Application of Plasma Phenomena

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Institute of Space and Plasma Sciences, National Cheng Kung University

2024 spring semester

Tuesday 9:10-12:00

Materials:

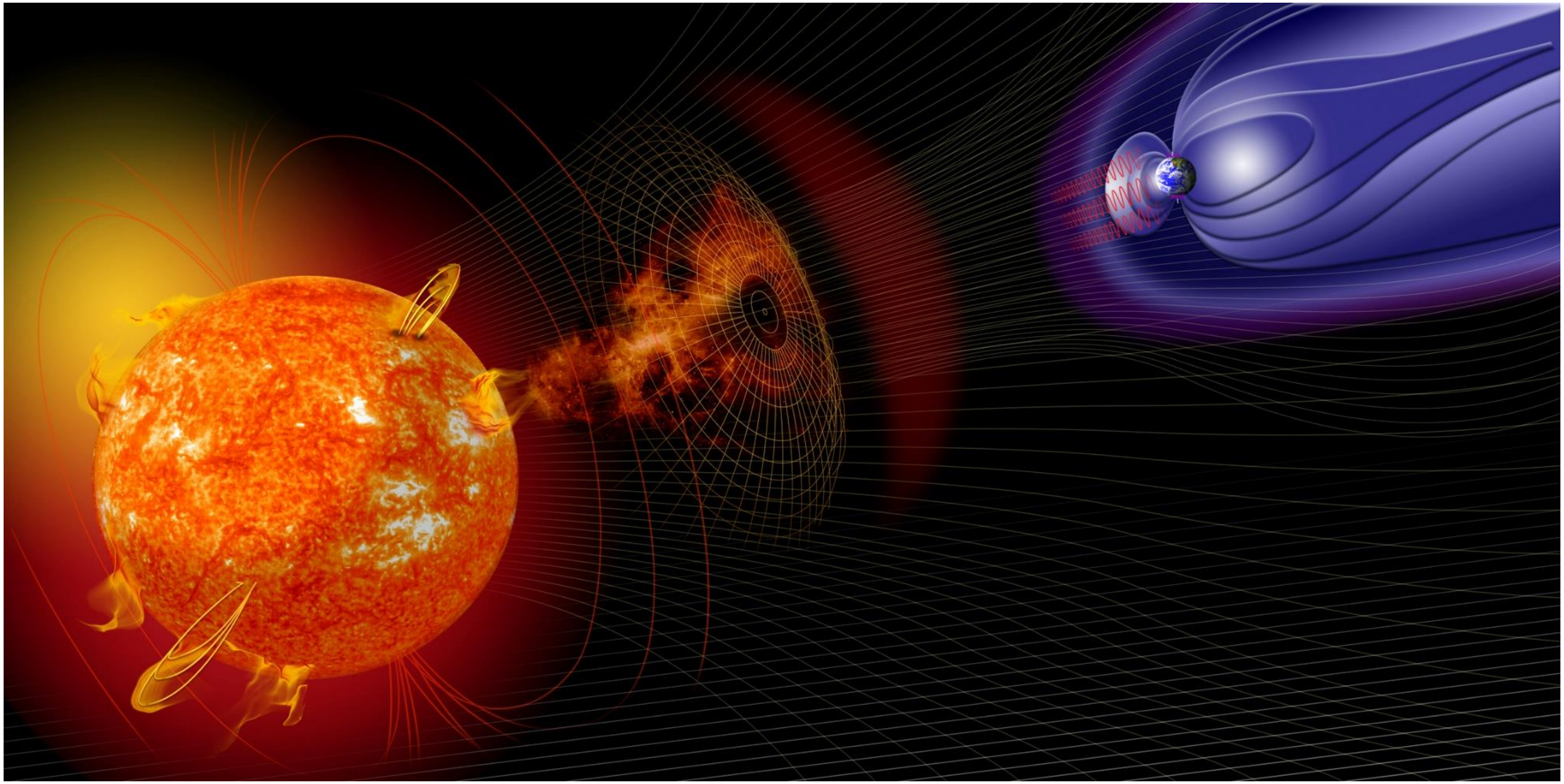
https://capst.ncku.edu.tw/PGS/index.php/teaching/

Online courses:

https://nckucc.webex.com/nckucc/j.php?MTID=m4082f23c59af0571015416f6e58dd803

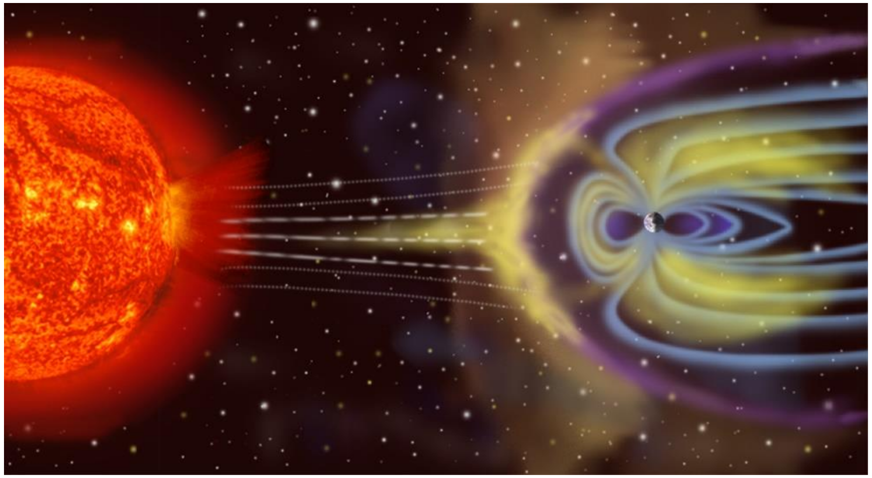
2024/2/20 updated

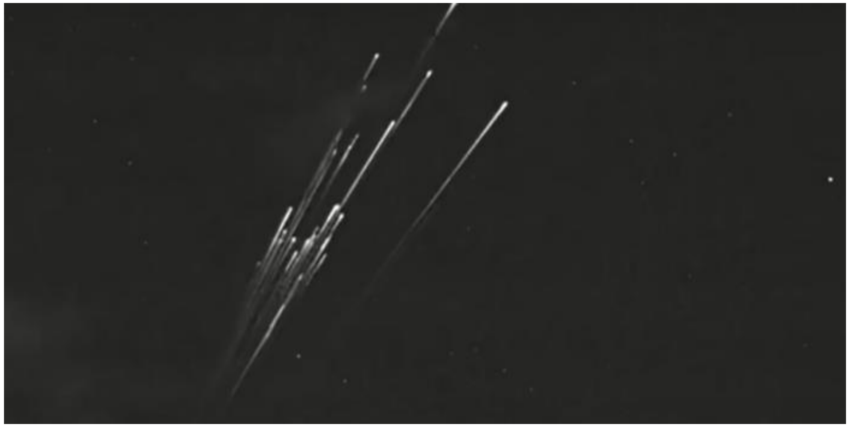
Most of the material in space is plasma



https://www.nasa.gov/mission_pages/sunearth/spaceweather/index.html

Forty SpaceX’s Starlink satellites were destroyed by a geomagnetic storm on 2022/2/4



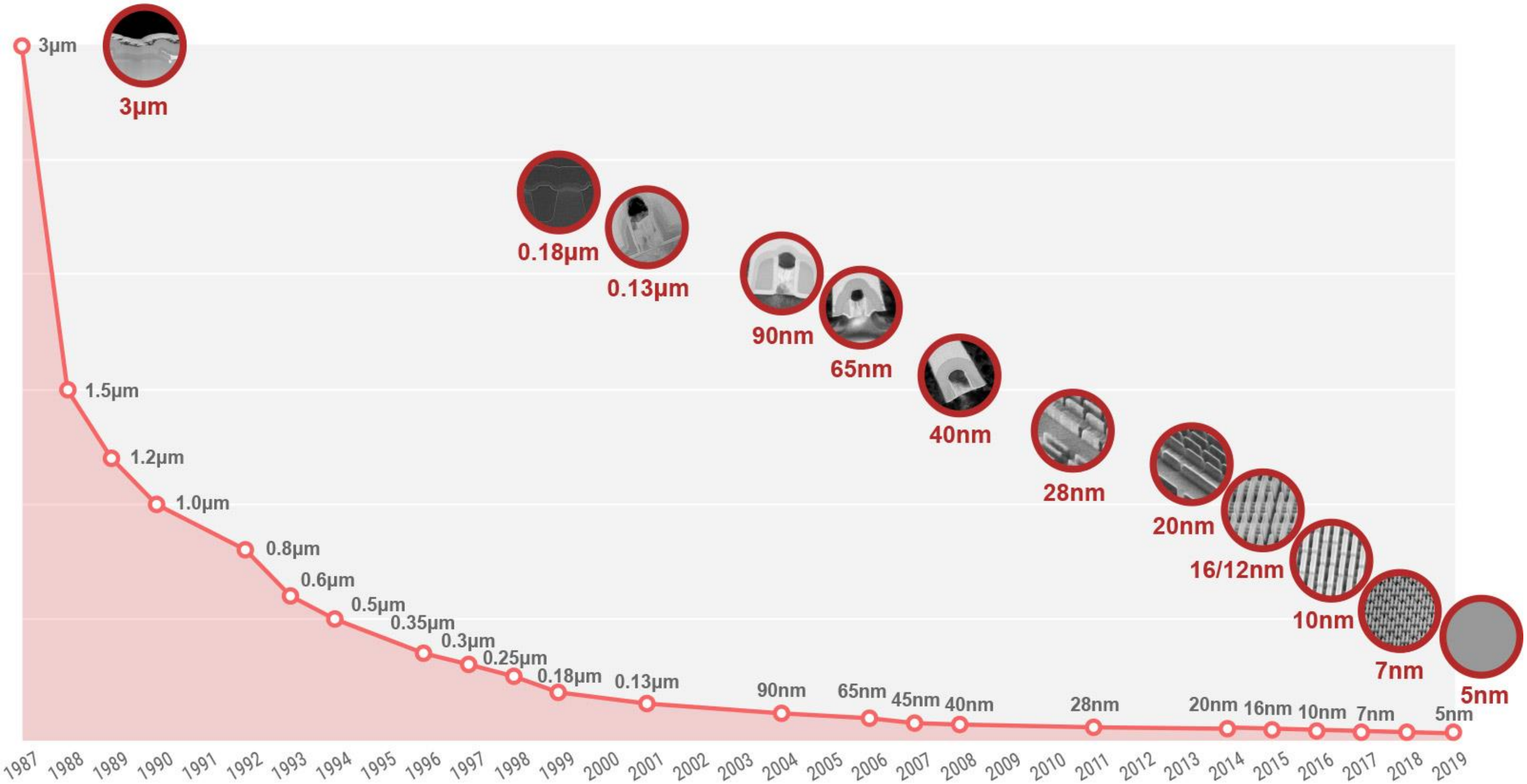


- **Geomagnetic storms occur when intense solar wind near Earth spawns shifting currents and plasmas in Earth's magnetosphere. This interaction can warm Earth's upper atmosphere and increase atmospheric density high enough above the planet to affect satellites in low orbits like SpaceX's new Starlink craft.**

https://en.wikipedia.org/wiki/Geomagnetic_storm
https://wonderfulengineering.com/watch-the-40-starlink-satellites-destroyed-by-a-geomagnetic-storm-burn-up-in-the-sky/

