$J = J_0 \frac{e^{\alpha d}}{1 - \gamma(e^{\alpha d} - 1)} \text{ (A/m}^2\text{)}$$

The Townsend condition for ignition (avalanche grows)



$$J = J_0 \frac{e^{\alpha d}}{1 - \gamma(e^{\alpha d} - 1)} \text{ (A/m}^2\text{)}$$

- The Townsend condition for ignition or called avalanche grows occurs when

$$1 - \gamma(e^{\alpha d} - 1) = 0$$

$$\gamma e^{\alpha d} = \gamma + 1 \quad \text{or} \quad \ln\left(1 + \frac{1}{\gamma}\right) = \alpha d$$

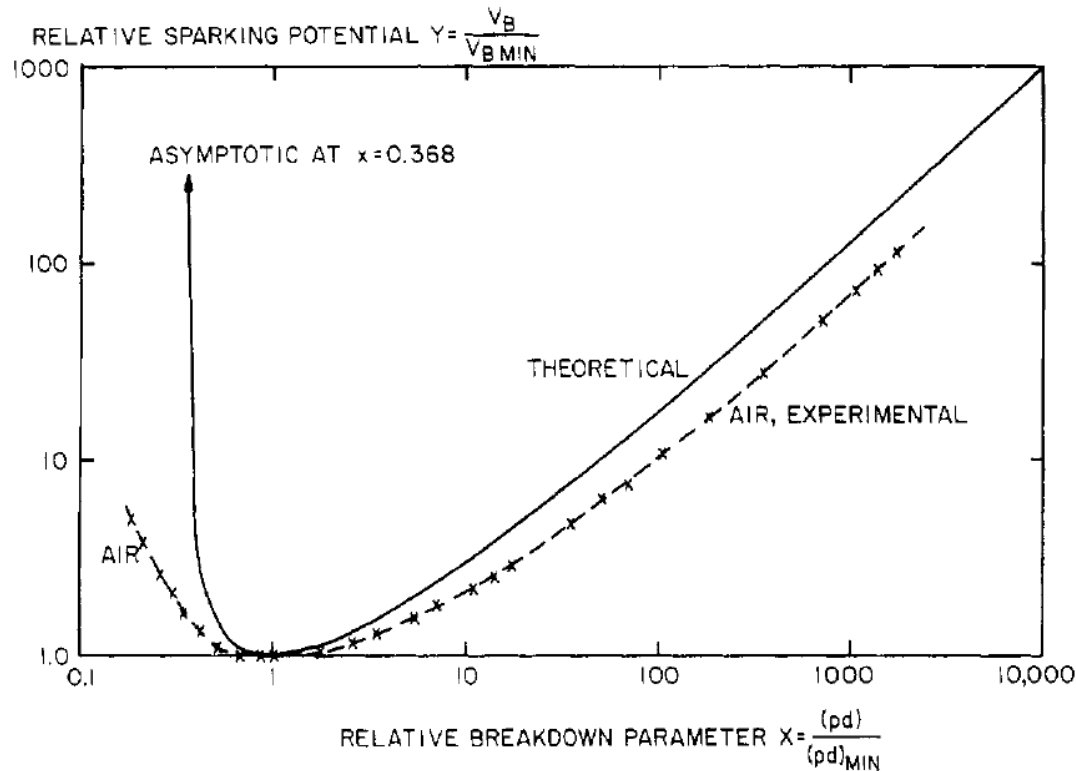
$$A p d \exp\left(-\frac{C p d}{V_B}\right) = \ln\left(1 + \frac{1}{\gamma}\right)$$

$$\frac{\alpha}{p} = A \exp\left(-\frac{C}{E/p}\right) \quad E_B = \frac{V_B}{d}$$

$$V_B = \frac{C p d}{\ln\left[\frac{A p d}{\ln\left(1 + \frac{1}{\gamma}\right)}\right]} = f(pd)$$

$$(pd)_{\min} = \frac{e}{A} \ln\left(1 + \frac{1}{\gamma}\right) = \frac{2.718}{A} \ln\left(1 + \frac{1}{\gamma}\right) \quad V_{B,\min} = 2.718 \frac{C}{A} \ln\left(1 + \frac{1}{\gamma}\right)$$

Universal Paschen's curve

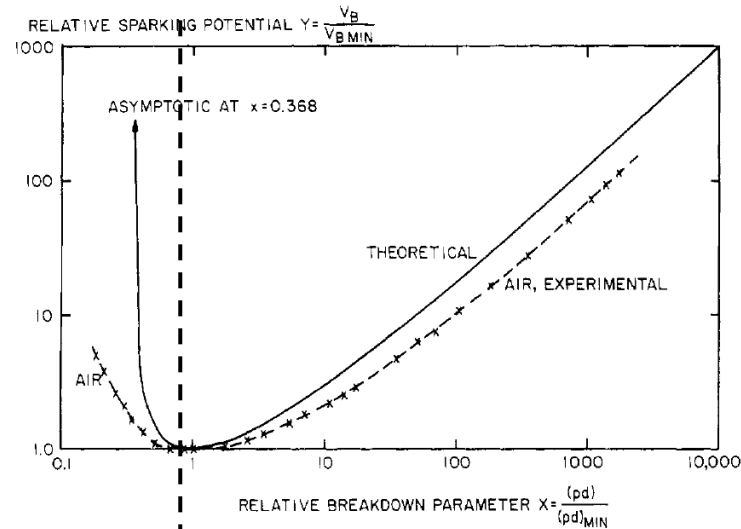


$$V_B = \frac{Cpd}{\ln \left[\frac{Apd}{\ln \left(1 + \frac{1}{\gamma} \right)} \right]} = f(pd)$$

$$Y \equiv \frac{V_B}{V_{B,\min}} \quad X \equiv \frac{pd}{(pd)_{\min}}$$

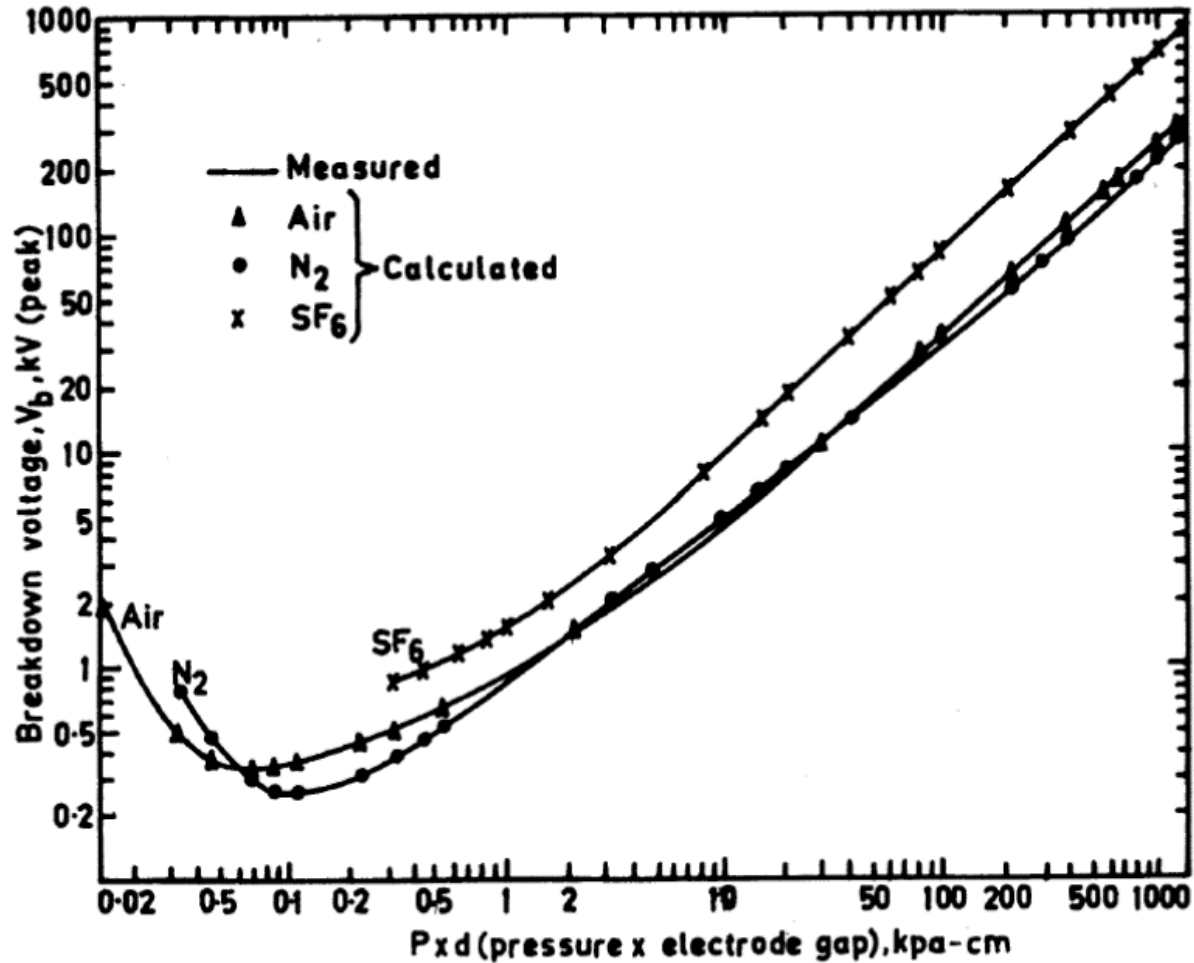
$$Y = \frac{V_B}{V_{B,\min}} = \frac{X}{1 + \ln X}$$

Collision frequency and electron energy gained from electric field are both important to electrical breakdown



- Collision is not frequent enough even the electrons gain large energy between each collision.
- Electrons do not gain enough energy between each collision even collisions happen frequently.
- The minimum of the Paschen's curve corresponds to the Stoletow point, the pressure at which the volumetric ionization rate is a maximum.

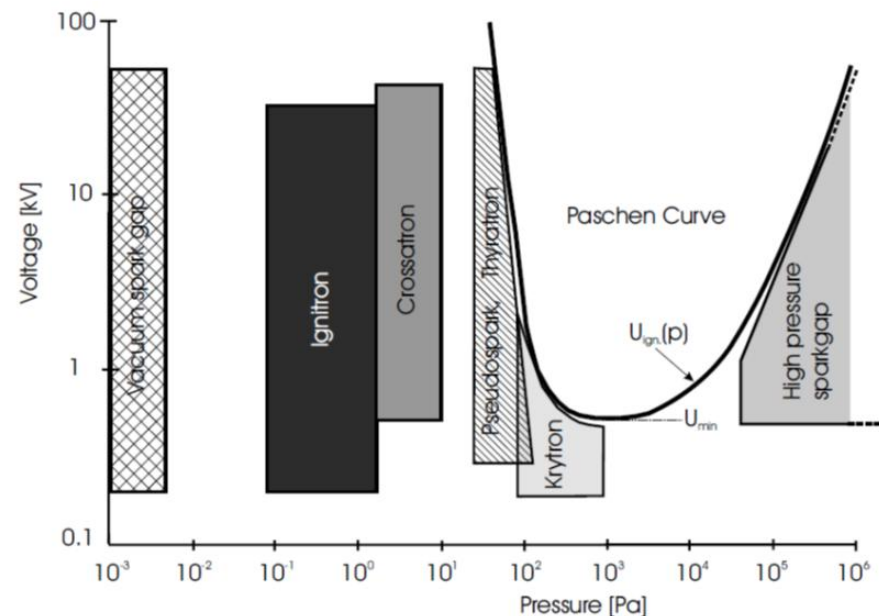
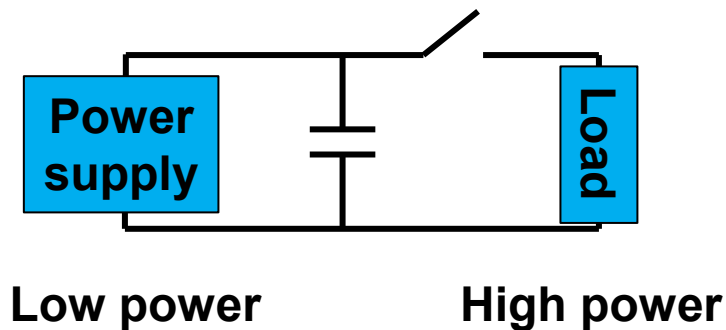
Experimental Paschen's curve



Paschen's curve is used to design different high voltage high current switches in pulsed-power system



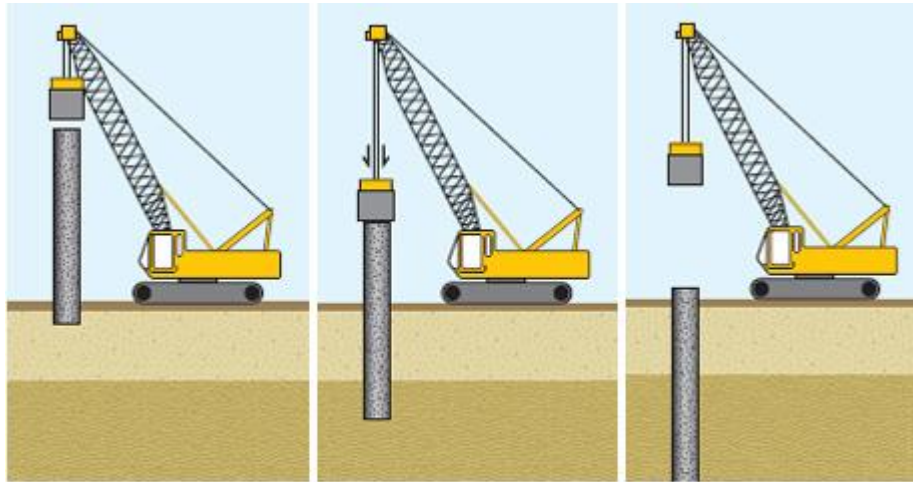
- Pulsed-power system



Driven piles - prefabricated steel, wood or concrete piles are driven into the ground using impact hammers



- Driven piles



PLACEMENT OF PILE

INSTALLATION OF PILE

REPETITION OF PROCESS

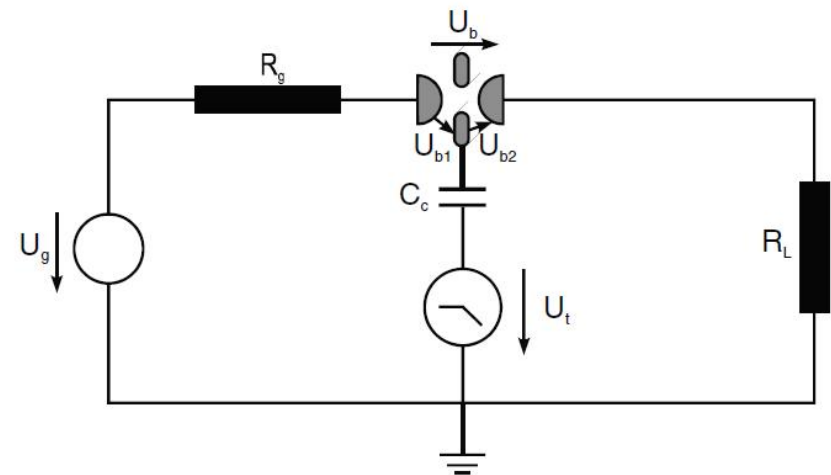
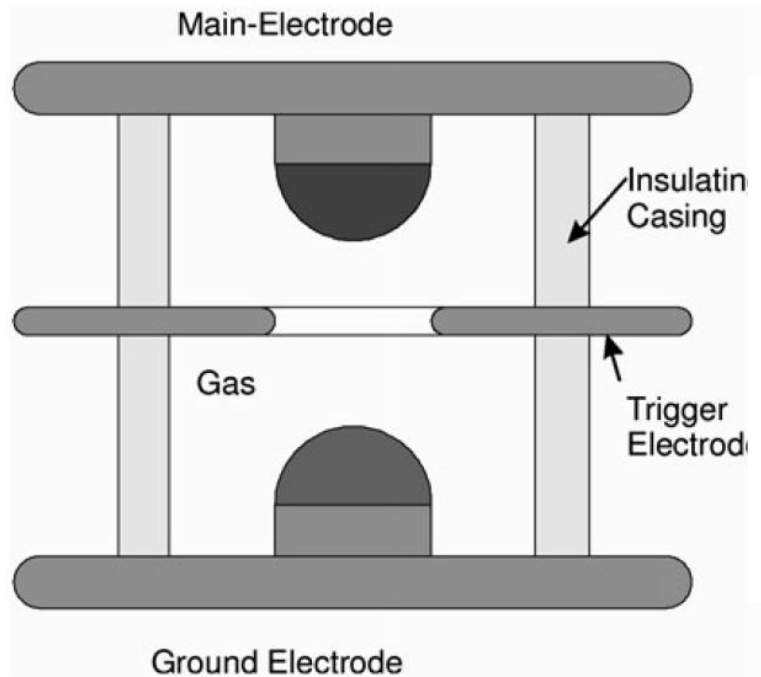
- Hammer



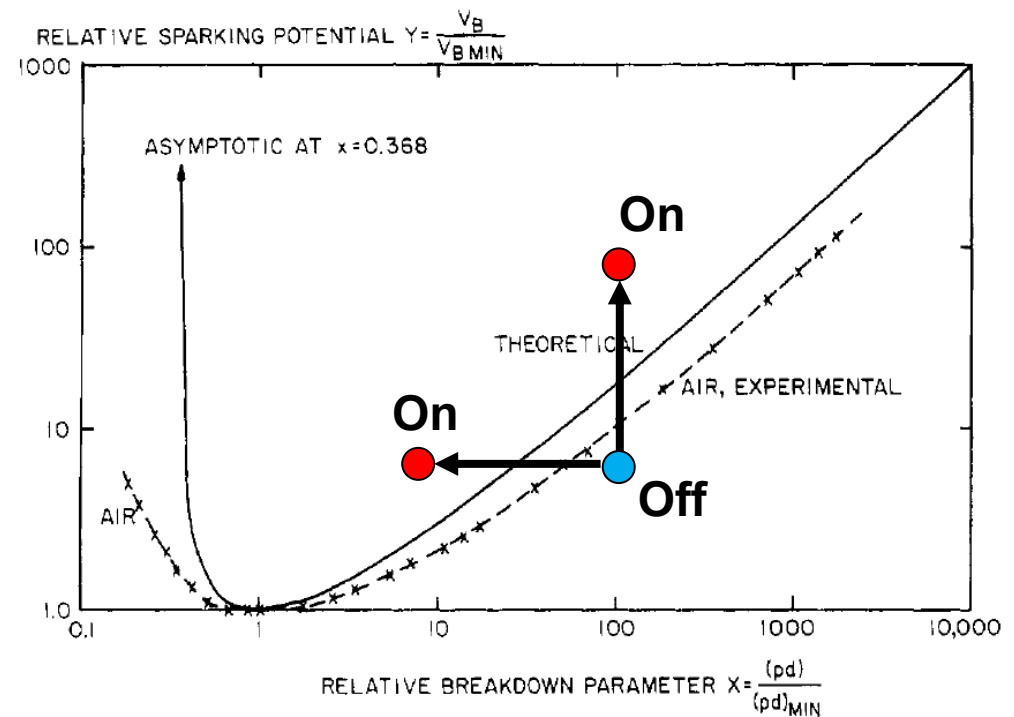
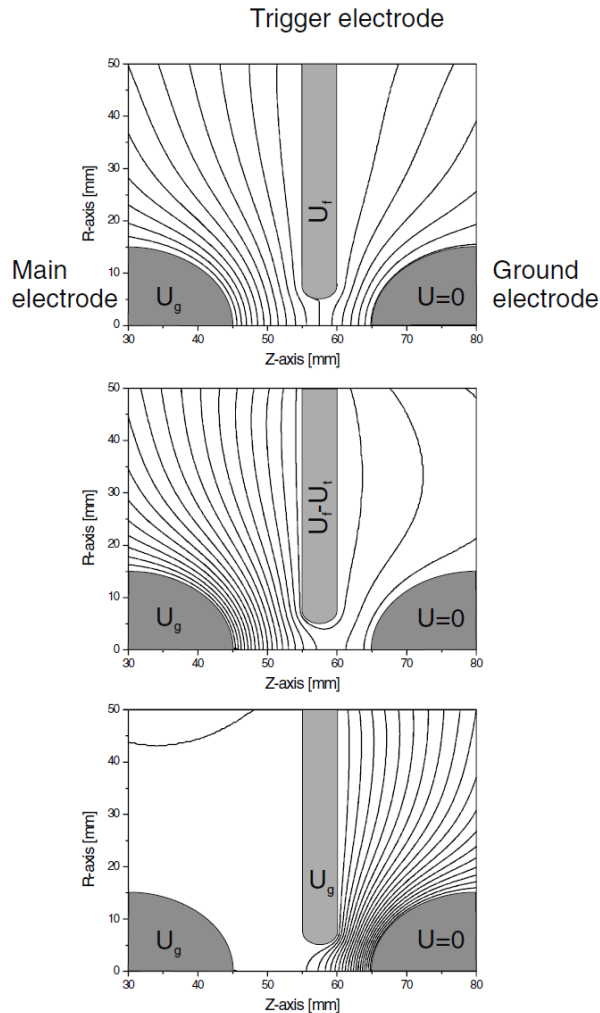
Example of short pulses with a controllable repetition rate



Spark-gap switch



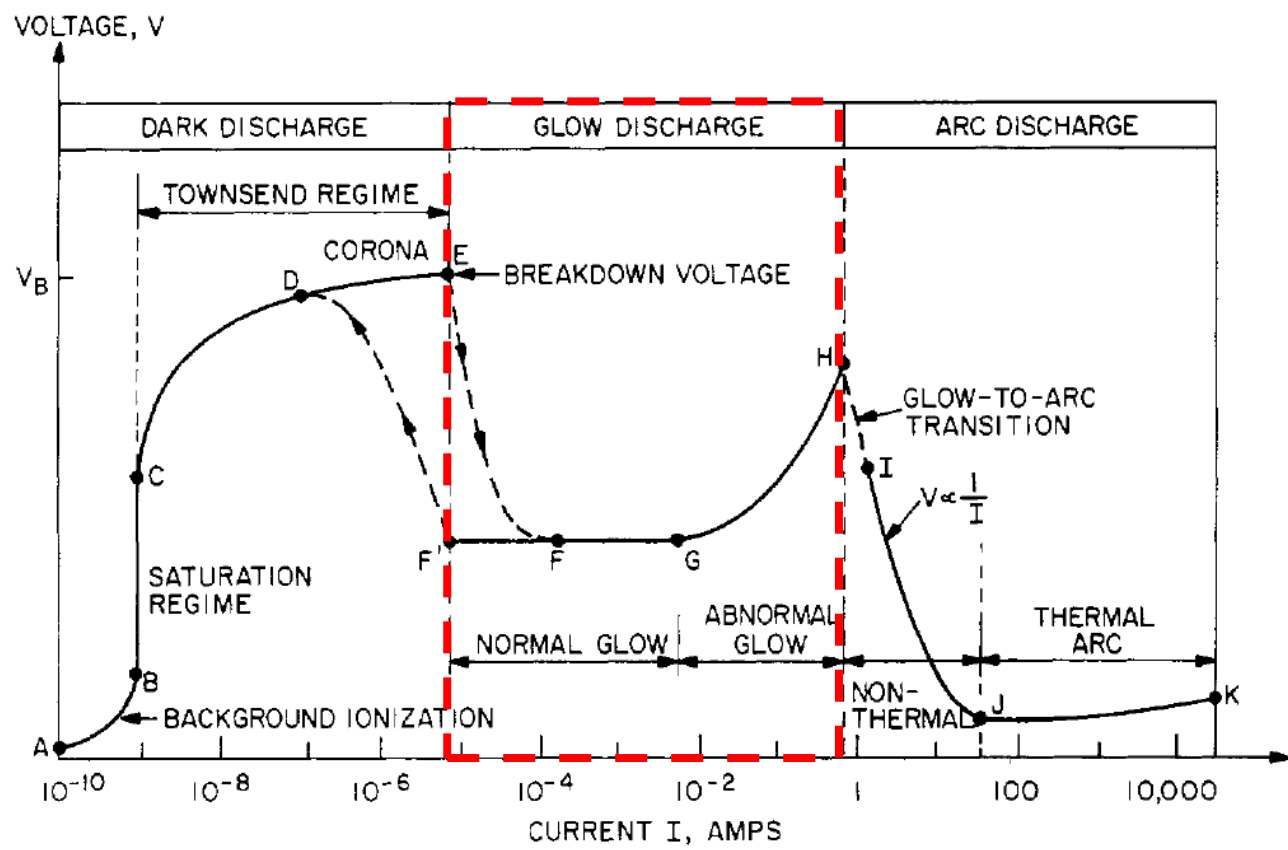
A spark gap switch is closed when electron breakdown occurs