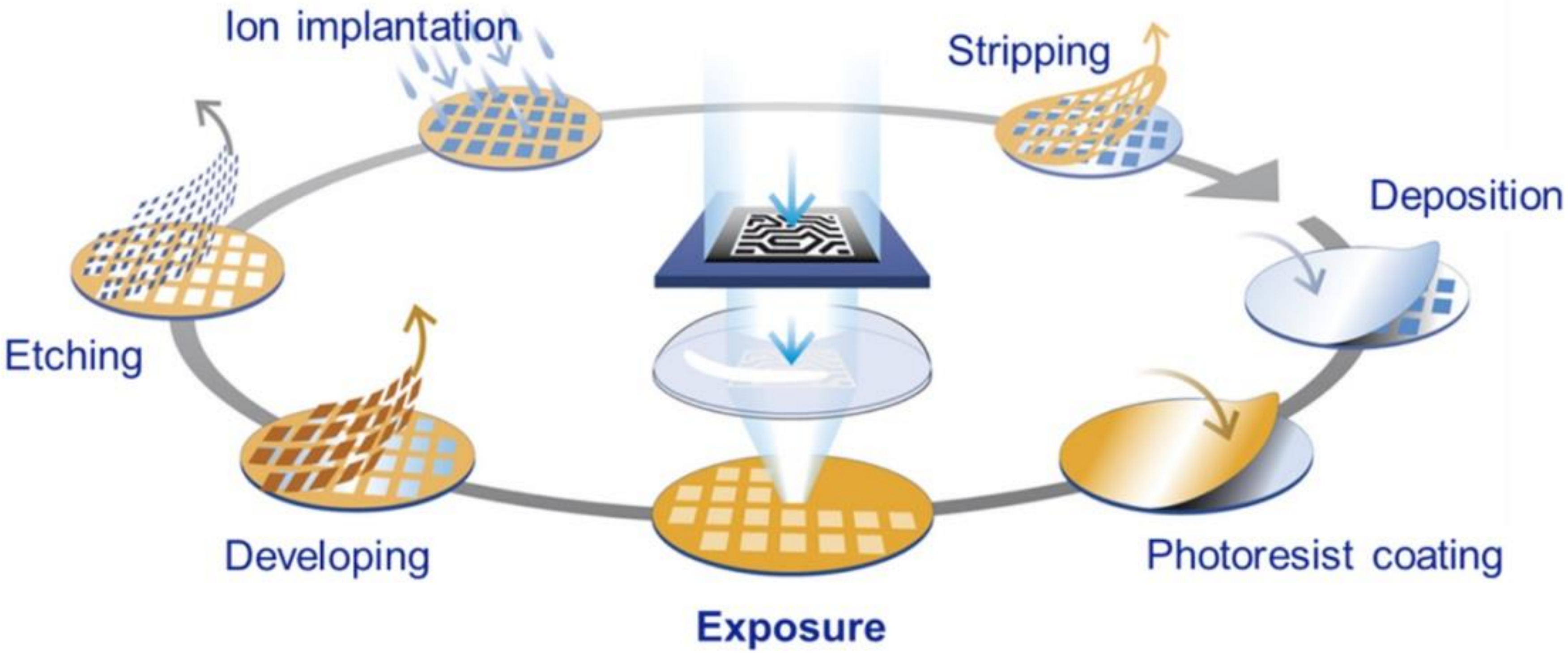
Plasma plays an important role on semiconductor manufacturing

- **The process technology of Taiwan Semiconductor Manufacturing Company Limited (TSMC):**



https://www.tsmc.com/chinese/dedicatedFoundry/technology/logic.htm

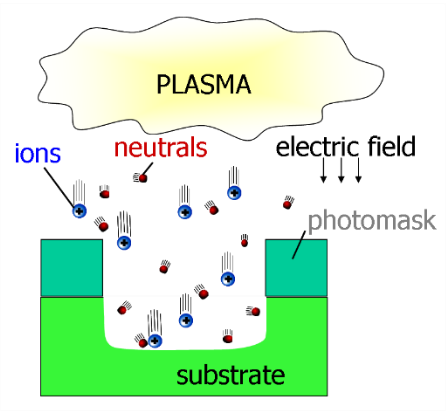
A semiconductor device is fabricated by many repetitive production process



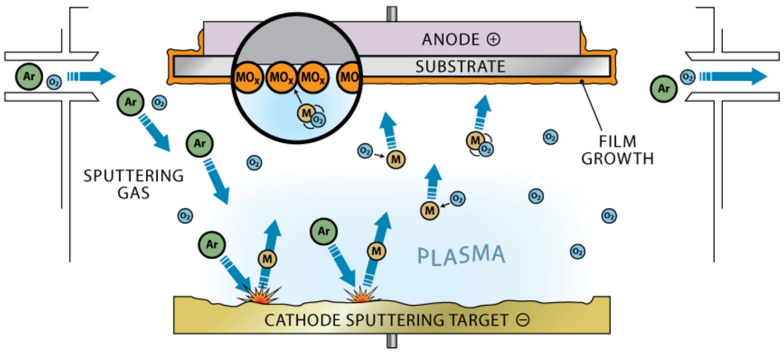
Surf. Topogr.: Metrol. Prop. **4** (2016) 023001

Plasma is widely used in semiconductor fabrication

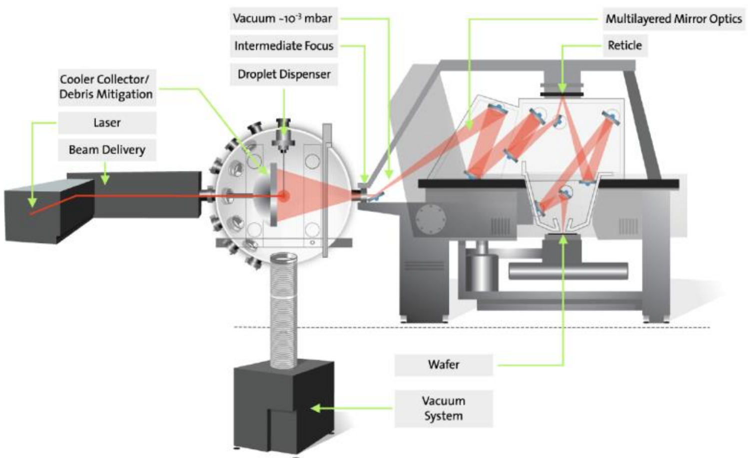
- **Etching**



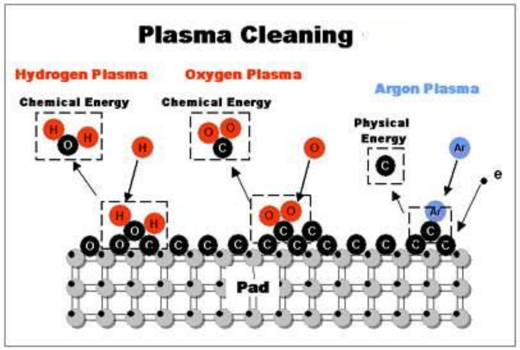
- **Deposition**



- **EUV light source**



- **Plasma cleaning**

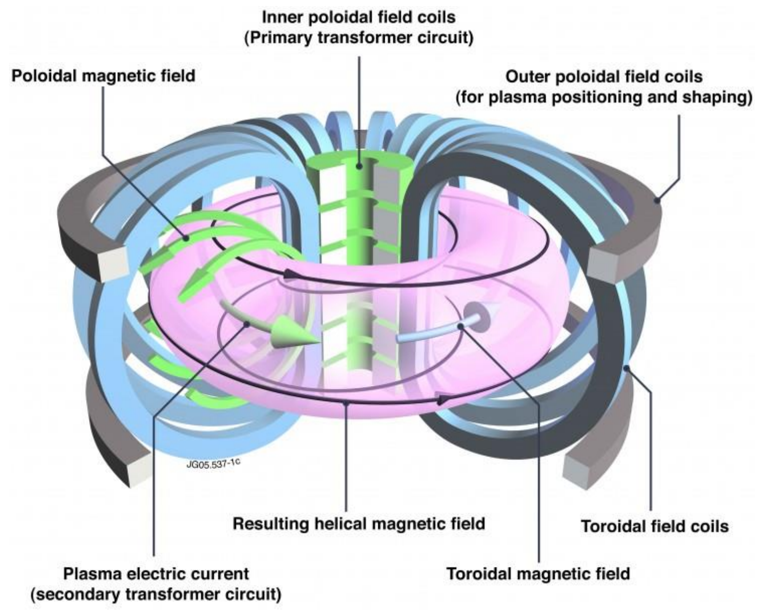


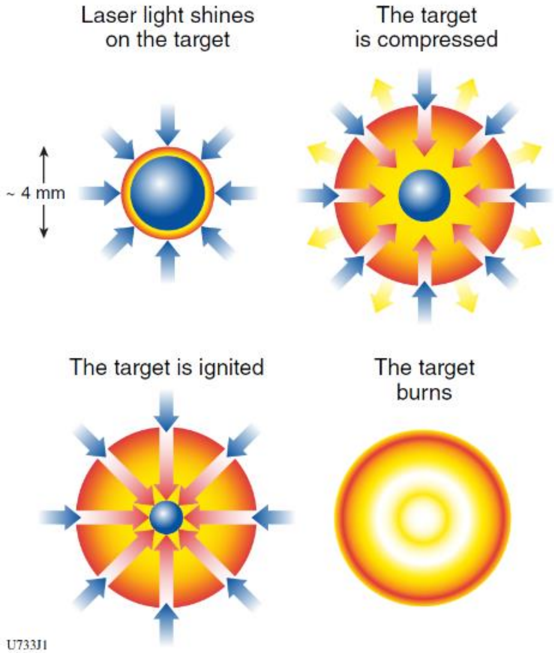
https://www.scorec.rpi.edu/research_plasmaetchmodeling.php
http://lnf-wiki.eecs.umich.edu/wiki/Sputter_deposition

R. S. Abhari, etc., J. Micro/Nanolithography, MEMS, and MOEMS, 11, 021114 (2012)
馗鼎奈米科技股份有限公司
https://www.ecplaza.net/products/plasma-cleaning_111807