DC electrical glow discharges in gases



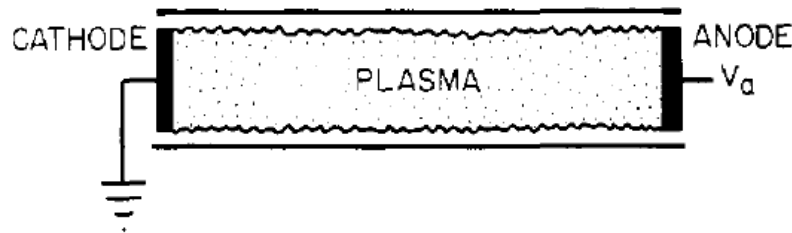
- The internal resistance of the power supply is relatively low, then the gas will break down at the voltage V_B , and the discharge tube will move from the dark discharge regime into the low pressure normal glow discharge regime.



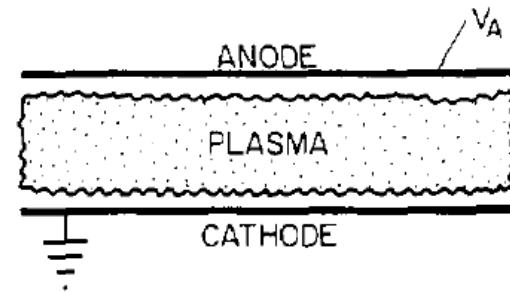
The plasma is luminous in the glow discharge regime



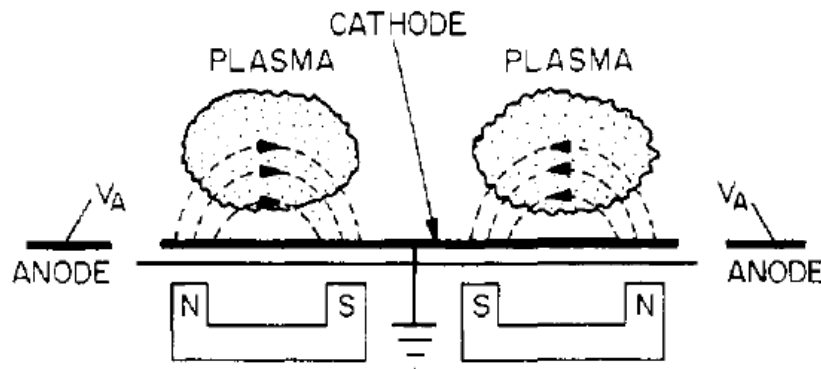
- The luminosity arises because the electron energy and number density are high enough to generate visible light by excitation collisions.



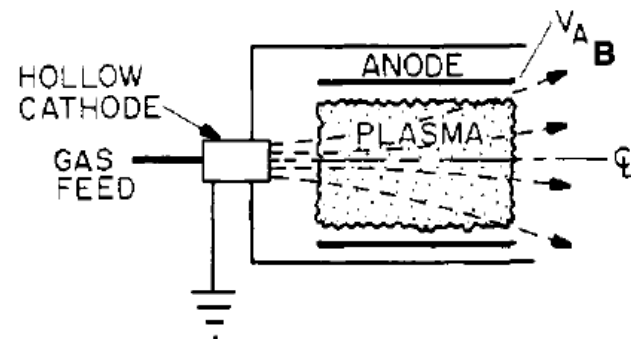
CLASSICAL DC ELECTRICAL DISCHARGE TUBE



PARALLEL PLATE PLASMA REACTOR

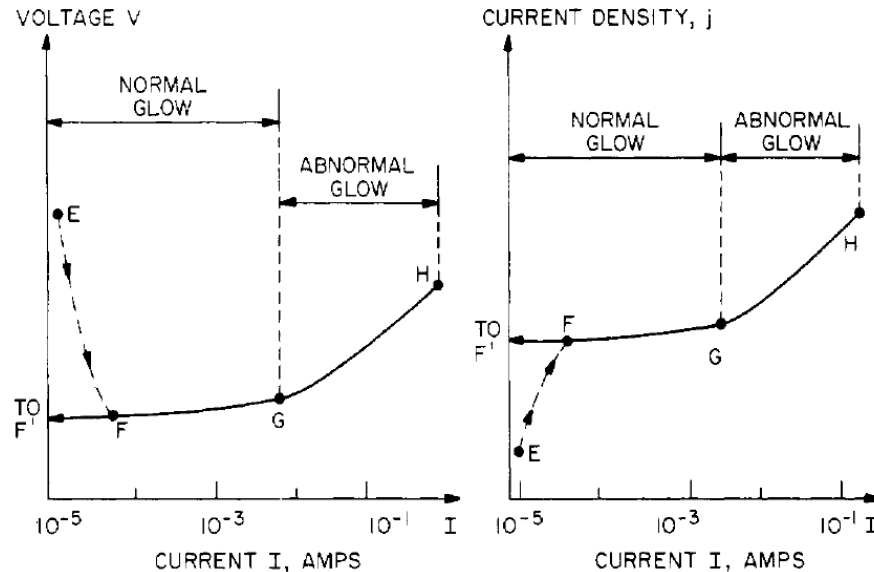


COPLANAR MAGNETRON REACTOR



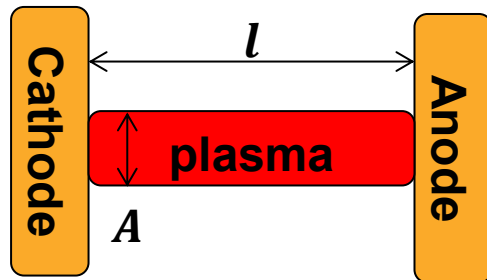
COAXIAL ELECTRON BOMBARDMENT DISCHARGE CHAMBER

Abnormal glow discharge occurs when the cross section of the plasma covers the entire surface of the cathode

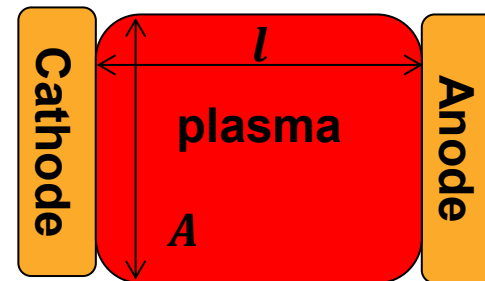


$$R = \eta \frac{l}{A}$$

- Normal glow discharge:

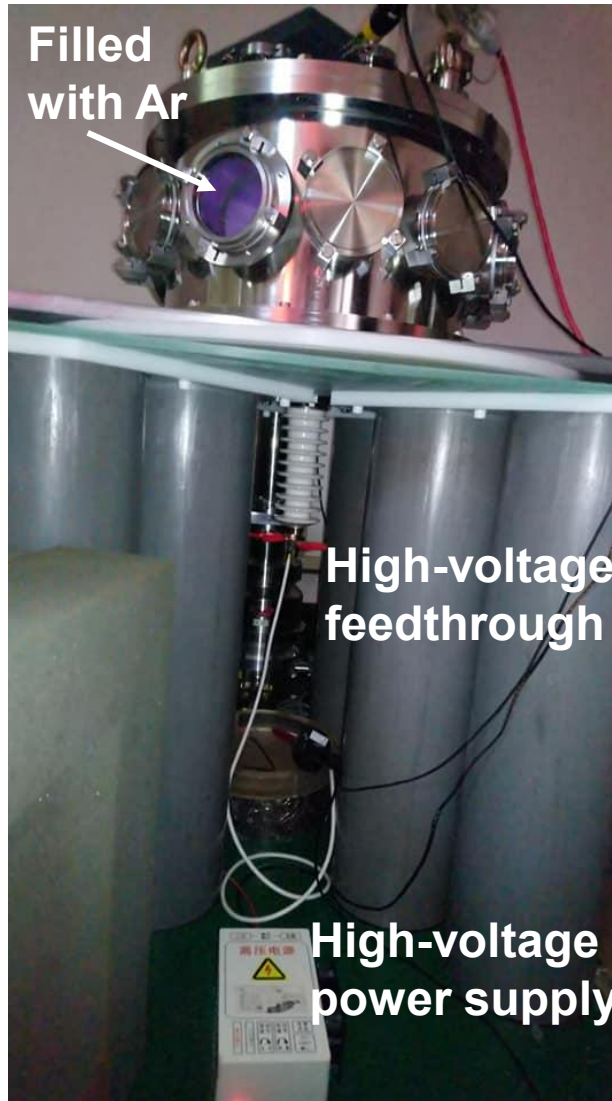


- Abnormal glow discharge:



- Surface cleaning using plasma needs to work in the abnormal glow discharge region.

Plasma cleaning needs to work in the regime of abnormal glow discharge



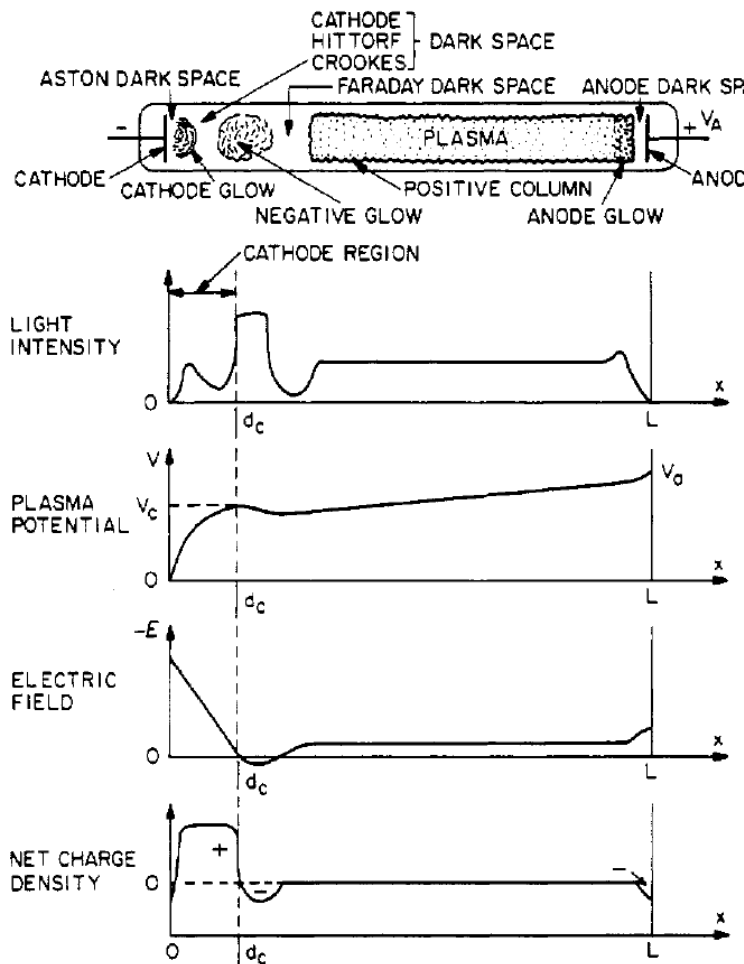
- Top view



- Side view

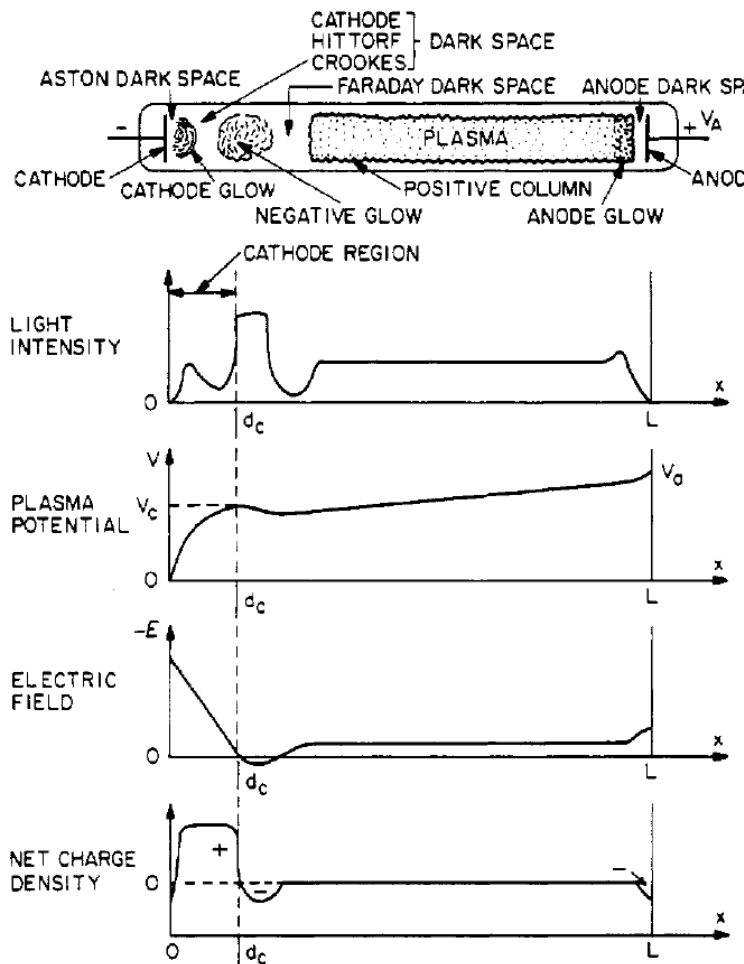


Low pressure normal glow discharge



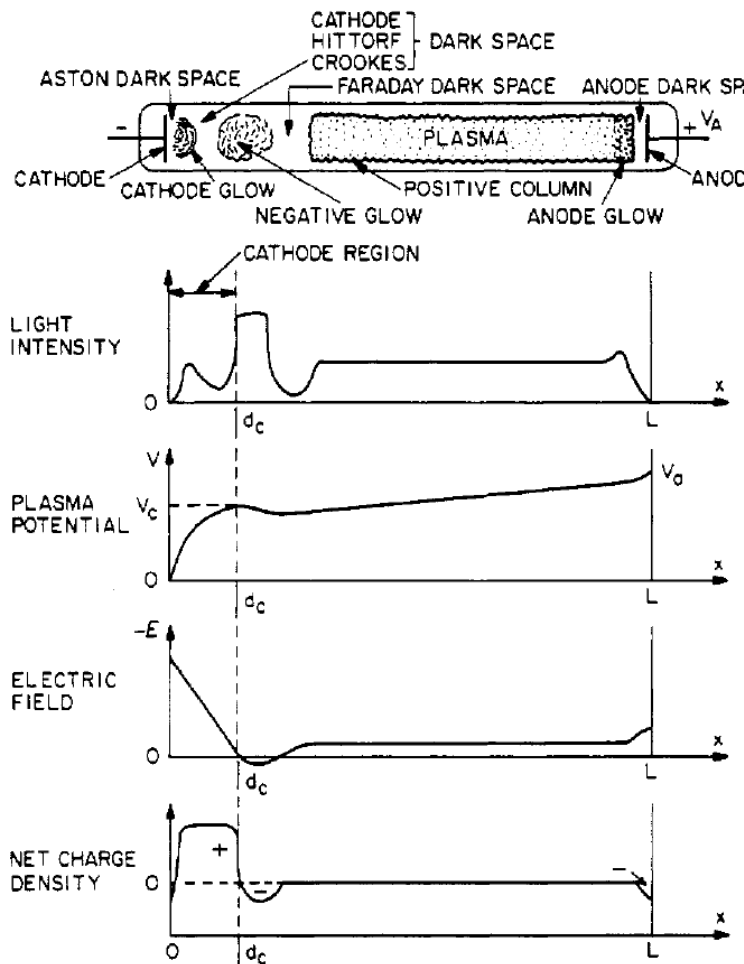
- **Cathode**: made of an electrically conducting metal with 2nd e⁻ emission γ , of which has a significant effect on the operation of the discharge tube.
- **Aston dark space**: a thin region with a strong electric field and a negative space charge. The electrons are of too low a density and/or energy to excite the gas, so it appears dark.
- **Cathode glow**: has a relatively high ion number density. The length depends on the type of gas and the gas pressure.
- **Cathode (Crookes, Hittorf) dark space**: has a moderate electric field, a positive space charge, and a relatively high ion density.

Low pressure normal glow discharge



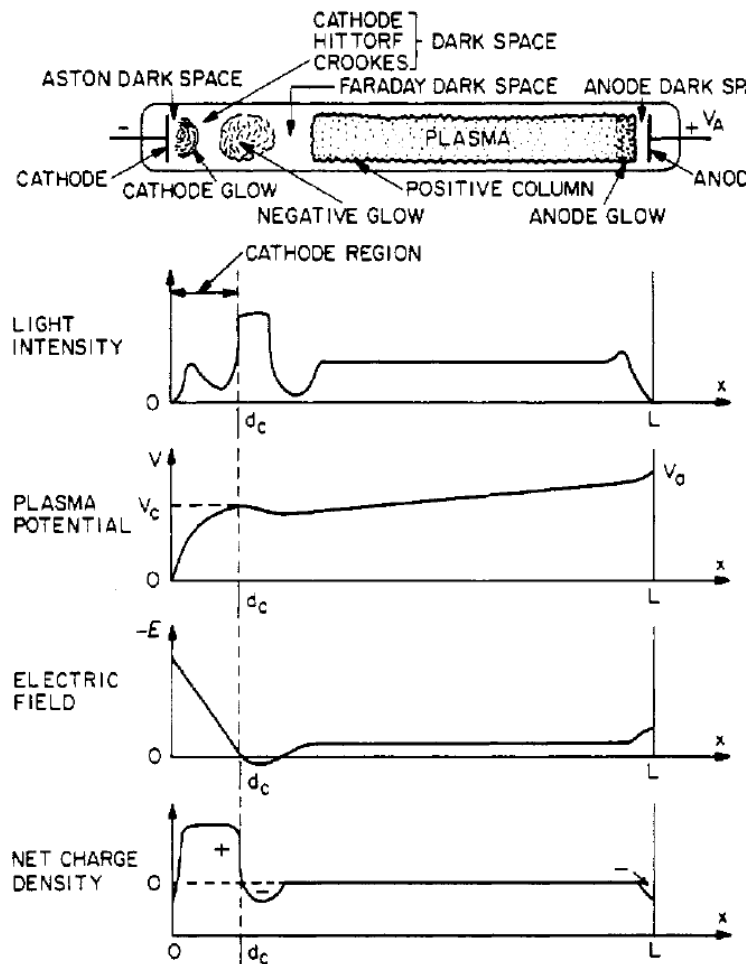
- **Cathode region:** most of the voltage drop (cathode fall) across the discharge tube appears between the cathode and the boundary between the cathode dark space and the negative glow. Electrons are accelerated to energies high enough to produce ionization and avalanching in this region. The axial length will adjust itself such that $d_c p \sim (dp)_{\min}$ where (dp) is the Paschen minimum.

Low pressure normal glow discharge



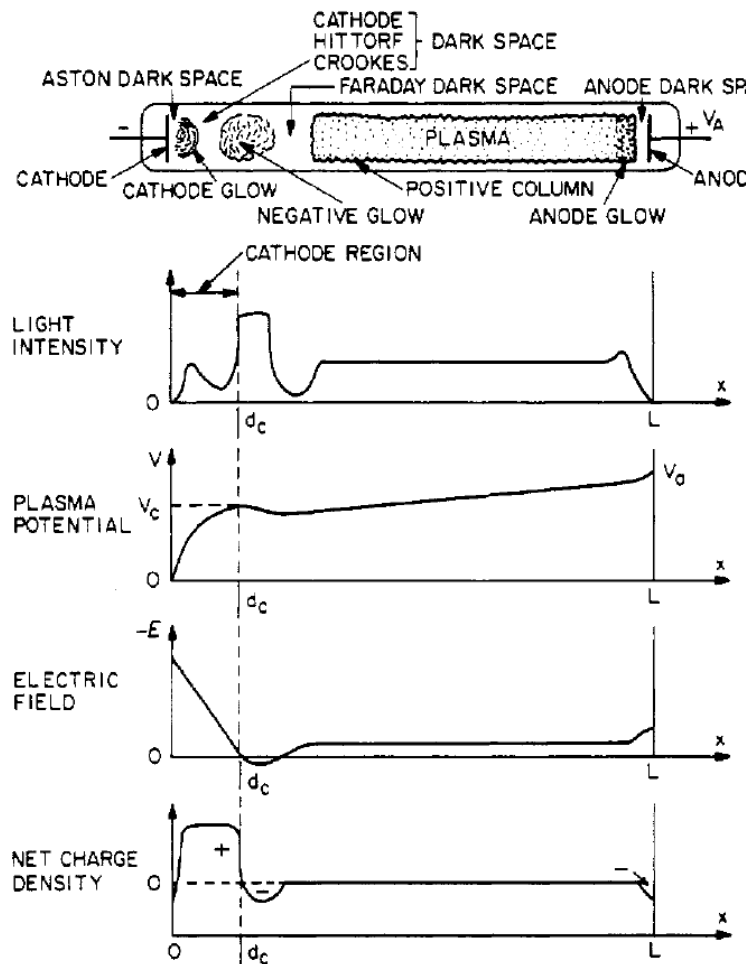
- **Negative glow**: the brightest light intensity in the entire discharge. It has a relatively low electric field and is usually long compared to the cathode glow. Electrons carry almost the entire current in the negative glow region. Electrons which have been accelerated in the cathode region produce ionization and intense excitation in the negative glow, hence the bright light output observed.