Nuclear fusion as an energy source is being developed

- **Magnetic confinement fusion (MCF)**
- **Inertial confinement fusion (ICF)**

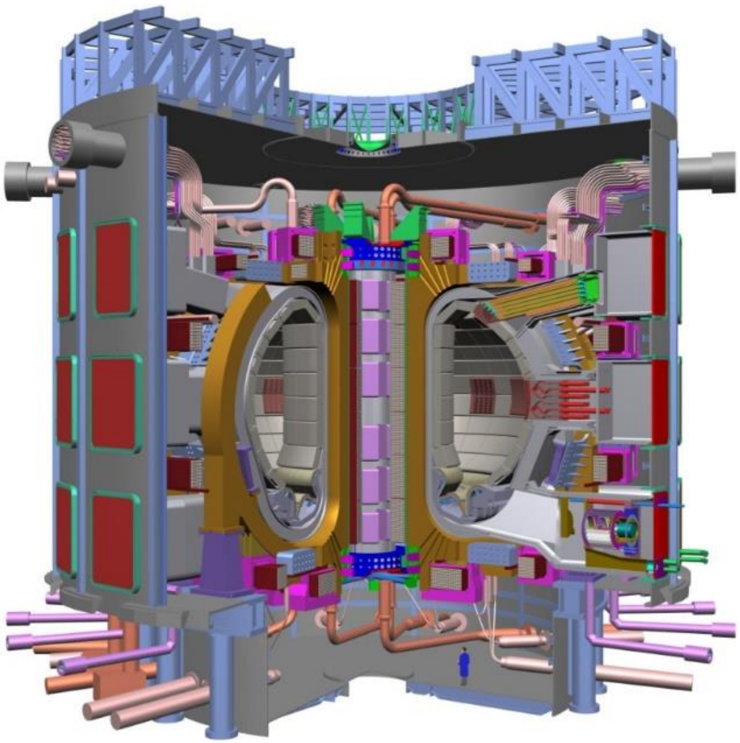




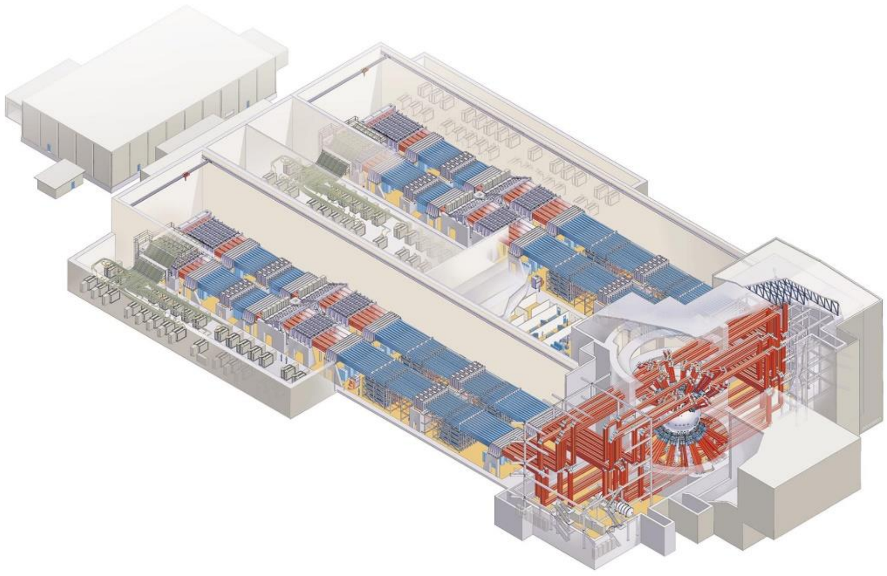
https://www.euro-fusion.org/2011/09/tokamak-principle-2/
Inertial confinement fusion: an introduction, Laboratory for Laser Energetics, University of Rochester

Nuclear fusion as an energy source is being developed

- **Magnetic confinement fusion (MCF)**



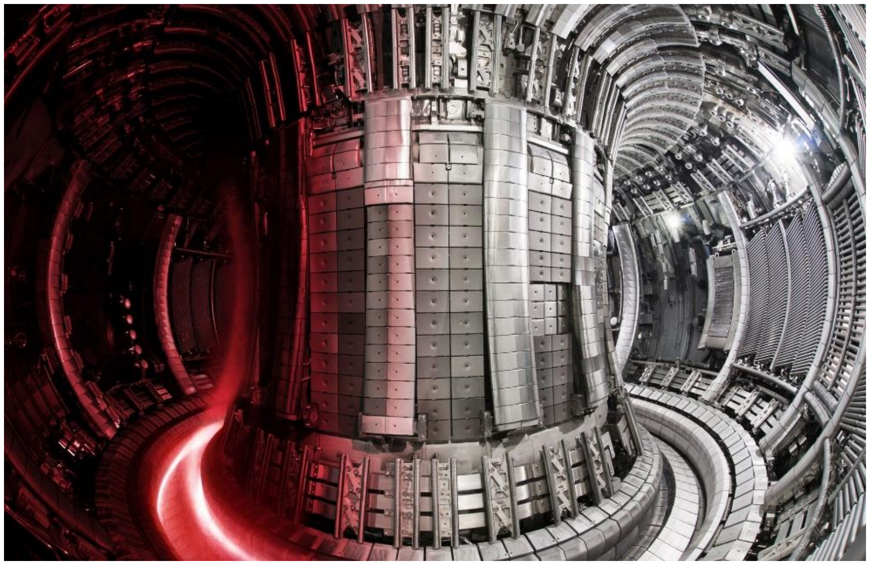
- **Inertial confinement fusion (ICF)**



https://www.iter.org
https://zh.wikipedia.org/wiki/國家點火設施

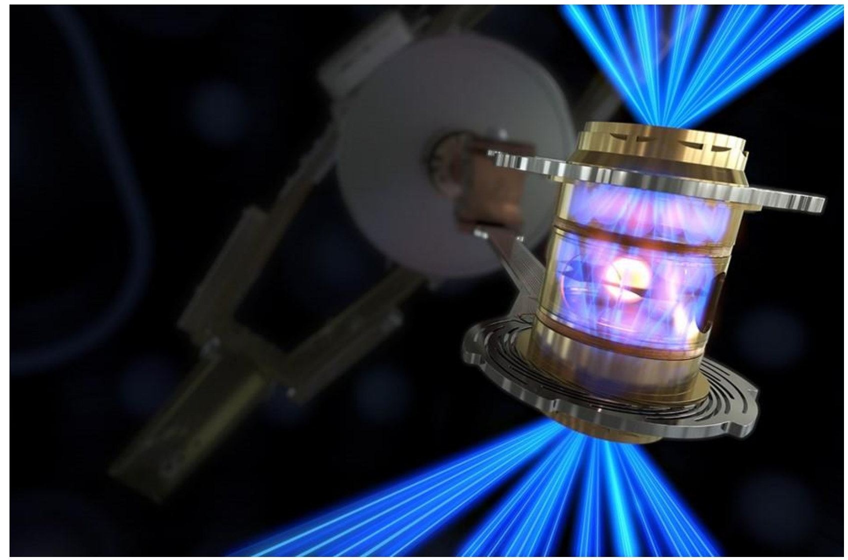
Significant breakthrough is achieved recently

- Magnetic confinement fusion (MCF)



- On 2024/2/(8), record-breaking 69.26 megajoules of sustained fusion energy in Joint European Torus (JET) facility in Oxford demonstrates powerplant potential and strengthens case for ITER.

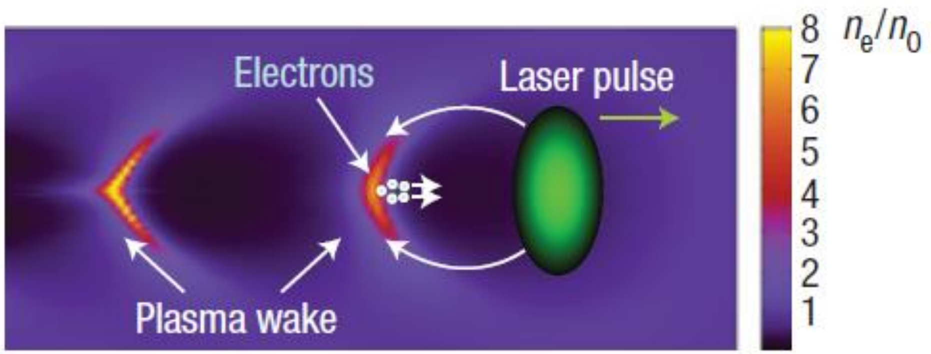
- Inertial confinement fusion (ICF)

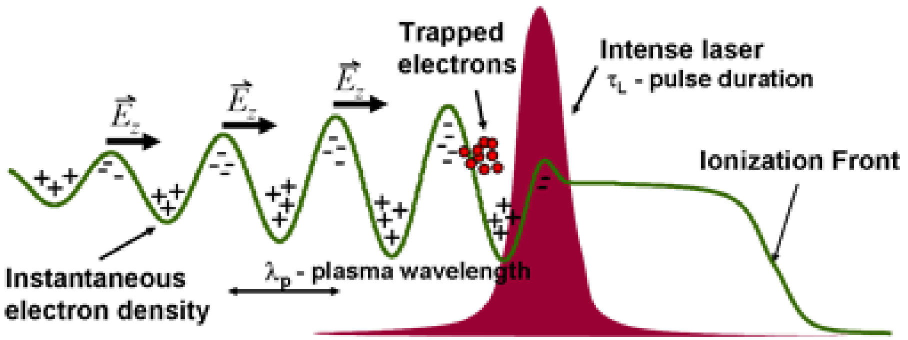


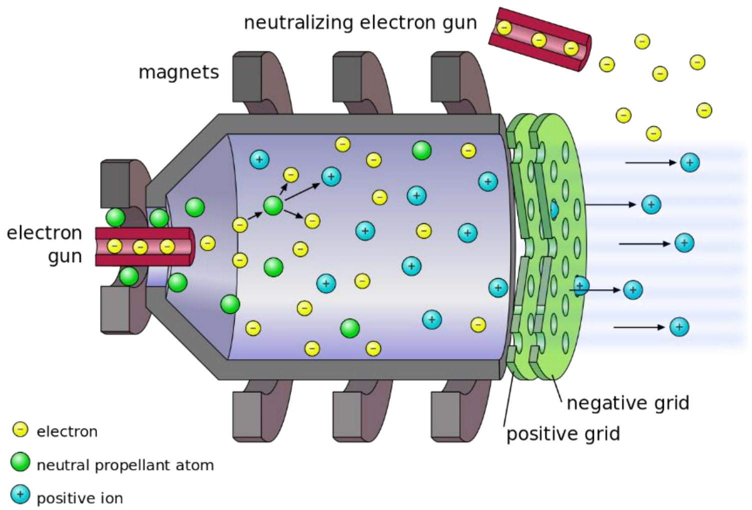
- National Ignition Facility (NIF) demonstrated a gain grater than 1 for the first time on 2022/12/5. The yield of 3.15 MJ from the 2.05-MJ input laser energy, i.e., Q=1.5.

https://ccfe.ukaea.uk/resources/#gallery
https://www.science.org/content/article/historic-explosion-long-sought-fusion-breakthrough

Plasma can be used as particle accelerators and thrusters

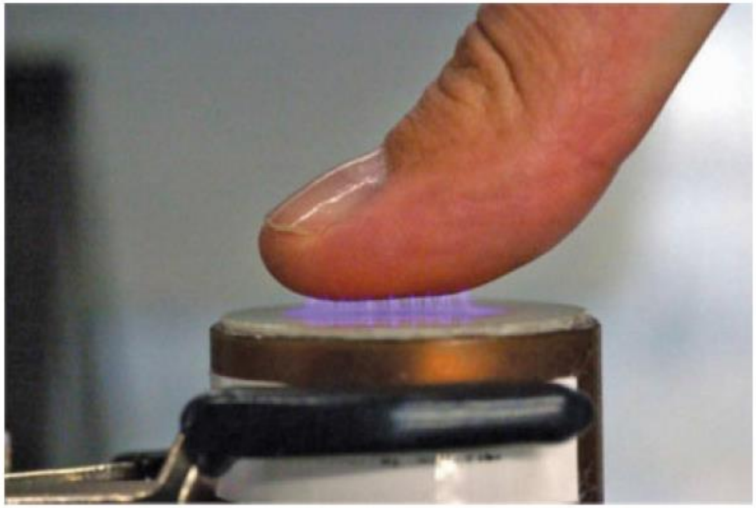


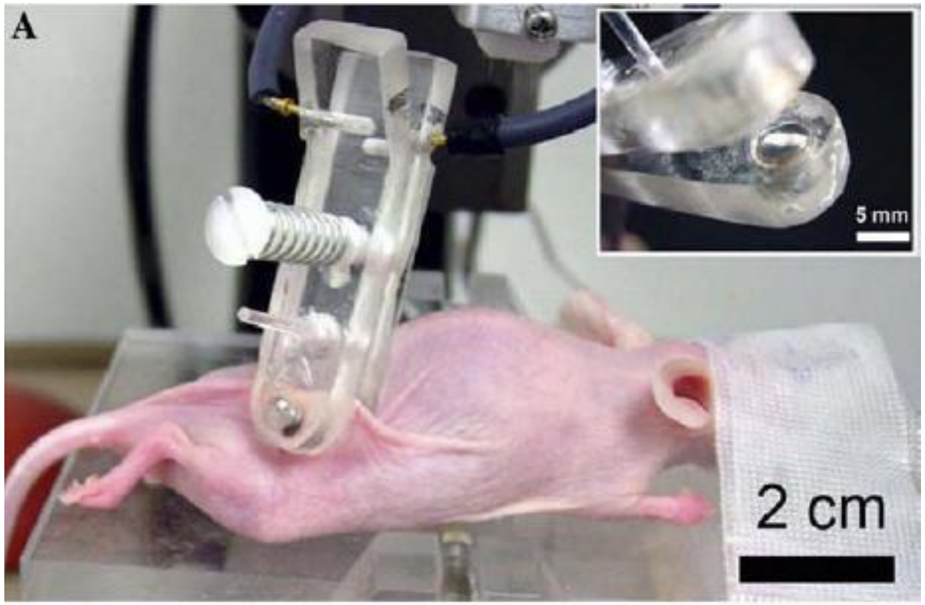




V. Malka, *et al.*, Nature Physics **4**, 447 (2008)
http://cuos.engin.umich.edu/researchgroups/hfs/research/laser-wakefield-acceleration/
https://zh.wikipedia.org/wiki/File:Electrostatic_ion_thruster-en.svg

Low temperature plasma is used in medical applications





Plasma medicine, by Alexander Fridman and Gary Friedman
Biochem Biophys Res Commun. 2006 May 5; 343(2): 351–360.

Course Outline

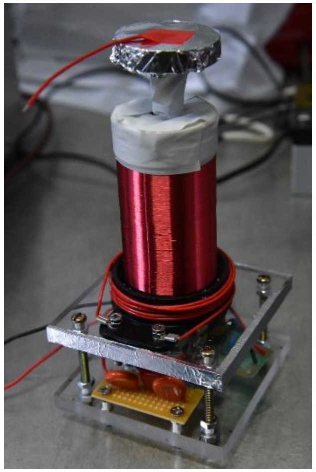
1. What is Plasma?
2. Methods for Plasma Production
3. Planeterrella - Artificial aurora demonstration
4. Plasma Diagnostics

Course Outline

5. Applications
    1. Material Processing
    2. Magnetron sputtering demonstration
    3. Plasma cleaning
    4. Light source and display systems
    5. Controlled thermonuclear fusion
    6. Magnetic mirror demonstration
    7. Plasma propulsion
    8. high energy particle accelerator
    9. biomedical application
    10. DBD discharge demonstration
6. Student presentations

Course Outline - Demonstration

a. Planeterrella
b. Magnetron sputtering
c. Dielectric barrier discharge (DBD)
d. Magnetic mirror
e. Tesla coil

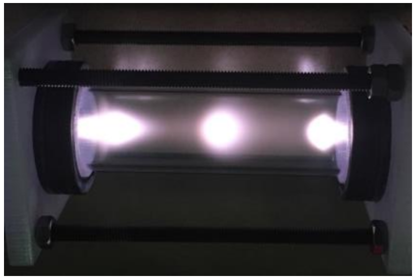


Tesla Coil

Planeterrella

Magnetron sputtering

DBD plasma



Magnetic mirror

Grading

- **Quizzes 50 % (2-min Q&A at the beginning of each class)**
- **Presentations 50 % (10-min presentation on any plasma applications or phenomena)**

- **No class on 3/5!**

Reference for the section “What is Plasma?”

- Introduction to plasma theory, by Dwight R. Nicholson
- Introduction to plasma physics and controlled fusion, by Francis F. Chen
- Principles of plasma physics for engineers and scientists, by Umran S. Inan and Marek Golkowski
- The physics of plasma, by T. J. M. Boyd and J. J. Sanderson
- Principles of plasma physics, by Krall and Trivelpiece
- NRL Plasma Formulary, Naval Research Laboratory, 2013 by J. D. Huba

Course Outline

1. What is Plasma?
2. Methods for Plasma Production
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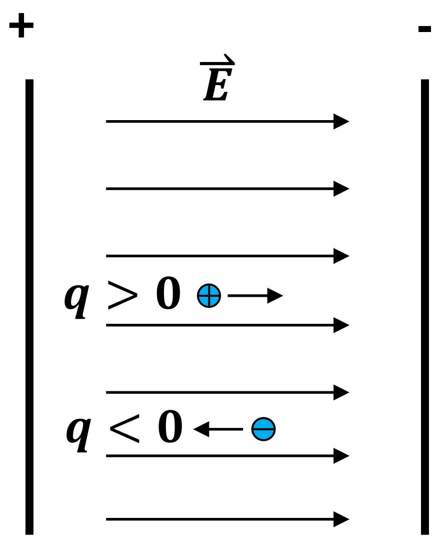
4. Plasma Diagnostics

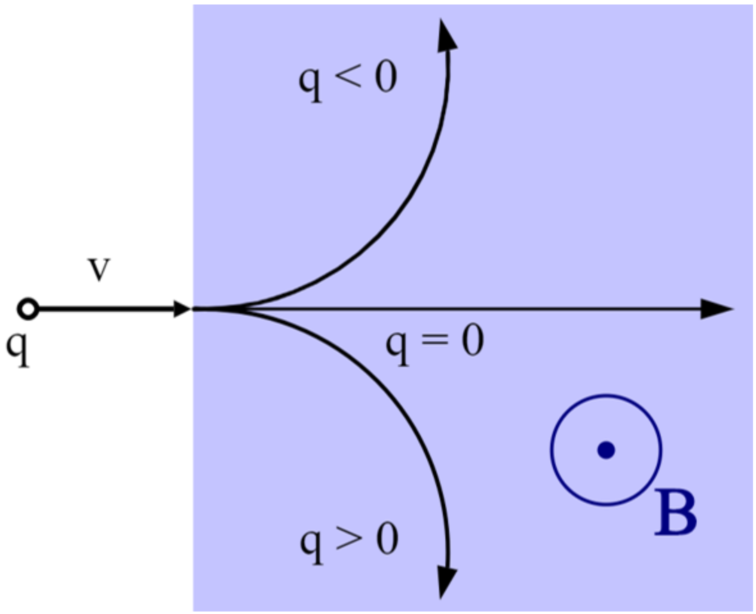
Charged particles are accelerated due to Lorentz force under electromagnetic fields

- **Lorentz force:** $\vec{F} = q\,\vec{E} + q\,\vec{v} \times \vec{B}$

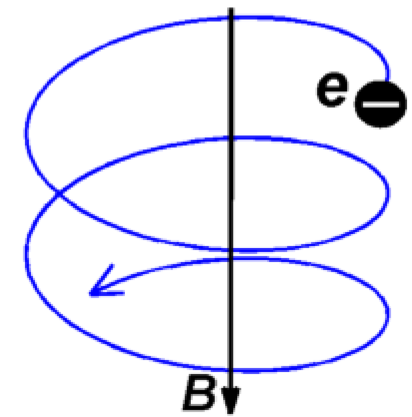
- **Force under electric fields**

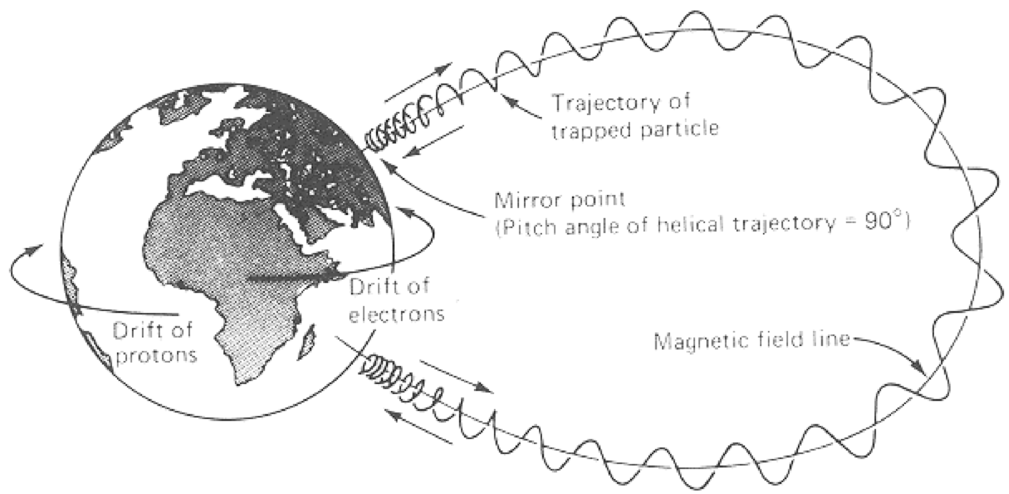


- **Force under magnetic fields**



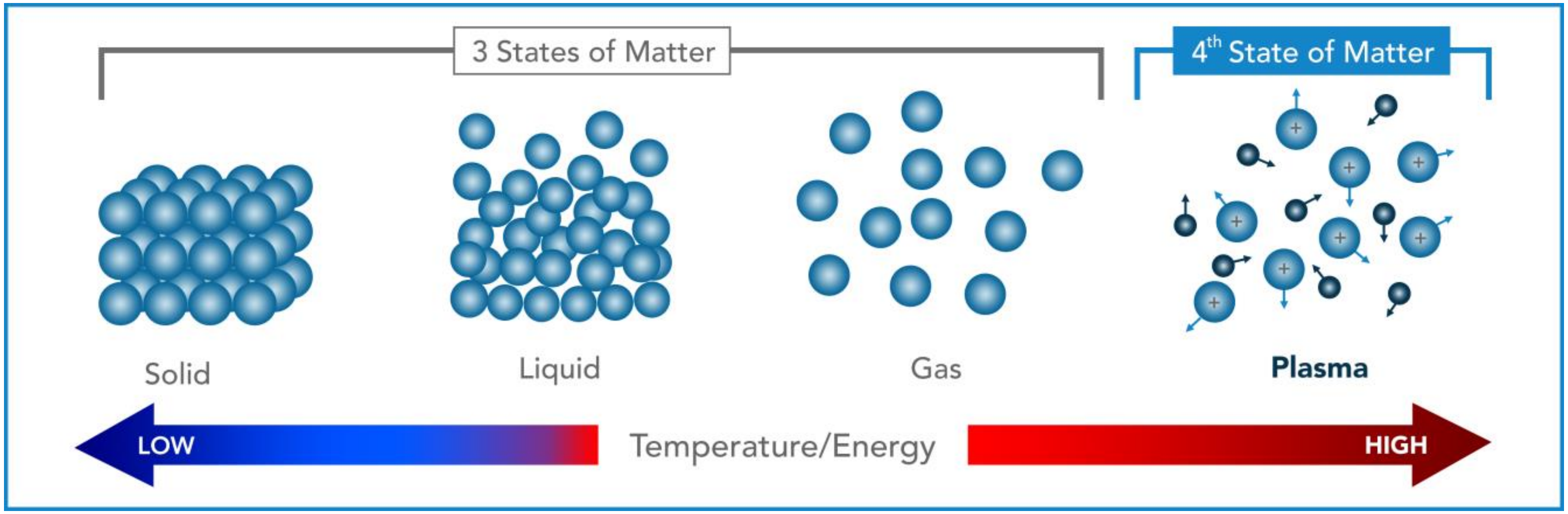
Charged particles gyro around magnetic field lines