Low pressure normal glow discharge



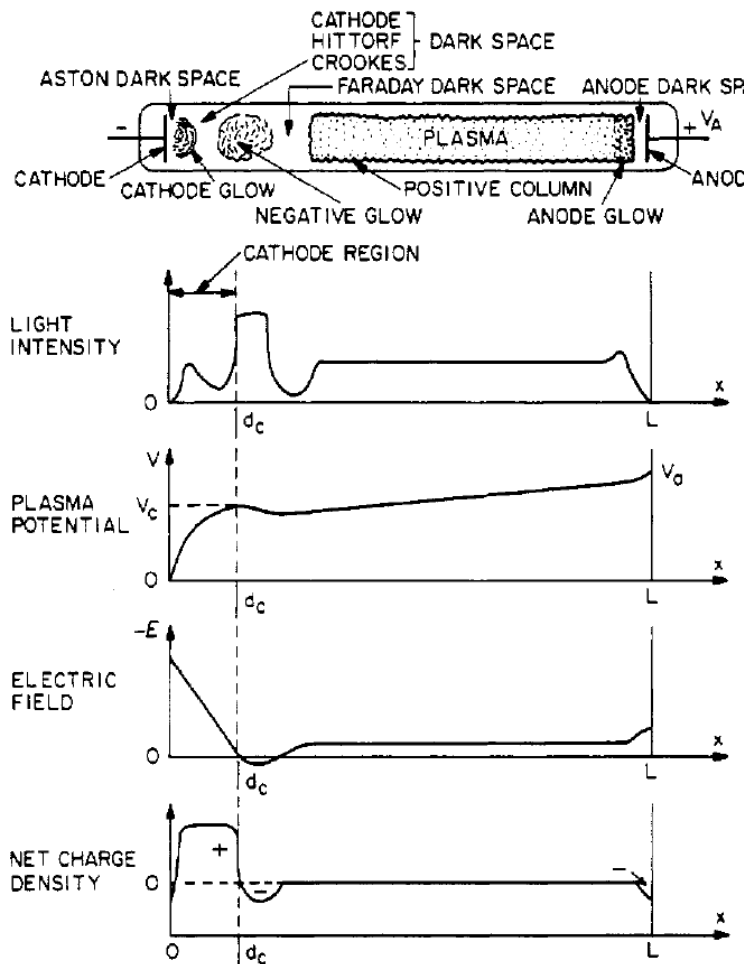
- **Faraday dark space:** the electron energy in it is low as a result of ionization and excitation interactions in the negative glow. The electron number density decreases by recombination and radial diffusion, the net space charge is very low, and the axial electric field is relatively small.

Low pressure normal glow discharge



- **Positive column:** quasi-neutral, the electric field is small and is just large enough to maintain the required degree of ionization at its cathode end. Since the length of cathode region remains constant, the positive column lengthens as the length of the discharge tube is increased.

Low pressure normal glow discharge

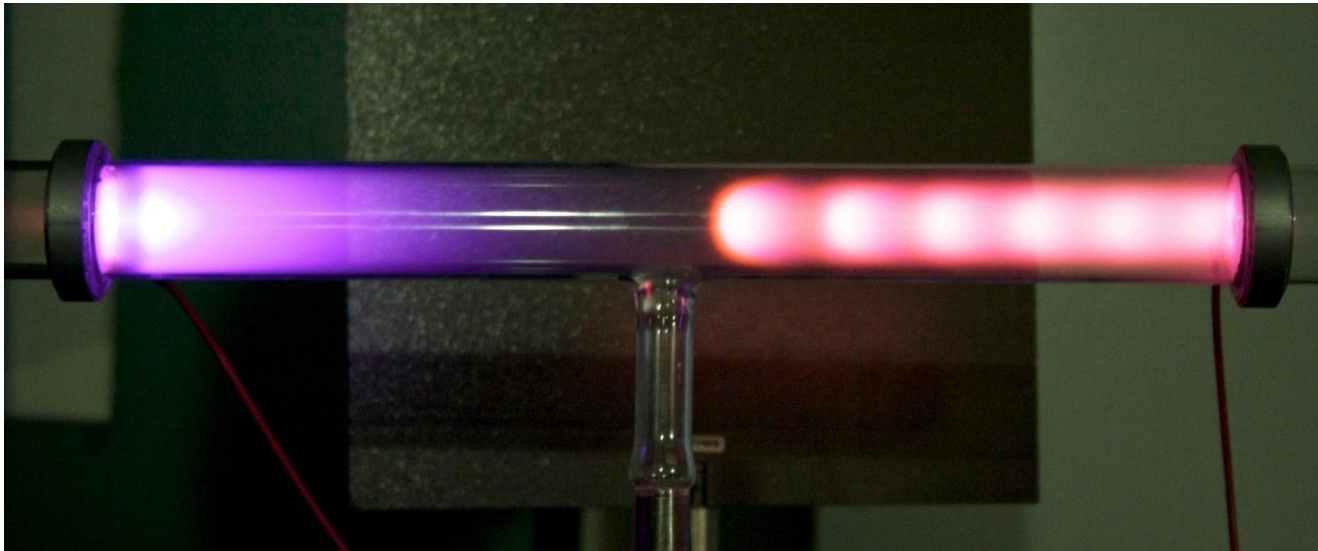


- **Anode glow**: the boundary of the anode sheath, slightly more intense than the positive column.
- **Anode dark space**: has a negative space charge due to electrons traveling from the positive column to the anode and a higher electric field than the positive column. The anode pulls electrons out of the positive column and acts like a Langmuir probe in electron saturation in this respect.

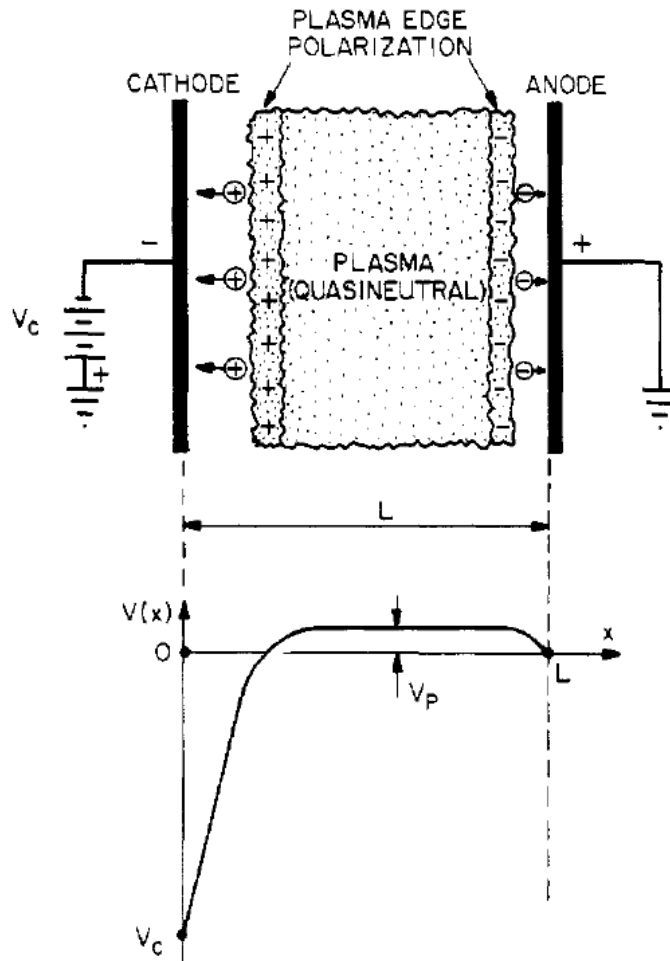
Striated discharges



- Moving or standing striations are, respectively, traveling waves or stationary perturbations in the electron number density which occur in partially ionized gases, including the positive columns of DC normal glow discharge tubes.
- <https://youtu.be/Be4RIjMTOWE>



Obstructed discharges



$$L < d_c$$

at the Paschen minimum, i.e., $(pd_c)_{\min}$

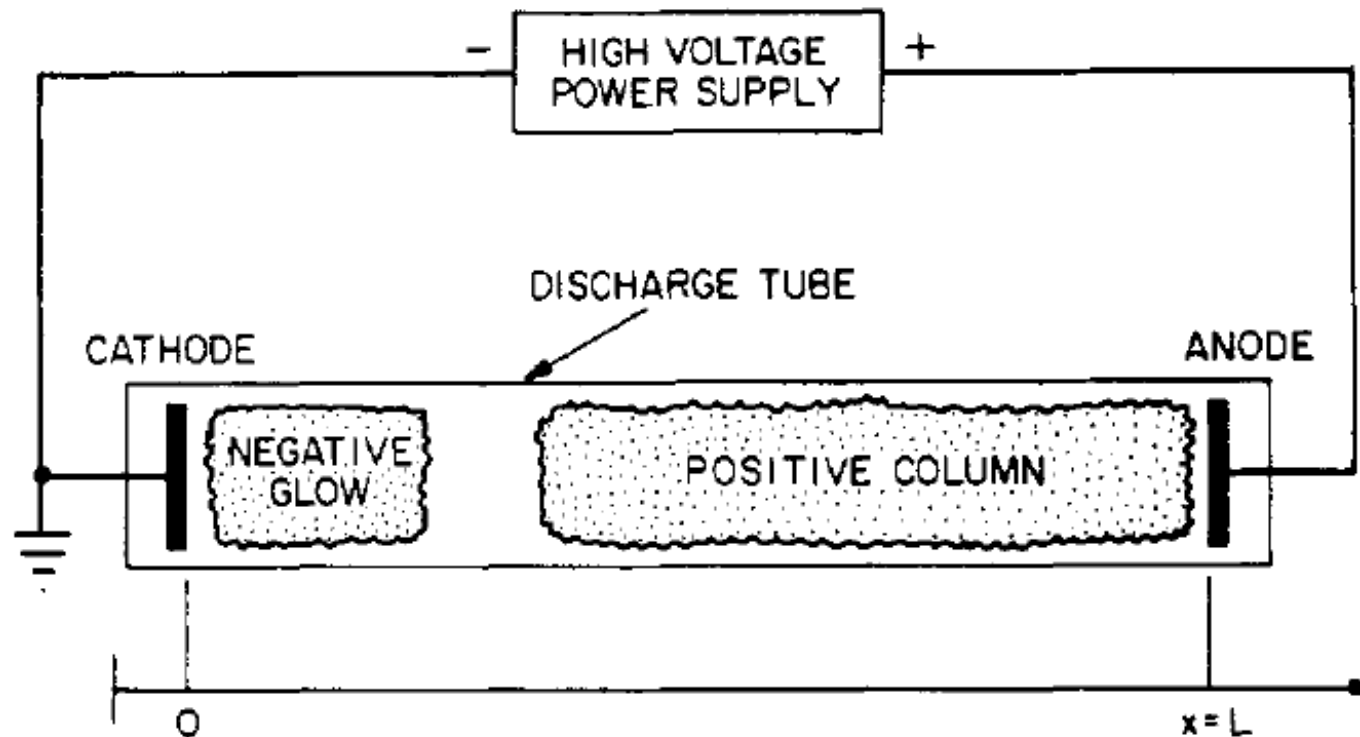
$$V_c > V_{\text{Paschen}}$$

- The obstructed glow discharge finds many uses in industry, where the high electron number densities generated by such discharge are desired. It will operate with a higher anode voltage. Such high voltage drops are sometimes desirable to accelerate ions into a wafer for deposition or etching purposes.

Cylindrical glow discharge sources



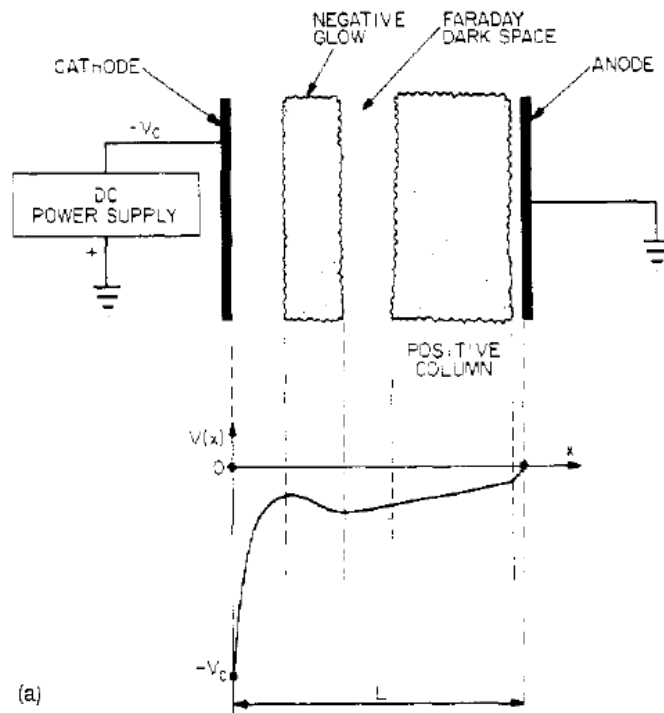
- This configuration is used in lighting devices, such as fluorescent lights and neon advertising signs.