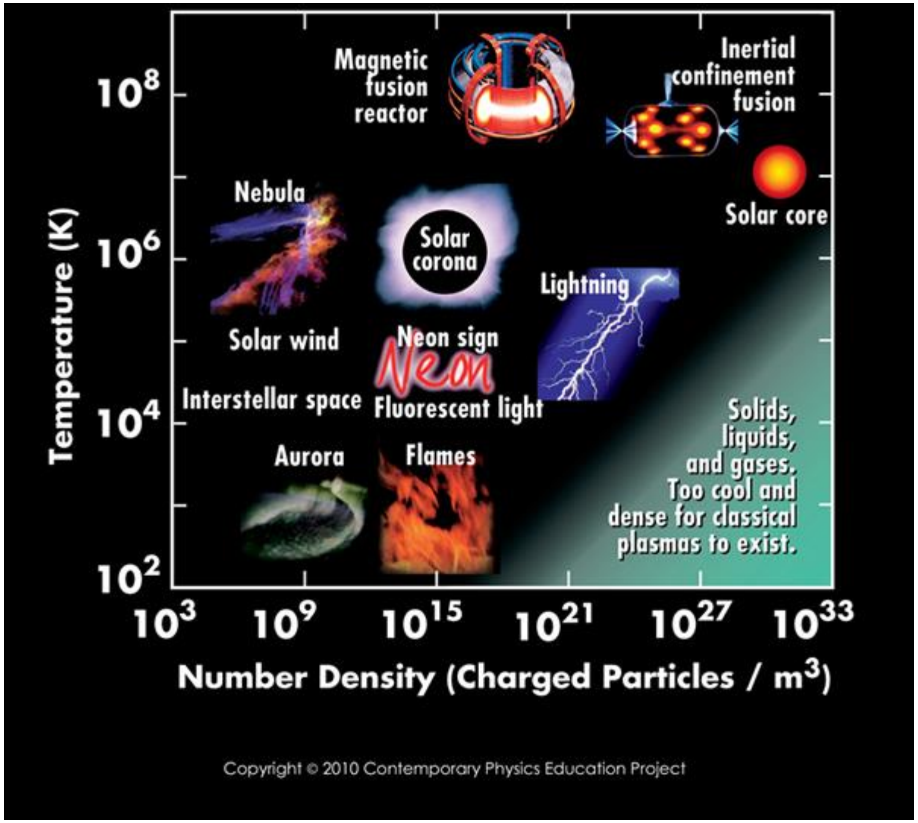
http://www.ipp.cas.cz/vedecka_struktura_ufp/tokamak/tokamak_compass/diagnostics/mikrovlnne-diagnostiky/ece-ebw-radiometr.html
http://www-ssg.sr.unh.edu/tof/Smart/Students/lees/periods.html

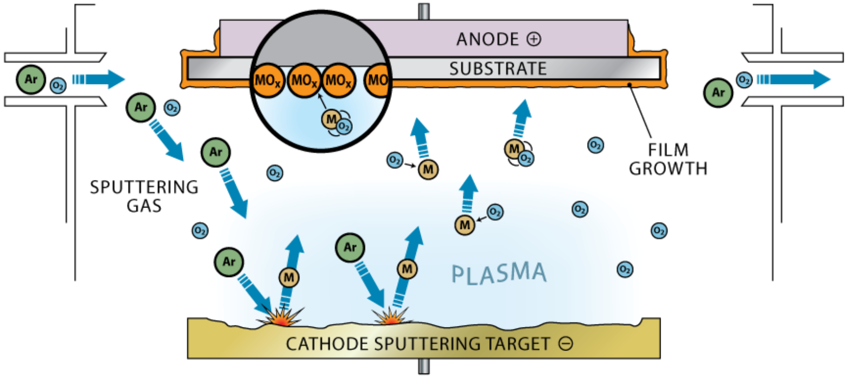
Plasma is the 4th state of matter



http://tetronics.com/our-technology/what-is-plasma/

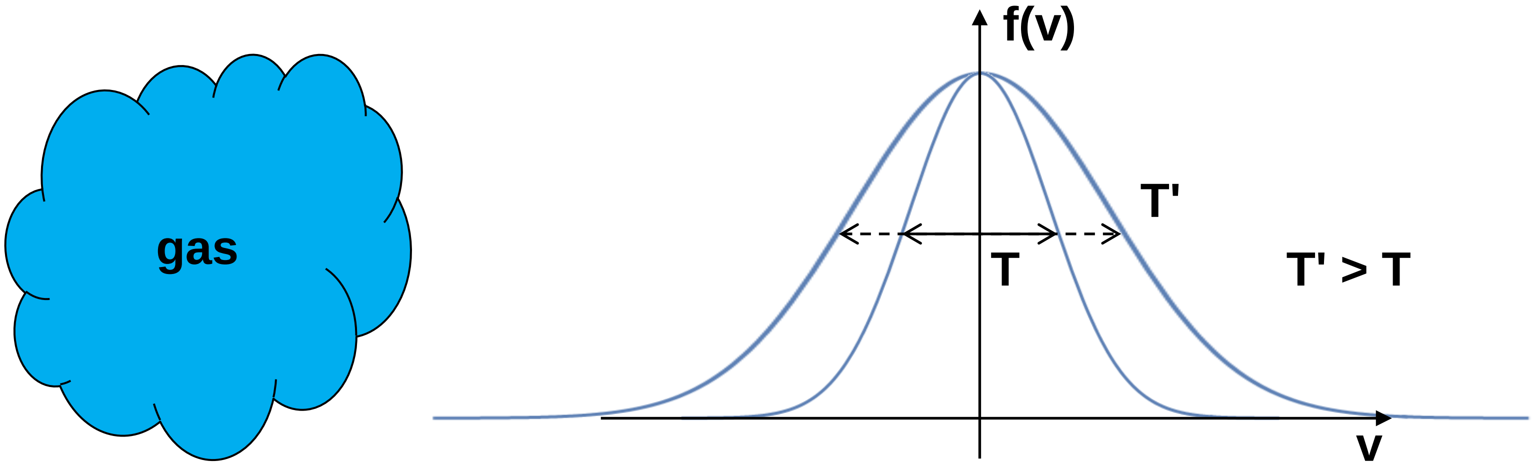
Plasma is everywhere

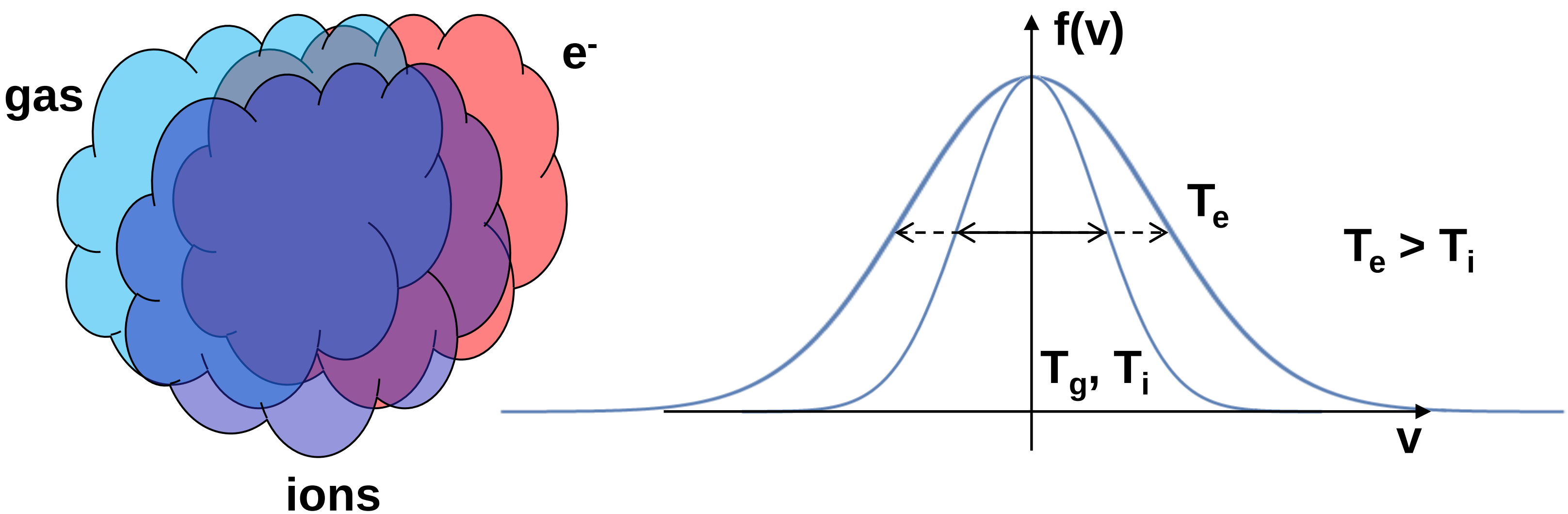




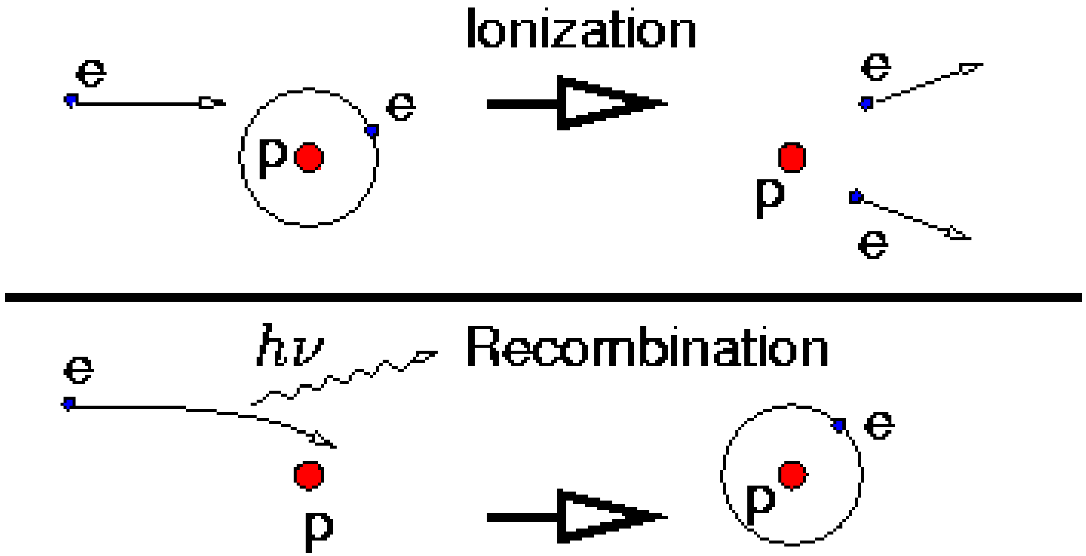
https://lasers.llnl.gov/science/understanding-the-universe/plasma-physics
http://lnf-wiki.eecs.umich.edu/wiki/Sputter_deposition
https://simple.wikipedia.org/wiki/Fluorescent_lamp

In plasma, there are ions, electrons, and neutral gas



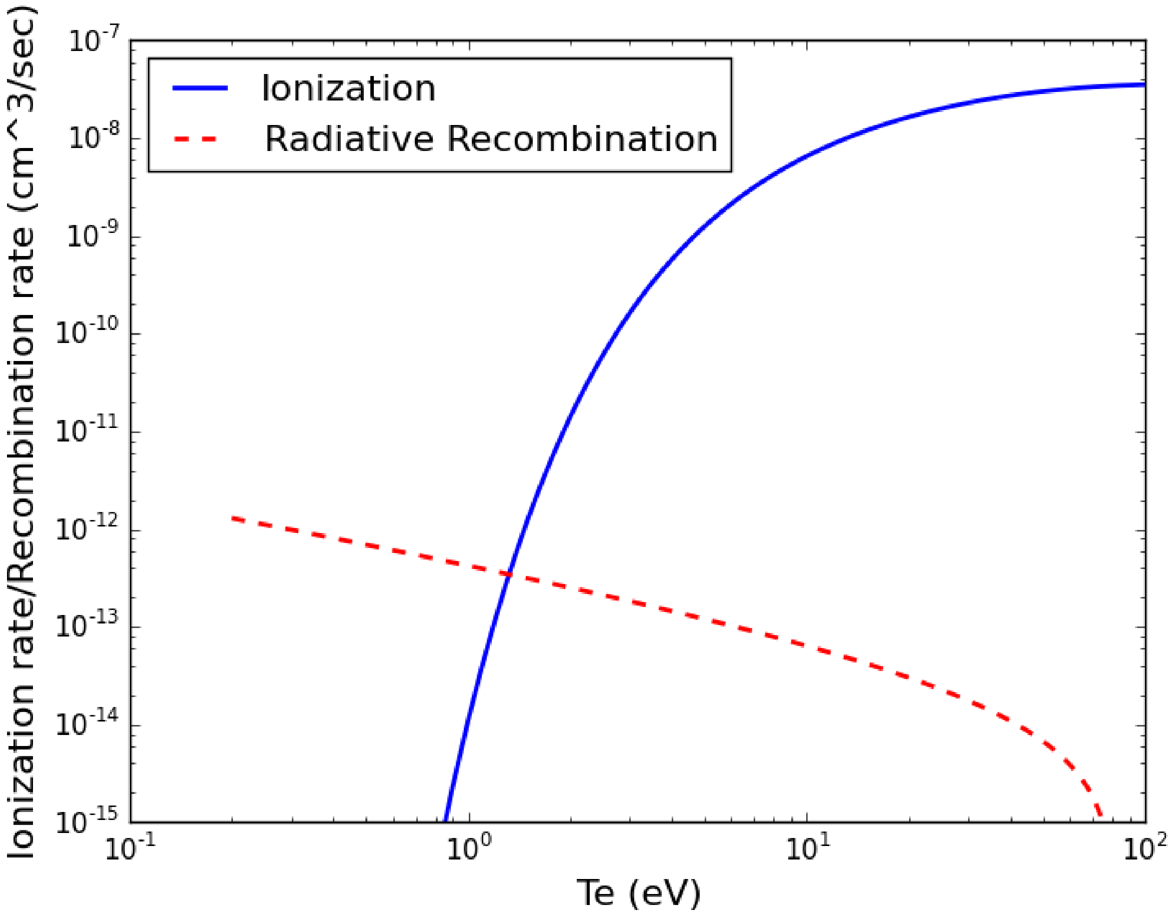


A plasma is a gas in which an important fraction of the atoms is ionized so that the electrons and ions are separated freely



http://ocw.mit.edu/courses/nuclear-engineering/22-611j-introduction-to-plasma-physics-i-fall-2003/lecture-notes/

A plasma can be created when the ionization rate is higher than the recombination rate

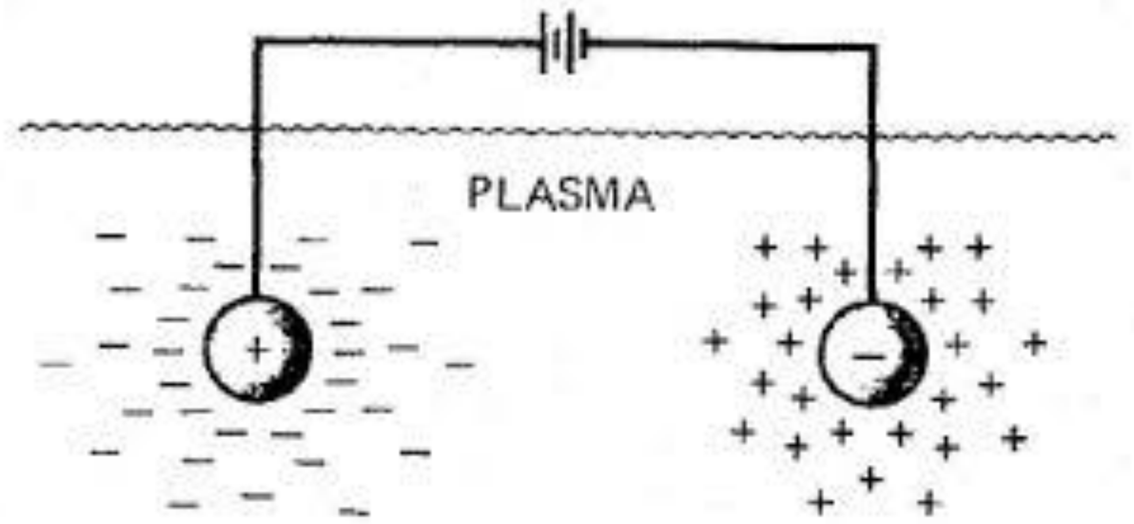


J. D. Huba \NRL Plasma Formulary", Naval Research Laboratory, 2013

There are several Important plasma parameters that need to be considered

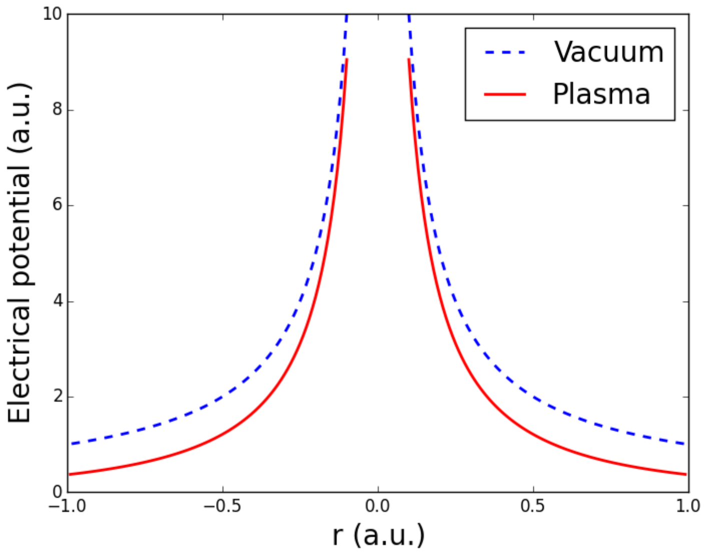
- **Debye length** $$\lambda_D \equiv \left(\frac{KT_e}{4\pi n e^2}\right)^{1/2}$$
- **Plasma parameter** $$\Lambda \equiv n\frac{4\pi}{3}{\lambda_D}^3$$
- **Plasma frequency** $$\omega_{pe} \equiv \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2}$$
- **Collision time** $$\tau_e \equiv \frac{3\sqrt{m_e}(KT_e)^{3/2}}{4\sqrt{2\pi} n e^4 \ln\Lambda}$$
- **Hall parameter** $\chi \equiv \omega_{ce}\tau_e$ **, where** $\omega_{ce} \equiv \frac{eB}{m_e c}$ **is the electron gyrofrequency**
- **Plasma beta** $\beta \equiv \frac{P}{P_B}$ **, where** $P_B \equiv \frac{B^2}{8\pi}$ **is the magnetic pressure**

A test ion in the plasma gathers a shielding cloud that tends to cancel its own charge



Francis F. Chen, \Introduction to plasma physics and controlled fusion"

Debye shielding is a phenomenon such that the potential due to a test charge in a plasma falls off much faster than in vacuum



- **Vacuum potential:**

$$\phi = \frac{\phi_0}{r}$$

- **Poisson's equation:**

$$\nabla^2 \phi = 4\pi e(n_e - n_i) - 4\pi q_T \delta(\vec{r})$$

- **Density profile:**

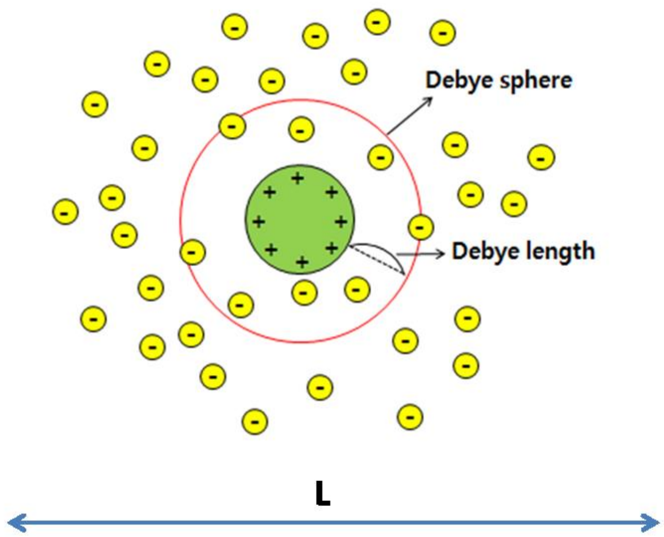
$$n_e = n_0 \exp\left(\frac{e\phi}{KT_e}\right) \quad , \quad n_i = n_0$$

- **For** $\vec{r} \neq 0$ **and assuming** $\frac{e\phi}{T_e} \ll 1$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = \frac{4\pi n_0 e^2}{KT_e}\phi$$

$$\phi = \frac{\phi_0}{r}\exp\left(-\frac{r}{\lambda_D}\right) \qquad \lambda_D \approx \left(\frac{KT_e}{4\pi n e^2}\right)^{1/2}$$

Plasma parameter Λ is the number of particles in a sphere with radius of λ_D



- **Plasma parameter:**

$$\Lambda \equiv n \frac{4\pi}{3} {\lambda_D}^3$$

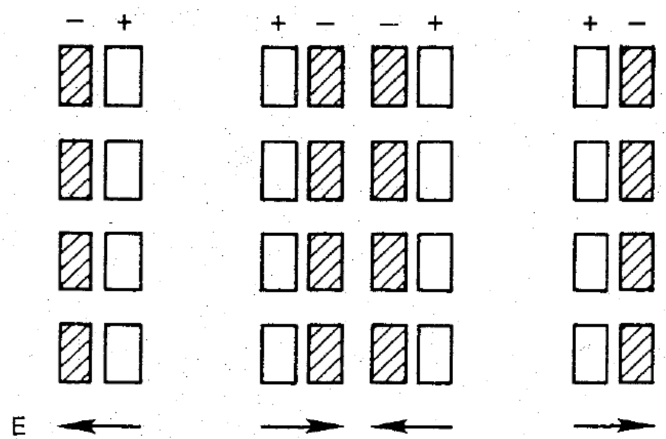
- **Criterion for an ionized gas to be plasma:**

$$\lambda_D \ll L$$

- **Requirement of "collective behavior":**

$$\Lambda \gg 1$$

Electron plasma frequency is the characteristic frequency such that electrons oscillate around their equilibrium positions



Mechanism of plasma oscillations.