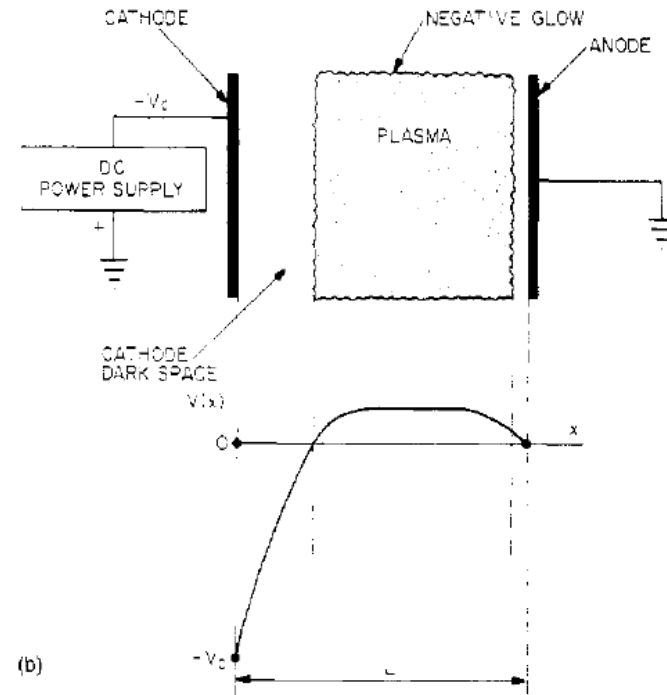
Parallel plate sources are widely used for plasma processing and plasma chemistry applications



- Unobstructed operation

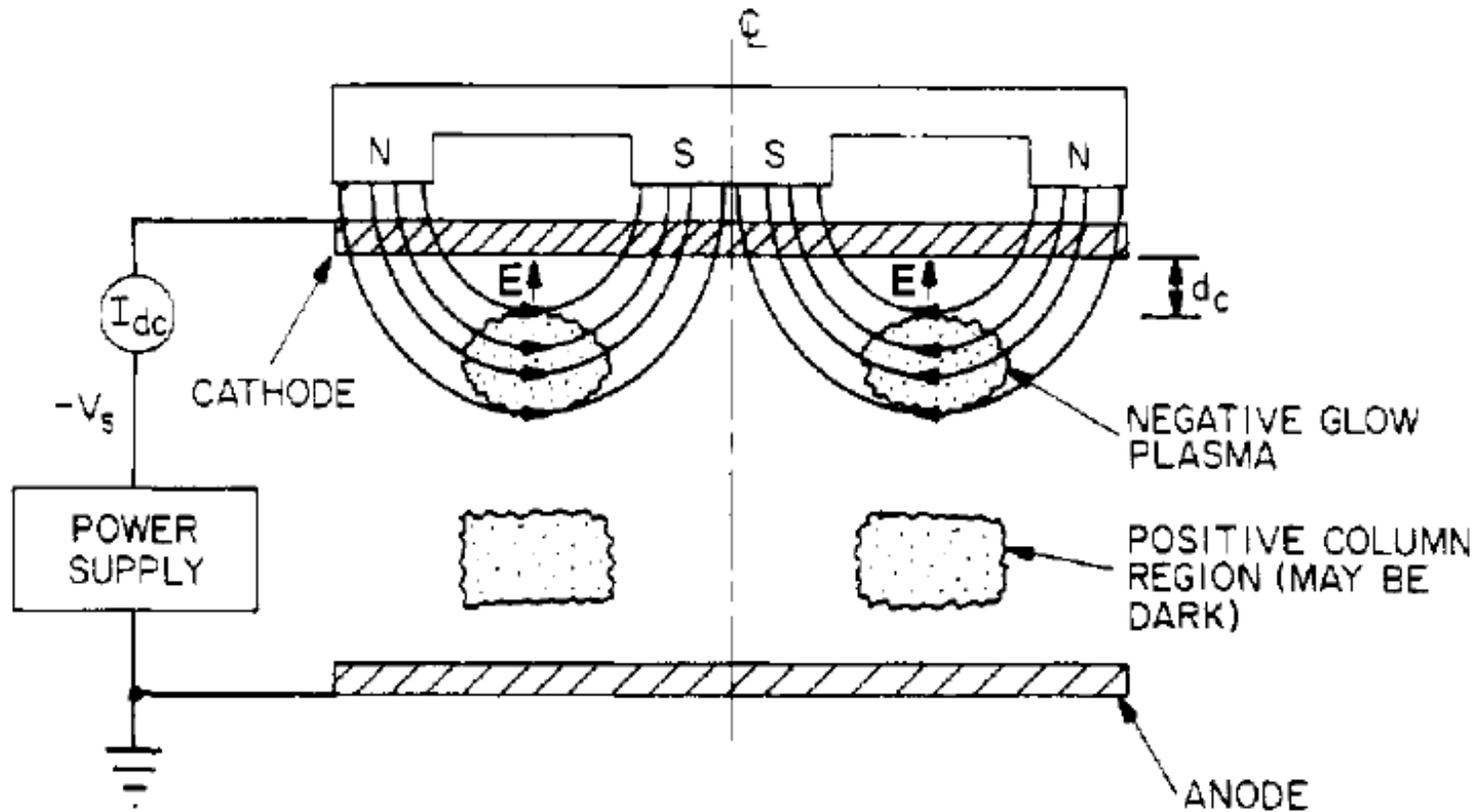


- Obstructed operation



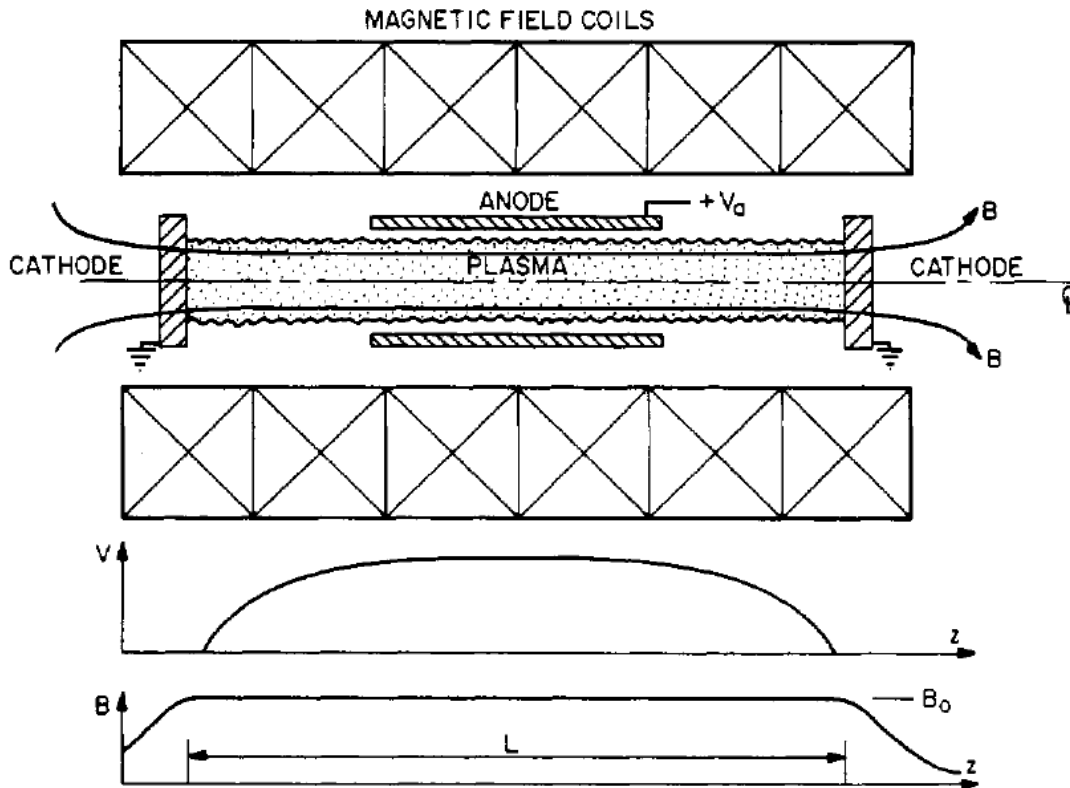
- The obstructed configuration is used for plasma processing, where high ion energies bombarding the cathode, over large areas and at vertical incidence, are desired.

Magnetron plasma source are used primarily for plasma-assisted sputtering and deposition



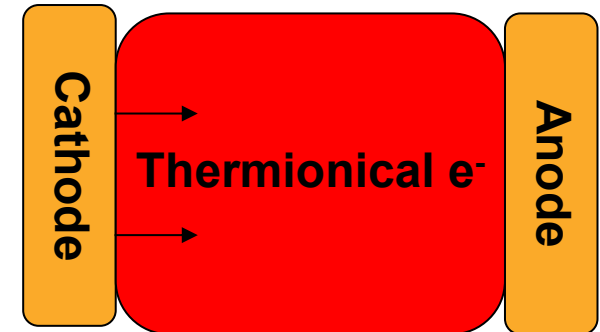
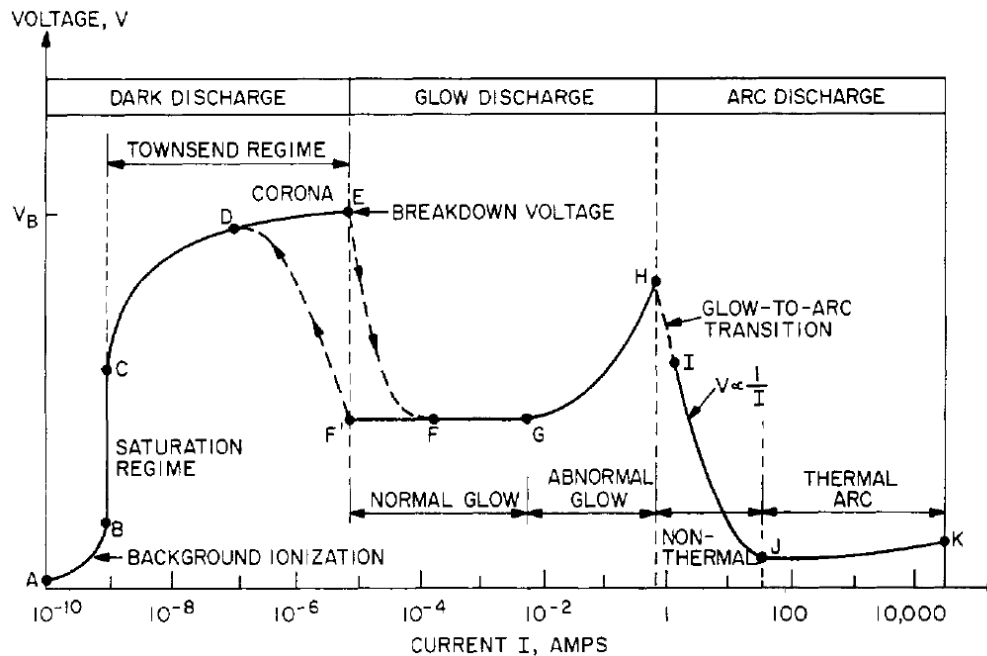
- When several hundred voltages are applied between the parallel plates, a glow discharge will form, with a negative glow plasma trapped in the magnetic mirrors above the magnet pole pieces.

Penning discharge plasma sources produce a dense plasma at pressures far below than most other glow discharges



- Strong axial magnetic fields: to prevent electrons from intercepting the anode.
- Axial electric fields: electrons are reflected by opposing cathodes.
- Multiple reflection of the electrons along axis.

Discharge may enter glow-to-arc transition region if the cathode gets hot enough to emit electrons thermionically



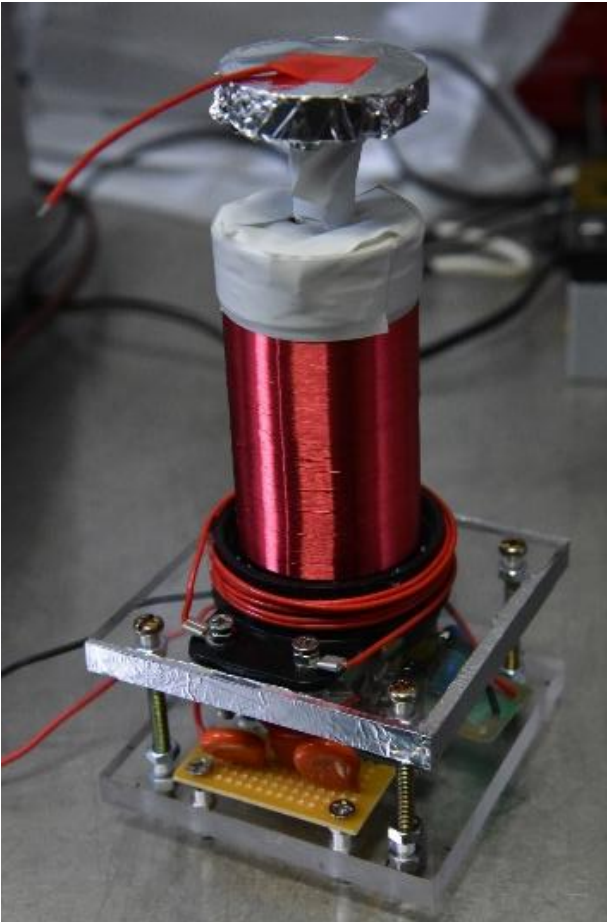
- If the cathode gets hot enough to emit electrons thermionically and the internal impedance of the power supply is sufficiently low, the discharge will make a transition into the arc regime.

Methods of plasma production

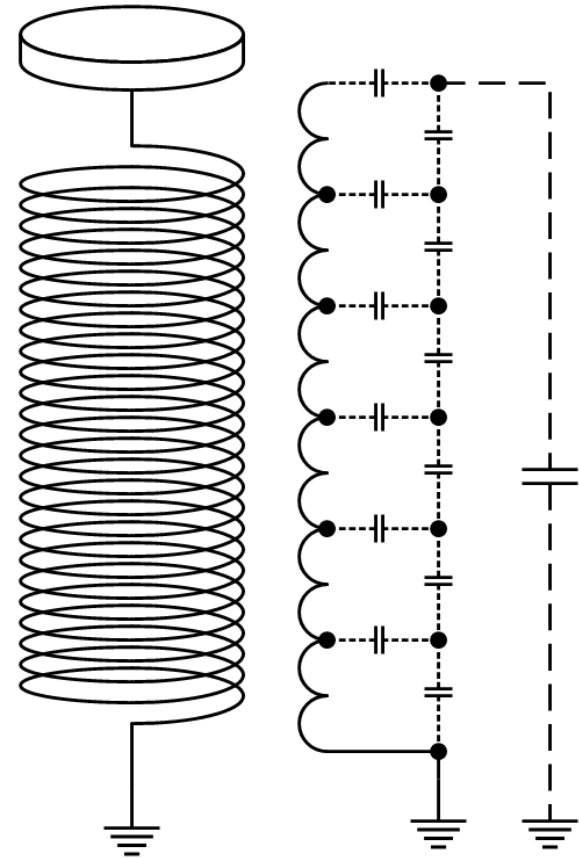
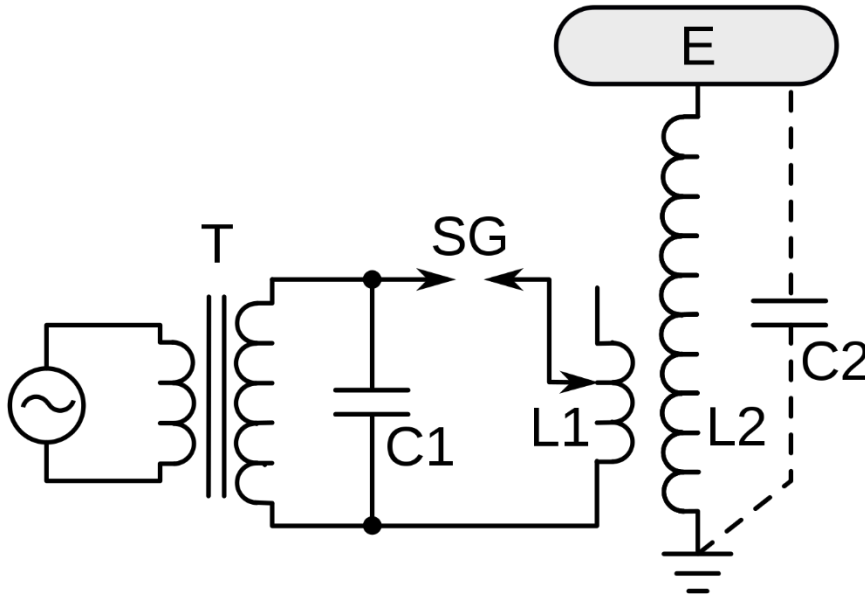


- **DC electrical discharges**
 - **Dark electrical discharges in gases**
 - **DC electrical glow discharges in gases**
 - **DC electrical arc discharges in gases**
- **AC electrical discharges**
 - RF electrical discharges in gases
 - Microwave electrical discharges in gases
 - Dielectric-barrier discharges (DBDs)
- **Other mechanism**
 - Laser produced plasma
 - Pulsed-power generated plasma

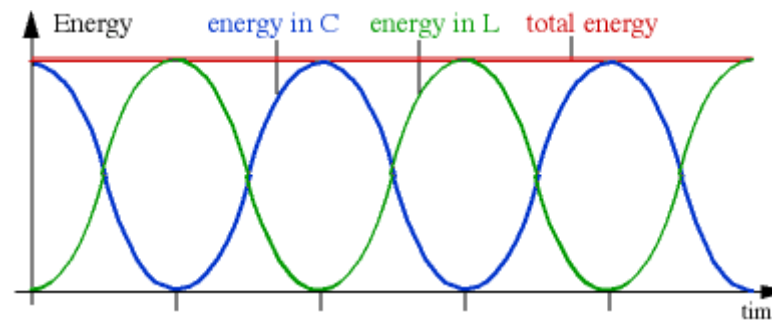
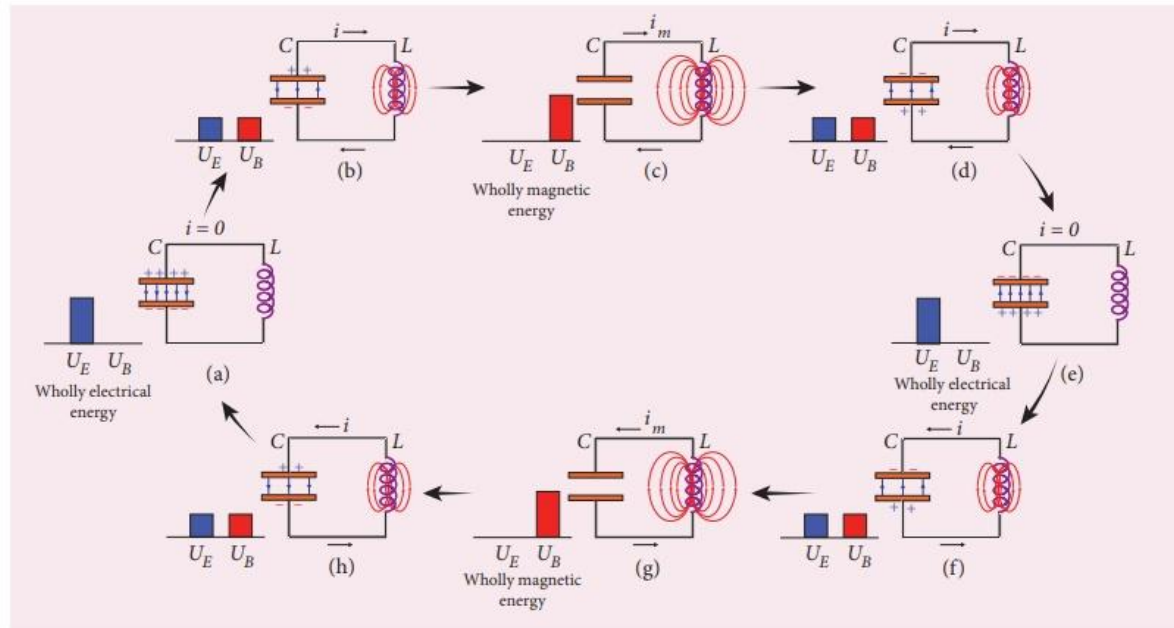
Tesla coil can generate high voltage



The high voltage is generated by two resonant LC circuits



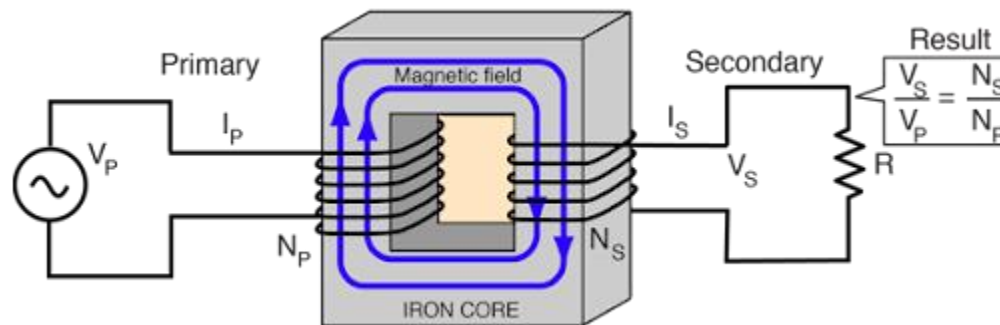
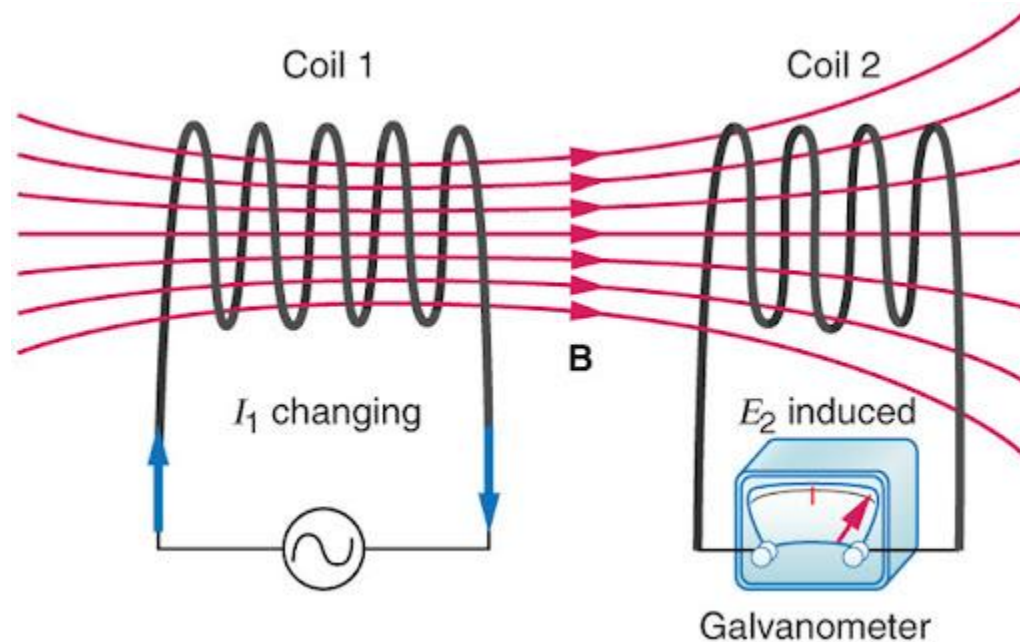
Energy is oscillating between the capacitor and the inductor