- **Assumption:**

$$\vec{\nabla} \equiv \hat{x}\frac{\partial}{\partial x}\ ,\ \vec{E} = \hat{x}E\ ,\ \ \vec{v}_{\mathrm{e}} = \hat{x}v\ \ ,$$

$$\vec{\nabla} \times \vec{E} = 0\ ,\ \vec{E} = -\vec{\nabla}\phi$$

- **Continuity and momentum equation for electron:**

$$\frac{\partial n_{\mathrm{e}}}{\partial t} + \vec{\nabla}\cdot(n_{\mathrm{e}}\vec{v}_{\mathrm{e}}) = 0$$

$$m_{\mathrm{e}}n_{\mathrm{e}}\left[\frac{\partial \vec{v}_{\mathrm{e}}}{\partial t} + \left(\vec{v}_{\mathrm{e}}\cdot\vec{\nabla}\right)\vec{v}_{\mathrm{e}}\right] = -en_{\mathrm{e}}\vec{E}$$

- **Gauss' law:**

$$\frac{\partial E}{\partial x} = 4\pi\mathrm{e}(n_{\mathrm{i}} - n_{\mathrm{e}})$$

Electron plasma frequency is obtained by linearizing the hydrodynamic equations

- **The oscillation is assumed to be small:**

$$n_{\mathrm{e}} = n_0 + n_1,\ \vec{E} = \vec{E}_0 + \vec{E}_1,\ v_{\mathrm{e}} = v_0 + v_1$$

where

$$\frac{\partial n_0}{\partial x} = v_0 = \vec{E}_0 = 0$$

$$\frac{\partial n_0}{\partial t} = \frac{\partial v_0}{\partial t} = \frac{\partial \vec{E}_0}{\partial t} = 0$$

- **Linearization:**

$$m_{\mathrm{e}} \frac{\partial v_1}{\partial t} = -\mathrm{eE}_1$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} = 0$$

$$\frac{\partial E_1}{\partial x} = -4\pi \mathrm{en}_1$$

- **Plane wave solution:**

$$\eta_1 = \hat{\eta}\exp[i(kx - \omega t)]$$

$$\eta_1 = v_1, n_1, E_1$$

- **Substitute into the previous equations:**

$$-im_{\mathrm{e}}\omega v_1 = -eE_1$$

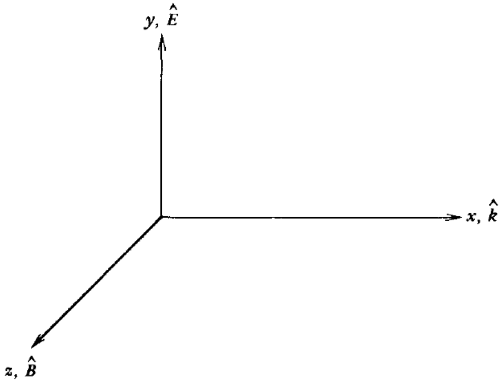
$$-i\omega n_1 = -n_0 ikv_1$$

$$ikE_1 = -4\pi e n_1$$

- **Electron plasma frequency is obtained by eliminating n_1 and E_1:**

$$\omega_{\mathrm{pe}} \equiv \omega = \left(\frac{4\pi n_{\mathrm{e}} e^2}{m_{\mathrm{e}}}\right)^{1/2}$$

Electromagnetic wave can not propagate in a plasma with density higher than critical density



$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\partial_t \vec{B}$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\partial_t \vec{E}$$

$$\vec{j} = -en_e \vec{v}_e$$

$$m_e n_e \partial_t \vec{v}_e + m_e n_e (\vec{v}_e \cdot \vec{\nabla})\vec{v}_e = -\vec{\nabla} p_e - en_e \vec{E} - \frac{en_e \vec{v}_e}{c} \times \vec{B}$$

$$\partial_t n_e + \vec{\nabla} \cdot (n_e \vec{v}_e) = 0$$

- **Linearization:**

$$\vec{\nabla} \times \vec{E}_1 = -\frac{1}{c}\partial_t \vec{B}_1$$

$$\vec{\nabla} \times \vec{B}_1 = \frac{4\pi e}{c} n_0 \vec{v}_e + \frac{1}{c}\partial_t \vec{E}_1$$

$$m_e n_0 \partial_t \vec{v}_1 = -en_0 \vec{E}_1$$

- **Eliminate** $\partial_t \vec{v}_1$ **and** $\vec{B}_1$

$$-c\,\vec{\nabla} \times (\vec{\nabla} \times \vec{E}_1) = \frac{1}{c}\frac{4\pi n_0 e^2}{m_e}\vec{E}_1 + \frac{1}{c}\frac{\partial^2 \vec{E}_1}{\partial t^2}$$

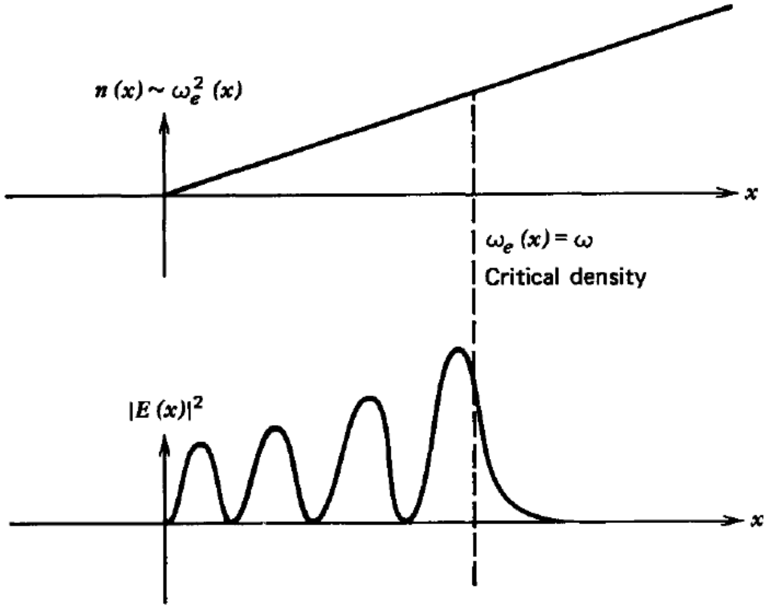
- **Take** $\vec{\nabla} \times (\vec{\nabla} \times) \to k^2$ **and** $\partial_t \to \omega$

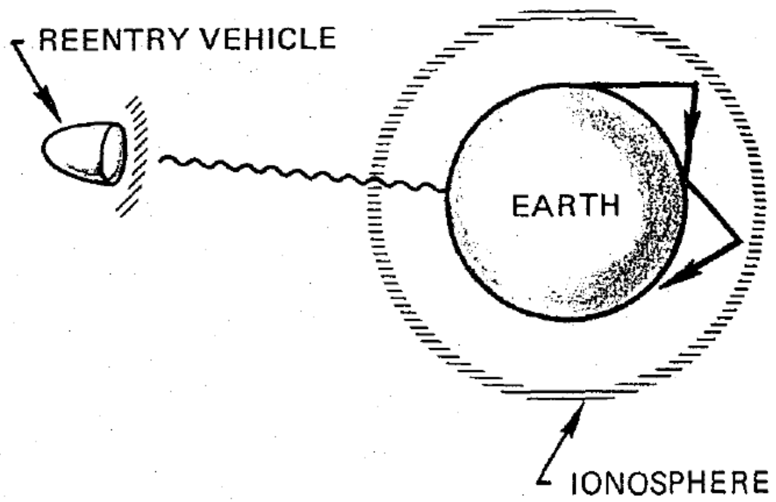
$$(\omega^2 - k^2c^2 - \omega_e{}^2)\vec{E}_1 = 0$$

$$\omega^2 - k^2c^2 - \omega_e{}^2 = 0$$

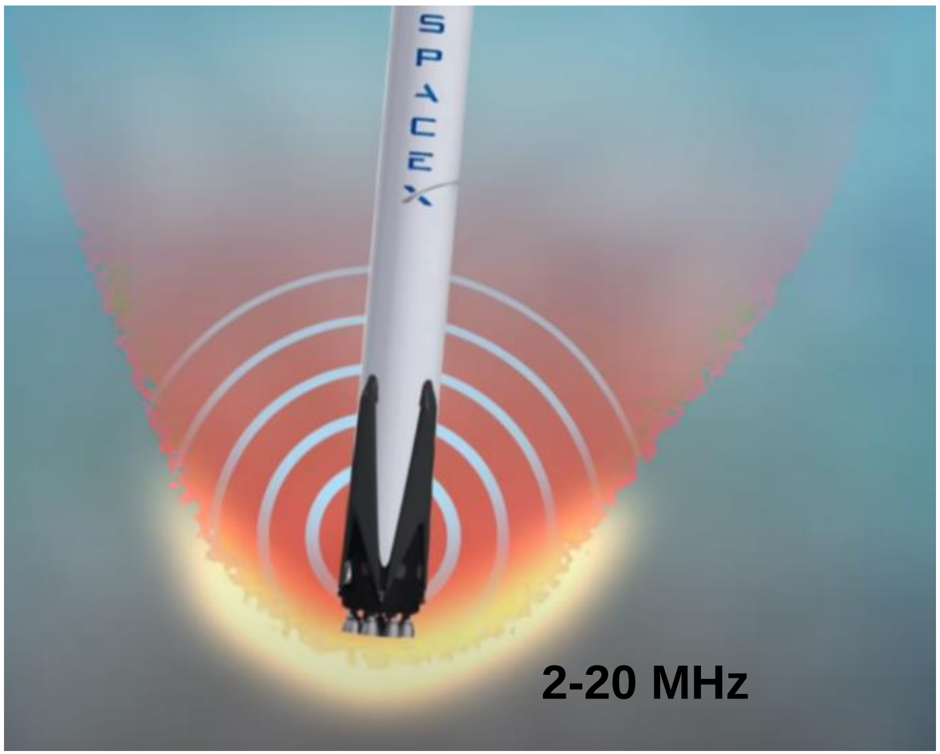
- **Wave number k becomes imaginary when** $\omega < \omega_e$ **.**

The cutoff of the electromagnetic wave is important in laser fusion and in the interaction of radio waves with the ionosphere