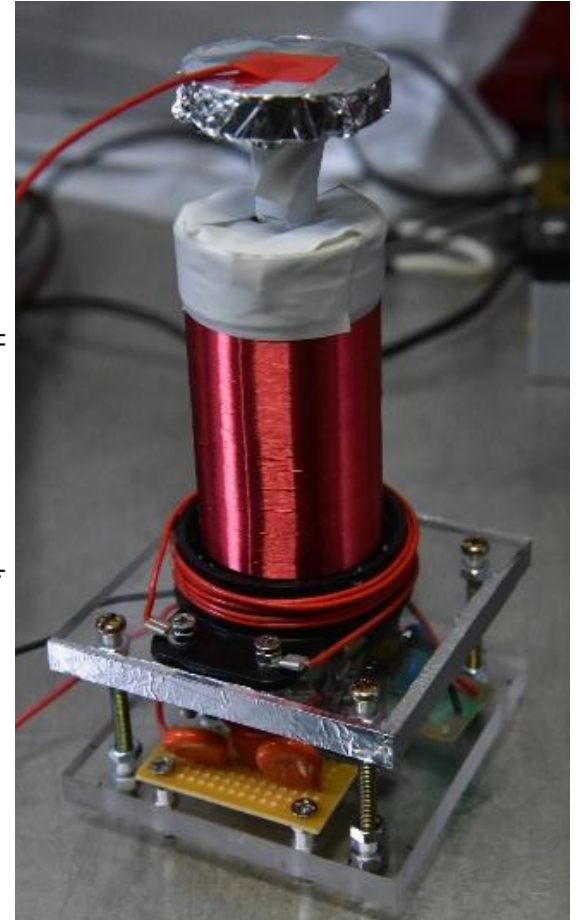
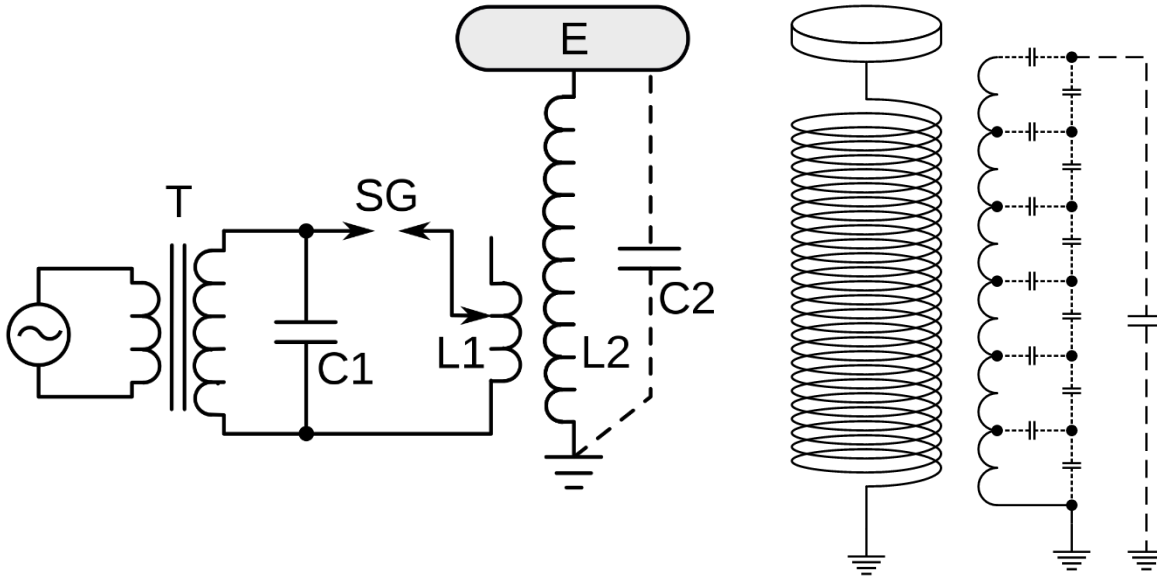
https://www.brainkart.com/article/Energy-conversion-during-LC-oscillations_38532/

http://ffden-2.phys.uaf.edu/webproj/211_fall_2016/Mark_Underwood/mark_underwood/Primary.html

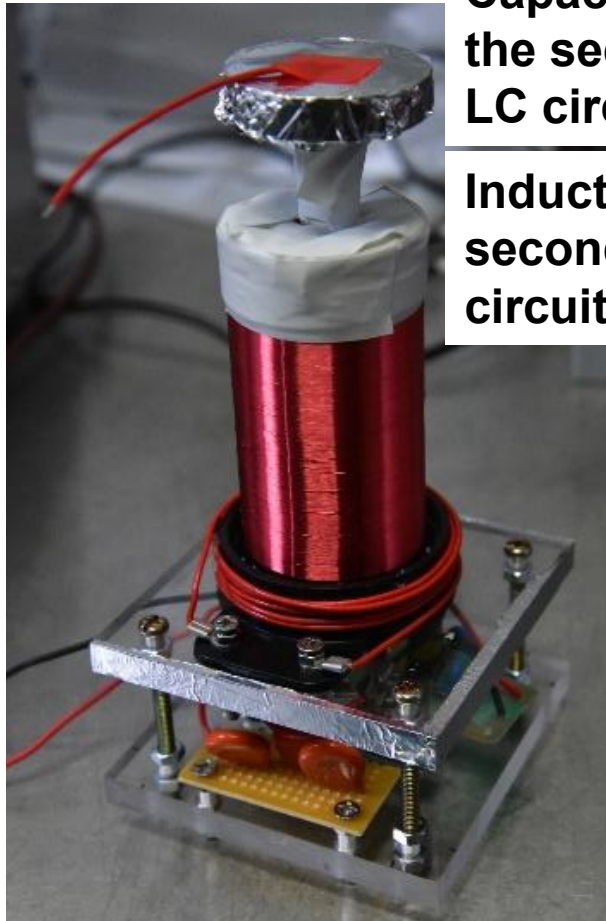
Voltage of two separated coils can be transferred by mutual inductance between two coils



The high voltage is generated by two resonant LC circuits

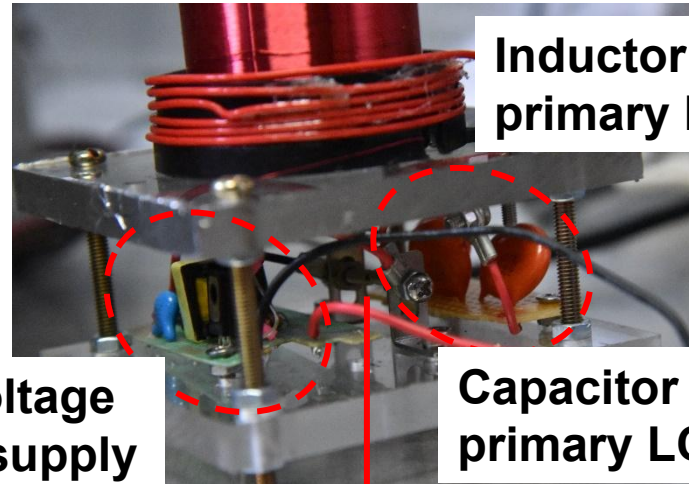


Components of the tesla coil



Capacitor of the secondary LC circuit

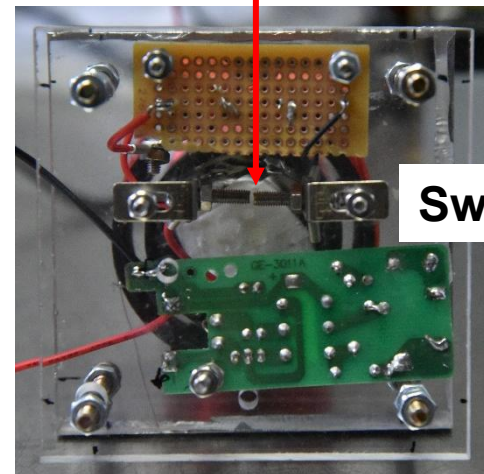
Inductor of the secondary LC circuit



Inductor of the primary LC circuit

Capacitor of the primary LC circuit

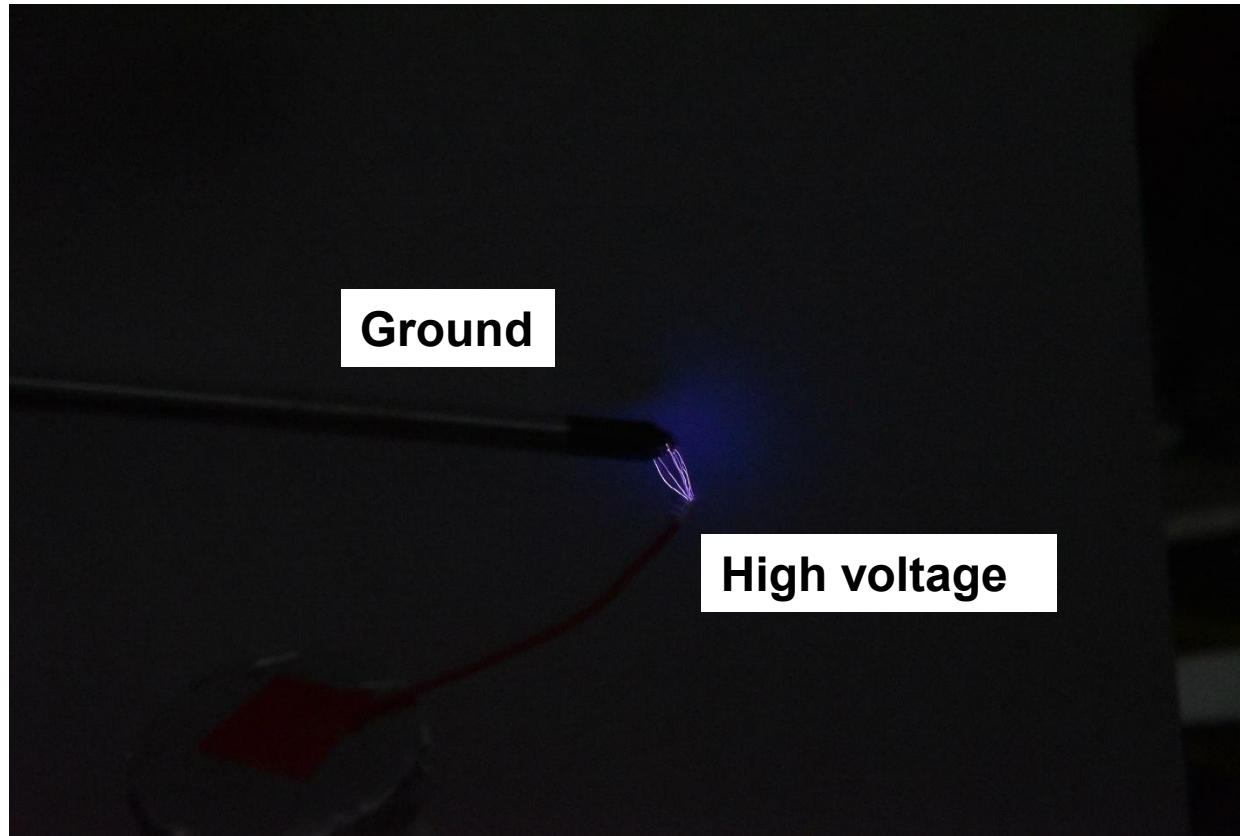
High voltage power supply



Switch

Show video.

Arc discharge occur between the high voltage and a grounded electrode



Corona discharge occurs when the electric field drops below a certain value