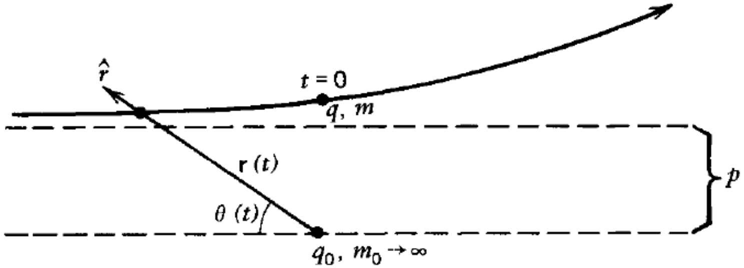


SpaceX moves their S-band transmitter to the top of their rocket to avoid communication blackout



https://www.youtube.com/watch?v=Bf4rPsS_fwA

Charged particles collide with each other through coulomb collisions



$$mv_\perp = \int_{-\infty}^{\infty} dt\, F_\perp(t)$$

- **Coulomb force:**

$$m\ddot{\vec{r}} = \frac{qq_0}{r^2}\hat{r}$$

$$F_\perp = \frac{qq_0}{p^2}\sin^3\theta$$

- **Relation between θ and t is**

$$x = -r\cos\theta = -\frac{p\cos\theta}{\sin\theta} = v_0 t$$

- **Therefore,**

$$v_\perp = \frac{qq_0}{mv_0 p}\int_0^{\pi} d\theta \sin\theta = \frac{2qq_0}{mv_0 p} \equiv \frac{v_0 p_0}{p}$$

where $p_0 \equiv \dfrac{2qq_0}{m{v_0}^2}$

- **Note that this is valid only when** $v_\perp \ll v_0$ **, i.e.,** $p \gg p_0$ **.**

Cumulative effect of many small angle collisions is more important than large angle collisions

- Consider a variable Δx that is the sum of many small random variables Δx_i, i=1,2,3,…,N,

$$\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 + \cdots + \Delta x_N = \sum_{i=1}^{N} \Delta x_i$$

- Suppose $< \Delta x_i > = < \Delta x_i \Delta x_j >_{i \neq j} = 0$

$$\langle (\Delta x)^2 \rangle = \left\langle \left(\sum_{i=1}^{N} \Delta x_i \right)^2 \right\rangle = \sum_{i=1}^{N} \langle (\Delta x_i)^2 \rangle = N \langle (\Delta x_i)^2 \rangle$$

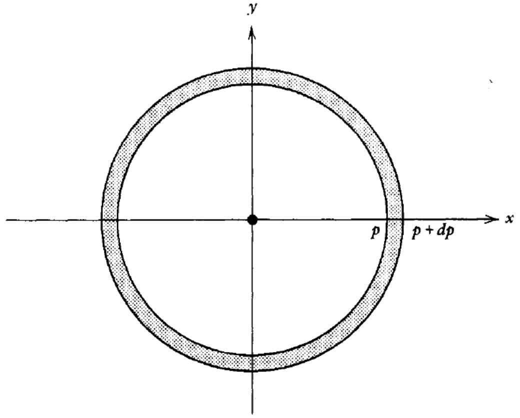
- For one collision:

$$\langle {v_\perp}^2 \rangle = \langle (\Delta v_x)^2 \rangle + \left\langle \left(\Delta v_y\right)^2 \right\rangle = \frac{{v_0}^2 {p_0}^2}{p^2} \qquad \langle (\Delta v_x)^2 \rangle = \left\langle \left(\Delta v_y\right)^2 \right\rangle = \frac{1}{2} \frac{{v_0}^2 {p_0}^2}{p^2}$$

- The total velocity in $\hat{x}$

$$\left\langle \left(\Delta {v_x}^{tot}\right)^2 \right\rangle = N \langle (\Delta v_x)^2 \rangle = \frac{N}{2} \frac{{v_0}^2 {p_0}^2}{p^2}$$

The collision frequency can be obtained by integrating all the possible impact parameter



- Number of collisions in a time interval:

$$dN = n_0 2\pi p\, dp\, v_0\, dt$$

i.e., $$\frac{dN}{dt} = 2\pi p\, dp\, n_0 v_0$$

- Therefore

$$\frac{d}{dt}\left\langle \left(\Delta v_x^{\text{tot}}\right)^2 \right\rangle = \frac{1}{2}\frac{{v_0}^2{p_0}^2}{p^2}\frac{dN}{dt}$$

$$= \pi\, n_0 {v_0}^3 {p_0}^2 \frac{dp}{p}$$

$$\frac{d}{dt}\left\langle \left(\Delta_\perp^{\text{tot}}\right)^2 \right\rangle = 2\frac{d}{dt}\left\langle \left(\Delta v_x^{\text{tot}}\right)^2 \right\rangle$$

$$= 2\pi n_0 {v_0}^3 {p_0}^2 \int_{p_{\min}}^{p_{\max}} \frac{dp}{p}$$

$$= 2\pi n_0 {v_0}^3 {p_0}^2 \ln\left(\frac{p_{\max}}{p_{\min}}\right)$$

$$\approx 2\pi n_0 {v_0}^3 {p_0}^2 \ln\left(\frac{\lambda_D}{|p_0|}\right)$$

$$\approx 2\pi n_0 {v_0}^3 {p_0}^2 \ln\Lambda$$

- Note that

$$\lambda_D \approx \left(\frac{KT_e}{4\pi n_0 e^2}\right)^{1/2}$$

$$\frac{\lambda_D}{|p_0|} \approx \frac{\lambda_D m_e {v_e}^2}{2e^2} \approx \frac{\lambda_D K T_e}{e^2} \approx 4\pi n_0 {\lambda_D}^3$$

$$\approx \Lambda$$

Comparison between the mean free path and the system size L determines the regime of the plasma

- A reasonable definition for the scattering time due to small angle collisions is the time it takes $\left\langle \left(\Delta v_\perp{}^{\text{tot}}\right)^2 \right\rangle$ to equal v_0^2. The collision frequency ν_c due to small-angle collisions:

$$\frac{d}{dt}\left\langle \left(\Delta_\perp{}^{\text{tot}}\right)^2 \right\rangle \approx 2\pi n_0 v_0{}^3 p_0{}^2 \ln\Lambda \approx v_0{}^2 \nu_c, \quad p_0 \equiv \frac{2qq_0}{m_e v_0{}^2} \Rightarrow \nu_c = \frac{8\pi n_0 e^4 \ln\Lambda}{m_e{}^2 v_0{}^3}$$

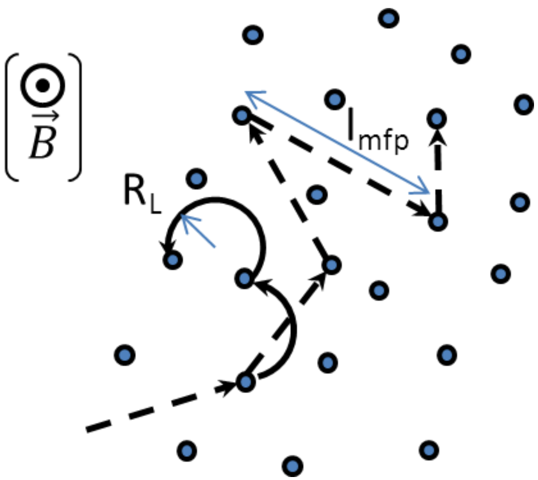
- With more careful derivation, the collisional time is obtained:

$$\tau_e{}^{-1} = \nu_c = \frac{4\sqrt{2\pi} n e^4 \ln\Lambda}{3\sqrt{m_e}(KT_e)^{3/2}}$$

- Mean free path: $l_{\text{mfp}} = v_e \tau_e$

$$\begin{cases} l_{\text{mfp}} < L & \text{Fluid Theory} \\ l_{\text{mfp}} > L & \text{Kinetic Theory} \end{cases}$$

Thermal conduction perpendicular to the magnetic field can be suppressed when the plasma is magnetized



- Plasma is magnetized when

$$\frac{R_L}{l_{mfp}} = \frac{v_e}{\omega_{ce}} \frac{1}{v_e \tau_e} < 1$$

i.e., the hall parameter

$$\chi \equiv \omega_{ce}\tau_e > 1$$

$$m_e \frac{d\vec{v}}{dt} = -\frac{e}{c}\vec{v} \times \vec{B}$$

- Assuming $\vec{B} = B\hat{z}$ and the electron oscillates in x-y plane

$$m_e \dot{v}_x = -\frac{e}{c} B v_y \qquad m_e \dot{v}_z = 0$$

$$m_e \dot{v}_y = \frac{e}{c} B v_x$$

$$\ddot{v}_x = -\frac{eB}{m_e c}\dot{v}_y = -\left(\frac{eB}{m_e c}\right)^2 v_x$$

$$\ddot{v}_y = -\frac{eB}{m_e c}\dot{v}_x = -\left(\frac{eB}{m_e c}\right)^2 v_y$$

- Therefore

$$\omega_{ce} = \frac{eB}{m_e c}$$

Plasma β is the ratio between hydro pressure and magnetic pressure

- **Momentum equation in Magnetohydrodynamics (MHD) approach:**

$$\rho \frac{d\vec{v}}{dt} + \rho\left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} = -\vec{\nabla} p + \frac{1}{c}\vec{j} \times \vec{B}$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{j}$$

$$\vec{j} \times \vec{B} = \frac{c}{4\pi}\left(\vec{\nabla} \times \vec{B}\right) \times \vec{B} = \frac{c}{4\pi}\left[\left(\vec{B} \cdot \vec{\nabla}\right)\vec{B} - \frac{1}{2}\vec{\nabla} B^2\right]$$

$$\rho \frac{d\vec{v}}{dt} + \rho\left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} = -\vec{\nabla}\left(p + \frac{B^2}{8\pi}\right) + \frac{1}{4\pi}\left(\vec{B} \cdot \vec{\nabla}\right)\vec{B}$$

- **Magnetic pressure:** $\frac{B^2}{8\pi}$

$$\beta \equiv \frac{p}{B^2/8\pi}$$

- **Magnetic tension:** $\frac{1}{4\pi}\left(\vec{B} \cdot \vec{\nabla}\right)\vec{B}$

There are several Important plasma parameters that need to be considered

- **Debye length** $\boldsymbol{\lambda_{\mathrm{D}} \equiv \left(\frac{KT_{\mathrm{e}}}{4\pi n e^2}\right)^{1/2}}$
- **Plasma parameter** $\boldsymbol{\Lambda \equiv n \frac{4\pi}{3} \lambda_{\mathrm{D}}{}^3}$
- **Plasma frequency** $\boldsymbol{\omega_{\mathrm{pe}} \equiv \left(\frac{4\pi n_{\mathrm{e}} e^2}{m_{\mathrm{e}}}\right)^{1/2}}$
- **Collision time** $\boldsymbol{\tau_{\mathrm{e}} \equiv \frac{3\sqrt{m_{\mathrm{e}}}(KT_{\mathrm{e}})^{3/2}}{4\sqrt{2\pi} n e^4 \ln\Lambda}}$
- **Hall parameter** $\boldsymbol{\chi \equiv \omega_{\mathrm{ce}} \tau_{\mathrm{e}}}$ **, where** $\boldsymbol{\omega_{\mathrm{ce}} \equiv \frac{eB}{m_{\mathrm{e}} c}}$ **is the electron gyrofrequency**
- **Plasma beta** $\boldsymbol{\beta \equiv \frac{P}{P_{\mathrm{B}}}}$ **, where** $\boldsymbol{P_{\mathrm{B}} \equiv \frac{B^2}{8\pi}}$ **is the magnetic pressure**